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A Novel Tuning Method of PD With Gravity Compensation Controller for Robot Manipulators

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ABSTRACT Proportional-Derivative (PD) control is one of the most widely used controllers, especially for robot manipulators. When the robot presents gravitational terms, PD control cannot guarantee position convergence, therefore compensation is required such as PD with gravity compensation, PD+G. PD+G control requires knowledge of the gravitational term and there exist several results that prove global asymptotic stability. However, there is no method to tune the PD gains. In this work, a novel method to tune the PD+G controller is proposed. The tuning method is obtained using the global asymptotic stability result of the La Salle's theorem and robot dynamics properties. A comparison between previous works is realized via simulations and experiments to verify our approach. The results show fast and smooth convergence to the desired reference without overshoots.

INDEX TERMS Global asymptotic stability, Lyapunov function, PD control, gravity compensation, tuning.

I. INTRODUCTION

Proportional-Derivative (PD) control is one of the simplest controllers for robot manipulators control [1], [2]. When the robot is not affected by gravitational terms then the PD controller guarantees global asymptotic stability (GAS) by choosing strictly positive gains [3], [4]. Nevertheless, PD control cannot achieve convergence to the desired control task if the robot presents gravitational terms.

To satisfy the control objectives, it has been developed different control techniques, such as PID [7]–[9], sliding mode control (SMC) [10]–[12], neural networks [13], intelligent techniques [14], [15] or even linear controllers [2], [16]. Each algorithm is capable to compensate the gravitational term and robustify the control law. However, the above controllers need knowledge of the complete robot dynamics in order to apply their tuning methods.

The PD+ controller [5], [6] is an alternative of the classical PD controller, where it is added additional terms to compensate dynamics or disturbances. Such is the case of the PD control with gravity compensation, PD+G, which assumes a partial knowledge of the robot dynamics. There exist different references that guarantee global asymptotic stability of the closed-loop system dynamics using Lyapunov stability theory [5], [17], [18]. The most popular result is obtained from La Salle's theorem, which guarantees global asymptotic stability of the closed-loop system but does not give any information of how the controller gains can be tuned and only meets the sufficient condition of choosing strictly positive gains [6].

For linear systems, there exists a broad theory to tune controllers [19], [20] and to use those techniques it is required to linearize the robot dynamics [21]. One of the most popular controllers and wide used is the PID control [22]–[24] which has been used to develop different tuning methods [7], [25], [26] using robot dynamics bounds. Non-linear controllers like sliding mode control require knowledge of the upper bound of the disturbance to compensate it [10], [27], [28]. Both PID and sliding mode control require knowledge of the complete robot dynamics which is not always available.

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In the literature there exists a tuning method for a PD+G controller variation called PD with desired gravity compensation: PD+$G(q_d)$, where the gravity compensation of the controller depends on a desired joint position [6], [29], [30]. Other approaches consist in novel controllers where the design of the PD+G controller changes like in [31] where the position error vector is modified by a vector whose components are polynomials of each element of the position error vector; and [32] where both position and velocity error vectors are normalized, which helps for the global stability proof using a strict Lyapunov function. However, the tuning method of the classical PD+G controller is still an open control problem.

In this paper it is proposed a novel and simple method to tune the PD+G controller that only requires the gravitational torques vector bound and the global asymptotic stability result using La Salle's theorem. Our approach is verified and compared with bounds of the controllers gains obtained from strict Lyapunov functions using two different planar robots and a 2-DOF four bar mechanism.

The outline of the paper is as follows: first it is given the robot dynamics and some useful properties for stability proof. Then the PD+G control law is given where its gains are tuned according to the global asymptotic stability bounds using Lyapunov theory. Different tuning methods are obtained using different Lyapunov functions. Simulation studies are shown using two different planar robots. A real time experiment is given to validate our approach in a 2-DOF mechanism. Finally conclusions and future work conclude the paper.

II. ROBOT MANIPULATORS DYNAMIC MODEL

Consider a serial robot manipulator of n-degrees of freedom (DOFs), i.e, $q \in \mathbb{R}^n$, without friction and disturbances. According to the Denavit-Hartenberg (DH) [4] convention, the final homogeneous transformation matrix is given by: $T_n = A_1A_2\cdots A_n = \begin{bmatrix} R_n(q) & o_n(q) \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}$, where $A_i \in \mathbb{R}^{4\times 4}$, $i = 1, \cdots, n$, are transformation matrices, $R_n(q) \in \mathbb{R}^{3\times 3}$ is the rotation matrix and $o_n(q) = [x_n, y_n, z_n]^\top \in \mathbb{R}^3$ is the robot position. Let consider that all joint DOFs q are revolute. Let define $z_{i-1} = R_i\vec{k}$, so z_{i-1} w.r.t. the base frame are given by the first three elements in the third column of T_i. Since all joints are revolute, the i-th column of the linear velocity Jacobian J_v is $J_v = [J_{v_1} \cdots J_{v_n}]$, $J_{v_i} = z_{i-1} \times (o_i - o_i - 1)$, here $\times$ denotes the vector cross product. The i-th column of the angular velocity Jacobian J_ω is $J_\omega = [J_{\omega_1} \cdots J_{\omega_n}]$, $J_{\omega_i} = z_{i-1}$.

The dynamics of a serial robot manipulator of n-DOFs includes the translational kinetic energy $K_T = \frac{1}{2}\dot{q}^\top \left[\sum_{i=1}^n m_i J_{v_i}^\top(q) J_{v_i}(q)\right]\dot{q}$, the rotational kinetic energy $K_R = \frac{1}{2}\dot{q}^\top \left[\sum_{i=1}^n J_{\omega_i}^\top(q) R_i(q) I_i R_i^\top(q) J_{\omega_i}(q)\right]$ and the potential energy $U = \sum_{i=1}^n m_i g y_i$; where $m_i \in \mathbb{R}$ and $I_i \in \mathbb{R}^{3\times 3}$ are the mass and inertia tensor of link i, respectively, $\dot{q} \in \mathbb{R}^n$ is the joint velocity and g is the gravitational acceleration.

