IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received May 16, 2020, accepted June 9, 2020, date of publication June 15, 2020, date of current version June 26, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3002714

Design of Fractional Swarm Intelligent Computing With Entropy Evolution for Optimal Power Flow Problems

YASIR MUHAMMAD[1], RAHIMDAD KHAN[1], MUHAMMAD ASIF ZAHOOR RAJA[2,3], FARMAN ULLAH[3], NAVEED ISHTIAQ CHAUDHARY[4], (Member, IEEE), AND YIGANG HE[5]

[1]Department of Electrical and Computer Engineering, COMSATS University Islamabad-Wah Campus, Wah Cantt 47040, Pakistan
[2]Future Technology Research Center, National Yunlin University of Science and Technology, Douliou 64002, Taiwan
[3]Department of Electrical and Computer Engineering, COMSATS University Islamabad-Attock Campus, Attock 43600, Pakistan
[4]Department of Electrical Engineering, International Islamic University, Islamabad 44000, Pakistan
[5]School of Electrical Engineering and Automation, Wuhan University, Wuhan 430072, China

Corresponding author: Yigang He (18655136887@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 51977153, Grant 51977161, and Grant 51577046, in part by the State Key Program of National Natural Science Foundation of China under Grant 51637004, in part by the National Key Research and Development Plan Important Scientific Instruments and Equipment Development under Grant 2016*YFF*010220, and in part by the Equipment Research Project in Advance under Grant 41402040301.

ABSTRACT Optimal reactive power dispatch (ORPD) problems in power system have been solved by using several variants of traditional nature inspired particle swam optimization (PSO) with aim to achieve a promising solution for a given objective such as line loss, voltage deviation and overall cost minimization. Several schemes have been designed to improve the performance of the optimization technique in tunning the operational variables and analyzed by evaluating the final results. In this article, a different method is designed to solve ORPD problems, by introducing Shannon entropy based diversity in the fractional order PSO dynamics, i.e., FOPSO-EE. The results show that synergy of both, the Shannon entropy and the fractional calculus can be used as the useful tools for enhancing the optimization strength of algorithm while solving the ORPD problems in standard IEEE 30 and 57 bus power systems. The performance of the design FOPSO-EE is further validated through results of statistical interpretations in terms of histogram analysis, box plot illustration, quantile-quantile probability plot and empirical probability distribution function.

INDEX TERMS Computational intelligence, optimal power flow, fractional calculus, Shannon entropy, particle swarm optimization.

I. INTRODUCTION

A. MOTIVATION AND INCITEMENT

Optimal reactive power dispatch (ORPD) aims to improve the performance of a power system by reducing the voltage deviation, transmission line losses, operational cost, improving system security, stability index, and line capacity. Most of these objectives are achieve by tunning the operational variables of the network such as the transformers tap positions, generator output voltages, capacitor banks and solid state flexible AC transmission systems (FACTS) devices [1]. However, the optimal tunning of these variables is a complex problem due to the multi-modal, non linear and discrete nature of the power systems.

B. LITERATURE REVIEW

In the literature, several arithmetic, stochastic, evolutionary, social-based and meta-heuristic optimization techniques are developed since last few decades including the differential evolution [2], genetic algorithm [3]–[8], particle swarm optimization (PSO) etc. A summary of the proposed optimization techniques for ORPD is documented in Table 1. Since the development of the canonical PSO, a considerable number of its variants based on different numerical tools have been proposed in order to enhance the algorithm performance, and significantly applied in a plethora of applications [9]–[14].

The associate editor coordinating the review of this manuscript and approving it for publication was Xiangtao Li.

TABLE 1. Summary of proposed algorithms for ORPD.

Ref.	Algorithm	Objectives	Year
[15]	Evolutionary programming	f_1	1995
[3]	Adaptive genetic algorithm	f_1	1998
[16]	PSO	f_1	2000
[10]	Multi agent PSO	f_1	2005
[4]	Improved GA	f_1, f_1	2005
[17]	GA-interior point method	f_1	2006
[18]	Modified PSO	Stability	2007
[19]	Self adaptive real coded GA	f_1	2009
[20]	Turbulent crazy PSO	f_1, f_2	2009
[9]	Comprehensive learning PSO	f_1	2010
[21]	Ant colony optimization	f_1	2011
[22]	MNSGA-II	f_1, stability	2011
[23]	Biogeography-based optimization	f_1, f_2	2011
[24]	Harmony search algorithm	f_1, f_2, f_3	2011
[25]	Adaptive approaches	f_1, f_2	2012
[26]	HFMOEA	f_1, stability	2013
[27]	Opposition based GSA	f_1, f_2, stability	2013
[28]	MICA-IWO	f_1	2014
[29]	Differential evolution (DE)	f_1	2015
[30]	Enhanced firefly algorithm	f_1, f_2	2015
[31]	Gray wolf optimizer (GWO)	f_1, f_2	2015
[32]	Teaching learning optimization	f_1	2015
[33]	Hybrid firefly algorithm	f_1, f_2	2015
[34]	Quasi-oppositional DE	f_1, f_2, stability	2016
[35]	Two-point estimate method	f_1, f_2	2016
[36]	Chaotic krill herd	f_1, f_2	2016
[37]	Exchange market algorithm	f_1, f_2, stability	2016
[38]	Backtracking search algorithm	f_1, f_2	2016
[39]	Oppositional krill herd	f_1, f_2	2016
[40]	Moth-flame optimization	f_1	2017
[41]	GBWC	f_1, f_2	2017
[42]	Whale optimization	f_1	2018
[43]	Chemical reaction optimization	f_1, f_2, stability	2018
[44]	ABC-FF	f_1, f_2, stability	2018
[45]	Sine cosine algorithm	f_1, f_2	2019
[46]	ALC-PSO algorithm	f_1, f_2	2019
[47]	Moth Swarm Algorithm	f_1, f_2	2019
[48]	Lightning Attachment Procedure	f_1	2019
[49]	Enhanced GWO	f_1, f_2	2019
[50]	Arificial bee colony	f_1, stability	2020
[51]	Chaotic Bat Algorithm	f_1, f_2, stability	2020

$*f_1$ and f_2 are loss and deviation minimization, respectively.

The PSO consists on a machine learning procedure which is basically inspired by the behavior of social species such as fish schooling or birds flocking in search of food. Each fish or bird is represented by a particle that move on the search space for finding an optimal solution. The particle is characterized by two vectors, namely by its current velocity and position. A set of particles is known as swarm which evolves during several iterations constituting a powerful computational technique. Since 1995, many mathematical tools are presented to complement and/or refine the conventional PSO technique, namely by adopting integration with other evolutionary strategies and by analyzing the tuning parameters. In this regard, Machado and team [52], [53] presented a new method for improving the convergence rate of a canonical PSO based on fractional calculus (FC) theory and developed fractional order optimization technique, known as FO-PSO.

Afterwards, FO-PSO has been successfully designed for a number of fields such as computational biology, color image quantization, pattern recognition simulation and animation of natural flocks or swarms, computer graphics, and social modeling [54]–[68]. However, the strength of FC based optimization algorithms have not yet been explored in solving problems of energy and power sector specifically in the domain of ORPD. Besides FC, the Shannon entropy, a mathematical tool, has been tested in myriad of fields, such as biology, sociology, communications and economics among others in the recent years, however, its use has been overlooked in evolutionary computation.

C. CONTRIBUTION AND PAPER ORGANIZATION

Considering the discussed concepts, a new algorithm i.e. fractional order particle swarm optimization with entropy evolution, known as FOPSO-EE, is designed and tested on primary problems of ORPD such as the minimization of the transmission line losses, overall cost of operation and voltage deviation in the IEEE standard power system and results are compared with those of other counterpart algorithms.

Therefore, this work uses the fractional calculus as a tool to improve the convergence rate of the canonical PSO while entropy metric to avoid the sub optimal solution. The salient contributions of this research work are

- New diversity indices stimulated by biologic systems and physics based on entropy evolution are exploited to enhance the optimization strength of FOPSO.
- The better performance of designed FOPSO-EE is verified over its integer counterpart while solving the ORPD problems in standard power system considering the FACTS devices, namely, the TCSC and SVC.
- The results of statistical interpretations including the histogram analysis, box plot illustration, quantile-quantile probability and empirical probability distribution function are used to further endorse the accuracy, reliability and stability of proposed FOPSO-EE.
- Simplicity in the concept, smooth implementation and wider applicability in energy and power sector are other valuable perks of designed FOPSO-EE.

Bearing these ideas in mind, the organization of this paper is set as follows. Section 2 formulates the objective functions to be optimized. Section 3 develops the methodology followed in the work including a brief description of the fractional calculus, fractional PSO and entropy. Section 4 presents the experimental results for the

FOPSO-EE. Section 5 presents the statistical analysis. Finally, section 6 summarizes the main conclusions and discusses future recommendations.

II. MATHEMATICAL MODELS OF ORPD PROBLEMS

This section presents the objective functions that are considered during the tests of FOPSO-EE. The optimization functions consists in minimizing transmission line losses, voltage deviation index and overall cost of operation in standard power system, namely, IEEE-30 bus (Fig. 1), which consists of 4 tap changing transformers, 6 generators, 3 capacitor banks and 41 transmission lines [69].

A. TRANSMISSION LINE LOSS MINIMIZATION, P_{loss}

This function is mathematically expressed as:

$$Minimize\, f_1(z) : P_{loss}(z) + \lambda(z) \tag{1}$$

here,

$$P_{loss} = \sum_{r=1}^{nl} G_r \left[U_m^2 + U_n^2 - 2 \times U_m \times U_n \cos(\delta_m - \delta_n)\right] \quad z \in [U, T, Q] \tag{2}$$

hereafter, G_r is the conductance of r^{th} line between m and n bus, U_m and U_n are the voltage magnitudes at m^{th} and n^{th} bus respectively, δ_m and δ_n are the voltage angles at m^{th} and n^{th} bus respectively, nl represents no. of transmission lines. λ is the penalty factor for control variables such as reactive power compensators Q (capacitor bank), transformer tap positions T, and bus voltages U in the IEEE standard power system.

The general form of objective function based on penalty factor is expressed as:

$$F = P_{loss} + \sum r_{Qm}(Q_m - Q_m^{\lim})^2 + \sum r_{Gm}(U_m - U_m^{\lim})^2 + \sum r_{Tm}(T_m - T_m^{\lim}) \tag{3}$$

$$Q_{Gm}^{\lim} = \begin{cases} Q_m^{\max} & if \;\; Q_m > Q_m^{\max} \\ Q_m^{\min} & if \;\; Q_m > Q_m^{\min} \end{cases}, \quad m = 1, 2, \ldots, K_G \tag{4}$$

$$U_m^{\lim} = \begin{cases} U_m^{\max} & if \;\; U_m > U_m^{\max} \\ U_m^{\min} & if \;\; U_m > U_m^{\min} \end{cases}, \quad m = 1, 2, \ldots, K \tag{5}$$

$$T_m^{\lim} = \begin{cases} T_m^{\max}; & if \;\; T_m > T_m^{\max} \\ T_m^{\min}; & if \;\; T_m > T_m^{\min} \end{cases}, \quad m = 1, 2, \ldots, K_T \tag{6}$$

Here, Q is reactive power rating of capacitor bank, V is the voltage at generator bus, and T represents tap setting of transformer, K denotes total no. of buses, K_T is the number of transformer with tap changer and K_G is the number of generators in the power system. The minimum and maximum values of $Q_m^{\lim}$, $U_m^{\lim}$, and $T_m^{\lim}$ in above expressions represents the control variable's permissible limits.

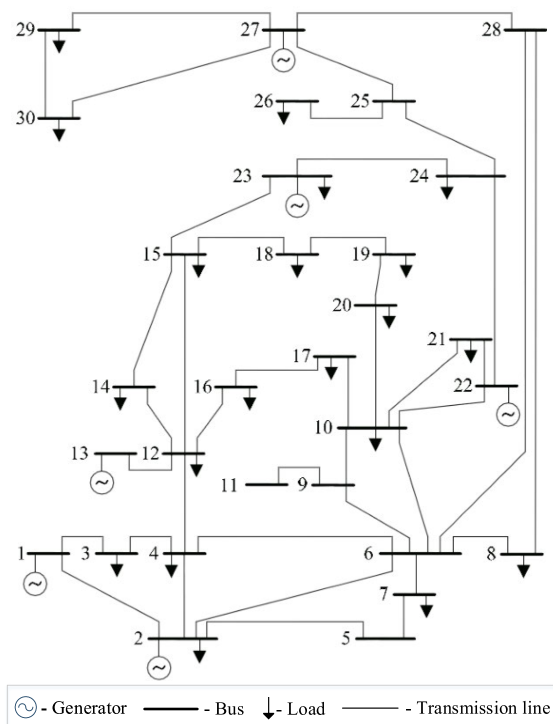


FIGURE 1. IEEE-30 bus system.

B. VOLTAGE DEVIATION, V_D

$$Minimize\, f_2 = V_D = \sum_{m=1}^{nload} |V_m - 1.0| \tag{7}$$

here, *nload* represents the total number of load-buses [70].

