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Impact Angle Constrained Distributed Cooperative Guidance Against Maneuvering Targets With Undirected Communication Topologies

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ABSTRACT The problem of impact angle constrained distributed cooperative guidance against maneuvering targets with undirected communication topologies is studied. A novel distributed cooperative guidance strategy is proposed to realize simultaneous attack with impact angle constraint. Firstly, the kinematic equations for the engagement are established. According to the engagement equations, the cooperative guidance problem is divided into two units, which include the cooperation of both impact time and impact angle. Secondly, the guidance algorithm is designed separately for simultaneous attack and dealing with the impact angle constraints. The problem is transformed into a consensus problem via disturbance compensation and feedback linearization technique. The convergence of the guidance algorithm is proven by using the consensus theory. Finally, numerical simulations are presented to verify the effectiveness of the proposed cooperative guidance law.

INDEX TERMS Maneuvering targets, distributed cooperative guidance law, extended state observer, multiple unmanned aerial vehicles.

I. INTRODUCTION

Cooperative guidance of multiple unmanned aerial vehicles (multi-UAVs) has been an attractive research area recently. Part of the reasons is that a group of low-costs UAVs usually provide better performance than a single expensive one. In order to penetrate the target defense effectively, a strategy of multi-UAVs simultaneous attack has been put forward, which is one of the most effective ways to attack the target with strong defensive capability.

In previous work, the multi-aircraft cooperative attack is mainly divided into two ways. One is individual homing [1]–[3], which can attack the target simultaneously by setting a unified impact time. Although individual homing can reach simultaneously attack, the problem of this method is that it is difficult to establish the attack time reasonably when the characteristics of the aircraft are inconsistent. If the attack time is unreasonable, then the cooperative attack will be difficult to achieve. The second way is cooperative guidance, in which UAVs communicate with neighbors to attack targets at a specific time without setting a fixed impact time. Cooperative guidance includes centralized cooperative guidance [4]–[9] and distributed cooperative guidance [5]–[34], of which centralized cooperative guidance needs real-time updating of global information, which will increase the difficulty of implementation and reduce the robustness of the system.

Global information is not needed in distributed cooperative guidance, so this method is more natural to implement in engineering. Wang *et al.* [10] proposed a two-stage control strategy. In the first step, a distributed time cooperative guidance law is designed by the second-order consensus theory. In the second stage, proportional guidance law is applied to realize the simultaneous attack. Zhao and Zhou [11] proposed a cooperative guidance law which is based on nonlinear model predictive control techniques and

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optimal control theory, in which the amount of calculation is reduced by updating the guidance command only in a specific time. In [12], the cooperative guidance model of multiple UAVs was obtained by taking derivation of the expression of the remaining time. Based on the guidance model and the first-order consensus theory, a cooperative guidance law was proposed. Wang and Tan [15] designed a cooperative guidance law, which can make the remaining flight time of multiple UAVs converge to consensus in a fixed-time. Zhang *et al.* [16] presented a novel distributed cooperative guidance strategy based on biased proportional navigation guidance. The guidance strategy can handle the field-of-view constraint and achieve simultaneous attacks under fixed or switching communication networks. In [17], a novel fault-tolerant cooperative guidance1law was presented, which is designed for disposing and uncertainty. The simultaneous arrival can be achieved in fixed-time under actuation failures. In [18], an integrated guidance and control algorithm was proposed by using the dynamic surface control theory. The effectiveness of the protocol is proved by comparison principle and Lyapunov stability theory, under the condition of unknown uncertainties and input saturation. In [19], [20], a two-stage way was adopted to design the cooperative guidance algorithm. The first step is based on the second-order consensus theory. In the second stage, proportional guidance method is adopted, which would not change the remaining flight time and make sure that different UAVs hit the target at the same time. In [21], two closed-loop cooperative guidance laws are proposed. The simultaneous attack under multiple constraints is achieved based on receding horizon control (RHC) strategy and convex optimization technique.

Most of the previous works focused on cooperative guidance against fixed targets. Research results on cooperative guidance for maneuvering targets were rare [26]–[34]. The target's acceleration is needed to be known in [26]–[31], which is difficult to be obtained accurately in engineering practice. An evading target was considered in [32], but the proposed cooperative guidance laws were not distributed. In [33], the problem of simultaneous attack against a maneuvering target is investigated, but the impact angle control part of [33] is in an individual way. In [34], a distributed cooperative guidance law is designed to realize simultaneous attack. But this guidance law cannot achieve cooperation on impact angle.

In this paper, a novel cooperative guidance algorithm is proposed against a maneuvering target. The algorithm consists of an extended state observer (ESO) and a cooperative guidance law. The cooperative guidance law is designed based on feedback linearization and disturbance compensation technic.

The advantages of our research can be summarized as follows: (1) The cooperative guidance law proposed in this paper can apply to a maneuvering target. The cooperative guidance problem against maneuvering targets is more complicated than stationary targets. (2) In our paper, a cooperative guidance law is designed without using the target's acceleration information. In [32]–[37] the target's acceleration needed to be measured, which is probably unavailable in engineering practice. (3) The cooperative guidance law we proposed can not only achieve simultaneously attack but also the coordination between impact angles. At the same time, the strategy proposed in this paper is distributed and does not need to set a specific attack angle, which will help to reduce energy consumption.

II. PRELIMINARIES

In the section, basic knowledge on graph theory is introduced, and an introduction to the ESO theory is presented.

