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Rational Bi-Quartic Spline With Six Parameters for Surface Interpolation With Application in Image Enlargement

SAMSUL ARIFFIN ABDUL KARIM
Fundamental and Applied Sciences Department, Universiti Teknologi PETRONAS, Seri Iskandar 32610, Malaysia
Centre for Smart Grid Energy Research (CSMER), Institute of Autonomous System, Universiti Teknologi PETRONAS, Seri Iskandar 32610, Malaysia
e-mail: samsul_ariffin@utp.edu.my

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ABSTRACT A new rational bi-quartic spline interpolation scheme is constructed in this study. This rational bi-quartic spline has six free parameters that can be used to refine the final interpolating surface. The degree of smoothness attained is C^1 continuity. Image interpolation is one of the methods in image processing that is used to rescale the images, either downscaling or upscaling. The proposed rational bi-quartic spline will be used to interpolate the missing pixel in image interpolation. We employ an algorithm that will speed up the process as well as be able to produce the quality upscaling images. We measure the quality of the proposed method by calculating peak-signal-to-noise-ratio (PSNR). Based on numerical comparison, the proposed scheme gives higher PSNR (consequently smaller RMSE) unlike some existing schemes. Thus, the new rational bi-quartic spline with six parameters is better than the existing scheme via an efficient algorithm. All numerical simulations and graphical results are presented using MATLAB.

INDEX TERMS Image interpolation, parameters, algorithm, bi-quartic spline, image quality.

I. INTRODUCTION

Surface interpolation is important in many aspects of sciences and engineering. The standard approach in surface interpolation is scattered data interpolation and constraint surface modeling on rectangular and triangular meshes. Draman *et al.* [5] have constructed a local method for scattered data interpolating using rational quartic triangular patches with three free parameters. Karim *et al.* [12] have discussed the surface constrained interpolation using partially blended rational bi-cubic spline defined on rectangular meshes. Besides that, image interpolation is a method that used to interpolate missing pixel during image enlargement or image zooming. Usually users are interested in downscaling or upscaling the images. This includes shrinking the images, zooming the images, images morphing and many more. The common method used in image interpolation is bicubic spline interpolation [4], [13], [18]. This method is well documented in MATLAB. However, bicubic spline has no free parameters for shape modification. As an alternative to the bicubic spline interpolation, many researchers have proposed various types of spline interpolation. For instance, from rational cubic spline, rational quartic spline to rational trigonometric Bezier splines. They all have the same characteristics i.e. to interpolate image by manipulating free parameters that exist in the description of the rational spline. They use tensor product to define the surface interpolation. One of the main objectives of image interpolation is to obtain a better image resolution when we are dealing the grayscale and RGB images with noise or artifact [4], [13], [19]. Abbas *et al.* [1] have constructed rational bi-cubic Ball interpolation with two parameters. They have extended the rational cubic Ball [3]. They use the surface for image interpolation. The optimal value of free parameters is obtained via genetic algorithm (GA). Abbas *et al.* [2] have used the same approach as in [1] but using rational bi-cubic spline interpolation (cubic/cubic) with two free parameters. So far there is no numerical comparison between [1] and [2] for image interpolation. Gao *et al.* [6], [7] have discussed various types of image interpolation using rational cubic spline (cubic/linear). Meanwhile, Yao *et al.* [20] and Zhang *et al.* [21], [22] also have constructed various types

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of rational cubic spline interpolation for image interpolation. However, their schemes also have no free parameters. Furthermore, their schemes are applicable if and only if the first partial derivatives are supplied. This is not possible since for real data or real images, the derivatives will not be given. Hussain *et al.* [10] have constructed quadratic trigonometric spline for image interpolation. Similar with [1] and [2], Hussain *et al.* [10] also utilized GA to find the optimal value of the free parameters. However, it is very hard to judge the advantages of their method. This is because, they only tested the scheme to three images. Karim and Saaban [11] have constructed a new rational bi-cubic Ball interpolation (cubic/quadratic) with two parameters. From numerical results, they found that, their scheme is better than bicubic spline interpolation. Zulkifli *et al.* [23] have constructed rational bi-cubic Ball interpolation (cubic/quadratic) with six parameters for image interpolation. From their results, it was found that, the proposed scheme is the best compared with some existing schemes. However, in this study, we found that their proposed scheme is no longer the best anymore. Majeed *et al.* [15], [16] have discussed image reconstruction using rational cubic Ball (cubic/cubic). The main difference approach in Majeed *et al.* [15], [16] and Abbas *et al.* [1], [2], Karim and Saaban [11] and Zulkifli *et al.* [23] is that they treated the images as curves on each corner on rectangular meshes. Meanwhile in [1], [2], [11], [23] they all have used tensor product approach. Image fusion also important in image interpolation. Wang and Tan [18] have utilized rational quartic spline with two parameters for image fusion. However, their method required the first partial derivatives to be modified if the interpolating curve is not monotonic. Therefore, Wang and Tan [18] cannot be used if the interpolation problem requires that the first derivatives must be interpolated. This happen for C^1 and C^2 continuities.

Another approach for image interpolation is applying different techniques such as wavelet transform and Fourier transform. Since in wavelet transform, there are two operators i.e. downscaling and upscaling. This concept is the same as image interpolation. Lakshman *et al.* [14] have discussed image interpolation using shearlet basis. Usually wavelets are used to cater the sharp edges as well as more complex geometries. However, the choice of the wavelet basis still an unsettle problems. Usually, researchers will do simulations by using different choices of wavelet basis functions. This is very tedious process since for single wavelets, for instance, Daubechies wavelets, we may have many wavelets with different filters length etc. This is the common practices in wavelets transform. But the main question is how to choose the best wavelets basis that will produce the best PSNR value?

As conjectured by Zulkifli *et al.* [23], scheme with more free parameters gives the better quality of image interpolation, in this study, we will propose a new rational bi-quartic spline interpolation (quartic/quadratic) with six free parameters. This function is defined on rectangular meshes. It is applicable for functional data i.e. scalar data sets. We believe that, the interpolating images have more quality than lower degree rational interpolant as they have more degree in numerator. The final rational interpolant has twelve degree i.e. octic numerator and quartic denominator. The main contribution of the present study is summarized below:

(a) In Zulkifli *et al.* [23], rational bi-cubic Ball interpolation has been used for image interpolation. In this study, we employ rational bi-quartic spline interpolation with six parameters. Since our interpolant has more degree, the resulting image interpolation is better than Zulkifli *et al.* [23].
(b) Our proposed algorithm is different than Zulkifli *et al.* [23] and Karim and Saaban [11].
(c) There are no free parameters to control the final image interpolation in the works of Gao *et al.* [6], [7], Saaban *et al.* [17], Yao *et al.* [20], Zhang *et al.* [21] and [22].
(d) The optimal parameters are obtained via numerical simulations and local control properties. Meanwhile Yao *et al.* [20], Abbas *et al.* [1], [2] and Hussain *et al.* [10] have used genetic algorithm (GA) to find the optimal value of the parameters. However, their methods are same for all papers except they used different types of spline.
(e) The proposed scheme does not require first derivative modification unlike Wang and Tan [18] method.
(f) In order to find the optimum PSNR value for each tested image, we only need to manipulate parameters values, unlike wavelets transform, in which we need to simulate various wavelets basis functions with different filter length [14]. Therefore, our proposed scheme is easier to be implemented than wavelets.
(g) In Zou *et al.* [24] the bivariate Thiele-Like rational interpolation continued fractions is used for image interpolation. Meanwhile in Tang and Zhu [25] a new version of C^1-continuous Coons patches over triangular domain is constructed for the application in image interpolation. Both schemes are not easy to be implemented compared with the proposed scheme which is direct and easier to be implemented.

