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Robust Disturbance Rejection in Uncertain Singular Systems Using Equivalent-Input-Disturbance Method Based on Output Feedback Control

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ABSTRACT A robust disturbance-rejection problem for an uncertain singular system is considered in this paper. An equivalent-input-disturbance (EID) method is applied to improve disturbance-rejection performance. At first, the disturbance estimation is obtained by the EID method; Then, the disturbance estimation is compensation to the control input channel to offset the adverse effect of external disturbance and uncertainties on the system. Two sufficient and necessary conditions using linear matrix inequality are obtained, which can guarantee the admissibility (regularity, non-impulsiveness, and stability) for the closed-loop control system. In addition, output-feedback control laws (static, dynamic) and observer gain are obtained using the singular value decomposition method. A numerical example and simulation studies prove the validity and feasibility of the presented method.

INDEX TERMS Linear matrix inequality, uncertain singular system, equivalent input disturbance (EID), output-feedback control laws.

I. INTRODUCTION

Singular systems contain differential equations and algebraic equations, which are more complicated and general form than standard state-space systems [1]–[3]. So, it can describe real physical systems such as electrical circuit networks, biological systems, and robotic systems [4]–[6]. Control problems of singular systems are harder to solve than the normal systems, which owing to the singular control systems need to consider regularity, impulses-free, and causality besides stability problem [7]–[9]. Up to now, control problems of singular systems have attracted considerable attention by scholars.

On the one hand, it is well known that disturbances can degrade control performance and instability of a control system. So, disturbance-rejection problem must be considered in the design of control systems. Numerous methods have been proposed to deal with disturbances for control systems, such as sliding-mode control method, the uncertainty-and-disturbance estimator, and disturbance observer [10]–[15]. Although these methods have been widely used in actual control systems, it is difficult to meet disturbance rejection performance and other control requirements (such as robustness performance) at the same time [16]–[18]. The equivalent-input-disturbance (EID) approach is effective active disturbance-rejection method [19]–[22]. The method rejects any kind of disturbance whether it's a matched or unmatched disturbances. And we do not need to know prior information about disturbances. On the other hand, uncertainties are inevitable exist in practical systems and influence control performance [23]–[25]. However, so far, a great number of researches are focused on the admissibility of singular systems, a few results published in the existing literature about the disturbance-rejection problem of singular systems [26], [27]. So, it is necessary

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to employ the EID method to reject the harmful effect of uncertainties and exogenous disturbances on the control system.

The EID method has been successfully applied to standard state-space systems to prove its excellent disturbance-rejection performance by using state-feedback control technique, such as linear systems [23], [24], time-delay systems [28], [29], and nonlinear systems [30], [31]. However, in control engineering practice, the reliability and the cost of implementation control of the system must be considered. So, it is more feasible if use output-feedback control technique rather than state feedback. Reference [32] consider a robust H_∞ control problem for an uncertain singular system based on static output-feedback control. However, it do not consider dynamic output-feedback control problem.

This paper deal with a disturbance-rejection problem of an uncertain singular system by using output-feedback control technique based on the EID method. Two sufficient and necessary conditions for admissibility are acquired for static and dynamic output-feedback control in term of linear matrix inequality. A singular value decomposition (SVD) method is used to acquire the output-feedback laws. A numerical simulation demonstrates the superiority and collectiveness of the EID-based control method.

The major contributions of the dissertation include: 1) Compared with the existing literature about EID, taking the cost of implementation control into account,we uses output-feedback control means in this paper; 2) Compared with previous control methods (such as SMC-based robust control method [33], [34], Neural network approach [35], [36], and H_∞ control method [37]), the EID-based control method does not need to know any prior information of disturbance and can reduces the conservatism of the system; 3) The parameters of the controller can be easy to designed by using SVD method instead of using a conservative method in [38]. And the configuration of uncertain singular control system is simple in this paper.

Notations : $\mathcal{Q} > 0$ denotes $\mathcal{Q}$ is positive definite matrix. $diag\{\cdots\}$ is a block-diagonal matrix, $\begin{bmatrix} \mathcal{X} & \mathcal{Y} \\ \star & \mathcal{Z} \end{bmatrix}$ represents $\begin{bmatrix} \mathcal{X} & \mathcal{Y} \\ \mathcal{Y}^T & \mathcal{Z} \end{bmatrix}$.

II. PROBLEM STATEMENT AND PRELIMINARY RESULT

An uncertain singular system was considered as follows.

$$\begin{cases} E\dot{x}(t) = [\mathcal{A} + \Delta\mathcal{A}(t)]x(t) + \mathcal{B}u(t) + \mathcal{B}_d d(t), \\ y(t) = \mathcal{C}x(t), \end{cases} \tag{1}$$

where $u(t) \in \mathbb{R}^m$ is the control input; $x(t) \in \mathbb{R}^n$ is the state; $d(t) \in \mathbb{R}^{n_d}$ is a disturbance; $y(t) \in \mathbb{R}^q$ is the output of the system. The matrix $E \in \mathbb{R}^{n\times n}$ satisfies $rankE = r \leq n$. $\mathcal{A}$, $\mathcal{B}_d$, $\mathcal{C}$, and $\mathcal{B}$ are constant matrices of appropriate dimensions.

Assume that the parameter uncertainty $\Delta\mathcal{A}(t)$ of the system (1) satisfies

$$\Delta\mathcal{A}(t) = \mathcal{D}F(t)\mathcal{N}, \tag{2}$$

where $\mathcal{N}$ and $\mathcal{D}$ are called real matrices.

$F(t)$ is a unknown matrix function and it satisfies

$$F^T(t)F(t) \leq I. \tag{3}$$

Definition 1: [39], [40] If $\det(sE - \mathcal{A}) \neq 0$, system $E\dot{x}(t) = \mathcal{A}x(t)$ is called regular.