Consider N UAVs participating in a cooperative attack. An interaction digraph can describe the information exchange among multiple UAVs. Let $\boldsymbol{G} = \{M, E, \boldsymbol{W}\}$ be an undirected graph, where $M = \{m_1, m_2, \ldots, m_N\}$ denotes the multi-UAVs. $\boldsymbol{E}$ denotes the edge set among multi-UAVs. $\boldsymbol{G} = [\omega_{ij}] \in \boldsymbol{R}^{N\times N}$ represents the weighted adjacency matrix. An edge in E is indicated as $e_{ij} = (m_i, m_j), (i \neq j)$. The adjacency element ω_{ij} in $\boldsymbol{G}$ satisfies $\omega_{ij} > 0$ if and only if information flows from m_j to m_i. In an undirected graph m_i and m_j can exchange information from each other $(i, e., \omega_{ij} = \omega_{ji} > 0)$.

Lemma 1 [35]: Consider the following high-order nonlinear system:

$$\begin{cases} \dot{x}_k(t) = x_{k+1}(t), & k = 1, 2, \ldots, n-1, \\ \dot{x}_n(t) = u(t) + f(x(t), t) \\ y(t) = x_1(t) \end{cases} \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \boldsymbol{R}^n$ are the states of (1), $u(t) \in \boldsymbol{R}$ is the input and $f(x(t), t)$ is the uncertainty and the disturbance. If $\mathrm{d}(f(x(t), t))/\mathrm{d}t$ is bounded, one can denote $x_{n+1}(t) = f(x(t), t)$ as an extended state. The ESO can be modeled as

$$\begin{cases} \dot{\hat{x}}_k(t) = \hat{x}_{k+1}(t) + \beta_{0k}(x_1(t) - \hat{x}_1(t)), \\ \qquad k = 1, 2, \ldots, n-1 \\ \dot{\hat{x}}_n(t) = u(t) + \hat{x}_{n+1}(t) + \beta_{0n}(x_1(t) - \hat{x}_1(t)) \\ \dot{\hat{x}}_{n+1}(t) = \beta_{0(n+1)}(x_1(t) - \hat{x}_1(t)) \\ y(t) = x_1(t) \end{cases} \tag{2}$$

where $\hat{\boldsymbol{x}}(t) = [\hat{x}_1(t), \hat{x}_2(t), \ldots, \hat{x}_{n+1}(t)]^T \in \boldsymbol{R}^{n+1}$ are the states of the observer (2). Choose $\beta = [\beta_{01}, \beta_{02}, \ldots, \beta_{0(n+1)}]^T$ to be a proper vector, then the estimation error $x_{n+1}(t) - \hat{x}_{n+1}(t)$ could be arbitrarily small.

III. PROBLEM DESCRIPTION

In this section, the problem description of the cooperative guidance is given.

A. SYSTEM MODEL

In this paper, we consider the problem that several UAVs attack a target with the arbitrary maneuver. In engineering practice, the following is a typical assumption when considering cooperative guidance problems.

Assumption 1.

(i) All of the UAVs and the target are treated as mass points.

(ii) Compared with the guidance loop, the autopilot dynamics of multiple UAVs are fast enough.

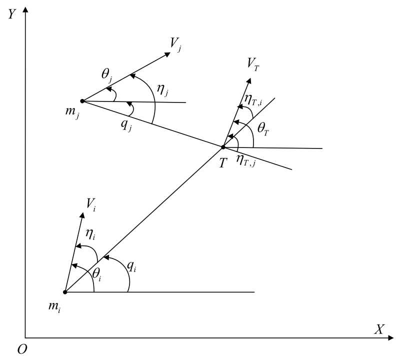

FIGURE 1. **Geometry relationship between multi-UAVs and target in cooperative guidance process.**

The homing guidance geometry is shown in Fig.1. Let $m_i(i \in \{1, 2, \ldots, N\})$ denote the ith UAV and T denote the target. The terms θ_i, η_i, q_i represent flight path angle, leading angle, and line-of-sight angle, of m_i. From Fig.1, one obtains

$$\eta_i\,(t) = q_i\,(t) - \theta_i\,(t) \tag{3}$$

Note that the target are maneuvering, the case $\eta_i(t) \in [0, -\pi]$, $(i, e., \theta_i(t) \geq q_i(t))$ can be treated as asymmetry of the one with $\eta_i(t) \in [0, \pi]$, $(i, e., \theta_i(t) \leq q_i(t))$. In this paper, the case $\eta_i\,(t) \in [0, \pi]$ is dealt with. The pursuit situation is given by

$$\begin{cases} \dot{R}_i(t) = V_T(t)\cos\eta_{Ti}(t) - V_i(t)\cos\eta_i(t) \\ R_i(t)\dot{q}_i(t) = V_i(t)\sin\eta_i(t) - V_T(t)\sin\eta_{Ti}(t) \\ \dot{\theta}_i(t) = n_{iy}(t)/V_i(t) \\ \dot{V}_i(t) = n_{ix}(t) \end{cases} \tag{4}$$

where $R_i(t)$ notes the range-to-go from m_i to the target; g represents the gravity acceleration; $V_i(t)$ is the axis velocity of m_i, $n_{ix}\,(t)$ and $n_{iy}\,(t)$ are the acceleration of the UAV in its velocity frame of m_i, adjusting the direction and magnitude of $V_i(t)$ respectively, and $V_T(t)$, $\eta_{Ti}(t)$ represent the axis velocity and line-of-sight angle of the target.