The dynamics of a n-link serial manipulator is derived from the Euler-Lagrange formulation as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ represent the joint position, velocity and acceleration, respectively; $M(q) = K_T + K_R \in \mathbb{R}^{n\times n}$ is a positive definite inertia matrix, $C(q,\dot{q}) = \{c_{kj}\} \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal forces matrix, $c_{kj} = \sum_{i=1}^n c_{ijk}\dot{q}_i$, $k, j = 1 \cdots n$, c_{ijk} are the Christoffel symbols $c_{ijk} = \frac{1}{2}\left(\frac{\partial m_{kj}}{\partial q_i} + \frac{\partial m_{ki}}{\partial q_j} - \frac{\partial m_{ij}}{\partial q_k}\right)$, m_{ij}, $i, j = 1 \ldots n$ are components of the inertia matrix $M(q)$, $G(q) = \frac{\partial}{\partial q}U(q) \in \mathbb{R}^n$ is the gravitational torques vector and $\tau \in \mathbb{R}^n$ is the vector of driven torque. In this paper we only need knowledge of the potential energy to compute the gravitational torques vector $G(q)$. The robot dynamics satisfies the following properties [6]:

P1 The inertia matrix $M(q)$ is symmetric and positive definite, and:

$$0 < \lambda_{\min}(M(q)) \le \|M(q)\| \le \lambda_{\max}(M(q)) \le \beta < \infty \tag{2}$$

where $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ stands for the minimum and maximum eigenvalue of the matrix $M \in \mathbb{R}^{n\times n}$. The norm $\|M\| = \sqrt{\lambda_{\max}(M^\top M)}$ represents the induced Frobenius norm.

P2 For the Coriolis matrix $C(q,\dot{q})$, there exists a number $k_c > 0$ such that

$$\|C(q,\dot{q})\dot{q}\| \le k_c\|\dot{q}\|^2 \tag{3}$$

and $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e. for any vector $x \in \mathbb{R}^n$:

$$x^\top \left[\dot{M}(q) - 2C(q,\dot{q})\right] x = 0 \tag{4}$$

also

$$\dot{M}(q) = C(q,\dot{q}) + C^\top(q,\dot{q}) \tag{5}$$

P3 The gravitational torques vector $G(q)$ is Lipschitz:

$$\|G(q_1) - G(q_2)\| \le k_g\|q_1 - q_2\|, \quad k_g > 0 \tag{6}$$

for any $q_1, q_2 \in \mathbb{R}^n$.

The bounds k_c and k_g can be obtained as:

$$\begin{aligned} k_c &= n^2\left(\max_{k,i,j,q} |c_{ijk}(q)|\right) \\ k_g &= n\left(\max_{i,j,q}\left|\frac{\partial G_i(q)}{\partial q_j}\right|\right) \end{aligned} \tag{7}$$

III. PD+G CONTROLLER

The main objective of a position controller is to evaluate the torques that must be applied at the joint angles q such that they tend to a desired joint position $q_d \in \mathbb{R}^n$ accurately in presence of gravitational terms. In this paper q_d is a constant reference. The PD controller with gravity compensation is able to satisfy the control objective.

It is defined the position error as:

$$\tilde{q} = q_{\text{d-q}} \tag{8}$$

The PD+G control law is given by the next expression:

$$\tau = K_p\tilde{q} - K_d\dot{q} + G(q) \tag{9}$$

where $K_p, K_d > 0 \in \mathbb{R}^{n\times n}$ are the proportional and derivative diagonal matrices gains, respectively. Because $\dot{q}_d = 0$ and $\dot{\tilde{q}} = -\dot{q}$, then the closed-loop system of the robot dynamics (1) and the controller (9) is written as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = K_p\tilde{q} - K_d\dot{q} \tag{10}$$

In matrix form results in:

$$\frac{d}{dt}\begin{bmatrix}\tilde{q}\\ \dot{q}\end{bmatrix} = \begin{bmatrix}-\dot{q}\\ M^{-1}(q)\left(K_p\tilde{q} - K_d\dot{q} - C(q,\dot{q})\dot{q}\right)\end{bmatrix}. \tag{11}$$

The unique equilibrium point of (11) is $\left[\tilde{q}^\top, \dot{q}^\top\right]^\top = [0,0]^\top \in \mathbb{R}^{2n}$. Lyapunov stability theory [6] is used to demonstrate the global asymptotic stability (GAS) of the closed-loop dynamics (11) at the equilibrium point.

A. GLOBAL ASYMPTOTIC STABILITY USING LaSalle's INVARIANCE PRINCIPLE

Consider the following Lyapunov function:

$$V(\tilde{q},\dot{q}) = \frac{1}{2}\dot{q}^\top M(q)\dot{q} + \frac{1}{2}\tilde{q}^\top K_p\tilde{q} \tag{12}$$

where the first term is the robot kinetic energy and the second term is a virtual potential energy due the proportional term of the controller. The time derivative of (12) is:

$$\dot{V}(\tilde{q},\dot{q}) = \dot{q}^\top M(q)\ddot{q} + \frac{1}{2}\dot{q}^\top\dot{M}(q)\dot{q} + \tilde{q}^\top K_p\dot{\tilde{q}} \tag{13}$$

By means of property **P2** and substituting the closed-loop dynamics (11) in $\dot{V}$ yields:

$$\dot{V}(\tilde{q},\dot{q}) = -\dot{q}^\top K_d\dot{q} \le 0 \tag{14}$$

Therefore, the function $\dot{V}(\tilde{q},\dot{q}) \le 0$ for all $\tilde{q}$ and $\dot{q}$, in consequence the origin is stable and all the solutions of $\tilde{q}$ and $\dot{q}$ are bounded. Using the LaSalle's invariance principle, the global asymptotic stability (GAS) can be concluded as follows:

$$\Omega = \left\{\left[\tilde{q}^\top, \quad \dot{q}^\top\right]^\top \in \mathbb{R}^{2n} : \dot{V}(\tilde{q},\dot{q}) = 0\right\} \tag{15}$$

$\dot{V}(\tilde{q},\dot{q}) = 0$ if and only if $\dot{q} = 0$, then it holds that $\ddot{q} = 0$ for all time $t \ge 0$. From (11) it can be concluded that if $\tilde{q}, \dot{q} \in \Omega$ then:

$$0 = M^{-1}(q)K_p\tilde{q} \tag{16}$$

that means that $\tilde{q} = 0$ for all $t \ge 0$ and guarantees GAS of the origin $\left[\tilde{q}^\top, \dot{q}^\top\right]^\top = 0 \in \mathbb{R}^{2n}$.

B. GLOBAL ASYMPTOTIC STABILITY USING STRICT LYAPUNOV FUNCTIONS

The previous stability analysis shows that the origin is GAS by means of the LaSalle's invariance principle, however there is no information about how the gains of the PD controller must be selected. There exists other results for GAS using strict Lyapunov functions where it is found bounds for the controller gains.