Definition 2: [39], [40] If $\deg(\det(sE - \mathcal{A}))$ is equal to rank E, system $E\dot{x}(t) = \mathcal{A}x(t)$ is called impulse-free.

Definition 3: [39], [40] If system $E\dot{x}(t) = \mathcal{A}x(t)$ is stable, impulse-free, and regular, it is called admissible.

The Fig. 1 is a block diagram of uncertain singular control system based on the EID method. The control system is composed of the plant, output-feedback controllers, an EID estimator, and a state observer. In this paper, two control strategies are considered. One is static output-feedback control (Case 1) and the other is dynamic output-feedback control (Case 2). In this study, the external disturbances and the uncertainties of the uncertain singular system can be regarded as load disturbances. As explained in [19], there exists an EID belongs to the set Ψ on the control input channel, which produces the same output that caused by the disturbance belongs to the set.

$$\Psi = \left\{ \sum_{i=0}^{m} q_i(t)\sin(\omega_i t + \varphi_i) \right\}, \quad i = 0, 1, \cdots, m, \; m < \infty, \tag{4}$$

where ω_i and φ_i are constants, and $q_i(t)$ denotes any polynomials in time t.

Therefore, EID $d_e(t)$ on the control input channel and $\Delta\mathcal{A}(t)x(t)+\mathcal{B}_d d(t)$ have the same impact on the output based on the EID concept [19].

Hence, the uncertain singular system (1) can be re-description as

$$\begin{cases} E\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{B}d_e(t), \\ y(t) = \mathcal{C}x(t). \end{cases} \tag{5}$$

The state-space equation of the singular system observer is given, which is applied to estimate the EID and reconstructed the state of the plant.

$$\begin{cases} E\dot{\hat{x}}(t) = \mathcal{A}\hat{x}(t) + \mathcal{B}u_f(t) + \mathcal{L}[y(t) - \hat{y}(t)], \\ \hat{y}(t) = \mathcal{C}\hat{x}(t), \end{cases} \tag{6}$$

where $\hat{x}(t)$ is the reconstructed state of $x(t)$, $\mathcal{L}$ is gain of the observer,$u_f(t)$ is control input of the observer.

As explained in [19], the disturbance estimation is given as

$$\hat{d}_e(t) = \mathcal{B}^+\mathcal{L}\mathcal{C}[x(t) - \hat{x}(t)] + u_f(t) - u(t), \tag{7}$$

where $\mathcal{B}^+$ is a pseudo inverse of $\mathcal{B}$.

An estimate of a disturbance does not really need to be exactly the same as the disturbance. Since a plant is usually a

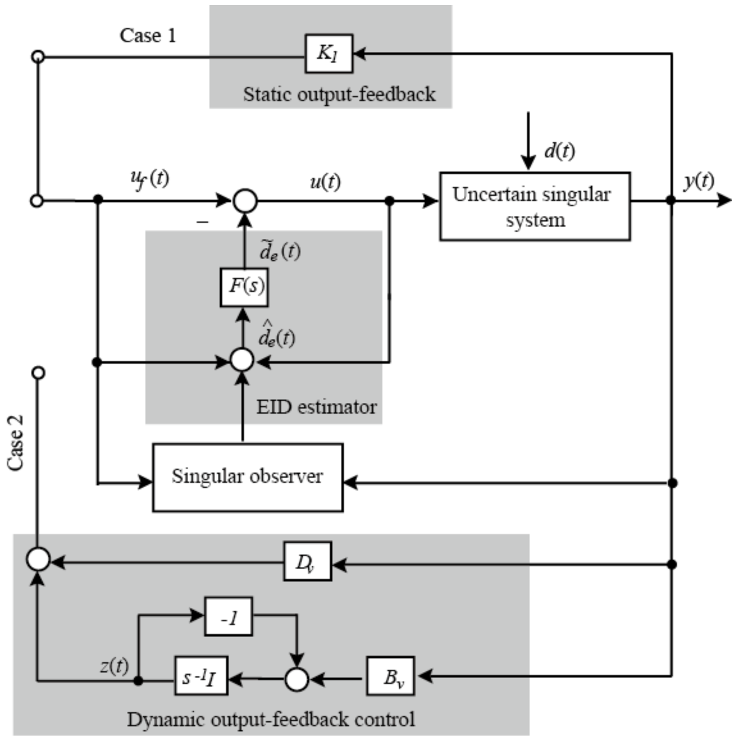

FIGURE 1. Configuration of EID-based uncertain singular control system using output-feedback control.

low-pass one, a disturbance at very high frequency does not influence the output of the plant very much. So, we only need to estimate the disturbance in a given low-frequency band to guarantee the disturbance-rejection performance in control engineering practice. To ensure this, we used a low-pass filter, $F(s)$, to select the frequency band for the disturbance estimation in this study.

$$F(s): \begin{cases} \dot{x}_F(t) = \mathcal{A}_{\mathcal{F}} x_F(t) + \mathcal{B}_{\mathcal{F}} \hat{d}_e(t), \\ \tilde{d}_e(t) = \mathcal{C}_{\mathcal{F}} x_F(t). \end{cases} \tag{8}$$

It meets

$$F(j\omega) \approx I, \forall \omega \in [0, \omega_r], \tag{9}$$

where ω_r for disturbance estimation is the highest angular frequency.

The relationship between the order of the filter and the disturbance rejection performance have been studied in [19]. And from the [19] show that a first-order low-pass filter is the best choice to filter the noise out of the estimate.

So, according to the above first-order low-pass filter, the filtered disturbance $\tilde{d}_e(t)$ is obtained by the following equation

$$\tilde{D}_e(s) = F(s)\hat{D}_e(s), \tag{10}$$

where $\hat{D}_e(s)$ and $\tilde{D}_e(s)$ are the Laplace transforms of $\hat{d}_e(t)$ and $\tilde{d}_e(t)$, respectively.

