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An Event-Based Interaction Sampled-Control for Consensus of Multi-Agents With Multiple Time-Varying Delays

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ABSTRACT An event-based consensus filtering control scheme for multi-agents with multiple mixing delays is proposed in the paper. Firstly, a piece-wise sampling model with transmission delay defined from sensors to controllers is built, and the effect of time-varying delay on sampling is analyzed. Secondly, a self-triggered scheme is proposed to take into consideration of reducing redundant data and complexity. Thirdly, to fully utilize the available information, by employing an improved generalized free-weighting matrix inequality, a novel Lyapunov-Krasovskii functional approach is proposed to achieve global asymptotically synchronization. At last, an example of multiple unmanned aerial vehicles is offered to show the effectiveness of proposed method.

INDEX TERMS Multi-agents, event-triggered, synchronization, time delay, sampled-data control.

I. INTRODUCTION

Multi-agent systems have received considerable attention since the 1990s. Based on the study of networked control system theory and computer science, while with analysis of characteristics such as clustering and connecting weights, a huge scale network could be simplified to some smaller subones. Researchers could only focus on the finite sub-networks instead of the whole system, and use networks to communicate with each other. Therefore, the study faced its boom era, numerous applications emerged in a variety of scenarios, unmanned aerial and ground vehicles, networking satellites, medical robots and some other networked electronic products to name a few [1]–[4].

But ubiquitous transmission delays caused by sensors, controllers, actuators and network transmission, also with inherent system delay make the developing of researches and applications slow down. Especially for most existing systems, due to the widely digital signal processing, the acquisition and transmission of information all need to be sampled. That means the sampled-data is only available information that can be used for analysis, processing and control. The accumulated time-delay will significantly increase the risk of system instability, also make the control scheme and processing challenging. Therefore, it is of great significance to model the multi-agent systems with mixing time delay and formulate a feasible control scheme.

To handle the problem, proposing a reasonable sampler is one of the important issues. In digital signal processing, the most classical method is Shannon sampling, which has a fixed periodic minimum sampling interval. This method had provided tremendous help for the development of the digital information era, and been widely used in the analysis and application of multi-agents. Bamieh and [6] proposed a lifting system with a finite-dimensional state-space to describe the continuous-time behavior of sampled-data systems, which could be applied for analysis of linear periodic sampling systems. Sufficient conditions were offered to guarantee the

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stability of linear plant with a sector bounded nonlinear and possibly time-varying. Some other synchronization and system stability works could be seen in [7]–[11]. However, the interval of sampling data is always assumed to be fixed. With the enlargement of multi-agent networks, the variability of signals requires the entire system to have a sufficiently high sampling frequency. Therefore, the fixed periodic sampling method requires a lot of computing resources to adapt. In fact, in most application systems, the operation of the system is usually required to be stable. In a sampling period, the signal is stable in most of the time, merely a few special moments will change drastically, and requires extremely high sampling frequency. Therefore, it is not indispensable to maintain an absolutely high sampling frequency in order to completely obtain the information in the system. The minimum sampling interval is only required when the signal changes very fast. In this way, using periodic sampling methods, actuators and sensors need to be kept at a relatively high frequency, which wastes computer power. For this reason, based on the rate of change of the signal, the time-varying sampling method is thought to be an alternative solution and attracted plenty of attention. For a continuous-time system with a piece-wise continuous input delay, Emilia Fridman proposed a sufficient robust sampled-data control approach in the literature [15]. For systems with sampling intervals of a certain upper boundary $h > 0$, the system stabilization could be guaranteed, even with polytopic type uncertainties. With this technology, samplers are not needed to follow the minimum periodic sampling interval approach now, and the sampling intervals could be time-varying. Consequently, this application can effectively reduce the number of samples and save computing resources. But how to choose the optimal sampling points is still challenging. Besides, the maximum h is also too small to meet the application. Wang *et al.* [16] modeled a continuous dynamic network with discrete-time communications and proposed a synchronization criterion through the aperiodic sampled method. To better make the utmost of hybrid information, in [17], an exponential synchronization criterion in discrete-time communications for CDNs is proposed, with a larger sampling interval, and the number of decision variables is decreased, thereby reducing the computational burden. Recent works on increasing the average sampling intervals could be also seen in [18]–[23] and so on.

Among the above works, one of the significant drawbacks is that, although the stability and robustness for the time-varying sampled-data control can be improved, sampling decision rules that take into account the integrity of the information and less computational burden are formidable to obtain. In recent years, the event-triggered sampling method received considerable attention. By setting specific rules, this sampler can monitor signal changes in real-time, and only sample the required signals, avoiding a lot of redundant information. Compared with other sampled-data approaches, the technique demonstrates better robustness, meanwhile synchronization is not required within transmission instants. This practical method has been applied in many fields and achieved a series of results [24]–[30]. But one important restriction for real-time monitor is that, continuous computation of time-varying thresholds and states errors are required. One solution is so-called ''periodic event-triggered control'' [32]–[36]. It puts the state and error measurement at the instant of cycle time, replacing the continuous calculation we mentioned before. It is designed that the particular system trajectories are independent of the lower-bounds on the inter-event intervals. Therefore Zeno behavior could be avoided automatically. Another solution is self-triggered control, which also received extraordinary attention. This strategy only needs the currently available information of a single agent to estimate future behavior. As a result, self-triggering control is more suitable for real-time execution of distributed controllers with additional energy savings. However, due to the design scheme, one important drawback is that the sampling interval with self-triggered control is smaller than that one based on event-based sampled control strategy, as shown in [38]–[41]. Yue Dong modeled an event-triggered H_∞ sampler for reducing the communication load of multi-systems, with a time-delay from sensors to controllers [42]. The properties of event-based and the effect of H_∞ is utilized on the system. Compared with some existing event-triggered control, the sampler achieved a larger average period, avoiding the Zeno phenomenon. The event generator is also implemented in [43]–[46]. By applying linear matrix inequalities(LMIs) and dissipation inequalities, as shown in [47]–[49], the self-triggered control scheme could be further improved, and more constraints are needed to meet the requirement of applications.

