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Performance Assessment of FO-PID Temperature Control System Using a Fractional Order LQG Benchmark

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ABSTRACT In this paper, a fractional order LQG benchmark is proposed for the control performance assessment of fractional order control systems. Similar to the conventional LQG benchmark, the fractional order LQG performance benchmark curve is determined by the numerical calculation method, which avoids the calculation of the complex interaction matrix. The fractional order process model is discretized via fractional order calculus. Meanwhile, the fractional order integral is introduced into the conventional LQG cost function. Then solving the linear quadratic Gaussian problem under the fractional order model and fractional control, the optimal input and output variances are determined for different weighting factors and the performance curves can be achieved. The comparison between fractional order LQG and the conventional LQG shows the improvement of the proposed benchmark under the same condition. The proposed benchmark can provide a more direct and superior reference standard to evaluate the performance of fractional order control system. Finally, a case study of fractional order PID(FO-PID) controller in industrial heating furnace temperature control experiment with model matching and model mismatch conditions is used to verify the effectiveness of the proposed benchmark.

INDEX TERMS Fractional order system, fractional order linear quadratic Gaussian (FO-LQG), control performance assessment, FO-PID. control.

I. INTRODUCTION

The research on control performance assessment of control loops can be traced back to 1970. Devries and Wu first proposed the idea of performance assessment in 1978. Their work laid a theoretical foundation for control performance assessment of control loops. In 1989, Harris [2] proposed a performance index based on minimum variance control, which uses the minimum variance controller as the upper limit for evaluating the performance of univariate control loops. In 1993, Stanfelj *et al.* [3] extended the performance evaluation method based on minimum variance control to the univariate feedforward-feedback control loops. In 1995, Tyler and Morari [4] revised and generalized the Harris indicator. they applied it to unstable systems and non-minimum phase systems. In 1996, Harris *et al.* [5] introduced the univariate minimum variance control benchmark into the multivariate control system. In 1999, Huang [6] proposed the use of linear quadratic Gaussian (LQG) optimal control as

a performance assessment benchmark, which considers both the input and output variance. This provides a practical lower bound of performance. Under the condition of changing the weighting factor, a tradeoff curve is obtained by calculating the optimal LQG control law and five performance indicators are defined. In 1999, Ko and Edgar [7] proposed performance assessment of constrained model predictive control systems. And Chen *et al.* [8] also proposed performance evaluation methods for cascade control loops. Lee *et al.* [9] proposed an economic performance assessment method for constrained model predictive control. In 2000, Huang *et al.* [10] extended the evaluation method based on minimum variance control to multivariate feedforward-feedback control systems. Grimble [11] proposed the generalized minimum variance (GMV) benchmark, which introduced the control signal amplitude as a penalty term into the objective function. Li and Evans *et al.* [12], Li *et al.* [13] proposed generalized minimum variance control for linear time-varying systems. A method for evaluating the performance of control loops with time-varying disturbances is proposed in [14], [15]. In 2007, Harris and Yu [16] extended the minimum variance

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control benchmark to a class of nonlinear univariate systems and presented a Volterra sequence approximation method for estimating the upper bound of performance from operational data.

Kadali and Huang [17] proposed a data driven subspace approach to calculate the LQG benchmark under closed-loop conditions with certain external excitations. To improve the accuracy of the LQG benchmark, Danesh Pour *et al.* [18] proposed a noise uniformity estimation for closed-loop subspace identification in 2009. In 2010, Danesh Pour *et al.* [19] extended the LQG benchmark to the cascade control loop. In 2011, a numerical equigrad algorithm was introduced to improve the traditional LQG benchmark by Liu *et al.* [20]. In 2014-2018, Wei and Wang [21], Wang *et al.* [22], Zhang and Wang [23] extended LQG benchmark from one dimension to two dimensions and proposed a two dimensional LQG benchmark for control performance assessment of ILC-Controlled batch processes in a 2-D System framework. Moreover, they developed a novel data-driven control performance assessment method under two dimensions. In 2019, an improved entropy benchmark for performance assessment of common cascade control system was proposed by Zhang *et al.* [24], which combined entropy with output mean value and deal with the inconsistency of the minimum variance benchmark in evaluating non-Gaussian disturbance systems.

PID control or model predictive control optimization based PID are the most widely used technology in the control system. Because of its simple structure and strong robustness, it is widely used in industrial process and has strong vitality [25], [26]. It has important practical significance to evaluate the performance of PID controller. Sendjaja and Kariwala *et al.* [27] used the MVC benchmark to evaluate the performance of the control loop by calculating the minimum variance that can be achieved by the PID controller. Horton *et al.* [28] has put forward that an optimal PI regulator was recommended as the default performance standard for industrial level control loops. In 2017, Fang *et al.* [29] evaluated the IMC-PID controller by a LQG benchmark in case of the model matching and the model mismatch on the heating furnace. Škarda *et al.* [30] proposed a new performance evaluation index based on the sensitivity function of ideal shape. The corresponding optimal achievable performance is calculated for all processes belonging to the fractional model sets controlled by PID-type controller. This method is based on classical minimum variance theory that maximum performance is strongly influenced by the process normalized dead time and only used for evaluating the process controller with fixed structure. However, there exists some limits. Meneses [31] *et al* proposed a combined performance evaluation index to solve the performance trade-off between servo control and regulation control, which is to evaluate FO-PID controller and also based on the theory that the achievable performance is influenced by the process normalized dead time and subject to a robustness constraint. And this study of its performance index is just to guide the

selection whether to use fractional order controller or integer order controller when assessing a fractional order system. But its scope of use is limited.