So, the control law of the uncertain singular control system (Fig. 1) is

$$u(t) = u_f(t) - \tilde{d}_e(t). \tag{11}$$

III. ADMISSIBILITY ANALYSIS OF THE SYSTEM AND DESIGN OF CONTROLLER AND OBSERVER

Case 1 (static output-feedback control).

Static output-feedback control is considered first in this paper, the controller $u_f(t)$ is designed as follows

$$u_f(t) = K_1 y(t) = K_1 \mathcal{C} x(t). \tag{12}$$

Lemma 1 ([41]): For a known matrix

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{12}^T & \Upsilon_{22} \end{bmatrix}, \tag{13}$$

the next three conditions are equivalent:

(a) $\Upsilon < 0$;
(b) $\Upsilon_{11} < 0$ and $\Upsilon_{22} - \Upsilon_{12}^T \Upsilon_{11}^{-1} \Upsilon_{12} < 0$; and
(c) $\Upsilon_{22} < 0$ and $\Upsilon_{11} - \Upsilon_{12} \Upsilon_{22}^{-1} \Upsilon_{12}^T < 0$.

Suppose that the SVD of a matrix Λ is

$$\Lambda = \bar{M} \left[\bar{P} \; 0 \right] \bar{N}^T, \tag{14}$$

where $\bar{M}$ and $\bar{N}$ are unitary matrices, $\bar{P}$ is a positive definite matrix.

Lemma 2 ([42]): X can be decomposed as

$$X = \bar{N} \begin{bmatrix} \bar{X}_{11} & 0 \\ 0 & \bar{X}_{22} \end{bmatrix} \bar{N}^T \tag{15}$$

if and only if there exists a matrix $\bar{X} \in \mathbb{R}^{p\times p}$ such that $\Lambda X = \bar{X}\Lambda$ holds for a given matrix $\Lambda \in \mathbb{R}^{p\times n}$ with $rank(\Lambda) = p$ and any $X \in \mathbb{R}^{n\times n}$. Where $\bar{X}_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$, $\bar{X}_{11} \in \mathbb{R}^{p\times p}$, and $\bar{N} \in \mathbb{R}^{n\times n}$ is a unitary matrix.

Lemma 3 ([43], [44]): There exist matrix M and positive-definite matrix N such that

$$\mathcal{A}(EN + M\Phi^T)^T + (EN + M\Phi^T)\mathcal{A}^T < 0 \tag{16}$$

if and only if system $E\dot{x}(t) = \mathcal{A}x(t)$ is called admissible. Where $\Phi \in \mathbb{R}^{n\times(n-r)}$ satisfies $E\Phi = 0$, which is any matrix with full column rank.

Next, in order to analyze the admissibility of the closed-loop control system, the external disturbance $d(t)$ is set to zero.

It is easy to find that the control system contains three states: $x_F(t)$, $\hat{x}(t)$, and $x(t)$ from Fig. 1, we define

$$\psi(t) = \begin{bmatrix} x^T(t) & \hat{x}^T(t) & x_F^T(t) \end{bmatrix}^T \tag{17}$$

and have

$$\begin{cases} E\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u_f(t) - \mathcal{B}\mathcal{C}_{\mathcal{F}}x_F(t) + \Delta\mathcal{A}(t)x(t), \\ E\dot{\hat{x}}(t) = \mathcal{L}\mathcal{C}x(t) + (\mathcal{A} - \mathcal{L}\mathcal{C})\hat{x}(t) + \mathcal{B}u_f(t), \\ \dot{x}_F(t) = (\mathcal{A}_{\mathcal{F}} + \mathcal{B}_{\mathcal{F}}\mathcal{C}_{\mathcal{F}})x_F(t) + \mathcal{B}_{\mathcal{F}}\mathcal{B}^+\mathcal{L}\mathcal{C}[x(t) - \hat{x}(t)]. \end{cases} \tag{18}$$

Hence, in Fig. 1, the state-space equation of the closed-loop system is

$$E_1\dot{\psi}(t) = [\mathcal{A}_1 + \Delta\mathcal{A}_1(t)]\psi(t) + \mathcal{B}_1u_f(t), \tag{19}$$

where

$$E_1 = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Delta\mathcal{A}_1(t) = \begin{bmatrix} \Delta\mathcal{A}(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{A}_1 = \begin{bmatrix} \mathcal{A} & 0 & -\mathcal{B}\mathcal{C}_{\mathcal{F}} \\ \mathcal{L}\mathcal{C} & \mathcal{A} - \mathcal{L}\mathcal{C} & 0 \\ \mathcal{B}_{\mathcal{F}}\mathcal{B}^+\mathcal{L}\mathcal{C} & -\mathcal{B}_{\mathcal{F}}\mathcal{B}^+\mathcal{L}\mathcal{C} & \mathcal{A}_{\mathcal{F}} + \mathcal{B}_{\mathcal{F}}\mathcal{C}_{\mathcal{F}} \end{bmatrix},$$

$$\mathcal{B}_1 = \begin{bmatrix} \mathcal{B} \\ \mathcal{B} \\ 0 \end{bmatrix}.$$

The static output-feedback control law can be expressed as

$$u_f(t) = K\psi(t), \tag{20}$$

where

$$K = \begin{bmatrix} K_1\mathcal{C} & 0 & 0 \end{bmatrix}. \tag{21}$$

In addition, according to the (22), we assume that the SVD of the matrix $\mathcal{C}$ is

$$\mathcal{C} = \mathcal{U}\begin{bmatrix} \mathcal{S} & 0 \end{bmatrix}\mathcal{V}^T, \tag{22}$$

where $\mathcal{V}$ and $\mathcal{U}$ are unitary matrices. $\mathcal{S} > 0$.

