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Marginal Distribution Multi-Target Bayes Filter With Assignment of Measurements

ZONG-XIANG LIU, MIAN WU, AND JIE GAN
College of Electronic and Information Engineering, Shenzhen University, Shenzhen 518060, China

Corresponding author: Zong-Xiang Liu (liuzx@szu.edu.cn)

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ABSTRACT This work proposes a marginal distribution multi-target Bayes filter with assignment of measurements to track multiple targets in the presence of an unknown and variable number of targets, clutter, and missed detections. Mathematically, the association of the measurements with either a target or clutter may be established by maximizing the joint likelihood function of the measurement partition, which leads to a two-dimensional assignment problem. By the introduction of detecting label, a handling approach for missed detections is also developed and is applied to the proposed filter. This filter greatly reduces the number of hypothesized targets or Gaussian terms by selecting the predicted probability density of a target or one of its multiple updated probability densities as its state distribution at each time step. Experimental results indicate that the proposed filter requires a less computational load than the existing filters and performs better than the efficient implementations of the δ-generalized labeled multi-Bernoulli filter for multi-target tracking at low and moderate clutter densities.

INDEX TERMS Multi-target tracking, marginal distribution multi-target Bayes filter, generalized labeled multi-Bernoulli filter, two-dimensional assignment.

I. INTRODUCTION

The objective of multi-target tracking is to estimate the states of multiple objects at different times by using the measurements that are obtained by a sensor at different times [1]–[3]. This technique has an important application in sonar tracking, radar tracking, space surveillance and traffic control. The major problem in multi-target tracking is the uncertainty of detection, uncertainty of data association and the presence of clutter. Joint probabilistic data association (JPDA) [4], multiple hypotheses tracking (MHT) [5] and random finite set (RFS) [1], [6] provide an efficient solution for this problem.

As a solution paradigm of multi-target tracking problem, the RFS approach provides an elegant Bayesian framework for multi-target tracking and is also the theoretical foundation of the probability hypothesis density (PHD) filter [7], cardinalized PHD (CPHD) filter [8] and cardinality-balanced multi-Bernoulli (CBMeMber) filter [9]. The main difference between the PHD filter and CBMeMber filter is that the former propagates the posterior intensity in the filter recursion [1], [6]–[8], while the latter propagates the parameters of multi-Bernoulli distribution to approximate the posterior density [9]. The PHD filter and CBMeMber filter are efficient for multi-target tracking in case of high detecting probability. However, they have a weak memory and become inefficient in case of low detecting probability because they fail to provide state estimation of an existing target if this target is undetected at a time. Instead of maintaining the joint posterior probability density of multiple targets, the marginal distribution multi-target Bayes (MDMTB) filter was proposed in [10] and [11] to propagate the state distribution and existence probability of each target in the filter recursion. The advantage of the MDMTB filter over the PHD filter and CBMeMber filter is that it may provide the state estimations of missed targets in case of low detecting probability due to its strong memory. Despite a lack of trajectory of the target, these filters have been applied to autonomous vehicle [12], sonar tracking [13] and radar tracking [14]. Their extensions have been developed to track the maneuvering target [15]–[17], the extended target [18]–[21], the target with glint noise [22] and the target in possible subsequent missed detections [23].

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Detection-guided Bayesian filter [24] and parallel implementation of the sequential Monte Carlo PHD filter [25] were also designed to obviate the need for exact knowledge of birth objects and to improve the computational efficiency of the sequential Monte Carlo PHD filter, respectively.

Recently, the δ-generalized labeled multi-Bernoulli (δ-GLMB) filter was proposed in [26], [27]. The advantages of the δ-GLMB filter over the PHD filter and CBMeMber filter are that it may provide the target track and that it does not require high signal to noise ratio, therefore it is applicable to the case of high clutter density and low detecting probability [28]. However, the δ-GLMB filter requires enumerating an exponentially growing number of hypotheses, which leads to a high computational complexity [29]. Two efficient approximations of the δ-GLMB filter have been proposed to reduce the computational complexity. One is the labeled multi-Bernoulli (LMB) filter [28] and the other is the efficient implementation of the generalized labeled multi-Bernoulli (GLMB) filter [30] called the rapid GLMB (R-GLMB) filter in this paper. The extensions of the δ-GLMB filter for diverse applications have also been reported in [31]–[34]. Despite its efficiency, Vo's efficient implementation of the δ-GLMB filter [30] still requires a much larger computational load than the PHD filter or CBMeMber filter due to the fact that the number of hypothesized targets or tracks in this implementation is significantly greater than the number of real targets. The large computational load also restricts the application of the R-GLMB filter in many actual tracking systems where the number of targets in a surveillance region is up to hundreds. To address this problem, we propose an MDMTB filter with assignment of measurements (AM-MDMTB filter) in this study. The proposed filter uses the maximization of the joint likelihood function or minimization of the sum of negative log-likelihood function to assign each measurement to either a target or clutter, which leads to a two-dimensional (2-D) assignment problem. The Hungarian algorithm is employed to solve the 2-D assignment problem to obtain the association result of individual measurements and individual targets. Based on the association result, a handling approach for missed detections is also developed and is applied to the proposed filter. Similar to the R-GLMB filter, the proposed AM-MDMTB filter uses the track label to distinguish individual targets. Therefore it may provide the target track. Unlike the R-GLMB filter that searches the K best 2-D assignments to determine whether a target is died, or surviving and detected, or surviving and undetected; the proposed filter finds an optimal 2-D assignment to associate each measurement to either a target or clutter. Importantly, the proposed filter selects the predicted probability density of a target in case of missed detections or one of its multiple updated probability densities in other case as its state distribution at each time step. Therefore the number of hypothesized targets or state distributions in the proposed filter approximates the number of real targets. Besides, the AM-MDMTB filter inherits the merits of the MDMTB filter, it may provide the state estimations of missed targets in case of low detecting probability due to its strong memory.

The main contributions of this paper are as follows:

(1) We establish a mathematical model of the 2-D assignment by minimizing the sum of negative log-likelihood function of the measurement partition to assign each measurement to either an existing target or a birth target or clutter.

(2) Based on the established model of the 2-D assignment and the developed handling approach of missed detections, we propose an AM-MDMTB filter for a linear Gaussian system.

The paper is organized as follows. In Section II, we introduce the MDMTB filter and establish the mathematical model for assignment of measurements. The AM-MDMTB filter for a linear Gaussian system is described in Section III. The performance evaluation of the proposed filter is given in Section IV by simulation, and concluding remarks are drawn in Section V.

II. PROBLEM FORMATION

A. MDMTB FILTER

The MDMTB filter was proposed in [10] and [11] to propagate the state distribution (i.e., probability density) and existence probability of each target in filter recursion. It assumes that each target evolves and generates the measurement independently of one another. In this filter, state distribution of each target is used to model the uncertainty of the target state caused by the dynamic uncertainty and measurement uncertainty, and the existence probability of each target is employed to characterize the randomness of target appearance and disappearance. Let $\{\boldsymbol{x}_{i,k-1}\}_{i=1}^{N_{k-1}}$ denote the multi-target states at time step $k-1$ where $\boldsymbol{x}_{i,k-1}$ and N_{k-1} are the state vector of hypothesized target i and the number of hypothesized targets at time step $k-1$, respectively, let $\boldsymbol{y}_{1:k-1} = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_{k-1}\}$ denote all observations up to time step $k-1$, and suppose that the state distributions and existence probabilities of individual hypothesized targets at time step $k-1$ are given by

$$f_{i,k-1}(\boldsymbol{x}_{i,k-1}|\boldsymbol{y}_{1:k-1}); \quad i = 1, \cdots, N_{k-1} \tag{1}$$

$$\rho_{i,k-1}; \quad i = 1, \cdots, N_{k-1} \tag{2}$$

Using the prediction equation of the MDMTB filter to deal with each hypothesized target independently, we obtain the predicted probability density of individual hypothesized targets at time step k as

$$\begin{aligned} f_{i,k|k-1}&(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1}) \\ &= \int f(\boldsymbol{x}_{i,k}|\boldsymbol{x}_{i,k-1}) \\ &\quad \cdot f_{i,k-1}(\boldsymbol{x}_{i,k-1}|\boldsymbol{y}_{1:k-1}) d\boldsymbol{x}_{i,k-1}; \quad i = 1, \cdots, N_{k-1} \end{aligned} \tag{3}$$

