

Received June 8, 2020, accepted June 17, 2020, date of publication June 22, 2020, date of current version July 2, 2020. *Digital Object Identifier 10.1109/ACCESS.2020.3003919*

Quaternary Census Transform Based on the Human Visual System for Stereo Matching

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This work was supported in part by the S. LSI Division, Samsung Electronics Company Ltd., Hwaseong, South Korea.

ABSTRACT The census transform is a non-parametric local transform that is widely used in stereo matching. This transform encodes the structural information of a local patch into a binary code stream representing the relative intensity ordering of the pixels within the patch. Despite its high performance in stereo matching, the census transform often generates identical binary code streams for two different patches because it simply thresholds the pixels within the patch at the center pixel intensity. To overcome this problem, we introduce a quaternary census transform that encodes the local structural information into a quaternary code stream by employing both the relative intensity ordering and the minimum visibility threshold of the human eye known as the just-noticeable difference. Moreover, because the human eye activates different areas of the retina based on brightness, the patch size for the proposed quaternary census transform adaptively varies depending on the luminance of each pixel. Experimental results on well-known Middlebury stereo datasets prove that the proposed transform outperforms the other census transform-based methods in terms of the accuracy of stereo matching.

INDEX TERMS Census transform, depth estimation, disparity map, human visual system, similarity cost calculation, stereo image processing, stereo matching.

I. INTRODUCTION

Stereo matching is one of the most extensively studied topics in computer vision [1]–[3]. In general, stereo matching is composed of four steps: similarity cost calculation, cost aggregation, disparity selection, and disparity refinement [4], [5]. Among these steps, the similarity cost calculation is the most important, because the performance of the other three steps heavily depends on the similarity cost.

To achieve high-performance of stereo matching, various similarity cost calculation methods have been proposed such as sum of absolute difference [6], relative gradient [7], normalized cross-correlation [8], Mahalanobis distance crosscorrelation [9], and census transform [10]. Among these methods, the census transform has been popularly used for stereo matching because it recorded the highest performance [11]. The census transform [10] summarizes the local image structure of a square patch as a binary code stream that

The associate editor coordinating the [rev](https://orcid.org/0000-0003-0017-1398)iew of this manuscript and approving it for publication was Yue Zhang

represents the relative intensity ordering of the pixels in the patch by thresholding the pixels within the patch using the center pixel intensity. However, the census transform solely depends on the relative intensities of pixels rather than the intensity values themselves. This often gives rise to matching ambiguity that generates an identical binary code stream for flat and textured patches.

To alleviate this drawback, assorted variants of the census transform have been proposed. Mei *et al.* combined the census transform with the conventional pixel-wise absolute difference (AD-Census) to take the advantages of both [12]. Shi *et al.* applied the census transform in the relative gradient [7] domain instead of the conventional spatial domain (RG-Census) [13]. Chang *et al.* proposed a modified version of the census transform called a trinary cross color census transform (TCC-Census) [14]. The TCC-Census extends the census transform to a three-level transform by additionally encoding the pixel intensities in a certain margin around the center pixel intensity and utilizes the crosssquare shaped patch. Lee *et al.* presented a three-moded

census transform (3M-Census) that not only extends the census transform to the three-level transform as in the case of TCC-Census, but also combines it with other similarity cost calculation methods such as color and gradient differences [15]. These methods, the TCC-Census [14] and 3M-Census [15], apply the additional margin to alleviate the matching ambiguity caused by the census transform. However, the TCC-Census and 3M-Census result in another type of matching ambiguity in flat regions of the image because they discard the structural information of those regions. Recently, a four-moded census transform (4M-Census) [16], [17] that extends the census transform to a four-level transform by additionally exploiting mean values of patch has been proposed. However, the 4M-Census gets only a minor performance gain of stereo matching.

Taken together, the existing variants of the census transform can be classified into two approaches: applying the census transform with another method or modifying the census transform itself. The focus of the first approach is to find a method that can supplement the limitation of the census transform. However, this approach can be counterproductive. For example, the pixel-wise absolute difference in the AD-Census may undermine the strength of the census transform such as its invariance to monotonic global gray-level shift. The second approach, on the other hand, is more versatile than the first approach because, if necessary, it can be easily extended to the first approach.

In this study, we introduce a quaternary census transform (QCT) that adopts two properties of the human visual system to solve the limitations of the conventional census transform and other census transform-based methods. First, the proposed transform utilizes the just-noticeable difference (JND), which is the minimum visible threshold of the human eye, along with the conventional relative intensity ordering to generate a quaternary code stream that summarizes more detailed structural information. Second, based on the characteristic of the human retina where the area of the activated light receptors varies according to brightness, the proposed transform exploits a variable-sized patch (VSP) whose size changes depending on the luminance of each pixel. Experimental results on the Middlebury stereo datasets [18]–[21] demonstrate that the proposed similarity measure outperforms the conventional measures in terms of the accuracy of stereo matching.

The remainder of this paper is organized as follows. The census transform and its variations are presented in Section II. In Section III, we present the QCT and describe the detail of the proposed transform. Experimental results are presented in Section IV to validate the effectiveness of the proposed transform. In Section V, we conclude the study.

II. RELATED WORKS

A. CENSUS TRANSFORM

The census transform [10] encodes the local structural information of a pixel **u** in an input image *I* into a binary code stream that represents the relative intensity ordering of the

pixels within the patch as follows:

$$
C(\mathbf{u}) = \otimes_{\mathbf{v} \in W} f(I_{\mathbf{u}}, I_{\mathbf{v}}), \qquad (1)
$$

where the symbol ⊗ denotes concatenation, *W* represents the set of pixels in a fixed-size square patch around **u**, and *I***^v** represents the intensity at pixel **v**. In [\(1\)](#page-1-0), the binary encoding function (BEF) *f* is defined as follows:

$$
f(I_{\mathbf{u}}, I_{\mathbf{v}}) = \begin{cases} 1, & \text{if } I_{\mathbf{u}} < I_{\mathbf{v}}, \\ 0, & \text{otherwise.} \end{cases}
$$
 (2)

Then, the similarity between the pixels \mathbf{u}_l and \mathbf{u}_r in the left and right images, respectively, is computed as follows:

$$
S(\mathbf{u}_l, \mathbf{u}_r) = 1 - \frac{H(C(\mathbf{u}_l), C(\mathbf{u}_r))}{N(W)},
$$
\n(3)

where $N(\cdot)$ is the number of pixels inside the input patch except for the center pixel, and $H(\cdot, \cdot)$ returns the Hamming distance between the input binary code streams which stands for the number of entries at which the corresponding codes are different.