Let $\mathcal{V}$ be

$$\mathcal{V} = \begin{bmatrix} \mathcal{V}_1 & \mathcal{V}_2 \end{bmatrix}. \tag{23}$$

Then, admissibility condition of the system (19) is given in the following theorem.

Theorem 1: Control system (19) is admissible, when and only when there exist matrices Y_1, Y_2, Y_3, positive-definite matrices X_1, X_2, X_3, appropriate matrices W_1, W_2, and a scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ \star & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ \star & \star & \Pi_{33} & \Pi_{34} \\ \star & \star & \star & -\varepsilon I \end{bmatrix} < 0, \tag{24}$$

where

$$\begin{aligned} \Pi_{11} &= \mathcal{A}X_1E^T + \mathcal{A}\Phi_1Y_1^T + \mathcal{B}W_2\mathcal{C}E^T \\ &\quad + (\mathcal{A}X_1E + \mathcal{A}\Phi_1Y_1^T + \mathcal{B}W_2\mathcal{C}E^T)^T + \varepsilon\mathcal{D}\mathcal{D}^T, \\ \Pi_{12} &= \mathcal{A}\Phi_1Y_2^T + (W_1\mathcal{C}E^T + \mathcal{A}\Phi_2Y_1^T)^T + (\mathcal{B}W_2\mathcal{C}E^T)^T, \\ \Pi_{13} &= \mathcal{A}\Phi_1Y_3^T - \mathcal{B}\mathcal{C}_{\mathcal{F}}X_2 + (\mathcal{B}_{\mathcal{F}}\mathcal{B}^+W_1\mathcal{C}E^T)^T, \\ \Pi_{14} &= EX_1\mathcal{N}^T + Y_1\Phi_1^T\mathcal{N}^T, \\ \Pi_{22} &= \mathcal{A}\Phi_2Y_2^T - W_1\mathcal{C}E^T + \mathcal{A}X_1\mathcal{E}^T + \mathcal{B}W_2\mathcal{C}E^T \\ &\quad + (\mathcal{A}\Phi_2Y_2^T - W_1\mathcal{C}E^T + \mathcal{A}X_1E^T + \mathcal{B}W_2\mathcal{C}E^T)^T, \\ \Pi_{23} &= \mathcal{A}\Phi_2Y_3^T + (-\mathcal{B}_{\mathcal{F}}\mathcal{B}^+W_1\mathcal{C}E^T)^T, \\ \Pi_{24} &= Y_2\Phi_1^T\mathcal{N}^T, \\ \Pi_{33} &= (\mathcal{A}_{\mathcal{F}} + \mathcal{B}_{\mathcal{F}}\mathcal{C}_{\mathcal{F}})X_2 + X_2(\mathcal{A}_{\mathcal{F}} + \mathcal{B}_{\mathcal{F}}\mathcal{C}_{\mathcal{F}})^T, \\ \Pi_{34} &= Y_3\Phi_1^T\mathcal{N}^T. \end{aligned}$$

Moreover, let the SVD of X_1 be

$$X_1 = \begin{bmatrix} \mathcal{V}_1 & \mathcal{V}_2 \end{bmatrix}\begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix}\begin{bmatrix} \mathcal{V}_1^T \\ \mathcal{V}_2^T \end{bmatrix}. \tag{25}$$

Then, the gains of the observer and the static output-feedback controller are

$$\mathcal{L} = W_1\mathcal{U}\mathcal{S}X_{11}^{-1}\mathcal{S}^{-1}\mathcal{U}^T, \; K_1 = W_2\mathcal{U}\mathcal{S}X_{11}^{-1}\mathcal{S}^{-1}\mathcal{U}^T. \tag{26}$$

Proof 1: Combining (19) and (20) give the closed-loop system

$$E_1\dot{\psi}(t) = [\mathcal{A}_1 + \mathcal{B}_1K + \Delta\mathcal{A}_1(t)]\psi(t). \tag{27}$$

In terms of Lemma 3, there exist matrix Y and matrix $X > 0$ such that

$$\begin{aligned} &[\mathcal{A}_1 + \Delta\mathcal{A}_1(t) + \mathcal{B}_1K](E_1X + \mathcal{Y}\Phi^T)^T \\ &\quad + (E_1X + Y\Phi^T)[\mathcal{A}_1 + \Delta\mathcal{A}_1(t) + \mathcal{B}_1K]^T < 0, \end{aligned} \tag{28}$$

if and only if the system (27) is admissible. Where $\Phi \in \mathbb{R}^{(2n+1)\times(2n-2r)}$ satisfies $E_1\Phi = 0$ and Φ is any matrix with full column rank.

Because

$$\begin{aligned} \Delta\mathcal{A}_1(t) &= \mathcal{D}_1F(t)\mathcal{N}_1, && (29) \\ \mathcal{D}_1 &= \begin{bmatrix} \mathcal{D}^T & 0 & 0 \end{bmatrix}^T, && (30) \\ \mathcal{N}_1 &= \begin{bmatrix} \mathcal{N} & 0 & 0 \end{bmatrix}, && (31) \end{aligned}$$

and $F(t)$ satisfies(3), by the lemma 2.4 in [45], we know that if and only if there exists a constant $\varepsilon > 0$, the above matrix inequality (28) is equivalent to

$$[A_1 + B_1K](E_1X + Y\Phi^T)^T$$

$$+(E_1X+Y\Phi^T)[A_1+B_1K]^T$$
$$+\varepsilon^{-1}(E_1X+Y\Phi^T)N_1^TN_1(E_1X+Y\Phi^T)^T$$
$$+\varepsilon D_1D_1^T<0. \quad (32)$$

