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# Optimal Pilot Design and Error Rate Analysis of Block Transmission Systems in the Presence of Channel State Information (CSI) Estimation Error

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**ABSTRACT** This work proposes novel techniques toward the design of optimal pilot sequences to perform channel estimation in block transmission systems over wideband frequency selective wireless fading channels. The framework developed is based on minimization of the Bayesian Cramér-Rao bound (BCRB) for the mean squared error (MSE) of the channel state information (CSI) estimate. Optimal pilot signals are determined for the four predominant classes of block transmission systems, viz. single carrier zero padding (SC-ZP), multi-carrier zero padding (MC-ZP), single carrier cyclic prefix (SC-CP) and multi-carrier cyclic prefix (MC-CP) systems. This makes the techniques developed general in nature and thus applicable in a wide variety of block transmission systems. As part of this study, succinct expressions and results are also derived to characterize the error rate performance, incorporating also the effect of CSI estimation error resulting due to the proposed algorithms. Finally, numerical results obtained via Monte-Carlo simulation are presented to illustrate and compare the CSI acquisition performance of optimal pilot designs with that of conventional designs and also validate the theoretical analysis for the error rate performance.

**INDEX TERMS** Block transmission system, mean square error (MSE), bit error rate (BER), diversity, minimum mean square error (MMSE), maximum likelihood (ML), zero forcing (ZF).

## I. INTRODUCTION

The advent of 4G and 5G technologies coupled with the increase in popularity of multimedia rich applications is driving the current demand for high data rates in wireless networks. While this can be achieved via transmission over channels with ever increasing bandwidths, the high delay spreads and the resulting frequency selectivity of wireless channels, which in turn lead to significant distortion due to inter symbol interference (ISI), pose a severe challenge toward the practical implementation of such systems. In this context, block-transmission schemes, which are based on the principle of either zero padding (ZP) or the addition of cyclic prefix (CP) to the transmission frames, present an excellent solution to overcome this impediment as shown in related works such as [1]. For ease of study and analysis, these transmission techniques can be grouped under four major categories, based on the number of carriers and nature of redundancy employed, as follows: single carrier zero padding (SC-ZP), single carrier cyclic prefix (SC-CP), multi-carrier zero padding (MC-ZP) and multi-carrier cyclic prefix (MC-CP). Some of these techniques are already used in practice, with MC-CP, alternatively termed as orthogonal frequency division multiplexing (OFDM), enjoying immense popularity in 4G and 5G systems due to its low complexity and robustness to frequency selectivity. At the same time, it is important to also realize that OFDM has some shortcomings, the prominent among these being its sensitivity to carrier frequency offsets (CFO) and also amplifier non-linearity arising due to the unusually large peak to average power ratio (PAPR). Another failing along these lines is its inability to accurately decode symbols at the channel nulls due to the low output signal-to-noise power ratios (SNR) at the corresponding subcarriers. Given these drawbacks, single carrier block transmission, different aspects of which are explored in [2]–[4] and [5], which do not suffer from CFO and PAPR distortions, are excellent alternatives to OFDM that are well-suited for practical implementation. Thus, it is

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essential to design a comprehensive framework for the design and performance analysis of all the major block-transmission schemes listed above, which can then be used in appropriate systems. For instance, in LTE, while OFDM is used in the downlink, it is replaced by the SC-FDMA waveform in the uplink to avoid the PAPR problem. The central focus of this paper is to present a unified approach for block transmission systems. In particular, channel estimation is one of the key processes in such systems, which has a significant bearing on their accuracy of symbol decoding. This becomes even more challenging in frequency selective channels due to the wide bandwidth, which can lead to prohibitively high pilot overheads in the absence of efficient CSI estimation schemes. Therefore, the design of optimal pilot signalling schemes that minimize the mean squared error (MSE) and thus maximize the quality of channel estimation while not compromising the spectral efficiency, is of utmost importance in these systems. A concise summary of the associated works by various researchers in the existing literature in these areas is presented next.

### A. RELATED WORKS

Many works targeting pilot designs have been reported in prior research, with a good number of them focusing on OFDM [6] and MIMO-OFDM [7], [8] due to its widespread adoption as described earlier. Authors in [9] present techniques for SC-CP and MC-CP systems with affine precoded training for a general doubly selective, i.e. time and frequency selective channel. On similar lines, for SC-ZP systems, [10] develops efficient superimposed pilot signalling schemes. Comprehensive performance analyses of SC block transmission systems are presented in the treatises in [11], [12]. The frameworks therein are based on linear equalization and [12] focuses specifically on the bandwidth efficiency-diversity gain tradeoff, which impacts the average error rate of the system. An analytical performance comparison of SC-CP and OFDM systems is illustrated in [13] considering the MMSE receiver. A detailed and thorough comparison of the performance of OFDM and SC-ZP systems considering various metrics such as throughput, BER sensitivity and PAPR, is presented in [3]. Authors in [14] characterize the BER of systems with the CP redundancy, along with the diversity order, for the popular ZF, MMSE and ML receivers. While the works above have assiduously addressed several challenging problems for SC/MC block-transmission systems, some shortcomings still remain. Most importantly, none of the above works present a general framework for optimal pilot design that encompasses all the major block transmission systems. Furthermore, results are not presented for the BER performance in block transmission systems incorporating CSI estimation error. To address these issues that have not been tackled in the existing literature, this paper proposes a single framework for the design of optimal pilot signals in SC-ZP, SC-CP, MC-ZP and MC-CP block-transmission formats. Its general nature renders the approach presented in this paper applicable in a wide variety of systems.

### B. MOTIVATION AND CONTRIBUTIONS

A part of this work was presented in the conference paper in [15]. The significant additional contributions of this work are described below

- On top of proposed optimal pilot design, we have derived analytical results for the bit error rate performance of all the four block transmission systems corresponding to three different receivers viz. zero forcing (ZF), minimum mean square error (MMSE) and maximum likelihood (ML). Further, the theoretical BER analysis is significantly more involved and in-depth.
- Additionally, we have also studied the impact of channel estimation error on the BER analysis, which is lacking in [15]. The channel estimate is obtained by employing the proposed optimal pilot design scheme.
- To make the BER analysis tractable, in case of ZP systems, we have introduced Overlap-add (OLA) operation at the receiver. This significantly reduces the complexity of SC-ZP and MC-ZP BER analyses and renders it similar to that of their CP counterparts, which is not present in [15].

Among other achievements, an equivalence is determined between zero padded systems and ones employing a cyclic prefix. It is also shown that the diversity order of OFDM and MC-ZP systems equals unity in multipath Rayleigh fading channels. Also worth noting is the fact that while SC-CP transmission achieves the full multi path diversity only for large block sizes, SC-ZP systems, in stark contrast, are able to do so for an arbitrary block size.

### C. ORGANIZATION

The section-wise organization of the rest of the paper is described below. The system models for SC-CP, SC-ZP, MC-CP and MC-ZP systems are described in section II. The next section details the proposed technique for optimal pilot signal design in the above systems based on minimizing the BCRB for MSE of channel estimation. This is followed by the BER performance analysis of block transmission systems with channel estimation error for the ZF, MMSE and ML receivers in section IV. Section V and VI describe the results of simulation studies for the above systems and conclusions respectively.

*Notation:* Boldface small letters ($\mathbf{a}$) and boldface capital letters ($\mathbf{A}$) denote vectors and matrices, respectively. $(.)^H$ denotes the Hermitian of a matrix. Tr denotes the trace operator. The expectation with respect to a random variable $\mathbf{Y}$ is denoted by $\mathbb{E}_{\mathbf{Y}}\{.\}$. $\mathbf{I}_N$ and $\mathbf{0}_{N\times M}$ denote the $N\times N$ identity and $N\times M$ zero matrices, respectively. The standard Gaussian $Q$ function, defined as $Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(-\frac{u^2}{2}\right)du$, is denoted by $Q(x)$

## II. SYSTEM MODEL

Consider a block transmission-based system with the information symbols $p(n)$ blocked into $N\times 1$ vectors denoted by

$$\mathbf{p}(k) = [p(kN), \ldots, p(kN+N-1)]^T, \tag{1}$$

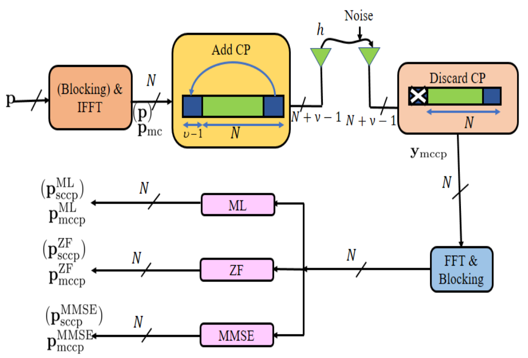


**FIGURE 1.** Block diagram of the transmit and receive processing in SC(MC)-CP systems.

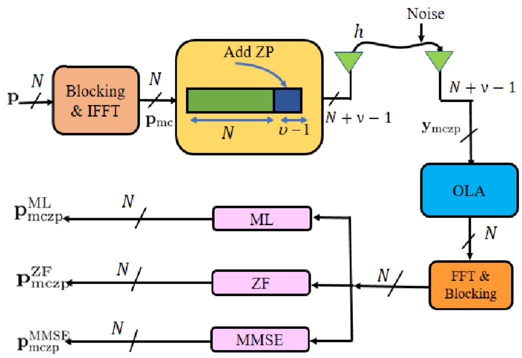


**FIGURE 2.** Block diagram of the transmit and receive processing in MC-ZP systems.

where $0 \leq n \leq (N-1)$ and $\mathbb{E}\left\{\left|p(kN+n)\right|^2\right\} = 1$. For ZP systems, the vector $\mathbf{p}(k)$ is multiplied by a $P\times N$ zero insertion matrix $\mathcal{T}_{zp} = \left[\mathbf{I}_N^H \quad \mathbf{0}_{\nu\times N}^H\right]^H$ prior to transmission, where $P = N+\nu$. As a result, a zero sequence of length $\nu$ is appended at the rear of every block for the removal of the effect of inter block interference due to multipath components. On the other hand, in CP systems, the vector $\mathbf{p}(k)$ is pre-multiplied with the $P\times N$ cyclic prefix inclusion matrix $\mathcal{T}_{cp} = \left[\mathbf{I}_{\nu\times N}^H \quad \mathbf{I}_N^H\right]^H$, which leads to the insertion of a prefix comprising of the trailing $\nu$ symbols of $\mathbf{p}(k)$ in the generated transmit symbol vector. The $L\times 1$ multipath channel vector $\mathbf{h} = [h(1), h(2), \ldots, h(L)]^T$ is assumed to be symmetric complex Gaussian with zero mean and covariance matrix $\mathbf{K}_h \in \mathbb{C}^{L\times L}$, such that $\mathbb{E}\{|h(l)|^2\} = \sigma_{h_l}^2$. The subsequent subsections describe further details regarding the transmit and receive processing for each of the block transmission systems viz. SC-CP, MC-CP, MC-ZP and SC-ZP.

### A. SC-CP SYSTEMS

At the receiver, after removal of the cyclic prefix using the truncation matrix $\Gamma = [\mathbf{0}_{N\times\nu} \quad \mathbf{I}_N]$, the resulting output symbol vector $\bar{\mathbf{y}}_{\text{sccp}} \in \mathbb{C}^{N\times 1}$ can be expressed in time domain as

$$\bar{\mathbf{y}}_{\text{sccp}}(k) = \underbrace{\Gamma\mathbf{H}\mathcal{T}_{\text{cp}}}_{\mathbf{W}}\mathbf{p}(k)+\mathbf{e}_{\text{cp}}(k). \tag{2}$$

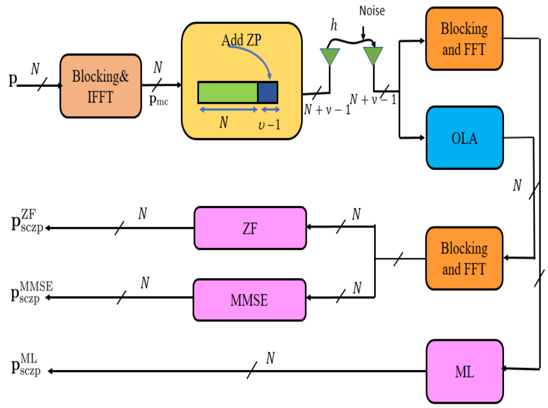


**FIGURE 3.** Block diagram of the transmit and receive processing in SC-ZP systems.

where $\mathbf{H}$ denotes a lower triangular Toeplitz matrix with the first row $[h(0), \mathbf{0}_{1\times(P-1)}]$ and the first column $[h(0), \ldots, h(L), \mathbf{0}_{(P-L-1)}]^T$. The $N$ point discrete Fourier transform (DFT) of the channel impulse response is given as $\boldsymbol{\omega} = [\omega_0, \omega_1, \ldots, \omega_{N-1}]$, where

$$\omega_n = \sum_{n=0}^{N-1} h_l e^{-j2\pi\frac{nl}{N}}. \tag{3}$$

It is well known that the resulting matrix $\mathbf{W} \in \mathbb{C}^{N\times N}$ is given as a circulant matrix with the first column $\left[\boldsymbol{\omega} \quad \mathbf{0}_{1\times(N-L)}\right]^T$. Let this be decomposed as $\mathbf{W} = \mathbf{F}_N^H\Omega\mathbf{F}_N$, where $\Omega = \text{Diag}(\boldsymbol{\omega})$ denotes the diagonal matrix of eigenvalues that are equal to the DFT coefficients of $\mathbf{h}$. Here, $\mathbf{F}_N$ represents an $N\times N$ discrete Fourier Transform (DFT) matrix with its $(i,j)$th entry given as $f_n^{(i-1)(j-1)}$ where $f_n = e^{-j\frac{2\pi}{N}}$. The circularly symmetric additive white Gaussian noise (AWGN) vector $\mathbf{e}_{\text{cp}} \in \mathbb{C}^{N\times 1}$ is assumed to be zero mean with covariance matrix $\mathbf{R}_{cp} \triangleq \mathbb{E}\left\{\mathbf{e}_{\text{cp}}(n)\mathbf{e}_{\text{cp}}^H(n)\right\} = \sigma_{e_{cp}}^2\mathbf{I}_N$. For ease of analyis, equation (2) can be alternatively expressed in frequency domain after demodulation with $\mathbf{F}_N$ as [1]

$$\mathbf{y}_{\text{sccp}}(k) = \mathbf{X}_{\text{sccp}}(\mathbf{p}(k))\,\mathbf{h}+\tilde{\mathbf{e}}_{\text{cp}}(k), \tag{4}$$

where $\mathbf{X}_{\text{sccp}}(\mathbf{p}(k)) = \sqrt{N}\text{Diag}(\mathbf{F}_N\mathbf{p}(k))\,\mathcal{F}_N \in \mathbb{C}^{N\times L}$ and $\mathcal{F}_N$ of size $N\times L$ denotes a truncated matrix created by choosing the first $L$ columns of $\mathbf{F}_N$ and $\tilde{\mathbf{e}}_{\text{cp}}(k) = \mathbf{F}_N\mathbf{e}_{\text{cp}}(k)$.