C. OVERALL COST MINIMIZATION, C_{total}

$$Minimize\, f_3 = C_{total} = C_{energy} + C_{cpex}, \tag{8}$$

where

$$C_{energy} = 0.06 \cdot 365 \cdot 24 \cdot P_{loss}$$

Here, days/year are 365, hours/day are 24, cost associated with power loss is 0.06 \$/kWh. The cost C_{cpex}, represents the capital cost of the reactive power compensators i.e., FACTS and taken from the Siemens AG database [7] as

$$C_{cpex} = \gamma + \beta(MVA_{rating}) + \alpha(MVA_{rating})^2 \tag{9}$$

here, γ, β and α represents the cost coefficients while MVA_{rating} is the operating range in mega volt ampere (MVA) of FACTS devices. The cost functions for SVC, TCSC and UPFC can be expressed respectively as:

$$C_{SVC} = 127.38 - 0.3051(MVA_{rating}) + 0.0003(MVA_{rating})^2 \tag{10}$$

$$C_{TCSC} = 153.75 - 0.7139(MVA_{rating}) + 0.0015(MVA_{rating})^2 \tag{11}$$

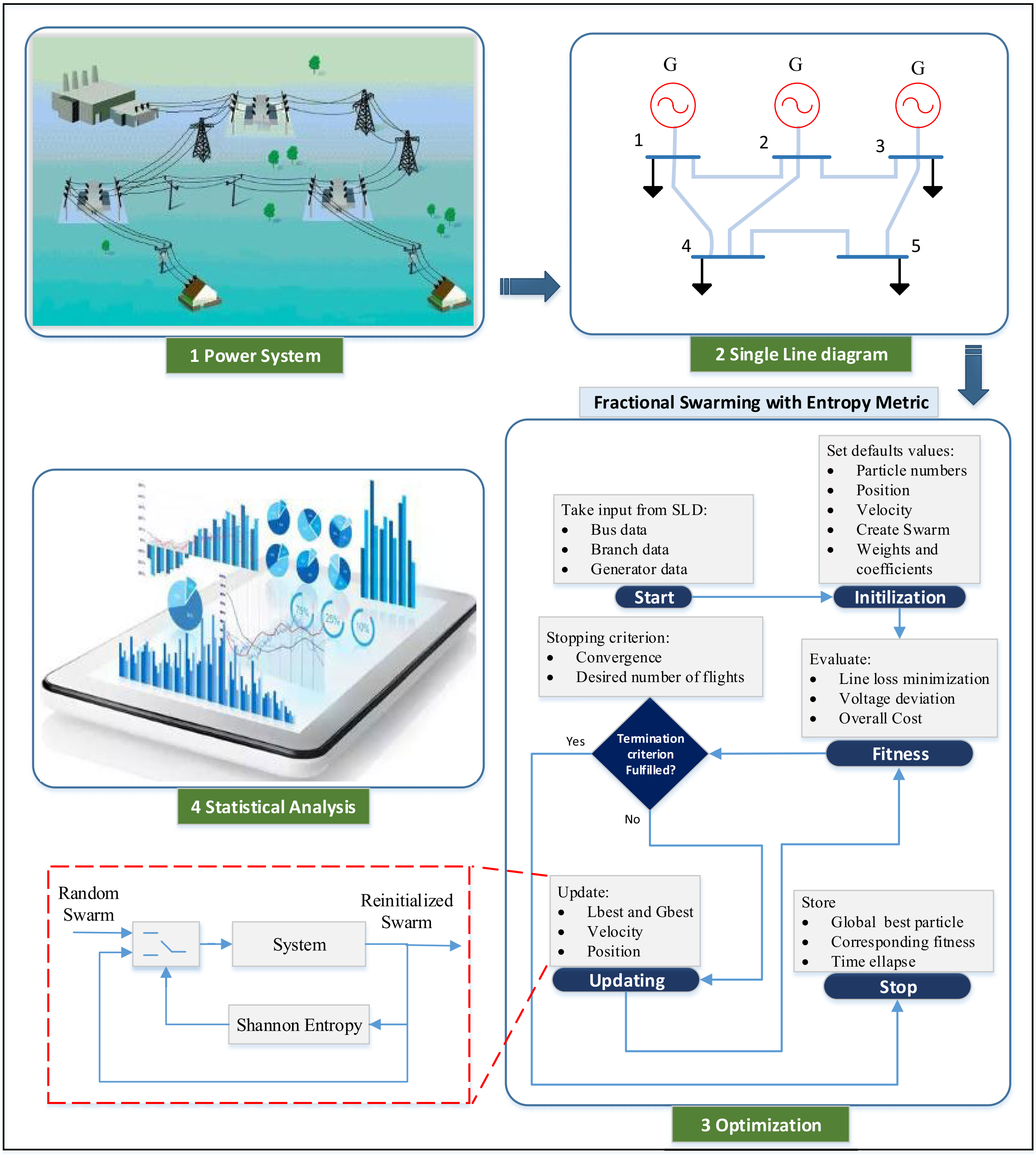


FIGURE 2. Graphical abstract.

$$C_{UPFC} = 188.22 - 0.2691(MVA_{rating}) + 0.0003(MVA_{rating})^2 \tag{12}$$

1) INEQUALITY CONSTRAINTS

The expressions for inequality constraints are given below:

- *Transformer boundaries*

$$T_{mn}^{\max} \leq T_i \leq T_{mn}^{\min}, \quad m = 1, 2, \ldots, K_T \tag{13}$$

- *Generator boundaries*

$$U_{Gm}^{\max} \leq U_{Gm} \leq U_{Gm}^{\min}, \quad m = 1, 2, \ldots, K_G \tag{14}$$

$$Q_{Gm}^{\max} \leq Q_{Gm} \leq Q_{Gm}^{\min}, \quad m = 1, 2, \ldots, K_G \tag{15}$$

- *Shunt-VAR boundaries*

$$Q_{cm}^{\max} \leq Q_{cm} \leq Q_{cm}^{\min}, \quad m = 1, 2, \ldots, N_c \tag{16}$$

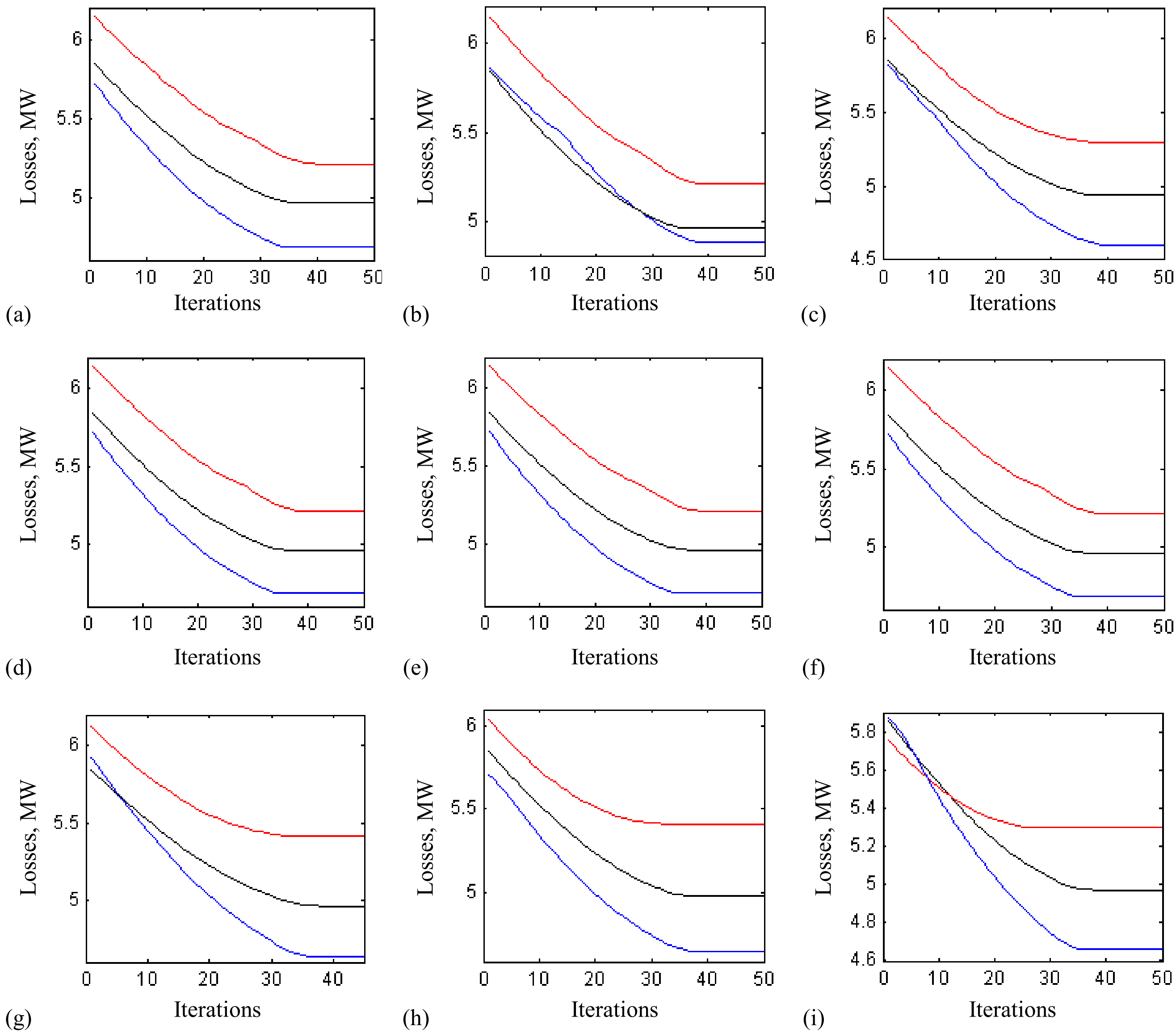


FIGURE 3. Best, average and worst learning curves for (a) $\alpha = 0.1$ (b) $\alpha = 0.2$ (c) $\alpha = 0.3$ (d) $\alpha = 0.4$ (e) $\alpha = 0.5$ (f) $\alpha = 0.6$ (g) $\alpha = 0.7$ (h) $\alpha = 0.8$ (i) $\alpha = 0.9$ during power loss minimization.

- *FACTS devices constraints*

$$-X^{max} \leq X_{TCSC} \leq X^{min} \tag{17}$$

$$-Q^{max} \leq Q_{SVC} \leq Q^{min} \tag{18}$$

Here, *max* and *min* defines the upper and lower bounds, respectively, N_C is the no. of buses where reactive power compensators are coupled, X is the per unit (p.u) reactance offered by the TCSC, Q is the reactive power provided by the SVC.

D. EQUALITY CONSTRAINTS

The equality constraints includes the power balance equations [70]. Mathematically, for any bus m, the real and reactive power balance equations are stated as:

$$-U_m \sum_{n=1}^{K} U_n \left[B_{mn} \sin(\delta_m - \delta_n) + G_{mn} \cos(\delta_m - \delta_n)\right] - P_{Dm} + P_{Gm} = 0 \tag{19}$$

$$-U_m \sum_{n=1}^{K} U_m \left[B_{mn} \cos(\delta_m - \delta_n) + G_{mn} \sin(\delta_m - \delta_n)\right] - Q_{Dm} + Q_{Gm} = 0 \tag{20}$$