The remainder of this paper is structured as follows. Section 2 deals with construction of rational bi-quartic spline interpolation with six parameters defined on rectangular meshes. An application in surface interpolation is discussed with several numerical examples. Later, this rational bi-quartic spline will be used for image interpolation via an efficient algorithm. Section 3 is devoted for this purpose. Results and discussion are given in Section 4. A Conclusion is given in the final section. Below we provide some lists of abbreviation used in this study.

List of Abbreviation:

CPU	*Central Processing Unit*
R^2	*Coefficient of Determination*
MSE	*Mean Square Error*

PSNR	*Peak-Signal-to-Noise Ratio*
RGB	*Red, Green or Blue channels*
α	*Alpha*
β	*Beta*
γ	*Gamma*
3D	*Three-Dimensional*
GA	*Genetic Algorithm*
A^T	*Transpose of matrix A*
m, n	Image pixels
$z_{i,j}$	Input pixel in grayscale intensity
$\bar{z}_{i,j}$	Interpolating missing pixels

II. RATIONAL BI-QUARTIC SPLINE INTERPOLATION

Let the rational bi-quartic functions is arranged over each rectangular patch $[x_i, x_{i+1}] \times [y_j, y_{j+1}]$, where $i = 0, 1, \ldots, n-1$ and $j = 0, 1, \ldots, m-1$, with the first partial derivative at the starting point, on x-direction, y-direction (see Figure 1), and the partial derivatives respectively denoted as $F_{i,j}$, $F^x_{i,j}$, $F^y_{i,j}$ and $F^{xy}_{i,j}$. Let, $h_i = x_{i+1} - x_i$, $\hat{h}_j = y_{j+1} - y_j$, and $\theta = \frac{x - x_i}{h_i}$ and $\emptyset = \frac{y - y_j}{\hat{h}_j}$, this resulting the local parameter $0 \leq \theta \leq 1$ and $0 \leq \emptyset \leq 1$. Thus, the rational bi-quartic spline with six parameters $\alpha_{i,j}, \beta_{i,j}, \gamma_{i,j}, \hat{\alpha}_{i,j}, \hat{\beta}_{i,j}, \hat{\gamma}_{i,j} > 0$, $i = 0, \ldots, n-1$ and $j = 0, \ldots, m-1$, is defined as below:

$$S(x, y) = S_{i,j}(x, y) = A_i(\theta) H_{i,j} A_j(\emptyset)^T \tag{1}$$

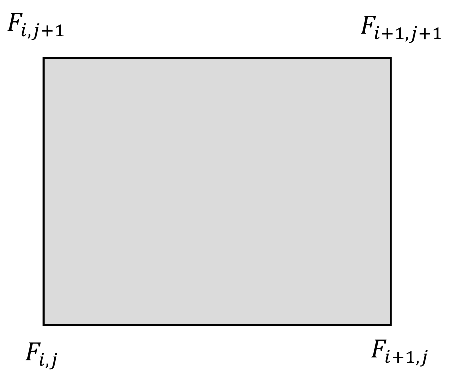


FIGURE 1. Rational bi-quartic spline defined on rectangular.

with

$$H_{i,j} = \begin{bmatrix} F_{i,j} & F_{i,j+1} & F^y_{i,j} & F^y_{i,j+1} \\ F_{i+1,j} & F_{i+1,j+1} & F^y_{i+1,j} & F^y_{i+1,j+1} \\ F^x_{i,j} & F^x_{i,j+1} & F^{xy}_{i,j} & F^{xy}_{i,j+1} \\ F^x_{i+1,j} & F^x_{i+1,j+1} & F^{xy}_{i+1,j} & F^{xy}_{i+1,j+1} \end{bmatrix}$$

$$A_i(\theta) = \begin{bmatrix} a_0(\theta) & a_1(\theta) & a_2(\theta) & a_3(\theta) \end{bmatrix}$$

$$A_j(\emptyset) = \begin{bmatrix} \hat{a}_0(\emptyset) & \hat{a}_1(\emptyset) & \hat{a}_2(\emptyset) & \hat{a}_3(\emptyset) \end{bmatrix}$$

where,

$$a_0(\theta) = \frac{\begin{bmatrix} \alpha_{i,j}(1-\theta)^4 + (2\alpha_{i,j} + \gamma_{i,j})\theta(1-\theta)^3 \\ + (\beta_{i,j} + \gamma_{i,j})\theta^2(1-\theta)^2 \end{bmatrix}}{q_i(\theta)}$$

$$a_1(\theta) = \frac{\begin{bmatrix} \beta_{i,j}\theta^4 + (2\beta_{i,j} + \gamma_{i,j})(1-\theta)\theta^3 \\ + (\alpha_{i,j} + \gamma_{i,j})\theta^2(1-\theta)^2 \end{bmatrix}}{q_i(\theta)}$$

$$a_2(\theta) = \frac{\alpha_{i,j} h_i (1-\theta)^3 \theta}{q_i(\theta)}$$

$$a_3(\theta) = \frac{-\beta_{i,j} h_i (1-\theta)\theta^3}{q_i(\theta)}$$

$$q_i(\theta) = \alpha_{i,j}(1-\theta)^2 + \gamma_{i,j}(1-\theta)\theta + \beta_{i,j}\theta^2$$

$$\hat{a}_0(\emptyset) = \frac{\begin{bmatrix} \hat{\alpha}_{i,j}(1-\emptyset)^4 + (2\hat{\alpha}_{i,j} + \hat{\gamma}_{i,j})\emptyset(1-\emptyset)^3 \\ + (\hat{\beta}_{i,j} + \hat{\gamma}_{i,j})\emptyset^2(1-\emptyset)^2 \end{bmatrix}}{q_j(\emptyset)}$$

$$\hat{a}_1(\emptyset) = \frac{\begin{bmatrix} \hat{\beta}_{i,j}\emptyset^4 + (2\hat{\beta}_{i,j} + \hat{\gamma}_{i,j})(1-\emptyset)\emptyset^3 \\ + (\hat{\alpha}_{i,j} + \hat{\gamma}_{i,j})(1-\emptyset)^2\emptyset^2 \end{bmatrix}}{q_j(\emptyset)}$$

$$\hat{a}_2(\emptyset) = \frac{\hat{\alpha}_{i,j}\hat{h}_j(1-\emptyset)^3\emptyset}{q_j(\theta)}$$

$$\hat{a}_3(\emptyset) = \frac{-\hat{\beta}_{i,j}\hat{h}_j(1-\emptyset)\emptyset^3}{q_j(\emptyset)}$$

$$q_j(\emptyset) = \hat{\alpha}_{i,j}(1-\emptyset)^2 + \hat{\gamma}_{i,j}(1-\emptyset)\emptyset + \hat{\beta}_{i,j}\emptyset^2$$

The proposed rational bi-quartic spline satisfies the following properties:

$$S(x_i, y_j) = F_{i,j}, \quad \frac{\partial S}{\partial x}(x_i, y_j) = F^x_{i,j},$$
$$\frac{\partial S}{\partial y}(x_i, y_j) = F^y_{i,j} \quad \text{and,} \quad \frac{\partial^2 S}{\partial x \partial y}(x_i, y_j) = F^{xy}_{i,j}.$$

This shows that the rational bi-quartic spline defined by Eq. (1) has C^1 continuity i.e. first-order parametric continuity

Some observations can be made as follows:

(a) The proposed scheme is symmetric i.e. $S_{i,j}(x, y) = S_{j,i}(y, x)$.