### B. MC-CP SYSTEMS

After removal of the cyclic prefix at the receiver the resulting output vector for MC-CP systems can be expressed as

$$\bar{\mathbf{y}}_{\text{mccp}}(k) = \mathbf{W}\mathbf{p}_{\text{mc}}(k)+\mathbf{e}_{\text{cp}}(k), \tag{5}$$

where $\mathbf{p}_{\text{mc}} = \mathbf{F}_N^H\mathbf{p}$ is the $N\times 1$ transmitted pilot vector obtained after the inverse discrete Fourier transform (IDFT) operation at the transmitter. An alternative formulation [15] for the above model after FFT demodulation at the receiver is given as

$$\mathbf{y}_{\text{mccp}}(k) = \mathbf{X}_{\text{mccp}}(\mathbf{p}(k))\,\mathbf{h}+\tilde{\mathbf{e}}_{\text{cp}}(k), \tag{6}$$

where $\mathbf{X}_{\text{mccp}}(\mathbf{p}(k)) = \sqrt{N}\text{Diag}(\mathbf{p}(k))\,\mathcal{F}_N \in \mathbb{C}^{N\times L}$.

### C. MC-ZP SYSTEMS

In an MC-ZP system, the received symbol vector $\bar{\mathbf{y}}_{\text{mczp}}$ can be expressed as

$$\bar{\mathbf{y}}_{\text{mczp}}(k) = \mathbf{H}\mathcal{T}_{zp}\mathbf{p}_{\text{mc}}(k) + \mathbf{e}_{\text{zp}}(k), \tag{7}$$

where $\mathbf{e}_{zp}$ is an AWGN noise vector of size $P\times 1$ with zero mean and covariance matrix $\mathbf{R}_{zp} \triangleq \mathbb{E}\left\{\mathbf{e}_{zp}(n)\mathbf{e}_{zp}^H(n)\right\} = \sigma_{e_{zp}}^2\mathbf{I}_P$. The received vector is pre-multiplied by the overlap-and-add (OLA) matrix $\mathbf{R}_{\text{oa}} = [\mathbf{I}_N \quad \mathcal{I}_\nu] \in \mathbb{C}^{N\times P}$, where $\mathcal{I}_\nu \in \mathbb{C}^{N\times \nu}$ comprises of the first $\nu$ columns of $\mathbf{I}_N$, which performs the overlap-add (OLA) operation. The resulting output vector can be expressed as

$$\mathbf{y}_{\text{mczp}}(k) = \underbrace{\mathbf{R}_{\text{oa}}\mathbf{H}\mathcal{T}_{\text{zp}}}_{\mathbf{W}}\mathbf{p}_{\text{mc}}(k) + \underbrace{\mathbf{R}_{\text{oa}}\mathbf{e}_{\text{zp}}}_{\mathbf{e}_{\text{oa}}}(k). \tag{8}$$

The OLA operation amounts to retaining the first $N$ entries of $\mathbf{y}_{\text{mczp}}$ while adding the last $\nu$ entries to its first $\nu$ entries. Further, the resulting noise vector $\mathbf{e}_{\text{oa}} = \mathbf{R}_{\text{oa}}\mathbf{e}_{\text{zp}}$ post the OLA operation at the receiver is given by

$$\mathbf{e}_{\text{oa}} = \begin{bmatrix} e_{\text{zp}_1} + e_{\text{zp}_{N+1}} \\ \vdots \\ e_\nu + e_{N+\nu} \\ e_\nu \\ \vdots \\ e_N \end{bmatrix}. \tag{9}$$

It can be easily shown that the vector $\mathbf{e}_{\text{oa}}$ is zero-mean colored noise that has the covariance matrix $\mathbf{R}_{\text{eoa}} \triangleq \mathbb{E}\left\{\mathbf{e}_{\text{oa}}(k)\mathbf{e}_{\text{oa}}^H(k)\right\} = \sigma_{\text{oa}}^2\mathbf{I}_N = (N-3\nu)\sigma_{e_{zp}}^2\mathbf{I}_N \;\forall N \geq 3\nu$. After demodulation with the $P$-point DFT matrix $\mathbf{F}_P$ with its $(i,j)$th entry given as $f_p^{(i-1)(j-1)}$ where $f_p = \mathrm{e}^{-j\frac{2\pi}{P}}$, (7) can be alternatively expressed as

$$\mathbf{y}_{\text{mczp}}(k) = \mathbf{X}_{\text{mczp}}(\mathbf{p}(k))\,\mathbf{h} + \underbrace{\mathbf{F}_P\mathbf{e}_{\text{zp}}}_{\tilde{\mathbf{e}}_{\text{zp}}}(k), \tag{10}$$

where the matrix $\mathbf{X}_{\text{mczp}}(\mathbf{p}(k)) \in \mathbb{C}^{P\times L}$ is defined as

$$\mathbf{X}_{\text{mczp}}(\mathbf{p}(k)) = \sqrt{P}\text{Diag}\left(\mathbf{Z}_{\text{zp}}\mathbf{F}_N^H\mathbf{p}(k)\right)\mathcal{F}_P \tag{11}$$

with $\mathbf{Z}_{\text{zp}} = \mathbf{F}_P\mathcal{T}_{zp} \in \mathbb{C}^{P\times N}$. The matrix $\mathcal{F}_P$ denotes the matrix formed by the last $L$ columns of $\mathbf{F}_P$.

### D. SC-ZP SYSTEMS

The output vector in an SC-ZP system, obtained after performing $N$-point DFT operation on the received vector can be expressed as

$$\mathbf{y}_{\text{sczp}}(k) = \mathbf{W}\mathbf{p}(k) + \mathbf{e}_{\text{oa}}(k). \tag{12}$$

The received vector without the OLA operation can also be expressed as

$$\bar{\mathbf{y}}_{\text{sczp}}(k) = \mathbf{X}_{\text{sczp}}(\mathbf{p}(k))\,\mathbf{h} + \tilde{\mathbf{e}}_{\text{zp}}(k), \tag{13}$$

where the matrix $\mathbf{X}_{\text{sczp}}(\mathbf{p}(k)) \in \mathbb{C}^{P\times(L)}$ is defined as

$$\mathbf{X}_{\text{sczp}}(\mathbf{p}(k)) = \sqrt{P}\text{Diag}\left(\mathbf{Z}_{\text{zp}}\mathbf{p}(k)\right)\mathcal{F}_P, \tag{14}$$

The procedure for optimal pilot design in each of the above systems is described next.

## III. PROPOSED OPTIMAL PILOT DESIGN

The proposed optimal pilot design algorithm is developed employing the BCRB metric, which corresponds to a lower bound on the mean squared error (MSE) of estimation wherein apriori information about the unknown parameter of interest is available. Since the BCRB is a lower bound on MSE that depends on the apriori pdf of the channel vector $\mathbf{h}$, it does not depend on any specific channel estimation technique. The BCRB corresponding to the multipath channel vector $\mathbf{h}$ is given as

$$\mathbb{E}\left\{(\hat{\mathbf{h}}-\mathbf{h})(\hat{\mathbf{h}}-\mathbf{h})^H\right\} \geq (\mathbf{J}_B)^{-1}, \tag{15}$$

where $\mathbf{J}_B \in \mathbb{C}^{L\times L}$ represents the Bayesian Fisher Information Matrix (BFIM) that can be expressed as [16]

$$\mathbf{J}_B = \mathbf{J}_T + \mathbf{J}_I. \tag{16}$$

The matrix $\mathbf{J}_T$ stands for the FIM corresponding to pilot symbols and $\mathbf{J}_I$ represents the FIM associated with prior information of $\mathbf{h}$. Using Equation (8.55) from [16], one can determine the $i,j$th element of$\mathbf{J}_D$ as Eqn. (17), as shown at the bottom of this page. Substituting $\mathbf{m}(\boldsymbol{\theta}) := \mathbf{m}(\mathbf{h}) = \mathbf{X}_{\text{sczp}}\mathbf{h}$ and $\mathbf{K}_x(\boldsymbol{\theta}) = \mathbf{K}_{\bar{\mathbf{y}}_{\text{sczp}}}(\mathbf{h})$ defined as

$$\begin{aligned}\mathbf{K}_{\bar{\mathbf{y}}_{\text{sczp}}}(\mathbf{h}) &= \mathbb{E}\left\{\left(\bar{\mathbf{y}}_{\text{sczp}} - \mathbf{m}(\mathbf{h})\right)\left(\bar{\mathbf{y}}_{\text{sczp}} - \mathbf{m}(\mathbf{h})\right)^H\right\} \\ &= \mathbb{E}\left\{\tilde{\mathbf{e}}_{\text{sczp}}\tilde{\mathbf{e}}_{\text{sczp}}^H\right\} = \sigma_{e_{zp}}^2\mathbf{I}_P,\end{aligned} \tag{18}$$

$\mathbf{J}_T$ can be determined as

$$\mathbf{J}_T = \frac{\mathbf{X}_{\text{sczp}}^H\mathbf{X}_{\text{sczp}}}{\sigma_{e_{zp}}^2}, \tag{19}$$

where $\mathbf{X}_{\text{sczp}} = \sqrt{N}\text{Diag}(\mathbf{F}_N\mathbf{p}(k))\mathcal{F}_N$. Using Eqn.(8.57) of [16], which states $\mathbf{J}_P = \mathbf{K}_{\boldsymbol{\theta}}^{-1}$, one can determine $\mathbf{J}_I = \mathbf{K}_{\mathbf{h}}^{-1}$. Further, given the probability density function $p_{\mathbf{h}}(\mathbf{h}) = \frac{1}{(2\pi)^{\frac{L}{2}}|\mathbf{K}_h|^{\frac{1}{2}}}\exp\left[-\frac{1}{2}\mathbf{h}^H\mathbf{K}_h^{-1}\mathbf{h}\right]$, the Fisher information matrix (FIM) $\mathbf{J}_I$ corresponding to a priori information can be determined as [16]

$$\mathbf{J}_I = -\mathbb{E}_{\mathbf{h}}\left[\nabla^2 \ln p_{\mathbf{h}}(\mathbf{h})\right] = \mathbf{K}_h^{-1}. \tag{20}$$

$$[\mathbf{J}_D]_{i,j} = \mathbb{E}_{\boldsymbol{\theta}}\left[\text{Tr}\left[\mathbf{K}^{-1}(\boldsymbol{\theta})\frac{\partial\mathbf{K}_{\mathbf{x}}(\boldsymbol{\theta})}{\partial\theta_i}\mathbf{K}^{-1}(\boldsymbol{\theta})\frac{\partial\mathbf{K}_{\mathbf{x}}(\boldsymbol{\theta})}{\partial\theta_j}\right] + 2\text{Re}\left[\frac{\partial\mathbf{m}^H(\boldsymbol{\theta})}{\partial\theta_i}\mathbf{K}_{\mathbf{x}}^{-1}(\boldsymbol{\theta})\frac{\partial\mathbf{m}(\boldsymbol{\theta})}{\partial\theta_j}\right]\right], \tag{17}$$

where $\mathbf{K}_h$ denotes the channel covariance matrix. The power delay profile of the channel, which represents the average power of the channel taps, is captured in the diagonal entries of the covariance matrix $\mathbf{K}_h$ defined as

$$\mathbf{K}_h = \mathbb{E}\left\{\mathbf{h}\mathbf{h}^H\right\}, \tag{21}$$

with $\mathbb{E}\left\{|\mathbf{h}(i)|^2\right\} = [\mathbf{K}_h]_{i,i}$. Substituting the above expressions for $\mathbf{J}_T$ and $\mathbf{J}_I$ in (16), one can express (15) as

$$\mathbb{E}\left\{(\hat{\mathbf{h}}-\mathbf{h})(\hat{\mathbf{h}}-\mathbf{h})^H\right\} \geq \left(\frac{\mathbf{X}^H_{\text{sczp}}(\mathbf{p}(k))\,\mathbf{X}_{\text{sczp}}(\mathbf{p}(k))}{\sigma^2_{e_{zp}}} + \mathbf{K}_h^{-1}\right)^{-1}. \tag{22}$$