Let $V_{ri}(t) = \dot{R}_i(t)$. Then one can obtain that

$$\begin{aligned} \dot{V}_{ri}(t) &= \dot{V}_T(t)\cos\eta_{Ti}(t) - V_T(t)\sin\eta_{Ti}(t)\dot{\eta}_{Ti}(t) \\ &\quad - \dot{V}_i(t)\cos\eta_i(t) + V_i(t)\sin\eta_i(t)\dot{\eta}_i(t) \\ &= \dot{V}_T(t)\cos\eta_{Ti}(t) + V_T(t)\sin\eta_{Ti}(t)\dot{\theta}_T(t) \\ &\quad - \dot{V}_i(t)\cos\eta_i(t) - V_i(t)\sin\eta_i(t)\dot{\theta}_i(t) \\ &\quad + R_i(t)\dot{q}(t)^2 \end{aligned}$$

Denote that

$$\omega_{ri}(t) = \dot{V}_T(t)\cos\eta_{Ti}(t) + V_T(t)\sin\eta_{Ti}(t)\cdot\dot{\theta}_T(t)$$

and

$$u_{ri}(t) = \dot{V}_i(t)\cos\eta_i(t) + V_i(t)\sin\eta_i(t)\cdot\dot{\theta}_i(t)$$

then one can further obtain that

$$\dot{V}_{ri}(t) = \omega_{ri}(t) - u_{ri}(t) + R_i(t)\dot{q}(t)^2 \tag{5}$$

Based on (3) and (4), one can obtain the following equation

$$\begin{aligned} \ddot{q}_i(t) = &\frac{1}{(R_i(t))^2}\cdot(R_i(t)\cdot(\dot{V}_i(t)\sin\eta_i(t) \\ &+ V_i(t)\cos\eta_i(t)\dot{\eta}_i(t) - \dot{V}_T(t)\sin\eta_{Ti}(t) \\ &- V_T(t)\cos\eta_{Ti}(t)\dot{\eta}_{Ti}(t)) - \dot{R}_i(t)R_i(t)\dot{q}_i(t)) \end{aligned}$$

Denote that

$$\omega_{qi}(t) = \dot{V}_T(t)\sin\eta_{Ti}(t) - V_T(t)\cos\eta_{Ti}(t)\cdot\dot{\theta}_T(t)$$

and

$$u_{qi}(t) = \dot{V}_i(t)\sin\eta_i(t) - V_i(t)\cos\eta_i(t)\cdot\dot{\theta}_i(t)$$

then one can get

$$\ddot{q}_i(t) = \frac{1}{R_i(t)}(u_{qi}(t) - \omega_{qi}(t)) - 2\cdot\frac{\dot{q}_i(t)\cdot V_{ri}(t)}{R_i(t)} \tag{6}$$

B. CONTROL OBJECTIVE

Illustrative Example. Fig.1 shows an example with two UAVs take part in a cooperative attack. Denote the target as T. Suppose that the multi-UAVs system consists n UAVs, then one can denote UAVs as $m_1, m_2, \cdots, m_n$. The task of cooperative guidance is to make multi-UAVs hit the target simultaneously, and ensure that UAVs attack the target according to the preset hit angle interval.

One can know that the time-to-go of m_i to T is determined by $R_i(t)$ and $V_{ri}(t)$. The simultaneous attack is achieved if $R_i(t)$ and $V_{ri}(t)$ achieve consensus. At the same time, one gets if $\lim_{t\to\infty}\dot{q}_i(t) = 0$ then m_i can hit the target T. Also, the impact angle is determined by $q_i(t)$ if

$$\lim_{t\to\infty}((q_i(t) - c_i) - (q_j(t) - c_j)) = 0$$

then the multi-UAVs system achieves cooperation on impact angle, where c_i and c_j are default constants. Then one can get the following definition.

Definition 1: Multi-UAVs system is said to achieve cooperative attack if the following equations hold in the meantime.

$$\begin{aligned} \lim_{t\to\infty}(R_i(t) - R_j(t)) &= 0 \\ \lim_{t\to\infty}(V_{ri}(t) - V_{rj}(t)) &= 0 \\ \lim_{t\to\infty}((q_i(t) - c_i) - (q_j(t) - c_j)) &= 0 \\ \lim_{t\to\infty}\dot{q}_i(t) = \lim_{t\to\infty}\dot{q}_j(t) &= 0 \end{aligned} \tag{7}$$

Then cooperative attack problem is transformed into the consensus problem of nonlinear systems with unknown disturbances. The main purposes of the paper are (i) How to design the guidance algorithm based on the neighboring communication and (ii) under what conditions the multi-UAVs system can achieve the cooperative attack against a maneuvering target.

IV. MAIN RESULTS

In this section, the cooperative guidance problem is divided into two parts. One of them is the cooperation on impact time, and another is the cooperation on impact angle. These two problems are transformed into the consensus problem of the linear system with unknown disturbance based on feedback linearization. Then, a distributed cooperative guidance law based on disturbance observer is projected to realize the cooperative attack of multiple UAVs. Finally, the necessary and sufficient conditions for the convergence of the guidance law are obtained by pole analysis.

From Definition 1, one can know that the purpose of the guidance law is to guarantee (7) holding simultaneously. By (5) and (6) it can be verified that $R_i(t)$, $V_{ri}(t)$ and $q_i(t)$, $\dot{q}_i(t)$ can be controlled independently. So the cooperation guidance law could be designed for $R_i(t)$, $V_{ri}(t)$ and $q_i(t)$, $\dot{q}_i(t)$ separately.

A. IMPACT TIME COORDINATION PART

From Definition 1 one can know that the multi-UAVs system achieves cooperative attack if $R_i(t)$, $V_{ri}(t)$ achieve consensus. The first step is designing an ESO to estimate the unknown target's maneuvering. Let $\hat{R}_i(t)$, $\hat{V}_{ri}(t)$, $\hat{\omega}_{ri}(t)$ be the estimation of $R_i(t)$, $V_{ri}(t)$, $\omega_{ri}(t)$ respectively. Then the following observer is established

$$\begin{cases} \dot{\hat{R}}_i(t) = \hat{V}_{ri}(t) + \alpha_1(R_i(t) - \hat{R}_i(t)) \\ \dot{\hat{V}}_{ri}(t) = \hat{\omega}_{ri}(t) - u_{ri}(t) + R_i(t)\dot{q}_i(t)^2 + \alpha_2(R_i(t) - \hat{R}_i(t)) \\ \dot{\hat{\omega}}_{ri}(t) = \alpha_3(R_i(t) - \hat{R}_i(t)) \end{cases} \tag{8}$$

where $\alpha_1, \alpha_2, \alpha_3$ are parameters. Assume that $\tilde{R}_i(t)$, $\tilde{V}_{ri}(t)$, $\tilde{\omega}_{ri}(t)$ are the estimation errors. It then follows that

$$\begin{cases} \dot{\tilde{R}}_i(t) = -\alpha_1\tilde{R}_i(t) + \tilde{V}_{ri}(t) \\ \dot{\tilde{V}}_{ri}(t) = -\alpha_2\tilde{R}_i(t) + \tilde{\omega}_{ri}(t) \\ \dot{\tilde{\omega}}_{ri}(t) = -\alpha_3\tilde{R}_i(t) \end{cases} \tag{9}$$