Here,

P_{Dm} and P_{Gm}, are the demanded and injected real powers at m^{th} bus,

Q_{Dm} and Q_{Gm}, are the demanded and injected reactive powers at m^{th} bus,

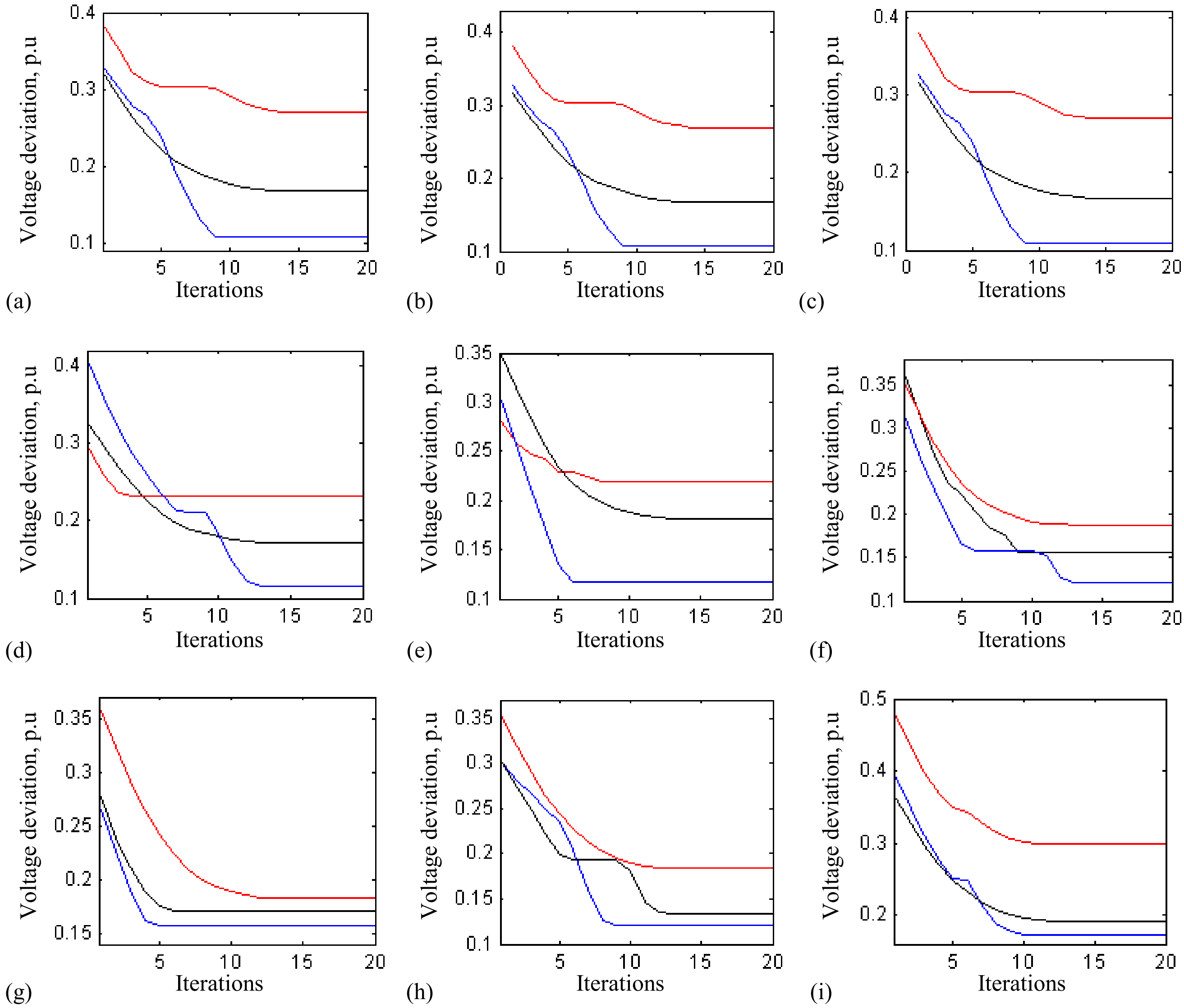


FIGURE 4. Best, average and worst learning curves for (a) $\alpha = 0.1$ (b) $\alpha = 0.2$ (c) $\alpha = 0.3$ (d) $\alpha = 0.4$ (e) $\alpha = 0.5$ (f) $\alpha = 0.6$ (g) $\alpha = 0.7$ (h) $\alpha = 0.8$ (i) $\alpha = 0.9$ during voltage deviation.

B_{mn} and G_{mn}, are line susceptance and conductance between m^{th} and n^{th} bus, respectively.

The incorporation of the TCSC between two buses, namely, m and n, alters the power balance equations as [13];

$$P_{mn} = +U_m^2 G_{mn} - U_m U_n G_{mn} \cos(\delta_m - \delta_n) - U_m U_n B_{mn} \sin(\delta_m - \delta_n) \tag{21}$$

$$Q_{mn} = -U_m^2 B_{mn} - U_m U_n G_{mn} \sin(\delta_m - \delta_n) + U_m U_n B_{mn} \sin(\delta_m - \delta_n) \tag{22}$$

Here, the conductance and susceptance of the transmission line are given by $G_{mn} = \frac{R}{R^2+(X-X_{TCSC})^2}$ and $B_{mn} = \frac{-X-X_{TCSC}}{R^2+(X-X_{TCSC})^2}$, respectively.

III. METHODOLOGY

This section presents the fundamental concepts about the computational tools, integrated to develop FOPSO-EE, such as the fractional calculus, particle swarm optimization and entropy.

A. FRACTIONAL CALCULUS

Fractional calculus (FC) is a generalization of the ordinary integer integration and differentiation to a non-integer order. FC was an important topic in the last few centuries and many mathematicians, such as Weyl, Riemann and Liouville contributed to its development. Fractional calculus has attained the focus of research community, being applied in various scientific fields such as irreversibility, modeling,

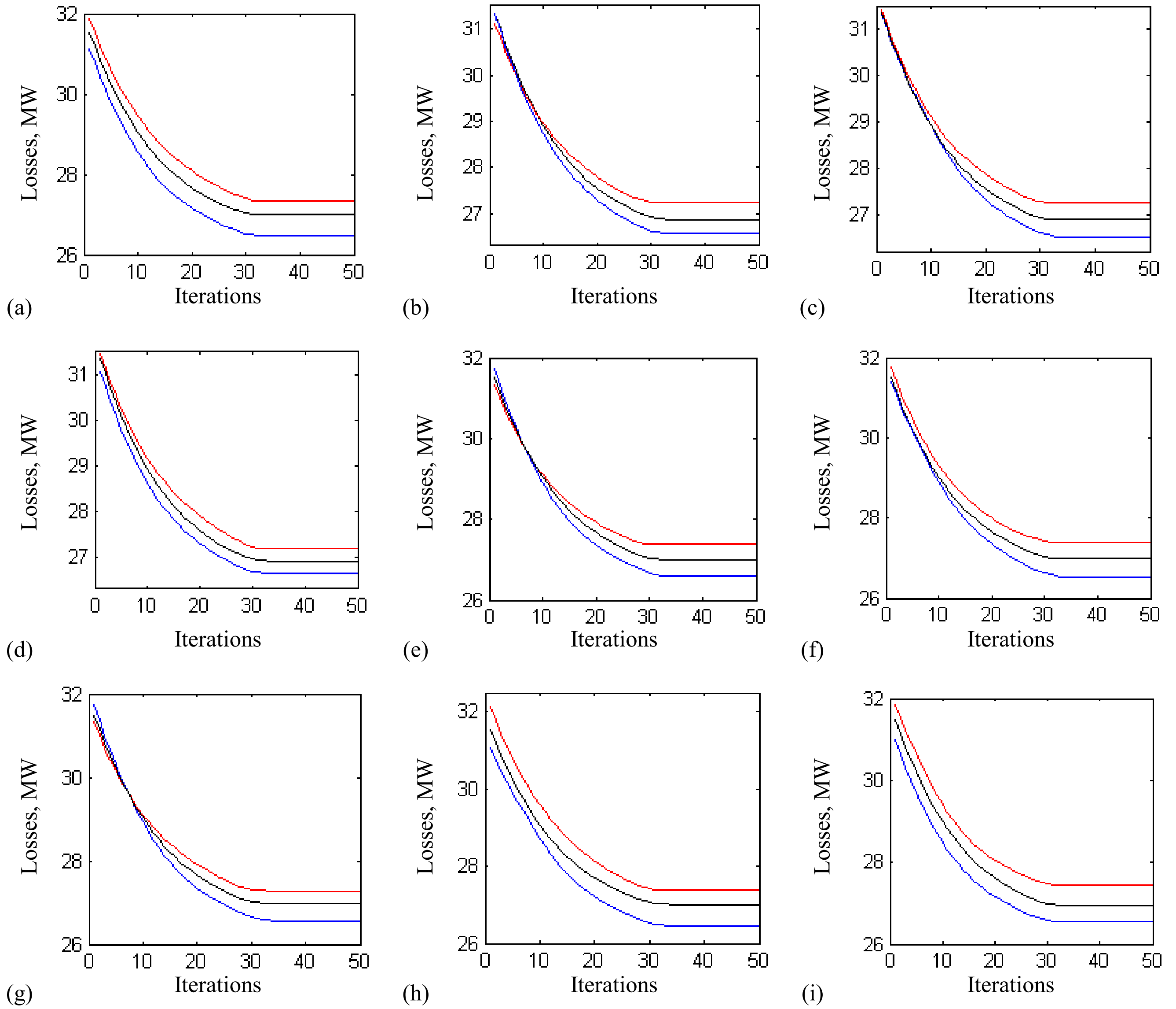


FIGURE 5. Best, average and worst learning curves for (a) $\alpha = 0.1$ (b) $\alpha = 0.2$ (c) $\alpha = 0.3$ (d) $\alpha = 0.4$ (e) $\alpha = 0.5$ (f) $\alpha = 0.6$ (g) $\alpha = 0.7$ (h) $\alpha = 0.8$ (i) $\alpha = 0.9$ during power loss minimization in IEEE-57 bus system.

wave propagation, viscoelasticity, electronics, fractals, chaos, signal processing, control, biology, percolation, diffusion, and physics. There are number of alternate interpretations of fractional derivatives, one of the most important being, the Grunwald-Letnikov(GL), is based on the concept of fractional differential with order $\alpha \epsilon C$ of a general signal $f(z)$, given by the equation [52], [71]:

$$D^{\alpha}[f(z)] = \lim_{h \to 0}\left[\frac{1}{h^{\alpha}}\sum_{w=0}^{\infty}\frac{(-1)^{w}\Gamma(\alpha+1)f(z-wh)}{\Gamma(w+1)\Gamma(\alpha-w+1)}\right], \tag{23}$$

Here, h represents the step time augmentation while $\Gamma()$ is the gamma function. This, Grunwald-Letnikov, interpretation reveal an important property that the integer derivative just infers a finite series, while the fractional derivative involves an infinite numeral of terms, thats is, they hold implicitly, a memory of previous events.

A discrete time implementation of the GL expression can be approximated as;

$$D^{\alpha}[f(z)] = \frac{1}{T^{\alpha}}\sum_{w=0}^{r}\frac{(-1)^{m}\Gamma(\alpha+1)f(z-wT)}{\Gamma(w+1)\Gamma(\alpha-w+1)}, \tag{24}$$

Here r is the truncation order and T denotes sampling period.

The inherent memory property of fractional order systems make them much suitable to define phenomena such as chaos and irreversibility. Therefore, the random behavior of particle's movement during search evolution constitute a scenario where FC tool fit appropriately.

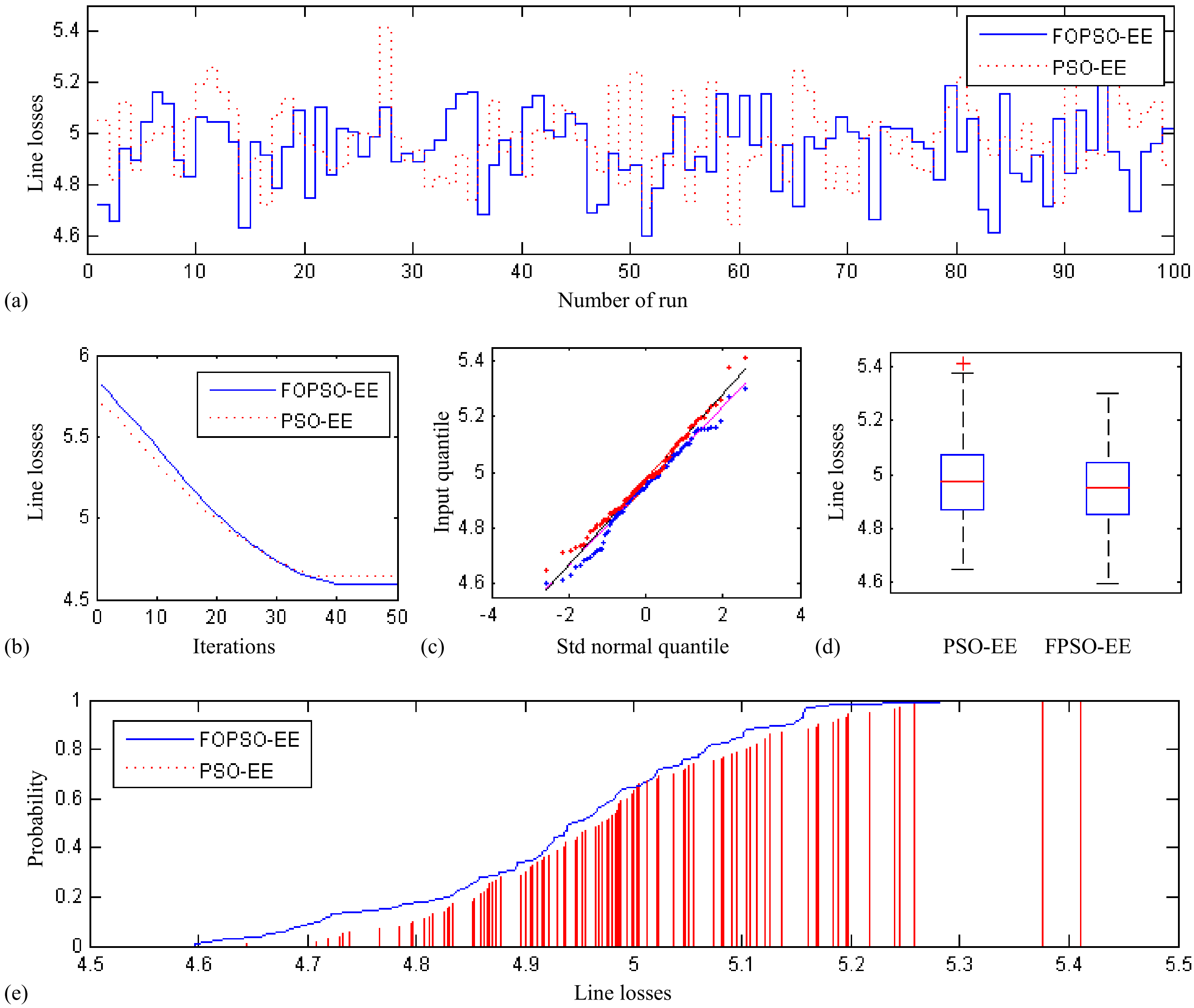


FIGURE 6. FOPSO-EE comparison with PSO-EE during f_1 minimization for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

B. PSO ALGORITHM

The traditional PSO algorithm, proposed originally by Kennedy and Eberhart in 1995, is meta heuristic computational technique inspired by the movement of the particles in a swarm both for, finding global solution and as a defensive approach. This movement is represented by two vectors, specifically by its position x and velocity v.