B. TRINARY CROSS COLOR CENSUS TRANSFORM (TCC-CENSUS)

The TCC-Census [14] utilizes the trinary encoding function (TEF) f_T to encode the pixels within a fixed-sized crosssquare patch (CSP) into a binary code stream as follows:

$$
C_T(\mathbf{u}) = \otimes_{\mathbf{v} \in W_T} f_T(I_{\mathbf{u}}, I_{\mathbf{v}}), \qquad (4)
$$

where W_T refers to the set of pixels in the fixed-sized CSP and the TEF is defined as

$$
f_T(I_{\mathbf{u}}, I_{\mathbf{v}}) = \begin{cases} 10, & \text{if } I_{\mathbf{u}} + \alpha < I_{\mathbf{v}}, \\ 01, & \text{if } I_{\mathbf{v}} < I_{\mathbf{u}} - \alpha, \\ 00, & \text{otherwise.} \end{cases} \tag{5}
$$

In [\(5\)](#page-1-1), α indicates the margin proportional to the intensity of the center pixel, which is computed as

$$
\alpha = \left[\frac{I_u}{\beta}\right],\tag{6}
$$

where [·] denotes the nearest integer function and β is a user-defined constant. As in the case of the census transform, the similarity between \mathbf{u}_l and \mathbf{u}_r is computed using the Hamming distance in [\(3\)](#page-1-2).

C. THREE-MODED CENSUS TRANSFORM (3M-CENSUS)

The 3M-Census [15] computes the similarity for stereo matching by using the distance of intensity and gradient as well as the Hamming distance. To calculate the similarity cost, the 3M-Census integrates the three distances as

$$
S_{3MCT}(\mathbf{u}_{l}, \mathbf{u}_{r}) = 3 - \exp\left(-\frac{H(C_{T}(\mathbf{u}_{l}), C_{T}(\mathbf{u}_{r}))}{\gamma_{H}}\right) - \exp\left(-\frac{\Delta I_{\mathbf{u}_{l}, \mathbf{u}_{r}}}{\gamma_{I}}\right) - \exp\left(-\frac{\Delta G_{\mathbf{u}_{l}, \mathbf{u}_{r}}}{\gamma_{G}}\right), \tag{7}
$$

116502 VOLUME 8, 2020

FIGURE 1. Input stereo image pair and resultant disparity maps of ''Books'' from the Middlebury stereo datasets. The magnified parts of the images are shown in the bottom row. (a) Pixel \mathbf{p}_1 in the left image; (b) \mathbf{p}'_1 in the right image that corresponds to \mathbf{p}_1 , and \mathbf{p}'_2 in the right image that dose not corresponds to **p**¹ ; (c) ground truth disparity map; (d) resultant disparity map obtained using the census transform; (e) resultant disparity map obtained using the proposed method.

		226 226 228			222 223 226 180 184 214				$\mathbf{0}$																IV.
$ 225 _{(227)}^{\mathbf{p}_1}$	\mathbf{D}_1			(223)	I 44.		\mathbf{p}_2	$-1(185)$ 2141	$\bf{0}$						\mathbf{n}'			\mathbf{p}_1						\mathbf{D}'_2	١V
					226 228 228 222 222 227 1	'791		187 215	$\bf{0}$				M								ΙV				w
(a)						(b)																			

F<mark>IGURE 2.</mark> 3 × 3 patches with center pixels p₁, p₁, and p₂ and their encoded patches. For the encoded patches with center pixels p₁ and p₂, neighboring
pixels that have a Hamming distance value of "1" as compare center pixel of p_1 , p'_1 , and p'_2 ; (b) encoded patches of (a) using the BEF; (c) encoded patches of (a) using the proposed QEF.

where $\Delta I_{\mathbf{u}_l, \mathbf{u}_r}$ and $\Delta G_{\mathbf{u}_l, \mathbf{u}_r}$ represent the intensity and gradient distance between \mathbf{u}_l and \mathbf{u}_r , respectively, and γ_H , γ_I , and γ ^{*G*} are empirical parameters.

D. LOCAL BINARY PATTERNS AND ITS VARIANTS

The local binary pattern (LBP) [22] is widely employed in the field of texture classification, whereas the census transform is successfully adopted for stereo matching. The LBP encodes binary code stream, which represents the local structural pattern same as the census transform. In stereo matching, binary code streams are used with the Hamming distance to calculate the similarity between two pixels in the horizontal scanline. However, in texture classification, all binary code streams of an image are gathered to generate the histogram, which is used as the image descriptor. In similar way to other variants of the census transform, the LBP has been modified to achieve the better performance. Similar to the TCC-Census and 3M-Census that use multi-level encoding function, local ternary pattern (LTP) [23] and elongated quinary pattern (EQP) [24] were proposed. There were also attempts to change the shape or size of the patch [25], [26]. Similar to the RG-Census, there were several LBP-based methods that perform the domain transformation before encoding the binary code stream [26]–[28]. However, owing to the difference between stereo matching and texture classification, these methods [26]–[28] adopt the domain transformation to make the image descriptor rotation-invariant, which is not desired for stereo matching.

III. PROPOSED METHOD

Precise similarity calculation is necessary for robust stereo matching. To overcome the limitations of the conventional

census transform-based methods discussed in Section II, both accurate encoding function and support of adaptive patch sizes are required. In this section, we present a QCT that adopts a JND-based quaternary encoding function (QEF) and VSP that automatically changes its size depending on the luminance of each pixel.

A. JND-BASED QCT

The census transform produces a binary code stream by using the BEF. Because the BEF employs relative intensity ordering and simply thresholds the neighboring pixels using the center pixel intensity, the BEF generates the identical code ''1'' for a pixel having a higher intensity than the center pixel irrespective of the magnitude of the intensity difference.

In stereo matching, the census transform often fails to distinguish a flat patch consisting of pixels having intensities similar to the center pixel from a textured patch consisting of pixels having intensities noticeably different from the center pixel. Figs. 1 and 2 illustrate an example in which the census transform fails to distinguish the pixel in the flat region from the pixel in the textured region in ''Books'' from the Middlebury datasets $[18]$ – $[21]$. For a pixel p_1 in the left image of Fig. [1\(](#page-2-0)a), pixels \mathbf{p}'_1 and \mathbf{p}'_2 in the right image of Fig. [1\(](#page-2-0)b) are corresponding pixel and not-corresponding pixel to \mathbf{p}_1 , respectively. Fig. [2\(](#page-2-1)a) shows 3×3 patches at the center pixels \mathbf{p}_1 , \mathbf{p}'_1 , and \mathbf{p}'_2 . As shown in Fig. [2\(](#page-2-1)a), \mathbf{p}'_1 belongs to the same flat area as \mathbf{p}_1 , where the center and neighboring pixels have similar intensities. On the other hand, \mathbf{p}'_2 is located in a textured area around a vertical edge, where the center pixel and the three right most pixels show noticeable intensity differences. Using the BEF,

FIGURE 3. Input stereo image pair and resultant disparity maps of "Bowling1" from the Middlebury stereo datasets. The magnified parts of the images are shown in the bottom row. (a) Pixel **p**₃ in the left image; (b) p'_3 in the right image that corresponds to p_3 , and p'_4 in the right image that dose not corresponds to **p**³ ; (c) ground truth disparity map; (d) resultant disparity map obtained using the TCC-Census; (e) resultant disparity map obtained using the proposed method.