(b) When the parameters $\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = 2$, the proposed rational bi-quartic spline interpolation is reduced to standard bi-quartic polynomial interpolation.

(c) $a_0(\theta) + a_1(\theta) = 1$ and $\hat{a}_0(\emptyset) + \hat{a}_1(\emptyset) = 1$. This is partial partition of unity properties for the rational bi-quartic spline.

(d) For all $\alpha_{i,j}, \beta_{i,j}, \gamma_{i,j}, \hat{\alpha}_{i,j}, \hat{\beta}_{i,j}, \hat{\gamma}_{i,j} > 0$, $i = 0, \ldots, n-1$ and $j = 0, \ldots, m-1$, $a_0(\theta) \geq 0$, $a_1(\theta) \geq 0$, $a_2(\theta) \geq 0$. Similarly, $\hat{a}_0(\emptyset) \geq 0$, $\hat{a}_1(\emptyset) \geq 0$, $\hat{a}_2(\emptyset) \geq 0$.

(e) When $\gamma_{i,j} \to \infty$ or $\alpha_{i,j}, \beta_{i,j} \to 0$, the rational bi-quartic spline defined by (1) are reduced to bilinear surface i.e. flat surface since all four curves network are reducing to straight line (See Harim *et al.* [8], [9]). This is called as tension effect.

The first partial derivatives value $F^x_{i,j}$, $F^y_{i,j}$ and $F^{xy}_{i,j}$ can be estimated by using arithmetic mean method (AMM) that has been described in [10], [11].

Now, we show the application of the proposed rational bi-quartic spline to interpolate surface data. We choose two data sets. For the first example, we study the local control properties of the proposed rational b-quartic spline.

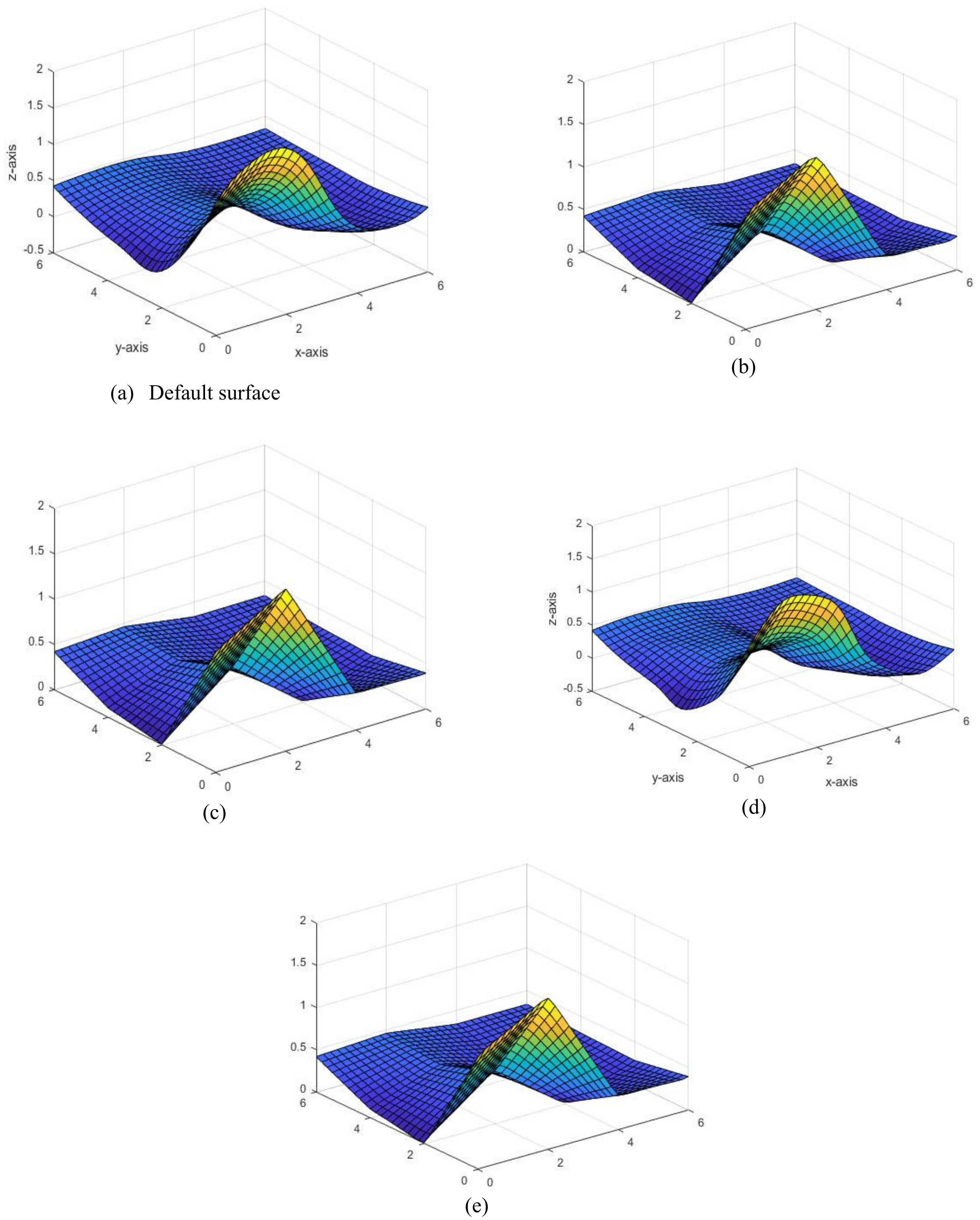
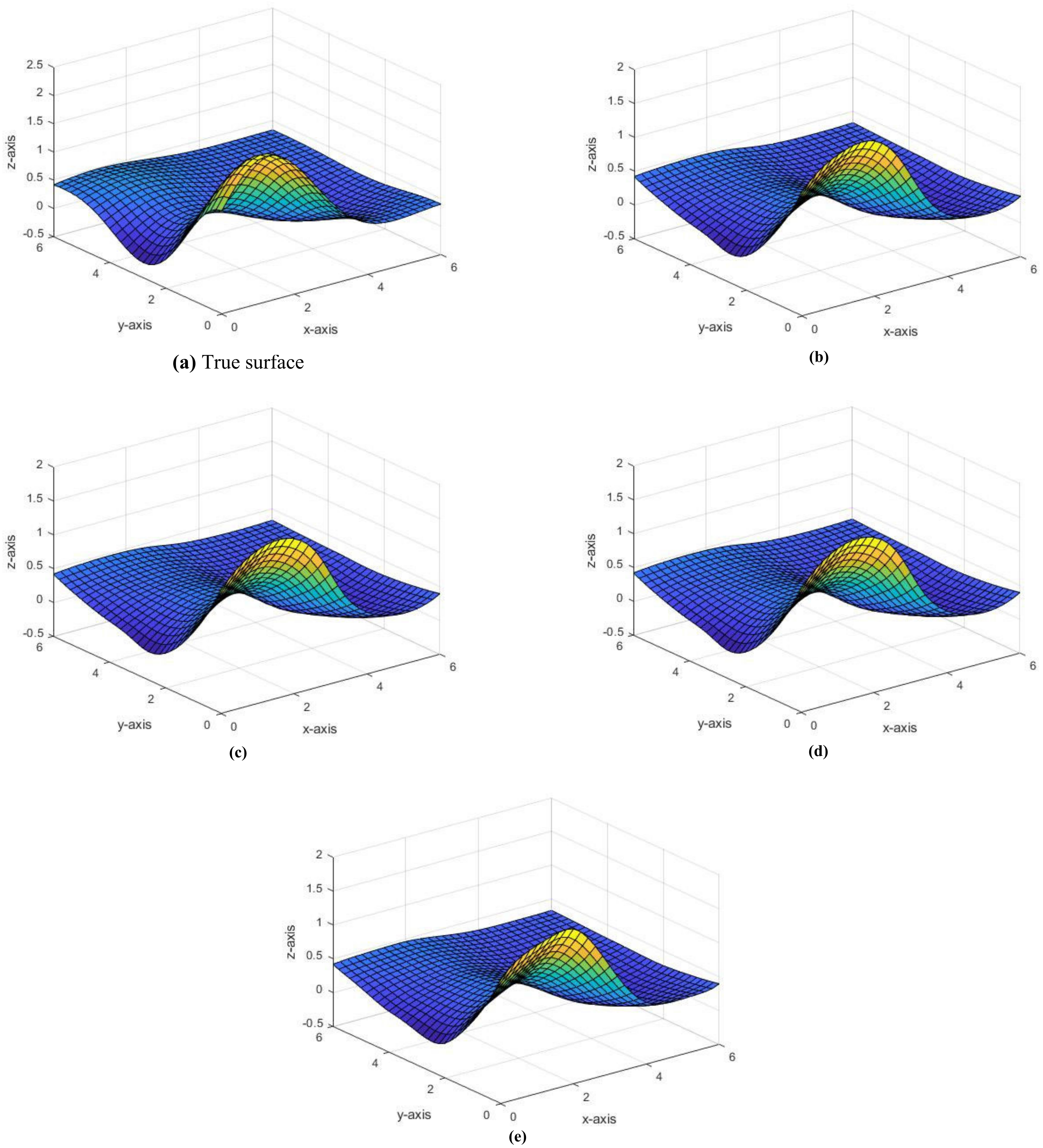