Therefore, the BCRB for the MSE associated with $\hat{\mathbf{h}}$ is lower bounded as $e_{\text{sczp}}(\mathbf{p}(k)) = \text{Tr}\left(\frac{\mathbf{X}^H_{\text{sczp}}(\mathbf{p}(k))\mathbf{X}_{\text{sczp}}(\mathbf{p}(k))}{\sigma^2_{e_{zp}}} + \mathbf{K}_h^{-1}\right)^{-1}$. The line of action for optimal pilot design is described next. In the following course of analysis, the index $k$ is dropped for notational ease. The optimization framework toward design of the pilot sequence for SC-ZP systems can be formulated as

$$\underset{\mathbf{p}}{\text{minimize}} \quad e_{\text{sczp}}(\mathbf{p}) \tag{23}$$

$$\text{subject to} \quad \|\mathbf{p}\|^2 \leq P_T, \tag{24}$$

where $P_T$ denotes the total transmit power. Further simplification of the optimization problem above can be performed as shown in the ensuing analysis. Since $\mathcal{T}_{\text{zp}}$ is a zero padding matrix, it follows that

$$\mathcal{T}_{\text{zp}}\mathbf{p} = \begin{bmatrix}\mathbf{p} & \mathbf{0}^H_{\nu\times N}\end{bmatrix}^H. \tag{25}$$

Therefore $\|\mathcal{T}_{zp}\mathbf{p}\| = \|\mathbf{p}\|$. Further, $\|\mathbf{F}_P\mathcal{T}_{\text{zp}}\mathbf{p}\| = \|\mathcal{T}_{\text{zp}}\mathbf{p}\| = \|\mathbf{p}\|$, where $\mathbf{F}_P$ is a unitary DFT matrix of size $P\times P$. Let $\mathbf{s} = \mathbf{F}_P\mathcal{T}_{\text{zp}}\mathbf{p} \in \mathbb{C}^{P\times 1}$ and $\rho_p = |s_p|^2$, where $\rho_p$ and $s_p \forall 1 \leq p \leq P$ denote the $p$th elements of $\boldsymbol{\rho}$ and $\mathbf{s}$. Hence, (24) can be modified as

$$\|\mathbf{s}\|^2 = \sum_{p=1}^{P}\rho_p \leq P_T. \tag{26}$$

Next, $e_{\text{sczp}}(\mathbf{p})$ in (23) is expressed as

$$e_{\text{sczp}}(\mathbf{p}) = \text{Tr}\left(\mathbf{K}_h^{-1} + \frac{P}{\sigma^2_{e_{zp}}}\mathcal{F}_P{}^H\underbrace{\text{Diag}(\mathbf{s})^H\text{Diag}(\mathbf{s})}_{\text{Diag}(\boldsymbol{\rho})}\mathcal{F}_P\right)^{-1}. \tag{27}$$

Let the eigenvalue decomposition of the inverse of the channel covariance matrix $\mathbf{K}_h^{-1}$ be given as

$$\mathbf{K}_h^{-1} = \mathbf{U}\Lambda\mathbf{U}^H, \tag{28}$$

where $\mathbf{U}$ represents a unitary matrix of size $L\times L$ and $\Lambda$ denotes a diagonal matrix of size $L\times L$ consisting of the eigenvalues $\lambda_l$, $1 \leq l \leq L$. Substituting this in (27), one can simplify the expression for $e_{\text{sczp}}(\mathbf{p})$ as

$$e_{\text{sczp}}(\mathbf{p}) = \text{Tr}\left(\Lambda + \frac{P}{\sigma^2_{e_{zp}}}\underbrace{(\mathcal{F}_P\mathbf{U})^H}_{\mathbf{A}^H}\text{Diag}(\boldsymbol{\rho})\underbrace{(\mathcal{F}_P\mathbf{U})}_{\mathbf{A}}\right)^{-1} \tag{29}$$

$$= \text{Tr}\,(\Lambda + \Theta)^{-1}, \tag{30}$$

where the matrix $\Theta \in \mathbb{C}^{L\times L}$ is defined as

$$\Theta = \frac{P}{\sigma^2_{e_{zp}}}\mathbf{A}^H\text{Diag}(\boldsymbol{\rho})\,\mathbf{A}. \tag{31}$$

Applying Schwartz's inequality [17], it follows that

$$\left[\mathbf{Y}^{-1}\right]_{i,i} \geq \frac{1}{[\mathbf{Y}]_{i,i}} \quad \forall\, 1 \leq i \leq N, \tag{32}$$

where $[\mathbf{Y}]_{i,i}$ denotes the $i$th diagonal entry of the matrix $\mathbf{Y}$. Employing this inequality, the quantity $e_{\text{sczp}}$ can be lower bounded as

$$e_{\text{sczp}}(\mathbf{p}) \geq \sum_{l=1}^{L}\frac{1}{[\Lambda+\Theta]_{l,l}}. \tag{33}$$

Let $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_l, \ldots, \theta_L]^T \in \mathbb{C}^{L\times 1}$ where $\theta_l = [\boldsymbol{\Theta}]_{l,l}\,\forall 1 \leq l \leq L$. From (31), it follows that

$$\boldsymbol{\theta} = \underbrace{\frac{P}{\sigma^2_{e_{zp}}}\begin{bmatrix}|[\mathbf{A}]_{1,1}|^2 & \ldots & |[\mathbf{A}]_{P,1}|^2\\ \vdots & \vdots & \vdots\\ |[\mathbf{A}]_{1,L}|^2 & \ldots & |[\mathbf{A}]_{P,L}|^2\end{bmatrix}}_{\tilde{\mathbf{A}}_{L\times P}}\underbrace{\begin{bmatrix}\rho_1\\ \vdots\\ \rho_P\end{bmatrix}}_{\boldsymbol{\rho}}, \tag{34}$$

where $\tilde{\mathbf{A}}$ can be expressed as

$$\tilde{\mathbf{A}} = \frac{P}{\sigma^2_{e_{zp}}}\begin{bmatrix}|[\mathbf{A}]_{1,1}|^2 & \ldots & |[\mathbf{A}]_{P,1}|^2\\ \vdots & \vdots & \vdots\\ |[\mathbf{A}]_{1,L}|^2 & \ldots & |[\mathbf{A}]_{P,L}|^2\end{bmatrix}. \tag{35}$$

The quantity $\boldsymbol{\rho}$ can be replaced by its least squares estimate

$$\boldsymbol{\rho} = \tilde{\mathbf{A}}^{\dagger}\boldsymbol{\theta}, \tag{36}$$

where $\tilde{\mathbf{A}}^{\dagger} = \left(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}}\right)^{-1}\tilde{\mathbf{A}}^H$ denotes the pseudo inverse of $\tilde{\mathbf{A}}$. Therefore, the constraint in (26) can be expressed in terms of $\boldsymbol{\theta}$ as

$$\sum_{p=1}^{P}\rho_p = \sum_{p=1}^{P}\sum_{l=1}^{L}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}\theta_l \leq P_T. \tag{37}$$

Substituting the bound determined in (33) for $e_{\text{sczp}}$, the problem in (23) can be expressed as

$$\underset{\boldsymbol{\theta}}{\text{minimize}} \quad \sum_{l=1}^{L}\frac{1}{\lambda_l+\theta_l}$$

$$\text{subject to} \quad \sum_{p=1}^{P}\sum_{l=1}^{L}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}\theta_l \leq P_T. \tag{38}$$

The Lagrangian function for the optimization problem in (34) can be formulated as

$$\mathcal{L}(\boldsymbol{\theta},\mu)=\sum_{l=1}^{L}\frac{1}{\lambda_l+\theta_l}+\mu\sum_{p=1}^{P}\sum_{l=1}^{L}\left(\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}\theta_l-P_T\right). \quad (39)$$

The KKT framework described in [18] states that the saddle point of the Lagrangian function $\nabla\mathcal{L}(\boldsymbol{\theta},\mu)$, obtained by setting $\nabla\mathcal{L}(\boldsymbol{\theta},\mu)=0$, is an optimal solution for the convex minimization problem (34). Using this property, the optimal solution can be determined as shown in the following steps. Differentiating $\mathcal{L}(\boldsymbol{\theta})$ respect to $\theta_l$, one obtains

$$\frac{\partial\mathcal{L}(\boldsymbol{\theta})}{\partial\theta_l}=\frac{-1}{(\lambda_l+\theta_l)^2}+\mu\sum_{p=1}^{P}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}. \quad (40)$$

Setting $\frac{\partial\mathcal{L}(\boldsymbol{\theta})}{\partial\theta_l}$ to zero determines $\theta_l$ as follows

$$\theta_l=\frac{1}{\sqrt{\mu\sum_{p=1}^{P}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}}}-\lambda_l. \quad (41)$$

where $\mu$ can be evaluated by employing the power constraint as shown below

$$\sum_{l=1}^{L}\sum_{p=1}^{P}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}\theta_l=P_T$$

$$\implies\sum_{l=1}^{L}\sum_{p=1}^{P}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}\left(\frac{1}{\sqrt{\mu\sum_{p=1}^{P}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}}}-\lambda_l\right)=P_T \quad (42)$$

$$\implies\mu=\left(\frac{\sum_{l=1}^{L}\sqrt{\sum_{p=1}^{P}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}}}{P_T+\sum_{l=1}^{L}\lambda_l\sum_{p=1}^{P}\left[\tilde{\mathbf{A}}^{\dagger}\right]_{p,l}}\right)^2. \quad (43)$$

Further $\boldsymbol{\rho}$ can be obtained as $\boldsymbol{\rho}=\tilde{\mathbf{A}}^{\dagger}\boldsymbol{\theta}$. $s_p^*$ can be expressed as

$$s_p^*=\sqrt{\rho_p},\qquad 1\le p\le P. \quad (44)$$

Let $\mathbf{s}=\mathbf{F}_P\mathcal{T}_{\mathrm{zp}}\mathbf{p}=\mathbf{Z}_{\mathrm{zp}}\mathbf{p}$. Finally, from the definition of $\mathbf{s}$ above, the optimal pilot sequence $\mathbf{p}^*_{\mathrm{sczp}}$ for SC-ZP block systems is derived as

$$\mathbf{p}^*_{\mathrm{sczp}}=\mathbf{Z}^{\dagger}_{\mathrm{zp}}\mathbf{s}^*. \quad (45)$$

Furthermore, it can be noted that the proposed optimization technique for pilot design is generic and can be readily extended to MC-ZP, SC-CP and MC-CP systems by appropriately defining $\mathbf{s}$ as follows. For MC-ZP systems, since $\mathbf{s}$ is given by Eqn. (11) as $\mathbf{s}=\mathbf{Z}_{\mathrm{zp}}\mathbf{F}_N^H\mathbf{p}_{\mathrm{mczp}}$, the optimal pilot for such a system follows as

$$\mathbf{p}^*_{\mathrm{mczp}}=\left(\mathbf{Z}_{\mathrm{zp}}\mathbf{F}_N^H\right)^{\dagger}\mathbf{s}^*. \quad (46)$$

For SC-CP systems, since $\mathbf{s}$ is given from the description in Eqn. (4) as $\mathbf{s}=\mathbf{F}_N\mathbf{p}_{\mathrm{sccp}}$, it follows that optimal pilot sequence can be obtained as

$$\mathbf{p}^*_{\mathrm{sccp}}=\mathbf{F}_N^{-1}\mathbf{s}^*. \quad (47)$$

Finally for MC-CP systems, since $\mathbf{s}$ is obtained from Eqn. (6) as $\mathbf{s}=\mathbf{F}_N\mathbf{p}_{\mathrm{mccp}}$, the optimal pilot can be determined as

$$\mathbf{p}^*_{\mathrm{mccp}}=\mathbf{s}^*. \quad (48)$$

## IV. PERFORMANCE ANALYSIS OF ZF, MMSE AND ML RECEIVERS FOR BLOCK TRANSMISSION SYSTEMS

This section begins with a brief characterization of the channel estimation error, followed by its effect on the performance of block transmission systems. Let $\Delta h(l)$ denote the estimation error corresponding to $h(l)$ that can be characterized as having zero mean with variance $\mathbb{E}\{|\Delta h(l)|^2\}=\sigma^2_{\Delta h_l}$. One can express $\hat{\omega}_n$, the $n$th DFT co-efficient of the estimated channel vector $\hat{\mathbf{h}}$, as

$$\hat{\omega}_n=\sum_{l=0}^{L-1}\hat{h}(l)\,\mathrm{e}^{-j2\pi\frac{nl}{N}}=\sum_{l=0}^{L-1}(h(l)+\Delta h(l))\,\mathrm{e}^{-j2\pi\frac{nl}{N}} \quad (49)$$

$$=\omega_n+\underbrace{\sum_{l=0}^{L-1}\Delta h(l)\mathrm{e}^{-j2\pi\frac{nl}{N}}}_{\Delta\omega_n}, \quad (50)$$

where $\hat{h}(l)=h(l)+\Delta h(l)$. From (50), assuming $\omega_n$ and the estimation error $\Delta\omega_n$ to be independent of each other, the variance corresponding to $\hat{h}(l)$ and $\hat{\omega}_n$ can be evaluated respectively as

$$\mathbb{E}\left\{\left|\hat{h}(l)\right|^2\right\}=\sigma^2_{\hat{h}_l}=\sigma^2_{h_l}+\sigma^2_{\Delta h_l}, \quad (51)$$

$$\mathbb{E}\{|\hat{\omega}_n|^2\}=\bar{\alpha}_n=\alpha_n+\Delta\alpha_n, \quad (52)$$

where $\alpha_n=\mathbb{E}\{|\omega_n|^2\}$ and $\Delta\alpha_n=\mathbb{E}\{|\Delta\omega_n|^2\}$ denote the variances of the DFT channel co-efficient $\omega_n$ and the error $\Delta\omega_n$ respectively. The expression for $\alpha_n$ can be derived as

$$\alpha_n=\mathbb{E}\left\{|\omega_n|^2\right\}=\mathbb{E}\left\{\omega_n\omega_n^*\right\}$$

$$=\mathbb{E}\left\{\sum_{l=0}^{L-1}\mathbf{h}(l)\frac{1}{\sqrt{N}}\mathrm{e}^{-j\frac{2\pi nl}{N}}\sum_{m=0}^{L-1}\mathbf{h}^*(m)\frac{1}{\sqrt{N}}\mathrm{e}^{j\frac{2\pi nm}{N}}\right\} \quad (53)$$