162 163 163 161 162 162 179 179 174		00 [°]			00 00 00	00	 00 00 	00	$\overline{00}$								H^{2}		
$\left 161 \right _{(162)}^{p_3} \left 163 \right \left 161 \right _{(161)}^{p'_3} \left 163 \right \left 180 \right _{(178)}^{p'_4} \left 176 \right $		00 ¹	\mathbf{p}_3		\parallel 00 \parallel 00 \mid p ₃		00 00	p_4'	$\overline{00}$			\mathbf{p}_3			\mathbf{p}'_3	$\mathbf{2}$	\mathbb{F}_2	p_4' v	
162 163 162 160 163 162 180 175 181		00	$\bf{00}$	100 H 00		00	00 00	00	$00\,$							$\sqrt{2}$	$\sqrt{2}$	$\mathsf{N}1$	$\mathsf{N2}$
(a)	(b)																		

FIGURE 4. 3 × 3 patches with center pixels p₃, p₃, and p₄ and their encoded patches. For the encoded patches with center pixels p₃ and p₄, neighboring
pixels that have a Hamming distance value of "1" as compared center pixel of p₃, p/₃, and p/₄; (b) encoded patches of (a) using the TEF; (c) encoded patches of (a) using the proposed QEF.

these three patches are encoded to binary patches as shown in Fig. [2\(](#page-2-1)b). Thus we obtain the similarities between **p**¹ and \mathbf{p}'_1 as $S(\mathbf{p}_1, \mathbf{p}'_1) = 0.875$ and between \mathbf{p}_1 and \mathbf{p}'_2 as $S(\mathbf{p}_1, \mathbf{p}_2') = 1$. As a result, \mathbf{p}_1 is mismatched to \mathbf{p}_2' , and the disparity map obtained using the census transform as shown in Fig. [1\(](#page-2-0)d) has the disparity error compared to the ground truth disparity map in Fig. [1\(](#page-2-0)c).

Although the TCC-Census does not have the aforementioned mismatching problem of the census transform, it faces difficulty in discriminating between different flat patches. This is because the TCC-Census adopts the TEF that encodes pixels having intensities similar to the center pixel into a single code and discards the fine structural information of flat patches. Figs. 3 and 4 demonstrate the failure case of the TCC-Census in ''Bowling1'' from the Middlebury datasets. Figs. 3(a) and (b) show pixel **p**³ in the left image and pixels \mathbf{p}'_3 and \mathbf{p}'_4 in the right image, respectively. These three pixels belong to the flat patches as shown in Fig. [4\(](#page-3-0)a), and their encoded patches obtained by using the TEF are identical as in Fig. [4\(](#page-3-0)b). Since $S(\mathbf{p}_3, \mathbf{p}_3') = S(\mathbf{p}_3, \mathbf{p}_4') = 1$, \mathbf{p}_3 can be mismatched to \mathbf{p}'_4 . As a result, the disparity map obtained using the TCC-Census in Fig. [3\(](#page-3-1)d) has the disparity error as compared to the ground truth disparity map in Fig. [3\(](#page-3-1)c).

To encode a more detailed structure of the patch and alleviate the aforementioned problems, we propose an intuitive and simple encoding function that stores distinct local structural information by allocating code values based on the visibility of the intensity difference. Motivated by the conventional methods [14], [15], [23], [24], we define our multi-level encoding function as follow:

$$
f'_{Q}(I_{\mathbf{u}}, I_{\mathbf{v}}) = \begin{cases} 3, & \text{if } I_{\mathbf{u}} + x < I_{\mathbf{v}}, \\ 2, & \text{if } I_{\mathbf{u}} < I_{\mathbf{v}} \leq I_{\mathbf{u}} + x, \\ 1, & \text{if } I_{\mathbf{u}} - x < I_{\mathbf{v}} \leq I_{\mathbf{u}}, \\ 0, & \text{otherwise}, \end{cases} \tag{8}
$$

where x is the minimum visibility threshold. To correctly define the minimum visibility threshold *x*, we exploit the property of the human visual system known as JND. The JND is the minimum visibility threshold of the human eye that is inversely proportional to brightness [29], [30]. The ratio of the JND to brightness *B* is modeled as follows:

$$
\frac{\text{JND}}{B} = k,\tag{9}
$$

where *k* represents the Weber fraction. To apply the JND to the image, *B* can be replaced by the background luminance obtained by applying Gaussian filtering to the image [31]. The JND of pixel **u**, denoted as JND**u**, can be written as

$$
JND_{\mathbf{u}} = k \cdot I_{\mathbf{u}}^{B},\tag{10}
$$

where $I_{\mathbf{u}}^B$ is the intensity of the background luminance at pixel **u**. Using [\(8\)](#page-3-2) and [\(10\)](#page-3-3), we define the JND-based QEF f_Q as follows:

$$
f_Q\left(I_\mathbf{u}^B, I_\mathbf{v}\right) = \begin{cases} 3, & \text{if } I_\mathbf{u}^B + \text{JND}_\mathbf{u} < I_\mathbf{v}, \\ 2, & \text{if } I_\mathbf{u}^B < I_\mathbf{v} \le I_\mathbf{u}^B + \text{JND}_\mathbf{u}, \\ 1, & \text{if } I_\mathbf{u}^B - \text{JND}_\mathbf{u} < I_\mathbf{v} \le I_\mathbf{u}^B, \\ 0, & \text{otherwise.} \end{cases} \tag{11}
$$

FIGURE 5. Two patches of the conventional methods and proposed VSP. (a) Square patch with a radius of 4 (Census transform); (b) cross-square patch (TCC-Census); (c) VSP with a radius ranging from 1 to 5.

Fig. [2\(](#page-2-1)c) illustrates that the proposed QEF successfully alleviates the problem of the BEF. Unlike the BEF that produces indistinguishable encoded patches, the proposed QEF generates encoded patches which distinguish \mathbf{p}'_2 from **p**¹ by employing both the JND and relative intensity ordering. By using the QEF, the similarity values are obtained as $S(\mathbf{p}_1, \mathbf{p}'_1) = 0.875$ and $S(\mathbf{p}_1, \mathbf{p}'_2) = 0.625$, and thus \mathbf{p}_1 can be correctly matched to \mathbf{p}_1 [']. As a result, the disparity map obtained using the proposed method no longer suffers from the disparity error in that region as shown in Fig. [1\(](#page-2-0)e). The QEF also alleviates the limitation of the TEF producing identical encoded patches for different flat patches. With the QEF, the similiarity values are obtained as \hat{S} (\mathbf{p}_3 , \mathbf{p}'_3) = 0.875 and $S(\mathbf{p}_3, \mathbf{p}'_4) = 0.125$, as shown in Fig. [4\(](#page-3-0)c). By preserving the fine structural information of the flat region, the QEF generates more similar encoded patches for the corresponding pixel pair, \mathbf{p}_3 and \mathbf{p}'_3 . Thus, the proposed method successfully produces the disparity map, as shown in Fig. [3\(](#page-3-1)e).