Let consider the next strict Lyapunov function [6]:

$$V_1(\tilde{q},\dot{q}) = \frac{1}{2}\dot{q}^\top M(q)\dot{q} + \frac{1}{2}\tilde{q}^\top K_p\tilde{q} - \gamma_1\tanh(\tilde{q})^\top M(q)\dot{q} \tag{17}$$

where the third term of (17) is a cross term between the position error $\tilde{q}$ and the joint velocity $\dot{q}$. The term $\tanh(\cdot)$ stands to the hyperbolic tangent function which has useful properties for the Lyapunov stability analysis. The function $\tanh(x)$ satisfies $|x| \ge |\tanh(x)|$ and $1 \ge |\tanh(x)|$ for all $x \in \mathbb{R}^n$, therefore the norm of $\|\tanh(x)\|$ satisfies

$$\|\tanh(x)\| \le \begin{cases}\|x\| & \forall\, x \in \mathbb{R}^n\\ \sqrt{n} & \forall\, x \in \mathbb{R}^n.\end{cases} \tag{18}$$

The term $\gamma_1 > 0$ is a small constant that satisfies the following inequalities [6]:

$$V_1(\tilde{q},\dot{q}) \ge \frac{1}{2}\begin{bmatrix}\|\tilde{q}\|\\ \|\dot{q}\|\end{bmatrix}^\top \begin{bmatrix}\lambda_{\min}(K_p) & -\gamma\beta\\ -\gamma\beta & \lambda_{\min}(M(q))\end{bmatrix}\begin{bmatrix}\|\tilde{q}\|\\ \|\dot{q}\|\end{bmatrix} \tag{19}$$

$$\dot{V}_1(\tilde{q},\dot{q}) \le -\gamma_1\begin{bmatrix}\|\tanh(\tilde{q})\|\\ \|\dot{q}\|\end{bmatrix}^\top Q\begin{bmatrix}\|\tanh(\tilde{q})\|\\ \|\dot{q}\|\end{bmatrix} \tag{20}$$

with

$$Q = \begin{bmatrix}\lambda_{\min}(K_p) & -\frac{1}{2}\lambda_{\max}(K_d)\\ -\frac{1}{2}\lambda_{\max}(K_d) & \frac{1}{\gamma}\lambda_{\min}(K_d) - \sqrt{n}k_c - \beta\end{bmatrix}$$

where $\lambda_{\min}(K_p)$ is the minimum eigenvalue of the matrix gain K_p and $\lambda_{\min}(K_d)$, $\lambda_{\max}(K_d)$ stand to the minimum and maximum eigenvalues of the derivative matrix gain K_d, respectively. From matrix Q and V_1 it is obtained the following bounds:

$$\lambda_{\min}(K_p) > 0 \tag{21}$$

$$\gamma_1 < \frac{\sqrt{\lambda_{\min}(K_p)\lambda_{\min}(M(q))}}{\beta} \tag{22}$$

$$\gamma_1 < \frac{4\lambda_{\min}(K_p)\lambda_{\min}(K_d)}{\lambda^2_{\max}(K_d) + 4\lambda_{\min}(K_p)\left[\sqrt{n}k_c + \beta\right]} \tag{23}$$

Note that in (21) is obtained a simple condition for the proportional gain K_p that not depends of the robot dynamics, nevertheless it is a lazy condition that is not useful for a tuning method. There is not an explicit expression for the derivative gain K_d.

Another strict Lyapunov function that gives bounds for the proportional and derivative gains is the following:

$$V_2(\tilde{q},\dot{q}) = \frac{1}{2}\dot{q}^\top M(q)\dot{q} + \frac{1}{2}\tilde{q}^\top K_p\tilde{q} - \gamma_2\dot{q}^\top M(q)\frac{\tilde{q}}{1+\tilde{q}^\top\tilde{q}} \tag{24}$$

where the third term is a cross term which facilitates the global asymptotic stability analysis by using a normalized position error. Here γ_2 is a small positive constant that satisfies:

$$\gamma_2 \leq \left(1 + \|\tilde{q}\|^2\right)\sqrt{\lambda_{\min}(K_p)} \tag{25}$$

The time-derivative of (24) is:

$$\begin{aligned}\dot{V}_2(\tilde{q}, \dot{q}) \leq & -\gamma_2\|\dot{q}\|^2\left(\frac{\lambda_{\min}(K_d)}{\gamma_2} - \frac{1}{2} - k_c - 3\beta\right)\\ & - \frac{\gamma_2\|\tilde{q}\|^2}{1+\|\tilde{q}\|^{\prime 2}}\left(\lambda_{\min}(K_p) - \frac{\lambda_{\min}^2(K_d)}{2}\right)\end{aligned} \tag{26}$$

One way to tune the proportional and derivative gains is obtained from the negativity condition of the time derivative of the Lyapunov function V_2 in (26):

$$\lambda_{\min}(K_d) \geq \gamma_2\left(\frac{1}{2} + k_c + 3\beta\right) \tag{27}$$

$$\lambda_{\min}(K_p) \geq \frac{\lambda_{\min}^2(K_d)}{2} \tag{28}$$

From the conditions (25),(26) and (27) we can obtain two different tuning methods. By means of the GAS result (26), we know that $\tilde{q} = 0$, then (25) is rewritten as $\gamma_2 = \sqrt{\lambda_{\min}(K_p)}$. If the user proposes the proportional gain K_p, then substituting γ_2 in (27) yields the tuning Method 1

$$\begin{aligned} k_{p_i} &= val\\ k_{d_i} &= \sqrt{k_{p_i}}\left(\frac{1}{2} + k_c + 3\beta\right).\end{aligned} \tag{29}$$

where $val > 0$ is proposed by the user. If the user proposes the derivative gain, then from (27) it is obtained the tuning Method 2

$$\begin{aligned} k_{p_i} &= \frac{k_{d_i}^2}{2}\\ k_{d_i} &= val\end{aligned} \tag{30}$$

The previous methods consist of proposing values either for the proportional gain (Method 1) or the derivative gain (Method 2).

From (30) it can be seen that if $k_{d_i} < 2$, then the proportional gain will be smaller than the derivative gain and the system will have an overdamped response. Nevertheless, to satisfy the conditions of method 1 (29) or method 2 (30) the user must propose a value for either the proportional gain K_p or derivative gain K_d since they are strongly correlated. It is clear that in practice is not difficult to fulfill the above conditions, however the tuning method is still an open problem.

C. OUR TUNING APPROACH

In the previous sections, it has been addressed some methods to establish GAS of the robot dynamics under the PD+G controller. Strict Lyapunov functions give some bounds for the proportional and derivative gains, however they are correlated and depends of full dynamic knowledge. In this section is shown our approach that is a simple method to tune the proportional and derivative gains based on the GAS result from LaSalle's invariance principle.