FIGURE 2. **Local control properties.**

Example 1: A data from the following function is truncated to five decimal places.

$$F_1(x, y) = e^{-(x^2+y^2)/15}(sin(x) + cos(y)) + 0.33, \quad 0 \leq x, y \leq 6$$

Figure 2 shows the local control properties of the rational bi-quartic spline defined in (1). Figure 2(a) shows the default surface i.e. standard quartic polynomial when parameters $\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = 2$. Figure 2(b) shows the surfaces tend to be flat i.e. bilinear when $\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = 10$. Figure 2(c) shows the interpolating surface when $\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = 100$. Figure 2(e) shows the flat surface when



(a) True surface

(b)

(c)

(d)

(e)

FIGURE 3. Various interpolating surfaces for Example 1.

$\alpha_{i,j} = \beta_{i,j} = 0.02 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = 1$. Finally Figure 2(d) shows the surface when $\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = -0.5$.

Since we have six free parameters for shape modification, it is interesting so see the capability of the proposed scheme for data interpolation. Figure 3 shows the examples of various interpolating surfaces by varying free parameters.

Figure 3(a) shows the true surface. Figure 3(b) shows the interpolating surface when parameters are $\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = 0.5$. Meanwhile Figure 3(c) shows the surface when $\alpha_{i,j} = \beta_{i,j} = 2.5 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and

TABLE 1. Data from function $F_1(x, y)$.

y/x	0	2	4	6
0	1.33000	0.011261	0.10505	0.41710
2	1.79240	0.619300	0.39739	0.45990
4	0.41370	0.020814	0.16294	0.33635
6	0.39537	0.281670	0.30087	0.33560

TABLE 2. Data from function $F_3(x, y)$.

y/x	-3	-2	-1	0	1	2	3
-3	0.040	0.040	0.175	1.040	0.175	0.040	0.040
-2	0.058	0.058	0.193	1.058	0.193	0.058	0.058
-1	0.407	0.408	0.543	1.407	0.543	0.408	0.407
0	1.040	1.040	1.175	2.040	1.175	1.040	1.040
1	0.407	0.408	0.543	1.407	0.543	0.408	0.407
2	0.058	0.058	0.193	1.058	0.193	0.058	0.058
3	0.040	0.040	0.175	1.040	0.175	0.040	0.040

$\gamma_{i,j} = \hat{\gamma}_{i,j} = 0.5$. In Figure 3(d) the surface is obtained when parameters $\alpha_{i,j} = \beta_{i,j} = 4 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = 0.5$. Finally Figure 3(e) shows the interpolating surface by using Wang and Tan [18] scheme.

Example 2: A data from the following function is truncated to three decimal places.

$$F_2(x, y) = e^{-x^2} + e^{-2y^2} + 0.04, \quad -3 \leq x, y \leq 3$$

Figure 4 shows the surface produced using the proposed rational bi-quartic spline interpolation. The true surface is shown in Figure 4(a). Figure 4(b) is bi-quartic polynomial spline surface interpolation when the parameters are $\boldsymbol{\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}}$, and $\boldsymbol{\gamma_{i,j} = \hat{\gamma}_{i,j} = 2}$. Meanwhile, Figure 4(c) is obtained when $\boldsymbol{\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}}$, and $\boldsymbol{\gamma_{i,j} = \hat{\gamma}_{i,j} = 0.5}$. Figure 4(d) shows the interpolating surface when $\boldsymbol{\alpha_{i,j} = \beta_{i,j} = 2.5 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}}$, and $\boldsymbol{\gamma_{i,j} = \hat{\gamma}_{i,j} = 0.5}$. Finally, Wang and Tan [18] scheme is shown in Figure 4(e).

In order to measure the quality of the surface interpolation, we calculate the root mean square error (RMSE) and coefficient of determination (R^2). Table 3 provides all the values for Figures 3 and 4, respectively.

From Table 3, we can see that, the proposed scheme is the best for both examples. Wang and Tan [18] scheme has higher error to interpolate both data sets. Although Wang and Tan [18] and our study have similar quartic numerator however Wang and Tan [18] has linear denominator while our proposed scheme has quadratic denominator. In addition, our scheme is better than [18] because we have more free parameters to control the final interpolating surfaces.

TABLE 3. Error measurements.

Figures	Error Measurements	
	RMSE	R^2
3(b)	0.1431	0.8839
3(c)	0.1432	0.8839
3(d)	0.1432	0.8838
3(e)	0.1487	0.8747
4(b)	0.0368	0.9941
4(c)	0.0322	0.9955
4(d)	0.0322	0.9955
4(e)	0.0381	0.9937

We also can reduce the error of the interpolating surface using smaller step sizes on both directions. Table 4 shows the RMSE and R^2 for Example 2 when $\boldsymbol{\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}}$, and $\boldsymbol{\gamma_{i,j} = \hat{\gamma}_{i,j} = 0.5}$. As the step sizes are reduced, we can see that, the RMSE is approaching to zero and R^2 is equal to 1. This shows that, the proposed scheme is highly accurate to interpolate surface data. However, computation time will be increased to reconstruct the final interpolating surfaces when step sizes are smaller.

TABLE 4. Error measurements for different step sizes.