B. VARIABLE-SIZED PATCH (VSP)

To compute an accurate similarity cost using census transform-based methods, a patch with a proper size is as crucial as a precise encoding function. However, only a few studies have been conducted to find proper patch sizes for these methods. The census transform [10], AD-Census [12], and RG-Census [13] simply apply a square patch with a fixed size as shown in Fig. [5\(](#page-4-0)a). TCC-Census [14] and 3M-Census [15] also utilize a fixed-sized patch with a different shape, namely a cross-square shaped patch, as shown in Fig. [5\(](#page-4-0)b).

In order to find an appropriately patch size for similarity cost computation, we exploit another property of the human visual system. In the field of texture classification, there has already been successful examples that observe micro and macro patterns simultaneously by mimicking human retina characteristics [25]. According to [32], [33], the human retina contains two types of light receptors: rods and cones. Rods are located throughout the retina and function at low brightness, whereas cones are densely concentrated at the center of the retina and function at high brightness. Therefore, as the brightness level decreases, the periphery of the retina becomes more sensitive than the central region of the retina because of the distribution of rods and cones.

Based on this property of the retina, we carried out an experiment, wherein under varying background luminance conditions, the impact of patch size on the accuracy of stereo matching was investigated. The stereo matching that employs the proposed encoding function using various sized patches with a radius ranging from 1 to 5 was applied to five stereo image pairs from the Middlebury stereo datasets [18]–[21] ("Baby1," "Flowerpots," "Wood1," "Teddy," and ''Tsukuba''). Every pixel of these images was classified as one of the four groups according to the intensity of its background luminance. Then, the stereo matching accuracy was computed in terms of the average percentage of bad pixels (APBP) for each group, which is defined as follows:

APBP(
$$
\%
$$
) = $\frac{100}{N(G)} \sum_{\mathbf{p} \in G} I\left(|D_{\mathbf{p}} - d_{\mathbf{p}}| > 1 \right)$, (12)

where *G* represents one of the four groups previously mentioned, and $N(G)$ returns the number of pixels in each group. In addition, $I(\cdot)$ stands for the indicator function, and D_p and *d***^p** represent the ground truth disparity value and resultant disparity value at pixel **p**, respectively.

Table [1](#page-5-0) indicates that when the background luminance is low, the stereo matching using a larger patch tends to record a higher accuracy. Based on this observation, the proposed transform exploits a VSP whose patch size is inversely proportional to the intensity of the background luminance. A radius of the VSP at pixel **u** is defined as follows:

$$
r_{\mathbf{u}} = \left[r_{\text{max}} \cdot \exp\left(-\frac{I_{\mathbf{u}}^B}{c} \right) \right],\tag{13}
$$

where $\lceil \cdot \rceil$ denotes the ceiling function, r_{max} is the maximum radius of the VSP, and *c* is a constant to adjust the sensitivity of the background luminance. Therefore, the proposed quaternary code stream of **u** is obtained as follows:

$$
C_Q(\mathbf{u}) = \otimes_{\mathbf{v} \in W_Q} f_Q\left(I_\mathbf{u}^B, I_\mathbf{v}\right),\tag{14}
$$

where W_O is the set of pixels in the VSP. The quaternary code stream for a pixel in the target image is obtained in the same manner. Finally, the similarity between pixels **u***^l* and **u***^r* in the left and right images, respectively, is calculated as:

$$
S_Q(\mathbf{u}_l, \mathbf{u}_r) = 1 - \frac{H\left(C_Q(\mathbf{u}_l), C_Q(\mathbf{u}_r)\right)}{N\left(W_Q\right)},\tag{15}
$$

where $N(\cdot)$ is the number of pixels inside the input patch except for the center pixel, and $H(\cdot, \cdot)$ returns the Hamming distance between the input quaternary code streams.

IV. EXPERIMENTAL RESULTS

To evaluate the performance of the proposed transform, 23 stereo image pairs and their corresponding groundtruth disparity maps available in the Middlebury stereo datasets [18]–[21], [34] were used. Each image was converted into a *lab* color space, and *l* channel was utilized as input data. The proposed QCT was compared with the popular

The best and second-best results are boldfaced and underlined, respectively.

TABLE 2. APBP (%) on the 23 Stereo Image Pairs and the Average APBP (%).

				non-occlusion			all								
Data	Census	TCC	4M	QCT_{SP}^{-1}	$\mathrm{QCT}_{\mathrm{CSP}}{}^2$	QCT^3	Census	TCC	4M	QCT_{SP}^{-1}	$\mathrm{QCT}_{\mathrm{CSP}}{}^2$	QCT^3			
	transform	Census	Census				transform	Census	Census						
Art	13.48	12.55	24.22	12.65	11.45	11.71	30.84	30.52	39.44	30.50	29.50	29.73			
Baby1	4.33	4.37	8.44	4.04	3.85	3.82	11.97	11.84	15.61	11.38	11.12	11.10			
Baby2	4.95	7.94	8.67	4.48	4.77	4.31	12.68	15.05	16.56	12.17	12.35	11.93			
Books	13.79	13.24	16.76	13.11	12.70	12.17	23.55	22.97	25.49	22.96	22.58	22.09			
Bowling1	9.15	9.60	14.60	8.54	8.10	8.17	24.44	24.76	28.96	23.63	23.21	23.25			
Cloth1	2.76	2.30	6.35	2.18	2.17	2.13	12.36	11.88	14.88	11.58	11.62	11.57			
Cloth ₂	5.91	5.48	12.17	5.57	5.42	5.44	18.84	18.46	24.04	18.57	18.38	18.39			
Cloth3	3.63	3.18	7.42	3.00	2.91	2.91	13.73	13.18	16.41	12.99	12.92	12.90			
Cloth4	2.62	2.49	6.63	2.45	2.32	2.36	17.38	17.26	20.03	17.01	16.80	16.84			
Dolls	8.24	6.97	15.69	7.00	6.82	6.82	19.81	18.55	26.08	18.53	18.35	18.43			
Flowerpots	12.31	12.31	16.26	11.51	10.89	10.89	32.05	31.28	35.42	31.42	30.99	30.98			
Midd1	42.43	46.95	46.34	42.18	43.23	41.42	47.94	51.90	51.13	47.67	48.68	47.04			
Moebius	11.10	11.65	17.05	10.19	9.70	9.84	21.62	22.00	26.93	20.94	20.37	20.64			
Plastic	36.56	51.66	38.19	35.38	38.14	35.30	42.86	56.70	44.76	42.37	44.62	42.18			
Reindeer	12.39	14.23	21.77	10.49	10.54	10.20	26.73	28.09	34.51	25.37	25.28	25.17			
Rocks1	6.49	6.11	9.99	5.59	5.55	5.46	17.67	17.28	19.94	16.56	16.53	16.46			
Rocks2	4.07	3.51	7.28	3.52	3.45	3.47	17.33	16.76	19.42	16.76	16.70	16.72			
Wood1	8.60	9.77	15.04	8.49	7.91	7.71	20.43	21.27	26.30	21.02	20.36	20.26			
Wood2	5.06	8.07	7.95	4.85	4.49	4.47	17.02	19.62	19.41	17.21	16.74	16.78			
Cones	5.33	4.35	9.46	4.52	4.29	4.25	17.75	16.86	20.90	17.03	16.78	16.80			
Teddy	9.03	9.85	15.23	8.58	8.44	8.31	19.63	20.43	24.85	19.36	19.20	19.12			
Tsukuba	4.35	5.96	4.38	3.77	4.61	3.41	23.55	24.85	23.52	23.28	23.86	22.87			
Venus	2.02	2.17	2.87	1.98	1.79	1.84	5.11	5.18	6.24	5.00	4.77	4.90			
Average	9.94	11.07	14.47	9.31	9.28	8.97	21.53	22.46	25.25	21.01	20.94	20.70			