First, from (16) it is observed that:

$$\begin{aligned} 0 &= K_p\tilde{q} = K_p(q_d - q)\\ 0 &\leq \lambda_{\min}(K_p)\|q_d - q\| \leq \lambda_{\max}(K_p)\|q_d - q\|\end{aligned} \tag{31}$$

The above expression is the same Lipschitz condition that satisfies the gravitational torques vector in **P3**, i.e.,

$$\|G(q_d) - G(q)\| \leq k_g\|q_d - q\| \tag{32}$$

then

$$0 \leq \lambda_{\min}(K_p)\|\tilde{q}\| \leq \lambda_{\max}(K_p)\|\tilde{q}\| = k_g\|\tilde{q}\| \tag{33}$$

A simple way to tune the proportional gain is that the maximum eigenvalue of the proportional gain K_p is equal to or less than k_g:

$$\lambda_{\max}(K_p) = k_g \tag{34}$$

The equality (34) gives an easy method to tune the proportional gain using the previous knowledge of the gravitational torques vector. When the gravitational torques vector is small, then the proportional gain will be small too and the transient time trajectory could be very slow. To guarantee a fast response, (34) is modified to:

$$\lambda_{\max}(K_p) = \alpha k_g. \tag{35}$$

where $\alpha \geq 1$. The above expression only accelerates the solution convergence. This tuning method is valid since the gravitational torques vector is known for the controller design. Note that:

$$k_g = \max_i \begin{bmatrix} k_{g_1}\\ \vdots\\ k_{g_i}\end{bmatrix}, \quad i = 1, \cdots, n \tag{36}$$

where $k_{g_i} = n\left(\max_{j,q}\left|\frac{\partial G_i(q)}{\partial q_j}\right|\right)$. Then the proportional gain is tuned as:

$$K_p = \alpha\begin{bmatrix} k_{g_1} & 0 & \cdots & 0\\ 0 & k_{g_2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & k_{g_n}\end{bmatrix}. \tag{37}$$

The matrix (37) verifies the equality (35). The derivative gains were chosen to satisfy $\lambda_{\min}(K_d) \leq \lambda_{\min}(K_p)$ and $\lambda_{\max}(K_d) \leq \lambda_{\max}(K_p)$. By practical experience, the derivative gain is tuned as follows:

$$\lambda_{\max}(K_d) = \lambda_{\max}(K_p) \cdot 25\% \tag{38}$$

The tuning method (38) adjust the derivative gains to put low under-damped response. Also (38) avoids knowledge of the complete robot dynamics such that the gravitational torques vector bound is the only parameter for the tuning design. It is worth mentioning that the derivative gain must be smaller than the proportional gain to guarantee a rapid response without overshoots and oscillations. The derivative matrix gain is written as

$$K_d = \frac{1}{4}\begin{bmatrix} k_{p_1} & 0 & \cdots & 0 \\ 0 & k_{p_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & k_{p_n} \end{bmatrix}. \tag{39}$$

where $K_p = \text{diag}\{k_{p_1}, k_{p_2}, \cdots, k_{p_n}\}$. Hence, it is obtained two new tuning methods denominated as method 3 and method 4, respectively. Method 3 is given by:

$$\begin{aligned} k_{p_i} &= k_{g_i} \\ k_{d_i} &= \frac{1}{4}k_{p_i} \end{aligned} \tag{40}$$

and method 4 is:

$$\begin{aligned} k_{p_i} &= \alpha k_{g_i} \\ k_{d_i} &= \frac{1}{4}k_{p_i}. \end{aligned} \tag{41}$$

Both methods use (37) and (39) for the tuning of each gain. Unlike the tuning methods (29) and (30), our tuning approach (40) and (41) finds an upper bound of the proportional and derivative gains according to the gravitational torques vector upper bound.

IV. SIMULATION STUDIES

To verify our approach, it is tested the four tuning methods (29), (30), (40) and (41). The simulations are done with Matlab/Simulink using two different planar robots. Each simulation last 10 seconds.

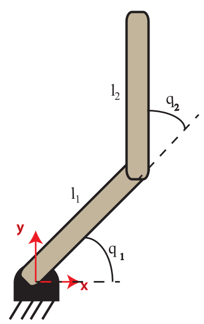


FIGURE 1. Planar robot of 2 GDL.

A. 2-DOF ROBOT SIMULATION

It is used a 2-DOF robot (see Figure 1) whose dynamic model in the form of (1) is:

$$\tau_1 = M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + C_{11}\dot{q}_1 + C_{12}\dot{q}_2 + G_1 \tag{42}$$

$$\tau_2 = M_{12}\ddot{q}_1 + M_{22}\ddot{q}_2 + C_{21}\dot{q}_2 + G_2 \tag{43}$$

where $M_{11} = (m_1 + m_2)l_1^2 + 2m_1l_1l_2c_2 + m_2l_2^2$, $M_{12} = m_2l_1l_2c_2 + m_2l_2^2$, $M_{22} = m_2l_2^2$, $C_{11} = -m_2l_1l_2s_2\dot{q}_2$, $C_{12} = -m_2l_1l_2s_2(\dot{q}_1 + \dot{q}_2)$, $G_1 = (m_1 + m_2)gl_1c_1 + m_2gl_2c_{12}$ and $G_2 = m_2gl_2c_{12}$ with $c_i = \cos(q_i)$, $s_i = \sin(q_i)$ and $c_{12} = \cos(q_1 + q_2)$, $i = 1, 2$. The kinematic and dynamic parameters are given in Table 1:

TABLE 1. 2-DOF robot Kinematic and dynamics parameters.

Symbol	Parameter	Value
l_1	Length of link 1	0.2 m
l_2	Length of link 2	0.18 m
m_1	Mass of link 1	0.5 kg
m_2	Mass of link 2	0.2 kg

In Table 2 shows a summary of the tuning methods that are given at this paper.

TABLE 2. Tuning methods.

Method	Gain k_{p_i}	Gain k_{d_i}
1	val	$\sqrt{k_{p_i}}\left(\frac{1}{2} + k_c + 3\beta\right)$
2	$\frac{k_{d_i}^2}{2}$	val
3	k_{g_i}	$\frac{1}{4}k_{p_i}$
4	αk_{g_i}	$\frac{1}{4}k_{p_i}$

where val is an arbitrarily value provided by the user, $K_p = \text{diag}\{k_{p_i}\}$ and $K_d = \text{diag}\{k_{d_i}\}$. According to the parameters of Table 1, the minimum and maximum eigenvalues of the inertia matrix are:

$$\begin{aligned} \lambda_m(M(q)) &= 0.0024 \\ \beta = \lambda_M(M(q)) &= 0.0529 \end{aligned}$$

meanwhile the upper bounds of the Coriolis matrix and the gravitational torques vector are:

$$\begin{aligned} k_c &= n^2m_2l_1l_2 = 0.0288 \\ k_g &= n\left((m_1 + m_2)l_1 + m_2l_2\right)g = 3.4531 \end{aligned}$$

Also k_g can be written as:

$$k_g = \max_{1,2}\left(\begin{bmatrix} k_{g_1} \\ k_{g_2} \end{bmatrix}\right) = \max_{1,2}\left(\begin{bmatrix} 3.4531 \\ 0.7063 \end{bmatrix}\right).$$

