

Received May 30, 2020, accepted June 8, 2020, date of publication June 12, 2020, date of current version June 24, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3002020

# Modeling Trajectories and Trajectory Variation of Turning Vehicles at Signalized Intersections

CHARITHA DIAS<sup>1</sup>, MIHO IRYO-ASANO<sup>2</sup>, (Member, IEEE), MUHAMMAD ABDULLAH<sup>3</sup>,  
TAKASHI OGUCHI<sup>4</sup>, AND WAEI ALHAJYASEEN<sup>1</sup>

<sup>1</sup>Qatar Transportation and Traffic Safety Center, College of Engineering, Qatar University, Doha, Qatar

<sup>2</sup>Department of Environmental Engineering and Architecture, Graduate School of Environmental Studies, Nagoya University, Nagoya 464-8601, Japan

<sup>3</sup>Department of Civil Engineering, School of Engineering, University of Management and Technology, Lahore 54700, Pakistan

<sup>4</sup>Institute of Industrial Science, The University of Tokyo, Meguro 153-8505, Japan

Corresponding author: Charitha Dias (cdias@qu.edu.qa)

This work was supported in part by the Japan Society for the Promotion of Science (JSPS) under Grant 19H02261.

**ABSTRACT** Information on the trajectories of turning vehicles at signalized intersections can be used in numerous applications, such as movement planning of autonomous vehicles, realistic representation of surrounding vehicle movements in driving simulator and virtual reality applications, and in microscopic simulation tools. However, no proper framework is currently available to realistically model and estimate trajectories of turning vehicles reflecting the intersection geometries, which is critical for the reliability of simulation models. This study explores the applicability of the minimum-jerk principle, which has been initially applied in neuroscience and robotics domains, to model and simulate free-flow trajectories of turning vehicles. The modeling method is validated by comparing model outputs with empirical trajectories collected at several signalized intersections in Nagoya, Japan. The capability of the model in realistically capturing the variations in turning trajectories based on intersection geometry (e.g., intersection angle and turning radius) is also explained. Further, the applicability of the modeling framework at intersections with different geometric features under different speeds and accelerations are also discussed.

**INDEX TERMS** Autonomous vehicles, motion planning, numerical simulation, path planning, predictive models, traffic control, trajectory optimization.

## I. INTRODUCTION

Signalized intersections with relatively higher traffic demands could often be accident blackspots, not only because of the large traffic volume, but also due to the complex movement patterns of road users and their interactions. Besides, due to the limitations of the available spaces in urban areas, the intersection geometries, e.g., intersection angle, curb radius, and location of crosswalks may not always be the ideal ones. The improper settings of geometries at these intersections may cause human errors and such errors may cause serious conflicts as well as accidents [1], [2]. To apply countermeasures effectively, it is necessary to understand and evaluate vehicle maneuvers particularly affected by intersection geometries. Especially, turning vehicle trajectories (including two-dimensional paths, speeds, and accelerations)

The associate editor coordinating the review of this manuscript and approving it for publication was Keli Xiao.

and their variation are important to be analyzed because of their complexity and the frequency of potential conflicts with other vehicles and pedestrians [3].

There are various applications of the models which can accurately represent detailed turning vehicle trajectories and their variation. In recent years, driving simulator (DS) tools including virtual reality (VR) applications and microscopic simulation tools have emerged as useful tools to study driver behavior and safety at intersections [4]–[6]. Those tools enable researchers and practitioners to analyze the impact of surrounding vehicle maneuver on the driver behavior at critical locations, e.g., accident blackspots. To investigate the realistic reactions of the subject drivers, behaviors of surrounding vehicles should be realistically represented. Furthermore, advances in autonomous driving also demand models particularly for representing accurate speed and acceleration profiles [7]–[9]. Autonomous vehicles (AVs) are required to react to the maneuvers of human-driven vehicles,

which are influenced by intersection geometry. For such purposes, reliable models to predict trajectories of human-driven vehicles are strongly required.

Currently, no proper modeling framework is available to represent driver behavior while negotiating turns. Most of the existing microscopic simulation tools, even those used in estimating surrogate safety measures such as time-to-collision, simplify turning maneuvers of vehicles inside the intersection [10]. As a result of these simplifications, properties such as variation in speed and acceleration may not be represented reflecting the impact of geometry information. It is questionable whether such simulation models are suitable for safety assessments at intersections for geometry improvement.

To overcome existing limitations, recent studies have developed dedicated microscopic simulation models for safety evaluation at signalized intersections [4]. However, these approaches have considered vehicle paths and speed profiles from different models. Tan *et al.* [4] considered the vehicle turning path model by Alhajyaseen *et al.* [11] and speed profile models by Wolfermann *et al.* [12] without considering the physical integrity of these models. Combining different models does not guarantee the spatial and temporal consistency between the location and speed of a turning vehicle. Further, the kinematic properties of a turning vehicle might not have been accurately represented. Wei *et al.* [13] also presented a model that can estimate the paths of left-turning vehicles. Speed and acceleration characteristics of the turning trajectories have not been described in this study. Further, generalization of this model for different intersections may be difficult due to some parameters, such as the vertical distance from the outermost lane boundary of the west entrance ( $d_s$ ) and the instantaneous turning angle ( $\theta$ ).

Several previous studies modified the classical social force model, which is being primarily used to simulate pedestrian behavior, to simulate right-turning vehicle trajectories (right-side traffic) [14], [15]. The influence of intersection geometry (e.g., intersection angle, curve radii) on the calibrated model was not discussed in these studies. Thus, re-calibration of the model may be necessary for intersections with different geometric features. Xu *et al.* [16] reproduced turning vehicle paths at intersections using potential fields. Although turning paths and variations are described in their model, speed and acceleration variations along the turning course are not considered. Several other works presented data-driven approaches using deep learning methods (e.g., neural networks) to predict trajectories of turning vehicles [17], [18]. Such approaches require video data for learning the model and therefore, generalization of the method to new cases may be difficult if the data is not available.

Wang *et al.* [19] explained a trajectory prediction method for left-turning vehicles that integrates geographic information system, global positioning system (GPS) and other sensors to provide real-time position, velocity and acceleration. Double-Kalman filter was used in prediction algorithm. Prediction accuracy of this method primarily depends on the accuracy of GPS-based data and reliability

of sensors. Further, onboard systems are required to be installed to obtain such data. In addition, behavioral aspects and effect of geometrical features of the intersection are not considered in this study. Jiang *et al.* [20] proposed an eco-approach for simulating left-turning vehicles at signalized intersections under mixed traffic, i.e., human driven and connected and automated, environments. They explained that the proposed method improves the traffic safety by providing a safe speed, enhances the fuel efficiency and reduces the emissions. A recent study deployed optimal control method, assuming that drivers maximize their utility, to model vehicle movements at intersections [21]. This approach also requires empirical data for calibration. Therefore, generalizing the model for intersections with different geometrical features may be hard.

In order to overcome the limitations of one-dimensional models, Ma *et al.* [22] proposed a method that is called plan-decision-action framework to simulate turning vehicle trajectories. In the desired trajectory model, which is a key element of their model, the angular velocity and acceleration were assumed as constants throughout the curved segment. Such assumptions could degrade the reliability of the model. Further, empirical data are needed to calibrate such parameters for a range of conditions. Several other studies used trajectory data extracted using computer vision-based tracking tools, e.g., BriskLUMINA [23], to explore pedestrian-vehicle interactions and surrogate safety measures at intersections [24], [25]. Even though such tracking tools can extract vehicle and pedestrian trajectories with a high accuracy, they do not describe behavioral characteristics of drivers that could be useful in modelling and estimating trajectories.

In microscopic simulation models, which were developed for safety evaluation, kinematic information, e.g., speed and acceleration, of turning vehicles must be precisely represented. The realistic representation of vehicle turning maneuvers is also useful for 3-D representation of vehicle maneuvers in DS and VR applications. The reliability of such applications may be dependent on accurate representation of vehicle trajectories. Considering such important applications, the objective of this study is to formulate and test a novel modelling approach for estimating trajectories (i.e., mainly, paths, speed, and acceleration profiles) of turning vehicles at intersections. The proposed approach is based on minimum-jerk concept, which was initially used in neuroscience domain to study human-like movements. We demonstrate that the proposed method can estimate the trajectories of turning vehicles with remarkable accuracy utilizing real-world trajectory data extracted from videos collected at several signalized intersections.