$$=\frac{1}{N}\mathbb{E}\left\{\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\mathbf{h}(l)\,\mathbf{h}^*(m)\,\mathrm{e}^{-j\frac{2\pi nl}{N}}\mathrm{e}^{j\frac{2\pi nm}{N}}\right\} \quad (54)$$

$$=\frac{1}{N}\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\mathbb{E}\left\{\mathbf{h}(l)\,\mathbf{h}^*(m)\right\}\mathrm{e}^{-j\frac{2\pi nl}{N}}\mathrm{e}^{j\frac{2\pi nm}{N}} \quad (55)$$

$$=\frac{1}{N}\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\mathbf{K}_h(l,m)\,\mathrm{e}^{-j\frac{2\pi nl}{N}}\mathrm{e}^{j\frac{2\pi nm}{N}} \quad (56)$$

$$=\frac{1}{N}\mathbf{f}_N^H(n)\,\mathbf{K}_h\mathbf{f}_N(n) \quad (57)$$

where $\mathbf{f}_N(n) = \left[1\ e^{-j2\pi\frac{n}{N}} \ldots e^{-j2\pi\frac{(L-1)n}{N}}\right]^T$. Similarly the variance of $\Delta\omega_n$, denoted by $\Delta\alpha_n$, can be determined as follows

$$\begin{aligned}\Delta\alpha_n &= \mathbb{E}\left\{|\Delta\omega_n|^2\right\} = \mathbb{E}\left\{\Delta\omega_n\Delta\omega_n^*\right\}\\ &= \mathbb{E}\left\{\sum_{l=0}^{L-1}\Delta\mathbf{h}(l)\frac{1}{\sqrt{N}}e^{-j\frac{2\pi nl}{N}}\sum_{m=0}^{L-1}\Delta\mathbf{h}^*(m)\frac{1}{\sqrt{N}}e^{j\frac{2\pi nm}{N}}\right\}\\ &= \frac{1}{N}\mathbb{E}\left\{\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\Delta\mathbf{h}(l)\mathbf{h}^*(l)e^{-j\frac{2\pi nl}{N}}e^{j\frac{2\pi nm}{N}}\right\}\\ &= \frac{1}{N}\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\mathbb{E}\left\{\Delta\mathbf{h}(l)\mathbf{h}^*(m)\right\}e^{-j\frac{2\pi nl}{N}}e^{j\frac{2\pi nm}{N}}\\ &= \frac{1}{N}\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}\mathbf{J}_B(l,m)e^{-j\frac{2\pi nl}{N}}e^{j\frac{2\pi nm}{N}}\\ &= \frac{1}{N}\mathbf{f}_N^H(n)\mathbf{J}_B\mathbf{f}_N(n)\end{aligned}$$

where $\mathbf{f}_N(n) = \left[1\ e^{-j2\pi\frac{n}{N}} \ldots e^{-j2\pi\frac{(L-1)n}{N}}\right]^T$. Let $g_n \triangleq |\omega_n|^2$ and $\hat{g}_n \triangleq |\hat{\omega}_n|^2$. Since the MMSE estimator is being employed, the error $\Delta h(l)$ follows a symmetric complex Gaussian distribution. Hence, $\hat{g}_n$ is exponentially distributed with the probability density function $f_{\hat{G}_n}(\hat{g}_n)$ that can be obtained as

$$f_{\hat{G}_n}(\hat{g}_n) = \frac{1}{\bar{\alpha}_n}\exp\left(\frac{-\hat{g}_n}{\bar{\alpha}_n}\right), \quad g_n \geq 0. \tag{58}$$

Employing the above results, the following sub-sections now develop the theoretical BER expressions in the presence of channel estimation error for SC-ZP, SC-CP, MC-CP and MC-ZP block transmission systems described in sections IV-A, IV-B, IV-C and IV-D respectively for various linear as well as non-linear receivers considering binary phase shift keying (BPSK) modulation.

### A. BER ANALYSIS OF SC-ZP SYSTEMS

#### 1) ZF RECEIVER

Let $\hat{\Omega}$ denote the estimate of $\Omega = \text{Diag}(\boldsymbol{\omega})$. Further, let $\Delta\Omega$ be defined as $\text{Diag}(\Delta\boldsymbol{\omega})$, where $\Delta\boldsymbol{\omega} = [\Delta\omega_1, \ldots, \Delta\omega_N]^T$. The ZF based symbol vector estimate for SC-ZP systems in the presence of channel estimation error can be expressed as

$$\begin{aligned}\hat{\mathbf{p}}^{\text{ZF}} &= \mathbf{F}_N^H\hat{\Omega}^{-1}\mathbf{F}_N\mathbf{y}_{\text{sczp}} &(59)\\ &= \mathbf{F}_N^H\hat{\Omega}^{-1}\mathbf{F}_N\left(\mathbf{F}_N^H\Omega\mathbf{F}_N\mathbf{p}+\mathbf{e}_{\text{oa}}\right) &(60)\\ &= \mathbf{F}_N^H\hat{\Omega}^{-1}\mathbf{F}_N\left(\mathbf{F}_N^H\left(\hat{\Omega}-\Delta\Omega\right)\mathbf{F}_N\mathbf{p}+\mathbf{e}_{\text{oa}}\right) &(61)\\ &= \mathbf{p}+\underbrace{\mathbf{F}_N^H\hat{\Omega}^{-1}\mathbf{F}_N\mathbf{e}_{\text{oa}}-\mathbf{F}_N^H\hat{\Omega}^{-1}\Delta\Omega\mathbf{F}_N\mathbf{p}}_{\tilde{\mathbf{e}}_{\text{oa}}}. &(62)\end{aligned}$$

The covariance of the noise vector $\tilde{\mathbf{e}}_{\text{oa}} \in \mathbb{C}^{N\times 1}$ can be computed as

$$\begin{aligned}&\mathbf{R}_{\tilde{e}_{\text{oa}}}\\ &= \mathbb{E}\{\tilde{\mathbf{e}}_{\text{oa}}\tilde{\mathbf{e}}_{\text{oa}}^H\} &(63)\\ &= \sigma_{\text{oa}}^2\mathbf{F}_N^H\hat{\Omega}^{-1}\hat{\Omega}^{-H}\mathbf{F}_N+P_T\mathbf{F}_N^H\hat{\Omega}^{-1}\Delta\Omega\Delta\Omega^H\hat{\Omega}^{-H}\mathbf{F}_N &(64)\\ &= \sigma_{\text{oa}}^2\left(\mathbf{F}_N^H\underbrace{\begin{bmatrix}\frac{1+\gamma_{\text{zp}}|\Delta\omega_1|^2}{|\hat{\omega}_1|^2} & 0\ldots & 0\\ \vdots & \ddots & \vdots\\ 0 & \ldots & \frac{1+\gamma_{\text{zp}}|\Delta\omega_N|^2}{|\hat{\omega}_N|^2}\end{bmatrix}}_{\Xi}\mathbf{F}_N\right), &(65)\end{aligned}$$

where $\gamma_{\text{zp}} = \frac{P_T}{\sigma_{\text{oa}}^2}$. It has been assumed that the symbol vector $\mathbf{p}$ and the noise vector $\mathbf{e}_{\text{oa}}$ are uncorrelated. The noise co-variance matrix above is a circulant matrix as it can be expressed in the form $\mathbf{R}_{\tilde{e}_{\text{oa}}} = \sigma_{\text{oa}}^2\mathbf{F}_N^H\Xi\mathbf{F}_N$, where $\Xi$ is the diagonal matrix of eigenvalues, all of which are equal to the component wise noise variance

$$\frac{1}{N}\sum_{n=1}^{N}\frac{\sigma_{\text{oa}}^2\left(1+\gamma_{\text{zp}}|\Delta\omega_n|^2\right)}{|\hat{\omega}_n|^2}, \quad \forall 1\leq n\leq N. \tag{66}$$

As the quantity $\Delta\omega_n$ is small, one can ignore its variation across the block of $N$ symbols. Let $|\Delta\omega_n|^2 \triangleq \breve{\omega}$, $1\leq n\leq N$. Therefore, the instantaneous signal-to-noise ratio (SNR) for the ZF receiver in SC-ZP systems with channel estimation error can be expressed as

$$\text{SNR}_{\text{sczp}}^{\text{zf}} = \frac{P_T}{\frac{1}{N}\sum_{n=1}^{N}\frac{\sigma_{\text{oa}}^2(1+\gamma_{\text{zp}}\breve{\omega})}{|\hat{\omega}_n|^2}} = \frac{1}{\frac{1}{N}\sum_{n=1}^{N}\frac{(1+\gamma_{\text{zp}}\breve{\omega})}{\gamma_{\text{zp}}\hat{g}_n}}. \tag{67}$$

The instantaneous bit error rate (BER) for the above system with BPSK modulation [14] can be determined as

$$\begin{aligned}\text{BER}_{\text{sczp}}^{\text{zf}} &= Q\left(\sqrt{\frac{2N}{\sum_{n=1}^{N}\frac{(1+\gamma_{\text{zp}}\breve{\omega})}{\gamma_{\text{zp}}\hat{g}_n}}}\right)\\ &= Q\left(\sqrt{\frac{2N}{\sum_{n=1}^{N}\frac{1}{\breve{\gamma}_{\text{zp}}\hat{g}_n}}}\right),\end{aligned} \tag{68}$$

where $\breve{\gamma}_{\text{zp}} \triangleq \frac{\gamma_{\text{zp}}}{1+\gamma_{\text{zp}}\breve{\omega}}$. Further, the harmonic mean inequality [19] has been employed in the last expression (68) to lower bound the instantaneous BER i.e.

$$Q\left(\sqrt{2N\breve{\gamma}_{\text{zp}}\hat{g}_n}\right) \geq Q\left(\sqrt{2N\breve{\gamma}_{\text{zp}}\min_{\substack{n\\1\leq n\leq N}}\hat{g}_n}\right). \tag{69}$$

In order to compute the average BER, consider

$$G_m = \min_{\substack{n\\1\leq n\leq N}}\hat{g}_n. \tag{70}$$

The cumulative distribution function of $X$ can be evaluated as

$$F_{G_m}(g_m) = \Pr(G_m \leq g_m) = 1-\Pr\left(\min_{\substack{n\\1\leq n\leq N}}\hat{g}_n \geq g_m\right) \tag{71}$$

$$= 1-\prod_{n=1}^{N}\Pr\left(\hat{g}_n \geq g_m\right) \tag{72}$$

$$= 1-\prod_{n=1}^{N} \exp\left(-\frac{g_m}{\bar{\alpha}_n}\right) \tag{73}$$

$$= 1-\exp\left(-g_m \sum_{n=1}^{N} \frac{1}{\bar{\alpha}_n}\right). \tag{74}$$

The simplification from (71) to (72) involves the assumption that $\hat{g}(0), \hat{g}(1), \ldots, \hat{g}(N-1)$ are uncorrelated i.e. for the scenario when $L = N$. Let $\tilde{\alpha} \triangleq \sum_{n=1}^{N} \frac{1}{\bar{\alpha}_n}$. The corresponding pdf $f_{G_m}(g_m)$ can be evaluated as

$$f_{G_m}(g_m) = \frac{dF_{G_m}(g_m)}{dg_m} = \tilde{\alpha}\exp(-g_m\tilde{\alpha}). \tag{75}$$

Hence, the average BER for the ZF-based symbol estimate in SC-ZP systems can be lower bounded as

$$\overline{\mathrm{BER}}^{\mathrm{zf}}_{\mathrm{sczp}}(\breve{\gamma}_{\mathrm{zp}})$$

$$= \int_0^\infty Q\left(\sqrt{2N\breve{\gamma}_{\mathrm{zp}} g_m}\right) f_{G_m}(g_m)\, dg_m \tag{76}$$

$$= \int_0^\infty Q\left(\sqrt{2N\breve{\gamma}_{\mathrm{zp}} g_m}\right) \tilde{\alpha}\exp(-g_m\tilde{\alpha}) dg_m \tag{77}$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty \int_{\sqrt{2N\breve{\gamma}_{\mathrm{zp}} g_m}}^\infty \exp\left(\frac{-y^2}{2}\right) \tilde{\alpha}\exp(-g_m\tilde{\alpha}) dy dg_m$$

$$= \frac{1}{2}\left(1-\sqrt{\frac{\frac{\breve{\gamma}_{\mathrm{zp}}N}{\tilde{\alpha}}}{\frac{\breve{\gamma}_{\mathrm{zp}}N}{\tilde{\alpha}}+1}}\right). \tag{78}$$

Substituting

$$\tilde{\alpha} \triangleq \sum_{n=1}^{N} \frac{1}{\bar{\alpha}_n} = \sum_{n=1}^{N} \frac{1}{\alpha_n+\Delta\alpha_n} \tag{79}$$

and

$$\breve{\gamma}_{\mathrm{zp}} \triangleq \frac{\gamma_{\mathrm{zp}}}{1+\gamma_{\mathrm{zp}}\breve{\omega}} = \frac{\frac{P_T}{\sigma^2_{\mathrm{oa}}}}{1+\frac{P_T}{\sigma^2_{\mathrm{oa}}}} = \frac{\frac{P_T}{(N-3\nu)\sigma^2_{e_{zp}}}}{1+\frac{P_T}{(N-3\nu)\sigma^2_{e_{zp}}}} \tag{80}$$

in eqn. (78), the resulting expression can be further simplified as

$$\frac{1}{2}\left(1-\sqrt{\frac{NP_T}{P_T+\sum_{n=1}^{N}\frac{1}{\bar{\alpha}_n}\left((N-3\nu)\,\sigma^2_{e_{zp}}+P_T\breve{\omega}\right)}}\right). \tag{81}$$

As a special case, the average BER in the absence of CSI estimation error can be obtained by setting $\Delta\alpha_n$ and $\breve{\omega}$ to zero in the above equation and is determined as

$$\frac{1}{2}\left(1-\sqrt{\frac{NP_T}{P_T+\sum_{n=1}^{N}\frac{1}{\alpha_n}(N-3\nu)\,\sigma^2_{e_{zp}}}}\right). \tag{82}$$