Algorithm 1 illustrates a canonical PSO mechanism. The cognitive learning of each particle is incorporated by considering the distance between its best position found up to now LB_t^n and the current position x_t^n, while, the social learning of each particle, is obtained by taking the distance between the swarm global best position obtained up to now GB_t^n and its current position x_t^n. Both learning factors are assigned a randomly generated weight ρ_1 and ρ_2, respectively.

C. PSO WITH FRACTIONAL VELOCITY

In 2010, Machado and team [52], introduced a new technique to improve the convergence of conventional PSO by integrating the concept of fractional derivative in velocity update equation of conventional PSO. At first, the original velocity expression is reshuffled to alter the coefficient of the velocity derivative, specifically

$$v_{t+1}^n = \omega v_t^n, +\rho_1 r_1(LB_t^n - x_t^n) + \rho_2 r_2(GB_t^n - x_t^n), \quad (25)$$

Hereafter, x denotes the particle's position, t represents the flight index, n is the particle index with corresponding velocity v, r_1 and r_2 are the random numbers between 0-1, ρ_1 is the local while ρ_2 is the global acceleration coefficients, LB represents the local and GB denotes the global best particle while ω is inertial weight.

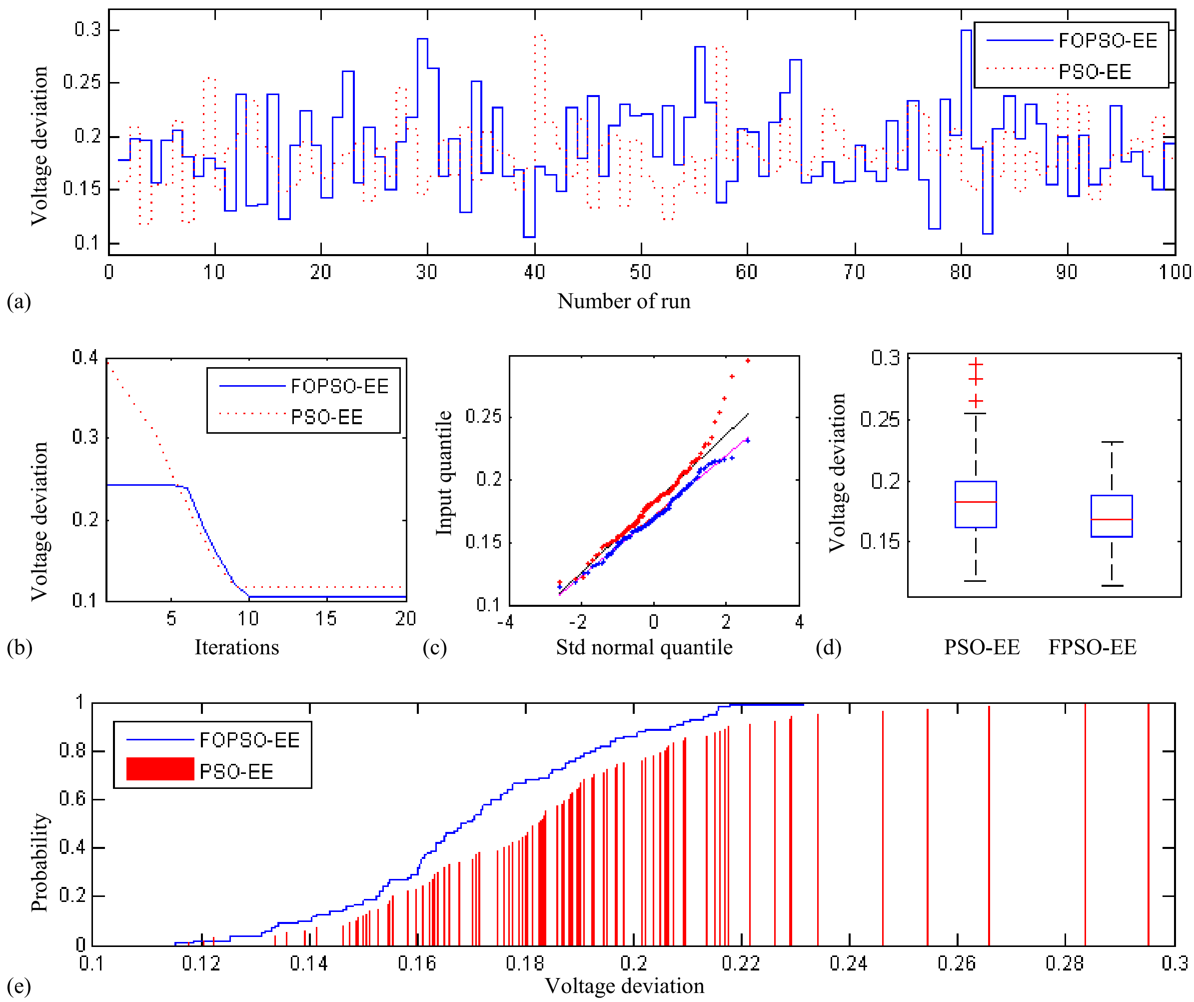


FIGURE 7. FOPSO-EE comparison with PSO-EE during f_2 minimization for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

TABLE 2. Function parameters.

Type	iter	P flights	Parameters [U, T, Q]	Range ($z\epsilon$) min-max	ρ_1, ρ_2	ω
f_1	50	30	[6, 4, 3]	[0.9-1.1, 0.9-1.05, 0-5]	0.9 - 0.1	0.9 - 0.4
f_2	20	20	[6, 4, 3]	[0.9-1.1, 0.9-1.05, 0-5]	0.9 - 0.1	0.9 - 0.4
f_3	20	20	[6, 4, 3]	[0.9-1.1, 0.9-1.05, 0-5]	0.9 - 0.1	0.9 - 0.4

Considering the inertial influence, $\omega = 1$, equation (25) can be rearranged as:

$$v_{t+1}^n - v_t^n = \rho_1 r_1(LB_t^n - x_t^n) + \rho_2 r_2(GB_t^n - x_t^n) = \nabla(v_{t+1}^n) \tag{26}$$

while assuming $T = 1$, the expression $v_{t+1}^n - v_t^n$ is the discrete form of the derivative with coefficient $\alpha = 1$, implies to the following relation:

$$D^\alpha(v_{t+1}^n) = \rho_1 r_1(LB_t^n - x_t^n) + \rho_2 r_2(GB_t^n - x_t^n) \tag{27}$$

Considering (24) with first four terms i.e. $r = 4$, (27) can be written as:

$$\begin{aligned} v_{t+1}^n - \alpha v_t^n - \frac{1}{2}\alpha(1-\alpha)v_{t-1}^n - \frac{1}{6}\alpha(1-\alpha)(2-\alpha)v_{t-2}^n \\ - \frac{1}{24}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v_{t-3}^n \\ = \phi_1 r_1(LB_t^n - s_t^n) + \phi_2 r_2(GB_t^n - s_t^n) \end{aligned} \tag{28}$$

or

$$\begin{aligned} v_{t+1}^n = \alpha v_t^n + \frac{1}{2}\alpha(1-\alpha)v_{t-1}^n + \frac{1}{6}\alpha(1-\alpha)(2-\alpha)v_{t-2}^n \\ + \frac{1}{24}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v_{t-3}^n \\ + \phi_1 r_1(LB_t^n - s_t^n) + \phi_2 r_2(GB_t^n - s_t^n) \end{aligned} \tag{29}$$

The coefficient α can be generalized to a real number $0 \leq \alpha \leq 1$, if the fractional calculus viewpoint is adopted, leading to a longer memory effect and smoother variation. It can

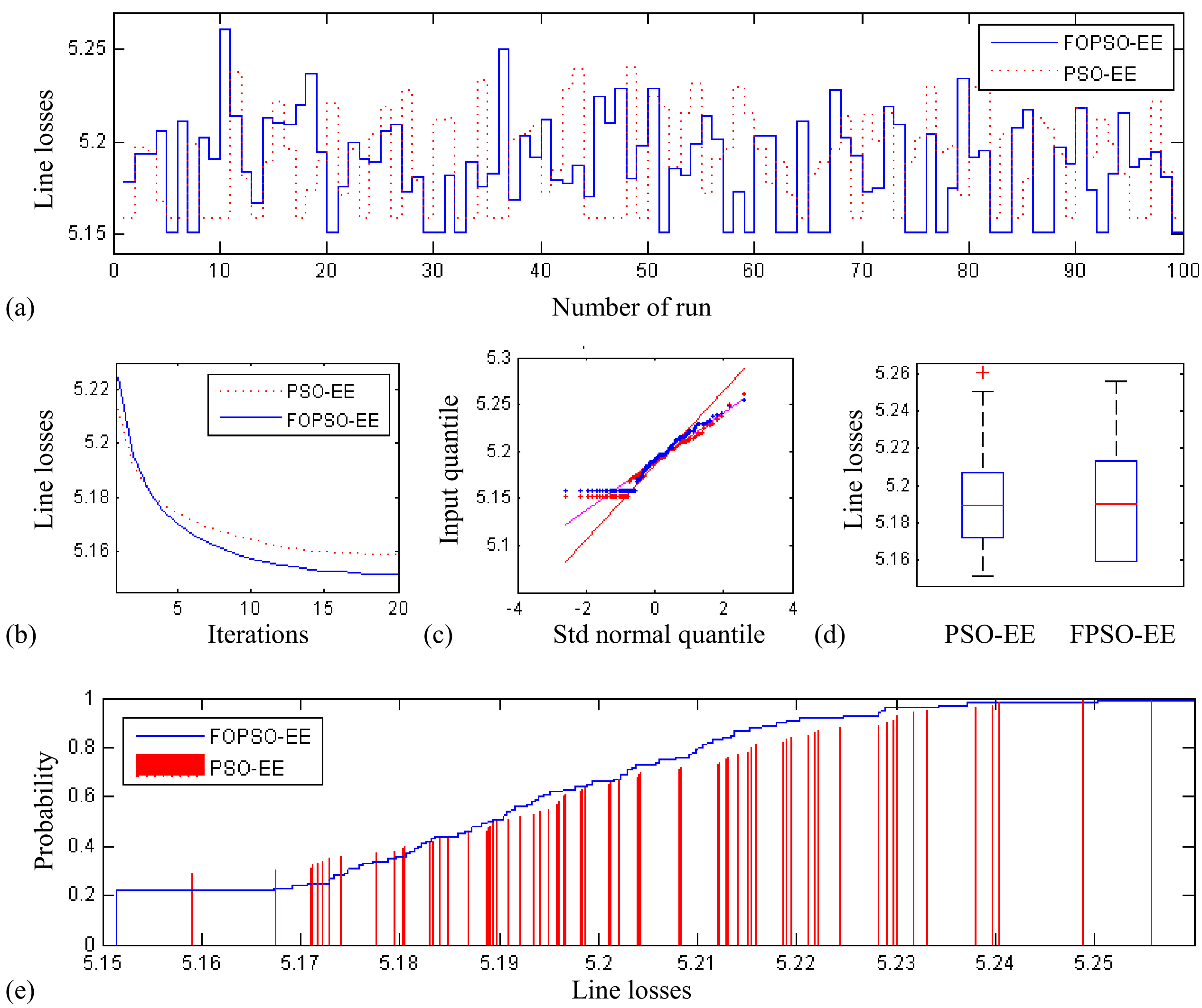


FIGURE 8. FOPSO-EE comparison with PSO-EE during minimization of f_1 in power system with FACTS for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

TABLE 3. Comparison of results for case 1.

