Received June 8, 2020, accepted June 16, 2020, date of publication June 23, 2020, date of current version July 7, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3004340

Distributed Finite-Time Secondary Voltage Restoration of Droop-Controlled Islanded Microgrids

XUEQIANG SHEN[1], HAIQING WANG[1], DEZHEN ZHANG[2], JIAN LI[1], (Member, IEEE), RENSHU WANG[3], AND QINGYU SU[1], (Member, IEEE)

[1]School of Automation Engineering, Northeast Electric Power University, Jilin City 132012, China
[2]School of Control Science and Engineering, Shandong University, Jinan 250061, China
[3]Electric Power Research Institute, State Grid Fujian Electric Power Company Ltd., Fuzhou 350007, China

Corresponding author: Qingyu Su (suqingyu@neepu.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61873057, Grant 61703384, and Grant 61703091, in part by the Natural Science Foundation of Jilin Province under Grant 20180520211JH and Grant 20190303107SF, and in part by the Jilin City Science and Technology Bureau under Grant 201831727 and Grant 201831731.

ABSTRACT In this paper, we investigate the distributed finite-time secondary voltage control for the islanded microgrid with droop-controlled, inverter-based distributed generations. By introducing an error function, a nonsingular terminal sliding manifold is established, then the secondary voltage control is converted to a tracking consensus problem of the distributed nonsingular terminal sliding mode control. In the proposed control scheme, each distributed generation unit only requires its local information and the information from neighbors, which means only a simple communication network is needed. Furthermore, the distributed controller can adjust the voltage magnitude to its reference value in despite of dynamic uncertainties and bounded external disturbances. Numerical simulations are presented to demonstrate the effectiveness of the proposed control protocol.

INDEX TERMS Islanded microgrids, secondary voltage control, nonsingular terminal sliding mode control.

I. INTRODUCTION

Microgrids (MGs) that integrate clusters of distributed generations (DGs), loads and storages, are small-scale power systems, which can operate in both grid-connected and islanded operating modes [1]–[3]. Normally, the MG is connected to the main grid. When a disturbance occurs, the MG works in the islanded mode [4], [5]. Recently, a hierarchical control structure for islanded MGs has been addressed to endow smartness and flexibility to MGs, which usually consists of three layers including primary, secondary and tertiary control [6]–[9]. The primary control is implemented locally at each DG unit. However, it will result in voltage deviations from their reference points. The secondary control level can compensate for the deviations caused by the primary control and restore the voltage to its reference value [10], [11]. Tertiary control is used toeconomic dispatch and power flow optimization. The main content of this paper is secondary control level.

The associate editor coordinating the review of this manuscript and approving it for publication was Guangdeng Zong.

With the development of control theory, more and more new technologies are considered to solve the complex problems in power system. For example, The theory of Markovian jump system can be used to analyze and solve the situation that some part of the power system suddenly breaks down or the state may suddenly change in the process of operation [12]. The power system can be regarded as both continuous and discrete cyber-physical systems (CPS). Therefore, the switching system theory and network are used in power system modeling, so as to better analyze its stability and other performance [13], [14]. Resilient control, which is a combination of computerized and digital control systems, is used in the design of control systems in malignant and uncertain environments [15]. Moreover, many other technologies are addressed to solve the control problems of complex systems.

MGs are highly complex, nonlinear dynamical networks by nature that present many theoretical and practical challenges. To address the issues of MGs monitoring and control, making the system more intelligent and self-healing, the following technologies are adopted: 1) Some observers (e.g., Luenberger observer, sliding-mode observer and H_-/H_∞ observer) are designed to detect the system faults [16]–[18]; and 2) Some intelligent filtering technologies (e.g., finite-time filtering and exponential H_∞ filtering) are addressed to suppress interference signal to control better [19], [20].

Existing secondary control methods consist of two structures: centralized and distributed structure [6], [10]–[27]. The former conventionally assumes a centralized controller, which means a complex, bidirectional, and fully connected communication network is required. In [6], [21], centralized controllers are proposed to resolve the secondary control issue of the MG, which cause a single point-of-failure and adversely affects the reliability of the entire system. To conquer the aforementioned drawbacks, a multi-agent system-based distributed control strategy provides suitable remedies to the problem of a complicated communication network and hard to achieve the plug-and-play function [22]–[27]. A novel approach based on the general distributed technique is addressed to restore the frequency and voltage of the MG in [22]. However, the scheme requires that each local controller communicates with all the other controllers in the entire system, which is not as efficient as a centralized controller mentioned above. An input-output feedback linearization approach is used for the distributed cooperative control of multi-agent systems in [23]. More and more intelligent control technologies, such as Neural Networks, L_1 adaptive control, H_∞ control or finite-time control, applied to the distributed cooperative control system [24], [26]. In the framework of distributed multi-agent strategy, the MG is considered as a multi-agent system, in which each DG is regarded as an agent. Such distributed control only requires a sparse communication network by which each DG only exchanges information with its neighboring DGs, reducing communication costs and improving reliability. Using this approach, the secondary voltage control is transformed into a linear second-order tracking synchronization problem, which over-relies on the control parameters of the MG. The existing distributed control approach reported in [27], whose convergence is realized in an infinite time range, which severely limits the convergence rate of synchronization. Similarly in [16], an infinite-time horizon based on a distributed integral controller is proposed in [28], which dynamically regulates the system frequency in the presence of a time-varying load. However, voltage restoration is not considered.