In short, many scholars have studied the performance evaluation of PID controller. But most of their researches on the performance evaluation of control system is aimed at the integer order controller's model. At the same time, from the perspective of development of the above performance assessment techniques, no matter which assessment benchmark, the process model they are targeting are nearly integer order models. We know that when encountering more complicated processes, the integer order model is not enough to accurately describe the process dynamics, the extra fractional order integral and differential may lead to more flexible and accurate process models for representing the dynamic characteristics of practical processes [32]–[34]. Fractional order controller is an extension of the concept of traditional integral order controller. And FO-PID controller can achieve better performance than integral order PID controller in controlling fractional order system [35]–[38]. Therefore, the performance evaluation of FO-PID controller and the performance assessment techniques is also worth studying. The existing evaluation methods for fractional order controllers have disadvantages of insufficient application scope. The most important thing is that their evaluation indicators are one-sided, only from the perspective of process output. However, we know that the LQG benchmark not only takes into account the input variance for the system, but also the output variables, which is more practice assessment benchmark for actual industrial processes. Unfortunately, its application is until now only for the system of integer order process model and the problem of integer order control. Consequently, on the basis of conventional LQG benchmark(based on the integer order model), we can consider a novel LQG benchmark that based on fractional order process model and fractional order control to achieve the performance assessment problem under fractional order systems with fractional order controller. This benchmark proposed in this paper can provide a higher reference standard for control performance assessment of fractional order system and any linear fractional order controller. Based on the benchmark proposed in this paper, a case study of heating furnace controlled by a FO-PID controller is used to verify the validity of this benchmark. At the same time, the performance of FO-PID has studied under the model matching and the model mismatch.

II. FRACTIONAL ORDER CALCULUS

Fractional calculus is a branch of calculus, which extends the order of calculus operator from integer order to fractional order. The commonly used definitions of fractional calculus are Grunwald-Letnikov(GL), Riemann-Liouville (RL) and Caputo [39].

The GL definition of fractional order calculus is:

$${}^c D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{i=0}^{\lfloor (t-c)/h \rfloor} (-1)^i \binom{\alpha}{i} f(t - ih) \quad (1)$$

where, $f(t)$ is a continuous function, c denotes the initial time, α denotes fractional order, h denotes the sampling step, $[(t - c)/h]$ denotes the integer part of $(t - c)/h$, ω_i^α is the weight coefficient, which is calculated by the following recurrence equation, $\omega_i^\alpha = (-1)^i \binom{\alpha}{i}$ is a polynomial coefficient, which can be directly calculated by the following recurrence formula:

$$\omega_0^\alpha = 1, \omega_i^\alpha = \left(1 - \frac{\alpha + 1}{i}\right) \omega_{i-1}^\alpha, \quad i = 1, 2, \dots, \quad (2)$$

The RL definition of fractional order calculus is:

$${}_c^{RL}D_t^\alpha f(t) = \frac{1}{\Gamma(m - \alpha)} \frac{d^m}{dt^m} \int_c^t \frac{f(\tau)}{(t - \tau)^{\alpha - m + 1}} d\tau \quad (3)$$

where, $m - 1 < \alpha < m$, $m \in N$, $\Gamma(\bullet)$ denotes Euler-gamma function.

The Caputo definition of fractional order calculus is:

$${}_c^{Ca}D_t^\alpha f(t) = \frac{1}{\Gamma(m - \alpha)} \int_c^t \frac{f^m(\tau)}{(t - \tau)^{\alpha - m + 1}} d\tau \quad (4)$$

The Laplace transform of the RL fractional order derivative with zero initial condition is given by:

$$L \left\{ {}_0^{RL}D_t^\alpha f(t) \right\} = s^\alpha F(s) \quad (5)$$

On the basis of general first order plus time delay model and the above Laplace transform in (5), a simple fractional order transfer function model with time delay will be considered:

$$G(s) = \frac{T}{Ks^\alpha + 1} e^{-\tau s} \quad (6)$$

where, τ denotes the time delay, T denotes the time constant, K denotes gains for process model.

In the time domain, Eq.(6) can be discretized to a fractional order differential equation by GL numerical approximated method, the validity of its numerical solution is demonstrated [40], [41] (see some details for Appendix A):

$$y(k) + \mu \sum_{j=1}^L w_j^\alpha y(k - j) = Hu(k - d) \quad (7)$$

where, $\mu = Th^{-\alpha}(1 + Th^{-\alpha})^{-1}$, $H = K(1 + Th^{-\alpha})^{-1}$ and the time delay $d = \tau/h$, L is the approximated memory length of fractional order operators, $y(k)$, $u(k)$ is the output and input of fractional order system, respectively.

$$w_0^\alpha = 1, w_j^\alpha = \left(1 - \frac{\alpha + 1}{j}\right) w_{j-1}^\alpha, \quad j = 1, 2, \dots, L.$$

III. FRACTIONAL ORDER LQG BENCHMARK

A. SOLUTION FOR FRACTIONAL ORDER LQG BENCHMARK

The LQG performance benchmark is an extension of the MVC benchmark, which takes into account the variance of the system output and the control input. Therefore, the LQG performance benchmark can provide more information on the controller performance. The computational burden of the

Riccati equation is very large when the LQG is used as the performance benchmark. As refer to the literature [42], the LQG problem can be solved via the infinite MPC(model predictive control) and a finite value of predictive horizon and control horizon is usually enough to achieve the approximate infinite horizon LQG solution via the MPC approach. Therefore, the LQG benchmark is solved by the approximation of solving the MPC, which can simplify the acquisition of the tradeoff curve. The solution of LQG problem based on fractional order model is similar to that based on integer order model. Therefore, when the process model is extended to fractional order, we can use fractional order MPC to solve the LQG problem based on fractional order model.