Variables	MFO [28]	GWO [31]	M-IWO [28]	HSA [70]	IWO [75]	ICA [28]	PSO [70]	GA [9]	DE [69]	FO-DPSO	PSO-EE	FOPSO-EE
V1	1.1	1.1	1.07972	1.0726	1.06965	1.0785	1.0313	1.0721	1.095319	ă1.01	1.1	1.1
V2	1.0946	1.096149	1.07055	1.0625	1.06038	1.06943	1.0114	1.063	1.085946	1.04231	1.1	1.1
V5	1.0756	1.080036	1.04836	1.0399	1.03692	1.06943	1.0221	1.0377	1.062628	1.0401	1.0844	1.0833
V8	1.772	1.080444	1.04865	1.0422	1.03864	1.04714	1.0031	1.0445	1.065076	1.0956	1.0820	1.08533
V11	1.0868	1.093452	1.07518	1.0318	1.02973	1.03485	0.9744	1.0132	1.0266	1.011	1.0834	1.0931
V13	1.1	1.1	1.07072	1.0681	1.05574	1.07106	0.9987	1.0898	1.014253	1.0491	1.0936	1.1
T6-9	1.0411	1.04	1.03	1.01	1.05	1.08	0.97	1.0221	1.017796	1.061	1.0278	1.0434
T6-10	0.95007	0.95	0.99	1	0.96	0.95	1.02	0.9917	0.979277	0.9295	1.0602	1.0294
T4-12	0.95541	0.95	1	0.99	0.97	1	1.01	0.9964	0.977843	0.9665	1.0813	1.0752
T27-28	0.95754	0.95	0.98	0.97	0.97	0.97	0.99	0.971	1.008938	0.9555	1.0362	1.0210
Qc3	7.1032	12	-7	34	8	-6	17	5.3502	20.22359	8.4272	7.5420	4.2822
Qc10	30.796	30	23	12	35	36	13	36	9.584327	25.1542	2.8600	2.6762
Qc24	9.8981	8	12	10	11	11	23	12.4175	13.02992	9.2331	9.4543	6.6747
P_{loss}	4.608	4.613	4.846	5.109	4.92	4.849	5.8815	4.8775	4.888081	4.606	4.6448	**4.5971**

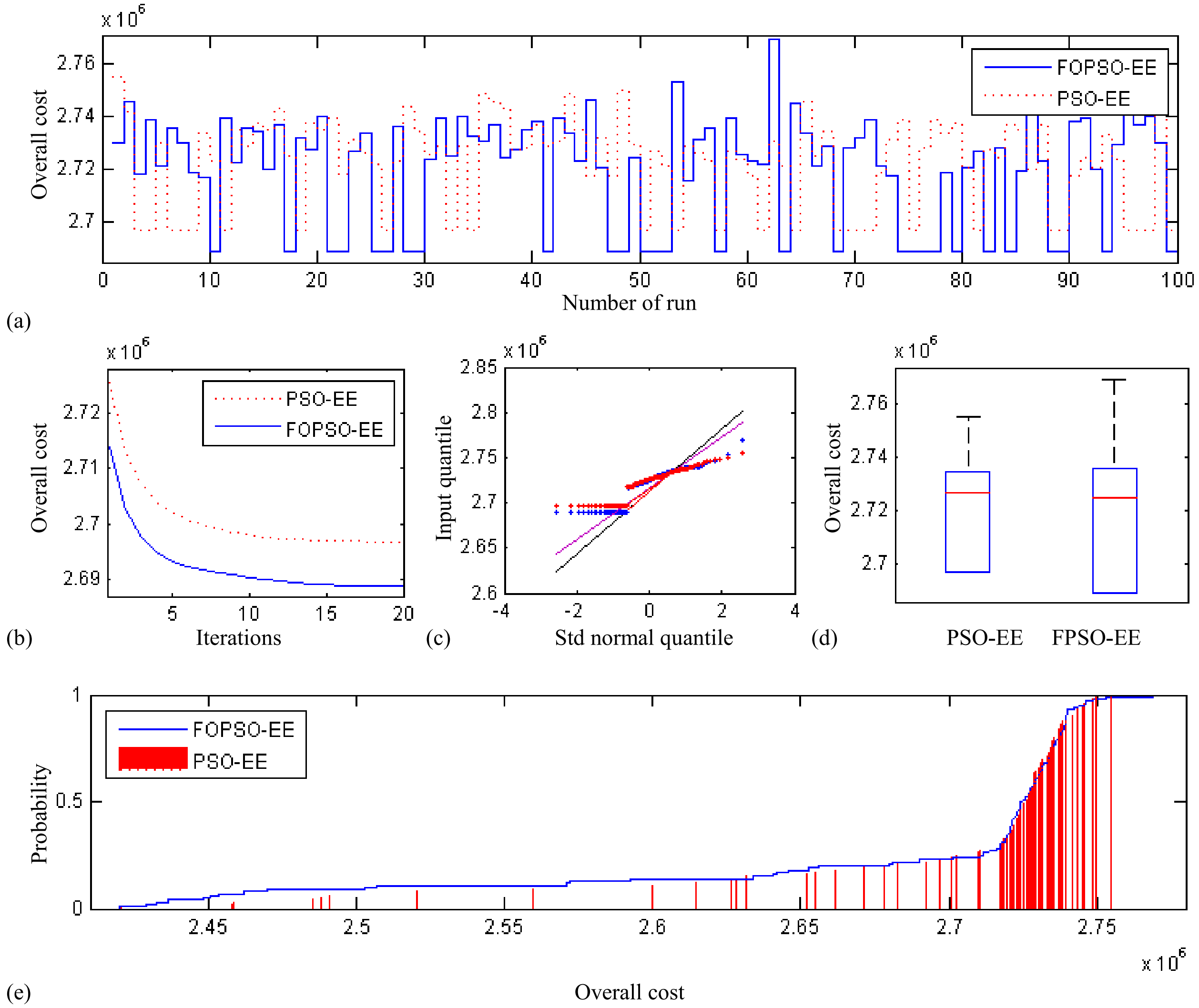


FIGURE 9. FOPSO-EE comparison with PSO-EE during minimization of f_3 in power system with FACTS for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

TABLE 4. Percentage line loss reduction in test system.

Parameter	initial value	DE	GA	HSA	M-IWO	GWO MFO	FO-DPSO	PSO-EE	FOPSO-EE
P_{loss} (MW)	5.663	4.88808	4.8775	5.109	4.846	4.613 4.608	4.606	4.6448	4.5971
Loss reduction (%)	-	13.68	13.87	9.78	14.44	18.54 18.64	18.66	17.97	18.82

be seen from expression (29) that the traditional PSO is a special scenario of the fractional PSO with $\alpha = 1$. Because, the FOPSO integrates the fractional calculus tool to control the particle convergence, the fractional order α must needs to be identified to guarantee a high level of exploration during the evolution of search. The additional literature of basic FOPSO can be seen in [53], [71]–[74].

D. ENTROPY

Several entropy definitions have been presented over the years, such as information, freedom spreading, mixing, chaos and disorder. The foremost interpretation of entropy was presented by Boltzmann as transformation of the system from ordered to disordered states. Lewis interpreted that, during the impulsive expansion of gas in an isolated system, the uncertainty or, missing information increases while knowledge about particles location decreases. Guggenheim used spreading to show the evolution in volume of an energy system from a small scale to a large scale. Shannon introduced the information theory for the quantization of the information loss during any message transmission in a communication network and concentrated on statistical and physical constraints that restrict the signal processing. Shannon described H as a degree of uncertainty, information

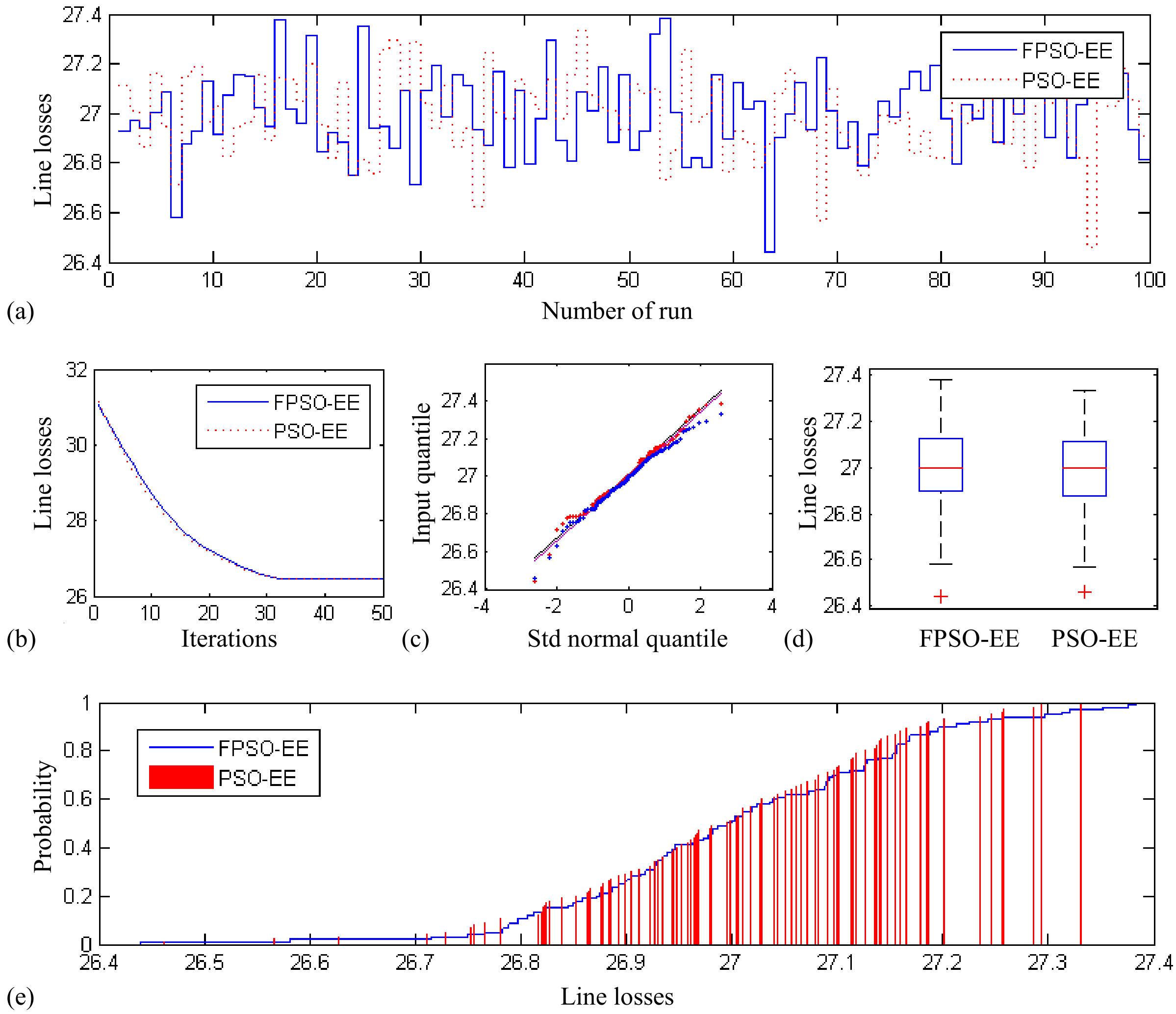


FIGURE 10. FOPSO-EE comparison with PSO-EE during f_1 minimization in IEEE 57 bus system for 100 independent runs (a) minimum fitness (b) convergence curve (c) probability plot (d) boxplot illustration (e) CDF.

TABLE 5. Comparative results of f_2 optimization.

Methods	V_D (p.u)	% improvement	Methods	V_D	% improvement
Base case	1.1606	-	-	-	-
OGSA	0.8540	26.41	KHA	0.2963	74.4
ALC-PSO	0.3001	74.14	CKHA	0.3524	69.6
CLPSO	0.245	78.89	NGBWCA	0.3003	74.1
PSO	0.2424	79.11	SGA	0.7369	36.5
PSO-EE	0.1177	89.8	-	-	-
FOPSO-EE	0.1057	90.8	-	-	-

and choice:

$$H(X) = -K \sum_{x \epsilon X} p_i(x) \log p_i(x) \tag{30}$$

For random variables $(x, y) \epsilon (X, Y)$

$$H(X, Y) = -K \sum_{x \epsilon X} \sum_{y \epsilon Y} p_i(x, y) \log p_i(x, y) \tag{31}$$

where, K is the constant parameter, normally equal to 1, $x \epsilon X$ is a discrete variable and $p(x)$ is the probability distribution.

E. FOPSO WITH ENTROPY EVOLUTION

The method followed in this study is inspired by the need to explore and exploit the entropy during the fractional particle swarm optimizer time evolution and to adopt this

TABLE 6. Comparative results of case 3 and 4.

Variable	SPSO [13]	WOA [76]	QOGWO [76]	QODE [76]	PSO-EE	FOPSO-EE
Q_G (2)	0.6	0.6	0.0339	0.3535	7.2287	1.6343
Q_G (5)	0.0	0.6250	-0.0027	0.2365	2.4845	3.3546
Q_G (8)	0.0	0.5	0.2907	0.4462	5.3979	0.3094
Q_G (11)	0.4	0.0029	0.0515	0.3497	4.0767	0.4432
Q_G (13)	0.0	0.0177	0.2342	0.2460	5.4464	7.1187
T (11)	0.9	0.9	0.9	0.9012	0.9803	0.9829
T (12)	0.9	0.9448	0.9452	0.9514	0.9791	0.9628
T (15)	0.9	0.9	0.9	0.9004	0.9683	0.9727
T (36)	0.9223	0.9285	0.9271	0.9278	0.9643	0.9660
TCSC (1)	0.1463(25)	0.08(25)	0.08(25)	0.0618(25)	0.1944	0.4184
TCSC (2)	0.0419(41)	0.08(41)	0.08(41)	0.1588(41)	0.8532	0.3075
TCSC (3)	0.1049(28)	0.08(28)	0.08(28)	0.1999(28)	0	0
TCSC (4)	0.1388(5)	0.08(5)	0.08(5)	0.0116(5)	0	0
SVC (1)	0.0(7)	0.0524(22)	0.0589(22)	0.0797(22)	0.2536	0.2308
SVC (2)	0.0(15)	0.1544(04)	0.12(04)	0.08(04)	0.1887	0.2253
SVC (3)	0.0(17)	0.2(28)	0.20(28)	0.0789(28)	0.2048	0.1393
SVC (4)	0.0840(21)	0.0145(20)	0.0091(20)	0.08(20)	0.1456	0.1248
P_{Loss}	0.05198	0.06333	0.06331	0.0528	5.1675	5.1673
$C_{overall}$	2.7324E+06	3.3289E+06	3.3279E+06	2.7755E+06	2.6969+06	2.6809+06

TABLE 7. Comparative results of loss reduction and overall cost reduction.