In the note, we focus on the event-triggered synchronization control for multi-agent systems. A direct model for multi-agents with discrete-time communications is proposed in the model, network transmission delay and system delay are both considered. a self-triggered event generator is proposed for system analysis and control design. Based on the model, A Lyapunov functional structure is built to better take full advantage of available information. Moreover, to the authors' knowledge, the mixed time-delay for self-triggered control scheme has not been investigated. We proposed a simulation to verify efficiency. Compared with some existing results, the average sampling interval would be increased in the example.

A. NOTATION

Some common symbols are explained in this part. The symbol $\otimes$ represents Kronecker product. $diag\{a_1, a_2, \ldots, a_m\}$ means diagonal matrix with $a_1, a_2, \ldots, a_m$ as central elements or matrices. $A + A^T$ is represented as $Sym\{A\}$.

II. THE STRATEGIES FOR EVENT-TRIGGERED GENERATORS AND PRELIMINARY KNOWLEDGE

Consider a directed full connected graph $\mathcal{G}(\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N} = \{n_1, n_2, \ldots, n_N\}$ represents N notes in the directed graph, $\mathcal{A} = [a_{ij}]_{N\times N}$ is the adjacency matrix, and $\mathcal{E}$ is a set of directed edges and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$. e_{ij} represents a directed edge in $\mathcal{G}(\mathcal{N}, \mathcal{E}, \mathcal{A})$ of the ordered pair of nodes n_i and n_j. If node n_j communicate the information with node n_i, then $a_{ij} > 0$, otherwise, $a_{ij} = 0$. For diagonal element a_{ii} is defined as $a_{ii} = -\sum_{i=1,j\neq i}^{N} a_{ij}$. A directed graph is strongly connected if and only if there is a directed path between any two district nodes. For any $i \neq j$; $i, j = 1, 2, \ldots, N$, the Laplacian matrix $L = [\iota_{ij}]_{N\times N}$ of (G) is defined as

$$\iota_{ij} = -a_{ij}, \iota_{ii} = -a_{ii} \tag{1}$$

The equation $\sum_{j=1}^{N} \iota_{ij} = 0$ holds in all directed full connected graph.

Consider a nonlinear system of single isolated node n_{ij} described by

$$\dot{x}_{ij}(t) = Ax_{ij}(t) + A_d x_{ij}(t-d) + f(x_{ij}(t)) + Bu_{ij}(t) \tag{2}$$

where $i = 1, \ldots, M$ is the sequence number of networks, $j = 1, \ldots, N$ is the number of node. $x_{ij}(t) = [x_{ij1}(t), x_{ij2}(t), \ldots, x_{ijn}(t)]^T \in \mathbb{R}^{n\times 1}$ is the system state vector of node j in the ith networks. At the time $t < 0$, $x_{ij}(t-d)$ holds at the initial condition. n is the dimension of node. In the paper, to make the representation simple, the vector dimension of each node is set to be the same. $u_{ij}(t) \in \mathbb{R}^{\rho\times 1}$ is the control input vector to be designed. A and B are matrix with appropriate dimension. The state of the ith network is $x_i = [x_{i1}(t), x_{i2}(t), \ldots, x_{im}(t)]^T \in \mathbb{R}^{nm\times 1}$ accordingly, and $u_i(t) \in \mathbb{R}^{\rho m\times 1}$. $f(x_{ij}(t)) = [f(x_{ij1}(t)), f(x_{ij2}(t)), \ldots, f(x_{ijn}(t))]^T$ is a nonlinear dynamic vector-valued continuous activation function, and following assumption is satisfied:

Assumption 1: Each element of vector $\|f(x_i)\|$ is Lipschitz continuous.

The sampled data controller is shown as following:

$$u_i(t) = \delta K \sum_{j=1,j\neq i}^{N} a_{ij}(x_j(t_\kappa) - x_i(t_\kappa)), t_\kappa \leq t < t_{\kappa+1} \tag{3}$$

where $K > 0$ is the controller gain, coupling strength $\delta > 0$ is a positive parameter. Throughout this paper, it is assumed that the transfer delay exists between sensors and controllers. The aim is to build a suitable K to achieve global synchronization of multi-agents in the directed networks. In the paper, we build a self-triggered samplers with the following condition:

$$e_\kappa^T(\kappa h)\Omega e_\kappa(\kappa h) > \xi x^T((\kappa+j)h)\Omega x((\kappa+j)h) \tag{4}$$

where $0 \leq \xi < 1$ and Ω is a symmetric positive definite matrix. κ is a positive integer to be chosen. The information from sensors to controllers should all be picked by the event-triggered sampler. Between the latest transmitted and the current sampling instant, the errors is represented as $e_\kappa(\kappa h) = [x((\kappa+j)h) - x(\kappa h)]$.