Consider a single-input single-output fractional order discrete time model:

$$Y_k = G_p(z^{-1})U_k + G_d(z^{-1})\xi_k \quad (8)$$

where, U_k , Y_k are the measured output of the process and the manipulated input of the process respectively. z^{-1} denotes back-shift operator. $G_p(z^{-1})$, $G_d(z^{-1})$ are the transfer function of process and disturbance, then it can be defined in details as follows:

$$G_p = \frac{B(z^{-1})}{A(z^{-1})}, \quad G_d = \frac{T(z^{-1})}{\Delta(z^{-1})} \quad (9)$$

where, $A(z^{-1})$, $B(z^{-1})$ are the numerator and denominator of the model transfer function respectively, both of two can be obtained from Eq.(7). $\{\xi_k\}$ is a zero-mean discrete white noise sequence of variance σ_ξ^2 . $T(z^{-1})$ is a prefilter to improve the system robustness rejecting disturbance and noise. Δ is the increment operator. $\Delta = 1 - z^{-1}$. Polynomial $A(z^{-1})$, $B(z^{-1})$, $T(z^{-1})$ are defined as follows:

$$\begin{aligned} A(z^{-1}) &= A_1 + A_2z^{-1} + \dots + A_rz^{-r} \\ B(z^{-1}) &= B_1 + B_2z^{-1} + \dots + B_sz^{-s} \\ T(z^{-1}) &= T_1 + T_2z^{-1} + \dots + T_qz^{-q} \end{aligned} \quad (10)$$

For an objective function of finite horizon model predictive control can be described as follows:

$$J_{MPC} = \sum_{j=1}^P (Y_{k+j|k} - r_{k+j|k})^2 + \rho \sum_{j=1}^M (\Delta U_{k+j-1})^2 \quad (11)$$

where, M is the control horizon, P is the predictive horizon, ρ is weighting factor of control input. $r_{k+j|k}$ is the reference value of j step at future instant k . ΔU_{k+j-1} is the control input value of j step at future instant $k - 1$. $\hat{Y}_{k+j|k}$ is the predictive output value of j step at future instant k , which can calculate by the nominal disturbance model:

$$Y_k = G_{p0}(z^{-1})U_k + G_{d0}(z^{-1})\xi_k = \frac{B_0(z^{-1})}{A_0(z^{-1})}U_k + \frac{1}{\Delta}\xi_k \quad (12)$$

where, the subscript '0' in this expression denotes nominal values used in the controller design.

Configuring parameter with $r_{k+j|k} = 0$, $P = M$ and $P \rightarrow \infty$, then Eq.(11) can be rewritten as:

$$J_{MPC} = \sum_{j=1}^P (Y_{k+j|k})^2 + \rho \sum_{j=1}^P (\Delta U_{k+j-1})^2 \quad (13)$$

The MPC objective function can converges to the LQG objective function, which is shown as follows:

$$\frac{1}{P} J_{MPC} \rightarrow J_{LQG} = \text{var}\{Y_k\} + \rho \text{var}\{\Delta U_k\} \quad (14)$$

Thus, the solution of LQG objective function can be achieved by solving the following performance indexes:

$$\begin{aligned} J_{LQG} &= \lim_{P \rightarrow \infty} \frac{1}{P} \left\{ \sum_{j=1}^P (Y_{k+j|k})^2 + \rho \sum_{j=1}^P (\Delta U_{k+j-1})^2 \right\} \\ &= \lim_{P \rightarrow \infty} \left\{ \frac{1}{(P-1)T_s} \int_{T_s}^{PT_s} [(Y_k)^2 + \rho(\Delta U_{k-1})^2] dk \right\} \end{aligned} \quad (15)$$

where T_s denotes the sampling step.

In order to achieve the optimization of fractional order control, according to [43], the fractional definite integral operator is added to the objective function (15) to obtain:

$$\begin{aligned} J_{FLQG} &= \varepsilon_1 I_{T_s}^{\varepsilon_1} (Y_k)^2 + \varepsilon_2 I_{T_s}^{\varepsilon_2} \rho (\Delta U_{k-1})^2 \\ &= \int_{T_s}^{PT_s} D^{1-\varepsilon_1} (Y_k)^2 dk + \rho \int_{T_s}^{PT_s} D^{1-\varepsilon_2} (\Delta U_{k-1})^2 dk \end{aligned} \quad (16)$$

where, ${}^a I_a^b f(t) = \int_a^t [D^{1-\alpha} f(t)] dt$, $\varepsilon_1, \varepsilon_2$ are arbitrary order integral.

Using the definition of GL fractional order calculus, Eq.(16) can be changed to discrete form, then, according to [44], [45], the following be derived:

$$\begin{aligned} J_{FLQG} &= T_s^{\varepsilon_1} [\omega_0^{-\varepsilon_1} Y_{k+p} + \omega_1^{-\varepsilon_1} Y_{k+p-1} + \dots + (\omega_{p-1}^{-\varepsilon_1} - \omega_0^{-\varepsilon_1}) Y_{k+1} \\ &\quad + (\omega_p^{-\varepsilon_1} - \omega_1^{-\varepsilon_1}) Y_k + (\omega_{p+1}^{-\varepsilon_1} - \omega_2^{-\varepsilon_1}) Y_{k-1} + \dots] \\ &\quad + T_s^{\varepsilon_2} \rho [\omega_0^{-\varepsilon_2} \Delta U_{k+p-1} + \omega_1^{-\varepsilon_2} \Delta U_{k+p-2} + \dots \\ &\quad + (\omega_{p-1}^{-\varepsilon_2} - \omega_0^{-\varepsilon_2}) \Delta U_k + (\omega_p^{-\varepsilon_2} - \omega_1^{-\varepsilon_2}) \Delta U_{k-1} + \dots] \\ &= T_s^{\varepsilon_1} Y^T \Gamma_1 Y + T_s^{\varepsilon_2} \Delta U^T \Gamma_2 \Delta U \\ &= (T_s^{\varepsilon_1} Y^T \Gamma_1 Y + T_s^{\varepsilon_2} \Delta U^T \Gamma_2 \Delta U) \\ &\quad + (T_s^{\varepsilon_1} Y^T \Gamma Y + T_s^{\varepsilon_2} \Delta U^T \Gamma_2 \Delta U) \end{aligned} \quad (17)$$

where, the symbols \leftarrow, \rightarrow denote the past and the future value, Γ_1 and Γ_2 are infinite-dimensional square real weighting matrices which depend, by construction, on ε_1 and ε_2 , respectively. $x = \varepsilon_1, \varepsilon_2$,

$$\forall j > 0, \omega_j^{-x} = (-1)^{-j} \binom{-x}{j}, \forall j < 0, \omega_j^{-x} = 0.$$