where $f(\boldsymbol{x}_{i,k}|\boldsymbol{x}_{i,k-1})$ is the transition probability between state vectors $\boldsymbol{x}_{i,k-1}$ and $\boldsymbol{x}_{i,k}$. The predicted existence probability of each hypothesized target is given by

$$\rho_{i,k|k-1} = \rho_{i,k-1}; \quad i = 1, \cdots, N_{k-1} \tag{4}$$

Besides the existing targets, the birth targets may appear at time step k. Let $\left\{\boldsymbol{x}_{i,k}^{b}\right\}_{i=1}^{N_k^b}$ denote the states of hypothesized birth targets at time step k where $\boldsymbol{x}_{i,k}^{b}$ and N_k^b are the state vector of hypothesized birth target i and the number of hypothesized birth targets at time step k, respectively, and suppose that the probability densities and existence probabilities of individual hypothesized birth targets at time step k are given by

$$f_{i,k}(\boldsymbol{x}_{i,k}^{b}); \quad i=1,\cdots,N_k^b \tag{5}$$

$$\rho_{i,k}^{b}; \quad i=1,\cdots,N_k^b \tag{6}$$

The multi-target state at time step k consists of the existing target state and birth target state and it may be given by

$$\left\{\boldsymbol{x}_{i,k}\right\}_{i=1}^{N_{k|k-1}} = \left\{\boldsymbol{x}_{i,k}\right\}_{i=1}^{N_{k-1}} \bigcup \left\{\boldsymbol{x}_{i,k}^{b}\right\}_{i=1}^{N_k^b} \tag{7}$$

where $N_{k|k-1} = N_{k-1} + N_k^b$ denotes the predicted number of hypothesized targets at time step k. The predicted probability densities and existence probabilities of individual hypothesized targets at time step k may be given by

$$\begin{aligned}&\left\{f_{i,k|k-1}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1})\right\}_{i=1}^{N_{k|k-1}}\\ &\quad = \left\{f_{i,k|k-1}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1})\right\}_{i=1}^{N_{k-1}} \bigcup \left\{f_{i,k}(\boldsymbol{x}_{i,k}^{b})\right\}_{i=1}^{N_k^b}\end{aligned} \tag{8}$$

$$\begin{aligned}&\left\{\rho_{i,k|k-1}\right\}_{i=1}^{N_{k|k-1}}\\ &\quad = \left\{\rho_{i,k|k-1}\right\}_{i=1}^{N_{k-1}} \bigcup \left\{\rho_{i,k}^{b}\right\}_{i=1}^{N_k^b}\end{aligned} \tag{9}$$

Let $\boldsymbol{y}_k = \left\{z_{j,k}\right\}_{j=1}^{M_k}$ denote the measurement at time step k where M_k is the number of measurements at time step k. The MDMTB filter exploits the Bayes rule to deal with measurement $z_{j,k}$ and predicted probability density $f_{i,k|k-1}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1})$ to obtain the updated probability density of target i corresponding to observation $z_{j,k}$ as

$$f_{ij,k}(\boldsymbol{x}_{i,k}|z_{j,k}) = \frac{f(z_{j,k}|\boldsymbol{x}_{i,k})f_{i,k|k-1}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1})}{\int f(z_{j,k}|\boldsymbol{x}_{i,k})f_{i,k|k-1}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1})d\boldsymbol{x}_{i,k}} \tag{10}$$

where $f(z_{j,k}|\boldsymbol{x}_{i,k})$ is the probability density that state vector $\boldsymbol{x}_{i,k}$ generates observation $z_{j,k}$.

Since measurement $z_{j,k}$ originates from clutter, target i or other targets, the updated existence probability of target i corresponding to observation $z_{j,k}$ may be given by

$$\rho_{ij,k} = \frac{\eta_{ij}}{\lambda_c + \sum_{e=1}^{N_{k|k-1}} \eta_{ej}} \tag{11}$$

where λ_c is the clutter density and η_{ij} is the probability that measurement $z_{j,k}$ originates from target i. This probability may be given by

$$\eta_{ij} = p_{D,k}\rho_{i,k|k-1}\int_S f(z_{j,k}|\boldsymbol{x}_{i,k})f_{i,k|k-1}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1})d\boldsymbol{x}_{i,k} \tag{12}$$

where S denotes the entire state space and $p_{D,k}$ is the detecting probability.

Assume that clutter follows a Poisson distribution and is uniform in the surveillance field. The probability density of clutter is given by

$$\lambda_c = \frac{N_c}{\Phi_S} \tag{13}$$

where N_c is the average number of clutter and Φ_S is the surveillance field.

Based on the predicted and updated existence probabilities of a hypothesized target, the MDMTB filter selects either its predicted probability density or one of its multiple updated probability densities as its state distribution at time step k. A pruning step is also required in the MDMTB filter to discard the hypothesized target with sufficiently small existence probability. The state distributions and existence probabilities of residual hypothesized targets after pruning may be denoted as $\left\{f_{i,k}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k}),\ \rho_{i,k}\right\}_{i=1}^{N_k}$ where N_k is the number of hypothesized targets at time step k. For more details, we refer the reader to [10] and [11].

B. ASSIGNMENT OF MEASUREMENTS

Each measurement originates from either a target or clutter. An optimal solution for assigning each measurement to either a target or clutter is to maximize the joint likelihood function of an assignment of measurements as

$$\max \prod_{j=1}^{M_k}\left\{\left(\prod_{i=1}^{N_{k|k-1}} s_{ij}\eta_{ij}\right)\left(\prod_{i=1}^{M_k} s'_{ij}\lambda_c\right)\right\} \tag{14}$$

subject to

$$\sum_{i=1}^{N_{k|k-1}} s_{ij} + \sum_{i=1}^{M_k} s'_{ij} = 1 \quad \text{for } j=1,\cdots,M_k \tag{15}$$

$$\sum_{j=1}^{M_k} s_{ij} \le 1 \quad \text{for } i=1,\cdots,N_{k|k-1} \tag{16}$$

$$\sum_{j=1}^{M_k} s'_{ij} \le 1 \quad \text{for } i=1,\cdots,M_k \tag{17}$$

where s_{ij} and s'_{ij} are the binary variables, and their values are either 1 or 0. $s_{ij}=1$ and $s'_{ij}=1$ indicate that $z_{j,k}$ originates from target i and that $z_{j,k}$ is clutter, respectively.

The maximization of the joint likelihood function in (14) is equivalent to the minimization of the sum of negative log-likelihood function. This problem can be converted to a 2-D assignment problem as

$$\min \sum_{j=1}^{M_k}\left(\sum_{i=1}^{N_{k|k-1}} s_{ij}c_{ij} + \sum_{i=1}^{M_k} s'_{ij}c'_{ij}\right) \tag{18}$$

subject to (15), (16) and (17). The c_{ij} and c'_{ij} in (18) are the association costs. Cost c_{ij} is given by the negative log-likelihood that measurement $z_{j,k}$ originates from target i as

$$c_{ij} = -\ln(\eta_{ij}) \tag{19}$$

$$
\begin{array}{cccc}
 & \text{targets} & & \text{clutter} \\
 & \overbrace{\mathrm{T}_1 \quad \cdots \quad \mathrm{T}_{N_{k-1}}}^{\text{existing targets}} & \overbrace{\mathrm{T}_{N_{k-1}+1} \quad \cdots \quad \mathrm{T}_{N_{k|k-1}}}^{\text{birth targets}} & \overbrace{c_1 \quad c_2 \quad \cdots \quad c_{M_k}}^{\text{clutter}}
\end{array}
$$

$$
\begin{array}{c}
\boldsymbol{z}_{1,k} \\ \boldsymbol{z}_{2,k} \\ \vdots \\ \boldsymbol{z}_{M_k,k}
\end{array}
\left[\begin{array}{cccccc}
-\ln\eta_{11} & \cdots & -\ln\eta_{N_{k-1},1} & -\ln\eta_{N_{k-1}+1,1} & \cdots & -\ln\eta_{N_{k|k-1},1} \\
-\ln\eta_{12} & \cdots & -\ln\eta_{N_{k-1},2} & -\ln\eta_{N_{k-1}+1,2} & \cdots & -\ln\eta_{N_{k|k-1},2} \\
\vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\
-\ln\eta_{1,M_k} & \cdots & -\ln\eta_{N_{k-1},M_k} & -\ln\eta_{N_{k-1}+1,M_k} & \cdots & -\ln\eta_{N_{k|k-1},M_k}
\end{array}\right]
\left[\begin{array}{cccc}
-\ln\lambda_c & \infty & \cdots & \infty \\
\infty & -\ln\lambda_c & \cdots & \infty \\
\vdots & \vdots & \ddots & \vdots \\
\infty & \infty & \cdots & -\ln\lambda_c
\end{array}\right]
$$

FIGURE 1. Cost matrices C and C' for the optimal 2-D assignment.