A crucial factor of the consensus problem is the convergence rate of the synchronization process towards the consensus value. In [27]–[30] the consensus of the multi-agent system is asymptotic where the consensus can be reached when the time tends to infinity. However, in practical systems, we hope that the consensus will be achieved not only at a fast speed but in finite time. In addition, finite-time control can achieve better transient behavior, high-precision control performance, robustness against uncertainties, and better disturbance rejection properties [31], [32]. In recent years, most of efforts concerned the linear distributed finite-time consensus problem, with little effort being devoted to the nonlinear system [33], [34]. Reference [33] solves the finite-time consensus problem of the second-order non-linear multi-agent system. Reference [34] proposes a distributed finite-time consensus controller for the non-linear multi-agent system with undirected communication topology. Distributed finite-time secondary control of MGs is reported in [35]–[37]. Reference [35] proposes a finite-time frequency controller that synchronizes the MG frequency to the nominal frequency and shares the active power among DGs. A distributed MPC-based secondary voltage control scheme is proposed in [36] for autonomous droop-controlled MGs. By incorporating predictive mechanisms into distributed generations, the secondary voltage control is transformed into a tracker consensus problem of the distributed model predictive control. Reference [37] only considers the finite-time voltage restoration control, while the finite-time frequency control is not discussed. Most of previous methods cannot guarantee robust stability when facing parametric uncertainties and external disturbances.

In this paper, we propose a distributed cooperative secondary control based on the nonsingular terminal sliding mode (NTSM) control for the voltage restoration of an islanded MG in finite-time. Therefore, the secondary voltage control of the MG is converted to a distributed NTSM synchronization procedure by establishing an error function, together with a faster convergence rate of synchronization compared with conventional methods. In the control scheme, each DG acts as an agent that only uses its own information and the information from neighboring agents. Our proposed algorithm is robust to system uncertainties and input disturbances. Therefore, the proposed scheme only partly requires the information of the control input. The main features of our study are as follows.

(1) The distributed NTSM method is first used to restore the secondary voltage of the islanded MG in finite-time, and robustness analysis for parametric uncertainties and external disturbances is given. We will establish the voltage error function first. Then the reaching and sliding phase of the voltage consensus error will be discussed, together with theoretic stability analysis to guarantee the proposed approach can drive the voltage magnitudes to their reference values for all DGs.

(2) Distributed cooperative control of multi-agent systems is implemented to construct a reliable distributed secondary control structure for the islanded MG, which only requires local information and the information from neighbors. Hence, the proposed method is fully distributed.

(3) A finite-time protocol is used for the voltage restoration of the islanded MG, which more rapidly reaches consensus than do asymptotic controllers.

II. PRELIMINARIES

In general, an islanded MG with a total of N DGs and related control scheme is shown in Fig.1. An MG can be considered as a multi-agent system, where each DG is an agent communicating through a spares communication link. A physical

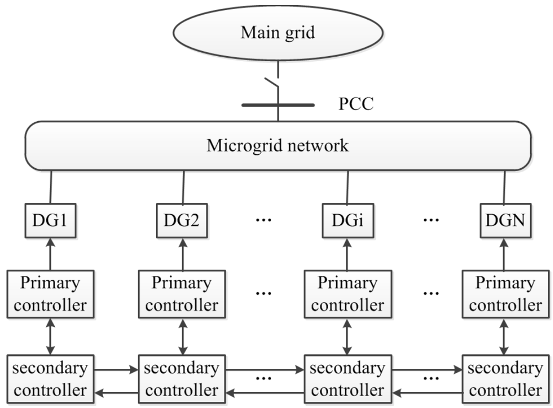

FIGURE 1. Control scheme of MG.

network integrates all grid components, including each DG and its respective load. In this part, we will make a brief introduction of the MG hierarchical control and the physical network. Then, the dynamic model of the islanded MG with parametric uncertainties and disturbances will be presented.

Notations: $\mathbb{R}_{>0}$: the set of positive real Numbers; $N_{\geq 1}$: the set of positive integers; $\overline{N}$: the set of network nodes; $\mathbb{R}_{<0}$: the set of negative real Numbers.

A. PRIMARY CONTROL

As shown in Fig.2, an inverter-based DG contains a dc energy source, the voltage source converter, an LCL filter and the connection part. In primary control, the power controller is only concerned because of that the dynamics of the voltage and current control loops faster than those of the power control loop.

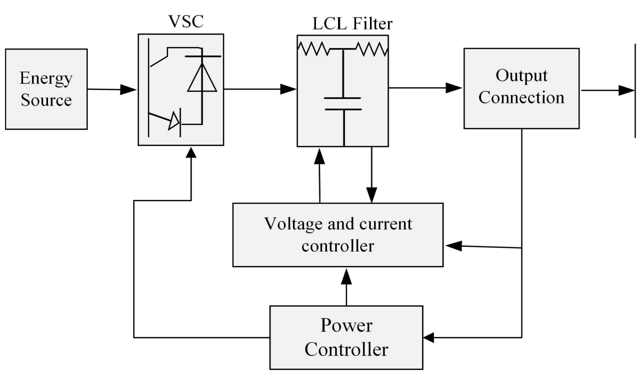

FIGURE 2. Primary control structure of an inverter-based DG.

Motivated by the widely use of the droop control in large conventional power systems, this technique is adapted to the power controller here. By analyzing the physical characteristics of inverters, the model of droop-controlled inverter-based microgrids is established [38]. By using the locally measured real power and reactive power information, we can regulate the frequency ω and voltage amplitude V of the inverter, respectively. According to [3], the frequency and voltage droop of the ith DG resembles a tracking modeled as:

$$\omega_i = \omega^d - k_{P_i}(P_i^m - P_i^d) \quad (1)$$

$$k_{V_i}\dot{V}_i = (V^d - V_i) - k_{Q_i}(Q_i^m - Q_i^d) \quad (2)$$

where $\omega^d \in \mathbb{R}_{>0}$ is the desired (nominal) frequency, $V^d \in \mathbb{R}_{>0}$ is the the desired (nominal) voltage amplitude. $k_{V_i} \in \mathbb{R}_{>0}$ is the voltage control gain, $k_{P_i} \in \mathbb{R}_{>0}$ and $k_{Q_i} \in \mathbb{R}_{>0}$ are the frequency and voltage droop gains, respectively, P_i^m and Q_i^m are the measured real and reactive power, respectively, and P_i^d and Q_i^d are the respective desired real and reactive power, which are the desired setpoints.