At the instant k , the output of the instant $k - 1$, the input and output of the previous instant are known, the objective

function (17) can be further converted into the following function cost:

$$J_{FLQG} = Y^T \underset{\rightarrow k}{\Lambda}(\varepsilon_1, T_s) Y + \Delta U^T \underset{\rightarrow k-1}{\Lambda}(\varepsilon_2, T_s) \Delta U \quad (18)$$

where,

$$\begin{aligned} \Lambda(\varepsilon_1, T_s) &= T_s^{\varepsilon_1} \Gamma_1 \\ &= T_s^{\varepsilon_1} \begin{bmatrix} \Gamma & 0 \\ \leftarrow^{-1} & \Gamma \end{bmatrix} \\ &= T_s^{\varepsilon_1} \text{diag}(\dots \omega_p^{-\varepsilon_1} - \omega_{-1}^{-\varepsilon_1} | \omega_{p-1}^{-\varepsilon_1} - \omega_0^{-\varepsilon_1} \dots \omega_1 \omega_0) \\ \Lambda(\varepsilon_2, T_s) &= T_s^{\varepsilon_2} \Gamma_2 \\ &= T_s^{\varepsilon_2} \begin{bmatrix} \Gamma & 0 \\ \leftarrow^{-2} & \Gamma \end{bmatrix} \\ &= \rho T_s^{\varepsilon_2} \text{diag}(\dots \omega_p^{-\varepsilon_2} - \omega_{-1}^{-\varepsilon_2} | \omega_{p-1}^{-\varepsilon_2} - \omega_0^{-\varepsilon_2} \dots \omega_1 \omega_0) \end{aligned}$$

$$Y = \begin{bmatrix} Y \\ \leftarrow^{-k} \\ Y \\ \rightarrow^{-k} \end{bmatrix} = \begin{bmatrix} \vdots \\ Y_{k-2} \\ Y_{k-1} \\ Y_k \\ Y_{k+1} \\ Y_{k+2} \\ \vdots \\ Y_{k+p} \end{bmatrix},$$

$$\Delta U = \begin{bmatrix} U \\ \leftarrow^{-k-1} \\ U \\ \rightarrow^{-k-1} \end{bmatrix} = \begin{bmatrix} \vdots \\ \Delta U_{k-3} \\ \Delta U_{k-2} \\ \Delta U_{k-1} \\ \Delta U_k \\ \Delta U_{k+1} \\ \vdots \\ \Delta U_{k+p-1} \end{bmatrix}$$

In the absence of constraints, the minimization of this cost function (18) under the constraints, the minimization of this cost function (18) under the closed-loop system leads to a control law [46]:

$$U_k = -G_c(z^{-1}) Y_k = -\frac{S(z^{-1})}{\Delta R(z^{-1})} Y_k \quad (19)$$

where, $S(z^{-1}), R(z^{-1})$ are the controller's polynomial, which can be obtained from Eq.(20). $\Phi_i, F_i, i = 1, 2, \dots, P$ are two polynomials obtained from the resolution of two Diophantine equations (see [46] for more details).

$$R(z^{-1}) = \frac{T(z^{-1}) + \sum_{i=1}^P k_i \Phi_i}{\sum_{i=1}^P k_i z^{-P+i}}, \quad S(z^{-1}) = \frac{\sum_{i=1}^P k_i F_i}{\sum_{i=1}^P k_i z^{-P+i}} \quad (20)$$

The specific expression of MPC controller can be obtained by solving LQG optimization problem with

spectral decomposition:

$$G_c(z^{-1}) = \frac{S(z^{-1})}{\Delta R(z^{-1})} = \frac{A_0(z^{-1})}{\bar{\gamma}(z^{-1}) - B_0(z^{-1})} \quad (21)$$

where, the polynomial $\bar{\gamma}(z^{-1})$ is the invertible factor of the spectral factorization Eq.(22).

$$\bar{\gamma}(z^{-1})\bar{\gamma}(z) = B_0(z^{-1})B_0(z) + \rho(1 - z^{-1})A_0(z^{-1})A_0(z)(1 - z) \quad (22)$$

The following closed loop relationships may be developed by inserting the control law (19) into the true system description of Eq.(8). For convenience, the operator z^{-1} is omitted from the following formula. According to Eq.(8),(19), and we can obtain:

$$Y_k = \frac{TAR}{BS + A\Delta R} \xi_k \quad (23)$$

$$U_k = -\frac{TAS}{\Delta(BS + A\Delta R)} \xi_k \quad (24)$$

Dropping the argument for convenience. Applying Parseval's theorem to Eq.(23),(24) enables the variances of the process output and differenced input to be computed as [47]:

$$Var(Y_k^{opt}) = \sigma_Y^2 = \frac{\sigma_\xi^2}{2\pi j} \oint_{|z|=1} \left| \frac{TAR}{BS + A\Delta R} \right|^2 \frac{dz}{z} \quad (25)$$

$$Var(U_k^{opt}) = \sigma_U^2 = \frac{\sigma_\xi^2}{2\pi j} \oint_{|z|=1} \left| \frac{TAS}{\Delta(BS + A\Delta R)} \right|^2 \frac{dz}{z} \quad (26)$$

B. TRADEOFF CURVE AND PERFORMANCE INDEX

By changing the ρ between $[0, \infty]$, then take the optimal input variance as the horizontal axis and the optimal output variance as the vertical axis, By drawing the performance limit curve, the variance performance indexes of the linear controller can be obtained, such as Figure. 1. And the performance indexes for assessment are defined in Figure. 1.