Based on the ESO (8) one can design the cooperative guidance protocol as

$$\begin{aligned} u_{ri}(t) = {}& _i(t)\dot{q}_i(t)^2 \\ & - \sum_{j=1}^{N} a_{ij}(k_1(R_i(t) - R_j(t)) + k_2(V_{ri}(t) - V_{rj}(t))) \\ & - k_3\text{sgn}(s_i) - k_4 s_i + \hat{\omega}_{ri}(t) \end{aligned} \tag{10}$$

where

$$\begin{aligned} & s_i(t) \\ & = V_{ri}(t) - V0 \\ & \quad - \int_0^t \left\{ \sum_{j=1}^{N} a_{ij}(k_1(R_i(t) - R_j(t)) + k_2(V_{ri}(t) - V_{rj}(t))) \right\} dt \end{aligned} \tag{11}$$

Under algorithm (10) the system (5) can be transformed into

$$\begin{cases} \dot{R}_i(t) = V_{ri}(t) \\ \dot{V}_{ri}(t) = \sum_{j=1}^{N} a_{ij}(k_1(R_i(t) - R_j(t)) + k_2(V_{ri}(t) - V_{rj}(t))) \\ \qquad + \tilde{\omega}_{ri}(t) + k_3 sgn(s_i(t)) + k_4 s_i(t) \end{cases} \tag{12}$$

In the following, the stability of protocol (10) is given.

Theorem 1: Multi-UAVs system (5) achieves simultaneously attack under protocol (10) if the following conditions hold simultaneously:

$$k_1 < 0, k_2 < 0, k_3 < -\tilde{\omega}_{r_r}(t), k_4 < 0 \tag{13}$$

Proof: Taking the derivative of equation (11), one can get

$$\dot{s}_i(t) = \tilde{\omega}_{ri}(t) + k_3\text{sgn}(s_i(t)) + k_4 s_i(t)$$

Choose the Lyapunov function as

$$V_1(t) = \frac{1}{2}s_i^2(t) \tag{14}$$

Taking the derivative of equation (14), one gets

$$\begin{aligned} \dot{V}_1(t) &= s_i(t)\dot{s}_i(t) \\ &= s_i(t)\left[k_3 sgn\left(s_i(t)\right) + k_4 s_i(t) + \tilde{\omega}_{ri}(t)\right] \\ &= k_3\left|s_i(t)\right| + \tilde{\omega}_{ri}(t)s_i(t) + k_4 s_i(t)^2 \\ &\leq \left(k_3 + \tilde{\omega}_{ri}(t)\right)\left|s_i(t)\right| + k_4 s_i(t)^2 \\ &\leq \sqrt{2}\left(k_3 + \tilde{\omega}_{ri}(t)\right)V_1(t)^{1/2} + 2k_4 V_1(t) \end{aligned} \tag{15}$$

It follows

$$\lim_{t\to T_i} s_i(t) = 0 \tag{16}$$

where

$$T_i \leq -\frac{1}{k_4}\ln\frac{2k_4 V_{1i}^{1/2}(0) + \sqrt{2}\left(k_3 + \tilde{\omega}_{ri}(t)\right)}{\sqrt{2}\left(k_3 + \tilde{\omega}_{ri}(t)\right)} \tag{17}$$

Let $T_1 = \max\{T_1, T_2, \ldots, T_N\}$. Then form (15) one has

$$s_i(t) = 0, \quad \forall t > T_1 \tag{18}$$

Taking the derivative of (18), one can obtain

$$\dot{s}_i(t) = 0, \quad \forall t > T_1 \tag{19}$$

then system (12) can be transformed into

$$\begin{cases} \dot{R}_i(t) = V_{ri}(t) \\ \dot{V}_{ri}(t) = \sum_{j=1}^{N} a_{ij}\left(k_1\left(R_i(t) - R_j(t)\right) + k_2\left(V_{ri}(t) - V_{ij}(t)\right)\right) \end{cases} \tag{20}$$

Choose the Lyapunov candidate function as

$$V_2(t) = -\frac{k_1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(R_i(t) - R_j(t))^2 + \frac{1}{2}\sum_{i=1}^{N} V_{ri}(t)^2 \tag{21}$$

Taking the derivative of (21), one has

$$\begin{aligned}\dot{V}_2(t) &= -\frac{k_1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(R_i(t)-R_j(t))(V_{ri}(t)-V_{rj}(t)) \\ &\quad + \sum_{i=1}^{N} V_i(t)\sum_{j=1}^{N} a_{ij}\left(k_1\left(R_i(t)-R_j(t)\right)\right. \\ &\quad \left. + k_2\left(V_{ri}(t)-V_{rj}(t)\right)\right) \\ &= -k_1\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}V_i(t)(R_i(t)-R_j(t)) \\ &\quad + k_1\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}V_i(t)(R_i(t)-R_j(t)) \\ &\quad + k_2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}V_{ri}(t)\left(\left(V_{ri}(t)-V_{rj}(t)\right)\right) \\ &= \frac{k_2}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\left(V_{ri}(t)-V_{rj}(t)\right)\right)^2 \le 0 \end{aligned} \tag{22}$$