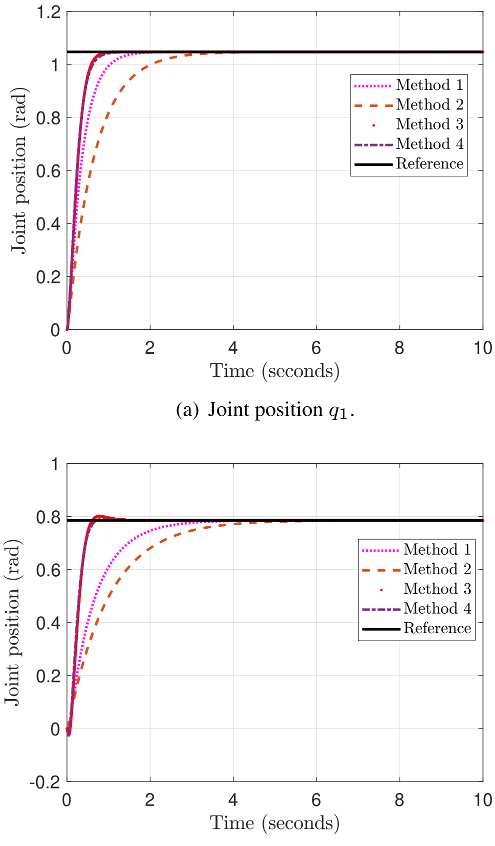
For method 1 it is proposed a proportional matrix gain of $K_p = \text{diag}\{3.5, 1\}$. For method 2 it is proposed a derivative matrix gain of $K_d = \text{diag}\{3, 2\}$. For method 3 the proportional matrix gain is $K_p = \text{diag}\{3.4531, 0.7063\}$. Method 4 increases the proportional matrix gain of method 3 by 20%, i.e, $\alpha = 1.2$. The proposed values satisfy the inequalities (27) and (28). The control gains are summarized in Table 3.

TABLE 3. Setting for the PD+G controller.

Method	Gain K_p	Gain K_d
1	diag$\{3.5, 1\}$	diag$\{1.2862, 0.6875\}$
2	diag$\{4.5, 2\}$	diag$\{3, 2\}$
3	diag$\{3.4531, 0.7063\}$	diag$\{0.8633, 0.1766\}$
4	diag$\{4.1437, 0.8476\}$	diag$\{1.0359, 0.2119\}$

The desired position is $q_d = [\pi/3, \pi/4]^\top$. The results are given in Figure 2.

The above results show that both tuning methods achieve the control objective with different responses. Method 1 requires full dynamic knowledge to tune the derivative gain,



(a) Joint position q_1.

(b) Joint position q_2.

FIGURE 2. **2-DOF robot: Joint position results.**

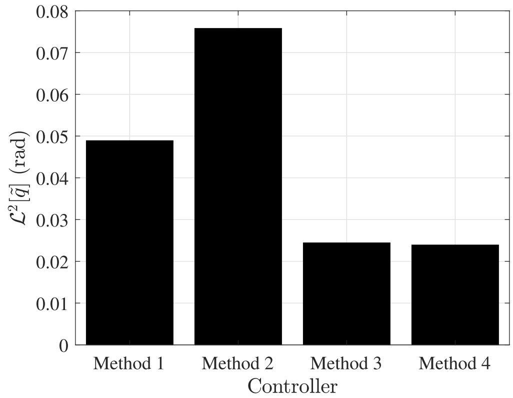


FIGURE 3. **Indexes of performance for the evaluated tuning methods. 2-DOF planar robot case.**

meanwhile method 2 needs that $k_{d_i} \geq 2$ to avoid an overdamped response. Both methods show a relative large transient time in comparison to method 3 and method 4. On the other hand, method 3 and method 4 show a fast convergence to the desired reference without big overshoots by using only knowledge of the gravitational torques vector instead of the complete robot dynamics.

It is used the scalar-valued $\mathcal{L}^2$ norm as an objective numerical measure for an entire error curve. The $\mathcal{L}^2[\tilde{q}]$ norm measures the true root-mean-square (RMS) 'average' of the position error $\tilde{q}$, which is given by:

$$\mathcal{L}^2[\tilde{q}] = \sqrt{\frac{1}{t_f - t_0}\int_{t_0}^{t_f} \|\tilde{q}\|^2 d\tau}, \tag{44}$$

where $t_0, t_f \in \mathbb{R}^+$ are the initial and final time, respectively. The performance indexes of the PD+G controller under each tuning method are summarized at the bar plot of Figure 3.

The $\mathcal{L}^2[\tilde{q}]$ shows that our approach (method 3 and method 4) has best performance in comparison to the results of method 1 and method 2. Our methods does not have large transient performance and adequately balance the control gains to avoid overshoots, oscillations and overdamped responses. Another advantages of our methods are that they do not need to propose any gain value to compute the other one and they not require knowledge of the complete robot dynamics. The parameter α serves to accelerate the transient time convergence and to overcame modeling error or disturbances at the robot dynamics.

Remark 1: Our approach does not consider friction. Nevertheless, the parameter α helps to adjust the controller gains such that they can compensate the friction term. It is worth mentioning that the derivative gain of our approach must be tuned manually in order to obtain a desired response. This is because the friction can help or affect the amount of damping in the closed-loop system.

It is used the norm of the torque vector $\|\tau\| = \sqrt{\tau^\top \tau}$ as an evaluation metric. The bar plot is given in Figure 4. The control torque norm shows balanced results since the control gains of Table 3 are similar, nevertheless, it is observed that our tuning methods (method 3 and method 4) present less $\|\tau\|$ in comparison to the others methods. Since α increases the control gains, then method 4 is slightly bigger than method 3.

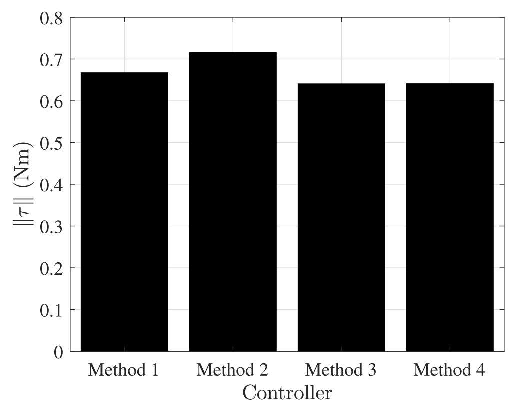


FIGURE 4. **Control torque performance. 2-DOF planar robot case.**

B. 6-DOF PLANAR ROBOT

To further illustrate our approach it is used a 6-DOF planar robot (see Figure 5). This robot is affected by the gravitational torques vector at each link, which makes difficult to control it when there is no gravity compensation.

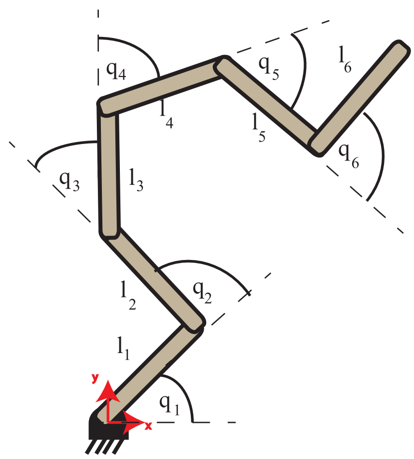


FIGURE 5. 6-DOF planar robot.