The measured P_i^m and Q_i^m can be obtained through the following first-order low-pass filters with $\tau_{P_i} \in \mathbb{R}_{>0}$, $\tau_{Q_i} \in \mathbb{R}_{>0}$ as the respective time constant.

$$\tau_{P_i}\dot{P}_i^m = -P_i^m + P_i \quad (3)$$

$$\tau_{Q_i}\dot{Q}_i^m = -Q_i^m + Q_i \quad (4)$$

where P_i and Q_i are the real and reactive power outputs of the ith DG.

Obviously, the measured P_i^m and Q_i^m can be deduced from (3) and (4). Substituting (3) and (4) into (1) and (2) respectively yields

$$\tau_{P_i}\dot{\omega}_i + \omega_i - \omega^d + k_{P_i}(P_i - P_i^d) = 0 \quad (5)$$

$$\tau_{Q_i}k_{V_i}\ddot{V}_i + (\tau_{Q_i} + k_{V_i})\dot{V}_i + V_i - V^d + k_{Q_i}(Q_i - Q_i^d) = 0 \quad (6)$$

Remark 1: In the modeling of inverters, we assume that the frequency regulation is instantaneous, but the voltage control happens with a delay that, following standard practice, is represented by a first order filter. The DC source of the DGs is assumed to be able to provide sufficient power for the DGs.

B. SECONDARY CONTROL

In order to compensate for the frequency and voltage amplitude deviations caused by droop control, secondary control is usually needed. Conventionally, the secondary control is implemented for each DG using a centralized controller, which has a complex communication structure. Since this technique is potentially unreliable, a distributed cooperative control structure is addressed in this paper, as shown in Fig.1. Using the distributed cooperative control of multi-agent systems knowledge, the secondary control is translated into a tracking synchronization problem.

Here, we define the secondary control input $u_i = [u_i^\omega, u_i^V]^T$, and add it into the primary control model, then (5) and (6) can be rewritten as:

$$\tau_{P_i}\dot{\omega}_i + \omega_i - \omega^d + k_{P_i}(P_i - P_i^d) + u_i^\omega = 0 \quad (7)$$

$$\tau_{Q_i}k_{V_i}\ddot{V}_i + (\tau_{Q_i} + k_{V_i})\dot{V}_i + V_i - V^d + k_{Q_i}(Q_i - Q_i^d) + u_i^V = 0 \quad (8)$$

C. PHYSICAL NETWORK

Kron reduction is frequently used in power system analysis, since it allows to equivalently represent a system of differential-algebraic equations as a set of pure ordinary differential equations. Kron reduction is also a standard tool employed in modeling, analysis, and control of power systems. In our paper, we consider a generic meshed MG and, following the classical approach in conventional power system studies, assume that loads are modeled by constant impedances. This leads to a set of nonlinear differential algebraic equations. Then, Kron reduction is carried out to eliminate all algebraic equations corresponding to loads and obtain a set of differential equations [39]. Using this representation, an islanded MG can be formed by $N_{\geq 1}$ nodes and a set of network nodes by $\overline{N}$, each of which represents a DG. We also assume that the admittances in the MG are purely inductive, therefore, the power transmission lines of the MG network are lossless. The overall active and reactive powers $\hat{P}_i$ and $\hat{Q}_i$ injected to the network at node $i \in \overline{N}$ are obtained as

$$\hat{P}_i = \sum_{k \in \overline{N}_i} V_i V_k |B_{ik}| \sin(\delta_{ik}) \tag{9}$$

$$\hat{Q}_i = V_i^2 \sum_{k \in \overline{N}_i} |B_{ik}| - \sum_{k \in \overline{N}_i} V_i V_k |B_{ik}| \cos(\delta_{ik}) \tag{10}$$

where $B_{ik} \in \mathbb{R}_{<0}$ is the susceptance, $\delta_{ik} = \delta_i - \delta_k$ are the angle differences.

Remark 2: Note that $k \in \overline{N}_i$ is the adjacent nodes of the node i, and $k \neq i$.

The assumption is that local loads are connected to each DG, and the apparent power flow is given by $S_i = P_i + jQ_i$. The ZIP load model is presented to connect various types of loads. The static characteristics of the load can be classified into constant impedance loads P_{1_i}, Q_{1_i}, constant current loads P_{2_i}, Q_{2_i} and constant power loads P_{3_i}, Q_{3_i}, depending on the power relation to the voltage V_i, expressed as

$$P_{L_i} = P_{1_i} V_i^2 + P_{2_i} V_i + P_{3_i} \tag{11}$$

$$Q_{L_i} = Q_{1_i} V_i^2 + Q_{2_i} V_i + Q_{3_i} \tag{12}$$

According to the relationship of the power conservation law, the outputs real and reactive power are expressed as

$$P_i = P_{L_i} + \hat{P}_i \tag{13}$$

$$Q_i = Q_{L_i} + \hat{Q}_i \tag{14}$$

As we are mainly concerned with dynamics of DG units, we express all power flows in generator convention. Combining (7)-(14), we can get the whole system dynamics.