Applying the Lemma (1) to (32), yields

$$\begin{bmatrix} \Omega & (E_1X+Y\Phi^T)\mathcal{N}_1^T \\ \star & -\varepsilon I \end{bmatrix}<0, \quad (33)$$

where

$$\Omega=[\mathcal{A}_1+\mathcal{B}_1K](E_1X+Y\Phi^T)^T$$
$$+(E_1X+Y\Phi^T)[\mathcal{A}_1+\mathcal{B}_1K]^T+\varepsilon\mathcal{D}_1\mathcal{D}_1^T. \quad (34)$$

Assume that $X=diag\{X_1,X_1,X_2\}$,

$$Y=\begin{bmatrix} Y_1 \\ Y_2 \\ Y_2 \end{bmatrix}, \quad (35)$$

and

$$\Phi=\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ 0 \end{bmatrix}, \quad (36)$$

where $X_1>0$ and $X_2>0$, Y_1, Y_2 and Y_3 are matrices with suitable dimensions, $\Phi_i\in\mathbb{R}^{n\times(2n-2r)}$, $(i=1,2)$. In this paper, we assume that $C\Phi_i=0$ and $E\Phi_i=0$.

Applying Lemma 2 to (22) and substituting (19) into (33), we have

$$\bar{X}_1=\mathcal{U}\mathcal{S}X_{11}\mathcal{S}^{-1}\mathcal{U}^T, \quad (37)$$

with

$$\mathcal{C}X_1=\bar{X}_1\mathcal{C}. \quad (38)$$

So, letting

$$W_1=\mathcal{L}\bar{X}_1, \quad K_1\bar{X}_1=W_2, \quad (39)$$

yield (24).

This completes the proof.

By the Theorem 1, the static output-feedback controller and the observer are designed based on the following algorithm.

1) Choose appropriate low-pass filter that meets (9),
2) Choose $\Phi_1$and Φ_2, that satisfies $E\Phi_i=0$ and $C\Phi_i=0$,
3) Calculate the unitary matrices $\mathcal{V},\mathcal{U}$, and matrix $\mathcal{S}$, that satisfies (22),
4) Find a feasible solution to LMI (24) and calculate $\mathcal{L}$ and $\mathcal{K}_1$ from Eq. (26).

Case 2 (dynamic output-feedback control).

When the control law is the dynamic output-feedback control, we have

$$\begin{cases} \dot{z}(t)=-z(t)+B_vy(t), \\ u_f(t)=z(t)+D_vy(t), \end{cases} \quad (40)$$

where $z(t)\in\mathbb{R}^l$.

So, based on the Eq. (40), the closed-loop control system is

$$E_2\dot{\psi}_1(t)=[\mathcal{A}_2+\Delta\mathcal{A}_2(t)]\psi_1(t), \quad (41)$$

where

$$\psi_1(t)=\begin{bmatrix} \psi(t) \\ z(t) \end{bmatrix},$$
$$E_2=\begin{bmatrix} E_1 & 0 \\ 0 & I \end{bmatrix},$$
$$\mathcal{A}_2=\begin{bmatrix} \mathcal{A}_1+\mathcal{B}_1D_V\bar{\mathcal{C}} & \mathcal{B}_1 \\ B_v\bar{\mathcal{C}} & -I \end{bmatrix},$$
$$\Delta\mathcal{A}_2(t)=\begin{bmatrix} \Delta\mathcal{A}_1(t) & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{\mathcal{C}}=\begin{bmatrix} \mathcal{C} & 0 & 0 \end{bmatrix}.$$

We have

$$\Delta\mathcal{A}_2(t)=\mathcal{D}_2F(t)\mathcal{N}_2, \quad (42)$$
$$\mathcal{D}_2=\begin{bmatrix} \mathcal{D}_1^T & 0 \end{bmatrix}^T, \quad (43)$$
$$\mathcal{N}_2=\begin{bmatrix} \mathcal{N}_1 & 0 \end{bmatrix}. \quad (44)$$

Then, we give the admissible condition for the system (41) in the following theorem.

Theorem 2: For control system (41), if and only if there exist matrices Y_{11}, Y_{12}, Y_{13}, $\bar{Y}_2$, positive-definite matrices P_2, $\bar{X}_2$, a scalar $\xi>0$, $\bar{X}_1$, and matrices W_1, W_2, and W_3 such that

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\ \star & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ \star & \star & \Pi_{33} & 0 & \Pi_{35} \\ \star & \star & \star & \Pi_{44} & \Pi_{45} \\ \star & \star & \star & \star & -\xi I \end{bmatrix}<0, \quad (45)$$