Step size	Error Measurements	
	RMSE	R^2
$\boldsymbol{h_i = \hat{h}_j = 2}$	0.1432	0.8839
$\boldsymbol{h_i = \hat{h}_j = 1}$	0.0196	0.9978
$\boldsymbol{h_i = \hat{h}_j = 0.5}$	3.0285e-04	1.000
$\boldsymbol{h_i = \hat{h}_j = 0.2}$	2.499e-17	1.000

According to Renka and Brown [26], the coefficient of determination is interpreted as follows: $R^2 = 0.9$ is considered a fair fit; $R^2 = 0.95$ indicates a good fit; $R^2 = 0.99$ is very good; $R^2 = 0.999$ is excellent; and $R^2 = 0.9999$ implies essentially no error in empirical data. Based on this, when $\boldsymbol{h_i = \hat{h}_j = 0.5}$ or $\boldsymbol{h_i = \hat{h}_j = 0.2}$, the proposed scheme is reproducing the surface exactly i.e. no error in the interpolation.

III. APPLICATION IN IMAGE PROCESSING

In this section, an efficient algorithm to apply the proposed image interpolation scheme for grayscale image upscaling is discussed in more detail. We begin with some definition on image processing techniques. The algorithm is comprising of several steps which are elaborated in a flowchart, as shown in Algorithm 1.

A. IMAGE INTERPOLATION

Image interpolation is a method that can be used to enlarge images (Kok and Tam [13]). Basically, image interpolation is used to interpolate missing data when image enlargement took places. The common methods i.e. bilinear and nearest neighbor interpolations may result some overshoot and oscillation around pixels, which will degrade the quality of image interpolation. To reduce the overshoot and oscillation artifacts in image interpolation, piecewise continuous interpolation such as cubic spline interpolation is commonly used [4], [13], [19]. Until today, bicubic spline interpolation still one of the best methods for image interpolation [13].

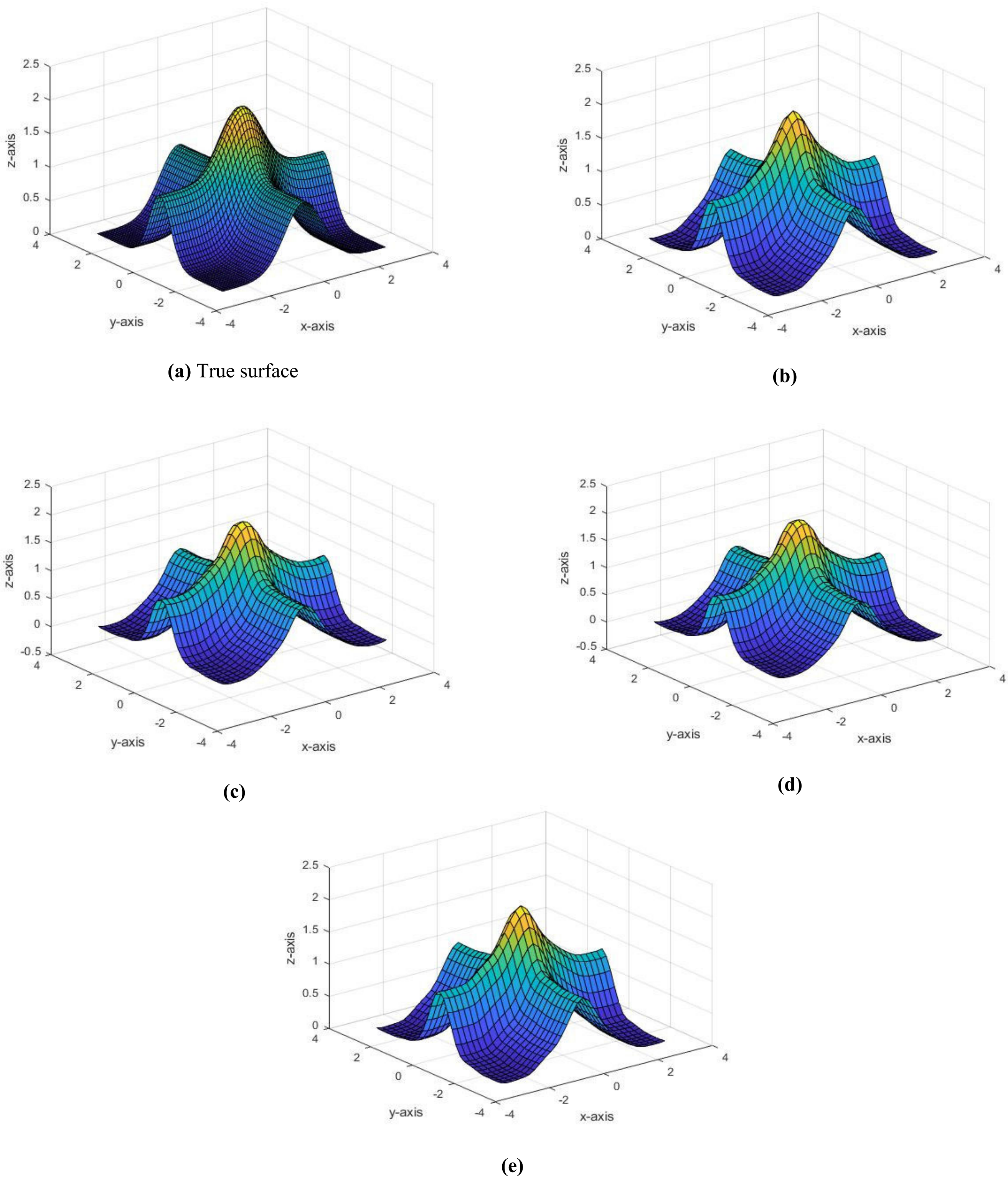


FIGURE 4. Interpolating surfaces for Example 2.

However, there is no free parameters that can be used to refine the final image interpolation unlike our proposed scheme. Furthermore, bicubic spline interpolation involves convolution operator while our proposed scheme only uses tensor product.

Figure 5 shows the examples of the kernel obtained from rational bi-quartic spline defined in (1).

After numerical simulations, we found that the best possible value for the parameters are when $\gamma_{i,j} = \hat{\gamma}_{i,j} = -1.5$. The kernel is shown in Figure 5(b).

Algorithm 1 Image Interpolation

Input: Image with m by n pixel and parameters $\alpha_{i,j}$, $\beta_{i,j}$, $\hat{\alpha}_{i,j}$, $\hat{\beta}_{i,j}$, $\gamma_{i,j}$ and $\hat{\gamma}_{i,j}$.

Step 1

Let (i, j), $i = 0, 1, 2, \ldots m$, $j = 0, 1, 2, \ldots, n$ represents the input pixel indexes and z_{ij} (0-255) are grayscale intensity. Where $\bar{z}_{ij} = F(i, j)$ is the function that will interpolate the given input pixels.

Step 2

Construct rectangular mesh for a given input pixels using the proposed rational bi-quartic spline surface defined by (1). This will produce 4×4 matrix and the kernel is shown in Figure 5(b).

Step 3

Let:
arbitrary input pixel as (x, y):
and (x', y') as arbitrary output pixel
k: scaling factor at pixel (1,1) $(k = 2)$

Step 4

Identify the rectangles where the missing grayscale intensity values. The original pixels' and its intensity values of input image are at vertex of rectangular mesh. Estimate the derivative at each vertex of rectangular (input pixels) by method discussed in [11]. Here we employ rational quartic spline to interpolate the missing pixel on the convex hull.