D. UNCERTAINTIES ANALYSIS

In this paper, we also consider robustness against uncertainties, and better disturbance rejection properties. To simplify notation we define

$$\begin{aligned} P &:= col(P_i) \in \mathbb{R}^n, \quad Q := col(Q_i) \in \mathbb{R}^n, \\ P^d &:= col(P_i^d) \in \mathbb{R}^n, \quad Q^d := col(Q_i^d) \in \mathbb{R}^n, \\ \omega &:= col(\omega_i) \in \mathbb{R}^n, \quad V := col(V_i) \in \mathbb{R}^n, \\ \tau_P &:= diag(\tau_{P_i}) \in \mathbb{R}^{n\times n}, \quad \tau_Q := diag(\tau_{Q_i}) \in \mathbb{R}^{n\times n}, \\ K_P &:= diag(k_{p_i}) \in \mathbb{R}^{n\times n}, \quad K_Q := diag(k_{q_i}) \in \mathbb{R}^{n\times n}, \\ K_V &:= diag(k_{V_i}) \in \mathbb{R}^{n\times n} \end{aligned} \tag{15}$$

We only consider the secondary voltage restoration here, and the global form of (8) with uncertainties and disturbances can be formulated as the following n DG networked system

$$M(V)\ddot{V} + C(V)\dot{V} + G(V)V + B + D(t) + U^V = 0 \tag{16}$$

Some terms above are defined as follows

$$\begin{aligned} M(V) &= M_0(V) + \Delta M(V) \\ C(V) &= C_0(V) + \Delta C(V) \\ G(V) &= G_0(V) + \Delta G(V) \end{aligned}$$

where $M_0(V), C_0(V), G_0(V)$ are the known terms, and $\Delta M_0(V), \Delta C_0(V), \Delta G_0(V)$ are the uncertain terms. B is the known constant term. D(t) is bounded input disturbances. Then, the dynamic equation of the secondary voltage can be written in the following form:

$$M_0(V)\ddot{V} + C_0(V)\dot{V} + G_0(V)V + B + U^V = \rho(t) \tag{17}$$

with

$$\rho(t) = -\Delta M_0(V)\ddot{V} - \Delta C_0(V)\dot{V} - \Delta G_0(V)V - D(t)$$

Assumption 1: $\rho(t)$ is bounded, and satisfies $\|\rho(t)\| < a_2\|\dot{V}\|^2 + a_1\|V\| + a_0$, and $a_0, a_1, a_2$are positive numbers.

III. SECONDARY CONTROL BASED ON DISTRIBUTED COOPERATIVE CONTROL

A. NOTATION

In this section, based on nonsingular terminal sliding-mode (NTSM) theory, we design a distributed finite-time consensus algorithm for the DG that utilizes its own information and its neighbors information to ensure each DG agent tracks the leader DG in finite time, which can also overcome the singularity problem compared to the conventional terminal sliding mode control.

Fig.1 shows the structure of voltage distributed secondary cooperative control for the MG. Different from the MGCC control strategy, our secondary controller is applied locally with communication with its neighboring controllers.

Lemma 1 [40]: Consider the following continuous nonlinear system

$$\dot{x} = f(x), f(0) = 0, \quad x \in \mathbb{R}^n \tag{18}$$

Suppose that there exists a continuous function $V : U \longrightarrow \mathbb{R}^n$, such that the following conditions hold:

(1) V is positive definite;

(2) there exist real numbers $c > 0$ and $\alpha \in (0, 1)$ and an open neighbourhood $U_0 \subset U$ of the origin, such that

$$\dot{V}(x) + c_0 V^{\alpha}(x) \leqslant 0 \tag{19}$$

then the origin of the system is finite-time stable, and the convergence time

$$T(x) \leqslant \frac{1}{c(1-\alpha)} V(0)^{1-\alpha} \tag{20}$$

The voltage dynamic (8) with uncertainties and disturbances is rewritten as state space expression for convenience. Then, the V_i is defined as variable x_1 and $\dot{V}_i$ as x_2.

$$\begin{cases} \dot{x}_{1_i} = x_{2_i} \\ \dot{x}_{2_i} = f_i(x_i, x_k) + \rho_i + \dfrac{1}{\tau_{Q_i} k_{V_i}} u_i^V, i, k \in \overline{N} \end{cases} \tag{21}$$

where $f_i(x_i, x_k)$ is (see (22)), with $|\rho_i| < a_2|\dot{x}_{1_i}|^2 + a_1|x_{1_i}| + a_0$.

$$\begin{aligned} f_i(x_i, x_k) = &-\frac{\tau_{Q_i} + k_{V_i}}{\tau_{Q_i} k_{V_i}} \dot{V}_i - \frac{k_{Q_i}(Q_{1_i} + \sum_{k \in N_i} |B_{ik}|)}{\tau_{Q_i} k_{V_i}} V_i^2 \\ &+ \frac{k_{Q_i}}{\tau_{Q_i} k_{V_i}} \sum_{k \in N_i} |B_{ik}| V_i V_k \cos(\delta_i - \delta_k) \\ &- \frac{1 + k_{Q_i} Q_{2_i}}{\tau_{Q_i} k_{V_i}} V_i - \frac{k_{Q_i}(Q_{3_i} - Q_i^d) - V^d}{\tau_{Q_i} k_{V_i}} \end{aligned} \tag{22}$$

B. DISTRIBUTED FINITE-TIME VOLTAGE RESTORATION OF THE ISLANDED MG

We select the tracking error $e_{1_i} = x_{1_i} - V^{ref}$, $e_{2_i} = \dot{x}_{1_i} - \dot{V}^{ref}$, so the error equation can be expressed as

$$\begin{cases} \dot{e}_{1_i} = e_{2_i} \\ \dot{e}_{2_i} = f_i(x_i, x_k) + \rho_i + \dfrac{1}{\tau_{Q_i} k_{V_i}} u_i^V \end{cases} \tag{23}$$

Remark 3: V^{ref} is the reference voltage value,which only the ith DG can be available generally a step signal. And $\dot{V}^{ref}$ is the derivative of reference voltage.