The best and second-best results are boldfaced and underlined, respectively.

¹ QCT_{SP} – QEF with the SP with a radius of 4

 2 QCT_{CSP} – QEF with the CSP

 3 QCT – QEF with the VSP

census transform [10], AD-Census [12], RG-Census [13], TCC-Census [14], and 4M-Census [17] proposed recently.

Because the proposed transform is composed of the QEF and the VSP, we conducted experiments on three versions of the proposed transform to validate the effectiveness of each. The first version, QCT_{SP} , exploits the proposed QEF with a fixed-sized square patch (SP) applied to the census transform, and the second version, QCT_{CSP} , utilizes the proposed QEF with a fixed-sized cross-square patch (CSP) applied to the TCC-Census. Finally, the last version, QCT, exploits the QEF

with the VSP. In case of the AD-Census and RG-Census that use the original census transform with other methods, we adopt parameter settings in their works. For the census transform and 4M-Census, the radius of the SP was set to 4, which records the highest accuracy with the census transform in the experimental data as shown in Fig. [6.](#page-6-0) In the case of the TCC-Census, the parameter for the TEF, β , was set to 50, and the radius of the CSP was set to 5. For the proposed QCT, *r*max and *c* were empirically set to 7 and 70 as shown in Fig. [7.](#page-6-1) The Weber fraction, *k*, was set to 0.14 according

FIGURE 6. Average APBP of the census transform with different patch radii on the 23 stereo image pairs. The radius varies from 1 to 8.

FIGURE 7. Average APBP of the proposed QCT with different parameters. (a) Average APBP of the QCT with different r_{max} ; (b) average APBP of the QCT with different c.

to [35]. Finally, the background luminance was obtained by applying Gaussian filtering with a standard deviation of 1.5.

The accuracy of stereo matching was evaluated in terms of the APBP in two types of regions: the non-occlusion region and all regions. Note that the similarity aggregation [6] was performed, and the disparity refinement was excluded from all the experiments for fair comparison. Tables 2 and 5 list the APBP of the stereo matching methods on various image pairs and the average APBP of all image pairs, where the best and second-best results are boldfaced and underlined, respectively. In Table [2,](#page-5-1) results of the aforementioned three versions of QCT, the census transform and conventional methods that modify the encoding function or patch shape of the census transform are listed. Note that all regions include the occlusion area where the disparity value could not be estimated. Thus, the APBP in all regions is much higher than that in the non-occlusion region.

A. BEF VS. QEF

To validate the effectiveness of the QEF as compared to the conventional BEF, the QCT_{SP} using the same sized SP was compared to the conventional census transform that utilizes the BEF. Because the QEF stores local structural information in the patch more precisely than the BEF does, the QCT_{SP} showed a 6.36% improvement in average APBP as compared to the census transform. In particular, the QCT_{SP} recorded a considerable improvement in accuracy as compared to the census transform in ''Cloth1,'' ''Cloth3,'' "Dolls," "Reindeer," and "Cones" as listed in the left two columns of Table [3.](#page-6-2) Fig. [8](#page-7-0) demonstrates the experimental

TABLE 3. Accuracy Improvement Ratio of the Proposed QEF (Top Five Data).

TABLE 4. Accuracy Improvement Ratio of the Proposed VSP (Top Five Data).

data and resultant disparity maps of ''Cones,'' where the BEF failed to distinguish between flat and textured areas. As shown in Figs. 8(d) and (e), the resultant disparity map of the census transform suffered from a disparity error in the enlarged region where the flat and textured areas appeared repeatedly. QCT_{SP} , on the other hand, successfully resolved the problem of the BEF and produced a resultant disparity map with reduced disparity errors in that region as shown in Figs. $8(f)$ and (g) .

B. TEF VS. QEF

The QCT_{CSP} that applies the QEF with the CSP was compared to the TCC-Census to verify that the QEF has an advantage over the conventional TEF. Unlike the TEF that generates an identical encoded patch for different flat patches, the proposed QEF produces distinct encoded patches. Therefore, the QCT_{CSP} showed an average performance improvement of 16.16% as compared to the TCC-Census. As listed in the right two columns of Table [3,](#page-6-2) in the ''Baby2,'' ''Plastic,'' "Reindeer," "Wood2," and "Tsukuba" which contain broad flat areas, the QCT_{CSP} showed an appreciable performance improvement as compared to the TCC-Census. Fig. [9](#page-7-1) illustrates the experimental data and resultant disparity maps of ''Wood2.'' At the bottom of each image, the area with low texture in the input left image is displayed. As shown in Figs. 9(d) and (e), the resultant disparity map of the TCC-Census suffered from an enormous disparity error in flat areas because the TEF could not distinguish flat areas as previously mentioned. However, Figs. 9(f) and (g) demonstrate that the QCT_{CSP} significantly reduced the disparity error.

C. SP & CSP VS. VSP

We compared the proposed VSP with the conventional SP and CSP to validate the effectiveness of the VSP. To accomplish this, the QCT was compared to QCT_{SP} and QCT_{CSP} . Compared to QCT_{SP} and QCT_{CSP} , the QCT showed an average performance improvement of 3.58% and 3.33%, respectively. Notably, because the VSP was used instead of SP,

FIGURE 8. Experimental data and results of ''Cones'' from the Middlebury stereo datasets. The magnified parts of the images are shown in the bottom row. (a) Input left image; (b) ground truth disparity map; (c) occlusion map; (d) disparity map obtained by the census transform; (e) error map of (d) in the non-occlusion area; (f) disparity map obtained by QCT_{SP} ; (g) error map of (f) in the non-occlusion area.

FIGURE 9. Experimental data and results of "Wood2" from the Middlebury stereo datasets. The magnified parts of the images with low texture area are shown in the bottom row. (a) Input left image; (b) ground truth disparity map; (c) occlusion map; (d) disparity map obtained by the TCC-Census; (e) error map of (d) in the non-occlusion area; (f) disparity map obtained by QCT_{CSP} ; (g) error map of (f) in the non-occlusion area.