Step 5

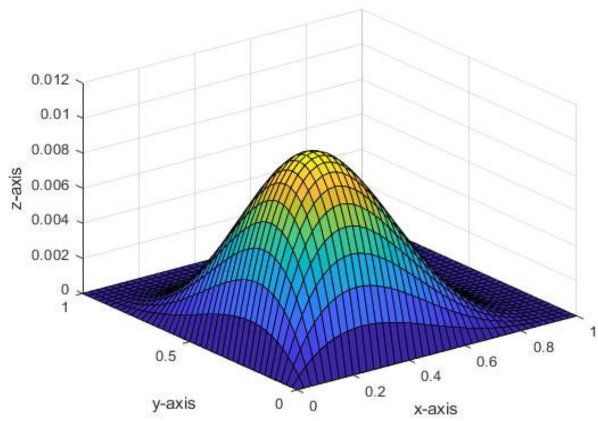
Estimate missing grayscale intensity value in Step 4 using the proposed rational bi-quartic spline surface defined by (1).

Output: Upscaling image and PSNR value including comparison with existing schemes.

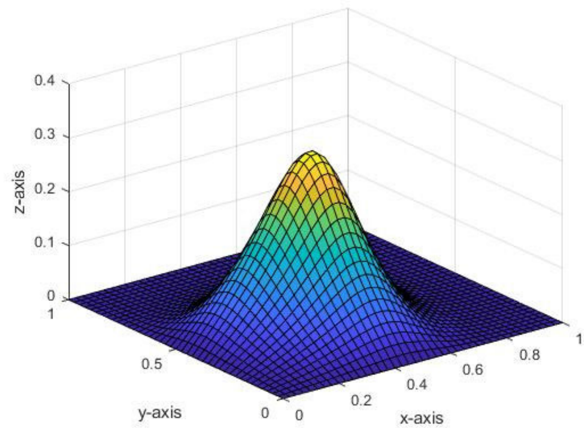
Note: Steps 1-4 will be repeated until we achieve the optimal value of the parameters that will give higher PSNR value for each scheme. Step 3 can be repeated for other scaling factor.

B. EFFICIENT ALGORITHM

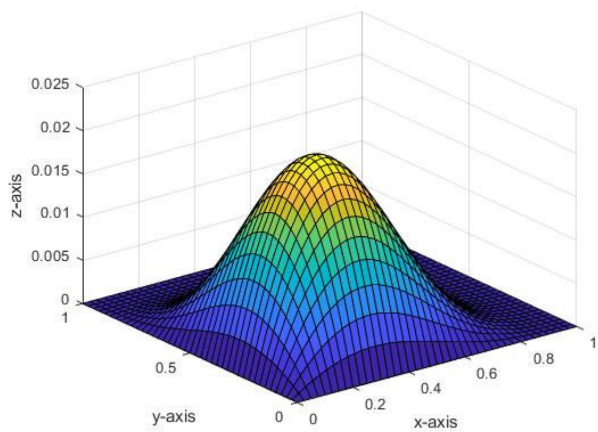
In order to apply the proposed rational bi-quartic spline scheme for image interpolation, we provide the following algorithm that is a bit modification to the main algorithm presented in Karim and Saaban [11] and Zulkifli *et al.* [23]. Our algorithm only consists of five important steps compared with Karim and Saaban [11] that required seven (7) steps. An m-file to define the proposed rational bi-quartic spline is given in Appendix.



(a) Quartic polynomial when $\gamma_{i,j} = \hat{\gamma}_{i,j}$=2



(b) Rational bi-quartic spline with $\gamma_{i,j} = \hat{\gamma}_{i,j}$=-1.5



(c) Rational bi-quartic spline with $\gamma_{i,j} = \hat{\gamma}_{i,j}$=0.5

FIGURE 5. The kernels of rational bi-quartic spline.

This Algorithm can be represented as flow chart as shown in Figure 6 below.

C. IMAGE QUALITY ASSESSMENT (IQA)

In this study, we measure the effectiveness of the proposed image interpolation techniques by calculating PSNR

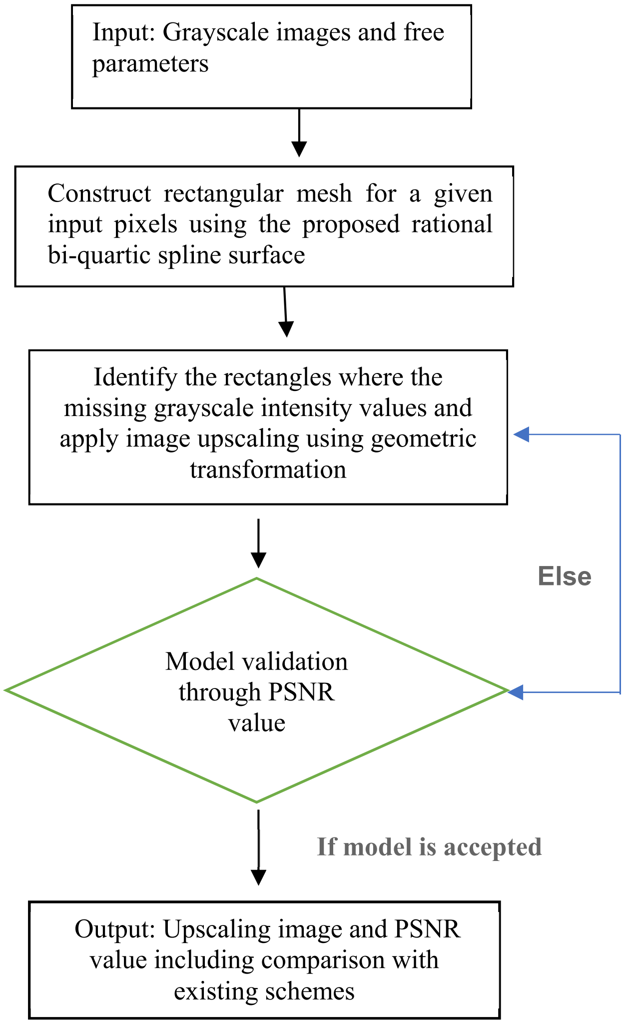


FIGURE 6. Flow chart of the algorithm.

value. The higher PSNR value, the better interpolating images [4], [13], [19]. Since RMSE is related with PSNR such that, if PSNR higher then RMSE is smaller and vice versa. This relationship can be seen in the formulation of PSNR.

The value of PSNR is defined as follows:

$$PSNR = 10\log_{10}\frac{255^2}{MSE}$$

where,

$$MSE = \frac{1}{mn}\sum_{i=0}^{n}\sum_{j=0}^{m}\left|z_{ij} - \bar{z}_{ij}\right|^2$$

RMSE is just simply a square root of MSE.

IV. RESULTS AND GRAPHICAL EXAMPLES

In this section, the proposed rational bi-quartic spline interpolation scheme is tested and compared with some existing schemes. Eight standard tested grayscale images with size 512 × 512 pixels were chosen and are shown in Figure 7. Firstly, the tested images are down scaling with factor half. This will give images with sizes 256 × 256. Then we apply various image interpolation methods to enlarge the images. Then we will obtain a new image (after image interpolation) with same size an original image i.e. 512 × 512. In this study, the scaling factor used is factor two.