Then one can obtain

$$\lim_{t\to\infty}(V_{ri}(t)-V_{rj}(t)) = 0 \tag{23}$$

From (20) and (23), one has

$$\lim_{t\to\infty}\dot{V}_{ri}(t) = \sum_{j=1}^{N} a_{ij}(k_1(R_i(t)-R_j(t))) \tag{24}$$

Because the communication topology is undirected so one has $a_{ij} = a_{ji}$, thus form (20) it can be verified that

$$\sum_{i=1}^{N}\dot{V}_{ri}(t) = 0 \tag{25}$$

From (23) and (25), it holds that

$$\lim_{t\to\infty}\dot{V}_{ri}(t) = 0 \tag{26}$$

By (24) and (26), one can further obtain

$$\lim_{t\to\infty}(R_i(t) = R_j(t)) \tag{27}$$

From (23) and (27), according to Definition 1 multi-UAVs system (5) achieves simultaneously attack. It completes the proof of Theorem 1.

B. Impact angle coordination part

By Definition 1, one can obtain that the purpose of impact angle coordination is

$$\begin{aligned}&\lim_{t\to\infty}((q_i(t)-c_i)-(q_j(t)-c_j)) = 0\\ &\lim_{t\to\infty}\dot{q}_i(t) = \lim_{t\to\infty}\dot{q}_j(t) = 0\end{aligned}$$

In other words, the purpose of impact angle coordination is to make the different values of the line-of-sight angles between UAVs equal to the values of expectation. Meanwhile, the cooperative guidance law should make each UAV's line-of-sight angle rate equal to zero, which enables the UAV to hit its target.

Let $\phi_{1i}(t) = q_i(t) - c_i$, $\phi_{2i}(t) = \dot{q}_i(t)$, $u_{2i}(t) = \frac{u_{qi}(t)}{R_i(t)}$, and $d_i(t) = -\frac{\omega_{qi}(t)}{R_i(t)}$. From (6), one can obtain

$$\begin{cases}\dot{\phi}_{1i}(t) = \phi_{2i}(t)\\ \dot{\phi}_{2i}(t) = -2\cdot\dfrac{\dot{q}_i(t)\cdot V_{ri}(t)}{R_i(t)} + d_i(t) + u_{2i}(t)\end{cases} \tag{28}$$

Based on the structure of (28), one can design the ESO to estimate the disturbance $d_i(t)$.

$$\begin{cases}\dot{\hat{\phi}}_{1i}(t) = \hat{\phi}_{2i}(t) + b_1(\phi_{1i}(t)-\hat{\phi}_{1i}(t))\\ \dot{\hat{\phi}}_{2i}(t) = -2\cdot\dfrac{\dot{q}_i(t)\cdot V_{ri}(t)}{R_i(t)} + \hat{d}_i(t)\\ \qquad\qquad + u_{2i}(t) + b_2(\phi_{1i}(t)-\hat{\phi}_{1i}(t))\\ \dot{\hat{d}}_i(t) = b_3(\phi_{1i}(t)-\hat{\phi}_{1i}(t))\end{cases} \tag{29}$$

One can get the estimation errors are

$$\begin{cases}\dot{\tilde{\phi}}_{1i}(t) = -b_1\tilde{\phi}_{1i}(t) + \tilde{\phi}_{2i}(t)\\ \dot{\tilde{\phi}}_{2i}(t) = -b_2\tilde{\phi}_{1i}(t) + \tilde{d}_i(t)\\ \dot{\tilde{d}}_i(t) = -b_3\tilde{\phi}_{1i}(t)\end{cases} \tag{30}$$

From Lemma 1 and (30) it can be verified that if $b_1 > 0$, $b_3 > 0$, $b_1b_2 > b_3$ are satisfied, one can obtain that $\lim_{t\to\infty}\tilde{d}_i(t) = 0$. Then based on the disturbance observer, the guidance strategy can be designed as

$$\begin{aligned}u_{2i}(t) &= 2\cdot\frac{\dot{q}_i(t)}{r_i(t)} - \hat{d}_i(t) + n_1\phi_{2i}(t)\\ &\quad + n_2\sum_{j=1}^{N} a_{ij}(\phi_{1i}(t)-\phi_{1j}(t))\end{aligned} \tag{31}$$

where n_1, n_2 are the feedback gains.

Remark 1: Compared with the guidance law in [33], algorithm (31) is distributed and does not need to preset an impact angle for each vehicle. At the same time, compared with the guidance law in [34], algorithm (31) can not only enable the aircraft to hit the target but also attack the target at a preset impact angle interval.

In the following, the stability of the algorithm (31) is proven.

Theorem 2: Multi-UAVs system (5) achieves impact angle cooperative attack under algorithm (31) if $n_1 < -\frac{1}{2}$, $n_2 < 0$.

Proof: Under protocol (31), system (28) can be rewritten as follows

$$\begin{cases}\dot{\phi}_{1i}(t) = \phi_{2i}(t)\\ \dot{\phi}_{2i}(t) = \tilde{d}_i(t) + n_1\phi_{2i} + n_2\displaystyle\sum_{j=1}^{N} a_{ij}(\phi_{1i}(t)-\phi_{1j}(t))\end{cases} \tag{32}$$

Consider a Lyapunov candidate function

$$V_3(t) = -\frac{n_2}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\phi_{1i}(t)-\phi_{1j}(t))^2 + \frac{1}{2}\sum_{i=1}^{N}\phi_{2i}^2 \tag{33}$$