The NTSM model is chosen as

$$s = e_{1_i} + \beta e_{2_i}^{p/q} \tag{24}$$

and the control law is designed as

$$u_i = \tau_{Q_i} k_{V_i} [f_i(x_i, x_k) + \beta \frac{q}{p} e_{2_i}^{2-p/q} + (\rho_i + \eta) sgn(s)] \tag{25}$$

where $\beta > 0$ is a design constant, and p and q are positive odd integers, which satisfies $1 < p/q < 2$. $\eta > 0$ is a constant satisfying

$$\frac{1}{2}\frac{d}{dt} s^2 < -\eta |s|$$

Theorem 1: For voltage dynamics system (21) with the NTSM model (24), if the controller is designed as (25), then the system tracking error $e_{1_i} = V_i - V^{ref}$ will converge to zero in finite time. Furthermore, using the designed control variable (25), the distributed finite-time voltage restoration problem in (8) with uncertainties and disturbances is solved.

Proof: The global form of (23) can be formulated as networked double-integrator systems

$$\begin{cases} \dot{\varepsilon}_1 = \varepsilon_2 \\ \dot{\varepsilon}_2 = f(x) + \rho + \dfrac{1}{\tau_Q k_V} U^V \end{cases} \tag{26}$$

where $\varepsilon_1 = [e_{1_1}, e_{1_2}, \ldots, e_{1_N}]^T$, $\varepsilon_2 = [e_{2_1}, e_{2_2}, \ldots, e_{2_N}]^T$, $f(x) = [f_1(x_1, x_k), f_2(x_2, x_k), \ldots, f_N(x_N, x_k)]^T$, $\tau_Q = [\tau_{Q_1}, \tau_{Q_2}, \ldots, \tau_{Q_N}]^T$, $k_V = [k_{V_1}, k_{V_2}, \ldots, k_{V_N}]$.

It can be conducted that systems (21) tends to be stable in finte time if and only if (26) is finite-time stable.

For the dynamic system (26), if the NTSM manifold is chosen as

$$s = \varepsilon_1 + C_1 \varepsilon_2^{p/q}, \tag{27}$$

where $C_1 = diag[c_1, \ldots, c_n]$ is a design matrix, and the NTSM control U^V is designed as follows, then the system tracking error $\varepsilon(t)$ will converge to zero in finite time.

$$U^V = U_0^V + U_1^V + U_2^V \tag{28}$$

where

$$U_0^V = -\tau_Q k_V f(x) \tag{29}$$

$$U_1^V = -\tau_Q k_V \frac{q}{p} C_1^{-1} \dot{\varepsilon}_1^{2-p/q} \tag{30}$$

$$\begin{aligned} U_2^V = &-\tau_Q k_V \frac{q}{p} \frac{[s^T C_1 diag(\dot{\varepsilon}_1^{p/q-1}) 1/\tau_Q k_V]^T}{\|s^T C_1 diag(\dot{\varepsilon}_1^{p/q-1}) 1/\tau_Q k_V\|} \\ &\times [\|s\| \|C_1 diag(\dot{\varepsilon}_1^{p/q-1}) 1/\tau_Q k_V\| (a_2|\dot{x}_1|^2 \\ &+ a_1|x_1| + a_0)] \end{aligned} \tag{31}$$

Consider the following Lyapunov function

$$V = \frac{1}{2} s^T s$$

Differentiating V with respect to time, and substituting (28)-(31) into it yields:

$$\begin{aligned} &\dot{V} \\ &= s^T \dot{s} = s^T (\dot{\varepsilon} + \frac{p}{q} C_1 diag(\varepsilon_2^{p/q-1}) \dot{\varepsilon}_2) \\ &= s^T (\dot{\varepsilon} + \frac{p}{q} C_1 diag(\varepsilon_2^{p/q-1}) 1/\tau_Q k_V (U_1^V + U_2^V + \rho(t))) \\ &= s^T (\frac{p}{q} C_1 diag(\varepsilon_2^{p/q-1}) 1/\tau_Q k_V (U_2^V + \rho(t))) \\ &= -\frac{p}{q} \|s\| \|C_1 diag(\dot{\varepsilon}_1^{p/q-1}) 1/\tau_Q k_V\| \times (a_2|\dot{x}_1|^2 + a_1|x_1| + a_0) \\ &\quad + s^T \frac{p}{q} C_1 diag(\varepsilon_2^{p/q-1}) 1/\tau_Q k_V \rho(t) \\ &\le -\frac{p}{q} \|s\| \|C_1 diag(\dot{\varepsilon}_1^{p/q-1}) 1/\tau_Q k_V\| \times (a_2|\dot{x}_1|^2 + a_1|x_1| + a_0) \\ &\quad + \frac{p}{q} \|s\| \|C_1 diag(\varepsilon_2^{p/q-1}) 1/\tau_Q k_V\| \|\rho(t)\| \\ &= -\frac{p}{q} \|C_1 diag(\varepsilon_2^{p/q-1}) 1/\tau_Q k_V\| \times (a_2|\dot{x}_1|^2 + a_1|x_1| + a_0 \\ &\quad - \|\rho(t)\|) \|s\| \end{aligned} \tag{32}$$

If we define $\eta(t) = \frac{p}{q} \|C_1 diag(\varepsilon_2^{p/q-1}) 1/\tau_Q k_V\| \times (a_2|\dot{x}_1|^2 + a_1|x_1| + a_0 - \|\rho(t)\|)$, which can be conducted that $\eta(t) > 0$. Hence, we can obtain $\dot{V} \le -\eta(t)\|s\| = -\eta(t) V^{\frac{1}{2}}$, with $\|s\| \neq 0$.