The paper is organized as follows: Section II discusses the trajectory data extraction method and the background of the modeling approach. Section III presents the verification of the proposed method. The sensitivity analysis is presented in Section IV, followed by a discussion. Finally, conclusions and directions for further studies are presented.

**TABLE 1. Geometric characteristics of the considered intersections.**

Intersection name and approach	Corner radius (m)	Intersection angle (°)
Nishi-Osu West approach	17	76.9
Taikotori West approach	17	94.1
Kawana North approach	17	106
Ueda South approach	13.5	119

## II. METHODS

### A. DATA

Video data were collected at several signalized intersections in Nagoya, Japan to verify and validate the proposed modeling approach. The geometric characteristics of the intersections considered in this study are summarized in Table 1. Note that all intersections operate in left-side traffic.

These intersections have different geometric characteristics and a shared green signal phase for left-turning vehicles and pedestrians/cyclists. Trajectories of turning vehicles at these intersections were extracted using the *TrafficAnalyzer* video image processing system [26]. Using this tool, the positions of vehicles, where the right-rear wheel is touching the ground, were obtained at 0.5 s intervals by manual tracking. By considering the dimensions of the vehicle, the observed right-rear wheel trajectory is transformed into a trajectory that corresponds to the center-front of the vehicle. Video coordinates (in pixels) are converted to global coordinates (in m) by using projective transformation. The transformed trajectories were smoothed using the Kalman smoothing method. A tracking example for a right-turning vehicle using *TrafficAnalyzer* is shown in Fig. 1.

Vehicle trajectories are affected by the existence of surrounding vehicles and pedestrians, traffic signal settings and intersection geometries. Meanwhile in this study, left and right turn trajectories only under free-flow conditions (no impact with other vehicles, pedestrians, or cyclists).

The trajectories generated by this analysis are expected as the base or intended trajectories of the turning vehicles which can be utilized for further analyses of interaction to other road users as well as to traffic signal control.

### B. MINIMUM JERK PRINCIPLE

The modeling method proposed in this study is based on the minimum-jerk principle, which was initially used to describe skilled human arm movements in a plane (2-D space). Flash and Hogan [27] demonstrated that the smoothness of skilled arm movement, e.g., reaching, writing, and drawing tasks, can be explained as a function of jerk. The cost function  $J$  that is to be minimized is the time integration of the square of the magnitude of the jerk vector when moving from a given initial location to a final location within a given time  $t_f$ . This cost function  $J$  can be given as;

$$J = \frac{1}{2} \int_0^{t_f} \left( \left( \frac{d^3x}{dt^3} \right)^2 + \left( \frac{d^3y}{dt^3} \right)^2 \right) dt \quad (1)$$

where  $t_f$  is the movement time from a known initial position to a specified final position.

In previous studies, features of human goal-oriented [28] and turning [29] movements have been described in terms of minimum-jerk theory. Further, this concept was utilized in motion planning of robot limbs [30], [31] and motion control problems in autonomous vehicles [32], [33]. In another study, the car following behavior was described using the minimum-jerk concept [34]. This is based on the analogy that the behavior of a following vehicle is comparable to the skilled reaching movements of the human arm. Dias *et al.* [35] explored the effect of curve radius and desired speed on drivers' speeding behavior on expressway curves using the minimum-jerk principle.

Flash and Hogan [27] proved that the solution of the minimization problem given in Eq. 1 can be obtained as a set of fifth-order polynomials of time expressed as;

$$x(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 \quad (2a)$$

$$y(t) = b_0 + b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5 \quad (2b)$$

where  $x(t)$  and  $y(t)$  are x- and y-coordinates of the location at time  $t$ , and  $a_j$  and  $b_j$  ( $j = \{0, \dots, 5\}$ ) are constants.

Eq. 2a and 2b indicate there are 12 unknowns, and therefore 12 boundary conditions are required to solve these equations. Location (in x- and y-coordinates), velocity and acceleration vectors (x- and y-components) at the initial and the final points provide the 12 boundary conditions. Initial and final locations of the trajectory can be obtained from geometry data of the intersection and its periphery. Speed and acceleration of a vehicle at entry and exit locations are dependent on the characteristics of the entry and exit links.

The movement time  $t_f$  is an unknown that is required to solve the equations. If the position and speed information of an intermediate location are known,  $t_f$  can be estimated using Eq. 2a and 2b. In this study, we use intermediate location information to estimate  $t_f$  and the constants in Eq. 2a and 2b.

### C. MODEL FRAMEWORK

Dias *et al.* [36] used location, velocity and acceleration at starting and final locations, and movement time between these locations ( $t_f$ ) as trajectory information and explained that trajectories of turning vehicles under free-flow conditions could be described using the minimum-jerk principle. Although they proved the applicability of minimum-jerk theory to turning vehicles, they did not represent the impact of intersection geometry. Another critical issue was that their model requires  $t_f$  as an input and this information is not available. The current study is an extension of the previous one. Instead of using movement time between initial and final points, we use information at an intermediate point to estimate the trajectory. Information at an intermediate point (minimum speed and location of minimum speed) is estimated using models presented in Wolfertmann *et al.* [12] to quantitatively estimate the impact of geometry and entry speed. Minimum-jerk theory and