Remark 1: Notice that equation (4) is to set a threshold. The current sampled-data will not be released to the controllers once if it exceeds the threshold of the event-based condition. It is easy to see that autonomy could be designed to take charge rather than passive reception of data. While with the event-samplers, it would reduce both computational and network transmission burden. As the decreasing of ξ, increasing available information will be sampled by the sensors. When $\xi = 0$, the sampler would be the form of normalized periodic sampler with the interval of κh.

In order to avoid various system errors caused by unsynchronized moments of initial conditions, as well as errors caused by the delay from sensors to the network, and from the network to the actuators, we have adopted the following project.

While setting $t_0 = 0$ as the initial condition, and $t_0\ h, t_1\ h, \ldots$ are the release instants, we have following equation

$$\begin{aligned} r_i h &= t_{i+1}h - t_i h \\ &= \min_{j>=1}\{jh \| e_\kappa^T(t)\Omega e_\kappa(t) > \xi x^T((\kappa+j)h)\Omega x((\kappa+j)h)\} \end{aligned} \tag{5}$$

$r_i h$ represents the intervals between the current and the next sampling instant. Therefore we have an inconsistent release period. We express the delay generated during the entire transmission process as τ_{t_κ} at the release time t_κ. Under the assumption $0 \leq \tau_m \leq \tau_{t_\kappa} \leq \tau_M$, where τ_m is the lower delay boundary and τ_M is the upper delay boundary. Then at the instants $t_0\ h + \tau_{t_0}, t_1\ h + \tau_{t_1}, \cdots$, the actuators would receive the information of the states $x(t_0\ h), x(t_1\ h), \cdots$.

Remark 2: In condition (4), notice the discrete-time minimum sampling interval is given, such that Zeno phenomenon could be avoided. But considering about communication delay coming from sensors and controllers, some small and redundant concomitant sampling intervals may still exist. Therefore suitable projection like min-interval threshold could be applied to eliminate the sampling intervals caused by sensors delay.

Then combined with system (2), and controller (3), the network model can be described as

$$\dot{x}_i(t) = Ax_i(t) + A_d x_i(t-d) + f(x_i(t)) + Bu_i(t_\kappa h), \tag{6}$$

where $t_\kappa h + \tau_{t_\kappa} \leq t \leq t_{\kappa+1}h + \tau_{t_{\kappa+1}}$. With the controller (3), the system (6) could be rewritten as

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + +A_d x_i(t-d) + f(x_i(t)) \\ &\quad + \delta BK \sum_{j=1,j\neq i}^{N} a_{ij}(x_j(t_\kappa h) - x_i(t_\kappa h)), \end{aligned} \tag{7}$$

Based on the results achieved before, the filtering error system could be described. Define delay function $\tau(t)$ as

$$\tau(t) = \begin{cases} t - t_\kappa h,\ t \in \left[t_\kappa h + \tau_{t_\kappa}, t_\kappa h + h + \tau_M\right) \\ t - t_\kappa h - h,\ t \in [t_\kappa h + h + \tau_M, t_\kappa h + 2h + \tau_M) \\ \vdots \\ t - t_\kappa h - d_M h, \\ t \in \left[t_\kappa h + \varrho_\kappa h + \tau_M, t_{\kappa+1}h + \tau_{t_{\kappa+1}}\right) \end{cases} \tag{8}$$

where $\varrho_\kappa := t_{\kappa+1} - t_\kappa - 1$ is the largest integer that satisfied $t_\kappa h + \varrho_\kappa h + \tau_M < t_{\kappa+1}h + \tau_{t_{\kappa+1}}$. Since $\tau_{t_\kappa} \le \tau_M$, ϱ_κ always exists.

For the case that $t_\kappa + h + \tau_M \ge t_{\kappa+1}h + \tau_{t_{\kappa+1}}$, the delay function (8) is

$$\tau(t) := t - t_\kappa h, t \in \left[t_\kappa h + \tau_{t_\kappa}, t_{\kappa+1}h + \tau_{t_{\kappa+1}}\right) \tag{9}$$

Then the error system is defined as

$$\epsilon_\kappa(t) := \begin{cases} x(t_\kappa h) - x(t_\kappa h) = 0, \ t \in \left[t_\kappa h + \tau_{t_\kappa}, t_\kappa h + h + \tau_M\right) \\ x(t_\kappa h) - x(t_\kappa h + h), \ t \in [t_\kappa h + h + \tau_M, t_\kappa h + 2h + \tau_M) \\ \vdots \\ x(t_\kappa h) - x(t_\kappa h + \varrho_\kappa h), \\ t \in [t_\kappa h + (\varrho_\kappa - 1)h + \tau_M, t_\kappa h + \varrho_\kappa h + \tau_M) \end{cases} \tag{10}$$

Depending on delay function (8) and error function (10), the system (6) could be rewritten as

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + A_d x_i(t-d) + f(x_i(t)) \\ &\quad + \delta BK \sum_{j=1, j\neq i}^{N} a_{ij}\left(x_j(t+ih) + \epsilon_{jk}(t) - (x_i(t+ih)\right. \\ &\quad \left. + \epsilon_{ik}(t))\right), \\ &= Ax_i(t) + A_d x_i(t-d) + f(x_i(t)) \\ &\quad + \delta BK \sum_{j=1, j\neq i}^{N} a_{ij}\left(x_j(t-\tau(t)) - x_i(t-\tau(t))\right) \\ &\quad + \delta BK \sum_{j=1, j\neq i}^{N} a_{ij}\left(\epsilon_{jk}(t) - \epsilon_{ik}(t)\right) \end{aligned} \tag{11}$$

where $t \in \left[t_\kappa h + \tau_{t_\kappa}, t_{\kappa+1} + \tau_{t_{\kappa+1}}\right)$. For $t \in [\tau_M, 0]$, $x(t)$ is assumed to be continuous and bounded. For every isolated nonlinear system, the aim is to design a distributed controller $u_i(t)$, such that with the information from other neighborhood systems, the consensus of multiple event-based networks could be achieved.