Similarly, cost c'_{ij} is given by the negative log-likelihood of clutter as

$$c'_{ij} = \begin{cases} -\ln(\lambda_c) & i = j \\ \infty & i \neq j \end{cases} \tag{20}$$

The cost matrices $\boldsymbol{C} = \left[c_{ij}\right]$ and $\boldsymbol{C}' = \left[c'_{ij}\right]$ for the optimal 2-D assignment are shown in Fig. 1.

The optimal 2-D assignment problem in (18) can be solved by the Hungarian algorithm.

III. AM-MDMTB FILTER FOR A LINEAR GAUSSIAN SYSTEM

In the linear Gaussian system, transition probability $f(\boldsymbol{x}_{i,k}|\boldsymbol{x}_{i,k-1})$ in (3) and probability density $f(z_{j,k}|\boldsymbol{x}_{i,k})$ in (10) and (12) are given by

$$f(\boldsymbol{x}_{i,k}|\boldsymbol{x}_{i,k-1}) = N\left(\boldsymbol{x}_{i,k}; \boldsymbol{F}_{k-1}\boldsymbol{x}_{i,k-1}, \boldsymbol{Q}_{k-1}\right) \tag{21}$$

$$f(z_{j,k}|\boldsymbol{x}_{i,k}) = N\left(z_{j,k}; \boldsymbol{H}_k\boldsymbol{x}_{i,k}, \boldsymbol{R}_k\right) \tag{22}$$

where $N(\cdot; \boldsymbol{m}, \boldsymbol{P})$ is a Gaussian distribution with mean vector $\boldsymbol{m}$ and covariance matrix $\boldsymbol{P}$, and $\boldsymbol{F}_{k-1}$, $\boldsymbol{Q}_{k-1}$, $\boldsymbol{H}_k$ and $\boldsymbol{R}_k$ are state transition matrix, covariance matrix of process noise, observation matrix and covariance matrix of observation noise, respectively.

To efficiently track the target in the presence of missed detections, we add a detecting label for each target. If a target is detected, its detecting label is 1; otherwise, its detecting label is 0. Besides, we also add a track label for each target to establish the target track. The proposed filter consists of the following steps.

A. PREDICTION

Suppose that the set consisting of state distributions, existence probabilities, detecting labels and track labels of individual hypothesized targets at time step $k-1$ is given by

$$\left\{f_{i,k-1}(\boldsymbol{x}_{i,k-1}|\boldsymbol{y}_{1:k-1}), \rho_{i,k-1}, l_{D,(i,k-1)}, \boldsymbol{l}_{T,(i,k-1)}\right\}_{i=1}^{N_{k-1}} \tag{23}$$

where $f_{i,k-1}(\boldsymbol{x}_{i,k-1}|\boldsymbol{y}_{1:k-1}) = N\left(\boldsymbol{x}_{i,k-1}; \boldsymbol{m}_{i,k-1}, \boldsymbol{P}_{i,k-1}\right)$ and $\boldsymbol{m}_{i,k-1}$, $\boldsymbol{P}_{i,k-1}$, $\rho_{i,k-1}$, $l_{D,(i,k-1)}$ and $\boldsymbol{l}_{T,(i,k-1)}$ denote the mean vector, covariance matrix, existence probability, detecting label and track label of target i, respectively. We use (3) to obtain the predicted probability density of each hypothesized target at time step k as

$$f_{i,k|k-1}(\boldsymbol{x}_{i,k}|\boldsymbol{y}_{1:k-1}) = N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k|k-1}, \boldsymbol{P}_{i,k|k-1}\right); \quad i = 1, 2, \cdots, N_{k-1} \tag{24}$$

where

$$\boldsymbol{m}_{i,k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{m}_{i,k-1} \tag{25}$$

$$\boldsymbol{P}_{i,k|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{P}_{i,k-1}(\boldsymbol{F}_{k-1})^T + \boldsymbol{Q}_{k-1} \tag{26}$$

The predicted existence probability, detecting label and track label of hypothesized target i are given by

$$\rho_{i,k|k-1} = \rho_{i,k-1}, \quad l_{D,(i,k|k-1)} = l_{D,(i,k-1)}, \quad \boldsymbol{l}_{T,(i,k|k-1)} = \boldsymbol{l}_{T,(i,k-1)} \tag{27}$$

Suppose that the set consisting of the probability densities, existence probabilities, detecting labels and track labels of individual hypothesized birth targets at time step k is given by

$$\left\{N\left(\boldsymbol{x}^b_{i,k}; \boldsymbol{m}^b_{i,k}, \boldsymbol{P}^b_{i,k}\right), \rho^b_{i,k}, l^b_{D,(i,k)}, \boldsymbol{l}^b_{T,(i,k)}\right\}_{i=1}^{N^b_k} \tag{28}$$

where $\boldsymbol{m}^b_{i,k}$, $\boldsymbol{P}^b_{i,k}$, $\rho^b_{i,k}$, $l^b_{D,(i,k)}$ and $\boldsymbol{l}^b_{T,(i,k)}$ are the mean vector, covariance matrix, existence probability, detecting label and track label of hypothesized birth target i, respectively. The detecting label $l^b_{D,(i,k)}$ and track label $\boldsymbol{l}^b_{T,(i,k)}$ may be given by

$$l^b_{D,(i,k)} = 0, \quad \boldsymbol{l}^b_{T,(i,k)} = \begin{bmatrix} k \\ i \end{bmatrix} \tag{29}$$

where i is the index of the birth target. This index is unique to distinguish the birth targets at time step k. We combine the predicted set of hypothesized targets with the set of hypothesized birth targets to generate an extended prediction set of hypothesized targets. The extended prediction set is given by

$$
\begin{aligned}
&\left\{N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k|k-1}, \boldsymbol{P}_{i,k|k-1}\right),\right. \\
&\qquad \left.\rho_{i,k|k-1}, l_{D,(i,k|k-1)}, \boldsymbol{l}_{T,(i,k|k-1)}\right\}_{i=1}^{N_{k|k-1}} \\
&= \left\{N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k|k-1}, \boldsymbol{P}_{i,k|k-1}\right),\right. \\
&\qquad \left.\rho_{i,k|k-1}, l_{D,(i,k|k-1)}, \boldsymbol{l}_{T,(i,k|k-1)}\right\}_{i=1}^{N_{k-1}} \\
&\qquad \bigcup \left\{N\left(\boldsymbol{x}^b_{i,k}; \boldsymbol{m}^b_{i,k}, \boldsymbol{P}^b_{i,k}\right), \rho^b_{i,k}, l^b_{D,(i,k)}, \boldsymbol{l}^b_{T,(i,k)}\right\}_{i=1}^{N^b_k}
\end{aligned} \tag{30}
$$

where $N_{k|k-1} = N_{k-1} + N^b_k$ is the number of hypothesized targets in the extended prediction set.