Therefore, according to Lemma 1, the states of system (26) can reach the sliding mode surface $s = 0$ in finite time and maintain on it.

Next, we will prove the consensus tracking of the MAS can be obtained on the sliding mode surface. Considering the

Lyapunov function $\overline{V} = \frac{1}{2}\varepsilon_1^T\varepsilon_1$, we can obtain

$$\dot{\overline{V}} = \varepsilon_1^T \varepsilon_2 \tag{33}$$

On the sliding mode surface we have $s = 0$, so $\varepsilon_2 = -C_1^{p/q}\varepsilon_1^{q/p}$, then it can be obtained

$$\dot{\overline{V}} = -C_1^{p/q}\varepsilon_1^T\varepsilon_1^{q/p} \leq -C_1^{p/q} 2^{\frac{1+p/q}{2p/q}} \overline{V}^{\frac{1+p/q}{2p/q}} \tag{34}$$

By Lemma 1, the consensus error $\varepsilon_1, \varepsilon_2$ defined in (26), will converge to zero in finite time. By the above analysis, the consensus of the MG can be obtained in finite time. Therefore, the proof is completed.

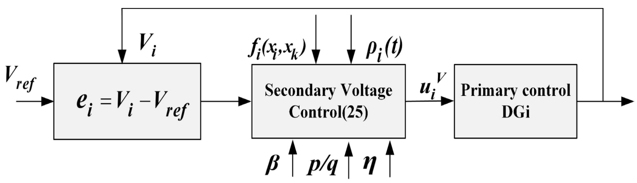

FIGURE 3. Schematic diagram of the proposed secondary voltage control.

Fig.3 shows the block diagram of the distributed secondary voltage control for the i DG, together with parametrs of the proposed distributed controller $\beta, p/q, \gamma$, and the untertainties $\rho_i(t)$.

Remark 2: The theorem above is proved under the assumption that the states of the leader are time-invariant. The topology in this paper is an undirected graph.

Remark 3: The nonlinear dynamics of each DG in (8) is transformed to networked double-integrator systems in (26). Then, solving the distributed finite-time voltage restoration problem in (8) is equivalent to stabilizing the double-integrator systems (26) in a finite time.

Remark 4: Note that, there is $sgn(s)$ in the finite-time terminal sliding mode approach, which will cause hopper problem. To overcome the problem, a saturation function $sat(s)$ is needed to replace the $sgn(s)$ in simulation. The relationship between the steady-state errors of the NTSM system and the width of the layer surrounding the NTSM manifold $s(t) = 0$ is as follows

$$|s(t)| \leq \gamma \Rightarrow |x(t)| \leq \gamma$$
$$and$$
$$|x(t)| \leq (2\beta\gamma)^{q/p}, \quad t \longrightarrow \infty$$

IV. CASE STUDY

An islanded microgrid system with 380 V, 50 Hz is builted to verify the effectiveness of the cooperative control strategy in MATLAB/Simulink environment. A test system consisting of four DGs connected local loads and three transmission lines is showned as Fig.4. There is only DG1 related to the reference voltage value, and the corresponding communication digraph with distributed secondary controllers is shown in Fig.5.

The parameters for the MG model and secondary control are summarized in Table 1 and Table 2. Note that to verify the robustness of the proposed controller with respect to the

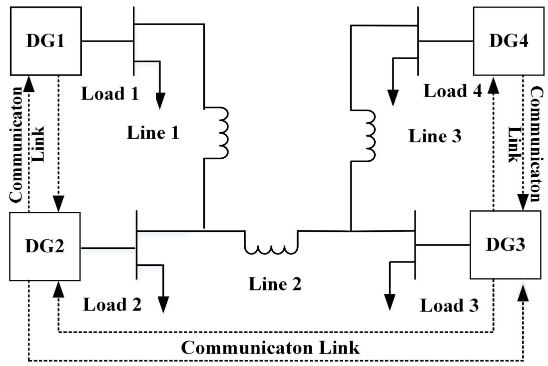

FIGURE 4. Islanded MG test system.

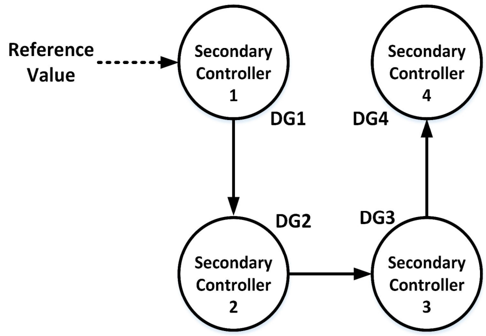

FIGURE 5. Topology of communication digraph.

TABLE 1. Parameters of the MG test system.

	DG1		DG2		DG3		DG4	
Model	τ_{Q_1}	0.016	τ_{Q_2}	0.016	τ_{Q_3}	0.016	τ_{Q_4}	0.016
	k_{V_1}	$1e^{-2}$	k_{V_2}	$1e^{-2}$	k_{V_3}	$1e^{-2}$	k_{V_4}	$1e^{-2}$
	k_{Q_1}	$4.2e^{-4}$	k_{Q_2}	$4.2e^{-4}$	k_{Q_3}	$4.2e^{-4}$	k_{Q_4}	$4.2e^{-4}$
Loads	Q_{1_1}	0.01	Q_{1_2}	0.01	Q_{1_3}	0.01	Q_{1_4}	0.01
	Q_{2_1}	1	Q_{2_2}	2	Q_{2_3}	3	Q_{2_4}	4
	Q_{3_1}	$1e^4$	Q_{3_2}	$1e^4$	Q_{3_3}	$1e^4$	Q_{3_4}	$1e^4$
Lines	$B_{12} = 10\Omega^{-1}$,$B_{23} = 10.67\Omega^{-1}$,$B_{34} = 9.82\Omega^{-1}$							
Reference	$V^{ref} = 380V$							

TABLE 2. Parameters of the proposed secondary voltage controller.