TABLE 4. 6-DOF robot Kinematic and dynamics parameters.

Symbol	Parameter	Value
$l_1 = l_3 = l_6$	Length of link 1,3,6	0.5 m
$l_2 = l_5$	Length of link 2,5	0.6 m
l_4	Length of link 4	0.4 m
m_1	Mass of link 1	2 kg
$m_2 = m_3$	Mass of link 2,3	1 kg
$m_4 = m_5 = m_6$	Mass of link 4,5,6	0.5 kg

The parameters of the robot are given in Table 4.

Here it is compared the tuning methods (29), (30), (40) and (41). Method 1 (29) is slightly modified as:

$$\begin{aligned} k_{p_i} &= val \\ k_{d_i} &= \sqrt{k_{p_i}}. \end{aligned} \tag{45}$$

This modification is due to the fact that the use of the upper bounds of the inertia matrix β and the Coriolis matrix k_c causes instability in the closed-loop system. The upper bound of the gravitational torques vector is:

$$k_g = ng\left[\sum_{i=1}^{6} m_i l_1 + \sum_{i=2}^{6} m_i l_2 + \sum_{i=3}^{6} m_i l_3 + \sum_{i=4}^{6} m_i l_4 + \sum_{i=5}^{6} m_i l_5 + m_6 l_6\right] = 444.393$$

As it can be seen the upper bound of the gravitational torques vector is large since each link has gravitational torques effect. For this kind of robot, the upper bound of each DOF can be obtained as follows:

$$\begin{aligned} k_{g_1} &= ng\left[\sum_{i=1}^{n} m_i l_1 + \sum_{i=2}^{n} m_i l_2 + \sum_{i=3}^{n} m_i l_3 + \cdots \right. \\ &\qquad \left. \cdots + \sum_{i=n-1}^{n} m_i l_{n-1} + m_n l_n\right] \\ k_{g_2} &= ng\left[\sum_{i=2}^{n} m_i l_2 + \sum_{i=3}^{n} m_i l_3 + \cdots \right. \\ &\qquad \left. \cdots + \sum_{i=n-1}^{n} m_i l_{n-1} + m_n l_n\right] \\ \vdots &= \qquad \vdots \\ k_{g_{n-1}} &= ng\left[\sum_{i=n-1}^{n} m_i l_{n-1} + m_n l_n\right] \\ k_{g_n} &= ngm_n l_n. \end{aligned} \tag{46}$$

For method 1 it is proposed a proportional matrix gain of $K_p = \{450, 300, 200, 100, 50, 25\}$. For method 2 it is proposed a derivative matrix gain of $K_d = \{30, 25, 20, 15, 10, 5\}$ since big values of k_{d_i} yields big values of k_{p_i} which makes the close-loop system unstable. For method 3 the proportional matrix gain is $K_p = \{444.36, 282.48, 158.88, 85.32, 49.98, 14.7\}$. Method 4 increases the proportional matrix gain of method 3 by 20%, i.e., $\alpha = 1.2$. The final control gains are given in Table 5.

The desired position is $q_d = [\pi/4, \pi/6, \pi/2, 0, \pi/3, \pi/3]^\top$. The comparisons are shown in Figure 6.

The results show that all methods present good results. Method 1 and method 2 have big proportional gain in comparison with the derivative gain, therefore its response presents overshoots. On the other hand, method 3 and 4 have smooth responses without overshoots, even more, the gains are well balanced in comparison to methods 1 and 2.

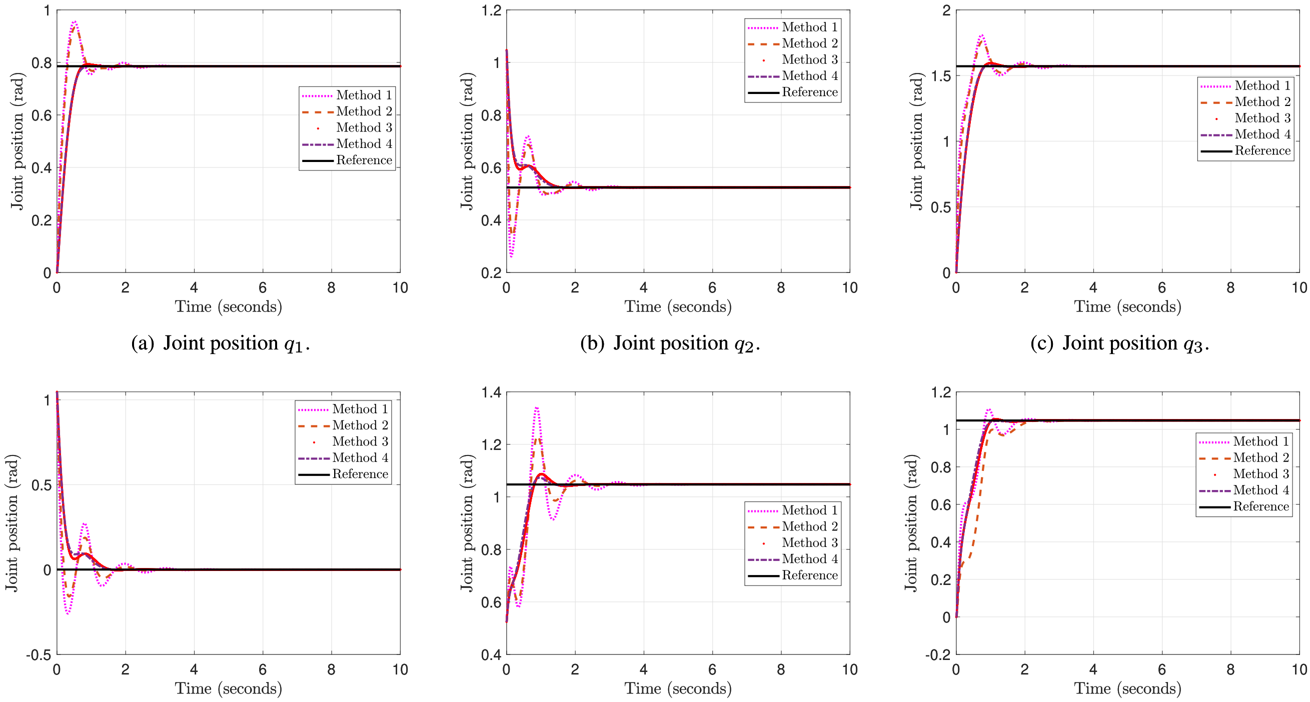
It is used the scalar-valued $\mathcal{L}^2$ norm (44) to compare the performance of our tuning approach. In Figure 7 is shown the performance indexes results. The $\mathcal{L}^2[\tilde{q}]$ shows that our approach presents less RMS error in comparison to the other tuning methods. Method 1 and method 2 require that the user proposes either the proportional or derivative gain, which translates into a trial and error procedure. Our approach takes advantage of the gravitational torques vector knowledge to tune the PD gains. The obtained control gains are balanced such that the system output achieves the desired reference with satisfactory results.