B. UPDATE

Based on (10), (11) and (12), the updated probability density and existence probability of target i corresponding to observation $z_{j,k}$, and probability that measurement $z_{j,k}$ originates from target i are as follows:

$$f_{ij,k}(\boldsymbol{x}_{i,k}|z_{j,k}) = N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{ij}, \boldsymbol{P}_{ij}\right) \tag{31}$$

$$\rho_{ij,k} = \frac{\eta_{ij}}{\lambda_c + \sum_{e=1}^{N_{k|k+1}} \eta_{ej}} \tag{32}$$

$$\eta_{ij} = p_{D,k}\rho_{i,k|k-1}N\left(z_{j,k}; \boldsymbol{H}_k\boldsymbol{m}_{i,k|k-1}, \boldsymbol{H}_k\boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^T + \boldsymbol{R}_k\right) \tag{33}$$

where

$$\boldsymbol{m}_{ij} = \boldsymbol{m}_{i,k|k-1} + \boldsymbol{A}_i \cdot (z_{j,k} - \boldsymbol{H}_k\boldsymbol{m}_{i,k|k-1}) \tag{34}$$

$$\boldsymbol{P}_{ij} = (\boldsymbol{I} - \boldsymbol{A}_i\boldsymbol{H}_k)\boldsymbol{P}_{i,k|k-1} \tag{35}$$

$$\boldsymbol{A}_i = \boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^T[\boldsymbol{H}_k\boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^T + \boldsymbol{R}_k]^{-1} \tag{36}$$

and updated detecting label $l_{D,(ij)}$ is given by

$$l_{D,(ij)} = 0 \tag{37}$$

C. ASSIGNMENT OF MEASUREMENTS

We first establish the cost matrices based on the association cost of measurements and targets in (19) and the association cost of measurements and clutter in (20), and use the Hungarian algorithm to solve the optimal 2-D assignment problem in (18) to obtain the association matrices $\boldsymbol{S} = \left[s_{ij}\right]$ and $\boldsymbol{S}' = \left[s'_{ij}\right]$.

Based on the association matrix $\boldsymbol{S} = \left[s_{ij}\right]$, we then adjust the updated existence probability of the target. $s_{ij} = 1$ indicates that measurement $z_{j,k}$ is assigned to target i and also implies that measurement $z_{j,k}$ originates from target i. In this case, we adjust the updated existence probability of the target as

$$\rho_{ej,k} = \begin{cases} 1 & \text{for } e = i \\ 0 & \text{for } e = 1, \cdots, i-1, i+1, \cdots, N_{k|k-1} \end{cases} \tag{38}$$

This adjustment is due to the fact that measurement $z_{j,k}$ originates from target i instead of any other target. At the same time, if $i \leq N_{k-1}$ then the detecting label of target i correlative with $z_{j,k}$ is set as

$$l_{D,(ij)} = 1 \tag{39}$$

This setting implies that target i is an existing target at time step k and is also detected at this time step.

D. HANDLING OF MISSED DETECTIONS

An existing target at time step k may be undetected at this time step. In this case, we use the predicted probability density of the target as its state distribution at time step k and decrease its existence probability.

To do this, we first judge whether an existing target is detected or not by using predicted detecting label $l_{D,(i,k|k-1)}$ and $b = \sum_{j=1}^{M_k} l_{D,(ij)}$. If the condition that $l_{D,(i,k|k-1)} = 1$ and $b = 0$ is satisfied, target i is undetected at time step k due to the fact that no measurement at time step k originates from it. Then we use the predicted probability density of target i as its state distribution at time step k as

$$N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k}, \boldsymbol{P}_{i,k}\right) = N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k|k-1}, \boldsymbol{P}_{i,k|k-1}\right) \tag{40}$$

Its existence probability and detecting label are given by

$$\rho_{i,k} = \eta_c \times \rho_{i,k|k-1}, \quad l_{D,(i,k)} = l_{D,(i,k|k-1)} \tag{41}$$

where η_c is the given decay factor, and its value range is $\eta_c \in [0, 1)$.

If the condition that $l_{D,(i,k|k-1)} = 1$ and $b = 0$ is not satisfied, we first find the index of the maximum updated existence probability from the set $\left\{\rho_{ie,k}|e = 1, \cdots, M_k\right\}$ as

$$a = \arg\max_{e\in[1,\cdots,M_k]} \left\{\rho_{ie,k}\right\} \tag{42}$$

The updated probability density, existence probability and detecting label with index a are used as the state distribution, existence probability and detecting label of target i at time step k as

$$N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k}, \boldsymbol{P}_{i,k}\right) = N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{ia}, \boldsymbol{P}_{ia}\right), \quad \rho_{i,k} = \rho_{ia,k}, \quad l_{D,(i,k)} = l_{D,(ia)} \tag{43}$$

The extended prediction track label of target i is used as its track label at time step k as

$$\boldsymbol{l}_{T,(i,k)} = \boldsymbol{l}_{T,(i,k|k-1)}; \quad i = 1, \cdots, N_{i,k|k-1} \tag{44}$$

E. PICKING OF STATE VECTORS AND TRACK LABELS

The existence probability of a hypothesized target indicates how likely it is a real target. If $\rho_{i,k} \geq 0.5$, we confirm that hypothesized target i is a real target, and then pick its mean vector and track label to generate the set of mean vectors and corresponding set of track labels, respectively. The generated sets of mean vectors and track labels are used as the output of the filter at this time step.

F. PRUNING

In this step, we prune the hypothesized targets with existence probability $\rho_{i,k} \leq \tau$ from the set $\left\{N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k}, \boldsymbol{P}_{i,k}\right), \rho_{i,k}, l_{D,(i,k)}, l_{T,(i,k)}\right\}_{i=1}^{N_{k|k-1}}$ where τ is the pruning threshold. The residual set $\left\{N\left(\boldsymbol{x}_{i,k}; \boldsymbol{m}_{i,k}, \boldsymbol{P}_{i,k}\right), \rho_{i,k}, l_{D,(i,k)}, l_{T,(i,k)}\right\}_{i=1}^{N_k}$ after pruning is transmitted to next time step and is exploited as the input of next recursion where N_k is the number of hypothesized targets at time step k.

Similar to the R-GLMB filter, the proposed filter affords the target track by using the track label to distinguish each target. Unlike the R-GLMB filter that searches the K best 2-D assignments to determine whether a target is died, or surviving and detected, or surviving and undetected; the proposed

TABLE 1. Initial state vectors of targets and their appearing and disappearing times.

Target	Initial state vector	Appearing time	Disappearing time
1	$[0,0,0,-10]^T$	$t=1$ s	$t=70$ s
2	$[400,-10,-600,5]^T$	$t=1$ s	$t=101$ s
3	$[-800,20,-200,-5]^T$	$t=1$ s	$t=70$ s
4	$[-200,-7,800,-5]^T$	$t=20$ s	$t=101$ s
5	$[400,-2.5,-600,10]^T$	$t=20$ s	$t=101$ s
6	$[0,7.5,0,-5]^T$	$t=20$ s	$t=101$ s
7	$[-800,12,-200,7]^T$	$t=40$ s	$t=101$ s
8	$[-200,15,800,-10]^T$	$t=40$ s	$t=101$ s
9	$[-800,3,-200,15]^T$	$t=60$ s	$t=101$ s
10	$[-200,-3,800,-15]^T$	$t=60$ s	$t=101$ s
11	$[0,20,0,-15]^T$	$t=80$ s	$t=101$ s
12	$[-200,15,800,-5]^T$	$t=80$ s	$t=101$ s

filter finds an optimal 2-D assignment to associate each measurement to either a target or clutter. Both the proposed filter and the R-GLMB filter need solving 2-D assignment problem at each time step, but the latter requires a larger computational load than the former. More importantly, the proposed filter selects either the predicted probability density of a target or one of its multiple updated probability densities as its state distribution at each time step. Therefore, the number of state distributions or hypothesized targets propagated to next time step in the proposed filter approximates that of real targets. Due to this fact, the proposed filter propagates significantly less Gaussian items than the GM-PHD filter, CBMeMber filter, R-GLMB filter and LMB filter. Similar to the MDMTB filter, the AM-MDMTB filter may afford a large existence probability for a missed target. It may provide state estimation of an existing target if this target is undetected at a time.

IV. SIMULATION RESULTS

In this section, we reveal the performance of the proposed AM-MDMTB filter by comparing this filter with the GM-PHD filter [7], CBMeMber filter [9], LMB filter [28], R-GLMB filter [30] and MDMTB filter [10], and exploit the OSPA distance [35] with $c = 100$ m and $p = 2$ as the assessment criteria.