### 2) ML RECEIVER

The symbol estimate $\mathbf{p}_{\mathrm{ml}}$ obtained using the ML receiver in an SC-ZP system with imperfect CSI can be expressed as

$$\mathbf{p}_{\mathrm{ml}} = \underset{\mathbf{p}}{\operatorname{argmin}} \left\|\bar{\mathbf{y}}_{\mathrm{sczp}} - \mathrm{Diag}\left(\sqrt{P}\mathcal{F}_P\mathbf{h}\right)\mathbf{Z}_{\mathrm{zp}}\mathbf{p}\right\|^2. \tag{83}$$

The pairwise error probability (PEP) $\Pr\left(\mathbf{p}\to\mathbf{p}'|\hat{\mathbf{h}},\gamma\right)$ for vectors $\mathbf{p}, \mathbf{p}'$ where the vector $\mathbf{p}'$ has been erroneously decoded in place of the transmitted vector $\mathbf{p}$ can be simplified as

$$Q\left(\sqrt{\frac{\gamma}{2}}\sqrt{\left\|\mathrm{Diag}\left(\sqrt{P}\mathcal{F}_P\hat{\mathbf{h}}\right)\mathbf{Z}_{\mathrm{zp}}\Big(\underbrace{\mathbf{p}-\mathbf{p}'}_{\mathbf{e}}\Big)\right\|^2}\right), \tag{84}$$

where $\gamma = \frac{P_T}{\sigma^2_{e_{zp}}}$ and the quantity $\mathrm{Diag}\left(\sqrt{P}\mathcal{F}_P\hat{\mathbf{h}}\right)\mathbf{Z}_{\mathrm{zp}}\mathbf{e}$ can be equivalently expressed as

$$\mathrm{Diag}\left(\sqrt{P}\mathcal{F}_P\hat{\mathbf{h}}\right)\mathbf{Z}_{\mathrm{zp}}\mathbf{e} = \underbrace{\mathrm{Diag}\left(\mathbf{Z}_{\mathrm{zp}}\mathbf{e}\right)}_{\mathbf{D}\mathrm{zp}}\underbrace{\sqrt{P}\mathcal{F}_P}_{\mathbf{V}}\hat{\mathbf{h}}. \tag{85}$$

Next, the square of the norm of the above quantity can be evaluated as

$$\left\|\mathrm{Diag}\left(\sqrt{P}\mathcal{F}_P\hat{\mathbf{h}}\right)\mathbf{Z}_{\mathrm{zp}}\mathbf{e}\right\|^2 = \left(\mathbf{D}_{\mathrm{zp}}\mathbf{V}\hat{\mathbf{h}}\right)^H\left(\mathbf{D}_{\mathrm{zp}}\mathbf{V}\hat{\mathbf{h}}\right) \tag{86}$$

$$= \hat{\mathbf{h}}^H\mathbf{V}^H\mathbf{D}^H_{\mathrm{zp}}\mathbf{D}_{\mathrm{zp}}\mathbf{V}\hat{\mathbf{h}} \tag{87}$$

$$= \hat{\mathbf{h}}^H\mathbf{U}_z{}^H\Psi\mathbf{U}_z\hat{\mathbf{h}} \tag{88}$$

$$= \hat{\mathbf{h}}^H_u\Psi\hat{\mathbf{h}}_u = \sum_{l=0}^{L-1}\psi_l\left|\hat{h}_{ul}\right|^2, \tag{89}$$

where $\hat{\mathbf{h}}_u = \mathbf{U}_z\mathbf{h}$. $\mathbf{U}_z^H\Psi\mathbf{U}_z$ denotes the eigenvalue decomposition of $\mathbf{V}^H\mathbf{D}^H_{\mathrm{zp}}\mathbf{D}_{\mathrm{zp}}\mathbf{V}$ and $\Psi = \mathrm{Diag}\left([\psi_0, \psi_1, \ldots, \psi_{L-1}]^T\right)$ is the diagonal matrix of eigenvalues. Since $\mathbf{U}_z$ is a unitary matrix, the modified channel tap vector $\hat{\mathbf{h}}_u$ has similar statistical characteristics as that of $\hat{\mathbf{h}}$. The PEP can now be expressed as

$$\underbrace{\Pr\left(\mathbf{p}\to\mathbf{p}'|\hat{\mathbf{h}},\gamma\right)}_{\mathrm{PEP}^{\mathrm{ml}}_{\mathrm{sczp}}} = Q\left(\sqrt{\frac{\gamma}{2}}\sqrt{\sum_{l=0}^{L-1}\psi_l\left|\hat{h}_{ul}\right|^2}\right). \tag{90}$$

The amplitudes of the channel taps $\left|\hat{h}_{ul}\right|$ are Rayleigh distributed with the probability density function

$$f_{\left|\hat{H}_{ul}\right|}\left(\left|\hat{h}_{ul}\right|\right) = \frac{\left|\hat{h}_{ul}\right|}{\sigma^2_{\hat{h}_l}}\exp^{-\frac{\left|\hat{h}_{ul}\right|^2}{2\sigma^2_{\hat{h}_l}}}. \tag{91}$$

Therefore the quantity $y_l = \psi_l\left|\hat{h}_{ul}\right|^2$ is exponentially distributed with the probability density function

$$f_{Y_l}(y_l) = \frac{1}{2\sigma^2_{\hat{h}_l}\psi_l}\mathrm{e}^{-\frac{y_l}{2\sigma^2_{\hat{h}_l}\psi_l}}.$$

The average PEP for vectors $\mathbf{p}$, $\mathbf{p}'$ can be determined by averaging the quantity in (90) over the joint pdf. Due to the independence assumption of $y_l$ s, the probability density can be simplified as

$$f_{Y_1,Y_2,\ldots,Y_L}(y_1, y_2, \ldots, y_L) = \prod_{l=1}^{L}\frac{1}{2\sigma^2_{\hat{h}_l}\psi_l}\mathrm{e}^{-\frac{y_l}{2\sigma^2_{\hat{h}_l}\psi_l}}. \tag{92}$$

The quantity in (90) can further be upper bounded using the Chernoff bound as

$$Q\left(\sqrt{\frac{\gamma\sum_{l=0}^{L-1}\psi_l\left|\hat{h}_{ul}\right|^2}{2}}\right)\leq\frac{1}{2}\exp\left(\frac{-\gamma\sum_{l=1}^{L}y_l}{4}\right). \quad (93)$$

Employing the pdf in (92), the upper bound for average PEP can be simplified as

$$\overline{\text{PEP}}^{\text{ml}}_{\text{sczp}}$$
$$\leq\frac{1}{2}\int_{y_1=0}^{\infty}\cdots\int_{y_L=0}^{\infty}\mathrm{e}^{\frac{-\gamma\sum_{l=1}^{L}y_l}{4}}\prod_{l=1}^{L}\frac{1}{2\sigma^2_{\hat{h}_l}\psi_l}\mathrm{e}^{-\frac{y_l}{2\sigma^2_{\hat{h}_l}\psi_l}}dy_l \quad (94)$$
$$=\frac{1}{2}\prod_{l=1}^{L}\int_{y_l=0}^{\infty}\mathrm{e}^{-\frac{1}{4}\gamma y_l}\frac{1}{2\sigma^2_{\hat{h}_l}\psi_l}\mathrm{e}^{-\frac{y_l}{2\sigma^2_{\hat{h}_l}\psi_l}}dy_l \quad (95)$$
$$=\prod_{l=1}^{L}\frac{1}{4\sigma^2_{\hat{h}_l}\psi_l}\int_{y_l=0}^{\infty}\mathrm{e}^{-\left(\frac{\gamma}{4}+\frac{1}{2\sigma^2_{\hat{h}_l}\psi_l}\right)y_l}dy_l \quad (96)$$
$$=\left(\frac{1}{2+\gamma\sigma^2_{\hat{h}_l}\psi_l}\right)^L. \quad (97)$$

Considering BPSK modulation and equi-probable symbols, the average BER for the ML receiver in SC-ZP systems can be simplified as

$$\overline{\text{BER}}^{\text{ml}}_{\text{sczp}}(\gamma)\leq\frac{1}{2^N}\left(\frac{1}{2+\gamma\sigma^2_{\hat{h}_l}\psi_l}\right)^L. \quad (98)$$

Therefore, it can be readily seen that the ML receiver in SC-ZP systems achieves diversity order $L$.

### 3) MMSE RECEIVER

The minimum mean square error (MMSE) receiver in presence of channel estimation error yields the symbol vector estimate as $\hat{\mathbf{p}}^{\text{mmse}}_{\text{sczp}}$ that can be expressed as

$$\hat{\mathbf{p}}^{\text{mmse}}_{\text{sczp}}=\hat{\mathbf{W}}_M\bar{\mathbf{y}}_{\text{sczp}}=\hat{\mathbf{W}}_M\mathbf{W}\mathbf{p}+\underbrace{\hat{\mathbf{W}}_M\tilde{\mathbf{e}}_{\text{zp}}}_{\mathbf{e}_{\text{sczp}}}, \quad (99)$$

where

$$\hat{\mathbf{W}}_M=\gamma_{\text{zp}}\hat{\mathbf{W}}^H\left(\gamma_{\text{zp}}\hat{\mathbf{W}}\hat{\mathbf{W}}^H+\mathbf{I}_N\right)^{-1} \quad (100)$$
$$=\mathbf{F}_N^H\begin{bmatrix}\frac{\gamma_{\text{zp}}\hat{\omega}_1^*}{\left(\gamma_{\text{zp}}|\hat{\omega}_1|^2+1\right)^2} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{\gamma_{\text{zp}}\hat{\omega}_N^*}{\left(\gamma_{\text{zp}}|\hat{\omega}_N|^2+1\right)^2}\end{bmatrix}\mathbf{F}_N \quad (101)$$

where $\hat{\mathbf{W}}=\mathbf{F}_N^H\hat{\Omega}\mathbf{F}_N$. The covariance matrix of the estimated symbol vector $\hat{\mathbf{p}}^{\text{mmse}}_{\text{sczp}}$ can be expressed as

$$\mathbb{E}\{\hat{\mathbf{p}}^{\text{mmse}}_{\text{sczp}}\left(\hat{\mathbf{p}}^{\text{mmse}}_{\text{sczp}}\right)^H\}=\mathbb{E}\left\{\left(\hat{\mathbf{W}}_M\bar{\mathbf{y}}_{\text{sczp}}\right)\left(\hat{\mathbf{W}}_M\bar{\mathbf{y}}_{\text{sczp}}\right)^H\right\} \quad (102)$$
$$=\hat{\mathbf{W}}_M\mathbb{E}\{\mathbf{y}_{\text{sczp}}\mathbf{y}^H_{\text{sczp}}\}\hat{\mathbf{W}}_M^H. \quad (103)$$

The right hand side term of the expression (103) can be further expanded as

$$\hat{\mathbf{W}}_M\mathbb{E}\{(\mathbf{W}\mathbf{p}+\tilde{\mathbf{e}}_{\text{zp}})\left(\mathbf{W}\mathbf{p}+\tilde{\mathbf{e}}_{\text{zp}}\right)^H\}\hat{\mathbf{W}}_M^H \quad (104)$$
$$=\mathbf{F}_N^H\begin{bmatrix}\frac{\gamma_{\text{zp}}\hat{\omega}_1^*\left(\gamma|\omega_1|^2+1\right)\hat{\omega}_1}{\left(\gamma_{\text{zp}}|\hat{\omega}_1|^2+1\right)^2} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{\gamma_{\text{zp}}\hat{\omega}_N^*\left(\gamma_{\text{zp}}|\omega_N|^2+1\right)\hat{\omega}_N}{\left(\gamma_{\text{zp}}|\hat{\omega}_N|^2+1\right)^2}\end{bmatrix}\mathbf{F}_N. \quad (105)$$

The simplification from (104) to (105) involves substitution of $\hat{\mathbf{W}}=\mathbf{F}_N^H\hat{\Omega}\mathbf{F}_N$, $\mathbf{W}=\mathbf{F}_N^H\Omega\mathbf{F}_N$, $\mathbb{E}\{\mathbf{p}\mathbf{p}^H\}=P_T$ and equation (101). The covariance matrix of the signal component in (99) i.e. $\hat{\mathbf{W}}_M\mathbf{W}\mathbf{p}$ can be evaluated as

$$\mathbb{E}\{\left(\hat{\mathbf{W}}_M\mathbf{W}\mathbf{p}\right)\left(\hat{\mathbf{W}}_M\mathbf{W}\mathbf{p}\right)^H\}$$
$$=\mathbb{E}\{\hat{\mathbf{W}}_M\mathbf{W}\mathbf{p}\mathbf{p}^H\mathbf{W}^H\hat{\mathbf{W}}_M^H\}$$
$$=\mathbf{F}_N^H\begin{bmatrix}\frac{\gamma^2_{\text{zp}}|\hat{\omega}_1|^2|\omega_1|^2}{\left(\gamma_{\text{zp}}|\hat{\omega}_1|^2+1\right)^2} & \cdots & \cdots & 0\\ \vdots & & \ddots & \vdots\\ 0 & & \cdots & \frac{\gamma^2|\hat{\omega}_N|^2|\omega_N|^2}{\left(\gamma_{\text{zp}}|\hat{\omega}_N|^2+1\right)^2}\end{bmatrix}\mathbf{F}_N. \quad (106)$$