FIGURE 10. Experimental data and results of ''Tsukuba'' from the Middlebury stereo datasets. The magnified parts of the images with disparity discontinuity area are shown in the bottom row. (a) Input left image. (b) ground truth disparity map; (c) disparity map obtained by the QCT; (d) error map of (c) in the non-occlusion area; (e) disparity map obtained by QCT_{SP}; (f) error map of (e) in non-occlusion area; (g) disparity map obtained by QCT_{CSP}; (h) error map of (g) in the non-occlusion area.

the QCT achieved an accuracy improvement of nearly 10% with "Wood1" and "Tsukuba," as shown in the left two columns of Table [4.](#page-6-3)

In addition, compared to QCT_{CSP} , the QCT showed a 25.93% performance improvement with ''Tsukuba'' as shown in the right two columns of Table [4.](#page-6-3) Fig. [10](#page-7-2) demonstrates the experimental data and results of ''Tsukuba,'' which showed significant accuracy improvements in both cases. At the bottom of each image, the area where depth values are discontinuous in the ground truth disparity map is displayed. As shown in Figs. 10(d) and (f), the VSP mitigated the disparity error in the red-boxed region as compared to the SP by properly adjusting the size of the patch for the QEF. In addition, Figs. 10(d) and (h) demonstrate that the VSP substantially alleviated the disparity error derived from the CSP in the blue-boxed region.

D. VARIANTS OF CENSUS TRANSFORM VS. QCT

In this subsection, the comparison between the proposed QCT and conventional census transform-based methods was made on extended stereo image pairs and the results are listed in Table [5.](#page-8-0) In comparison with the AD-Census and RG-Census that use the original census transform with other similarity measures, the proposed method improved the average accuracy by 8.03% and 27.37%, respectively. The AD-Census successfully improved the conventional census transform and even showed higher accuracy compared with the proposed QCT in ''Baby2'', ''Rocks2'' and ''Vintage''. However, the AD-Census suffered from numerous disparity error on the image pairs, where the left and right images were captured in different environments such as ''ArtL'', ''MotorcycleE'', and ''PianoL'' due to the absolute difference which is utilized with the census transform. In comparison with the

The best and second-best results are boldfaced and underlined, respectively.

 1 Stereo image pairs that right images were captured under different Lighting.

² Stereo image pair that right image was captured under different Exposure.

³ Stereo image pair that was Perfectly calibrated.

FIGURE 11. Experimental data and results of ''PlaytableP'' from the Middlebury stereo datasets. The magnified parts of the images with repetitive patterns are shown in the bottom row. (a) Ground truth disparity map and input left image; (b) disparity map obtained by the AD Census and its error map; (c) disparity map obtained by the RG Census and its error map; (d) disparity map obtained by the census transform and its error map; (e) disparity map obtained by the TCC Census and its error map; (f) disparity map obtained by the 4M Census and its error map; (g) disparity map obtained by the QCT and its error map.

census transform, the proposed QCT significantly improved the performance of 38.64% in ''PlaytableP,'' and exhibited an average performance improvement of 8.85%. In comparison with the TCC-Census, the proposed QCT improved the performance from at least 0.77% in ''Cloth2'' to over 45.69% in ''Baby2,'' and exhibited an average improvement of 16.42%. Compared to the the 4M-Census, the proposed QCT improved the accuracy from at least 7.57% in ''Plastic'' to over 66.51% in "Cloth1," and exhibited an average accuracy improvement of 31.23%. As shown in Fig 11, the pro-

		LBP & Variants of LBP									
Data	$_{\rm LBP}$ (CLBP _S)	LTP	LQP	$CLBP_M$	$CLBPM+S$	QEF					
Art	13.48	26.76	16.02	18.50	14.15	12.65					
Baby1	4.33	12.79	5.40	5.83	4.88	4.04					
Baby2	4.95	18.71	8.76	6.89	5.40	4.48					
Books	13.79	23.16	15.01	14.97	12.65	13.11					
Bowling ₂	9.15	45.84	13.64	10.60	9.33	8.54					
Cloth1	2.76	4.01	4.10	4.81	3.73	2.18					
Cloth ₂	5.91	11.70	7.17	8.16	6.83	5.57					
C loth 3	3.63	6.82	3.73	5.41	4.20	3.00					
Cloth4	2.62	6.77	2.46	4.68	3.10	2.45					
Dolls	8.24	16.35	9.27	11.07	8.70	7.00					
Flowerpots	12.31	38.57	18.41	12.94	12.13	11.51					
Midd1	42.43	60.44	48.54	52.88	42.15	42.18					
Moebius	11.10	21.67	12.63	12.34	10.12	10.19					
Plastic	36.56	71.88	56.59	50.89	38.58	35.38					
Reindeer	12.39	38.43	24.11	13.51	11.44	10.49					
Rocks1	6.49	8.75	6.45	7.61	6.76	5.59					
Rocks2	4.07	7.16	3.87	5.85	4.85	3.52					
Wood1	8.60	26.66	17.44	9.46	7.72	8.49					
Wood2	5.06	37.48	10.54	5.44	4.80	4.85					
Cones	5.33	9.03	4.65	7.89	5.86	4.52					
Teddy	9.03	19.19	11.35	11.91	9.74	8.58					
Tsukuba	4.35	6.30	3.92	4.40	4.29	3.77					
Venus	2.02	15.63	4.21	2.51	1.88	1.98					
Average	9.94	23.22	13.40	12.55	10.14	9.31					

TABLE 6. APBP (%) on the 23 Stereo Image Pairs and the Average APBP (%) in Non-Occlusion Region.

The best and second-best results are boldfaced and underlined, respectively.

posed method successfully alleviates disparity error on the floor where the other methods suffer from large disparity error due to repetitive patterns. Consequently, the proposed QCT exhibited promising accuracy consistently over the entire experimental data.

E. VARIANTS OF LBP VS. QEF

As mentioned in Section II, the LBP has been actively researched for texture classification. Among the variants of the LBP, some can be applied to stereo matching. In this subsection, we compared the proposed QEF with the LTP, local quinary patter (LQP) [24] and completed-LBP (CLBP) [36]. The CLBP consists of three methods: $CLBP_Sign$ ($CLBP_S$), $CLBP_Magnitude$ ($CLBP_M$), and $CLBP_C$ center gray level ($CLBP_C$). The $CLBP_C$ converts an input image to a binary image by global thresholding, resulting in excessive information loss for stereo matching. On the other hand, since the $CLBP_S$ and $CLBP_M$ use the same method of encoding the code stream as the census transform, these methods can be directly employed for stereo matching. Also, the $CLBP_S$ is identical to the LBP/c ensus transform, and thus we conducted experiments with two types of the CLBP: CLBP_M and CLBP_{M+S} whose output is obtained by concatenating the outputs of the $CLBP_M$ and $CLBP_S$. As in the previous experiments, we used a square patch of radius 4, the similarity aggregation method [6], and the encoding functions as follows: LBP, LTP, LQP, CLBP_M, CLBP_{M+S}, and QEF.