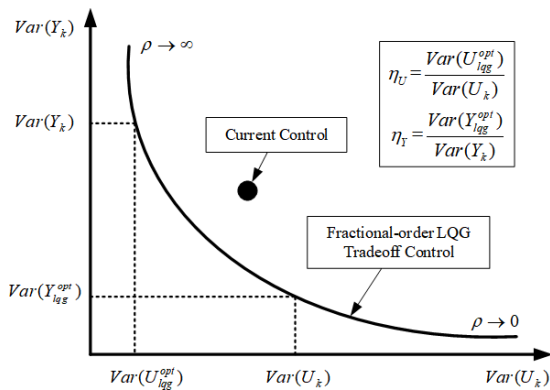


FIGURE 1. The performance limit curve based on fractional order LQG.

IV. SIMULATION CASE

In this section, we use an example of a heating furnace in literature [28] (Fang *et al.*, 2017) to develop a fractional order model of the heating furnace with the corresponding

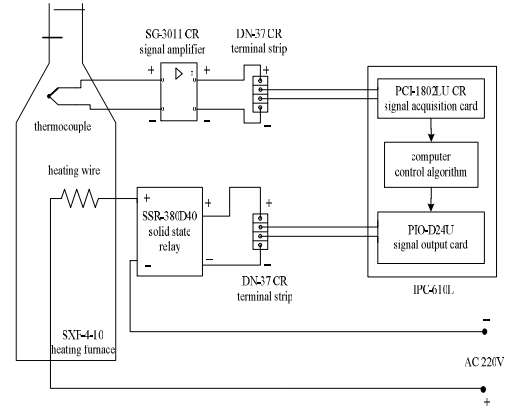


FIGURE 2. The electric heating furnace SXF-4-10.

measured data. Then we will introduce the performance evaluation results of FO-PID and the validity of fractional order LQG benchmark is verified through MATLAB simulation. The simulations are done on MATLAB 2014a, and the corresponding computer configurations are: operation system windows 10, CPU i7-8550U 1.8GHz, Memory 16.0 GB.

A. TEMPERATURE CONTROL SYSTEM DESCRIPTION

The heating furnace's schematic diagram is shown in Figure 2. Its rated working voltage is 220V and rated power is 4KW. The heating process circuit of the furnace is described as follows: from the positive pole of 220V AC to the positive pole on the left side of the resistive driver, the heating wire and the solid-state relay respectively, from the negative pole on the right side of the solid-state relay to the negative pole of 220V AC.

The FO-PID controller is used to control the temperature of the industrial heating furnace system modeled by a fractional order model. The control process block diagram is shown in Figure 3.

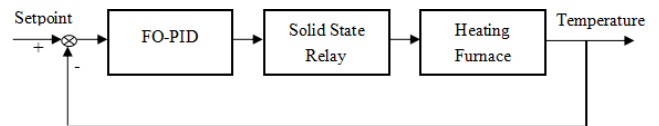


FIGURE 3. The control process block diagram of heating furnace.

B. PROCESS MODEL

The heating furnace model in literature [28] is

$$G_p = \frac{28.5}{735s + 1} e^{-100s} \quad (27)$$

On the basic of the values of $T = 735$, $K = 28.5$, $\tau = 100$, $\alpha = 1$ of the integer order model in Eq.(27), the variances between the step response output of model and practical process output of the heating furnace were utilized to adjust the parameters of fractional order model. Then it is obtained by using the new Luus-Jaakola(NLJ) method [48],

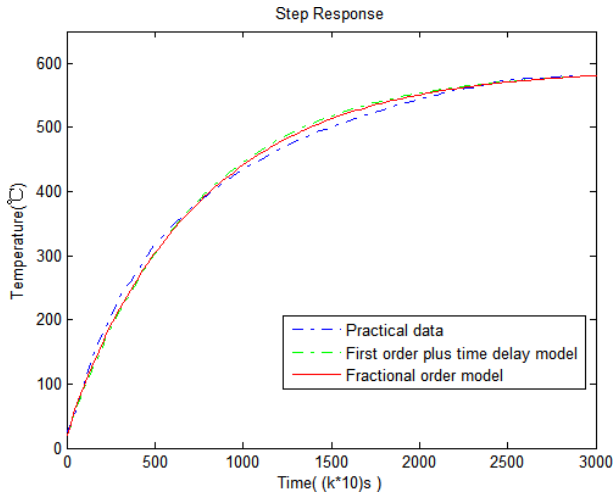


FIGURE 4. The practical temperature and open-loop step responses of models.

and its effectiveness for practical process has been testified on the heating furnace by several experiments.

The fractional order model is

$$G_p = \frac{31.8}{510s^{0.93} + 1} e^{-100s} \quad (28)$$

In Fig. 4, the practical temperature and the open-loop step response of the two models are shown.

In the course of the experiment, the disturbance transfer function model is simulated as:

$$G_d = \frac{1 + 0.2500z^{-1}}{1 - 0.9732z^{-1}} \quad (29)$$

Finally, the process model of the furnace can be selected as:

$$Y_k = G_p U_k + G'_d \xi_k \quad (30)$$

where, G'_d is the transfer function form of G_d .

C. FRACTIONAL ORDER LQG TRADEOFF CURVE

A gaussian white noise with a mean value of 0 and variance of 0.1 is added to the temperature control system. The order of LQG objective function are selected as $\epsilon_1 = 0.8$, $\epsilon_2 = 1.2$, the weighted value ρ is changed within [0.00000001,160000] since that the selected interval of ρ is enough to represent the whole trend of LQG curve. Through the fractional order LQG algorithm, the variance value of corresponding optimal input and output is calculated. Then, we can obtain the fractional order LQG Performance tradeoff curve, as shown in Figure. 5.

D. THE COMPARISON OF TRADEOFF CURVES

For the fractional order and Integer order process model obtained from the same heating furnace data, they use the same interference transfer function, the corresponding performance curve can be obtained by using a conventional LQG benchmark and a fractional order LQG benchmark, respectively. The result is shown in Figure. 6.