The control inputs are generated by the interconnection from event-generator of other networks. Denote

$$x(t) := \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}, \quad F(x(t)) := \begin{bmatrix} f(x_1(t)) \\ f(x_2(t)) \\ \vdots \\ f(x_N(t)) \end{bmatrix}$$

$$\epsilon(t_\kappa) := \begin{bmatrix} \epsilon_{1k}(t) \\ \epsilon_{2k}(t) \\ \vdots \\ \epsilon_{Nk}(t) \end{bmatrix}$$

Then the multiple systems could be described as

$$\begin{aligned} \dot{x}(t) &= (I_N \otimes A)x(t) + (I_N \otimes A_d)x(t-d) + F(x(t)) \\ &\quad - \delta(L \otimes BK)x(t-\tau(t)) - \delta(L \otimes BK)\epsilon_\kappa(t) \end{aligned} \tag{12}$$

where $t \in \left[t_\kappa h + \tau_{t_\kappa}, t_{\kappa+1}h + \tau_{t_{\kappa+1}}\right)$.

The structure of multiple networks is shown in Fig.1. It can be seen from the structure diagram that the information sent to the controllers is combined by two parts, the local network and the transmission from other systems of the event-based sampler.

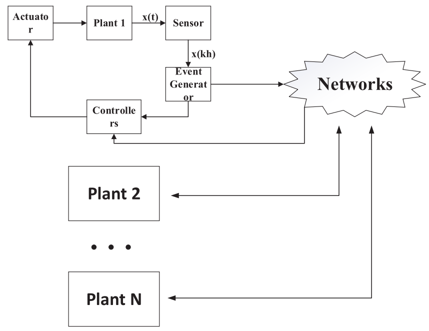

FIGURE 1. **Schematic diagram of multi-agent signal transmission.**

Following definitions and lemmas must be presented before the presentation of our main result.

Definition 1 [51]: For any initial conditions, if the multi-agent systems with the form of (2) satisfy:

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j = 1, \ldots, m \tag{13}$$

Then systems (2) are said to be consensus.

Lemma 1 [52]: For symmetric positive definite matrix $R \in \mathbb{R}^{n\times n}$, any matrices X_1, X_2, any vector $\omega : [a, b] \to \mathbb{R}^n$ such that integration concerned are well defined, then the following inequality holds

$$\begin{aligned} -\int_b^a \dot{\omega}^T(s)R\dot{\omega}(s)ds &\le Sym\{\beta_0^T X_1 \chi_1 + \beta_0^T X_2 \chi_2\} \\ &\quad + (b-a)\beta_0^T \left(\frac{3X_1R^{-1}X_1^T + X_2R^{-1}X_2^T}{3}\right)\beta_0 \end{aligned} \tag{14}$$

where

$$\begin{aligned} \chi_1 &= \int_a^b \dot{\omega}(s)ds \\ \chi_2 &= -\chi_1 + \frac{2}{b-a}\int_a^b \int_a^s \dot{\omega}(s)ds \\ &= \int_a^b \dot{\omega}(s)ds - \frac{2}{b-a}\int_a^b (\omega(s) - \omega(a))\, duds \\ &= \int_a^b \dot{\omega}(s)ds - \frac{2}{b-a}\int_a^b \omega(s)ds + 2\omega(a) \\ &= \begin{bmatrix} 2 & -2 & 1 \end{bmatrix} \varsigma \\ \varsigma &= \begin{bmatrix} \omega(a) & \frac{1}{b-a}\int_a^b \omega(s) & \int_a^b \dot{\omega}(s)ds \end{bmatrix}^T \end{aligned} \tag{15}$$

And β_0 is any vector free to choose. Such that we have

$$-\int_b^a \dot{\omega}^T(s)R\dot{\omega}(s)ds \le \begin{bmatrix} \beta_0 \\ \omega(a) \\ \int_a^b \omega(s)ds \\ \int_a^b \dot{\omega}(s)ds \end{bmatrix}^T M_1 \begin{bmatrix} \beta_0 \\ \omega(a) \\ \int_a^b \omega(s)ds \\ \int_a^b \dot{\omega}(s)ds \end{bmatrix} \tag{16}$$

where (17), as shown at the bottom of the next page.

This Lemma replaces the $\omega(s)$ with $\dot{\omega}(s)$ from [52]. And some general form could be seen in [53] and [54].