	DG1		DG2		DG3		DG4	
Controller	p	5	p	20	p	10	p	16
	q	3	q	11	q	7	q	9
	η	0.5	η	0.5	η	0.5	η	0.5
	β	1	β	0.8	β	2	β	1.7
	γ	0.001	γ	0.001	γ	0.001	γ	0.001

uncertainties and disturbances, the boundary parameters in (21) are assumed to be $a_0 = 8.6, a_1 = 3.5, a_2 = 2.6$. Suppose the tracking error $\tilde{e}_i = V_i - V^{ref}$, and the 1st tracking error $\tilde{e}_i = \dot{V}_i - \dot{V}^{ref}$ are to be $|\tilde{e}_i| \leq 0.001, |\dot{\tilde{e}}_i| \leq 0.024$. According to Remark 4, it is obtained that $|\tilde{e}_i| \leq \gamma_i$, and let $\gamma_i = 0.01$, the tracking error of the system$|\tilde{e}_i|$ can be guaranteed. can be guaranteed. We can also obtained that

$|\tilde{e}_i| \leq (2\beta_i\gamma_i)^{q/p}$, if we define $(2\beta_i\gamma_i)^{p/q} \leq 0.024$, then we have $\frac{q}{p} \leq \frac{\lg 0.024}{\lg(2\beta_i\gamma_i)}$. The detail parameter information of the controller can be found in Table 2.

The simulation results are presented in two scenarios,and all of the two scenarios use a fixed communication topology which is shown in Fig.4.

A. CASE 1: CONSTANT POWER LOAD

The simulation scenario proceeds are divided into four stages:

Stage 1 (0-1 s): Only the primary control is activated at t = 0 s.

Stage 2 (1 s-): The proposed secondary voltage control in (25) begins to work at t = 1 s.

Stage 3 (1 s-2.5 s): A constant load $L_c = 1 \times 10^4 + j4.5 \times 10^3 W$ is added to Load 4 at t = 2.5 s.

Stage 4 (2.5 s-4 s): Load L_c is removed at t = 4 s.

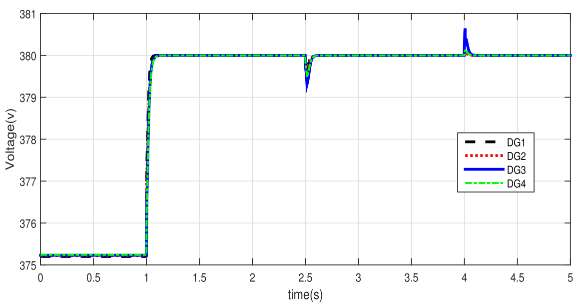

FIGURE 6. Output voltage magnitude of the proposed finite-time control in *Case 1*.

Fig.6 depicts the simulation results. At first, the MG goes through the islanded mode condition. From stage 1, we can conclude that the voltage amplitudes of four DGs change to different value owing to the droop characteristic of the primary control. Hence, a secondary control layer is needed to reduce the deviations between the output voltage and reference value. Once the secondary control works at stage 2, the output values caused by the droop-controller-produced are rapidly restored to their reference values in a finite time. Then, we show the the robust performance of the distributed secondary control in the presence of load changes at stage 3 and 4. A constant load L_c is suddenly connected and disconnected from the islanded microgrid at t = 2.5 s and t = 4 s, respectively. The result confirms that the distributed secondary control can restore the MG voltage after a short transient time, as Fig.6 shows.

B. CASE 2: COMPARING THE PROPOSED METHOD IN [37]

As mentioned in the introduction, we only consider the voltage restoration problem in this paper. We compare the proposed distributed voltage controller (25) with the one presented in [37]. The parameters of the MG test system in Table 1 are set the same as [37], note that to verify the robustness of the proposed controllers with respect to the parameters, part of the parameters are not given. For the sake of simplicity, we depict comparative evaluation of voltage responses with only one DG in Case 1. The results of Fig.7 show some advantages to us. Our proposed finite-time voltage protocol has a faster convergence rate than that of the control approach applied in [37], which can be obtained at stage 1 in Fig.7. As seen in Fig.7, the time that our mentioned control protocol settles down is about 0.5s, while the time in [37] is about 1.5s, which is approximately 120% longer than that the time we require. This fully proves that our proposed protocol ensures a faster convergence within finite time. Moreover, in this paper, the mentioned finite-time method behaves better disturbance rejection properties in the case of load change at stage 2 and 3.

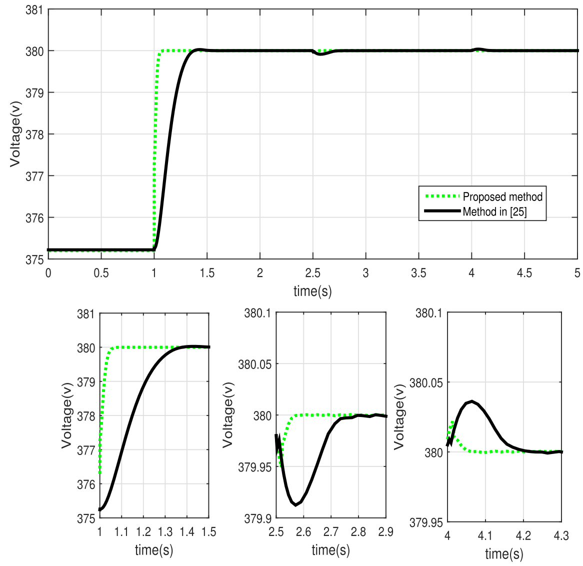

FIGURE 7. Performance comparison of DG1 response.