The covariance matrix of the effective noise vector $\mathbf{e}_{\text{sczp}}=\hat{\mathbf{W}}_M\tilde{e}_{\text{zp}}$ can be evaluated as

$$\mathbb{E}\{\left(\hat{\mathbf{W}}_M\tilde{\mathbf{e}}_{\text{zp}}\right)\left(\hat{\mathbf{W}}_M\tilde{\mathbf{e}}_{\text{zp}}\right)^H\} \quad (107)$$
$$=\sigma^2_{\text{zp}}\hat{\mathbf{W}}_M\hat{\mathbf{W}}_M^H \quad (108)$$
$$=\mathbf{F}_N^H\begin{bmatrix}\frac{\gamma_{\text{zp}}|\hat{\omega}_1|^2}{\left(\gamma|\hat{\omega}_1|^2+1\right)^2} & \cdots & \cdots & 0\\ \vdots & & \ddots & \vdots\\ 0 & & \cdots & \frac{\gamma_{\text{zp}}|\hat{\omega}_N|^2}{\left(\gamma|\hat{\omega}_N|^2+1\right)^2}\end{bmatrix}\mathbf{F}_N. \quad (109)$$

From (105), (106), (109), it can be observed that the covariance matrices are circulant in nature. Hence, the component-wise variance is equal for all $N$ symbols of the block. The instantaneous SINR for the $n$th symbol can be evaluated as

$$\mathrm{SINR}_{\mathrm{sczp}}^{\mathrm{mmse}} = \frac{\frac{1}{N}\sum_{n=1}^{N} \frac{\gamma_{\mathrm{zp}}{}^2|\hat{\omega}_n|^2|\omega_n|^2}{\left(\gamma_{\mathrm{zp}}|\hat{\omega}_n|^2+1\right)^2}}{\frac{1}{N}\sum_{n=1}^{N} \frac{\gamma|\hat{\omega}_n|^2}{\left(\gamma_{\mathrm{zp}}|\hat{\omega}_n|^2+1\right)^2}}. \quad (110)$$

Applying the Cauchy- Schwarz inequality in (110), the numerator can be upper bounded as

$$\sum_{n=1}^{N} \frac{\gamma_{\mathrm{zp}}^2|\hat{\omega}_n|^2|\omega_n|^2}{\left(\gamma_{\mathrm{zp}}|\hat{\omega}_n|^2+1\right)^2} \leq \gamma_{\mathrm{zp}}^2 \sum_{n=1}^{N} \frac{|\hat{\omega}_n|^2}{\left(\gamma_{\mathrm{zp}}|\hat{\omega}_n|^2+1\right)^2} \sum_{n=1}^{N} |\omega_n|^2.$$

Hence, the expression for $\mathrm{SINR}_{\mathrm{sczp}}^{\mathrm{mmse}}$ in (110) can be modified as

$$\mathrm{SINR}_{\mathrm{sczp}}^{\mathrm{mmse}} \leq \sum_{n=1}^{N} \gamma_{\mathrm{zp}}|\omega_n|^2 \leq \sum_{n=1}^{N} \gamma_{\mathrm{zp}} g_n.$$

The instantaneous BER corresponding to symbol detection using the MMSE estimate for SC-ZP systems is seen to be

$$\mathrm{BER}_{\mathrm{sczp}}^{\mathrm{mmse}} = Q\left(\sqrt{2\gamma_{\mathrm{zp}} \sum_{n=1}^{N} g_n}\right). \quad (111)$$

Let $g_{\mathrm{sum}} = \sum_{n=1}^{N} g_n$. The probability density function associated with the quantity $g_{\mathrm{sum}}$ can be expressed as

$$f_{G_{\mathrm{sum}}}(g_{\mathrm{sum}}) = f_{g_1+g_2+\ldots+g_N}(g_{\mathrm{sum}}) \quad (112)$$
$$= \left[\prod_{n=1}^{N} \frac{1}{\alpha_n}\right] \sum_{j=1}^{N} \frac{e^{-\frac{g_{\mathrm{sum}}}{\alpha_j}}}{\prod_{\substack{n=1\\ n\neq j}}^{N}\left(\frac{1}{\alpha_n}-\frac{1}{\alpha_j}\right)}. \quad (113)$$

The average BER corresponding to the MMSE symbol estimate of SC-ZP systems with channel estimation error can be evaluated as shown next

$$\overline{\mathrm{BER}}_{\mathrm{sczp}}^{\mathrm{mmse}} = \int_0^{\infty} Q\sqrt{\left(2\gamma_{\mathrm{zp}} g_{\mathrm{sum}}\right)} f_{G_{\mathrm{sum}}}(g_{\mathrm{sum}})\, dg_{\mathrm{sum}} \quad (114)$$
$$= \frac{1}{2}\sum_{n=1}^{N}\sum_{j=1}^{N} \frac{\prod_{\substack{n=1\\ n\neq j}}^{N} \frac{\alpha_j}{\alpha_n}}{\prod_{\substack{n=1\\ n\neq j}}^{N}\left(\frac{1}{\alpha_j}-\frac{1}{\alpha_n}\right)} \left(1-\sqrt{\frac{\gamma_{\mathrm{zp}}\alpha_j}{\gamma_{\mathrm{zp}}\alpha_j+1}}\right). \quad (115)$$

### *B. BER ANALYSIS FOR SC-CP SYSTEMS*

#### 1) ZF RECEIVER

The ZF-based symbol vector estimate $\hat{\mathbf{p}}_{\mathrm{sccp}}^{\mathrm{ZF}}$ in the presence of channel estimation error can be expressed as

$$\hat{\mathbf{p}}_{\mathrm{sccp}}^{\mathrm{zf}} = \mathbf{F}_N^H \hat{\Omega}^{-1} \mathbf{F}_N \mathbf{y}_{\mathrm{sccp}} \quad (116)$$
$$= \mathbf{F}_N^H \hat{\Omega}^{-1} \mathbf{F}_N \left(\mathbf{F}_N^H \Omega \mathbf{F}_N \mathbf{p} + \tilde{\mathbf{e}}_{\mathrm{cp}}\right) \quad (117)$$
$$= \mathbf{F}_N^H \hat{\Omega}^{-1} \mathbf{F}_N \left(\mathbf{F}_N^H \left(\hat{\Omega}-\Delta\Omega\right) \mathbf{F}_N \mathbf{p} + \tilde{\mathbf{e}}_{\mathrm{cp}}\right) \quad (118)$$
$$= \mathbf{p} \underbrace{-\mathbf{F}_N^H \hat{\Omega}^{-1} \Delta\Omega \mathbf{F}_N \mathbf{p} + \mathbf{F}_N^H \hat{\Omega}^{-1} \mathbf{F}_N \tilde{\mathbf{e}}_{\mathrm{cp}}}_{\mathbf{e}_{\mathrm{zs}}}. \quad (119)$$

Next, the covariance of the noise vector $\mathbf{e}_{\mathrm{zs}}$ can be evaluated as

$$\mathbf{R}_{e_{\mathrm{zs}}} = \mathbb{E}\{\mathbf{e}_{\mathrm{zs}}\mathbf{e}_{\mathrm{zs}}^H\} \quad (120)$$
$$= \sigma_{e_{cp}}^2 \mathbf{F}_N^H \hat{\Omega}^{-1}\hat{\Omega}^{-H} \mathbf{F}_N + P_T \mathbf{F}_N^H \hat{\Omega}^{-1} \Delta\Omega\Delta\Omega^H \hat{\Omega}^{-H} \mathbf{F}_N$$
$$= \sigma_{e_{cp}}^2 \mathbf{F}_N^H \begin{bmatrix} \frac{\left(1+\gamma_{\mathrm{cp}}|\Delta\omega_1|^2\right)}{\hat{g}_1} & 0\ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \frac{\left(1+\gamma_{\mathrm{cp}}|\Delta\omega_n|^2\right)}{\hat{g}_N} \end{bmatrix} \mathbf{F}_N, \quad (121)$$

where the quantity $\gamma_{\mathrm{cp}} = \frac{P_T}{\sigma_{e_{\mathrm{cp}}}^2}$. It can be observed that the noise covariance matrix is circulant in nature since it can be expressed in the standard form of a circulant matrix i.e. $\mathbf{A}^H\mathbf{D}\mathbf{A}$, where $\mathbf{A}$ and $\mathbf{D}$ are unitary and diagonal matrices respectively. Thus, the noise variance is identical for all the components and is given by $\frac{1}{N}\sum_{n=1}^{N} \frac{\sigma_{e_{\mathrm{cp}}}^2\left(1+\gamma_{\mathrm{cp}}|\Delta\omega_n|^2\right)}{\hat{g}_n}$. Similar to the discussion of SC-ZP systems, one can define $\breve{\omega} \triangleq |\Delta\omega_n|^2$. The instantaneous SNR for the ZF receiver for SC-CP systems in the presence of channel estimation error, can be expressed as

$$\mathrm{SNR}_{\mathrm{sccp}}^{\mathrm{zf}} = \frac{P_T}{\frac{1}{N}\sum_{n=1}^{N} \frac{\sigma_{e_{cp}}^2\left(1+\gamma_{\mathrm{cp}}\breve{\omega}\right)}{\hat{g}_n}}$$
$$= \frac{1}{\frac{1}{N}\sum_{n=1}^{N} \frac{1+\gamma_{\mathrm{cp}}\breve{\omega}}{\gamma_{\mathrm{cp}}\hat{g}_n}}. \quad (122)$$

Proceeding on similar lines to that of SC-ZP systems, the average BER can be determined as

$$\overline{\mathrm{BER}}_{\mathrm{sccp}}^{\mathrm{zf}} \geqslant \int_0^{\infty} Q\left(\sqrt{2N\frac{\gamma_{\mathrm{cp}}}{1+\gamma_{\mathrm{cp}}\breve{\omega}} g_m}\right) f_{G_m}(g_m)\, dg_m \quad (123)$$
$$= \frac{1}{2}\left(1-\sqrt{\frac{NP_T}{P_T+\sum_{n=1}^{N}\frac{1}{\alpha_n}\left(\sigma_{e_{\mathrm{cp}}}^2+P_T\breve{\omega}\right)}}\right), \quad (124)$$

where $G_m$ stands for the quantity defined in (70). As a special case, the average BER in absence of channel estimation error can be determined as

$$\frac{1}{2}\left(1-\sqrt{\frac{NP_T}{P_T+\sum_{n=1}^{N}\frac{1}{\alpha_n}\sigma_{e_{\mathrm{cp}}}^2}}\right). \quad (125)$$

#### 2) MMSE RECEIVER

On similar lines to that of SC-ZP systems, the average BER corresponding to MMSE symbol estimation for SC-CP systems in the presence of channel estimation error can be approximated by setting $\gamma_{cp} = \frac{P_T}{\sigma^2_{e_{cp}}}$ as

$$\overline{\text{BER}}^{\text{mmse}}_{\text{sccp}} = \int_0^{\infty} Q\left(\sqrt{2\gamma_{cp} g_{\text{sum}}}\right) f_{G_{\text{sum}}}(g_{\text{sum}})\, dg_{\text{sum}} \quad (126)$$

$$= \frac{1}{2}\sum_{n=1}^{N}\sum_{j=1}^{N} \frac{\prod_{\substack{n=1\\ n\neq j}}^{N} \frac{\alpha_j}{\alpha_n}}{\prod_{\substack{n=1\\ n\neq j}}^{N}\left(\frac{1}{\alpha_j}-\frac{1}{\alpha_n}\right)} \left(1-\sqrt{\frac{\gamma_{cp}\alpha_j}{\gamma_{cp}\alpha_j+1}}\right), \quad (127)$$

where, $g_{\text{sum}} = \sum_{n=1}^{N} g_n$.

### 3) ML RECEIVER

As shown in [14], the PEP corresponding to the symbol vectors $\mathbf{p}, \mathbf{p}'$ conditioned on the channel estimate $\hat{\mathbf{h}}$ can be simplified as

$$\Pr\left(\mathbf{p} \to \mathbf{p}' | \hat{\mathbf{h}}, \gamma_{cp}\right) = Q\left(\sqrt{\frac{\gamma_{cp}\left\|\hat{\mathbf{W}}(\mathbf{p}-\mathbf{p}')\right\|^2}{2}}\right) \quad (128)$$

$$= Q\left(\sqrt{\frac{\gamma_{cp}\hat{\mathbf{h}}^H \underbrace{\mathbf{V}^H_{cp}\mathbf{D}^H_{cp}\mathbf{D}_{cp}\mathbf{V}_{cp}}_{\tilde{\mathbf{V}}_{cp}} \hat{\mathbf{h}}}{2}}\right) \quad (129)$$

$$= Q\left(\sqrt{\frac{\gamma_{cp}\sum_{l=0}^{L-1}\phi_l |\hat{h}_{ul}|^2}{2}}\right), \quad (130)$$

where $\mathbf{V}_{cp} = \sqrt{N}\mathcal{F}_N, \mathbf{D}_{cp} = \text{Diag}\left(\mathbf{F}_N(\mathbf{p}-\mathbf{p}')\right)$ and $\phi_l$ is the $l$th eigenvalue of $\tilde{\mathbf{V}}_{cp} = \mathbf{V}^H_{cp}\mathbf{D}^H_{cp}\mathbf{D}_{cp}\mathbf{V}_{cp}$. As shown in [14], the average PEP can be determined by averaging over the channel pdf as

$$\Pr\left(\mathbf{p} \to \mathbf{p}' | \gamma_{cp}\right) = \int_0^{\infty} Q\left(\sqrt{\gamma_{cp} 2NL\chi}\right)\frac{1}{\alpha'}e^{-\frac{\chi}{\alpha'}}d\chi \quad (131)$$

$$= \frac{1}{2}\left(1-\sqrt{\frac{\gamma_{cp}NL\alpha'}{\gamma_{cp}NL\alpha'+1}}\right). \quad (132)$$

The average BER for the ML receiver in SC-CP systems can be further obtained as

$$\overline{\text{BER}}^{\text{ml}}_{\text{sccp}} = \frac{1}{2^{N-1}}\left(1-\sqrt{\frac{\gamma_{cp}NL\alpha'}{\gamma_{cp}NL\alpha'+1}}\right) \quad (133)$$