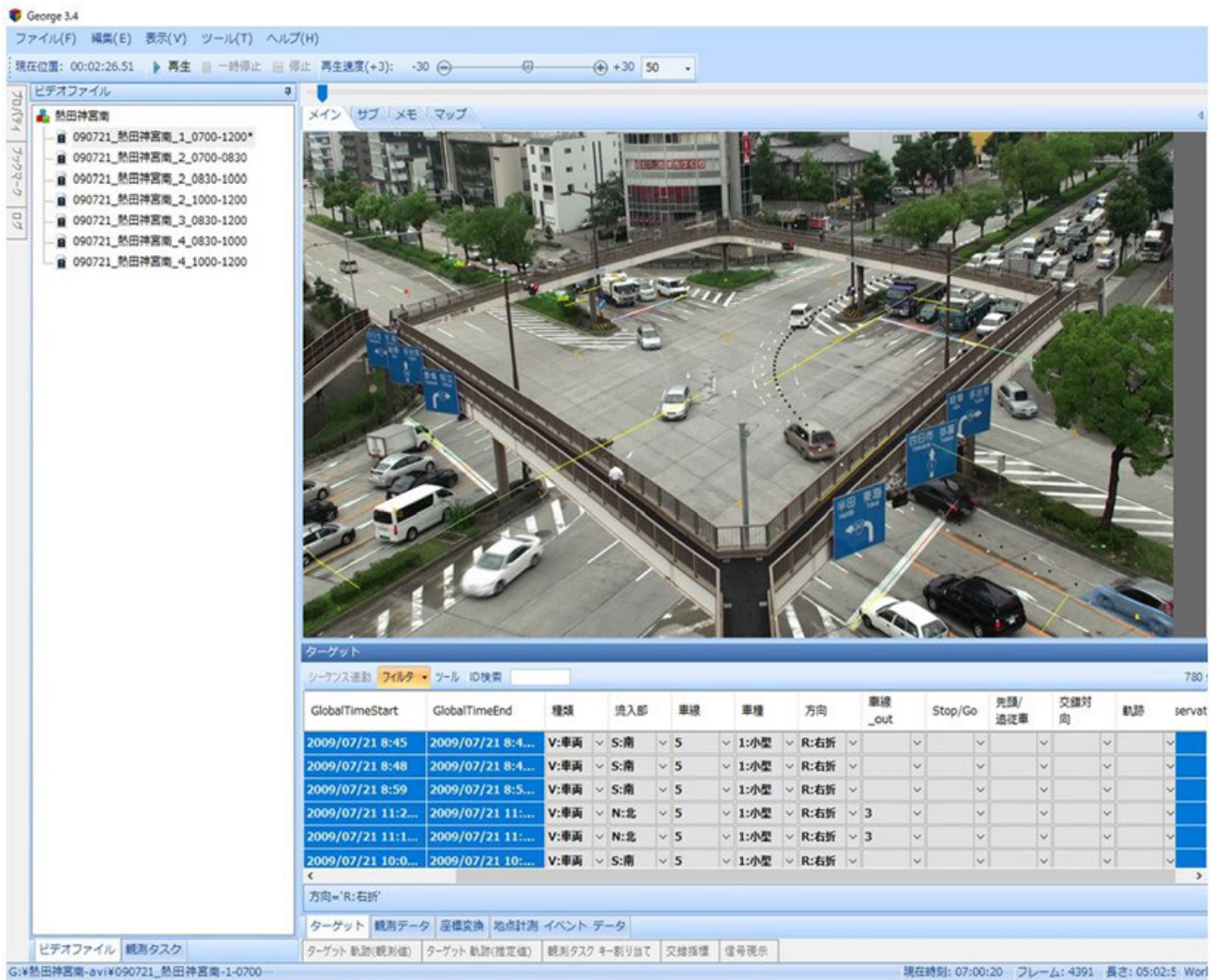


FIGURE 1. Trajectory data extraction using TrafficAnalyzer.

models presented in Wolfermann *et al.* [12] (to estimate minimum speed ( $v_{min}$ ) and location of minimum speed ( $s_{min}$ )) are combined in this study to estimate trajectories of turning vehicles.

The structure of the proposed model is shown in Fig. 2. The input variables are the conditions of a turning vehicle at entry and exit of the intersection (i.e., location, speed and acceleration), type of the vehicle (passenger car or heavy vehicle), and intersection geometry settings such as intersection angle, curb radius, lateral exit distance and hard nose distance, defined as in Fig. 3(b).

Considering the simulation application of this model, it is rational to assume the conditions of turning vehicles at entry are available when running the simulation. Meanwhile, it is also reasonable to assume the vehicle conditions at exit are determined by the design speed of the exit road sections. With given entering speed and vehicle type, minimum speed and location of minimum speed are probabilistically chosen using the normal distribution estimated by Wolfermann’s model.

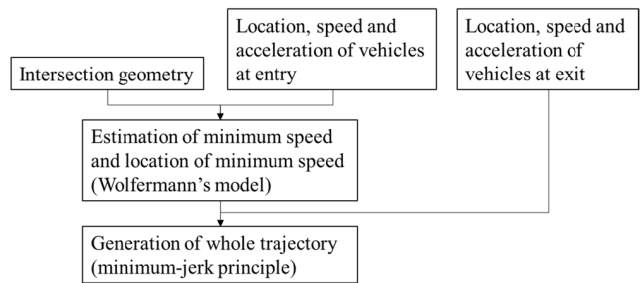
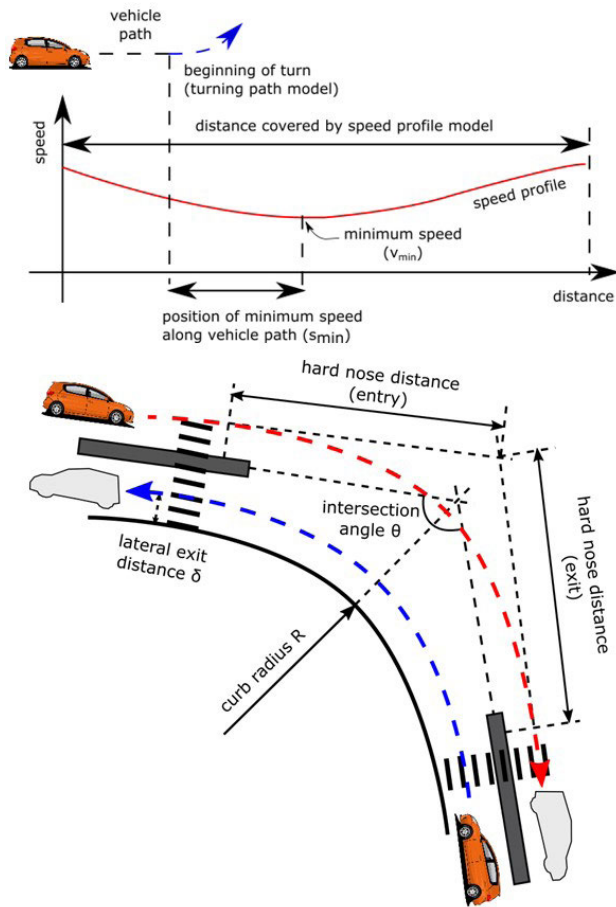


FIGURE 2. Flowchart of trajectory generation.

The minimum speed and location of minimum speed are used in minimum-jerk model to estimate the whole trajectory of a single turning vehicle.

The models by Wolfermann *et al.* [12] empirically estimated the minimum speed and the location of the minimum speed of a turning vehicle under ideal or free-flow conditions as functions of entry speed and several geometric properties



**FIGURE 3.** Schematic of (up) the position of minimum speed relative to the vehicle path, (down) the parameters in regression models (adopted from Wolfermann *et al.* [12], note: IP point is the crossing point of the two median extensions of entry and exit approaches).

of the intersection. Parameters of the probabilistic models for minimum speed ( $v_{min}$ ) and location of minimum speed ( $s_{min}$ ) are presented in Table 2. The models for  $v_{min}$  and  $s_{min}$  are represented as normal distributions; mean ( $\mu$ ) and standard deviation ( $\sigma$ ) are modeled as a function of entry speed and geometric characteristics of the intersection. Physical meaning of the minimum speed ( $v_{min}$ ) and location of minimum speed related to the trajectory ( $s_{min}$ ) are schematically illustrated in Fig. 3(a). Model parameters are depicted in Fig. 3(b).

Combining empirical models with the minimum-jerk principle, a series of simultaneous equations are obtained as explained below.

At the starting position of the trajectory;

$$x(t = 0) = a_0 \tag{3a}$$

$$\dot{x}(t = 0) = a_1 \tag{3b}$$

$$\ddot{x}(t = 0) = 2a_2 \tag{3c}$$

where  $x(t = 0)$ ,  $\dot{x}(t = 0)$ , and  $\ddot{x}(t = 0)$  are the x-components of the location, velocity, and acceleration vectors of the turning vehicle at the starting point, respectively. These are known, and for convenience the initial location is set as (0, 0).

**TABLE 2.** Parameters of minimum speed ( $v_{min}$ ) and location of minimum speed ( $s_{min}$ ) models for turning vehicles (Wolfermann *et al.* [12]).

Normal Distribution Parameters	Left-turning vehicles		Right-turning vehicles	
	$v_{min}$ N( $\mu, \sigma$ )	$s_{min}$ N( $\mu, \sigma$ )	$v_{min}$ N( $\mu, \sigma$ )	$s_{min}$ N( $\mu, \sigma$ )
Constant	-0.301	1.42	2.6508	7.346
Entering speed (m/s)	0.0908	-	0.1879	0.501
Corner radius (m)	0.0607	0.586	-	-
Intersection angle (deg)	0.0387	0.0896	0.0289	0.0776
Lateral exit distance (m)	0.233	0.577	-	-
$\mu$				
Heavy vehicle dummy (HV: 1, PC: 0)	-0.496	-	-	-
Distance from IP point to entering hard nose $\Delta H N_{in}$ (m)	-	-	-	0.288
Constant	0.665	0.135	1.4042	14.056
Entering speed (m/s)	-	-	-	-0.528
Corner radius (m)	-	0.144	-	-
Intersection angle (deg)	-	-	-0.0054	-
$\sigma$				
Lateral exit distance (m)	0.0419	0.336	-	-
Distance from IP point to entering hard nose $\Delta H N_{in}$ (m)	-	-	-	0.110

At the final location of the trajectory;

$$x(t = t_f) = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5 \tag{4a}$$

$$\dot{x}(t = t_f) = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4 \tag{4b}$$

$$\ddot{x}(t = t_f) = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3 \tag{4c}$$

where  $x(t = t_f)$ ,  $\dot{x}(t = t_f)$ , and  $\ddot{x}(t = t_f)$  are the x-components of the location, velocity, and acceleration vectors of the turning vehicle at the known final location or the exit point, respectively.