Lemma 2 [17]: The following equation holds:

$$\tilde{M}G = \tilde{M}G\mathcal{P}\tilde{M} \tag{18}$$

where

$$\tilde{M} = \begin{bmatrix} 1 & -1 & 0 & \cdots & & 0 \\ 1 & 0 & -1 & 0 & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 1 & 0 & \cdots & 0 & -1 & 0 \\ 1 & 0 & \cdots & & 0 & -1 \end{bmatrix} \in \mathbb{R}^{(N-1)\times N}$$

$$\mathcal{P} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ -1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & & \ddots & -1 & 0 \\ 0 & \cdots & 0 & 0 & -1 \end{bmatrix} \in \mathbb{R}^{N\times(N-1)}$$

and $G \in \mathbb{R}^{N\times N}$ is a matrix that satisfy the same sum for each row.

III. MAIN RESULTS

Consider the multi-agent systems in (6) with the sampled controller in equation (3), then under the self-triggered condition (4), we have following theorem holds.

Theorem 1: Consider the multi-agent systems with directed strongly connected graph. $\tau_M > 0$ is the upper delay bound, $R = R^T > 0 \in$, $Q > 0$, $P > 0$ are positive matrices, and matrices X_1, X_2 are with appropriate dimensions, then under the *Lemma 1*, if following inequality holds

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix} < 0 \tag{19}$$

where

$$\begin{aligned} \Theta_{11} &= \frac{-\tau_M\left(3X_1 R^{-1}X_1^T + X_2 R^{-1}X_2^T\right)}{3} \\ &\quad +\tau_M\mathcal{U}^T(R+P_2)\mathcal{U} + I_1 P_1\mathcal{U} + \mathcal{U}^T P_1^T I_1 + Q_1 \\ &\quad +(X_1+X_2)I_1 + I_1^T(X_1^T + X_2^T) \\ \Theta_{12} &= X_2 - X_1 \\ \Theta_{13} &= \frac{-2}{\tau_M}X_2 + I_1^T(X_1+X_2) \\ \Theta_{22} &= -Q + R + P_2 \\ \Theta_{23} &= X_2 - X_1 \\ \Theta_{23} &= -(t-\tau_M)P_2^T \\ \Theta_{33} &= \frac{-\tau_M\left(3X_1 P_2^{-1}X_1^T + X_2 P_2^{-1}X_2^T\right)}{3} - \frac{2}{\tau_M}(X_2 + X_2^T) \end{aligned}$$

$$\mathcal{U} := \begin{bmatrix} (I_{N-1}\otimes A)^T \\ (I_{N-1}\otimes A_d)^T \\ I \\ -\delta M(L\otimes BK)^T \\ -\delta M(L\otimes BK)^T \end{bmatrix}^T$$

Then the multi-agent systems (12) are consensus.

Proof: Define the error system

$$y(t) = \begin{bmatrix} x_1(t) - x_2(t) \\ x_1(t) - x_3(t) \\ \vdots \\ x_1(t) - x_N(t) \end{bmatrix} \tag{20}$$

Meanwhile,

$$y(t-d) = \begin{bmatrix} x_1(t-d) - x_2(t-d) \\ x_1(t-d) - x_3(t-d) \\ \vdots \\ x_1(t-d) - x_N(t-d) \end{bmatrix} \tag{21}$$

Then with *Lemma 2* we have $y(t) = Mx(t)$, where $M = \tilde{M}\otimes I_n$.

Such that error system is defined

$$\begin{aligned} \dot{y}(t) &= (I_{N-1}\otimes A)y(t) + (I_{N-1}\otimes A_d)y(t-d) + F(y(t)) \\ &\quad -\delta M(L\otimes BK)y(t-\tau(t)) - \delta M(L\otimes BK)\epsilon_\kappa(t) \end{aligned} \tag{22}$$

where $F(y(t)) = MF_1(y(t))$. Following Lyapunov functional is constructed

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) + V_3(t) + V_4(t), \\ &\qquad t \in \left[t_\kappa h + \tau_{t_\kappa}, t_{\kappa+1}h + \tau_{t_{\kappa+1}}\right) \end{aligned} \tag{23}$$

where

$$V_1(t) = y^T(t)P_1y(t) \tag{24}$$

$$V_2(t) = \int_{t-\tau_M}^{t} y^T(s)Qy(s)ds \tag{25}$$

$$V_3(t) = \int_{-\tau_M}^{0}\int_{t+v}^{t} \dot{y}^T(s)R\dot{y}(s)dsdv \tag{26}$$

$$V_4(t) = \int_{t-\tau_M}^{t}\int_{s}^{t} \dot{y}^T(s)P_2\dot{y}(s)dsdv \tag{27}$$

$$M_1 = \begin{bmatrix} -\left(\frac{(b-a)(3X_1 R^{-1}X_1^T + X_2 R^{-1}X_2^T)}{3}\right) & 2X_2 & \frac{-2}{b-a}X_2 & X_1+X_2 \\ 2X_2^T & 0 & 0 & 0 \\ \frac{-2}{b-a}X_2^T & 0 & 0 & 0 \\ X_1^T + X_2^T & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

Calculating the derivative of $V(t)$, following equations hold

$$\dot{V}_1(t) = 2y^T(t)P_1\dot{y}(t) \quad (28)$$
$$\dot{V}_2(t) = y^T(t)Qy(t) - x^T(t-\tau_M)Qy(t-\tau_M) \quad (29)$$
$$\dot{V}_3(t) = \tau_M\dot{y}^T(t)R\dot{y}(t) - \int_{t-\tau_M}^{t}\dot{y}^T(s)R\dot{y}(s)ds \quad (30)$$
$$\dot{V}_4(t) = \tau_M\dot{y}^T(t)P_2\dot{y}(t) - \int_{t-\tau_M}^{t}\dot{y}^T(s)P_2\dot{y}(s)ds \quad (31)$$