Methods	P_{Los} (p.u)	$C_{overall}$ (USD)	Loss reduction (%)	Cost reduction (%)
Base case	0.0711	3737016	-	-
WOA [76]	0.06333	3.33E+06	10.928	10.723
QOGWO [76]	0.06331	3.33E+06	10.956	10.723
SPSO [13]	0.05198	2.73E+06	26.891	26.809
QODE [76]	0.0528	2.78E+06	25.788	25.469
PSO-EE	0.051675	2.6969+06	27.320	27.832
FOPSO-EE	0.051673	2.6809+06	27.323	28.260

synergy to enhance the algorithm characteristics, namely, the convergence. For this reason, the Shannon entropy is evolved in the internal structure of the optimizer while adopting the ORPD problems. Since the FOPSO is a non-deterministic solver, therefore, a set of 100 independent runs is performed to develop a representative statistical data set. Mean while, the influence of entropy signal is observed in the behavior of algorithm, namely by the swarm reinitialization, along the FOPSO execution for enhancing its convergence.

Indeed, the entropy measure the changing tendency of a system energy i.e., the spreading of particles within the search space, during the present case. Bearing this idea in mind, a distance d_i is considered between i^{th} particle position and the best global particle. Then, probability p_i for each particle is given by the distance d_i to maximum possible distance, that is:

$$p_i = \frac{d_i}{d_{\max}} \tag{32}$$

For a n swarm size, and $k = 1$, the diversity index (30) of particle is quantified as:

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i \tag{33}$$

The overall workflow diagram is depicted in Fig. 2 while the procedural steps are illustrated in algorithm 2.

IV. RESULTS AND DISCUSSION

This section demonstrates the efficacy of FOPSO-EE over the other counter part algorithms for 4 different cases, each considering one objective function based on optimal reactive power dispatch scenarios defined previously using the parameters listed in Table 2.

Since particle swarm optimization is a stochastic method, every time it is applied it shows a different learning behavior. Therefore, a set of 100 independent trials was conducted, for each fractional order $\alpha = 0.1, \ldots, 0.9$ and the median, arithmetic mean, maximum and minimum values were taken as the final output.

A. CASE 1: MINIMIZATION OF P_{loss}

The first objective function to be adapted belongs to the minimization of transmission line loss, as described by

TABLE 8. Comparison of results for case 5.

Variables	Initial	NGBWCA	DE	CKHA	GSA	PSO	PSO-EE	FOPSO-EE
V1	1.04	1.06	1.0397	1.06	1.06	1.0834	1.0877	1.0915
V2	1.01	1.0591	1.0463	1.059	1.06	1.0849	1.0853	1.0908
V3	0.985	1.0492	1.0511	1.0487	1.06	1.0823	1.0800	1.0863
V6	0.98	1.0399	1.0236	1.0431	1.0081	1.0818	1.0806	1.0829
V8	1.05	1.0586	1.0538	1.06	1.0549	1.0887	1.0889	1.0875
V9	0.98	1.0461	0.9451	1.0447	1.009.8	1.0835	1.0798	1.0849
V12	1.015	1.0413	0.9907	1.041	1.0185	1.0796	1.0833	1.0898
T4-18	0.97	0.9712	1.02	0.9179	1.1	1.0429	1.0418	1.0327
T4-18	0.978	0.9243	0.91	1.0256	1.0826	1.0107	1.0273	1.0238
T21-20	1.043	0.9123	0.97	0.9	0.9219	1.0265	1.0260	1.0217
T24-26	1.043	0.9001	0.91	0.902	1.0167	1.0380	1.0273	1.0249
T7-29	0.967	0.9112	0.96	0.9104	0.9962	1.0120	1.0253	1.0345
T34-32	0.965	0.9004	0.99	0.9005	1.1	1.0290	1.0236	1.0196
T11-41	0.955	0.9128	0.98	0.9	1.0746	1.0394	1.0159	1.0203
T15-45	0.955	0.9	0.96	0.9	0.9543	1.0117	1.0342	1.0233
T14-46	0.9	1.0218	1.05	1.0797	0.9377	1.0456	1.0299	1.0333
T10-51	0.93	0.9902	1.07	0.9887	1.0167	1.0165	1.0093	1.0195
T13-49	0.895	0.9568	0.99	0.9914	1.0525	1.0269	1.0260	1.0360
T11-43	0.958	0.9	1.06	0.9	1.1	1.0345	1.0429	1.0165
T40-56	0.958	0.9	0.99	0.9002	0.9799	1.0282	1.0355	1.0375
T39-57	0.98	1.0118	0.97	1.0173	1.0246	1.0286	1.0222	1.0421
T9-55	0.94	1	1.07	1.0023	1.0373	1.0375	1.0402	1.0400
Qc18	0	0.0914	0	0.0994	0.0782	4.6205	3.9490	3.0492
Qc25	0	0.0587	0	0.059	0.0058	1.0404	3.1515	5.0487
Qc53	0	0.0634	0	0.063	0.0468	4.0080	6.8901	7.6275
P_{loss}	27.86	26.74	35.94	27.48	29.4	26.8986	26.4616	26.4390

equation (3). Equation (33) is considered to monitor the fractional PSO evolution. The learning curves are plotted in Fig. 3, showing the best, average and worst iterative updates of transmission line losses during 100 independent trials for $\alpha = 0.1, \ldots, 0.9$. In Table 3, the results can be seen for the P_{loss} function f_1 minimization, yielded by FOPSO-EE, along a comparison with other algorithms such as MFO, GWO, MICA-IWA, HSA, IWO, ICA, GA, DE, FO-DPSO, and PSO-EE. Observing Table 3, percentage line loss reduction in Table 4 and Fig. 3, one can conclude that, FOPSO-EE revealed a better behavior by computing minimum losses as compared to counter part algorithms, hence, the synergy of fractional calculus and entropy evolution contributes to an enhanced convergence dynamics.

Algorithm 1 Traditional PSO

1: **procedure** In steps with input and outputs
2: **Inputs**: Inertia weights, particle number, acceleration coefficients
3: **Output**: Global best solution
4: **Start of PSO**
5: **Initialization**: Swarm, random position x and velocity v
6: **Fitness evaluation**: Repeat until stopping criteria
- for all particles calculate fitness

7: **Updating mechanism**: for all particles update,
- *pbest*, *gbest* and *lbest*
- velocity based on

$$v_{t+1}^n = v_t^n, +\rho_1 r_1 (LB_t^n - x_t^n) + \rho_2 r_2 (GB_t^n - x_t^n),$$

- position based on

$$x_{t+1}^n = x_t^n + v_{t+1}^n$$

8: **Termination criterion**: Convergence, stagnation, iteration
9: **End PSO**

B. CASE 2: MINIMIZATION OF V_D

The second optimization function to be adopted is the minimization of voltage deviation index, as described by expression (7), to monitor the FOPSO-EE evolution. The results are demonstrated in Fig. 4 for archive size of 20 particles and order $\alpha = 0.1, \ldots, 0.9$, where best global solution is found at $\alpha = 0.9$. The comparison of results revealed by FOPSO-EE and that by other algorithms is documented in Table 5. It can be verified that FOPSO-EE yielded better results with respect to the counter part algorithms.

C. CASE 3: MINIMIZATION OF P_{loss} WITH FACTS

In third case, the transmission line losses are minimized considering FACTS devices, namely, the TCSC and SVC, as the

Algorithm 2 Pseudocode of FOPSO-EE for ORPD

1: **procedure** In steps with input and outputs

2: **Inputs**: Bus, branch and generator data for IEEE standard power system i.e., IEEE 30 bus.

3: **Output**: Minimum power loss as expressed by equation (1), Minimum voltage deviation as expressed by equation (7) and Minimum overall cost as expressed by equation (8).

4: **Start of FOPSO-EE**

5: **Initialization**: Initialize

- Position x and velocity v matrices with real values
- Swarm with set of possible solutions P, known as *particle* in n-dimensional search space as:

$$S_n^{\min} = \left[V_1^{\min}, V_2^{\min}, \ldots, V_n^{\min}, T_1^{\min}, T_2^{\min}, \ldots, T_n^{\min}, Q_1^{\min}, Q_2^{\min}, \ldots, Q_n^{\min} \right]$$
$$S_n^{\max} = \left[V_1^{\max}, V_2^{\max}, \ldots, V_n^{\max}, T_1^{\max}, T_2^{\max}, \ldots, T_n^{\max}, Q_1^{\max}, Q_2^{\max}, \ldots, Q_n^{\max} \right]$$

6: **Fitness evaluation**: Add exterior penalty function with fitness function to restrict the constraint violation as

$$Minimize : f(f_1/f_2/f_3) + \sum \lambda_V (V_i - V_i^{\lim})^2 + \sum \lambda_T (T_i - T_i^{\lim}) + \sum \lambda_Q (Q_i - Q_i^{\lim})^2 \tag{34}$$

with, $T_i^{\lim} = \begin{cases} T_i^{\max}; T_i > T_i^{\max} \\ T_i^{\min}; T_i < T_i^{\min} \end{cases}$, $Q_{Gi}^{\lim} = \begin{cases} Q_i^{\max}; Q_i > Q_i^{\max} \\ Q_i^{\min}; Q_i < Q_i^{\min} \end{cases}$ and $V_i^{\lim} = \begin{cases} V_i^{\max}; V_i > V_i^{\max} \\ V_i^{\min}; V_i < V_i^{\min} \end{cases}$.

7: **Updating mechanism**: FOPSO-EE is updated based on two mechanisms

- Velocity using equation (29) as:

$$v(p, k+1) = \alpha v(p, k) + \frac{1}{2}\alpha(1-\alpha)(p, k-1) + \frac{1}{6}\alpha(1-\alpha)(2-\alpha)v(p, k-2) + \frac{1}{24}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v(k-3)$$
$$+ \phi_1 r_1 (LB(p, k) - x(p, k)) + \phi_2 r_2 (GB(p, k) - x(p, k))$$

here, p denotes the particle, k is the flight index, LB is used for *pbest* and GB for *gbest*.

- Particle position using the expression:

$$x(p, k+1) = x(p, k) + v(p, k+1)$$

If current best particle i.e. $f(x(p, k+1)) >$ previous best particle i.e. $f(LB(p, k))$, then $LB(p, k+1) = x(p, k+1)$ else $LB(p, k+1) = x(p, k)$ Endif $f(LB(p, k+1)) > f(GB(p, k))$ then $GB(p, k+1) = LB(p, k+1)$ else $GB(p, k+1) = LB(p, k)$ and repeat the update for each particle in a swarm

8: **Termination criterion**: The algorithm will stop the searching process if saturation point is reached. Print the particle with latest *gbest*. Until termination criteria, repeat from step 6 with updated swarm.

9: **Storage**: The variables of global best particle are stored on the basis of minimum power losses, voltage deviation and overall cost.

10: **Analysis**: Repeat steps 5 to 9 for the given variations to produce a large dataset for comprehensive analysis of the FOPSO-EE performance:

- Different fractional orders α of the FOPSO-EE
- Perform 100 independent trials for each variant of the FOPSO-EE

11: **End of FOPSO-EE**

axillary reactive power sources. The incorporation of FACTS devices alters the equality constraints and adds additional constraints, as defined in section II. The best global particle and corresponding fitness value evaluated by FOPSO-EE is listed in Table 6, while the percentage line loss reduction is listed in Table 7. It can be seen that, the proposed scheme has outperformed other algorithms, namely, the SPSO, WOA, QOGWO, QODE, and PSO-EE.

D. CASE 4: MINIMIZATION OF OVERALL COST

The results for the overall cost minimization function f_3 are illustrated in Table 6. The percentage cost reduction, fourth column in Table 7, show that FOPSO-EE has computed a lower overall cost of operation in comparison with those obtained with the SPSO, WOA, QOGWO, QODE, and PSO-EE. We verify that the FOPSO-EE lead to a significantly better solution for ORPD problems.

E. CASE 5: VALIDATION OF FOPSO-EE IN LARGE SCALE TEST SYSTEM

The effectiveness of FOPSO-EE is further ascertained by testing it on large scale power system i.e., IEEE 57 bus by adopting line loss minimization as a fitness function. The evolution of proposed algorithm execution is monitored by using the Equation (33). The obtained learning curves for all the fractional orders i.e., $\alpha = [0.1, 0.2, \ldots, 0.9]$ are plotted in Fig. 5, depicting the best, average and worst iterative updates of transmission line losses during 100 independent trials. The optimum value of the operational variable with corresponding losses are documented in Table 8, along a comparison with other well-known algorithms such as NGB-WCA, DE, CKHA, GSA, PSO, and PSO-EE. Observing Table 8 and graphical illustrations in Fig. 5, one can conclude that, FOPSO-EE revealed a better behavior by computing minimum losses as compared to counterpart algorithms, hence, the synergy of fractional calculus and entropy evolution contributes to an enhanced convergence dynamics while endorsing a better optimization strength of FOPSO-EE for large scale power system.

V. STATISTICAL ANALYSIS

To analyze the consistency and reliability of the FOPSO-EE, a detailed statistical analysis has been carried out for all four cases, considering the best fractional order α in the set. In this line of thought, for all test cases, 100 independent trials were carried out and the median of the fitness evolution is taken as reference, for nominating the fractional order. The statistical assessment is based on the minimum fitness evolution in each independent simulation, convergence curves, quantile-quantile plots, box plot illustrations, and empirical cumulative distribution function, as depicted in Figs 6-10.