where

$$\Pi_{11}=\mathcal{A}\bar{X}_1E^T+\mathcal{A}\Phi_{11}Y_{11}^T+\mathcal{B}W_2\mathcal{C}E^T$$
$$+(\mathcal{A}\bar{X}_1E+\mathcal{A}\Phi_1Y_{11}^T+\mathcal{B}W_2\mathcal{C}E^T)^T+\xi\mathcal{D}\mathcal{D}^T,$$
$$\Pi_{12}=\mathcal{A}\Phi_{11}Y_{12}^T+(W_1\mathcal{C}E^T+\mathcal{A}\Phi_{12}Y_{11}^T)^T+(\mathcal{B}W_2\mathcal{C}E^T)^T,$$
$$\Pi_{13}=\mathcal{A}\Phi_{11}Y_{13}^T-\mathcal{B}\mathcal{C}_{\mathcal{F}}\bar{X}_2+(\mathcal{B}_{\mathcal{F}}\mathcal{B}^+W_1\mathcal{C}E^T)^T,$$
$$\Pi_{13}=\mathcal{A}\Phi_{11}Y_{13}^T-\mathcal{B}\mathcal{C}_{\mathcal{F}}\bar{X}_2+(\mathcal{B}_{\mathcal{F}}\mathcal{B}^+W_1\mathcal{C}E^T)^T,$$
$$\Pi_{14}=\mathcal{A}\Phi_{11}\bar{Y}_2^T+\mathcal{B}P_2+(W_3\mathcal{C}E^T)^T,$$
$$\Pi_{15}=E\bar{X}_1\mathcal{N}^T+Y_{11}\Phi_{11}^T\mathcal{N}^T,$$
$$\Pi_{22}=\mathcal{A}\Phi_{12}Y_{12}^T-W_1\mathcal{C}E^T+\mathcal{A}\bar{X}_1E^T+\mathcal{B}W_2\mathcal{C}E^T$$
$$+(\mathcal{A}\Phi_{12}Y_{12}^T-W_1\mathcal{C}E^T+\mathcal{A}\bar{X}_1E^T+\mathcal{B}W_2\mathcal{C}E^T)^T,$$
$$\Pi_{23}=\mathcal{A}\Phi_{12}Y_{13}^T+(-\mathcal{B}_{\mathcal{F}}\mathcal{B}^+W_1\mathcal{C}E^T)^T,$$
$$\Pi_{24}=\mathcal{A}\Phi_{12}\bar{Y}_2^T+\mathcal{B}P_2,$$
$$\Pi_{25}=Y_{12}\Phi_{11}^T\mathcal{N}^T,$$
$$\Pi_{33}=(\mathcal{A}_{\mathcal{F}}+\mathcal{B}_{\mathcal{F}}\mathcal{C}_{\mathcal{F}})\bar{X}_2+\bar{X}_2(\mathcal{A}_{\mathcal{F}}+\mathcal{B}_{\mathcal{F}}\mathcal{C}_{\mathcal{F}})^T,$$
$$\Pi_{35}=Y_{13}\Phi_{11}^T\mathcal{N}^T,$$
$$\Pi_{44}=-P_2-P_2^T,$$
$$\Pi_{45}=\bar{Y}_2\Phi_{11}^T\mathcal{N}^T,$$

the control system (41) is admissible.

Furthermore, let the SVD of $\bar{X}_1$ be

$$\bar{X}_1 = \begin{bmatrix} V_1 & V_2 \end{bmatrix} \begin{bmatrix} \tilde{X}_{11} & 0 \\ 0 & \tilde{X}_{22} \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}. \tag{46}$$

Then, the observer and the dynamic output-feedback controller are

$$\mathcal{L} = W_1 \mathcal{U} \mathcal{S} \tilde{X}_{11}^{-1} \mathcal{S}^{-1} \mathcal{U}^T,\ D_v = W_2 \mathcal{U} \mathcal{S} \tilde{X}_{11}^{-1} \mathcal{S}^{-1} \mathcal{U}^T,$$
$$B_v = W_3 \mathcal{U} \mathcal{S} \tilde{X}_{11}^{-1} \mathcal{S}^{-1} \mathcal{U}^T. \tag{47}$$

Proof 2: According to the Lemma 3, there exist matrices $\bar{X}$ and $\bar{Y}$ satisfying

$$[\mathcal{A}_2 + \Delta\mathcal{A}_2(t)](E_2\bar{X} + \bar{Y}\bar{\Phi}^T)^T + (E_2\bar{X} + \bar{Y}\bar{\Phi}^T)[\mathcal{A}_2 + \Delta\mathcal{A}_2(t)^T] < 0 \tag{48}$$

if and only if system (41) is admissible. Where full column rank matrix $\bar{\Phi} \in \mathbb{R}^{(2n+1+l)\times(2n-2r)}$ satisfies $E_2\bar{\Phi} = 0$.

Applying Lemma 2.4 in [45], yields

$$A_2(E_2\bar{X} + \bar{Y}\bar{\Phi}^T)^T + (E_2\bar{X} + \bar{Y}\bar{\Phi}^T)A_2^T + \xi D_2 D_2^T + \xi^{-1}(E_2\bar{X} + \bar{Y}\bar{\Phi}^T)N_2^T N_2 (E_2\bar{X} + \bar{Y}\bar{\Phi}^T)^T < 0. \tag{49}$$

By the Lemma (1) and (49), we get

$$\begin{bmatrix} \Xi & (E_2\bar{X} + \bar{Y}\bar{\Phi}^T)\mathcal{N}_2^T \\ \star & -\xi I \end{bmatrix} < 0, \tag{50}$$

where

$$\Xi = \mathcal{A}_2(E_2\bar{X} + \bar{Y}\bar{\Phi}^T)^T + (E_2\bar{X} + \bar{Y}\bar{\Phi}^T)\mathcal{A}_2^T + \xi\mathcal{D}_2\mathcal{D}_2^T \tag{51}$$

Assume that $\bar{X} = diag\{P_1, P_2\}$,

$$\bar{Y} = \begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \end{bmatrix}, \tag{52}$$

$P_1 = diag\{\bar{X}_1, \bar{X}_1, \bar{X}_2\}$,

$$\bar{Y}_1 = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{13} \end{bmatrix}, \tag{53}$$

$$\bar{\Phi} = \begin{bmatrix} \bar{\Phi}_1 \\ 0 \end{bmatrix}, \tag{54}$$

where $\bar{Y}_1$, $\bar{Y}_2$ are suitable dimensions matrices, P_1 and P_2 are undetermined positive definite matrices, and $\bar{\Phi}_1 \in \mathbb{R}^{(2n+1)\times(2n-2r)}$ meet

$$\bar{\Phi}_1 = \begin{bmatrix} \Phi_{11} \\ \Phi_{12} \\ 0 \end{bmatrix}, \tag{55}$$

$\Phi_{1i} \in \mathbb{R}^{n\times(2n-2r)}$, $(i = 1, 2)$.