Rewritten equation (12) as following

$$\begin{aligned}\dot{y}(t) &= (I_{N-1}\otimes A)y(t) + (I_{N-1}\otimes A_d)y(t-d) + F(y(t))\\ &\quad -\delta(L\otimes BK)x(t-\tau(t)) - \delta(L\otimes BK)\epsilon_\kappa(t)\\ &= \mathcal{U}\mu_1 \end{aligned} \quad (32)$$

where

$$\mathcal{U} := \begin{bmatrix}(I_{N-1}\otimes A)^T\\ (I_{N-1}\otimes A_d)^T\\ I\\ -\delta M(L\otimes BK)^T\\ -\delta M(L\otimes BK)^T\end{bmatrix}^T$$
$$\mu_1 := \begin{bmatrix}y^T(t) & y^T(t-d) & F^T(y(t)) & y^T(t-\tau(t)) & \epsilon_\kappa^T(t)\end{bmatrix}^T$$

Such that we have

$$y(t) = I_1\mu_1 \quad (33)$$

where

$$\begin{aligned}I_1 &= \begin{bmatrix}I_{(N-1)\times n} & 0 & 0 & 0 & 0\end{bmatrix} \in \mathbb{R}^{5(N-1)\times n}\\ I_2 &= \begin{bmatrix}0 & I_{(N-1)\times n} & 0 & 0 & 0\end{bmatrix} \in \mathbb{R}^{5(N-1)\times n}\\ I_3 &= \begin{bmatrix}0 & 0 & I_{(N-1)\times n} & 0 & 0\end{bmatrix} \in \mathbb{R}^{5(N-1)\times n}\\ I_4 &= \begin{bmatrix}0 & 0 & 0 & I_{(N-1)\times n} & 0\end{bmatrix} \in \mathbb{R}^{5(N-1)\times n}\\ I_5 &= \begin{bmatrix}0 & 0 & 0 & 0 & I_{(N-1)\times n}\end{bmatrix} \in \mathbb{R}^{5(N-1)\times n}\end{aligned} \quad (34)$$

Then we have

$$\begin{aligned}&\dot{V}_1(t) + \tau_M\dot{y}^T(t)R\dot{y}(t) + \tau_M\dot{y}^T(t)P_2\dot{y}(t)\\ &= \tau_M\mu_1^TX_1^T(R+P_2)X_1\mu_1\\ &\quad + y^T(t)P_1\dot{y}(t) + \dot{x}^T(t)P_1\,y(t)\\ &= \mu_1^T\left(\tau_M\mathcal{U}^T(R+P_2)\mathcal{U} + I_1\,P_1\mathcal{U} + \mathcal{U}^TP_1^TI_1\right)\mu_1\end{aligned} \quad (35)$$

From Lemma 1 we could have

$$-\int_{t-\tau_M}^{t}\dot{y}^T(s)R\dot{y}(s)ds \le \mu_2\Phi_1\mu_2 \quad (36)$$

where (37) and (38), as shown at the bottom of the next page.

We have $\int_{t-\tau_M}^{t}\dot{y}(s)ds = y(t) - y(t-\tau_M)$ here, then (36) could be rewritten as

$$-\int_{t-\tau_M}^{t}\dot{y}^T(s)R\dot{y}(s)ds \le \mu_2^T\Phi_2\mu_2 \quad (39)$$

where (40) and (41), as shown at the bottom of the next page.

At the same time

$$-\int_{t-\tau_M}^{t}\dot{y}^T(s)P_2\dot{y}(s)ds \le \mu_3^T\Phi_3\mu_3 \quad (42)$$

where μ_3 and Φ_3 are similar to μ_2 and Φ_2 are (43) and (44), as shown at the bottom of the next page.

By (35), (29), (30), (31) and (36), following equation would be held

$$\begin{aligned}&\dot{V}(t)\\ &= \dot{V}_1(t) + \tau_M\dot{x}^T(t)R\dot{y}(t) + \tau_M\dot{x}^T(t)P_2\dot{y}(t) + \dot{V}_2(t)\\ &\quad - \int_{t-\tau_M}^{t}\dot{y}^T(s)R\dot{y}(s)ds - \int_{t-\tau_M}^{t}\dot{y}^T(s)P_2\dot{y}(s)ds\\ &\le \mu_1^T\left(\tau_MX_1^T(R+P_2)X_1 + I_1\,PX_1 + X_1^TP^TI_1\right)\mu_1\\ &\quad + y^T(t)Qy(t) - x^T(t-\tau_M)Qy(t-\tau_M)\\ &\quad - \int_{t-\tau_M}^{t}\dot{y}^T(s)R\dot{y}(s)ds - \int_{t-\tau_M}^{t}\dot{y}^T(s)P_2\dot{y}(s)ds\\ &\le \mu_1^T\left(\tau_M\mathcal{U}^T(R+P_2)\mathcal{U} + I_1\,P_1\mathcal{U} + \mathcal{U}^TP_1^TI_1 + Q_1\right)\mu_1\\ &\quad - \mu_2^TI_2^TQI_2\mu_2 + \mu_2^T\Phi_2\mu_2 + \mu_3^T\Phi_3\mu_3\end{aligned} \quad (45)$$

where $Q_1 = I_1^TQI_1$.