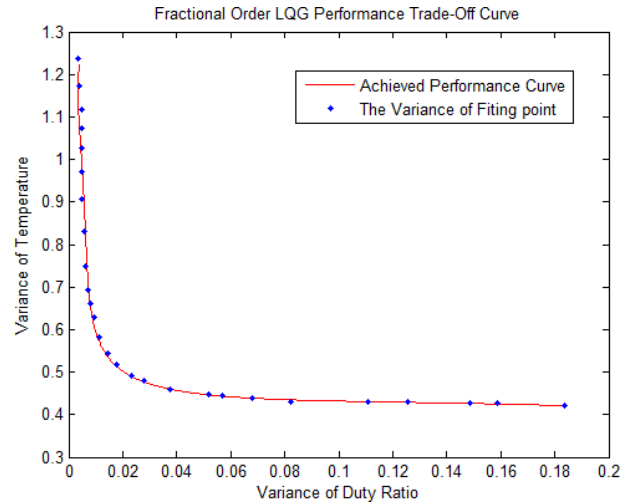


FIGURE 5. The fractional order LQG performance tradeoff curve.

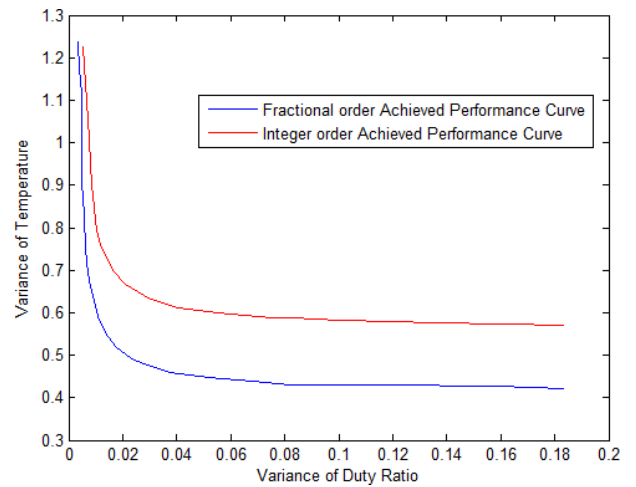


FIGURE 6. The comparison of two tradeoff curves.

As can be seen from Figure. 4, the performance tradeoff curve calculated using a fractional order LQG benchmark is better than that calculated using the conventional LQG benchmark under the same conditions. It means that this benchmark can provide a better reference standard for the performance assessment of system. Moreover, when evaluating the fractional order systems and the fractional order controllers, the conventional LQG algorithm can be available by an operation of model transformation. The fractional order LQG benchmark is directly calculated using a fractional order LQG algorithm, the operation of the model transformation is omitted, the model approximation error is reduced and the computation is smaller.

E. EXPERIMENTAL RESULTS

A class of fractional order PID controller [49] is used to the experiment of the heating furnace, the transfer function of which can be described as:

$$G_c(S) = k_p + K_i \frac{1}{S} + K_d S^\gamma \quad (31)$$

The proportion, integral and differential coefficients, fractional order γ of the corresponding fractional order PID controller are selected as follows:

$$\gamma = 0.3, K_p = 0.2011, K_i = 0.000132, K_d = 0.8956$$

At the same time, considering the actual situation, there are often some uncertain factors or unmeasured interference, which makes the established model have certain errors. Therefore, it is necessary to consider both the model matching and the model mismatch. Here, we select two sets of parameters of process model under model mismatch, so three sets of the process parameters are as follows:

Model 1: $\alpha = 0.93, T = 510, \tau = 100, K = 31.8$

Model 2: $\alpha = 0.87, T = 408, \tau = 80, K = 25.4$

Model 3: $\alpha = 0.92, T = 612, \tau = 120, K = 38.16$

Since we mainly evaluate the steady state performance of the furnace system, the setpoint of temperature are selected 600°C and 605°C respectively. Through many experiments, the actual temperature and duty ratio variance of the system are shown in Figure.7a-c. The actual the performance assessment results of the actual temperature and duty ratio variance of the system are shown in Figure. 8.

TABLE 1. Statistical results of steady state performance.

Setpoint	Model	$Var(u_k)$	$Var(y_k)$	$Var(u_{lag}^{opt})$	η_u (%)	$Var(y_{lag}^{opt})$	η_y (%)
600°C	Model 1	0.0267	0.8137	0.0063	23.59	0.4804	59.33
	Model 2	0.0218	0.6921	0.0074	33.94	0.4957	71.62
	Model 3	0.0306	0.8930	0.0057	18.62	0.4716	52.81
605°C	Model 1	0.0289	0.8684	0.0059	20.41	0.4751	54.70
	Model 2	0.0240	0.7546	0.0068	28.33	0.4879	64.65
	Model 3	0.0334	0.9430	0.0053	15.86	0.4664	49.45

According to the experimental results, the input variance and the output variance in the model matching and the model mismatch are shown in Table 1. The assessment of the performance of the control system has been performed according to the performance index defined above. When the heating furnace temperature setpoint is 600°C, the performance index of the match model (Model 1) is $\eta_u = 23.59\%$, $\eta_y = 59.03\%$, the performance index of the mismatch model (Model 2) is $\eta_u = 33.94\%$, $\eta_y = 71.62\%$, and the performance index of the mismatch model (Model 3) is $\eta_u = 18.62\%$, $\eta_y = 52.81\%$. When the heating furnace temperature setpoint is 605°C, the performance index of the match model (Model 1) is $\eta_u = 20.41\%$, $\eta_y = 54.70\%$, the performance index of the mismatch model (Model 2) is $\eta_u = 28.33\%$, $\eta_y = 64.65\%$, and the performance index of the mismatch model (Model 3) is $\eta_u = 15.86\%$, $\eta_y = 49.45\%$. On the basis of the above results, there is a large room for improvement in the control performance.