The considered scenario is a 2-D surveillance field [−1000 m, 1000 m]×[−1000 m, 1000 m]. Each target moves at a constant velocity with state vector $\boldsymbol{x}_{i,k} = [\eta^k_{i,x}, \dot{\eta}^k_{i,x}, \eta^k_{i,y}, \dot{\eta}^k_{i,y}]^T$ where $\eta^k_{i,x}$ and $\eta^k_{i,y}$ are its position component, and $\dot{\eta}^k_{i,x}$ and $\dot{\eta}^k_{i,y}$ are its velocity component.

Example 1: There are twelve targets occurring from four specific positions at different times in this example. The initial state vectors of these twelve targets and their appearing and disappearing times are shown in Table 1.

The moving of each target follows (21) where $\boldsymbol{F}_{k-1}$ and $\boldsymbol{Q}_{k-1}$ are given by

$$\boldsymbol{F}_{k-1} = \begin{bmatrix} 1 & \Delta t_k & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

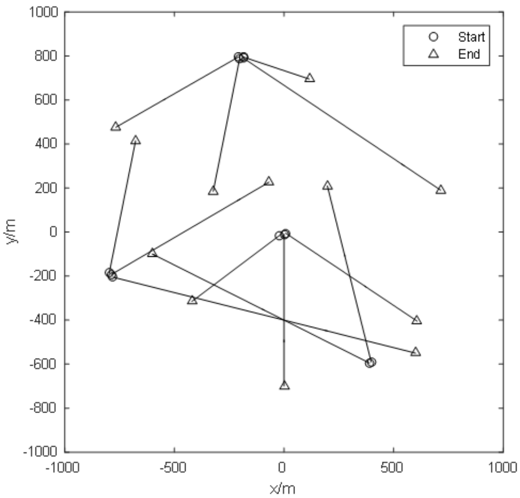

FIGURE 2. Real trajectories of the twelve targets.

$$\boldsymbol{Q}_{k-1} = \begin{bmatrix} \frac{\Delta t_k^4}{4} & \frac{\Delta t_k^3}{2} & 0 & 0 \\ \frac{\Delta t_k^3}{2} & \Delta t_k^2 & 0 & 0 \\ 0 & 0 & \frac{\Delta t_k^4}{4} & \frac{\Delta t_k^3}{2} \\ 0 & 0 & \frac{\Delta t_k^3}{2} & \Delta t_k^2 \end{bmatrix} \sigma_v^2 \quad (45)$$

where $\Delta t_k = t_k - t_{k-1}$ is the interval and $\Delta t_k = 1$ s, and σ_v is the standard deviation of process noise and $\sigma_v = 2$ ms^{-2}. The real trajectories of these twelve targets are shown in Fig. 2.

We consider two cases of linear observation and nonlinear observation in Example 1.

A. CASE OF LINEAR OBSERVATION

In this case, the observation is a noisy 2-D position vector of each target, and each observation follows (22) where $\boldsymbol{H}_k$ and $\boldsymbol{R}_k$ are given by

$$\boldsymbol{H}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{R}_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \sigma_w^2 \quad (46)$$

where $\sigma_w = 2$ m. The detecting probability and average clutter density are set to $p_{D,k} = 0.9$ and $\lambda_c = 6.25 \times 10^{-6}$ m^{-2} (i.e., $N_c = 10$), respectively.

To track the twelve targets, we use four birth models $\left\{N\left(\boldsymbol{x}^b_{i,k}; \boldsymbol{m}^b_{i,k}, \boldsymbol{P}^b_{i,k}\right)\right\}_{i=1}^4$ in the proposed filter, GM-PHD filter, CBMeMber filter, MDMTB filter, LMB filter and R-GLMB filter where $\boldsymbol{m}^b_{1,k} = [0, 0, 0, 0]^T$, $\boldsymbol{m}^b_{2,k} = [400, 0, -600, 0]^T$, $\boldsymbol{m}^b_{3,k} = [-800, 0, -200, 0]^T$, $\boldsymbol{m}^b_{4,k} = [-200, 0, 800, 0]^T$ and $\boldsymbol{P}^b_{i,k} = diag\left([30, 10, 30, 10]^T\right)^2$. The other parameters of the proposed AM-MDMTB filter are set to $\rho^b_{i,k} = 0.03$, $\tau = 10^{-5}$ and $\eta_c = 0.75$; and other corresponding parameters of the GM-PHD filter, CBMeMber filter, LMB filter, R-GLMB filter and MDMTB filter are identical to those in [7], [9], [28], [30] and [10], respectively.

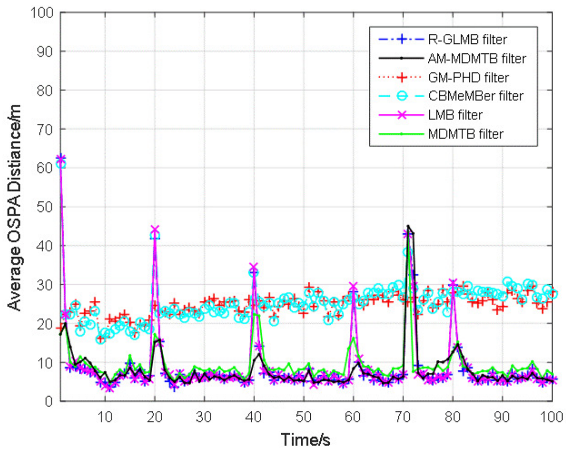

FIGURE 3. Average OSPA distance of the six filters in case of linear observation.

TABLE 2. Average OSPA distance (m) of different clutter densities in case of linear observation.

$\lambda_c(10^{-6}$ m) N_c	0.625 1	3.125 5	6.25 10	9.375 15	12.5 20	15.625 25
R-GLMB filter	6.3028	7.3883	9.0065	8.9171	9.2769	9.1797
AM-MDMTB filter	5.7574	7.0585	8.0521	8.7625	9.5498	9.7529
GM-PHD filter	23.4452	24.1794	24.6307	24.9137	26.8126	27.0933
CBMeMber filter	21.7262	23.5505	25.3934	26.2074	27.2284	27.8737
LMB filter	6.5806	7.3373	8.9823	8.7946	9.2537	9.3722
MDMTB	6.6951	8.3111	9.3137	10.0591	10.2565	10.8684

We perform the proposed AM-MDMTB filter, GM-PHD filter, CBMeMber filter, LMB filter, MDMTB filter and R-GLMB filter for 150 Monte Carlo runs. Fig. 3 shows the average OSPA distance over 150 Monte Carlo runs, which suggests that the AM-MDMTB filter performs best among the six filters at $p_{D,k} = 0.9$ and $\lambda_c = 6.25 \times 10^{-6}$ m^{-2} due to the fact that its average OSPA distance is the least at most times. Fig. 3 also shows that several peaks appear in the curves of the R-GLMB filter, LMB filter and AM-MDMTB filter because of excessive or inadequate state estimations.

To show the effect of clutter density on the performance of these filters, we perform the AM-MDMTB filter, GM-PHD filter, CBMeMber filter, R-GLMB filter, LMB filter and MDMTB filter for 150 Monte Carlo runs at different clutter densities and a constant detecting probability of 0.9. The average OSPA distance over 150 Monte Carlo runs and average performing time for a Monte Carlo run are shown in Table 2 and Table 3, respectively.

The average OSPA distance in Table 2 indicates the R-GLMB filter, LMB filter, MDMTB filter and AM-MDMTB filter perform better than the GM-PHD filter and CBMeMber filter because the latter two filters are prone to discard the information of missed targets due to their weak memory, which results in the consequence that their OSPA distances are obviously larger than those of the former four filters. According to the average OSPA distance, the AM-MDMTB filter performs better than the MDMTB

TABLE 3. Average performing time (s) of the six filters in case of linear observation.