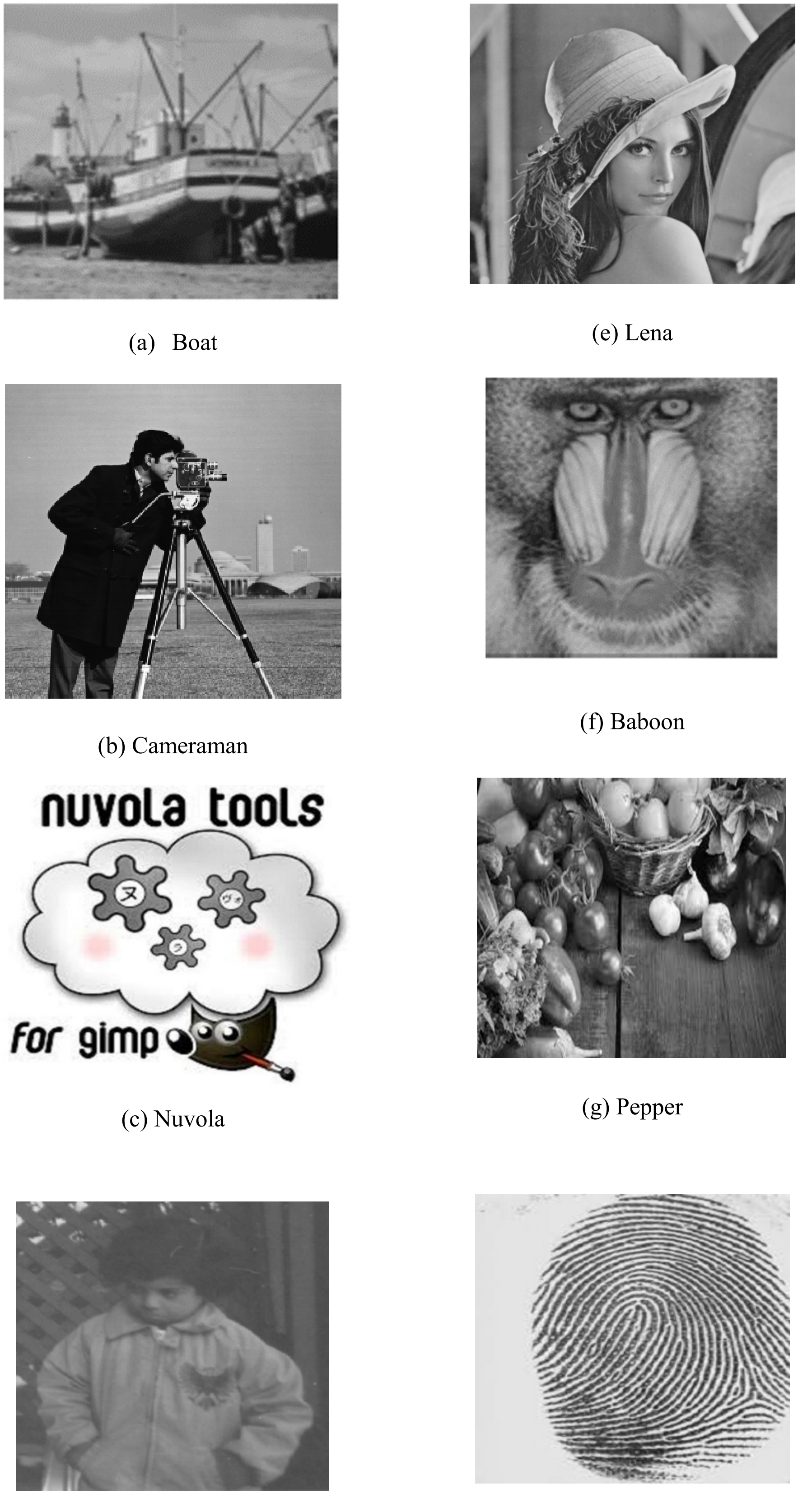
All computational, simulations as well as graphical results are obtained by using MATLAB Version 2019a installed on Windows 10 with processors AMD Ryzen 3 2200G equipped with built-in Radeon Vega Graphics 3.50 GHz. The stopping criteria used was a PSNR value. If the PSNR value is higher, then the scheme gives a better interpolation images [1], [23]. Overall, the CPU time (in seconds) require performing Algorithm 1 is about less than 0.2 seconds for all images.

The main question is how to choose the best parameters value that will give higher PSNR value? To answer this question, we have implemented numerical simulations (based on local control property) in order to find the best possible parameters combination. After several simulations, we found that, the proposed scheme with parameters $\alpha_{i,j} = \beta_{i,j} = 1 = \hat{\alpha}_{i,j} = \hat{\beta}_{i,j}$, and $\gamma_{i,j} = \hat{\gamma}_{i,j} = -1.5$, give the best results compared with other parameters values. We also found that, Zulkifli *et al.* [23] scheme will produce an artifact on the image interpolation if one of the parameters are negative. See Figure 8 to see this effect.

The comparison results in terms of PSNR between the proposed scheme (denoted as BQ) and some established schemes such as Abbas *et al.* [1], Abbas *et al.* [2], Zulkifli *et al.* [23], Karim and Saaban [11] denoted as AA, AB, BB and KS respectively. Meanwhile, BC stand for bicubic spline interpolation that well documented in MATLAB. We make no comparison with bilinear interpolation and nearest neighbor as both methods are not competent compared with the rest. We have written MATLAB coding for each scheme. Table 5 summarize the results. The proposed scheme (BQ) is the best for all images except for pout and pepper. Where for both images bicubic spline is the best. In fact, AA, AB, BB and KS schemes also second best for both images compared with bicubic spline. Taking average of all PSNR values, we found that, the proposed image interpolation scheme has the best PSNR with average value about 38.4825 followed by bicubic spline with 38.4588. The remaining schemes are not competent compared with the proposed scheme and bicubic spline. Karim and Saaban [11]

TABLE 5. Comparison results in terms of PSNR.

Image/ Method	Factor 2					
	AA	AB	BC	BB	KS	BQ
Boat	34.55	34.40	34.47	34.60	34.45	**34.71**
Camera man	40.18	40.06	39.87	40.21	40.08	**40.29**
Nuvola	36.92	36.65	36.73	37.01	36.75	**37.18**
Pout	48.86	48.58	49.93	48.92	48.92	**48.69**
Lena	36.84	36.61	37.00	36.92	36.64	**37.10**
Baboon	36.35	35.93	36.76	36.49	36.65	**37.14**
Pepper	36.21	35.87	37.15	36.32	36.20	**36.42**
Thumb print	35.95	35.79	35.76	36.01	35.82	**36.33**
Average	38.2325	37.9863	38.4588	38.31	36.3275	**38.4825**



(a) Boat

(e) Lena

(b) Cameraman

(f) Baboon

(c) Nuvola

(g) Pepper

(d) Pout

(h) Thumb print

FIGURE 7. Eight tested grayscale images.

(a) Zulkifli et al. [23] scheme producing an artifact

(b) The proposed scheme without any artifact

FIGURE 8. Image Interpolation by factor two for boat image.

scheme have lowest average value compared to the other schemes. According to Kok and Tam [13], bicubic spline interpolation for image interpolation still one of the best methods to enlarge any images. This is because, bicubic spline is C^2 continuous. Therefore, the bicubic kernel is more smooth than other method. However, based on the main finding in this study, we found that, the proposed rational bi-quartic spline scheme is the best compared with bicubic spline interpolation. This is significant result in image processing. However, we feel that, we still can improve the quality of the image's interpolation obtained in this study by integrating some neural network techniques.

The proposed scheme is applicable for RGB image interpolation. Basically, there are two ways we could interpolate RGB images. The first method is converting the RGB images to the grayscale images and then apply our proposed image interpolation algorithm discussed in Section 3. The final images then will be converted back to become RGB images. The second method is to apply the proposed image interpolation algorithm to three different color channels i.e. Red, Green and Blue. The second method is quite challenging since more MATLAB coding are needed.