At the location of minimum speed,

$$x(t = t_m) = a_0 + a_1 t_m + a_2 t_m^2 + a_3 t_m^3 + a_4 t_m^4 + a_5 t_m^5 \tag{5a}$$

$$\dot{x}(t = t_m) = a_1 + 2a_2 t_m + 3a_3 t_m^2 + 4a_4 t_m^3 + 5a_5 t_m^4 \tag{5b}$$

$$\ddot{x}(t = t_m) = 2a_2 + 6a_3 t_m + 12a_4 t_m^2 + 20a_5 t_m^3 \tag{5c}$$

where  $t_m$  represents the time to the location of minimum speed measured from the starting location, and  $x(t = t_m)$ ,  $\dot{x}(t = t_m)$ , and  $\ddot{x}(t = t_m)$  are the x-components of the location, velocity, and acceleration vectors of the turning vehicle, respectively, at the location of minimum speed. Distributions of  $x(t = t_m)$  and  $\dot{x}(t = t_m)$  can be estimated using the models reported in Wolfermann *et al.* [12]. Based on these distributions, values for  $x(t = t_m)$  and  $\dot{x}(t = t_m)$  are randomly chosen and substituted in Eq. 5(b) and 5(c).

However,  $t_m$  and  $\ddot{x}(t = t_m)$  are unknowns. Nevertheless, it may be assumed that  $\ddot{x}(t = t_m) = 0$ . If we consider

Eq. 3(a)-(c), Eq. 4(a)-(c), and Eq. 5(a)-(b) (ignoring Eq. 5(c)), there are eight unknowns ( $a_0, a_1, a_2, a_3, a_4, a_5, t_f$ , and  $t_m$ ) with eight simultaneous equations. Thus, solving these equations, a set of solutions for constants and movement times ( $t_f$  and  $t_m$ ) can be obtained. Using the estimated  $t_f$ , constants ( $b_0, b_1, b_2, b_3, b_4$ , and  $b_5$ ) for y-components of the trajectory can be obtained by solving the simultaneous equations formulated below.

At the starting location of the trajectory,

$$y(t=0) = b_0 \quad (6a)$$

$$\dot{y}(t=0) = b_1 \quad (6b)$$

$$\ddot{y}(t=0) = 2b_2 \quad (6c)$$

where,  $y(t=0)$ ,  $\dot{y}(t=0)$ , and  $\ddot{y}(t=0)$  are the y-components of the location, velocity, and acceleration vectors of the turning vehicle at the starting point, respectively.

At the final point of the trajectory,

$$y(t=t_f) = b_0 + b_1 t_f + b_2 t_f^2 + b_3 t_f^3 + a_4 t_f^4 + b_5 t_f^5 \quad (7a)$$

$$\dot{y}(t=t_f) = b_1 + 2b_2 t_f + 3b_3 t_f^2 + 4b_4 t_f^3 + 5b_5 t_f^4 \quad (7b)$$

$$\ddot{y}(t=t_f) = 2b_2 + 6b_3 t_f + 12b_4 t_f^2 + 20b_5 t_f^3 \quad (7c)$$

where  $y(t=t_f)$ ,  $\dot{y}(t=t_f)$ , and  $\ddot{y}(t=t_f)$  are the y-components of the location, velocity, and acceleration vectors of the turning vehicle at the final point, respectively. As  $t_f$  was estimated earlier,  $b_j$  ( $j = \{0, \dots, 5\}$ ) can also be estimated in a similar manner.

Due to the stochastic nature of  $v_{min}$  and  $s_{min}$ , a Monte Carlo simulation was conducted to verify the performance and validity of the proposed modeling approach.

### III. MODEL VALIDATION

The capability of the proposed model for estimating a trajectory with random boundary conditions was tested. A Monte Carlo simulation was performed with 100 different random seeds to choose an entry speed, an exit speed, an entry acceleration, and an exit acceleration from the speed and acceleration distributions at the initial and final locations obtained from empirical data.  $v_{min}$  and  $s_{min}$  were estimated based on the randomly chosen entry speed and geometric characteristics of the considered curve using the models presented in Table 2. For the simulations,  $(\mu \pm \sigma)$  ranges of the  $v_{min}$  and  $s_{min}$  models in Table 2 were used. It should be noted that only passenger cars (PC) were considered in this study and therefore heavy vehicle dummy (in Table 2) was ignored.

Resulting estimated path distributions for several right and left turn maneuvers at four intersections are compared with empirical paths in Fig. 4. It is observed that the average estimated paths do not deviate considerably from the average empirical paths. Further, estimated paths do not travel beyond the road boundaries. Statistical tests reveal that for Taikodori right, Nishi-Osu left, Kawana right, and Ueda left turns, the difference between averages of estimated and actual trajectories are not statistically significant. The t-statistic and p-value of t-test for Taikodori, Nishi-Osu, Kawana, and Ueda

were (0.84, 0.20), (0.62, 0.27), (0.24, 0.41), and (0.90, 0.18), respectively. Maximum deviations (approximately at the middle of the intersection) of average estimated trajectories from average empirical trajectories were 1.06 m, 0.81 m, 0.74 m, and 0.37 m for Taikodori, Nishi-Osu, Kawana, and Ueda, respectively. The maximum deviation between average estimated path and the average empirical path tends to increase with decreasing corner radius.

Estimated speed profiles and acceleration profiles for two intersection approaches are compared with empirical profiles in Fig. 5 and Fig. 6. It is evident that estimated speed and acceleration profiles are consistent with empirical profiles. Estimated speed and acceleration profiles do not significantly deviate from empirical profiles, as shown in Fig. 5(c)-(d) and Fig. 6(c)-(d). Thus, if initial and final states (locations, velocities, and accelerations) are known, trajectories of turning vehicles can be estimated by integrating minimum-jerk theory with regression models presented in Wolfermann *et al.* [12], which estimate minimum speed and location of minimum speed.

### IV. SENSITIVITY ANALYSIS

A sensitivity analysis was conducted to verify the effect of intersection geometry (intersection angle and turning radius) and boundary conditions (speed and acceleration at entry and exit). Details of the analysis are discussed in this section.