$\lambda_c(10^{-6}$ m) N_c	0.625 1	3.125 5	6.25 10	9.375 15	12.5 20	15.625 25
R-GLMB filter	29.9417	31.559	31.7699	32.7020	33.9420	34.0850
AM-MDMTB filter	0.3596	0.3583	0.3641	0.3858	0.4039	0.4159
GM-PHD filter	1.2418	1.2709	1.3467	1.4613	1.5381	1.6024
CBMeMber filter	1.7084	1.7232	1.7858	1.8705	1.9205	1.9445
LMB filter	37.8380	40.1062	40.5505	41.8002	43.9754	44.5298
MDMTB	0.5335	0.6207	0.7389	0.8546	0.9815	1.0540

filter at each clutter density. The main reason is that the former uses a 2-D assignment to associate individual measurements with either a target or clutter. The experimental result in Table 2 also reveals that the AM-MDMTB filter performs better at clutter densities of 0.625×10^{-6} m^{-2}, 3.125×10^{-6} m^{-2}, 6.25×10^{-6} m^{-2} and 9.375×10^{-6} m^{-2}, but slightly worse at clutter densities of 12.5×10^{-6} m^{-2} and 15.625×10^{-6} m^{-2} than the R-GLMB filter and LMB filter. This phenomenon implies that the AM-MDMTB filter is applicable to low and moderate clutter densities while the R-GLMB filter and LMB filter are applicable to a high clutter density.

The average performing time in Table 3 indicates that the R-GLMB filter and LMB filter require a significantly larger computational load than the MDMTB filter, AM-MDMTB filter, GM-PHD filter and CBMeMber filter at each clutter density, while the performing time of the AM-MDMTB filter is the least among the six filters. The main reasons are as follows:

(1) The number of hypothesized targets or state distributions in the proposed AM-MDMTB filter approximates the number of real targets, whereas the R-GLMB filter, LMB filter, GM-PHD filter and CBMeMber filter propagate much more hypothesized targets or Gaussian terms than real targets to next time step. The more the number of hypothesized targets or Gaussian terms is, the larger computational load is required.

(2) The R-GLMB filter, LMB filter and proposed AM-MDMTB filter need solving the 2-D assignment problem in the filter recursion. Solving the 2-D assignment problem to find the K best solutions in the R-GLMB filter and LMB filter can be accomplished by the optimizing Murty's algorithm [36] with a complexity of $\mathcal{O}\left(K(M_k + 2N_{k|k-1})^3\right)$, while solving the 2-D assignment problem to search an optimal solution in the proposed AM-MDMTB filter is accomplished by the Hungarian algorithm with a complexity of $\mathcal{O}\left((M_k + N_{k|k-1})^3\right)$. Due to a lower complexity, searching an optimal solution in the proposed AM-MDMTB filter requires a less computation than finding the K best solutions in the R-GLMB filter and LMB filter.

The recursion of the LMB filter consists of LMB prediction, conversion from LMB to δ-GLMB, δ-GLMB update and conversion from δ-GLMB to LMB [28]. The additional conversions from LMB to δ-GLMB and from δ-GLMB to

TABLE 4. Average OSPA distance at different observation noises.

Filter	R-GLMB	AM-MDMTB	GM-PHD	CBMeMber	LMB	MDMTB
$\sigma_w = 2$ m	9.0065	8.0521	24.6307	25.3934	8.923	9.3137
$\sigma_w = 4$ m	10.5186	9.9573	25.8684	26.8579	10.5318	11.9425
$\sigma_w = 6$ m	12.1049	11.8087	27.1603	28.4243	12.0883	14.4599
$\sigma_w = 8$ m	13.4587	12.9754	28.6041	29.7400	13.4599	16.8567

LMB in the LMB filter result in its performing time being larger than the R-GLMB filter.

To demonstrate the effect of the measurement noise on the tracking performance of these filters, we use different measurement noises and a clutter density of 6.25×10^{-6} m^{-2} to generate the measurement, and perform 150 Monte Carlo runs for each filter to obtain the average OSPA distance. The experimental result in Table 4 indicates that the average OSPA distance of the six filters increases with the increase of observation noise and that the proposed AM-MDMTB filter performs best at each observation noise because it provides a least OSPA distance at each measurement noise.

B. CASE OF NONLINEAR OBSERVATION

In this case, the measurement consists of the bearing and range, and is given by

$$z_{j,k} = \begin{bmatrix} \theta_{j,k} \\ r_{j,k} \end{bmatrix} = h(\boldsymbol{x}_{i,k}) + u_k = \begin{bmatrix} \arccos\left(\dfrac{\eta_{i,x}^k - s_x}{\sqrt{(\eta_{i,x}^k - s_x)^2 + (\eta_{i,y}^k - s_y)^2}}\right) \\ \sqrt{(\eta_{i,x}^k - s_x)^2 + (\eta_{i,y}^k - s_y)^2} \end{bmatrix} + u_k \quad (47)$$

where $(s_x, s_y) = (-500\text{ m}, -600\text{ m})$ is the position of a radar, u_k denotes the observation noise, and $\theta_{j,k}$ and $r_{j,k}$ denote the azimuth and range, respectively.

We exploit the linearization strategy of nonlinear function to deal with the nonlinearity of observation equation (47), and replace the predicted observation vector $\boldsymbol{H}_k \boldsymbol{m}_{i,k|k-1}$ and observation matrix $\boldsymbol{H}_k$ in (33), (34), (35) and (36) with the predicted observation vector $h(\boldsymbol{m}_{i,k|k-1})$ and Jacobian matrix $\boldsymbol{H}_{i,k}$ respectively, where

$$\boldsymbol{H}_{i,k} = \left.\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{m}_{i,k|k-1}} \quad (48)$$

The detecting probability and clutter density are set to $p_{D,k} = 0.9$ and $\lambda_c = 1.4105 \times 10^{-3}$ rad^{-1}m^{-1} (i.e., $N_c = 10$), and covariance matrix $\boldsymbol{R}_k$ in (33) and (36) is given by

$$\boldsymbol{R}_k = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix} \quad (49)$$

where $\sigma_\theta = 0.0087$ rad (i.e., $\sigma_\theta = 0.5°$) and $\sigma_r = 2$ m. We perform the proposed AM-MDMTB filter, GM-PHD filter, CBMeMber filter, LMB filter, MDMTB filter and R-GLMB filter for 150 Monte Carlo runs. The average OSPA distance and performing time are shown in Fig. 4 and Table 5, respectively. Similar to the conclusion in case of linear observation, the experimental results in Fig. 4 and Table 5 also suggest that the AM-MDMTB filter performs best among the six filters because it has a smallest average OSPA distance and takes a least performing time.

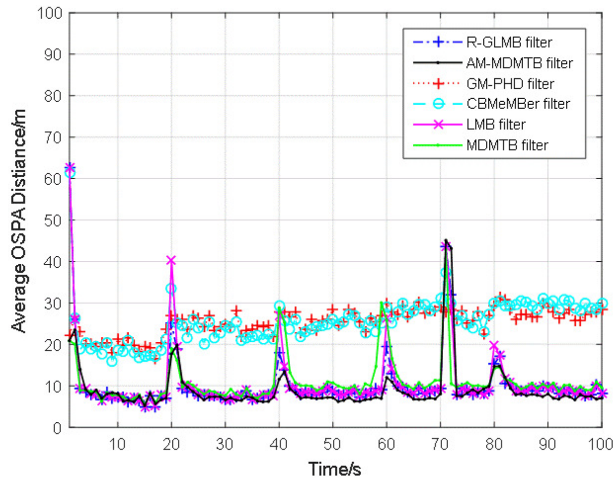

FIGURE 4. Average OSPA distance of the six filters in case of nonlinear observation.

TABLE 5. Average OSPA distance and performing time of the six filters in case of nonlinear observation.

Filter	R-GLMB	AM-MDMTB	GM-PHD	CBMeMber	LMB	MDMTB
Average OSPA distance (m)	10.3171	9.1521	25.4308	25.5585	10.7680	11.0209
Average performing time (s)	31.3275	0.4356	1.3555	2.0486	38.6293	0.7750

TABLE 6. Average OSPA$^{(2)}$ error of the three filters in case of nonlinear observation.