Choose $n_2 < 0$ then $V_3(t)$ is a continuous nonnegative function. Taking the derivative of $V_3(t)$ gives

$$\begin{aligned}\dot{V}_3(t) &= -\frac{n_2}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\phi_{1i}(t)-\phi_{1j}(t))(\phi_{2i}(t)-\phi_{2j}(t)) \\ &\quad+\sum_{i=1}^{N}\phi_{2i}(t)(\tilde{d}_i(t)+n_1\phi_{2i}(t) \\ &\quad+n_2\sum_{j=1}^{N} a_{ij}\phi_{2i}(t)((\phi_{1i}(t)-\phi_{1j}(t)))) \\ &= -n_2\sum_{j=1}^{N} a_{ij}\phi_{2i}(t)((\phi_{1i}(t)-\phi_{1j}(t))) \\ &\quad+n_2\sum_{j=1}^{N} a_{ij}\phi_{2i}(t)((\phi_{1i}(t)-\phi_{1j}(t))) \\ &\quad+\sum_{i=1}^{N}\phi_{2i}(t)(\tilde{d}_i(t)+n_1\phi_{2i}(t)) \\ &= n_1\sum_{i=1}^{N}\phi_{2i}(t)^2+\sum_{i=1}^{N}\phi_{2i}(t)\tilde{d}_i(t)\end{aligned} \tag{34}$$

From Lemma 1 one can get that the estimation error of ESO (30) will converge to a bounded set in a finite time. Suppose that when the guidance law is worked the ESO estimation error converges to bounded set $\left|\tilde{d}_i(t)\right| \le \varepsilon_d$, then one gets:

$$\begin{aligned}\sum_{i=1}^{N}\phi_{2i}(t)\tilde{d}_i(t) &\le \frac{1}{2}\sum_{i=1}^{N}(\phi_{2i}(t)^2+\tilde{d}_i(t)^2) \\ &\le \frac{1}{2}\sum_{i=1}^{N}\phi_{2i}(t)^2+\frac{N\varepsilon_d}{2}\end{aligned} \tag{35}$$

Then (34) can be transformed into

$$\begin{aligned}\dot{V}_3(t) &\le (n_1+\frac{1}{2})\sum_{j=1}^{N}\phi_{2i}(t)^2+\frac{1}{2}\tilde{d}_i(t)^2 \\ &\le (n_1+\frac{1}{2})\sum_{j=1}^{N}\phi_{2i}(t)^2+\frac{N\varepsilon_d}{2}\end{aligned} \tag{36}$$

For $n_1+\frac{1}{2} < 0$, according to the Lyapunov stability theory, it follows:

$$\lim_{t\to\infty}|\phi_{2i}(t)| \le \sqrt{\frac{N\varepsilon_d}{-2\left(n_1+\frac{1}{2}\right)}}$$

Because n_1 can be designed as large as possible, so $|\phi_{2i}(t)|$ will converge to a small set. In addition, by the definition of $V_3(t)$, it can be seen that the consensus error $\left|(\phi_{1i}(t)-\phi_{1j}(t))\right|$ will converge to a small set at the same time. Therefore, if the value of $|n_1|$, $|n_2|$ is larger, it guaranteed that the situation of Definition 1 is satisfying. Thus the cooperation of the attack angle is achieved.

Remark 2: The explanation of the convergence on consensus error $\left|(\phi_{1i}(t)-\phi_{1j}(t))\right|$ is as follows. $|\phi_{2i}(t)|$ will be arbitrarily small by setting n_1 as large as possible. At the same time the Lyapunov function $V_3(t)$ will converge to a small set. Which means $\dot{\phi}_{2i}$ will converge to a small set. Then from (32) one can get that the consensus error $\left|(\phi_{1i}(t)-\phi_{1j}(t))\right|$ will be arbitrarily small by setting n_2 as large as possible.

V. NUMERICAL SIMULATIONS

In this section, numerical simulations are given to demonstrate the effectiveness of the cooperative guidance law.

A. SIMULATION RESULTS

We consider an engagement scenario where four UAVs are expected to hit a maneuvering target simultaneously. The initial conditions of multiple UAVs are displayed in Table 1.

TABLE 1. Initial conditions of multiple UAVs.

UAV	$V_i(0)$ / $\mathrm{m \cdot s^{-1}}$	$x_i(0)$ /m	$y_i(0)$ /m	$q_i(0)$ /°
1	282	0	6000	-20
2	268	0	5000	-15
3	260	0	4000	-10
4	241	0	3000	-5

The initial conditions of the target are shown in Table 2.

TABLE 2. Initial conditions of the target.

V_t / $\mathrm{m \cdot s^{-1}}$	$x_t(0)$ /m	$y_t(0)$ /m	$q_t(0)$ /°	a_t / $\mathrm{m \cdot s^{-2}}$
220	12000	0	0	$20 \cdot \sin(t/10)$

The communication topology is shown in Fig.2.

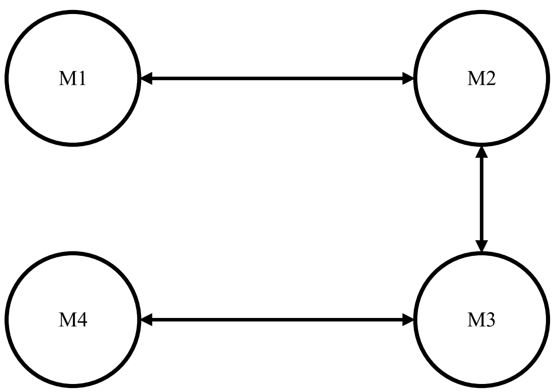

FIGURE 2. Communication topology among UAVs.