Table [6](#page-9-0) lists the APBP of the stereo matching methods using the LBP variants and QEF on the stereo image pairs. The proposed QEF recorded significantly better performance than LTP in the stereo matching. Compared with the LBP (CLBP_S), LQP, CLBP_M, and CLBP_{M+S}, the QEF showed higher accuracy of 6.36%, 25.81%, 30.55%, and 8.23%, respectively.

F. NOISE ROBUSTNESS OF QEF (vs. BEF & TEF)

To evaluate the noise robustness of the proposed QEF compared to the conventional BEF and TEF, we conduct additional experiments in noise environment. Following the experiment in [27], we considered different level of additive

TABLE 7. APBP (%) on the 23 Stereo Image Pairs with AWGN and the Average APBP (%) in Non-Occlusion Region.

		AWGN with $\sigma = 5$			AWGN with $\sigma = 10$			AWGN with $\sigma = 15$		AWGN with $\sigma = 25$			
Data	BEF	TEF	QEF	BEF	TEF	QEF	BEF	TEF	QEF	BEF	TEF	QEF	
Art	24.19	23.56	18.63	45.99	44.89	33.54	63.34	62.61	50.75	81.50	81.22	74.47	
Baby1	12.58	11.15	10.03	32.07	29.78	22.77	50.90	49.63	44.86	77.06	76.21	75.05	
Baby2	21.65	17.09	15.95	39.18	36.58	28.83	55.65	54.00	45.92	79.63	78.81	74.68	
Books	24.13	21.71	19.22	37.64	35.10	28.92	55.73	53.60	47.90	77.40	76.38	72.83	
Bowling2	36.45	34.21	29.91	56.53	54.43	47.58	71.52	69.91	62.32	87.38	86.79	80.96	
Cloth1	5.47	5.46	3.98	8.39	8.22	6.23	15.93	15.46	14.03	43.27	42.46	46.88	
Cloth ₂	11.25	10.13	9.32	22.86	21.11	18.06	43.16	40.76	36.91	75.63	74.22	70.94	
Cloth3	7.21	7.08	5.22	15.40	14.61	10.64	31.68	30.42	26.13	62.28	61.05	59.47	
Cloth4	6.28	6.28	4.81	12.81	11.96	9.35	24.51	23.26	19.32	54.26	52.48	51.43	
Dolls	19.17	18.52	13.92	32.84	31.94	23.13	46.96	46.09	36.28	70.13	69.35	61.73	
Flowerpots	43.23	39.50	31.51	67.68	66.55	51.65	82.20	81.47	67.66	91.72	91.63	82.11	
Midd1	58.28	57.87	55.76	69.53	68.70	63.78	77.36	76.84	72.91	86.14	85.93	84.55	
Moebius	20.52	19.45	17.65	35.73	34.84	27.80	57.59	56.63	48.31	80.00	79.48	72.77	
Plastic	70.21	66.49	63.88	80.19	77.96	72.85	86.74	85.90	80.19	93.14	92.85	89.15	
Reindeer	27.61	27.20	24.82	54.92	53.17	45.14	69.30	68.46	60.10	82.91	82.65	76.26	
Rocks1	10.99	10.79	9.67	22.05	21.46	17.94	40.17	39.45	34.87	71.60	71.04	68.88	
Rocks2	7.08	6.95	5.82	15.49	15.09	12.27	31.88	31.05	26.41	67.45	66.65	62.88	
Wood1	24.67	23.16	22.06	47.63	46.57	40.37	74.47	73.59	67.43	90.98	90.69	86.93	
Wood2	39.04	37.43	36.66	63.63	62.16	58.22	80.68	80.16	77.57	91.79	91.63	91.08	
Cones	7.90	7.59	5.86	16.70	15.63	12.59	32.93	31.54	27.18	62.13	61.06	57.98	
Teddy	17.65	16.95	15.29	36.86	35.67	28.05	53.86	52.92	46.75	74.33	73.69	70.34	
Tsukuba	11.14	10.25	6.95	20.91	20.12	14.06	34.61	33.93	28.47	51.18	51.07	50.17	
Venus	15.23	14.46	11.75	33.83	32.53	25.72	45.93	44.98	41.33	64.59	63.96	65.47	
Average	22.69	21.45	19.07	37.78	36.48	30.41	53.35	52.29	46.24	74.63	73.97	70.74	

The best and second-best results are boldfaced and underlined, respectively.

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FIGURE 12. Experimental data and disparity error images of ''No.79'', ''No.85'', and ''No.132'' from the KITTI2015 datasets. The disparity error images are visualized using the color scheme depicted in the legend. The disparity error images are displayed from top-to-bottom: result of AD-Census, RG-Census, Census transform, TCC-Census, 4M-Census, and the proposed method. (a) Input ''No.79'' left image; (b) input ''No.85'' left image; (c) input ''No.132'' left image; (d) disparity maps of (a); (e) disparity maps of (b); (f) disparity maps of (c).

white Gaussian noise (AWGN). For this experiment, we used a square patch of radius 4 and the similarity aggregation method [6] with BEF, TEF, and QEF.

Table [7](#page-9-1) lists the APBP of the stereo matching methods using the three encoding functions on the stereo image pairs with AWGN. In the case of AWGN with $\sigma = 5$, 10, and 15, the proposed QEF achieved 16.26% and 13.09% higher accuracy on average compared to the BEF and TEF, respectively. In the extreme case of AWGN with $\sigma = 25$, compared to the BEF and TEF, the QEF still showed higher accuracy of 5.21% and 4.37%, respectively.

V. CONCLUSION

This paper presented a quaternary census transform for similarity measure in stereo matching. To overcome the limitations of the conventional transforms, the proposed transform adopts two properties of the human visual system: the minimum visibility threshold of the human eye, the JND, and the varying area of the human retina activated depending on the luminance. Therefore, the proposed transform summarizes more detailed local structural information in a variable-sized patch by using both the JND and relative intensity ordering as compared to the conventional methods. Experimental results demonstrate that the proposed transform significantly improves the accuracy of stereo matching as compared to the conventional methods.