Then, we assume $E\Phi_{1i} = 0$ and $\mathcal{C}\Phi_{1i} = 0$.

By using Lemma 2 to (22), substituting (19), (41) into (50), yields

$$\tilde{X}_1 = \mathcal{U}\mathcal{S}\tilde{X}_{11}\mathcal{S}^{-1}\mathcal{U}^T, \tag{56}$$

with

$$\mathcal{C}\bar{X}_1 = \tilde{X}_1\mathcal{C}. \tag{57}$$

So, letting

$$\mathcal{L}\tilde{X}_1 = W_1,\ D_v\tilde{X}_1 = W_2,\ B_v\tilde{X}_1 = W_3. \tag{58}$$

yield (45)

This completes the proof.

By the Theorem 2, the dynamic output-feedback controller and the observer are designed based on the following algorithm.

1) Choose appropriate low-pass filter that meets (9),
2) Choose a $\Phi_{11}$and Φ_{12}, that satisfies $E\Phi_{1i} = 0$ and $\mathcal{C}\Phi_{1i} = 0$,
3) Calculate the unitary matrices $\mathcal{V}, \mathcal{U}$, and matrix $\mathcal{S}$, that satisfies (22),
4) Find a feasible solution to linear matrix inequality (LMI) (45) and calculate $\mathcal{L}$, D_v, and B_v from Eq. (47).

Remark 1: Theorem 1 and Theorem 2 give two necessary and sufficient conditions to guarantee the closed-loop control system to be admissible. The conditions are used to obtain the parameters of the output-feedback controllers and observers. Even though the LMI in Theorems 1 and 2 are large, it can easily be handled using the LMI toolbox in MATLAB. So, the matrix Y_1 and other parameters in Theorem 1 and Theorem 2 can easily be derived.

IV. NUMERICAL SIMULATION

In order to prove the the validity of proposed method, a numerical example is given as follows. For plant(1), we set the parameters as follows

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \mathcal{A} = \begin{bmatrix} 2 & 0 \\ 4 & -5 \end{bmatrix},\ \mathcal{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathcal{B} = \begin{bmatrix} 10 \\ 1 \end{bmatrix},\ \mathcal{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathcal{B}_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$\mathcal{N} = \begin{bmatrix} 0.5 & 0.4 \\ 0.5 & 0.4 \end{bmatrix}, F(t) = \begin{bmatrix} \sin(0.5\pi t) & 0 \\ 0 & \sin(0.5\pi t) \end{bmatrix}.$$

we choose the firs-order low-pass filter

$$F(s) = \frac{100}{s + 101}. \tag{59}$$

So, its state-space form was selected as follows

$$\mathcal{A}_{\mathcal{F}} = -101,\ \mathcal{B}_{\mathcal{F}} = 100,\ \mathcal{C}_{\mathcal{F}} = 1. \tag{60}$$

A step signal is introduced as the disturbance

$$d(t) = 0.5 \times 1(t). \tag{61}$$

Case 1 : First, for the case 1 where the static-output-feedback control, letting

$$\Phi_1 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{62}$$

and

$$\Phi_2 = \begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix}. \tag{63}$$

The scalar ε was get by applying Theorem 1

$$\varepsilon = 0.6473, \tag{64}$$

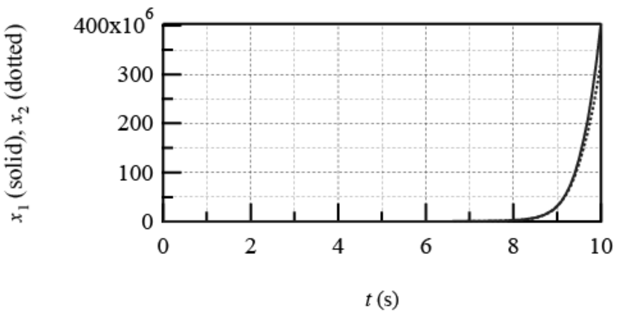

FIGURE 2. The state trajectories when $u = 0$ for system (1).

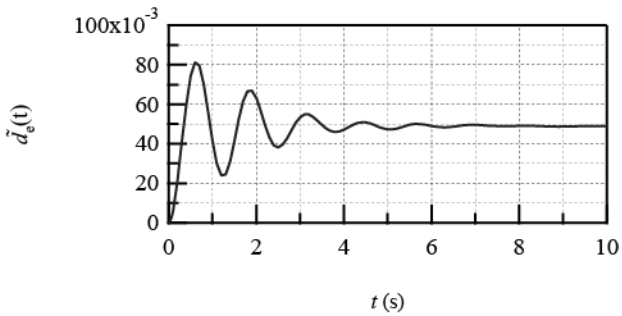

FIGURE 5. EID estimate, $\tilde{d}_e(t)$).

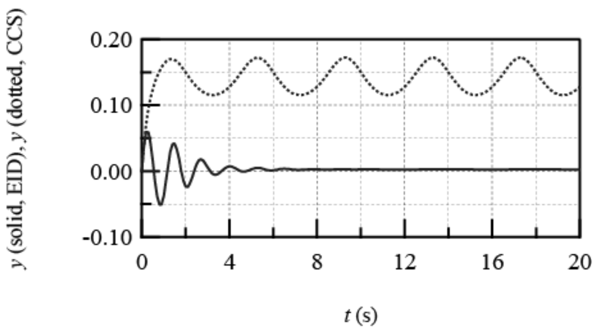

FIGURE 3. Output response for CCS and EID-based uncertain singular control system for case 1.).