According to the results of the assessment and the simulation results, here the parameter of the FOPID controller as well as the integral and the derivative coefficients are

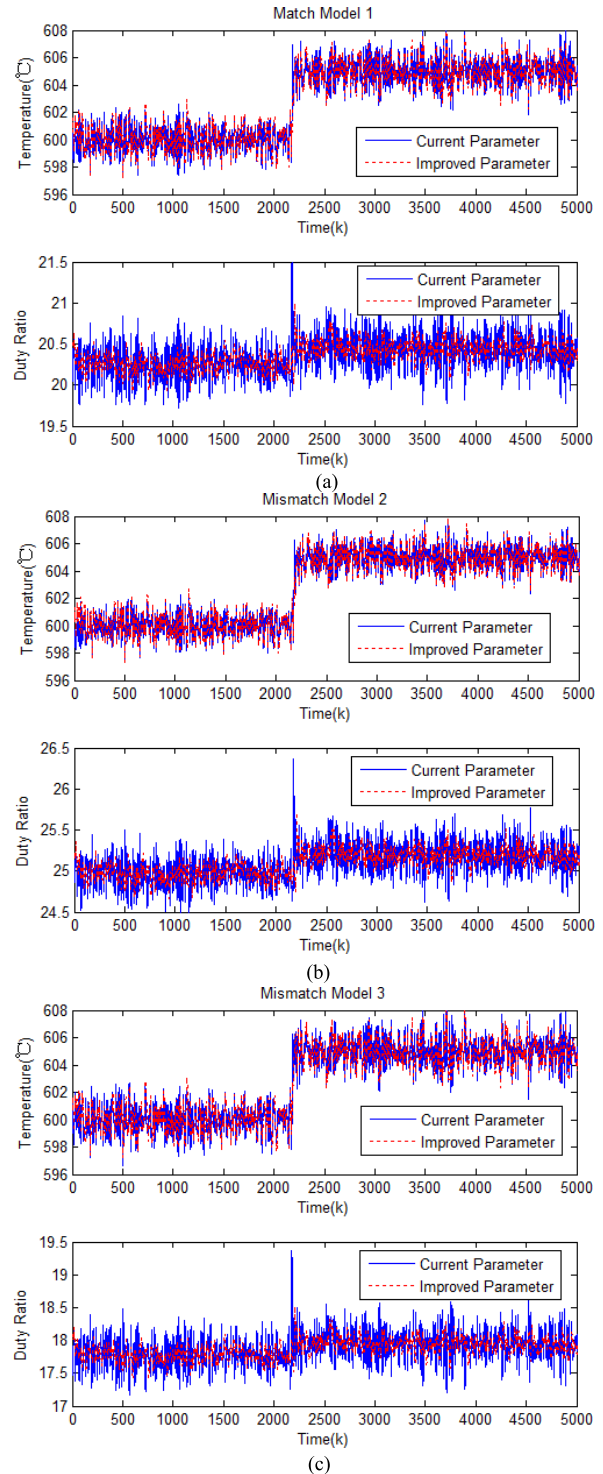


FIGURE 7. (a) Corresponding input and output of the different parameter (Model 1). (b) Corresponding input and output of the different parameter (Model 2). (c) Corresponding input and output of the different parameter (Model 3).

adjusted by the trial and error method. We choose one set of the controller parameters with the best control effect of the heating furnace, the parameters are:

$$\gamma = 0.6, K_p = 0.1011, K_i = 0.000126, K_d = 0.8956$$

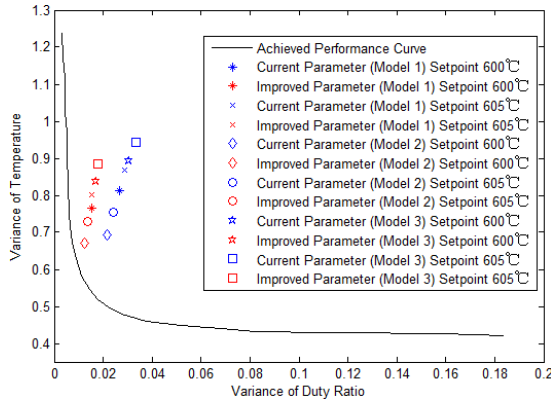


FIGURE 8. Limit curve based on fractional order LQG benchmark and actual variance of the system.

After adjustment and recalculation of the relevant data and performance index, the simulation results of the three different model parameters are further shown in Figure. 7a-c. When the heating furnace temperature setpoint is 600 °C, it shows that the performance index of the match model (Model 1) is improved to $\eta_u = 44.07\%$, $\eta_y = 69.37\%$, the performance index of the mismatch model (Model 2) is improved to $\eta_u = 62.60\%$, $\eta_y = 83.25\%$, and the performance index of the mismatch model (Model 3) is improved to $\eta_u = 36.25\%$, $\eta_y = 61.75\%$. When the heating furnace temperature setpoint is 605 °C, it shows that the performance index of the match model (Model 1) is improved to $\eta_u = 41.55\%$, $\eta_y = 66.02\%$, the performance index of the mismatch model (Model 2) is improved to $\eta_u = 51.09\%$, $\eta_y = 74.64\%$, and the performance index of the mismatch model (Model 3) is improved to $\eta_u = 32.58\%$, $\eta_y = 58.01\%$. By readjusting the parameters of FO-PID controller, the control performance is greatly improved for both model matching and model mismatch cases, statistical results of which is shown in Table 2.

TABLE 2. Statistical results of steady state performance.

Setpoint	Model	$Var(u_k)$	$Var(y_k)$	$Var(u_{lqg}^{opt})$	η_u (%)	$Var(y_{lqg}^{opt})$	η_y (%)
600°C	Model 1	0.0152	0.7650	0.0067	44.07	0.5307	69.37
	Model 2	0.0123	0.6719	0.0077	62.60	0.5594	83.25
	Model 3	0.0171	0.8388	0.0062	36.25	0.5180	61.75
605°C	Model 1	0.0154	0.8014	0.0064	41.55	0.5291	66.02
	Model 2	0.0137	0.7280	0.0070	51.09	0.5434	74.64
	Model 3	0.0178	0.8857	0.0058	32.58	0.5138	58.01

According to the results, in the case of model matching and model mismatch, the actual input variance and output variance of the system are obtained before and after changing the controller parameters, and the specific values are shown in Figure. 9, Figure. 10.