The control parameters are designed as follows

$$\begin{aligned}&k_1 = -0.1, k_2 = -0.5, k_3 = -0.05, k_4 = -0.05 \\ &n_1 = -0.8, n_2 = -0.1, a_1 = b_1 = 100, a_2 = b_2 = 2000 \\ &a_3 = b_3 = 30000, V0 = 262 \\ &c_1 = 0°, c_2 = 5°, c_3 = 10°, c_4 = 15°\end{aligned}$$

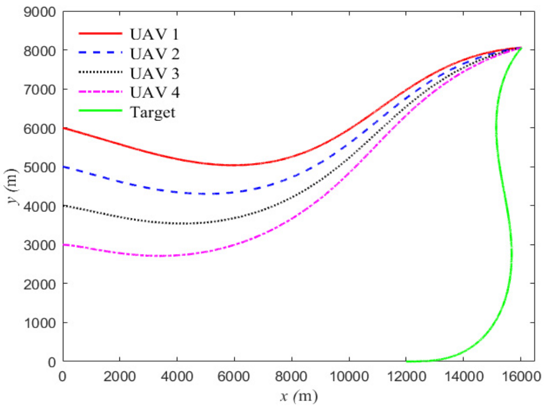

FIGURE 3. Trajectories of the UAVs.

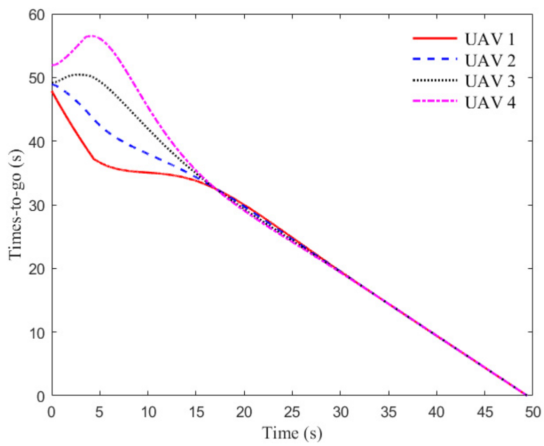

FIGURE 4. Time to go of the UAVs.

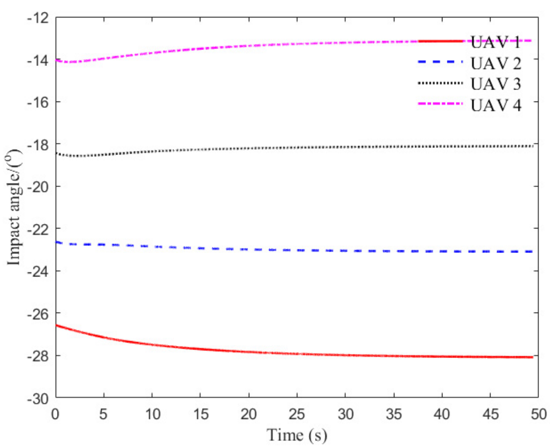

FIGURE 5. Impact angle of the UAVs.

The simulation results are presented in Figs.3-8. Fig.3 shows that the four UAVs under the proposed guidance law successfully hit the target along different trajectories. From Fig.4, it can be shown that four UAVs intercept the target simultaneously. In Fig.5, it can be seen that different UAVs hit the target with the impact angles of preset defaults.

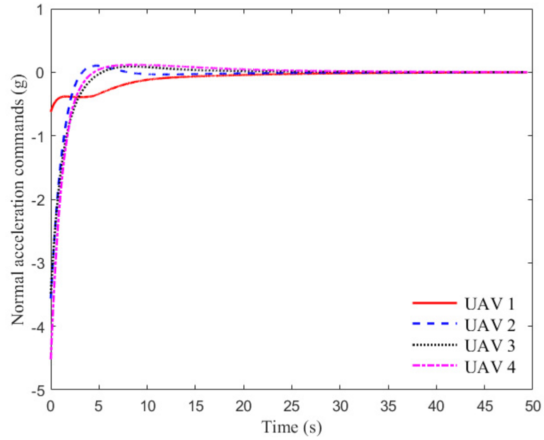

FIGURE 6. Normal acceleration command of the UAVs.

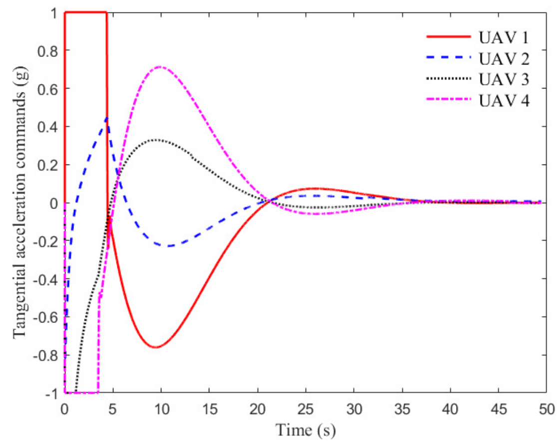

FIGURE 7. Tangential acceleration command of the UAVs.

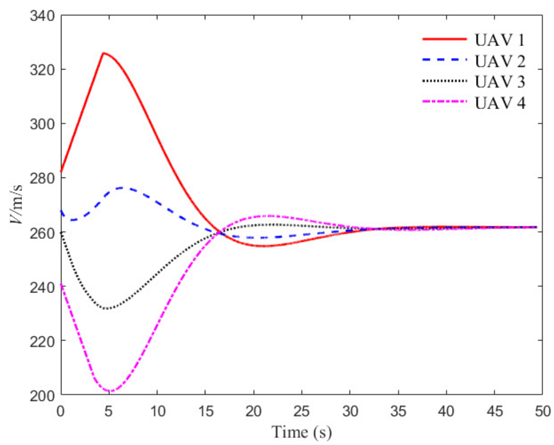

FIGURE 8. Velocity of the UAVs.

From Figs.6, it can be seen that the normal accelerations of multi-UAVs are smooth. As shown in Fig.7, the tangential accelerations of the UAVs are limited to 1g. The tangential accelerations of UAVs are mainly determined by thrust and resistance. In engineering practice, they are usually smaller

TABLE 3. Initial conditions of multiple UAVs.