The sub figures 6(a)-10(a) reveal that FOPSO-EE compute minimum fitness in most of the independent run when compared with PSO-EE. A considerable difference in learning behaviors can be verified through sub figures 6(b)-10(b) where in all charts the FOPSO-EE performed better than PSO-EE. It can be verified, through sub figures 6(c)-10(c), that the minimum fitness evolution versus the quantiles of a standard normal distribution is more ideal in case of FOPSO-EE. Sub figures 6(d)-10(d) indicates that median guages were always occurred at lower side for FOPSO-EE. The sub figures 6(e)-10(e) depicts that the probability of finding a best fitness through FOPSO-EE is on higher side than PSO-EE.

Bearing these results in mind, it is verified that both, the entropy and fractional calculus characterizes a natural tool which allows to design new variants of traditional algorithms, and leads to future promising advancements based on a new vantage point.

VI. CONCLUSION

A novel optimization approach FOPSO-EE is presented for solving the ORPD problems in the power system by exploitation of entropy diversity in fractional swarm intelligence. In the designed method FOPSO-EE, two important mathematical tools, namely the Shanon entropy and fractional calculus are integrated with traditional PSO algorithm. The proposed FOPSO-EE is viably implemented in standard IEEE 30 bus power system for minimizing the transmission line losses, voltage deviation and overall operational cost by tuning the operational variables such as transformer tap positions, bus voltages and reactive power compensators to near optimum-value. The results demonstrated that, the synergies of applying both, the Shanon entropy and fractional calculus concept, improved the performance of the optimizer in terms of fitness evolution and convergence rate during the proposed FOPSO-EE executions.

In this line of thought, both, the entropy evolution and fractional order dynamics will be considered in designing new integrated fractional swarming/evolutionary algorithms to solve significant optimization problems related to engineering sector in future research works such as the distributed generation [77], coordination of directional over current ralay [78] and parameter extraction [79] etc. Moreover, the proposed algorithm can be further investigated for slandered benchmark functions with performance evaluation in terms of Wilcoxon sign rank tests.

REFERENCES

[1] M. A. Taher, S. Kamel, F. Jurado, and M. Ebeed, "Optimal power flow solution incorporating a simplified UPFC model using lightning attachment procedure optimization," *Int. Trans. Electr. Energy Syst.*, vol. 30, no. 1, Jan. 2020, Art. no. e12170.

[2] W. S. Sakr, R. A. EL-Sehiemy, and A. M. Azmy, "Adaptive differential evolution algorithm for efficient reactive power management," *Appl. Soft Comput.*, vol. 53, pp. 336–351, Apr. 2017.

[3] Q. H. Wu, Y. J. Cao, and J. Y. Wen, "Optimal reactive power dispatch using an adaptive genetic algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 20, no. 8, pp. 563–569, Nov. 1998.

[4] S. Durairaj, P. S. Kannan, and D. Devaraj, "Application of genetic algorithm to optimal reactive power dispatch including voltage stability constraint," *J. Energy Environ.*, vol. 4, no. 63, p. 7, 2005.

[5] S.-C. Kim and S. R. Salkuti, "Optimal power flow based congestion management using enhanced genetic algorithms," *Int. J. Electr. Comput. Eng.*, vol. 9, no. 2, p. 875, Apr. 2019.

[6] M. S. Osman, M. A. Abo-Sinna, and A. A. Mousa, "A solution to the optimal power flow using genetic algorithm," *Appl. Math. Comput.*, vol. 155, no. 2, pp. 391–405, Aug. 2004.

[7] B. Bhattacharyya and V. K. Gupta, "Fuzzy genetic algorithm approach for the optimal placement of flexible AC transmission systems devices in a power system," *Electr. Power Compon. Syst.*, vol. 42, no. 8, pp. 779–787, Jun. 2014.

[8] B. Bhattacharyya and V. K. Gupta, "Fuzzy based evolutionary algorithm for reactive power optimization with FACTS devices," *Int. J. Electr. Power Energy Syst.*, vol. 61, pp. 39–47, Oct. 2014.

[9] K. Mahadevan and P. S. Kannan, "Comprehensive learning particle swarm optimization for reactive power dispatch," *Appl. Soft Comput.*, vol. 10, no. 2, pp. 641–652, Mar. 2010.

[10] B. Zhao, C. X. Guo, and Y. J. Cao, "A multiagent-based particle swarm optimization approach for optimal reactive power dispatch," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1070–1078, May 2005.

[11] M. Mehdinejad, B. Mohammadi-Ivatloo, R. Dadashzadeh-Bonab, and K. Zare, "Solution of optimal reactive power dispatch of power systems using hybrid particle swarm optimization and imperialist competitive algorithms," *Int. J. Electr. Power Energy Syst.*, vol. 83, pp. 104–116, Dec. 2016.

[12] E. Naderi, M. Pourakbari-Kasmaei, and H. Abdi, "An efficient particle swarm optimization algorithm to solve optimal power flow problem integrated with FACTS devices," *Appl. Soft Comput.*, vol. 80, pp. 243–262, Jul. 2019.

[13] B. Bhattacharyya and S. Raj, "Swarm intelligence based algorithms for reactive power planning with flexible AC transmission system devices," *Int. J. Electr. Power Energy Syst.*, vol. 78, pp. 158–164, Jun. 2016.

[14] Y. Muhammad, R. Khan, F. Ullah, A. U. Rehman, M. S. Aslam, and M. A. Z. Raja, "Design of fractional swarming strategy for solution of optimal reactive power dispatch," *Neural Comput. Appl.*, pp. 1–18, Nov. 2019.

[15] Q. H. Wu and J. T. Ma, "Power system optimal reactive power dispatch using evolutionary programming," *IEEE Trans. Power Syst.*, vol. 10, no. 3, pp. 1243–1249, Aug. 1995.

[16] H. Yoshida, K. Kawata, Y. Fukuyama, S. Takayama, and Y. Nakanishi, "A particle swarm optimization for reactive power and voltage control considering voltage security assessment," *IEEE Trans. Power Syst.*, vol. 15, no. 4, pp. 1232–1239, Nov. 2000.

[17] W. Yan, F. Liu, C. Y. Chung, and K. P. Wong, "A hybrid genetic algorithm–interior point method for optimal reactive power flow," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1163–1169, Aug. 2006.

[18] G. Cai, Z. Ren, and T. Yu, "Optimal reactive power dispatch based on modified particle swarm optimization considering voltage stability," in *Proc. IEEE Power Eng. Soc. Gen. Meeting*, Jun. 2007, pp. 1–5.

[19] P. Subbaraj and P. N. Rajnarayanan, "Optimal reactive power dispatch using self-adaptive real coded genetic algorithm," *Electr. Power Syst. Res.*, vol. 79, no. 2, pp. 374–381, Feb. 2009.

[20] P. K. Roy, S. P. Ghoshal, and S. S. Thakur, "Turbulent crazy particle swarm optimization technique for optimal reactive power dispatch," in *Proc. World Congr. Nature Biologically Inspired Comput. (NaBIC)*, 2009, pp. 1219–1224.

[21] A. A. Abou El-Ela, A. M. Kinawy, R. A. El-Sehiemy, and M. T. Mouwafi, "Optimal reactive power dispatch using ant colony optimization algorithm," *Electr. Eng.*, vol. 93, no. 2, pp. 103–116, Jun. 2011.

[22] S. Jeyadevi, S. Baskar, C. K. Babulal, and M. W. Iruthayarajan, "Solving multiobjective optimal reactive power dispatch using modified NSGA-II," *Int. J. Electr. Power Energy Syst.*, vol. 33, no. 2, pp. 219–228, Feb. 2011.

[23] P. K. Roy, S. P. Ghoshal, and S. S. Thakur, "Optimal reactive power dispatch considering flexible AC transmission system devices using biogeography-based optimization," *Electr. Power Compon. Syst.*, vol. 39, no. 8, pp. 733–750, Apr. 2011.

[24] S. Sivasubramani and K. S. Swarup, "Multi-objective harmony search algorithm for optimal power flow problem," *Int. J. Electr. Power Energy Syst.*, vol. 33, no. 3, pp. 745–752, Mar. 2011.

[25] R. Mallipeddi, S. Jeyadevi, P. N. Suganthan, and S. Baskar, "Efficient constraint handling for optimal reactive power dispatch problems," *Swarm Evol. Comput.*, vol. 5, pp. 28–36, Aug. 2012.

[26] A. Saraswat and A. Saini, "Multi-objective optimal reactive power dispatch considering voltage stability in power systems using HFMOEA," *Eng. Appl. Artif. Intell.*, vol. 26, no. 1, pp. 390–404, Jan. 2013.

[27] B. Shaw, V. Mukherjee, and S. P. Ghoshal, "Solution of reactive power dispatch of power systems by an opposition-based gravitational search algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 55, pp. 29–40, Feb. 2014.

[28] M. Ghasemi, S. Ghavidel, M. M. Ghanbarian, and A. Habibi, "A new hybrid algorithm for optimal reactive power dispatch problem with discrete and continuous control variables," *Appl. Soft Comput.*, vol. 22, pp. 126–140, Sep. 2014.

[29] Y. Amrane, M. Boudour, A. A. Ladjici, and A. Elmaouhab, "Optimal VAR control for real power loss minimization using differential evolution algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 66, pp. 262–271, Mar. 2015.

[30] R.-H. Liang, J.-C. Wang, Y.-T. Chen, and W.-T. Tseng, "An enhanced firefly algorithm to multi-objective optimal active/reactive power dispatch with uncertainties consideration," *Int. J. Electr. Power Energy Syst.*, vol. 64, pp. 1088–1097, Jan. 2015.

[31] M. H. Sulaiman, Z. Mustaffa, M. R. Mohamed, and O. Aliman, "Using the gray wolf optimizer for solving optimal reactive power dispatch problem," *Appl. Soft Comput.*, vol. 32, pp. 286–292, Jul. 2015.

[32] M. Ghasemi, M. Taghizadeh, S. Ghavidel, J. Aghaei, and A. Abbasian, "Solving optimal reactive power dispatch problem using a novel teaching–learning-based optimization algorithm," *Eng. Appl. Artif. Intell.*, vol. 39, pp. 100–108, Mar. 2015.

[33] A. Rajan and T. Malakar, "Optimal reactive power dispatch using hybrid Nelder–Mead simplex based firefly algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 66, pp. 9–24, Mar. 2015.

[34] M. Basu, "Quasi-oppositional differential evolution for optimal reactive power dispatch," *Int. J. Electr. Power Energy Syst.*, vol. 78, pp. 29–40, Jun. 2016.

[35] S. M. Mohseni-Bonab, A. Rabiee, B. Mohammadi-Ivatloo, S. Jalilzadeh, and S. Nojavan, "A two-point estimate method for uncertainty modeling in multi-objective optimal reactive power dispatch problem," *Int. J. Electr. Power Energy Syst.*, vol. 75, pp. 194–204, Feb. 2016.

[36] A. Mukherjee and V. Mukherjee, "Chaotic krill herd algorithm for optimal reactive power dispatch considering FACTS devices," *Appl. Soft Comput.*, vol. 44, pp. 163–190, Jul. 2016.

[37] A. Rajan and T. Malakar, "Exchange market algorithm based optimum reactive power dispatch," *Appl. Soft Comput.*, vol. 43, pp. 320–336, Jun. 2016.

[38] A. M. Shaheen, R. A. El-Sehiemy, and S. M. Farrag, "Optimal reactive power dispatch using backtracking search algorithm," *Austral. J. Electr. Electron. Eng.*, vol. 13, no. 3, pp. 200–210, Jul. 2016.

[39] S. Dutta, P. Mukhopadhyay, P. K. Roy, and D. Nandi, "Unified power flow controller based reactive power dispatch using oppositional krill herd algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 80, pp. 10–25, Sep. 2016.

[40] R. Ng Shin Mei, M. H. Sulaiman, Z. Mustaffa, and H. Daniyal, "Optimal reactive power dispatch solution by loss minimization using moth-flame optimization technique," *Appl. Soft Comput.*, vol. 59, pp. 210–222, Oct. 2017.

[41] A. A. Heidari, R. Ali Abbaspour, and A. Rezaee Jordehi, "Gaussian bare-bones water cycle algorithm for optimal reactive power dispatch in electrical power systems," *Appl. Soft Comput.*, vol. 57, pp. 657–671, Aug. 2017.

[42] K. B. O. Medani, S. Sayah, and A. Bekrar, "Whale optimization algorithm based optimal reactive power dispatch: A case study of the Algerian power system," *Electr. Power Syst. Res.*, vol. 163, pp. 696–705, Oct. 2018.

[43] S. Dutta, S. Paul, and P. K. Roy, "Optimal allocation of SVC and TCSC using quasi-oppositional chemical reaction optimization for solving multi-objective ORPD problem," *J. Electr. Syst. Inf. Technol.*, vol. 5, no. 1, pp. 83–98, May 2018.

[44] S. M. Shareef and R. Srinivasa Rao, "Optimal reactive power dispatch under unbalanced conditions using hybrid swarm intelligence," *Comput. Electr. Eng.*, vol. 69, pp. 183–193, Jul. 2018.

[45] S. Abdel-Fatah, M. Ebeed, and S. Kamel, "Optimal reactive power dispatch using modified sine cosine algorithm," in *Proc. Int. Conf. Innov. Trends Comput. Eng. (ITCE)*, Feb. 2019, pp. 510–514.