It is used the norm of the torque vector $\|\tau\| = \sqrt{\tau^\top \tau}$ to compare the control performance. The bar plot is shown in Figure 8. All the controllers show a similar values of $\|\tau\|$ due to the similarity of the control gains, nevertheless, our approach presents better control performance in terms of the position error $\tilde{q}$ and the applied torque.

The previous results show that our tuning methods present fast convergence to the desired reference without overshoots by using only the gravitational torques bound as tuning parameter. The other methods require knowledge of the complete robot dynamics which is not always available. However, those tuning methods are used to guarantee stability of the closed-loop dynamics and not guarantee good transient responses and fast convergence as it is shown in the 6-DOF robot case.

TABLE 5. Control gains of the 6-DOF planar robot.

Method	Gain K_p	Gain K_d
1	$\text{diag}\{450, 300, 200, 100, 50, 25\}$	$\text{diag}\{21.2132, 17.3205, 14.1421, 10, 7.0711, 5\}$
2	$\text{diag}\{450, 312.5, 200, 112.5, 50, 12.5\}$	$\text{diag}\{30, 25, 20, 15, 10, 5\}$
3	$\text{diag}\{444.36, 282.48, 158.88, 85.32, 49.98, 14.7\}$	$\text{diag}\{111.09, 70.62, 39.72, 21.33, 12.495, 3.675\}$
4	$\text{diag}\{533.232, 338.98, 190.66, 102.384, 59.98, 17.64\}$	$\text{diag}\{133.3080, 840744, 47.664, 25.60, 14.994, 4.41\}$



(a) Joint position q_1. (b) Joint position q_2. (c) Joint position q_3.

(d) Joint position q_4. (e) Joint position q_5. (f) Joint position q_6.

FIGURE 6. 6-DOF robot: Joint position results.

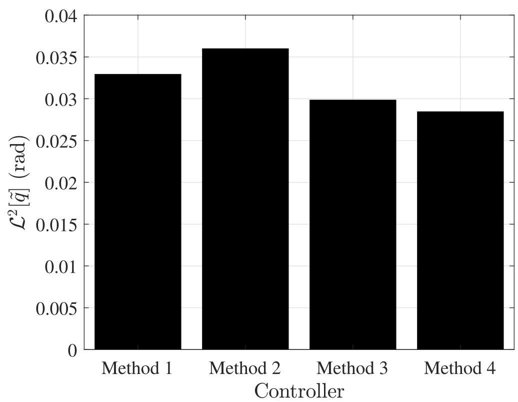


FIGURE 7. Indexes of performance for the evaluated tuning methods. 6-DOF planar robot case.

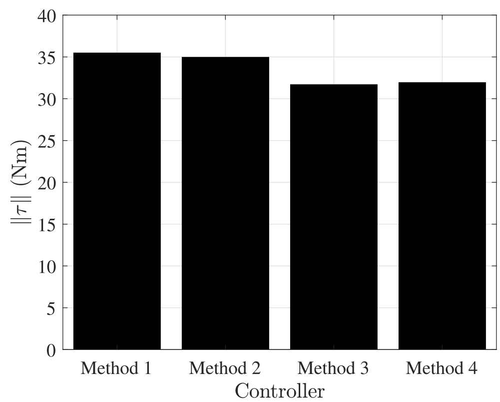


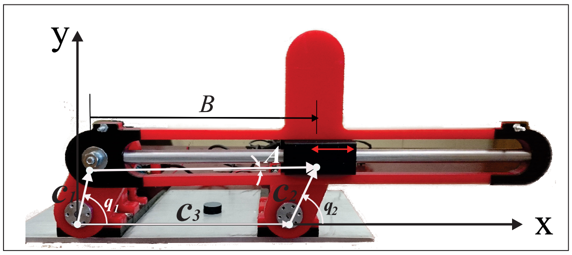
FIGURE 8. Control torque performance. 6-DOF planar robot case.

A large gravitational torques bound is obtained when the robot is affected by the gravity at each robot link, as in the 6-DOF planar robot case. This bound directly affects the first link as shown in the first element of the proportional gain of (40). The gravitational torques effect is decreased in the following links because the gravitational terms of previous links does not affect the subsequent links.

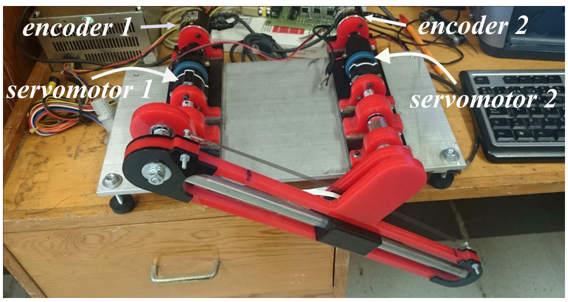
The use of the parameter α helps to accelerate the transient time and to compensate the modeling error of the gravitational torques vector $G(q)$ of the PD+G controller. Also the parameter α can compensate other disturbances and unmodeled dynamics such as friction. Nevertheless, the control gains obtained may have to be slightly tuned in order to obtain a desired response.

V. EXPERIMENTS

The proposed tuning approach holds for parallel robots. To verify this premise, it is used the 2-DOF four-bar mechanism of Figure 9. This mechanism has a slider inside the coupler; this slider is attached to a second actuated crank and its purpose is to give more workspace and the facility to change the orientation of the coupler. The mechanism is driven by two servomotors. Position information in obtained from Omron incremental encoders of 1024 pulses per revolution located on the motors. It is used Matlab/Simulink® 2012 and the Sensoray 626 real time board for the real-time interface.



(a) Frontal view



(b) Top view

FIGURE 9. **2-DOF mechanism prototype.**

The kinematic parameters of the 2-DOF mechanism are $C_1 = C_3 = 0.1$ m and $C_3 = 0.24$ m. The orientation of the coupler and the slider position are given by:

$$A = \arctan\left(\frac{C_2\sin(q_2) - C_1\sin(q_1)}{C_3 + C_2\cos(q_2) - C_1\cos(q_1)}\right)$$
$$B = \frac{C_3 + C_2\cos(q_2) - C_1\cos(q_1)}{\cos(A)} \tag{47}$$