## C. BER ANALYSIS FOR MC-CP SYSTEMS

### 1) ZF RECEIVER

The ZF-based symbol estimate for MC-CP systems in the presence of channel estimation error can be expressed as

$$\hat{\mathbf{p}}^{\text{zf}}_{\text{mccp}} = \hat{\Omega}^{-1}\mathbf{F}_N\mathbf{y}_{\text{mccp}} \quad (134)$$

$$= \hat{\Omega}^{-1}\mathbf{F}_N\left(\mathbf{F}^H_N\Omega\mathbf{p}+\tilde{\mathbf{e}}_{cp}\right) \quad (135)$$

$$= \hat{\Omega}^{-1}\Omega\mathbf{p}+\hat{\Omega}^{-1}\mathbf{F}_N\tilde{\mathbf{e}}_{cp} \quad (136)$$

$$= \hat{\Omega}^{-1}\left(\hat{\Omega}-\Delta\Omega\right)\mathbf{p}+\hat{\Omega}^{-1}\mathbf{F}_N\tilde{\mathbf{e}}_{cp} \quad (137)$$

$$= \mathbf{p}-\hat{\Omega}^{-1}\Delta\Omega\mathbf{p}+\hat{\Omega}^{-1}\mathbf{F}_N\tilde{\mathbf{e}}_{cp} \quad (138)$$

The system model for the $n$th subcarrier can be expressed as

$$\hat{\mathbf{p}}^{\text{zf}}_{\text{mccp}}(n) = \mathbf{p}(n) + \underbrace{\left(\frac{[\mathbf{F}_N]_{n,n}\tilde{\mathbf{e}}_{cp}(n) - \Delta\omega_n\mathbf{p}(n)}{\hat{\omega}_n}\right)}_{\mathbf{e}_{zm}(n)}. \quad (139)$$

The noise variance for the $n$th subcarrier can be determined as

$$\mathbb{E}\{|\mathbf{e}_{zm}(n)|^2\} = \frac{\sigma^2_{e_{cp}}+P_T\breve{\omega}}{|\hat{\omega}_n|^2} = \frac{\sigma^2_{e_{cp}}(1+\gamma\breve{\omega})}{\hat{g}_n}. \quad (140)$$

Employing the above result, the instantaneous SNR on $n$th subcarrier can therefore be evaluated as

$$\text{SNR}^{\text{zf}}_{\text{mccp}} = \frac{P_T}{\frac{\sigma^2_{e_{cp}}+P_T\breve{\omega}}{\hat{g}_n}} = \frac{\gamma_{cp}\hat{g}_n}{(1+\gamma_{cp}\breve{\omega})} = \breve{\gamma}_{cp}\hat{g}_n, \quad (141)$$

where $\breve{\gamma}_{cp} = \frac{\gamma_{cp}}{1+\gamma_{cp}\breve{\omega}}$. The average BER for the $n$th subcarrier with BPSK modulated symbols can be obtained as

$$\overline{\text{BER}}^{\text{zf}}_{\text{mccp}}(\breve{\gamma}_{cp}) = \frac{1}{N}\sum_{n=1}^{N}\int_0^{\infty} Q\left(\sqrt{2\breve{\gamma}_{cp}\hat{g}_n}\right)\frac{1}{\bar{\alpha}_n}\exp\left(-\frac{\hat{g}_n}{\bar{\alpha}_n}\right)d\hat{g}_n$$

$$= \frac{1}{2N}\sum_{n=1}^{N}\left(1-\sqrt{\frac{\breve{\gamma}_{cp}\bar{\alpha}_n}{1+\breve{\gamma}_{cp}\bar{\alpha}_n}}\right). \quad (142)$$

Hence, as seen from the above expression, the average BER of MC-CP based systems asymptotically achieves a diversity order equal to unity. In the absence of channel estimation error i.e. where $\breve{\omega} = 0$, the BER expression reduces to

$$\text{BER}^{\text{zf}}_{\text{mccp}}(\gamma_{cp}) \approx \frac{1}{2}\sum_{n=1}^{N}\left(1-\sqrt{\frac{\gamma_{cp}\alpha_n}{1+\gamma_{cp}\alpha_n}}\right). \quad (143)$$

### 2) MMSE RECEIVER

Replacing $\mathbf{p}$ by $\mathbf{p}_{mc} = \mathbf{F}^H_N\mathbf{p}$ and $\gamma_{zp} = \breve{\gamma}_{cp} = \frac{\gamma_{cp}}{1+\gamma_{cp}\breve{\omega}}$ in (99) for SC-ZP, the SINR for the $n$th subcarrier in MC-CP systems can be obtained as

$$\text{SINR}^{\text{mmse}}_{\text{mccp}} = \breve{\gamma}_{cp}g_n. \quad (144)$$

The average BER for BPSK modulated symbols on the $n$th subcarrier of the MC-CP system can be simplified as

$$\overline{\text{BER}}^{\text{mmse}}_{\text{mccp}}(\breve{\gamma}_{cp}) = \frac{1}{N}\sum_{n=1}^{N}\int_0^{\infty} Q\left(\sqrt{2\breve{\gamma}_{cp}g_n}\right)f_{G_n}(g_n) \quad (145)$$

$$= \frac{1}{2N}\sum_{n=1}^{N}\left(1-\sqrt{\frac{\breve{\gamma}_{cp}\alpha_n}{1+\breve{\gamma}_{cp}\alpha_n}}\right). \quad (146)$$

### 3) ML RECEIVER

The problem for maximum likelihood (ML) detection of the data symbol vector $\mathbf{p}$ using the received signal $\mathbf{y}_{\text{mccp}}$ for MC-CP systems can be expressed as

$$\hat{\mathbf{p}}^{\text{ml}}_{\text{mccp}} = \arg\min_{\mathbf{p}} \left(\hat{\Omega}^{-1}\mathbf{F}_N\mathbf{y}_{\text{mccp}} - \mathbf{p}\right)^H \hat{\Omega}^H\hat{\Omega}\left(\hat{\Omega}^{-1}\mathbf{F}_N\mathbf{y}_{\text{mccp}} - \mathbf{p}\right).$$

Since, $\hat{\Omega}^H\hat{\Omega}$ is a diagonal matrix with positive diagonal entries, it is evident from (147) that ML detection is equivalent to considering symbol detection on the individual subcarriers similar to the ZF receiver described in (142). Hence, the average BER performance of the ML receiver is similar to that of the ZF receiver given in (142) for MC-CP.

## D. BER ANALYSIS FOR MC-ZP SYSTEMS

### 1) ZF, MMSE AND ML RECEIVER

The ZF symbol estimate in MC-ZP systems in the presence of channel estimation error can be expressed as

$$\hat{\mathbf{p}}^{\text{zf}}_{\text{mczp}} = \hat{\Omega}^{-1}\mathbf{F}_N\mathbf{y}_{\text{mczp}} \tag{147}$$
$$= \hat{\Omega}^{-1}\mathbf{F}_N\left(\mathbf{F}_N^H\Omega\mathbf{p} + \mathbf{e}_{\text{oa}}\right) \tag{148}$$
$$= \mathbf{p} + \underbrace{\hat{\Omega}^{-1}\left(\mathbf{F}_N\mathbf{e}_{\text{oa}} - \Delta\Omega\mathbf{p}\right)}_{\tilde{\mathbf{e}}_{\text{oa}}}. \tag{149}$$

The noise affecting the $n$th data symbol $\mathbf{p}(n)$ transmitted on $n$th subcarrier has the variance

$$\mathbb{E}\{|\tilde{\mathbf{e}}_{\text{oa}}(n)|^2\} = \frac{\sigma^2_{\text{oa}} + P_T\breve{\omega}}{|\hat{\omega}_n|^2} = \frac{\sigma^2_{\text{oa}}(1+\gamma_{\text{zp}}\breve{\omega})}{\hat{g}_n}. \tag{150}$$

The instantaneous SNR on the $n$th subcarrier can therefore be evaluated as

$$\text{SNR}^{\text{zf}}_{\text{mczp}} = \frac{P_T}{\frac{\sigma^2_{\text{oa}} + P_T\breve{\omega}}{\hat{g}_n}} = \frac{\gamma_{\text{zp}}\hat{g}_n}{(1+\gamma_{\text{zp}}\breve{\omega})} = \breve{\gamma}_{\text{zp}}\hat{g}_n, \tag{151}$$

where $\breve{\gamma}_{\text{zp}} = \frac{\gamma_{\text{zp}}}{1+\gamma_{\text{zp}}\breve{\omega}}$. On similar lines to that of MC-CP systems, the average BER can be determined as

$$\overline{\text{BER}}^{\text{zf}}_{\text{mczp}}(\breve{\gamma}_{\text{zp}}) = \frac{1}{N}\sum_{n=1}^{N}\int_0^\infty Q\left(\sqrt{2\breve{\gamma}_{\text{zp}}\hat{g}_n}\right)\frac{1}{\bar{\alpha}_n}\exp\left(-\frac{\hat{g}_n}{\bar{\alpha}_n}\right)d\hat{g}_n$$
$$= \frac{1}{2N}\sum_{n=1}^{N}\left(1-\sqrt{\frac{\breve{\gamma}_{\text{zp}}\bar{\alpha}_n}{1+\breve{\gamma}_{\text{zp}}\bar{\alpha}_n}}\right). \tag{152}$$

In the absence of channel estimation error, the average BER for these systems can be readily obtained by setting $\breve{\omega} = 0$

$$\overline{\text{BER}}^{\text{zf}}_{\text{mczp}}(\breve{\gamma}_{\text{zp}}) = \frac{1}{2N}\sum_{n=1}^{N}\left(1-\sqrt{\frac{\breve{\gamma}_{\text{zp}}\alpha_n}{1+\breve{\gamma}_{\text{zp}}\alpha_n}}\right). \tag{153}$$

Once again, the average BER performance of the ZF receiver is similar to that of the ML receiver for MC-ZP based systems due to the diagonal nature of $\hat{\Omega}^H\hat{\Omega}$.

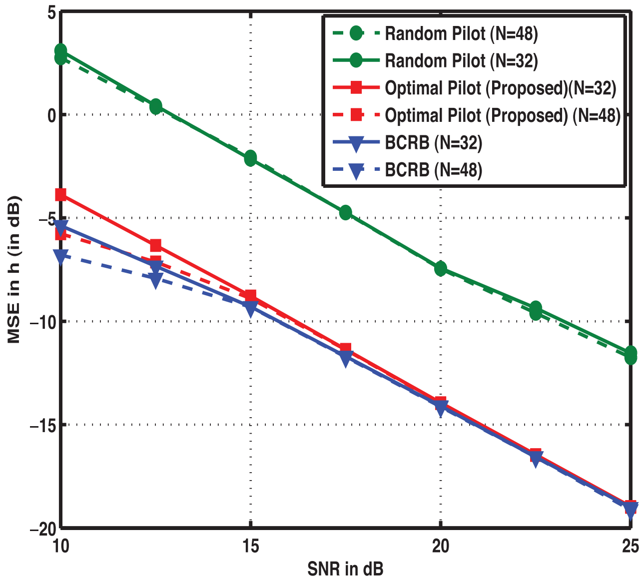


**FIGURE 4.** MSE comparison of the proposed pilot scheme with the standard random pilot scheme and BCRB for SC-ZP systems.

## V. SIMULATION RESULTS

This section presents results of a Monte-Carlo simulation-based study for the MSE and BER performance of the described optimal pilot design strategies toward channel estimation in SC-ZP and MC-CP systems. The setup considers a frequency selective Rayleigh fading channel, in which the channel taps are generated as independent circularly symmetric complex Gaussian random variables with mean equal to zero. The transmit symbols for various block-transmission systems are assumed to be BPSK modulated. Simulations are performed for two different pairs of $(\nu, N)$ given as (32, 32) and (16, 48). The second set is chosen from the reference [1]. Both these sets satisfy the requirement $\nu \geq L$ and also enable us to explore the effect of guard interval and block length on the estimation performance. The simulation results are averaged over 1000 independent channel realizations. Fig.4 and Fig.5 plot the output MSE of the linear minimum mean square errror (LMMSE) [20] channel estimate versus SNR for SC-ZP and MC-CP systems, respectively. For any zero mean parameter h, the LMMSE estimator is given by

$$\hat{\mathbf{h}} = \mathbf{R}_{\mathbf{hy}}\mathbf{R}^{-1}_{\mathbf{yy}}\mathbf{y}, \tag{154}$$

where $\mathbf{R}_{\mathbf{hy}} = \mathbb{E}\{\mathbf{h}\mathbf{y}^H\} = \mathbb{E}\left\{\mathbf{h}\left(\mathbf{X}_{\text{sczp}}\mathbf{h} + \tilde{\mathbf{e}}_{\text{zp}}\right)^H\right\} = \mathbf{K}_h\mathbf{X}^H_{\text{sczp}}$ and $\mathbf{R}_{\mathbf{yy}} = \mathbb{E}\{\mathbf{y}\mathbf{y}^H\} = \mathbf{K}_h\mathbf{X}_{\text{sczp}}\mathbf{X}^H_{\text{sczp}} + \sigma^2_{\text{zp}}\mathbf{I}_P$. Thus, the LMMSE estimate of $\mathbf{h}$ for our system model in Eqn. (13) can be evaluated as

$$\hat{\mathbf{h}} = \left(\mathbf{X}^H_{\text{sczp}}\mathbf{X}_{\text{sczp}}\right)^{-1}\mathbf{X}^H_{\text{sczp}}\bar{\mathbf{y}}_{\text{sczp}}, \tag{155}$$

where $\mathbf{X}_{\text{sczp}} = \sqrt{P}\text{Diag}\left(\mathbf{Z}_{\text{zp}}\mathbf{p}^*_{\text{sczp}}\right)\mathcal{F}_P$. These figures clearly demonstrate that the MSE achieved by the proposed optimal pilot design technique is substantially lower in both the systems in comparison to that of random pilot sequences.