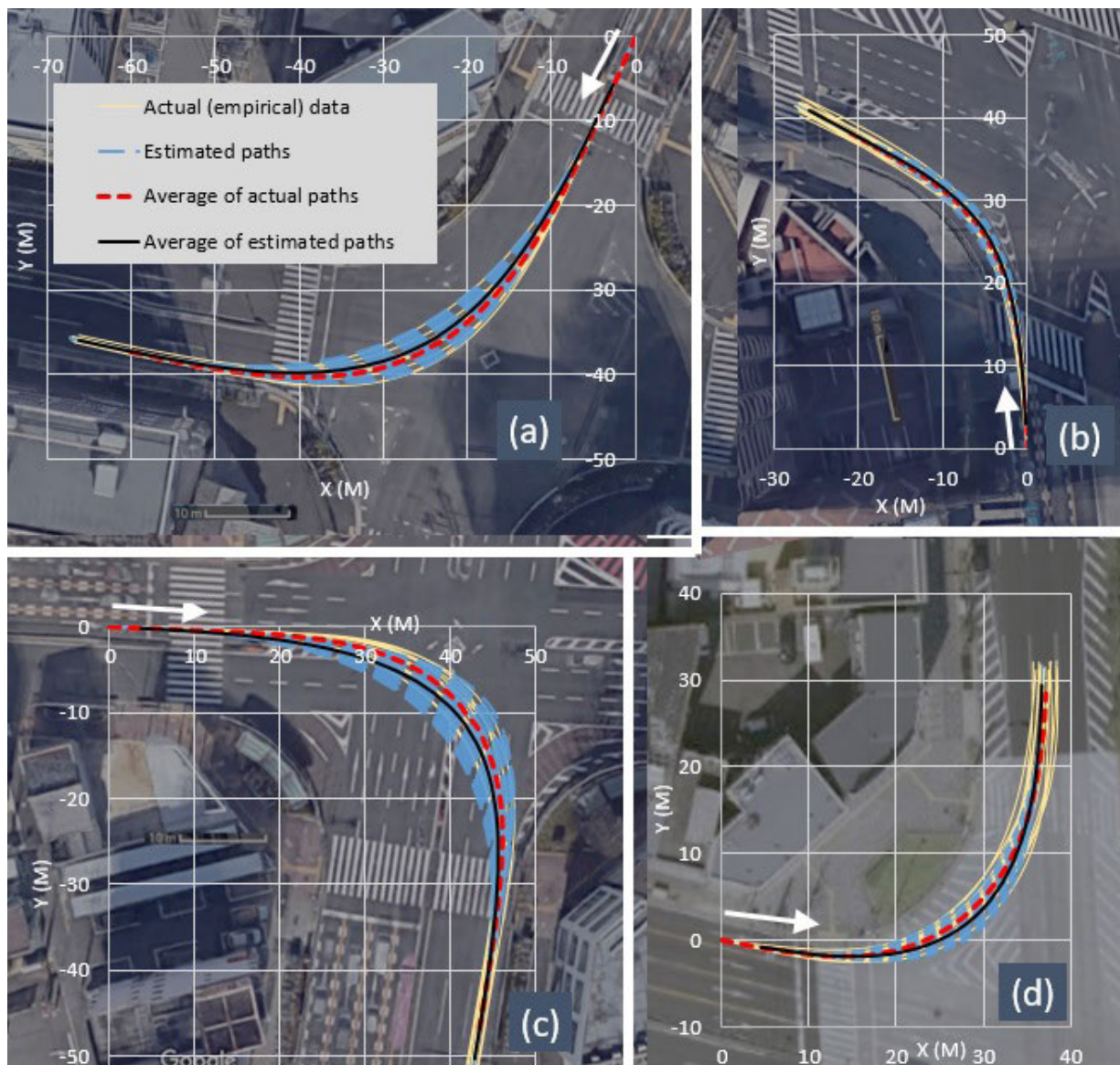
#### A. EFFECT OF INTERSECTION GEOMETRY

##### 1) CURVE RADII

Estimated path, speed, and acceleration profile distributions for two hypothetical intersections with corner radii of 15 m and 20 m are depicted in Fig. 7. Intersection angle was kept at  $90^\circ$  for both intersections. Entering and exit speeds were assumed to be the same and were set as 8.5 m/s. Entering and exit accelerations were set as  $-0.2 \text{ m/s}^2$  and  $0.2 \text{ m/s}^2$ , respectively. Distance from IP point to entering hard nose, which is required to estimate minimum speed and location of minimum speed, was set as 20 m. A Monte Carlo simulation was performed with 100 random seeds to randomize  $v_{min}$  and  $s_{min}$ . Standard deviations of maximum lateral deviation from the median path for 15 m and 20 m radii were estimated as  $\pm 0.73 \text{ m}$  and  $\pm 0.93 \text{ m}$ , respectively. These statistics indicate that intersections with larger radii tend to have larger trajectory variation than intersections with smaller radii. This result is consistent with the findings of Alhajyaseen *et al.* [11] who estimated the paths of vehicles using the Euler-Spiral approximation method. The variation may be mainly due to the variations in movement times (variance of estimated  $t_f$  was  $0.11 \text{ s}^2$  and  $0.20 \text{ s}^2$  for corner radii of 15 m and 20 m, respectively).

Speed profiles in Fig. 7 show that the radius of the curve does not affect the average speed and variation. This may be because the curve radius is not considered in estimating the value and location of minimum speed for right-turning vehicles (see Table 2). The estimated minimum speed ( $\pm \text{SD}$ )





**FIGURE 4.** Comparison of estimated and empirical (actual) paths for (a) Kawana right turns, (b) Ueda left turns, (c) Taikotori right turns, (d) Nishiosu left turns.

for 15 m and 20 m radii was  $5.81 (\pm 0.90)$  m/s and  $6.05 (\pm 0.85)$  m/s, respectively. Statistical tests confirmed that these minimum speeds are not statistically significant (Mann-Whitney U test z-score = 1.51, p-value = 0.13). However, due to differences in size or curve distance, movement time differs significantly.

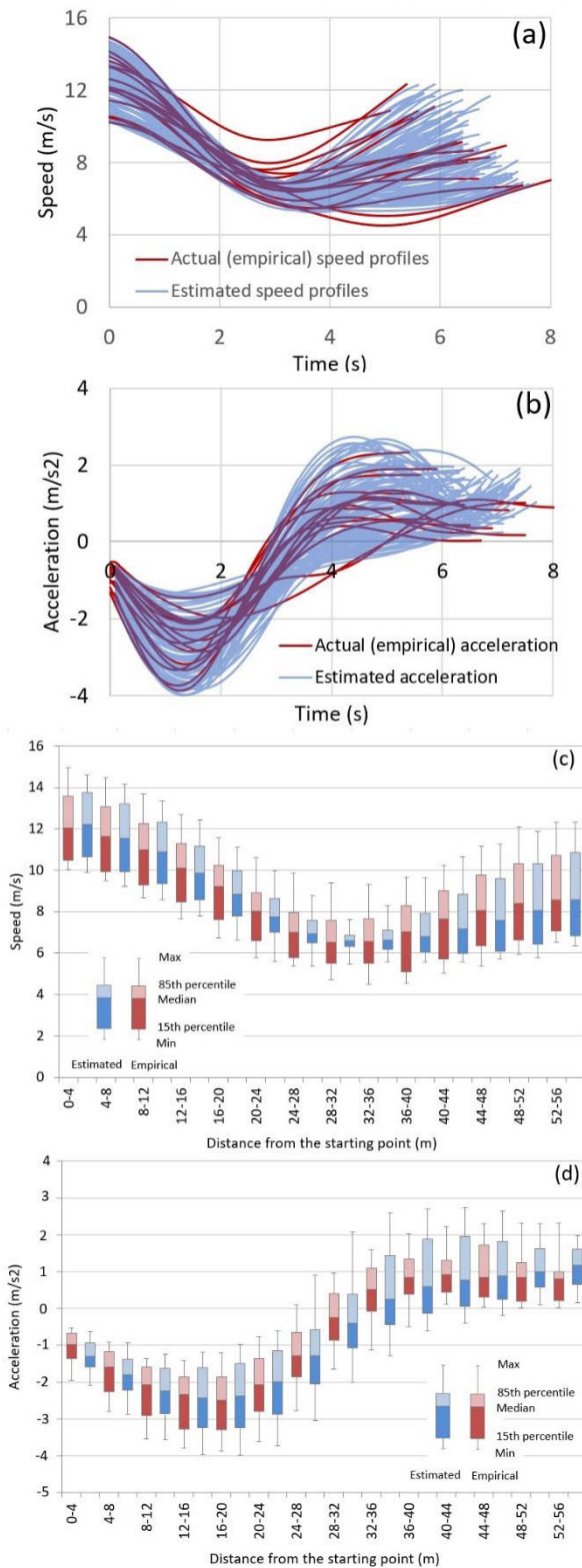
Estimated maximum acceleration ( $\pm$  SD) for 15 m and 20 m radii was  $1.48 (\pm 0.39)$  m/s<sup>2</sup> and  $1.29 (\pm 0.33)$  m/s<sup>2</sup>, respectively. Statistical tests confirmed that these maximum accelerations are statistically significant (Mann-Whitney U test z-score = 2.46, p-value = 0.01). This suggests that although speed profiles do not display significant variations, accelerations are sensitive to the radius of the turn. The reason, in this particular case, could be the movement time; as the travel path of the turning maneuver is shorter for  $R = 15$  m and the average minimum speed is the same for both radii

cases, larger decelerations and accelerations are necessary for  $R = 15$  m to maintain the speed pattern. Such behaviors are also realistically captured in the proposed model.