Filter	R-GLMB	AM-MDMTB	LMB
Average OSPA$^{(2)}$ error(m)	17.6539	15.8318	17.5728

To reveal the performance of the proposed AM-MDMTB filter on track label, we exploit the OSPA$^{(2)}$ error [37] with $c = 100$ m, $p = 2$ and scan window length $L_w = 5$ as the assessment criteria to compare this filter with the R-GLMB filter [30] and LMB filter [28]. Unlike the OSPA distance for evaluating the difference between the estimated and true multi-target states, the OSPA$^{(2)}$ error was proposed to assess the dissimilarity between the estimated and true sets of tracks [37]. The average OSPA$^{(2)}$ error of the R-GLMB, LMB and AM-MDMTB filters over 150 Monte Carlo runs at the detecting probability of 0.9 and clutter density of 1.4105×10^{-3} rad^{-1}m^{-1} in case of nonlinear observation is shown in Fig. 5 and Table 6. The results in Fig. 5 and Table 6 indicate that the AM-MDMTB filter performs best among these three filters because its average OSPA$^{(2)}$ error is the least.

Example 2: This example aims to test the performance of the six filters on tracking the close targets. In this example,

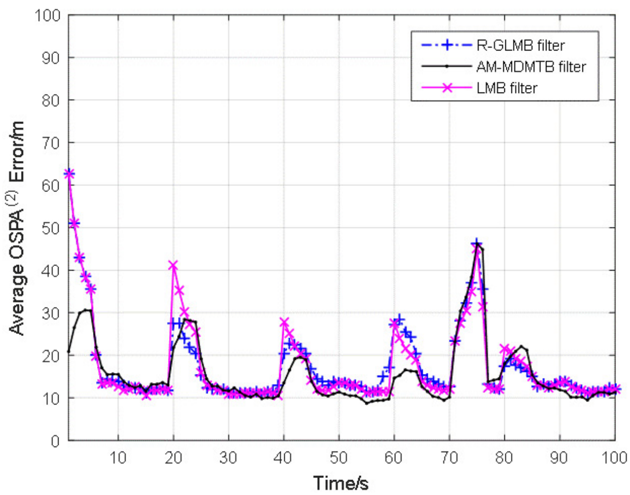

FIGURE 5. Average OSPA$^{(2)}$ error in case of nonlinear observation.

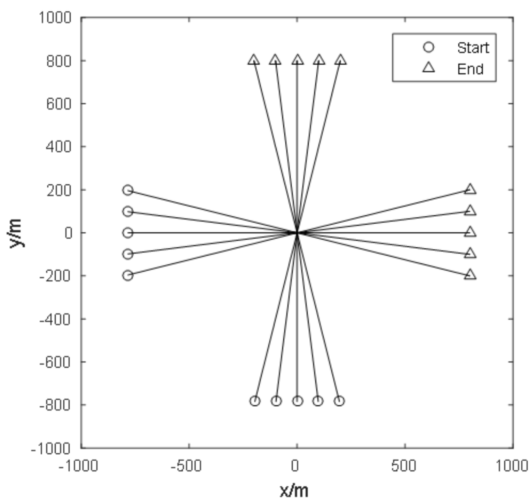

FIGURE 6. Real trajectories of the ten targets.

ten targets appear at $t = 1$ s and disappear at $t = 101$ s. They move closely and cross their paths at $t = 50$ s. The real trajectories of these ten targets are shown in Fig. 6.

Ten birth models $\left\{N\left(\boldsymbol{x}_{i,k}^{b}; \boldsymbol{m}_{i,k}^{b}, \boldsymbol{P}_{i,k}^{b}\right)\right\}_{i=1}^{10}$ are used in this example, where $\boldsymbol{m}_{1,k}^{b} = [-800, 0, 0, 0]^{T}$, $\boldsymbol{m}_{2,k}^{b} = [-800, 0, -100, 0]^{T}$, $\boldsymbol{m}_{3,k}^{b} = [-800, 0, 100, 0]^{T}$, $\boldsymbol{m}_{4,k}^{b} = [-800, 0, -200, 0]^{T}$, $\boldsymbol{m}_{5,k}^{b} = [-800, 0, 200, 0]^{T}$, $\boldsymbol{m}_{6,k}^{b} = [0, 0, -800, 0]^{T}$, $\boldsymbol{m}_{7,k}^{b} = [-100, 0, -800, 0]^{T}$, $\boldsymbol{m}_{8,k}^{b} = [100, 0, -800, 0]^{T}$, $\boldsymbol{m}_{9,k}^{b} = [-200, 0, -800, 0]^{T}$, and $\boldsymbol{m}_{10,k}^{b} = [200, 0, -800, 0]^{T}$. The other parameters of different filters are identical to those in Example 1.

The average OSPA distance and performing time of the six filters over 150 Monte Carlo runs are shown in Fig. 7 and Table 7. The result in Fig. 7 and Table 7 indicates that the MDMTB filter, R-GLMB filter and GM-PHD filter deteriorate in performance for tracking the close targets, while the AM-MDMTB filter performs best because its OSPA distance and performing time are the least.

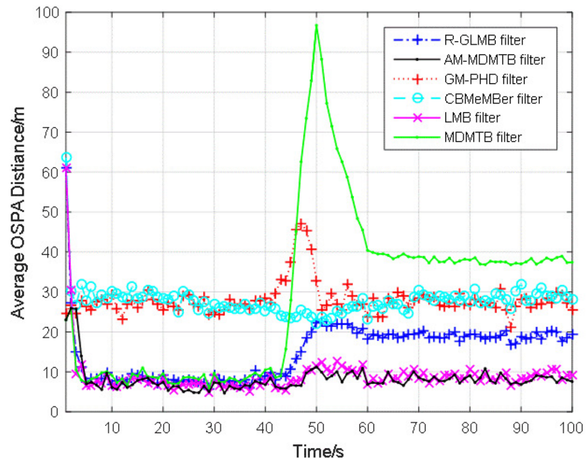

FIGURE 7. Average OSPA distance of the six filterss.

TABLE 7. Average OSPA distance and performing time of the six filters.

Filter	R-GLMB	AM-MDMTB	GM-PHD	CBMeMber	LMB	MDMTB
Average OSPA distance (m)	15.0644	8.1734	27.8155	27.8155	9.0240	29.4178
Average performing time (s)	62.7271	0.7114	3.7158	4.3485	84.1923	1.3142

V. CONCLUSIONS

In this study, an AM-MDMTB filter is presented by representing the association of the measurement and the target as an optimal 2-D assignment problem. In the presented AM-MDMTB filter, the Hungarian algorithm is employed to solve the 2-D assignment problem to assign each measurement to either a target or clutter. By the introduction of detecting label, a handling method for missed detections is also developed and is applied to this filter. Unlike the GM-PHD, CBMeMber, LMB and R-GLMB filters that propagate multiple state distributions or Gaussian items of each target to next time step, the proposed AM-MDMTB filter propagates a state distribution or Gaussian item of each target to next time step. The number of hypothesized targets or state distributions in the proposed AM-MDMTB filter approximates the number of real targets, therefore it requires a smaller numerical computation than the GM-PHD filter, CBMeMber filter, LMB filter and R-GLMB filter. Simulation results demonstrate that the proposed AM-MDMTB filter has a less average OSPA distance than the GM-PHD, CBMeMber and MDMTB filters, and a less average OSPA distance at low and moderate clutter densities and a slightly larger average OSPA distance at a high clutter density than the R-GLMB filter and LMB filter. This phenomenon suggests that the tracking performance of the proposed AM-MDMTB filter is the best at low and moderate clutter densities. The experimental results also illustrate that the proposed AM-MDMTB filter requires a less performing time than the other filters, which is important for reducing the delay of data processing. A future research scope of the proposed filter is to extend its application in real tracking systems.

REFERENCES

[1] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*. Norwood, MA, USA: Artech House, 2007.

[2] R. Mahler, *Advances in Statistical Multisource-Multitarget Information Fusion*. Boston, MA, USA: Artech House, 2014.

[3] Y. Bar-Shalom, *Multitarget-Multisensor Tracking: Applications and Advances*, vol. 3. London, U.K.: Artech House, 2000.

[4] S. Puranik and J. Tugnait, "Tracking of multiple maneuvering targets using multiscan JPDA and IMM filtering," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 1, pp. 23–35, Jan. 2007.

[5] S. S. Blackman, "Multiple hypothesis tracking for multiple target tracking," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 19, no. 1, pp. 5–18, Jan. 2004.