Recall that in *Lemma 1*, β_1 and β_2 is free to choose. Therefore for $\mu_2^T\Phi_2\mu_2$, we choose

$$\beta_1 = \mu_1 \quad (46)$$

Therefore

$$\mu = \begin{bmatrix}\mu_1\\ y(t-\tau_M)\\ \int_{t-\tau_M}^{t}y(s)ds\end{bmatrix}^T \quad (47)$$

Then we have

$$\mu_2^T\Phi_2\mu_2 \le \mu^T(\Pi_1 + \Delta_1)\mu \quad (48)$$

where

$$\Pi_1 = \begin{bmatrix}\frac{-\tau_M\left(3X_1\,R^{-1}X_1^T + X_2\,R^{-1}X_2^T\right)}{3} & X_2 - X_1 & \frac{-2}{\tau_M}X_2\\ * & R & 0\\ * & * & 0\end{bmatrix}$$
$$\Delta_1 = \begin{bmatrix}(X_1+X_2)I_1 + I_1^T(X_1^T+X_2^T) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Note that item $y(t)$ in μ_2 could be rewritten into $I_1\mu_1$ here.

For $\mu_3^T\Phi_3\mu_3$, let

$$\beta_2 = \int_{t-\tau_M}^{t}y(s)ds$$

from (47), we have

$$\mu_3^T\Phi_3\mu_3 \le \mu^T(\Pi_2 + \Delta_2)\mu \quad (49)$$

where

$$\Pi_2 = \begin{bmatrix}0 & 0 & I_1^T(X_1+X_2)\\ * & P_2 & X_2 - X_1\\ * & * & \frac{-\tau_M\left(3X_1\,P_2^{-1}X_1^T + X_2\,P_2^{-1}X_2^T\right)}{3}\end{bmatrix}$$
$$\Delta_2 = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\frac{2}{\tau_M}(X_2 + X_2^T)\end{bmatrix} \quad (50)$$

The same to the situation in μ_2, item $y(t)$ in μ_3 could be rewritten into $I_1\mu_1$. And β_2 is also the same as an item $\int_{t-\tau_M}^{t} y(s)ds$ in μ_3.

and define

$$\Lambda_1 = \begin{bmatrix} \tau_M \mathcal{U}^T(R+P_2)\mathcal{U} + I_1 P_1 \mathcal{U} + \mathcal{U}^T P_1^T I_1 + Q_1 & 0 & 0 \\ * & -Q & 0 \\ * & * & 0 \end{bmatrix} \tag{51}$$

Then combined all inequalities above, we have

$$\begin{aligned} \dot{V}(t) &\le \mu^T(\Lambda_1 + \Pi_1 + \Pi_2 + \Delta_1 + \Delta_2)\mu \\ &\le \mu^T \Theta \mu \end{aligned} \tag{52}$$

From inequality (19) we could conclude that

$$\dot{V}(t) < 0 \tag{53}$$

Then the multi-agent system (12) is said to achieve consensus. ■

Compared with the scheme of [42], the expression of the control method we proposed in this paper is more concise, and the parameters that are reused also make the number of decision variables significantly further reduced. The proof is also more flexible than some existed works. Numerical examples and simulation would show the effeteness.

Remark 3: With the improved free-weighting matrix inequalities, there are more expressions of Theorem 1, such that the self-triggered sampling-data control is more

$$\Phi_1 = \begin{bmatrix} -(\tau_M)\left(\frac{3X_1R^{-1}X_1^T + X_2R^{-1}X_2^T}{3}\right) & 2X_2 & \frac{-2}{\tau_M}X_2 & X_1+X_2 \\ 2X_2^T & 0 & 0 & 0 \\ \frac{-2}{\tau_M}X_2^T & 0 & 0 & 0 \\ X_1^T+X_2^T & 0 & 0 & 0 \end{bmatrix} \tag{37}$$

$$\mu_2' = \begin{bmatrix} \beta_0 \\ y(t-\tau_M) \\ \int_{t-\tau_M}^{t} y(s)ds \\ \int_{t-\tau_M}^{t} \dot{y}(s)ds \end{bmatrix} \tag{38}$$

$$\mu_2 = \begin{bmatrix} \beta_1 \\ y(t-\tau_M) \\ \int_{t-\tau_M}^{t} y(s)ds \\ y(t) \end{bmatrix} \tag{40}$$

$$\Phi_2 = \begin{bmatrix} \frac{-\tau_M\left(3X_1R^{-1}X_1^T + X_2R^{-1}X_2^T\right)}{3} & X_2-X_1 & \frac{-2}{\tau_M}X_2 & X_1+X_2 \\ X_2^T-X_1^T & R & 0 & 0 \\ \frac{-2}{\tau_M}X_2^T & 0 & 0 & 0 \\ X_1^T+X_2^T & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

$$\mu_3 = \begin{bmatrix} \beta_2 \\ y(t-\tau_M) \\ \int_{t-\tau_M}^{t} y(s)ds \\ y(t) \end{bmatrix} \tag{43}$$

$$\Phi_3 = \begin{bmatrix} \frac{-\tau_M\left(3X_1P_2^{-1}X_1^T + X_2P_2^{-1}X_2^T\right)}{3} & X_2-X_1 & \frac{-2}{\tau_M}X_2 & X_1+X_2 \\ X_2^T-X_1^T & P_2 & 0 & 0 \\ \frac{-2}{\tau_M}X_2^T & 0 & 0 & 0 \\ X_1^T+X_2^T & 0 & 0 & 0 \end{bmatrix} \tag{44}$$

diversely. In parameter adjustment, it would also have more advantages. At the same time, free matrices L, M and others could be further developed.