Final Remark: In this study, we have done comprehensive comparison between the proposed scheme with many existing schemes for image interpolation. PSNR value is used as measurement to determine the effectiveness of the proposed scheme. However, Zou *et al.* [24] also have discussed image interpolation using bivariate Thiele-Like rational continued fractional. They have tested their scheme to image interpolation of Lena's grayscale image. They indicate that, their scheme is the best through visually comparison. It is very hard to judge whether the scheme is better than the other schemes via visually comparison. In contrast to the present study, we have calculated PSNR value as a measurement to determine the goodness of the proposed scheme against many existing schemes. Meanwhile Tang and Zhu [25] have constructed a type of Coons patches on triangular domain. However, their scheme is more complicated to be implemented because each triangular patch is blended through three side-vertex on each triangle edges. This will increase the computation time as well as their method is not easy to be implemented. In contrast, our proposed scheme is easy to be implemented via efficient algorithm. Therefore, the main finding in this study has more merit than the work of Zou *et al.* [24] and Tang and Zhu [25] for the following reasons:

1. We provide an efficient algorithm for the computer implementation. MATLAB coding is provided in the Appendix.
2. We have done comprehensive comparison compared with [24] and [25].

V. CONCLUSIONS

In this study, we have constructed a new rational bi-quartic spline interpolation defined on rectangular meshes. This rational interpolant has six free parameters that can be used for shape modification. This scheme is applicable for functional data interpolation. The proposed scheme is tested to surface interpolation. From numerical results, the proposed scheme is better than Wang and Tan [18] scheme. Furthermore, as step sizes is decreased, the error is closed to zero and the coefficient of determination, R^2 is equal to 1. This shows that, the proposed rational bi-quartic spline interpolation is highly accurate even though with less interpolating data sets. We have also implemented the proposed scheme for image interpolation. An efficient algorithm is developed by a bit modification on the algorithm presented in Karim and Saaban [11] and Zulkifli *et al.* [23]. We have tested the scheme for eight grayscale images. Based on the comparison with Abbas *et al.* [1], [2], Karim and Saaban [11], Zulkifli *et al.* [23] and bicubic spline interpolation from MATLAB, we found that, the proposed scheme is the best with average PSNR value **38.4825** compared with the rest. One possible extension to the current study is to increase the continuity of the rational bi-quartic interpolant i.e. from C^1 to C^2. This will give a better image interpolation especially for scaling factor higher than two. An extension to the parametric cases is in progress. Finally, cybersecurity analysis is needed when the users want to share the images via online communication media such as internet, WhatsApp etc. This will involve cryptography and cybersecurity.

APPENDIX

```
% MATLAB coding to define rational bi-
quartic spline %interpolation given in
Equation (1).
% User can change it to define the
schemes of Abbas et al. [1, % 2], Karim
and Saaban [10] and Zulkifli et al. [23]
% alf,bet,ga,alfc,betc,gac are free
%parameters

function pi =
interp_rat(R,pt,mat,alf,bet,ga,alfc,betc
,gac)
%  Detailed explanation goes here
x0=pt(:,1);y0=pt(:,2);z0=pt(:,3);px=pt(:
,4);py=pt(:,5);
pxy=pt(:,6);
rt=R(:,1:4);
x=x0(rt(:,1:4));
y=y0(rt(:,1:4));
z=z0(rt(:,1:4));
fx=px(rt(:,1:4));
fy=py(rt(:,1:4));
fxy=pxy(rt(:,1:4));
h0=x(:,2)-x(:,1);
h1=y(:,4)-y(:,1);

L2=mat(:,2);
zn=z(L2,:);
fxn=fx(L2,:);
fyn=fy(L2,:);
fxyn=fxy(L2,:);
hij=[h0(L2) h1(L2)];
t1=mat(:,7);t2=mat(:,8);
    hi=hij(:,1); % step size
    hj=hij(:,2); % step size
    z1=zn(:,1);
    z2=zn(:,2);
    z3=zn(:,3);
    z4=zn(:,4);

% partial derivatives on x
    fx1=fxn(:,1);
    fx2=fxn(:,2);
    fx3=fxn(:,3);
    fx4=fxn(:,4);

% partial derivatives on y
    fy1=fyn(:,1);
    fy2=fyn(:,2);
    fy3=fyn(:,3);
    fy4=fyn(:,4);

% partial derivatives on xy
    fxy1=fxyn(:,1);
    fxy2=fxyn(:,2);
    fxy3=fxyn(:,3);
    fxy4=fxyn(:,4);

%The following are denominators on both
%directions

qi=alf.*(1-t1).^2 + (ga).*t1.*(1-
t1)+bet.*t1.^2;
qj=alfc.*(1-t2).^2 + (gac).*t2.*(1-
t2)+betc.*t2.^2;

%The following are all four basis
functions on %both directions

% x-direction
a0=(alf.*(1-t1).^4 + (2*alf+ga).*(1-
t1).^3.*t1+(bet+ga).*(1-
t1).^2.*(t1).^2)./qi;
a1= (bet.*t1.^4 + (2*bet+ga).*(1-
t1).*t1.^3+(alf+ga).*(1-
t1).^2.*(t1).^2)./qi;
a2= (alf*hi.*(1-t1).^3.*t1)./qi;
a3= -(bet*hi.*(1-t1).*t1.^3)./qi;

% y-direction
a0c=(alfc.*(1-t2).^4 + (2*alfc+gac).*(1-
t2).^3.*t2+(betc+gac).*(1-
t2).^2.*(t2).^2)./qj;
a1c= (betc.*t2.^4 + (2*betc+gac).*(1-
t2).*t2.^3+(alfc+gac).*(1-
t2).^2.*(t2).^2)./qj;
a2c= (alfc*hj.*(1-t2).^3.*t2)./qj;
a3c= -(betc*hj.*(1-t2).*t2.^3)./qj;

%Now we define the rational bi-quartic
%spline by expanding the matrix given in
%Eq. (1)

pi=floor((a0.*z1 + a1.*z2 +
a2.*fx1 + a3.*fx2).*a0c +...
    (a0.*z4 + a1.*z3 + a2.*fx4 +
a3.*fx3).*a1c + ...
    (a0.*fy1 + a1.*fy2 + a2.*fxy1 +
a3.*fxy2).*a2c + ...
    (a0.*fy4 + a1.*fy3 + a2.*fxy4 +
a3.*fxy3).*a3c);

% floor operator is used

end
% Ended definition of the proposed %
```

```
% rational bi-quartic spline
% User need MATLAB m-file to estimate
% partial derivatives,
% applying image enlargement etc.
```

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SAMSUL ARIFFIN ABDUL KARIM received the B.App.Sc., M.Sc., and Ph.D. degrees in computational mathematics and computer-aided geometric design (CAGD) from Universiti Sains Malaysia (USM). He has been with the department for more than 11 years. He had 20 years of experience using Mathematica and MATLAB software for teaching and research activities. He is a Senior Lecturer with the Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS (UTP), Malaysia. He has published four books with Springer. He has published more than 140 articles in journals and conferences as well as seven books, including four research monographs, and three edited conferences volume and 30 book chapters. His research interests include curves and surfaces designing, geometric modeling, wavelets applications in image compression and statistics, numerical analysis, differential equations, mathematical modeling, finance, optimization, and approximation theory. He was a Certified WOLFRAM Technology Associate, Mathematica Student Level. He was the recipient of the Effective Education Delivery Award and the Publication Award (journal articles and conference papers) at the UTP Quality Day from UTP, from 2010 to 2012.