C. CASE 3: ROBUSTNESS AGAINST UNCERTAINTIES AND INPUT DISTURBANCES

It is ideal that the controller is completely independent of the DG patameters, meanwhile, the controller performance should not be degraded by the variations owing to aging and thermal effects. Both of loads, the line impedances and DGs parameters are increased by 25% with regard to the values in Table 1 to test the controller robustness against

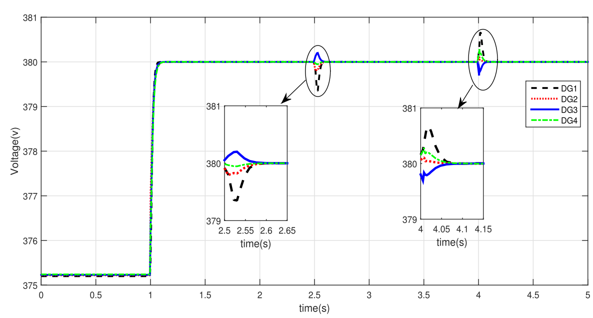

FIGURE 8. Response curves of the test DGs for Case 3.

uncertainties and input disturbances. An additional constant load $L_c = 1 \times 10^4 + j4.5 \times 10^3 W$ is added to DG2 at t = 2.5 s instead of DG4 in contrast to Case 1 and Case 2, and taken down at t = 4 s. It clear konws that the controller still drives the voltage to the nominal values in Fig.8. This illustrates the designed controller can accommodate the uncertainties and input disturbances.

V. CONCLUSION

The distributed finite-time restoration problem of voltage terms in islanded AC MGs with undirected communication networks is proposed in this paper. The distributed NTSM-based scheme is first applied to the voltage restoration of the islanded MG. By introducing the error function, a nonsingular terminal sliding manifold is established. Then the secondary voltage control is transformed to a tracking consensus problem of the distributed NTSM control in finite-time. Compared with the traditional sliding mode method, NTSM can make the system converge in finite time, and there is no singularity in the traditional terminal sliding mode method. The microgrid is regarded as a multi-agent system in which DG units communicate with one another through a directed sparse communication network without a central computing and communication unit. As for parametric uncertainties and disturbances, the addressed method is also robust through uncertainties analysis that changes with the voltage in MG system. Simulations verify the performance of the addressed secondary voltage control approach. Frequency control and power distribution are not involved in this paper. Our future work will focus on the accurate distribution of power, as well as on the coupling between frequence control and voltage control.

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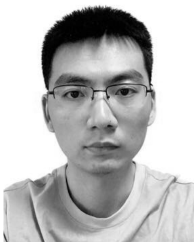
XUEQIANG SHEN received the M.Sc. degree from Northeast Electric Power University, China, in 2008, and the Ph.D. degree from North China Electric Power University, in 2014. From October 2015 to October 2016, he was a Visiting Scholar with the Intelligent Systems and Biomedical Robotics Group (ISR), University of Portsmouth, U.K. He is currently an Associate Professor with the School of Automation Engineering, Northeast Electric Power University. He has published three articles on SCI journals and five papers on international conferences. His research interests include process control and intelligent control.

HAIQING WANG is currently pursuing the master's degree with Northeast Electric Power University. Her research interest includes power cyber-physical systems.



DEZHEN ZHANG received the M.Sc. degree from Northeast Electric Power University, in 2018. He is currently pursuing the Ph.D. degree in biomedical engineering with Shandong University. His research interests include bioinformatics, machine learning, and control theory.

JIAN LI (Member, IEEE) received the B.Sc. and M.Sc. degrees in electrical automation and control theory and application from Liaoning Technical University, China, in 2005 and 2008, respectively, and the Ph.D. degree in control theory and application from Northeast University, China, in 2013. From July 2015 to August 2015, she was a Visiting Scholar with the Nonlinear Dynamics Group, Yeungnam University, South Korea. From October 2016 to October 2017, she was a Visiting Scholar with the National University of Singapore, Singapore. She is currently an Associate Professor with the School of Automation Engineering, Northeast Electric Power University, China. She has published 22 articles on SCI journals and nine papers on international conferences. Her research interests include fault detection, robust control, and power cyber-physical systems.

RENSHU WANG received the B.Sc. degree and the Ph.D. degree in control theory and control engineering from North China Electric Power University, in 2009 and 2014, respectively. He is currently a Postdoctoral Researcher of postdoctoral programme with State Grid Fujian Electric Power Company Ltd., and Southeast University. His research interests include intelligent recognition and control systems.

QINGYU SU (Member, IEEE) received the B.Sc. degree in electrical automation and the M.Sc. degree in control theory and application from Liaoning Technical University, China, in 2005 and 2008, respectively, and the Ph.D. degree in control theory and application from Northeastern University, China, in 2013. From October 2015 to October 2016, he was a Visiting Scholar with the Intelligent Systems and Biomedical Robotics Group (ISR), University of Portsmouth, U.K. He is currently an Associate Professor with the School of Automation Engineering, Northeast Electric Power University, China. He has published 21 articles on SCI journals and eight papers on international conferences. His research interests include switched systems, nonlinear control systems, and power cyber-phyiscal systems.