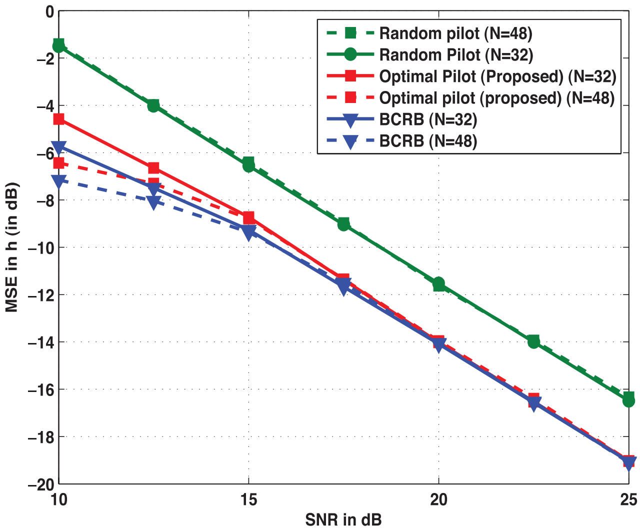


**FIGURE 5.** MSE comparison of the proposed pilot scheme with the standard random pilot scheme and BCRB for MC-CP systems.

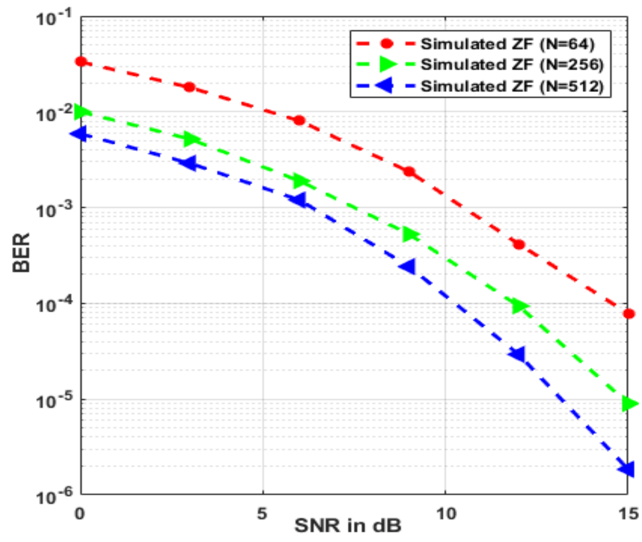


**FIGURE 7.** BER performance comparison of ZF receiver employing proposed pilot design approach for different values of $N = \{64, 256, 512\}$ in SC-ZP systems.

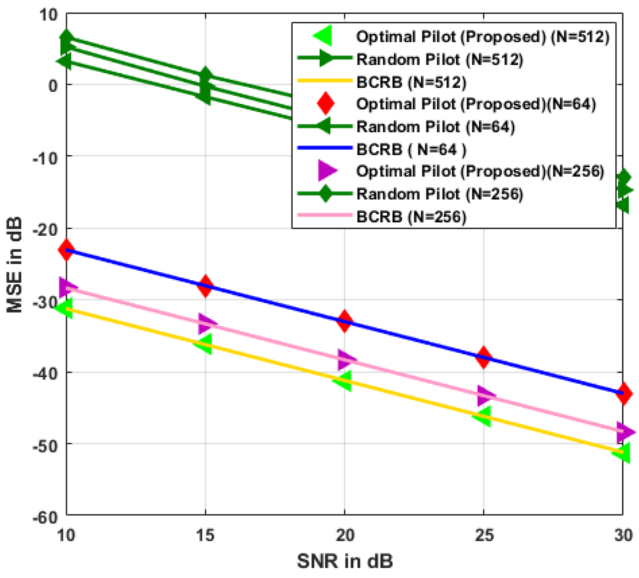


**FIGURE 6.** MSE and BCRB performance comparison of channel estimate employing the proposed pilot design scheme for different values of $N = \{64, 256, 512\}$ in SC-ZP systems.

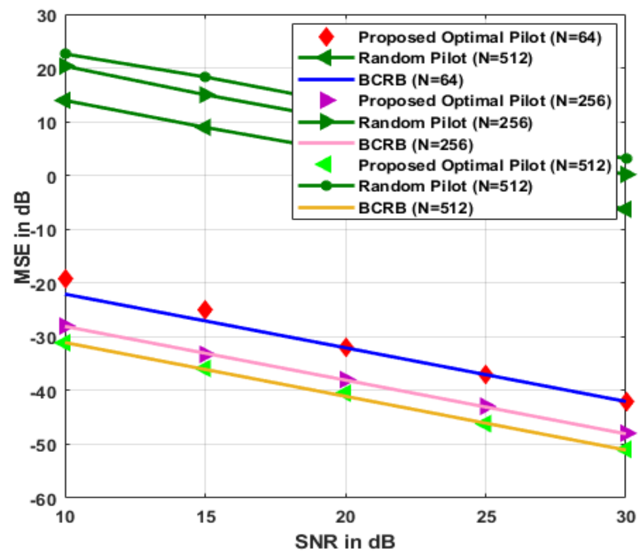


**FIGURE 8.** MSE and BCRB performance comparison of channel estimate employing the proposed pilot design scheme for different values of $N = \{64, 256, 512\}$ in MC-CP systems.

More specifically, one can observe a 6.62 dB improvement in MSE for the SC-ZP system, whilst that for its MC-CP counter part is 3.1 dB at SNR = 10 dB. The MSE is also seen to decrease when the block length increases from $N = 32$ to $N = 48$, as expected, since the estimation accuracy is higher for a pilot sequence of larger length. Another trend that is eminently clear from the figures is that the MSE of both the single and multi-carrier systems above achieves the BCRB for the proposed techniques. Yet, there is a slight deviation of the MSE from the ideal BCRB in the low SNR regime, which can be attributed to the inequality in (32) that is used to determine the optimal designs. This inequality becomes tighter at higher SNRs, which explains the observed deviation. We have also provided MSE and BER results for other $(\nu, N)$ pairs (16, 64), (16, 256), (16, 512), comprising of larger number of subcarriers and block lengths, as shown in Fig. 6, Fig. 8, Fig. 7 and Fig. 9. It can be observed from all the four figures that the estimation performance get significantly improved for larger values of $N$.

Results are now presented to validate the BER performance and the pertinent analysis carried out in section IV. For this purpose, the ZF, MMSE and ML receivers are variously used to estimate the transmit symbol vector. For these simulations, the number of symbols per block is set as $N = 16$ and the SNR is increased from 0 dB to 40dB. The BER performance of MC-ZP and MC-CP systems for such a setup is shown in Fig. 10 and fig.11, respectively. It can be readily seen

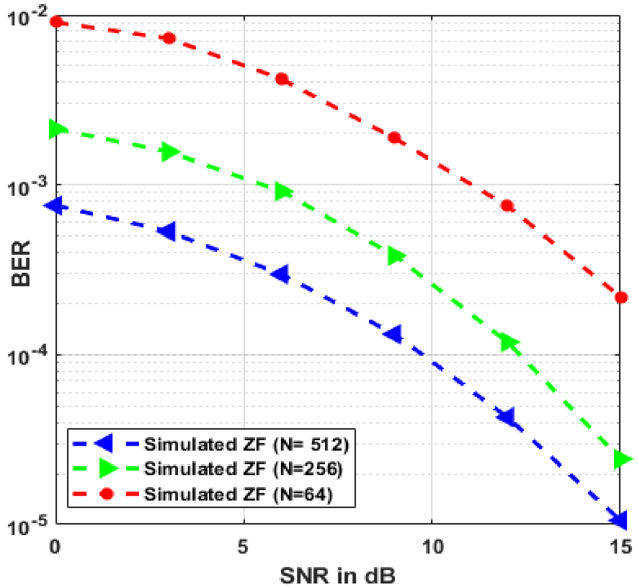


FIGURE 9. BER performance comparison of ZF receiver employing proposed pilot design approach for different values of $N = \{64, 256, 512\}$ in MC-CP systems.

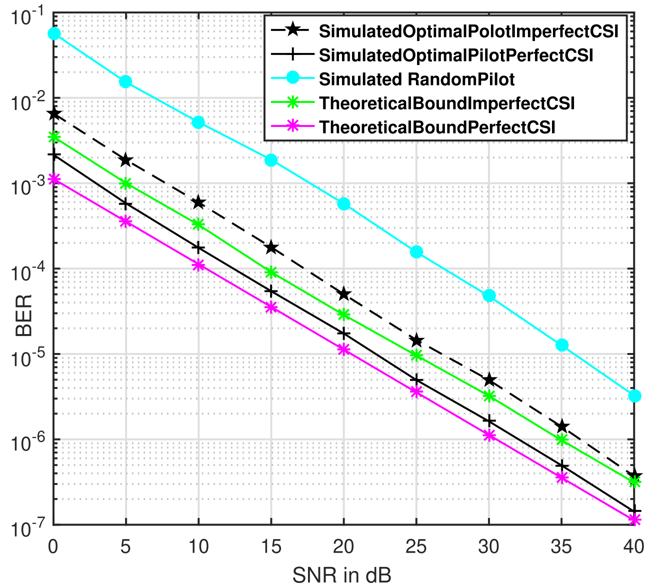


FIGURE 11. BER comparison of the proposed pilot scheme with the standard random pilot scheme and theoretical bounds for MC-CP systems.

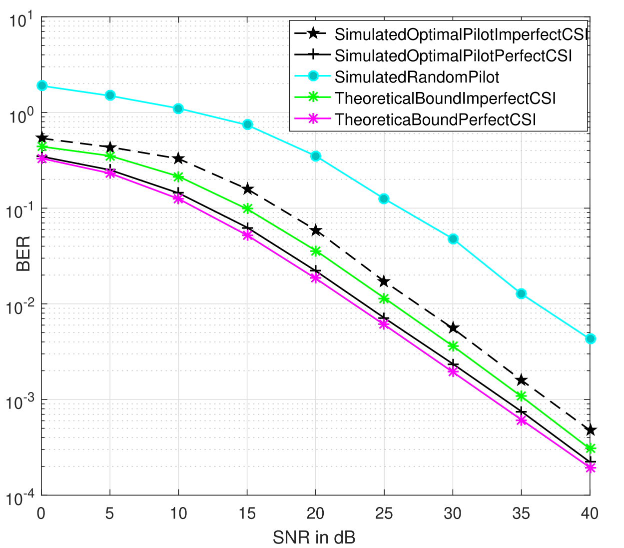


FIGURE 10. BER comparison of the proposed pilot scheme with the standard random pilot scheme and theoretical bounds for MC-ZP systems.

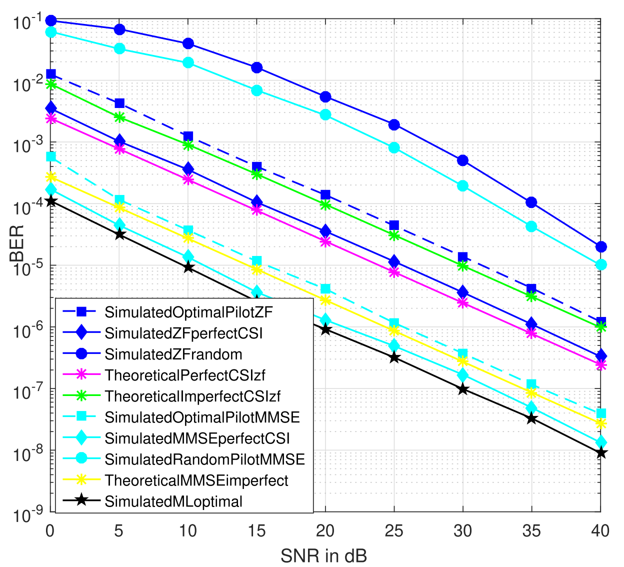


FIGURE 12. BER comparison of the proposed pilot scheme with the standard random pilot scheme and theoretical bounds for SC-ZP systems.

that similar to MSE, the BER of the proposed algorithms is significantly lower than that obtained using random pilot sequences. Moreover in Fig. 10, the BER performance is seen to be in good agreement with the corresponding analytical results derived in (152) and (153) for imperfect and perfect channel knowledge, respectively, which lends credence to analysis in section IV. It is also worth noting that both the figures attest to the fact that the ZF, MMSE and ML receivers considered in this study yield the same BER values. This is explained by the decoupled symbol detection process for each subcarrier. Fig.11 shows a similar trend for MC-CP systems. In contrast, Fig.12 that plots the BER performance of SC-ZP systems shows that the ML receiver yields the best response for these systems followed by the MMSE receiver, which is in turn better in comparison to that of the ZF receiver. Interestingly, this is due to the fact that the decoding across the subcarriers is not decoupled for this single-carrier system, unlike these multi-carrier systems whose performance has been shown in earlier figures. Finally, the BER performance of the optimal pilot design once again shows an improvement over that of the random pilot sequence.

## VI. CONCLUSION

This work has successfully developed a general framework for the construction of optimal pilot signals that is applicable in single as well as multi-carrier systems with cyclic prefix and zero padding redundancies for block transmission. Since the proposed algorithm is based on minimizing the BCRB that represents a lower bound on the MSE of channel estimation, it yields the best possible performance. Subsequently, analysis was carried out to present compact closed-form expressions for the output BER of all the four major classes of block transmission systems, viz. SC-ZP, MC-ZP, SC-CP and MC-CP, incorporating also the effect of CSI estimation error, with linear as well as non-linear equalization techniques employed at the receiver. In the end, a comprehensive Monte-Carlo-based simulation study was carried out to demonstrate the improved MSE and BER performance resulting from the proposed pilot signalling techniques over that of random pilot sequence. The results that emerged from this study not only explicitly demonstrated the improved MSE and BER of the optimal pilot designs but also validated the analytical results for the BER.

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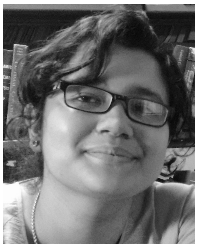

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