IV. NUMERICAL EXAMPLE

In this section, we present the simulation results obtained from the examples of unmanned aerial vehicles(UAVs) in [19], and effectiveness could be shown both in simulations and applications.

The topology of the networks is shown in Fig.2. It's the simplest full connected graph.

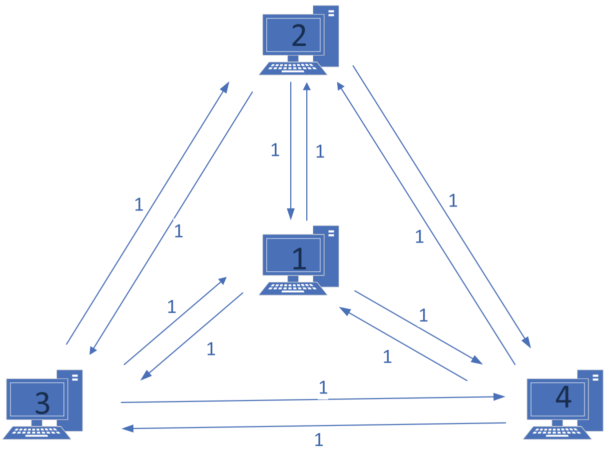

FIGURE 2. Full connected topology of networks.

For convenience, considering the state of the single network to be a two-dimensional vector, with the same natural property of the networks, the parameters of systems are given as follows:

$$A = \begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (54)$$

$$A_d = \begin{bmatrix} -10 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -10 & 0 \end{bmatrix} \quad (55)$$

The nonlinear part is given as frequency domain, $f(x_i(t)) = \left[0.5\ sin(x_{i1}(t))\ 0.5\ sin(x_{i2}(t))\right]^T$. And the initial states are

$x_1(0) = \begin{bmatrix} 0.7 & 0.5 & 0.05 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 3 & 0.2 & 0.15 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} 0 & 0.1 & -1.05 \end{bmatrix}^T$, $x_4(0) = \begin{bmatrix} -0.5 & -0.5 & 2 \end{bmatrix}^T$

The parameters in event-based sampling condition (4) are considered as follows:

$$\Omega = \begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.5 \end{bmatrix} \quad (56)$$

we set $h = 0.12$, $\xi = 0.005$, $\tau_M = 0.1$, and $\delta = 1.53$, with the initial conditions above, we have feedback gain matrix as

$$K = \begin{bmatrix} 0.5974 & -2.772 & 1.221 \\ -0.3116 & 0.5802 & 0 \\ 1.221 & 0 & 0.5 \end{bmatrix} \quad (57)$$

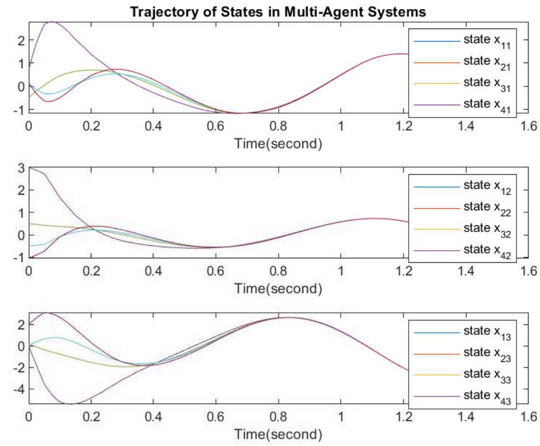

FIGURE 3. Trajectories of states in multi-agent.

The results are shown in in Fig 3, all trajectories of states achieve consensus.

To better express the effectiveness, we also provide the consensus errors in Fig.4, where it is defined as $r(t) = \sum_{j=2}^{N} \|x_1(t) - x_j(t)\|$. The results showed that after the influence of initial value, the accumulated error of the system gradually converges to 0.

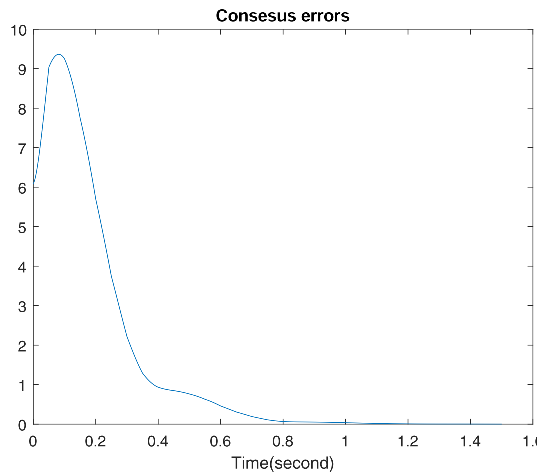

FIGURE 4. Cumulative synchronization error $r(t)$ of multi-agent systems.

V. CONCLUSION

In the paper, based on self-triggered sampled-control, a novel consensus criterion of multi-agent systems with multiple mixing delay is proposed. With the minimum sampling interval, the Zeno phenomenon is eliminated. In the system, delay from sensors to the network, and network to the actuators are considered, including system time-delay. A more concise global synchronization criterion for is achieved with considerable sampling interval. A numerical simulation example has shown that the event-triggered sampled-control can have an excellent synchronization performance.

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