APPENDIX A PERFORMANCE EVALUATION IN THE OUTDOOR ENVIRONMENTS

Since Middlebury stereo datasets were captured in the indoor environments, we conducted additional experiments on KITTI2015 datasets [37] to evaluate the performance of stereo matching methods in the outdoor environments. Unlike Middlebury stereo datasets that use an error threshold of 1 pixel, KITTI2015 stereo datasets count errors if the disparity exceeds 3 pixels and 5% of its true value. We use

FIGURE 13. Input stereo pairs, disparity maps and error maps of ''Adirondack'' from the Middlebury stereo datasets with various conditions. The disparity maps and error images are displayed from top-to-bottom: result of HD3, GWC, QCT, QCT*, and QCT⁺. (a) Original input image pair; (b) input image pair with AWGN; (c) input image pair that contrast of right image is adjusted; (d) input image pair with AWGN and contrast adjustment; (e) disparity and error maps of (a); (f) disparity and error maps of (b); (g) disparity and error maps of (c); (h) disparity and error maps of (d).

the development kit provided with KITTI2015 datasets to calculate the disparity error and visualize the disparity error maps. The experiments were conducted with the same setup as the previous experiments for Middlebury datasets, and the proposed QCT was compared with the census transformation, AD-Census, RG-Census, TCC-Census, and 4M-Census.

Table [8](#page-11-0) lists the average disparity error of the stereo matching methods on 200 stereo image pairs from KITTI2015 datasets. The proposed method still recorded the highest accuracy the same as Middlebury datasets, and other methods except for the AD-Census recorded similar rank compared to the previous experiments. The AD-Census which achieved the highest accuracy in the indoor environments except for the proposed method recorded the secondlowest accuracy in the outdoor environments. This is because the absolute difference which is used with the census transform degrades the performance of the AD-Census in the outdoor environments with large illumination variations. Fig. [12](#page-10-0) shows the disparity error and visual quality comparison of disparity images on KITTI2015 datasets obtained by using the provided development kit. Figs. 12(a), (b), and (c) show the input left images and Figs. $12(d)$, (e), and (f) show resultant disparity error images of the input images. The disparity error images were visualized using the color scheme depicted in the legend (bottom row in Fig. [12\)](#page-10-0). As shown in the last row in Figs. 12(d), (e), and (f), the proposed QCT **TABLE 8.** Average Disparity Error (%) on KITTI2015 Datasets.

The best and second-best results are boldfaced and underlined, respectively.

produced disparity maps with lower error compared to other methods.

APPENDIX B

COMPARISON OF THE GENERALIZATION ABILITY

Owing to a rapid development of neural networks, neural network-based stereo matching methods have shown promising results. However, these learning-based methods face a generalization problem when there is a domain gap between the training and test datasets. Although simple, the census transform is illumination-invariant and morphologically invariant, and thus the census transform-based stereo matching techniques have showed robustness to different environments [11], [38]. To compare the generalization ability of the proposed and neural network-based methods, we conducted additional experiments. In particular, we simulated two challenging conditions: noisy image pairs and image pairs with different global contrast. The accuracy of the neural networkbased methods trained on the KITTI datasets and the proposed QCT were evaluated on the Middlebury 2014 dataset.

TABLE 9. Average APBP (%) and RMSE on Middlebury datasets.

	APBP											
			non-occlusion				all					
Conditions	HD3	GWC	OCT	$\overline{\text{OCT}^*}$	HD3	GWC	\overline{OCT}	QCT^*				
N/A	32.52	29.55	19.05	18.47	45.82	42.16	32.97	32.39				
$\mathrm{N}_1{}^1$	38.68	34.69	31.21	29.26	52.27	47.67	45.48	43.49				
$\mathrm{N}_2{}^2$	44.99	40.96	41.88	36.96	58.79	54.52	56.40	51.40				
C_1^3	38.61	39.61	28.67	27.53	52.19	53.11	42.77	41.61				
C_2^4	45.66	51.73	38.67	37.09	59.64	66.01	52.85	51.28				
$N_1 \& C_1$	43.05	43.20	37.40	35.40	56.78	56.95	51.78	49.76				
$N_1 \& C_2$	49.30	53.86	45.16	43.02	63.33	68.26	59.65	57.50				
N_2 & C_1	50.39	48.31	44.12	39.77	64.43	62.57	58.68	54.25				
N_2 & C_2	55.78	57.49	50.11	45.86	70.07	72.15	64.74	60.45				
			non-occlusion		all							
Conditions	HD3	GWC	QCT	QCT^+	HD3	GWC	OCT	QCT ^{$+$}				
N/A	9.64	7.16	8.56	7.75	13.68	12.72	15.44	13.12				
$\rm N_1$	9.06	10.55	10.85	8.81	12.96	15.64	16.55	14.04				
N_2	11.59	10.80	14.09	12.47	15.20	15.74	19.33	18.42				
C_1	12.92	12.45	15.99	16.94	16.44	17.08	20.29	20.44				
C ₂	15.60	13.03	18.60	17.40	19.43	18.15	22.52	20.90				
$N_1 \& C_1$	10.73	13.07	15.16	10.83	14.37	17.81	20.10	15.68				
$N_1 \& C_2$	13.39	13.21	17.36	16.24	17.17	18.37	21.94	24.15				
N_2 & C_1	15.68	12.90	15.96	11.08	18.82	17.66	20.79	15.73				
$\mathrm{N}_2\ \&\ \mathrm{C}_2$	17.36	13.52	17.65	14.12	20.43	18.82	22.24	18.28				
The best and second best results are boldfoced and underlined, respectively												

he best and second-best results are boldfaced and underlined, respectively. ${}^{1}N_1$ – AWGN with σ = 5

 $^2\rm N_2$ – AWGN with $\sigma=10$

 3 C₁ – Contrast adjustment by saturating the bottom and top 20% of pixels.

 ${}^{4}C_{2}$ – Contrast adjustment by saturating the bottom and top 30% of pixels.

 $QCT^* - QCT$ with median filtering (11x11)

 $QCT^+ - QCT$ with SDR

The accuracy of the QCT with additional disparity refinements was also measured because the neural network-based methods implicitly perform the disparity refinement as well as cost computation and disparity estimation in an end-to-end manner. The accuracy of stereo matching was evaluated in terms of the average percent of bad pixel (APBP) and root mean square error (RMSE).

Table [9](#page-12-0) lists the APBP and RMSE of the hierarchical discrete distribution decomposition (HD3) [39], group-wise correlation stereo network (GWC) [40], QCT, and QCT with two disparity refinements: median filtering and segment based disparity refinement (SDR) [41]. For the generation of noisy image pairs, we added AWGN to left and right images. For the simulation of image pairs with different contrast, we changed the global contrast of the right image by linearly mapping the range between the top and bottom 20% (or 30%) of the pixels to the total dynamic range, i.e., [0, 255]. In terms of the APBP, the proposed QCT showed lower error rate compared to the HD3 and GWC. When combined with simple median filtering, the proposed method achieved the lowest error rate. In terms of the RMSE, the HD3 and GWC performed generally better than the proposed method, but the QCT with SDR showed the best and second-best results on several conditions. In other words, the QCT generated disparity maps with a higher number of accurately estimated pixels, whereas the HD3 and GWC generated disparity maps closer to the ground-truth on average. This can also be found in Fig. 13. As shown in Figs. 13 (e), (f) , (g) , and (h) , the resultant disparity maps of the QCT contain a fewer number of erroneous pixels, but the HD3 and GWC produce visually plausible results.

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