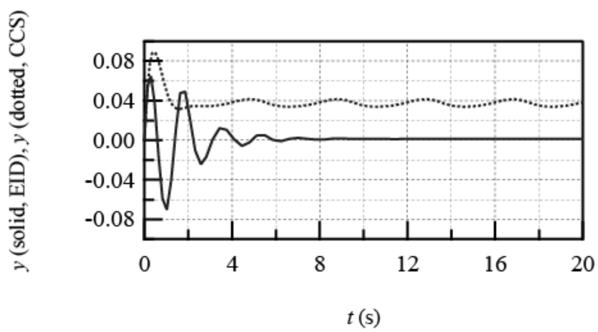

FIGURE 6. Output response for CCS, and EID-based uncertain singular control system for case 2.

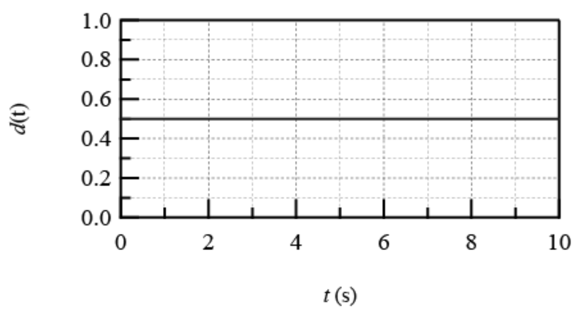

FIGURE 4. Disturbance, $d(t)$).

the gain of state observer was get by applying Theorem 1

$$\mathcal{L} = \begin{bmatrix} 2.4841 & -22.3580 \end{bmatrix}^T. \tag{65}$$

and the static output-feedback control law was

$$K_1 = -0.5657. \tag{66}$$

It is easy to verify the uncertain singular system are regular and impulse free. when $u = 0$, the state trajectories were shown in Fig2. We find that the open loop system is unstable.

The Fig.4 and Fig.5 gives waveforms of the disturbance $d(t)$ and the EID estimate. For the EID-based control method (Fig. 3), the simulation result indicate that the system was stable and the largest steady state error of the output was about 0.002.

For comparison, simulation was also carried out for the conventional control system (CCS) method. The CCS does not have EID estimator, compared with the our method. The largest steady state error of the output for CCS method was about 0.17. This shows the validity of our method.

Case 2 : Now consider the case 2, letting

$$\Phi_{11} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{67}$$

and

$$\Phi_{12} = \begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix}. \tag{68}$$

For simplicity, we consider $l = 1$, and applying Theorem 2 yielded

$$\xi = 402.0155, \tag{69}$$

$$B_v = -0.9973, \ D_v = -0.5528, \tag{70}$$

and

$$\mathcal{L} = \begin{bmatrix} 2.4586 & -23.2985 \end{bmatrix}^T. \tag{71}$$

The Fig. 6 implies that the largest steady state error of the output was about 0.001 and the largest steady state error of the output for CCS method was about 0.04. This shows the validity and feasibility of the presented method.

Due to the complexity of singular systems, very few studies have appeared on the problem of disturbance rejection for those systems. To better show the validity of our method, a comparison with conventional methods revealed the following points.

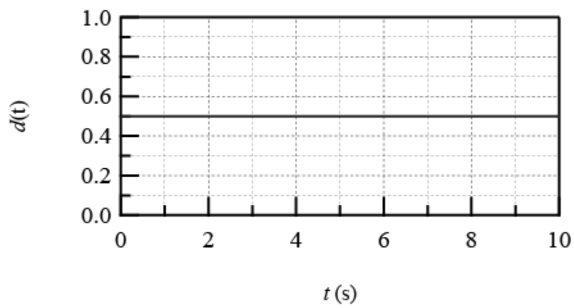

FIGURE 7. Disturbance, $d(t)$).

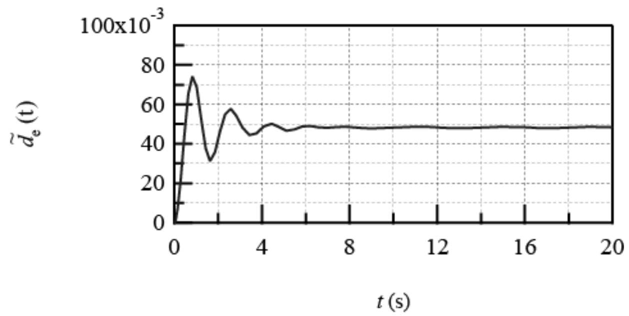

FIGURE 8. EID estimate, $\tilde{d}_e(t)$).

1) SMC-based robust control method [33], [34] required the system states are available. However, some information of the state variables is unavailable in most engineering practice. So, in this paper, we use output-feedback control technique to consider disturbance rejection problem.
2) Neural network approach and SMC method [35], [36] require that the exogenous disturbance is bounded with a known upper bound. However, EID-based method does not require a prior information about exogenous disturbances.
3) An observer-based H_∞ control SMC method was proposed for singular systems in [37]. Although people are known to H_∞ control method can obtain satisfactory control performance under the circumstance of worst disturbance, it is very conservative. At the same time, this may cause chattering by using SMC control method. In contrast, EID method is used to estimate the harmful effect of disturbance on the control system, which improves system performance and reduces the conservatism of the system. Moreover, it can not cause this chattering problem.

V. CONCLUSION

In this paper, a robust disturbance-rejection problem is considered by EID method based on two different feedback control. A disturbance-rejection design method was presented using EID method. Two necessary and sufficient conditions are given to guarantee the closed-loop control system to be admissible. And static output-feedback controller and the dynamic output-feedback controller are derived in terms of the LMI and SVD method. It has been shown via simulation studies that the proposed method is feasible and effective. And it can achieves satisfactory disturbance-rejection performance.In addition, research is still ongoing, the disturbance rejection for nonlinear singular system based on the EID method will be carried out in the future.

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