## 2) INTERSECTION ANGLE

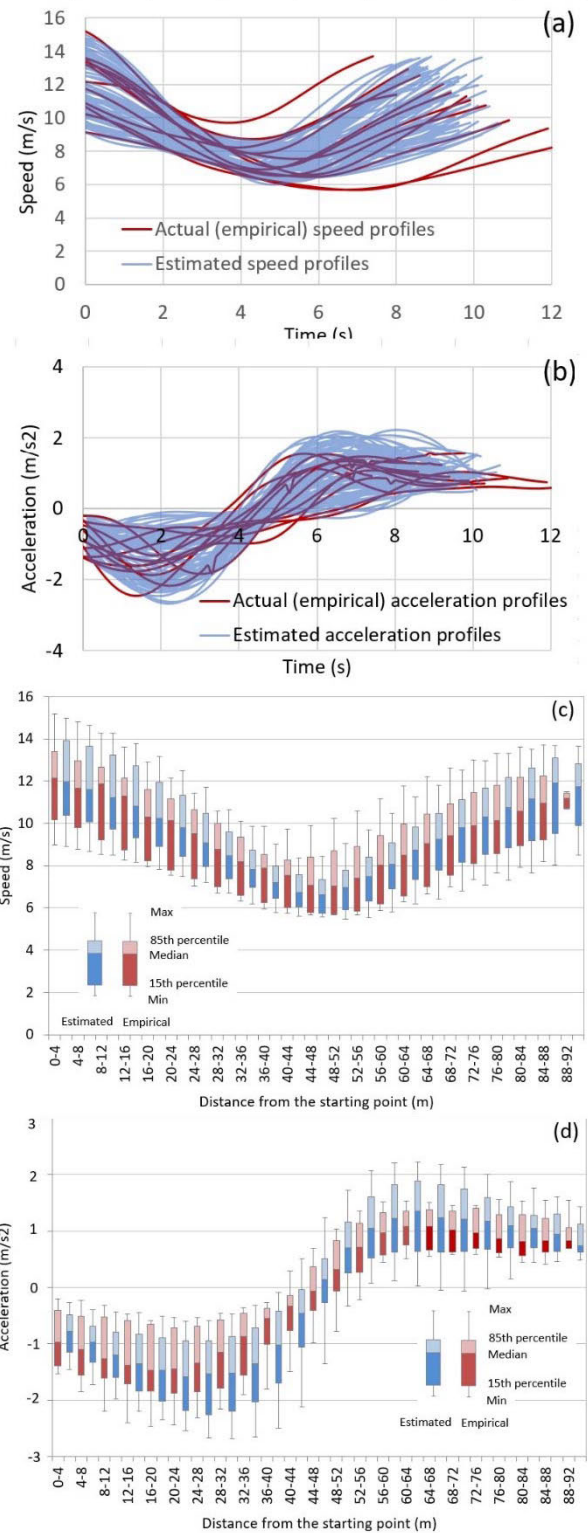
Estimated path distributions for three hypothetical intersections with intersection angles of  $60^\circ$ ,  $90^\circ$ , and  $120^\circ$  are shown in Fig. 8.

In these simulations, corner radius was assumed as 15 m for all intersections. Entering and exit speeds were assumed the same and were set as 8.5 m/s for all cases. Entering and exit accelerations were set as  $-1.0$  m/s<sup>2</sup> and  $1.0$  m/s<sup>2</sup>, respectively, for all cases. Lateral exit distance was set as 1.5 m. A Monte Carlo simulation was conducted with 100 random seeds to randomize the minimum speed and location of minimum speed.



**FIGURE 5.** Comparison of estimated and empirical (actual) (a) speed profiles, (b) acceleration profiles, (c) aggregated speed profiles, (d) aggregated acceleration profiles, for Ueda left turns.

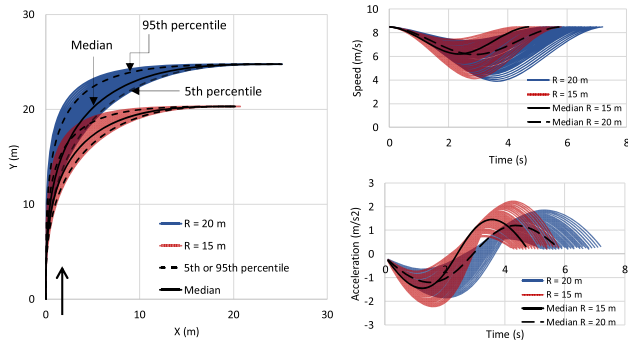
Standard deviations of maximum lateral deviation from the median path for 60°, 90°, and 120° angles were estimated as ± 0.90 m, ± 0.68 m, and ± 0.54 m, respectively.



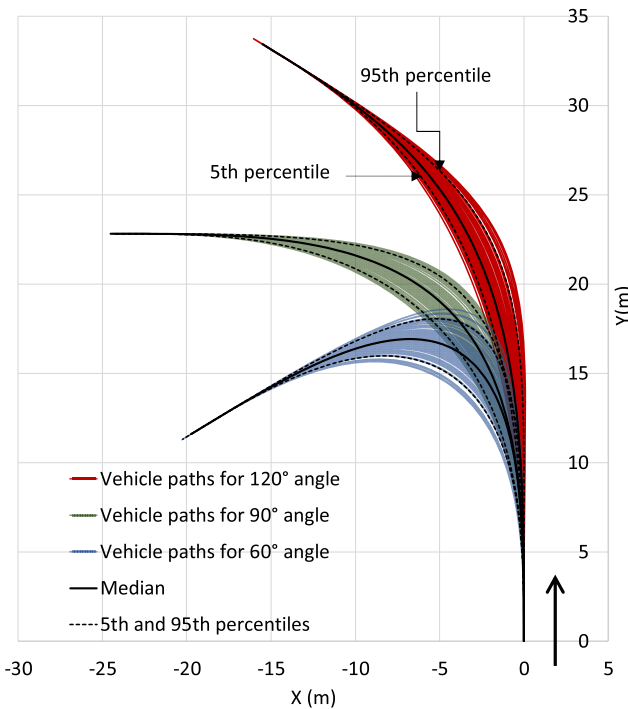
**FIGURE 6.** Comparison of estimated and empirical (actual) (a) speed profiles, (b) acceleration profiles, (c) aggregated speed profiles, (d) aggregated acceleration profiles, for Taktiotori right turns.

These statistics illustrate that as the intersection angle increases, vehicle paths become less varied or less distributed. This finding is consistent with the result obtained in Alhajjaseen *et al.* [11].





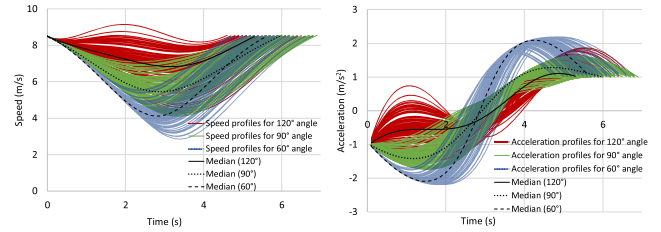
**FIGURE 7.** Sensitivity of right-turning vehicle paths, speed profiles, and acceleration profiles to curve radii.



**FIGURE 8.** Sensitivity of left-turning vehicle paths to intersection angle.

Fig. 10 shows boxplots of the estimated speed and acceleration profiles. In contrast to path distributions, speed and acceleration profiles with larger intersection angles display a larger variation, as shown in Fig. 8 and Fig. 9. Average minimum speed ( $\pm$  SD) for speed profiles shown in Fig 8 is  $4.2 (\pm 0.6)$  m/s,  $5.5 (\pm 0.7)$  m/s, and  $6.7 (\pm 0.9)$  m/s for  $60^\circ$ ,  $90^\circ$ , and  $120^\circ$  angles, respectively. Estimated minimum speed values using the Wolfermann *et al.* [12] model for those angles are  $4.2 (\pm 0.7)$  m/s,  $5.4 (\pm 0.7)$  m/s, and  $6.5 (\pm 0.7)$  m/s, respectively. These minimum speeds are very similar for all cases. However, the standard deviations of speed tend to be larger with increasing intersection angle.