[46] R. P. Singh, V. Mukherjee, and S. P. Ghoshal, "Optimal reactive power dispatch by particle swarm optimization with an aging leader and challengers," *Appl. Soft Comput.*, vol. 29, pp. 298–309, Apr. 2015.

[47] S. Abdel-Fatah, M. Ebeed, S. Kamel, and L. Nasrat, "Moth swarm algorithm for reactive power dispatch considering stochastic nature of renewable energy generation and load," in *Proc. 21st Int. Middle East Power Syst. Conf. (MEPCON)*, Dec. 2019, pp. 594–599.

[48] S. Abdel-Fatah, M. Ebeed, S. Kamel, and J. Yu, "Reactive power dispatch solution with optimal installation of renewable energy resources considering uncertainties," in *Proc. IEEE Conf. Power Electron. Renew. Energy (CPERE)*, Oct. 2019, pp. 118–123.

[49] S. Kamel, S. Abdel-Fatah, M. Ebeed, J. Yu, K. Xie, and C. Zhao, "Solving optimal reactive power dispatch problem considering load uncertainty," in *Proc. IEEE Innov. Smart Grid Technol. Asia (ISGT Asia)*, May 2019, pp. 1335–1340.

[50] M. Ettappan, V. Vimala, S. Ramesh, and V. T. Kesavan, "Optimal reactive power dispatch for real power loss minimization and voltage stability enhancement using artificial bee colony algorithm," *Microprocessors Microsyst.*, vol. 76, Jul. 2020, Art. no. 103085.

[51] S. Mugemanyi, Z. Qu, F. X. Rugema, Y. Dong, C. Bananeza, and L. Wang, "Optimal reactive power dispatch using chaotic bat algorithm," *IEEE Access*, vol. 8, pp. 65830–65867, 2020.

[52] E. J. Solteiro Pires, J. A. Tenreiro Machado, P. B. de Moura Oliveira, J. B. Cunha, and L. Mendes, "Particle swarm optimization with fractional-order velocity," *Nonlinear Dyn.*, vol. 61, nos. 1–2, pp. 295–301, Jul. 2010.

[53] M. S. Couceiro, R. P. Rocha, N. M. F. Ferreira, and J. A. T. Machado, "Introducing the fractional-order darwinian PSO," *Signal, Image Video Process.*, vol. 6, no. 3, pp. 343–350, Sep. 2012.

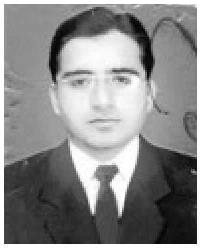
[54] P. Ghamisi, M. S. Couceiro, and J. A. Benediktsson, "A novel feature selection approach based on FODPSO and SVM," *IEEE Trans. Geosci. Remote Sens.*, vol. 53, no. 5, pp. 2935–2947, May 2015.

[55] A. Ates, B. B. Alagoz, G. Kavuran, and C. Yeroglu, "Implementation of fractional order filters discretized by modified fractional order darwinian particle swarm optimization," *Measurement*, vol. 107, pp. 153–164, Sep. 2017.

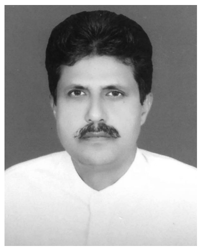
[56] E. S. Alaviyan Shahri, A. Alfi, and J. A. T. Machado, "Fractional fixed-structure H_∞ controller design using augmented lagrangian particle swarm optimization with fractional order velocity," *Appl. Soft Comput.*, vol. 77, pp. 688–695, Apr. 2019.

[57] K. Taş, J. A. T. Machado, and D. Baleanu, *Mathematical Methods in Engineering*. Springer, 2007.

[58] P. Ghamisi, M. S. Couceiro, F. M. L. Martins, and J. A. Benediktsson, "Multilevel image segmentation based on fractional-order darwinian particle swarm optimization," *IEEE Trans. Geosci. Remote Sens.*, vol. 52, no. 5, pp. 2382–2394, May 2014.
[59] A. Ates, G. Kavuran, B. B. Alagoz, and C. Yeroglu, "Improvement of IIR filter discretization for fractional order filter by discrete stochastic optimization," in *Proc. 39th Int. Conf. Telecommun. Signal Process. (TSP)*, Jun. 2016, pp. 583–586.
[60] F. Guo, H. Peng, B. Zou, R. Zhao, and X. Liu, "Localisation and segmentation of optic disc with the fractional-order darwinian particle swarm optimisation algorithm," *IET Image Process.*, vol. 12, no. 8, pp. 1303–1312, Aug. 2018.
[61] Q. Zhu, M. Yuan, Y. Chen, Y. Liu, and W. Chen, "Research and application on fractional-order darwinian PSO based adaptive extended Kalman filtering algorithm," *IAES Int. J. Robot. Autom.*, vol. 3, no. 4, p. 245, Dec. 2014.
[62] N. Yokoya and P. Ghamisi, "Land-cover monitoring using time-series hyperspectral data via fractional-order darwinian particle swarm optimization segmentation," in *Proc. 8th Workshop Hyperspectral Image Signal Process., Evol. Remote Sens. (WHISPERS)*, Aug. 2016, pp. 1–5.
[63] Y.-Y. Wang, H. Zhang, C.-H. Qiu, and S.-R. Xia, "A novel feature selection method based on extreme learning machine and fractional-order darwinian PSO," *Comput. Intell. Neurosci.*, vol. 2018, pp. 1–8, May 2018.
[64] K. K. Paliwal, S. Singh, and P. Gaba, "Feature selection approach of hyperspectral image using GSA-FODPSO-SVM," in *Proc. Int. Conf. Comput., Commun. Autom. (ICCCA)*, May 2017, pp. 1070–1075.
[65] P. Ghamisi, M. S. Couceiro, and J. A. Benediktsson, "Classification of hyperspectral images with binary fractional order darwinian PSO and random forests," in *Proc. 21st Image Signal Process. Remote Sens.*, Oct. 2013, Art. no. 88920S.
[66] S. Akbar, F. Zaman, M. Asif, A. U. Rehman, and M. A. Z. Raja, "Novel application of FO-DPSO for 2-D parameter estimation of electromagnetic plane waves," *Neural Comput. Appl.*, vol. 31, no. 8, pp. 3681–3690, Aug. 2019.
[67] A. Zameer, M. Muneeb, S. M. Mirza, and M. A. Z. Raja, "Fractional-order particle swarm based multi-objective PWR core loading pattern optimization," *Ann. Nucl. Energy*, vol. 135, Jan. 2020, Art. no. 106982.
[68] S. Lodhi, M. A. Manzar, and M. A. Z. Raja, "Fractional neural network models for nonlinear Riccati systems," *Neural Comput. Appl.*, vol. 31, no. S1, pp. 359–378, Jan. 2019.
[69] A. A. A. E. Ela, M. A. Abido, and S. R. Spea, "Differential evolution algorithm for optimal reactive power dispatch," *Electr. Power Syst. Res.*, vol. 81, no. 2, pp. 458–464, Feb. 2011.
[70] A. H. Khazali and M. Kalantar, "Optimal reactive power dispatch based on harmony search algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 33, no. 3, pp. 684–692, Mar. 2011.
[71] A. McBride, "Advances in fractional calculus: Theoretical developments and applications in physics and engineering," Tech. Rep., 2008.
[72] M. D. Ortigueira and J. A. Tenreiro Machado, "What is a fractional derivative?" *J. Comput. Phys.*, vol. 293, pp. 4–13, Jul. 2015.
[73] E. C. De Oliveira and J. A. T. Machado, "A review of definition for fractional derivatives ans integral, math," *Probl. Eng*, 2014.
[74] M. Davison and C. Essex, "Fractional differential equations and initial value problems," *Math. Scientist*, vol. 23, no. 2, pp. 108–116, 1998.
[75] B. Mandal and P. K. Roy, "Optimal reactive power dispatch using quasi-oppositional teaching learning based optimization," *Int. J. Electr. Power Energy Syst.*, vol. 53, pp. 123–134, Dec. 2013.
[76] S. Raj and B. Bhattacharyya, "Optimal placement of TCSC and SVC for reactive power planning using whale optimization algorithm," *Swarm Evol. Comput.*, vol. 40, pp. 131–143, Jun. 2018.
[77] A. Selim, S. Kamel, A. S. Alghamdi, and F. Jurado, "Optimal placement of DGs in distribution system using an improved harris hawks optimizer based on single- and multi-objective approaches," *IEEE Access*, vol. 8, pp. 52815–52829, 2020.
[78] A. Korashy, S. Kamel, T. Alquthami, and F. Jurado, "Optimal coordination of standard and non-standard direction overcurrent relays using an improved moth-flame optimization," *IEEE Access*, vol. 8, pp. 87378–87392, 2020.
[79] A. S. Menesy, H. M. Sultan, A. Korashy, F. A. Banakhr, M. G. Ashmawy, and S. Kamel, "Effective parameter extraction of different polymer electrolyte membrane fuel cell stack models using a modified artificial ecosystem optimization algorithm," *IEEE Access*, vol. 8, pp. 31892–31909, 2020.



YASIR MUHAMMAD received the M.Sc. degree in electrical engineering from UET Peshawar, Pakistan, in 2014. He is currently pursuing the Ph.D. degree with COMSATS University Islamabad, Wah Campus, Pakistan. He worked and collaborated on various projects funded by the Higher Education Commission and the Ministry of Defense, Pakistan. Before joining COMSATS University, he was a Lecturer with the Swedish College of Engineering and Technology, Wah Cantt, Pakistan. He was with the Energy and Power Department, KPK, Pakistan. He is a Lecturer with the Department of Electrical and Computer Engineering, COMSATS University Islamabad, Attock, Pakistan. His research interests include power systems, flexible AC transmission systems, economic dispatch, and optimization techniques.



RAHIMDAD KHAN received the M.Phil. degree in solid state physics from Punjab University, Pakistan, in 1988, and the Ph.D. degree from Shanghai University, China, in 1993. He has nine years pre Ph.D. and 22 years post Ph.D. experience of teaching and research. He is currently a Professor with the Department of Electrical and Computer Engineering, COMSATS University Islamabad, Wah Campus, Pakistan. His research interests include power systems, economic load dispatch, and optimization techniques. He is a member of the Pakistan Institute of Physics (PIP) and the Pakistan Physical Society (PPS).

MUHAMMAD ASIF ZAHOOR RAJA was born in Rawalpindi, Pakistan, in 1973. He received the M.Sc. degree in mathematics from the Forman Christen College, Lahore, Pakistan, in 1996, the M.Sc. degree in nuclear engineering from Quaid-e-Azam University, Islamabad, Pakistan, in 1999, and the Ph.D. degree in electronic engineering from International Islamic University, Islamabad, in 2011. He is currently an Assistant Professor with the Department of Electrical Engineering, COMSATS University Islamabad, Attock Campus, Attock, Pakistan. He is involved in research and development assignment of Engineering and Scientific Commission of Pakistan, from 1999 to 2012. He has developed the Fractional Least Mean Square Algorithm and Computational Platform formulated for the first time for solving fractional differential equation using artificial intelligence techniques during his Ph.D. studies. He has been an author of more than 140 publications, out of which more than 150 are reputed journal publications with impact factor more than 450. He was a resource person and gives invited talks on many workshops and conferences held at the national level. His research interests include solving linear and nonlinear differential equation of arbitrary order, active noise control systems, fractional adaptive signal processing, nonlinear system identification, direction of arrival estimation, and bioinformatics problem.

FARMAN ULLAH received the M.S. degree in computer engineering from CASE Islamabad, Pakistan, in 2010, and the Ph.D. degree from Korea Aerospace University, South Korea, in 2016. He worked and collaborated on various projects funded by the Ministry of Economy, the Korea Research Foundation, and ETRI, South Korea. He joined COMSATS, as an Assistant Manager of telemetry, AERO, Pakistan. He is currently an Assistant Professor with the Department of Electrical and Computer Engineering, COMSATS University Islamabad, Attock Campus, Pakistan. He has authored/coauthored more than 40 peer-reviewed publications. His research interests include complex networks, graph theory, optimization techniques, machine learning, the IoT, and WoO/WoTs.

NAVEED ISHTIAQ CHAUDHARY (Member, IEEE) was born in Rawalpindi, Pakistan, in 1988. He received the Ph.D. degree in electronic engineering from International Islamic University (IIUI), Islamabad, Pakistan, in 2018. He is currently an Assistant Professor with IIUI. He has published more than 40 research articles in highly reputed international journals. His recent research interests include machine learning, nonlinear system identification, fractional signal processing, and parameter estimation.

YIGANG HE received the M.Sc. degree in electrical engineering from Hunan University, Changsha, China, in 1992, and the Ph.D. degree in electrical engineering from Xian Jiaotong University, Xi'an, China, in 1996. He is currently the Vice-Head of the School of Electrical Engineering and Automation, Wuhan University, China. He has been an author of more than 300 publications and chapters in edited books. His teaching and research interests include power electronic circuit theory and its applications, testing and fault diagnosis of analog and mixed-signal circuits, electrical signal detection, smart grid, satellite communication monitoring, and intelligent signal processing. He was a recipient of a number of national and international awards, prizes, and honors. He received the national Outstanding Youth Science Fund and the China National Excellent Science and Technology Worker.