[6] R. P. S. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 4, pp. 1152–1178, Oct. 2003.

[7] B.-N. Vo and W.-K. Ma, "The Gaussian mixture probability hypothesis density filter," *IEEE Trans. Signal Process.*, vol. 54, no. 11, pp. 4091–4104, Nov. 2006.

[8] B.-T. Vo, B.-N. Vo, and A. Cantoni, "Analytic implementations of the cardinalized probability hypothesis density filter," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3553–3567, Jul. 2007.

[9] B.-T. Vo, B.-N. Vo, and A. Cantoni, "The cardinality balanced multi-target multi-Bernoulli filter and its implementations," *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 409–423, Feb. 2009.

[10] Z.-X. Liu, Y.-N. Zou, W.-X. Xie, and L.-Q. Li, "Multi-target Bayes filter with the target detection," *Signal Process.*, vol. 140, pp. 69–76, Nov. 2017.

[11] Z.-X. Liu, B.-J. Huang, Y.-N. Zou, and L.-Q. Li, "Multi-object Bayesian filter for jump Markov system under glint noise," *Signal Process.*, vol. 157, pp. 131–140, Apr. 2019.

[12] C. Lundquist, L. Hammarstrand, and F. Gustafsson, "Road intensity based mapping using radar measurements with a probability hypothesis density filter," *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1397–1408, Apr. 2011.

[13] D. E. Clark and J. Bell, "Bayesian multiple target tracking in forward scan sonar images using the PHD filter," *IEE Proc. Radar, Sonar Navigat.*, vol. 152, no. 5, pp. 327–334, Oct. 2005.

[14] M. Tobias and A. D. Lanterman, "Probability hypothesis density-based multitarget tracking with bistatic range and Doppler observations," *IEE Proc.-Radar, Sonar Navigat.*, vol. 152, no. 3, pp. 195–205, Jun. 2005.

[15] S. A. Pasha, B. N. Vo, H. D. Tuan, and W. K. Ma, "A Gaussian mixture PHD filter for jump Markov system models," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 45, no. 3, pp. 919–936, Jul. 2009.

[16] C. Ouyang, H.-B. Ji, and Z.-Q. Guo, "Extensions of the SMC-PHD filters for jump Markov systems," *Signal Process.*, vol. 92, no. 6, pp. 1422–1430, Jun. 2012.

[17] P. Dong, Z. Jing, D. Gong, and B. Tang, "Maneuvering multi-target tracking based on variable structure multiple model GMCPHD filter," *Signal Process.*, vol. 141, pp. 158–167, Dec. 2017.

[18] K. Granstrom, U. Orguner, R. Mahler, and C. Lundquist, "Extended target tracking using a Gaussian-mixture PHD s filter," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 53, no. 2, pp. 1055–1058, Oct. 2017.

[19] Q. Hu, H. Ji, and Y. Zhang, "A standard PHD filter for joint tracking and classification of maneuvering extended targets using random matrix," *Signal Process.*, vol. 144, pp. 352–363, Mar. 2018.

[20] Y. Zhang, H. Ji, and Q. Hu, "A fast ellipse extended target PHD filter using box-particle implementation," *Mech. Syst. Signal Process.*, vol. 99, pp. 57–72, Jan. 2018.

[21] Y. Zhang, H. Ji, X. Gao, and Q. Hu, "An ellipse extended target CBMeMBer filter using gamma and box-particle implementation," *Signal Process.*, vol. 149, pp. 88–102, Aug. 2018.

[22] W. Li, Y. Jia, J. Du, and J. Zhang, "PHD filter for multi-target tracking with glint noise," *Signal Process.*, vol. 94, pp. 48–56, Jan. 2014.

[23] M. Yazdian-Dehkordi and Z. Azimifar, "Refined GM-PHD tracker for tracking targets in possible subsequent missed detections," *Signal Process.*, vol. 116, pp. 112–126, Nov. 2015.

[24] Y. Wang, Z. Jing, S. Hu, and J. Wu, "Detection-guided multi-target Bayesian filter," *Signal Process.*, vol. 92, no. 2, pp. 564–574, Feb. 2012.

[25] T. Li, S. Sun, M. Bolic, and J. M. Corchado, "Algorithm design for parallel implementation of the SMC-PHD filter," *Signal Process.*, vol. 119, pp. 115–127, Feb. 2016.

[26] B.-T. Vo and B.-N. Vo, "Labeled random finite sets and multi-object conjugate priors," *IEEE Trans. Signal Process.*, vol. 61, no. 13, pp. 3460–3475, Jul. 2013.

[27] B.-N. Vo, B.-T. Vo, and D. Phung, "Labeled random finite sets and the Bayes multi-target tracking filter," *IEEE Trans. Signal Process.*, vol. 62, no. 24, pp. 6554–6567, Dec. 2014.

[28] S. Reuter, B. T. Vo, B. N. Vo, and K. Dietmayer, "The labeled multi-Bernoulli filter," *IEEE Trans. Signal Process.*, vol. 62, no. 12, pp. 3246–3260, Dec. 2014.

[29] Z. Lu, W. Hu, and T. Kirubarajan, "Labeled random finite sets with moment approximation," *IEEE Trans. Signal Process.*, vol. 65, no. 13, pp. 3384–3398, Jul. 2017.

[30] B.-N. Vo, B.-T. Vo, and H. G. Hoang, "An efficient implementation of the generalized labeled multi-Bernoulli filter," *IEEE Trans. Signal Process.*, vol. 65, no. 8, pp. 1975–1987, Apr. 2017.

[31] N. Q. Chen, H. B. Ji, and Y. C. Gao, "Efficient box particle implementation of the multi-sensor GLMB filter in the presence of triple measurement uncertainty," *Signal Process.*, vol. 162, pp. 316–317, Sep. 2019.

[32] B. Yang, J. Wang, and W. Wang, "An efficient approximate implementation for labeled random finite set filtering," *Signal Process.*, vol. 150, pp. 215–227, Sep. 2018.

[33] D. S. Bryant, B.-T. Vo, B.-N. Vo, and B. A. Jones, "A generalized labeled multi-Bernoulli filter with object spawning," *IEEE Trans. Signal Process.*, vol. 66, no. 23, pp. 6177–6189, Dec. 2018.

[34] Z.-X. Liu and B.-J. Huang, "The labeled multi-Bernoulli filter for jump Markov systems under glint noise," *IEEE Access*, vol. 7, pp. 92322–92328, 2019.

[35] D. Schuhmacher, B.-T. Vo, and B.-N. Vo, "A consistent metric for performance evaluation of multi-object filters," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3447–3457, Aug. 2008.

[36] M. L. Miller, H. S. Stone, and I. J. Cox, "Optimizing Murty's ranked assignment method," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, no. 3, pp. 851–862, Jul. 1997.

[37] M. Beard, B. T. Vo, and B.-N. Vo, "OSPA(2): Using the OSPA metric to evaluate multi-target tracking performance," in *Proc. Int. Conf. Control, Autom. Inf. Sci. (ICCAIS)*, Oct. 2017, pp. 86–91.

ZONG-XIANG LIU received the B.S. and M.S. degrees from the Department of Precision Instrument Engineering, Tianjin University, Tianjin, China, in 1985 and 1988, respectively, and the Ph.D. degree from the School of Electronic Engineering, Xidian University, Xi'an, China. He is currently a Professor with the ATR Key Laboratory, College of Electronic and Information Engineering, Shenzhen University, Shenzhen, China. His research interests include multi-sensor data fusion, intelligent information processing, and fuzzy information processing.

MIAN WU received the B.S. degree in electronics and information engineering from the Wuhan University of Science and Technology, Wuhan, China, in 2018. He is currently pursuing the M.S. degree with the ATR Key Laboratory, College of Electronic and Information Engineering, Shenzhen University, Shenzhen, China. His research interests include intelligent information processing and multi-sensor data fusion.

JIE GAN received the B.S. degree in communication engineering from Anhui Jianzhu University, Hefei, China, in 2018. She is currently pursuing the M.S. degree with the ATR Key Laboratory, College of Electronic and Information Engineering, Shenzhen University, Shenzhen, China. Her research interests include intelligent information processing and multi-sensor data fusion.