Findings of the sensitivity analysis further explain that the proposed modeling framework can be applied to estimate trajectories of turning vehicles at any intersection when geometric features of the intersection and expected entry and



**FIGURE 9.** Sensitivity of speed and acceleration profiles of left-turning vehicles to intersection angle.

exit conditions (speed and acceleration) are known. That is, empirical data are not required.

### B. EFFECT OF BOUNDARY CONDITIONS

As explained in Section II-C,  $v_{min}$  and  $s_{min}$  are modeled as functions of entry speed with geometric variables. The exit speed and entry and exit accelerations have not been considered in these models. However, such properties are inputs for the minimum-jerk model (Eq. 3 to 7). Thus, the effects of these variables on estimated paths, speed and acceleration profiles were also explored, and outcomes are presented in this section. In the simulations reported in this section, geometric characteristics of the intersections kept unchanged.

#### 1) EXIT SPEED

Trajectories for  $90^\circ$  left-turning vehicles were simulated using a constant entry speed (8.5 m/s) and different exit speeds (6 m/s, 8.5 m/s, and 11 m/s). Entry and exit accelerations were set as  $-0.5 \text{ m/s}^2$  and  $0.5 \text{ m/s}^2$ , respectively, for all cases. To randomize the minimum speed and location of minimum speed, a Monte Carlo simulation was conducted with 150 random seeds.  $v_{min}$  and  $s_{min}$  distributions are the same for all cases as the entry speeds are the same.

Resulting path, speed and acceleration profiles, and aggregated distributions for the three cases are compared in Fig. 11. The trajectories tend to shift to the inner side when the exit speed decreases, as shown in Fig. 11(a). Speed and acceleration profiles in Fig. 11(b) and 11(c) display a similar pattern until the minimum speed is achieved and then begin to deviate significantly. This is mainly because  $v_{min}$  and  $s_{min}$  distributions used in this study (proposed in Wolfermann *et al.* [12]) are independent of exit speed. Thus, further studies are necessary to explore the effect of exit speed on  $v_{min}$  and  $s_{min}$  distributions.

#### 2) ENTRY ACCELERATION

For this simulation, entry and exit speeds were set as 8.5 m/s for all cases. Curve radius and lateral exit distances were set as 15 m and 1.75 m, respectively. Exit acceleration was set as  $0.5 \text{ m/s}^2$  and three entry accelerations,  $0.0 \text{ m/s}^2$ ,  $-1.0 \text{ m/s}^2$ , and  $-2.0 \text{ m/s}^2$ , were considered. A Monte Carlo simulation was performed with 100 random seeds for each case to randomize the minimum speed and location of minimum speed. Resulting paths, speed profiles, and acceleration profiles for the three entry accelerations are compared in Fig. 12. It is

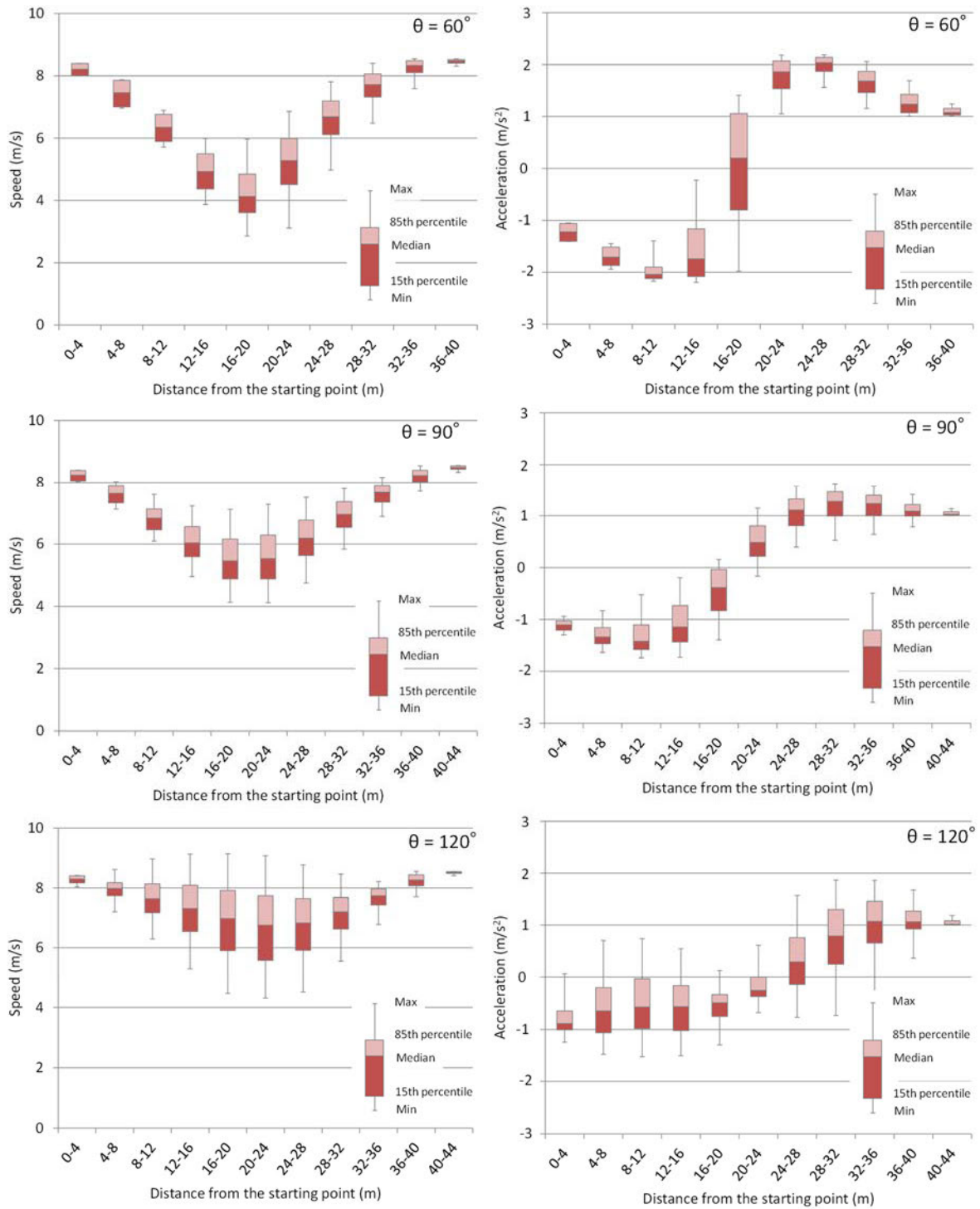
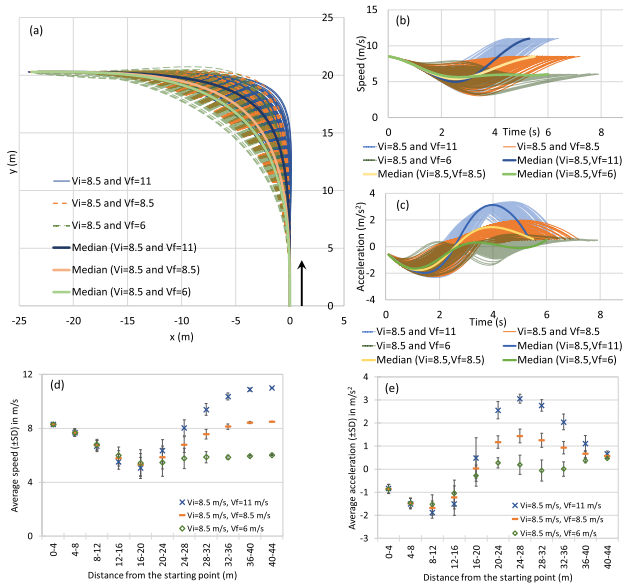