We can intuitively find that input variance and output variance can be reduced to a certain extent by adjusting

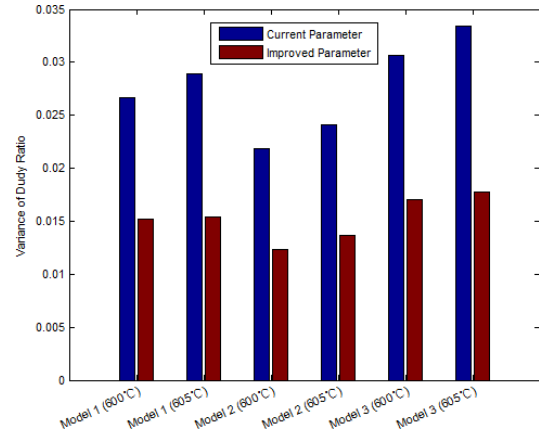


FIGURE 9. The output variance of system under the tuning of current/improved parameter.

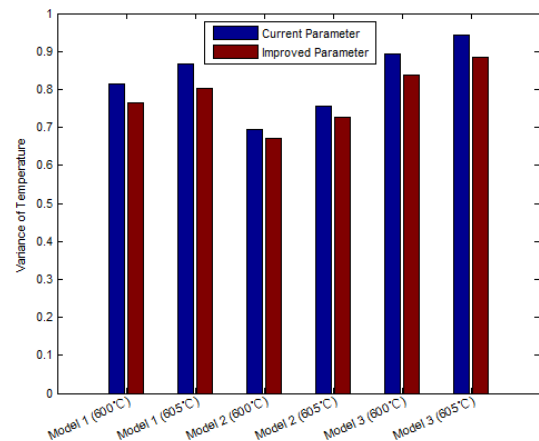


FIGURE 10. The output variance of system under the tuning of current/improved parameter.

fractional order PID parameters in the case of model matching and model mismatch, so that the performance point is close to FO-LQG performance tradeoff curve, this indicates that the control performance of the controller has been greatly improved after parameter adjustment as expected. In addition, the actual input variance and output variance of the system before and after the controller parameter adjustment were compared under the conditions of model matching and model mismatch for different heating furnace temperature setpoints, and the results were drawn as bar graphs in Figure 9, Figure 10. The results show that when the furnace temperature was set at 600°C, the input and output variances corresponding to model 1 decreased by 20.48% and 10.34%, respectively. The input and output variances corresponding to model 2 decreased by 28.66% and 11.62%, respectively. And the input and output variances corresponding to model 3 decreased by 24.63% and 8.94%, respectively. When the furnace temperature is set at 605°C, the input and output variances corresponding to the matching model 1 decreased by 20.48% and 10.34%, respectively. The input and output variances corresponding to the mismatch model 2 decreased by 21.14% and 11.32%, respectively. And the input and output variances corresponding to the mismatch

model 3 decreased by 16.72% and 8.56%, respectively. Compared with the output variance, the decrease of input variance is more obvious, and the system is in the control state of minimum energy. Through the redesign of the controller parameters after the system performance evaluation, the system control performance under the condition of model matching and model mismatch is improved effectively.

V. CONCLUSION

In this paper, A fractional order LQG benchmark is developed for the performance assessment on fractional order systems. By solving the linear quadratic Gaussian problem under the fractional order model and fractional control, the optimal input and output variances are determined for different weighting factors, the achieved performance curves and performance indicators of input and output variance are given. At the same time, the comparison between fractional order LQG and the conventional LQG shows the improvement of the proposed benchmark under the same condition. Finally, the fractional order LQG benchmark is used to evaluate the performance of the furnace system based on FO-PID controller with model matching and model mismatch. By solving the current control performance of FO-PID with current parameter, and then calculating the corresponding performance indexes, much room for improvement in control performance can be given. Furthermore, through the adjustment of parameters, the control performance of FO-PID controller is further improved. The furnace system is not only in the control state of minimum energy, but also more stable for control output. Besides, the lower performance limit of the linear fractional order controller is given. This allows us to effectively evaluate the control performance of fractional order controllers during furnace control experiments and to make adjustments in time.

APPENDIX A

CALCULATION OF THE REPRESENTATIVE OF FRACTIONAL ORDER DISCRETE MODEL

Step1: For simplicity, the following single-input single-output fractional order Laplace transform model with time delay is discussed:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{Ts^\alpha + 1} e^{-\tau s} \quad (\text{A.1})$$

Step2: The representative of the fractional differential equation (A.1) can be written in the following:

$$TS^\alpha Y(s) + Y(s) = e^{-\tau s} U(s) \quad (\text{A.2})$$

Then a inverse Laplace transform is applied into the Eq.(A.2), the following fractional differential equation is gained [34], [35]:

$${}_0D_t^\alpha y(t) + y(t) = u(t - d) \quad (\text{A.3})$$

Step3: Discretion of the fractional differential equation

According to the Grunwald-Letnikov (GL) definition, the discretization formula of fractional derivatives is defined

to be (A.4):

$${}_0D_t^\alpha y(t) = h^{-\alpha} \sum_{j=0}^L w_j^\alpha y(k - j) \quad (\text{A.4})$$

where h denotes the sampling instant, w_j^α is the weight coefficient, which is calculated by the following recurrence equation:

$$w_0^\alpha = 1, w_j^\alpha = \left(1 - \frac{\alpha + 1}{j}\right) w_{j-1}^\alpha \quad (\text{A.5})$$

Then the discretization fractional differential equation is obtained:

$$y(k) + \mu \sum_{j=1}^L w_j^\alpha y(k - j) = Hu(k - d) \quad (\text{A.6})$$

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