UAV	$V_i(0)$ / $\mathrm{m \cdot s^{-1}}$	$x_i(0)$ /m	$y_i(0)$ /m	$q_i(0)$ /°
1	282	0	6000	0
2	268	0	3000	0
3	260	0	2000	0
4	241	0	1500	0

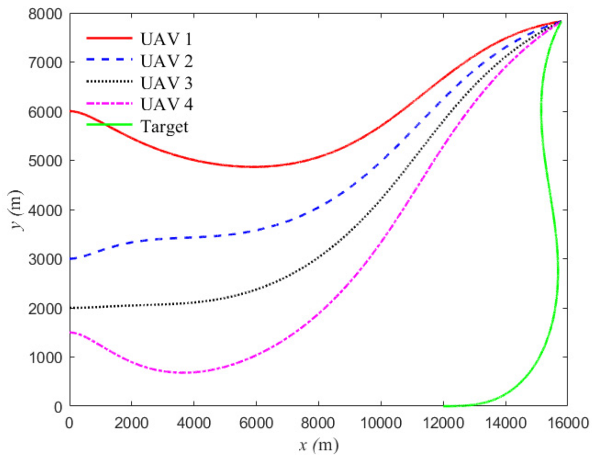

FIGURE 9. Trajectories of G1.

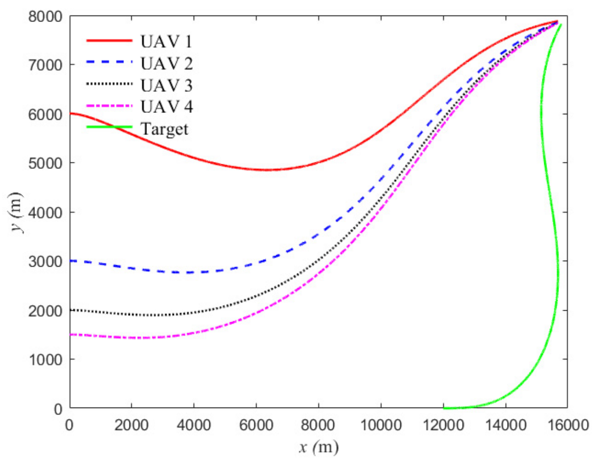

FIGURE 10. Trajectories of G2.

than the gravity of the vehicle. So the tangential accelerations of the UAVs are bounded by 1g. The simulation results show that our guidance law could achieve cooperative guidance under the strict limitation of the tangential acceleration. In Fig.8. it can be shown that the velocity of UAVs achieves consensus.

B. COMPARISON SIMULATION

Here a comparison simulation to the cooperative guidance law proposed in [34] is given, where the maneuvering target have also considered, however, the impact angle constriaint

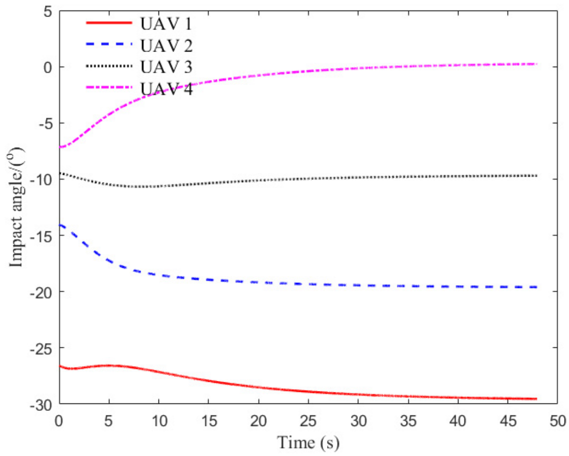

FIGURE 11. Impact angle of G1.

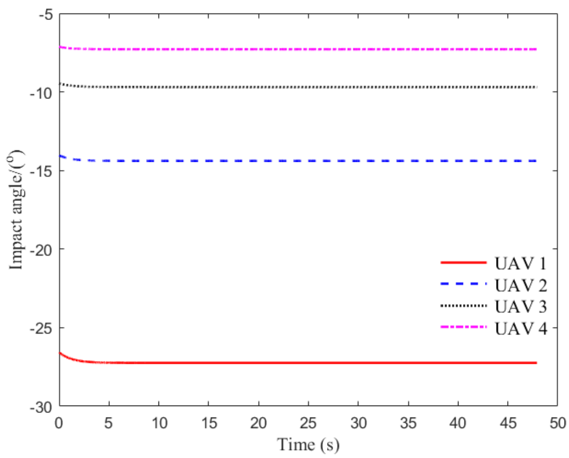

FIGURE 12. Impact angle of G2.

cannot be achieved. The initial conditions of multiple UAVs are displayed in Table 3. The initial conditions of the target is displayed in Table 2.

Group 1 (G1) is under the proposed guidance law, and Group 2 (G2) is under the guidance law in [34]. The impact angle parameter is as follows

$$c_1 = 0°, c_2 = 10°, c_3 = 20°, c_4 = 30°$$

From Figs.9-10, one can get that both of the guidance laws can achieve simultaneously attack. It can be verified from Figs.11-12 that the guidance law we proposed can satisfy the impact angle constraint, but the method in [34] cannot.

VI. CONCLUSIONS

In this paper, a novel cooperative guidance strategy was designed for salvo attack against a maneuvering target considering the impact angle constraint. Disturbance estimation technic and sliding mode control technic were used in the designing of the guidance law. It is proven that, under undirected communication topologies, all of the UAVs can hit the target simultaneously and satisfy the constraint of impact angle. The proposed guidance strategy could be applied to not only fixed targets but also maneuvering targets without using the target's acceleration information. The given cooperative guidance law can achieve both simultaneous attack and cooperation of the impact angle, which is usually very important for improving the striking efficiency.

Several questions still remain to be answered. (1) The issue of distributed cooperative guidance with directed topologies is an intriguing one that could be usefully explored in further research. (2) The issue of group cooperative guidance needs further research.

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