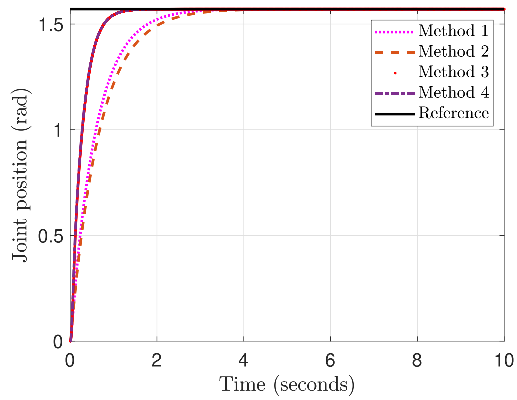
Each controller uses the following gravity compensation $G(q) = [g_1, g_2]^\top$:

$$g_1 = \frac{1.28Bc_1 + 0.078c_{A1}s_A - 0.32c_Ac_{A1}}{B}$$
$$g_2 = \frac{0.32c_A\cos_{A2} + 0.14Bc_2 - 0.078s_Ac_{A2}}{B}$$

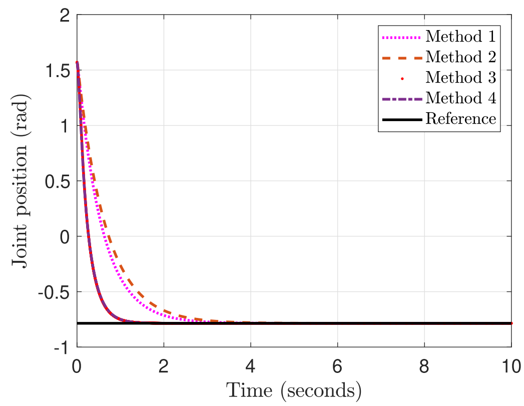
where $c_i = \cos(q_i)$, $s_i = \sin(q_i)$, $c_A = \cos(A)$, $s_A = \sin(A)$ and $c_{Ai} = \cos(A - q_i)$ with $i = 1, 2$. Notice that the coupler orientation A and the slider position B couple the mechanism dynamics making it a very non-linear system. In this experiment we compare the four methods given in this paper, where Method 1 is modified as in (45) because the dynamics parameters of the mechanism are unknown (except the gravitational torques vector).

TABLE 6. **Final control gains. 2-DOF mechanism case.**

Method	Gain K_p	Gain K_d
1	diag$\{3, 3\}$	diag$\{1.73, 1.73\}$
2	diag$\{4.5, 4.5\}$	diag$\{3, 3\}$
3	diag$\{2.9235, 2.9235\}$	diag$\{0.73, 0.73\}$
4	diag$\{5.85, 5.85\}$	diag$\{1.4618, 1.4618\}$



(a) Joint position q_1

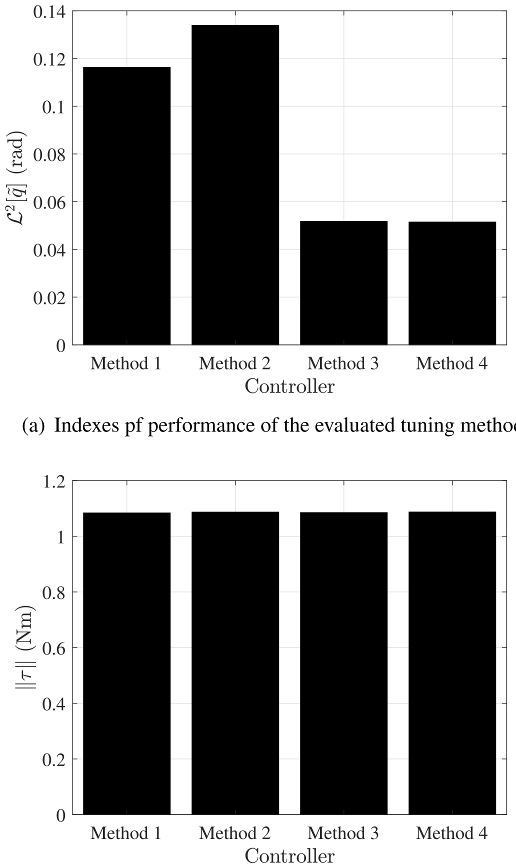


(b) Joint position q_2

FIGURE 10. **Tracking results.**

For parallel robots, the gravitational torques of each link affects the other ones since they are joined by the coupler. Therefore, it is not useful to separate the gravitational torques vector bound into separates bounds as it was done in the previous simulations. So, it is used the complete gravitational torques bound to obtain the proportional and derivative gains. These gains will be the same for each DOF. The gravitational torques vector bound is $k_g = 2.9235$, so for method 3 the proportional and derivative matrices gains are $K_p = 2.9235I$ and $K_d = \frac{2.9235}{4}I$, where I is a 2×2 identity matrix. For method 4 it is used a gain of $\alpha = 1.2$. For method 1 it is proposed a proportional matrix gain of $K_p = 3I$ and for method 2 it is proposed a derivative matrix gain of $K_d = 3I$. The final control gains are given in Table 6. The desired position is $q_d = [\pi/2, -\pi/4]^\top$ and the initial mechanism posture is $q_0 = [0, \pi/2]^\top$.

The tracking results are given in Figure 10. The mechanism presents small friction at the slider groove, however, this friction does not affect the control performance.



(a) Indexes pf performance of the evaluated tuning methods.

(b) Control input torque performance.

FIGURE 11. 2-DOF mechanism performance studies.

Our tuning methods (method 3 and method 4) have fast transient time without overshoots and any oscillations. Method 1 and method 2 have a slower response than our approach due to their derivative gains and the friction component, nevertheless their response is acceptable.

It is used the $\mathcal{L}^2[\tilde{q}]$ and $\|\tau\|$ norm to observe the performance of the tuning methods. The bar plots are given in Figure 11. The obtained results show that our tuning approach is fast and has minor RMS error. Even more, the norm of the control inputs for all methods are practically the same, which means that our tuning approach is effective when the gravitational torques vector is known. If the gravitational torques vector has modeling error, then the parameter α can compensate it.

VI. CONCLUSIONS

In this work, a novel tuning method for the PD+G controller is proposed. The method is obtained from the GAS result from LaSalle's invariance principle and robot dynamic properties. Both proportional and derivative gains are functions of the robot gravitational torques vector bound, which is a known element for the controller design. The tuning method does not require full knowledge of the robot dynamics.

Our approach is verified using two different planar robots and a 2-DOF mechanism and it is compared with other bounds obtained from the GAS result using strict Lyapunov functions. It is shown that our approach presents good results in comparison to the other tuning methods which require complete knowledge of the robot dynamics. The main advantage of the proposed tuning method is that we only require knowledge of the gravitational torques vector to tune the controller gains and the closed-loop dynamic presents fast and smooth convergence to the desired reference without overshoots and oscillations.

Recent results show that PD control can achieve position tracking for time variant references [27], however there is no stability proof that verifies the above fact. Our future research work focuses on finding Lyapunov functions candidates that verify position tracking of the PD, PD+G and PID controllers.

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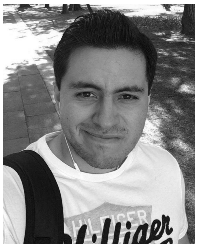

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