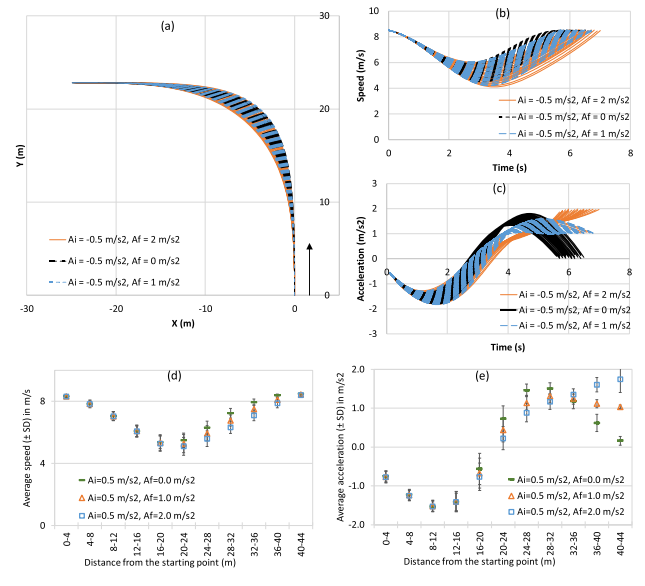
FIGURE 10. Boxplots for estimated speed and acceleration profiles for different intersection angles.

observed that simulated vehicle paths tend to shift to the inner corner of the turn with decreasing entry acceleration. Differences between medians of paths at the middle of the intersection were statistically significant (Kruskal-Wallis test H statistic = 148.88, p-value < 0.0001). Observing speed profiles, it is understood that minimum speeds tend to decrease

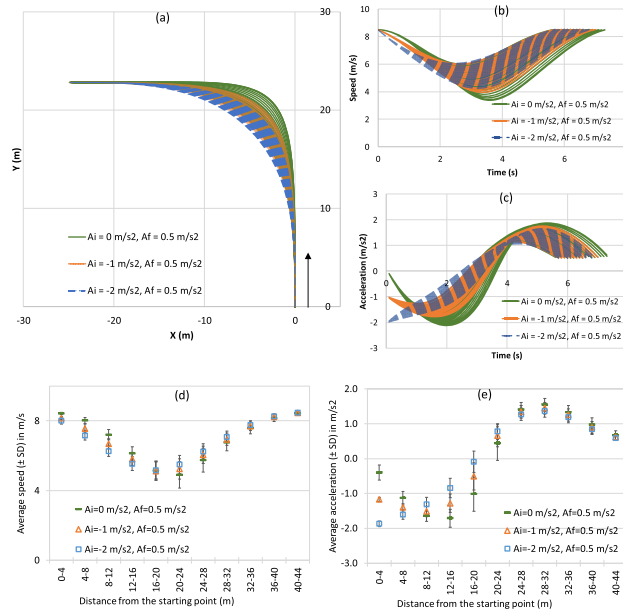
with decreasing entry acceleration. Average minimum speeds ( $\pm$  SD) were 4.87 ( $\pm$  0.77) m/s, 5.03 ( $\pm$  0.57) m/s, and 5.16 ( $\pm$  0.53) m/s for entry accelerations ( $A_i$ ) of 0 m/s<sup>2</sup>, 1 m/s<sup>2</sup>, and 2 m/s<sup>2</sup>, respectively. Statistical tests confirmed that the difference between minimum speeds for the three cases is statistically significant (Kruskal-Wallis test H



**FIGURE 11.** Comparison of (a) paths, (b) speed profiles, (c) acceleration profiles, (d) aggregated speed profiles, (e) aggregated acceleration profiles, for different exit speeds.



**FIGURE 13.** Comparison of (a) paths, (b) speed profiles, (c) acceleration profiles, (d) aggregated speed profiles, (e) aggregated acceleration profiles, for different exit accelerations.



**FIGURE 12.** Comparison of (a) paths, (b) speed profiles, (c) acceleration profiles, (d) aggregated speed profiles, (e) aggregated acceleration profiles, for different entry accelerations.

statistic = 9.68, p-value = 0.008). Estimated  $v_{min}$  for this particular case (using the model presented in Wolfermann *et al.* [12]) was  $5.27 (\pm 0.74)$  m/s. Acceleration distributions significantly deviate until the minimum speed is achieved and then follow a similar pattern.

### 3) EXIT ACCELERATION

As with entry acceleration, entry and exit speeds were set as 8.5 m/s for all exit acceleration cases. Curve radius and lateral exit distance were set as 15 m and 1.75 m, respectively.

Entry acceleration was set as  $-0.5 \text{ m/s}^2$  and three exit accelerations,  $0.0 \text{ m/s}^2$ ,  $1.0 \text{ m/s}^2$ , and  $2.0 \text{ m/s}^2$ , were considered. A Monte Carlo simulation was performed with 100 random seeds for each case to randomize the minimum speed and location of minimum speed. Resulting paths, speed profiles, and acceleration profiles for the three entry accelerations are compared in Fig. 13. It is clear that paths and speed profiles overlap for exit deceleration cases. Statistical tests confirm that differences between paths are not statistically significant (Kruskal-Wallis test H statistic = 4.63, p-value = 0.1). Average minimum speeds ( $\pm$  SD) were  $5.38 (\pm 0.40)$  m/s,  $5.29 (\pm 0.58)$  m/s, and  $5.20 (\pm 0.52)$  m/s for exit accelerations (Af) of  $0 \text{ m/s}^2$ ,  $1 \text{ m/s}^2$ , and  $2 \text{ m/s}^2$ , respectively. Statistical tests confirmed that the difference between minimum speeds for the three cases is statistically significant (Kruskal-Wallis test H statistic = 3.58, p-value = 0.06). Acceleration distributions follow a similar pattern until the minimum speed is achieved and then begin to deviate significantly.

## V. DISCUSSION

Currently, no proper framework is available in microscopic simulation models and VR tools to realistically model and estimate trajectories of turning vehicles, even under free-flow conditions taking into account the impact of intersection geometry. In existing models, the maneuvers and behaviors of drivers in terms of the path, speed, and acceleration are not described. Further, intersection geometry has not been properly incorporated. In this study, we proposed a modeling framework based on the minimum-jerk principle to estimate trajectories of turning vehicles at signalized intersections. The minimum-jerk principle is an empirically verified theoretical concept that has been previously applied to describe smooth human movements.



Previous studies combined vehicle paths and speed profiles generated from separate models [4]. Maintaining spatial and temporal consistency between paths and speed and acceleration profiles is extremely difficult as the location of the deceleration starting point (before making the turning maneuver) is generally unknown and depends on many factors including geometric characteristics of the intersection and desired speed of the vehicle. Although there are alternative approaches, e.g., data driven approaches [17]–[19], it is difficult to generalize such approaches for intersections with different geometric properties. This model is robust compared to previous studies as it; a) maintains the spatial and temporal consistency between paths, speeds, and higher order profiles, b) does not assume any pre-defined shape (or trend) for speed and acceleration profiles, c) can be applied to estimate trajectories at intersections with different geometric properties, and d) does not require empirical trajectory data for calibration. Further studies are needed to validate such advantages and to compare the performance of the proposed approach with existing approaches.

Section III and IV explained that the proposed modelling framework could realistically estimate paths and speed profiles for a given intersection even without empirical data. That is, variations of paths and speed profiles are consistent in estimates and corresponding empirical data even though randomized boundary conditions were used. However, acceleration (and higher order profiles) tend to display larger variations under such conditions. This explains the necessity of incorporating realistic boundary conditions if higher order profiles are to be estimated with higher accuracy.

Trajectories for vehicles yielding or stopping for pedestrians or cyclists were not examined in this study. Other interactions with preceding or following vehicles were also not considered. Under such non-free-flow conditions, drivers may not tend to maximize the smoothness of their motions. Nevertheless, accurate representation of free-flow trajectories in microscopic simulations or DS applications as the ‘desired trajectory’ or ‘ideal trajectory’ is important. For example, the desired trajectory (including paths, speed, and acceleration profiles), which is the output of the model described in this study, can be implemented in a driving simulator in a leading vehicle with subject drivers asked to follow the simulated vehicle to test their reactions. In addition to geometric features, road conditions (including pavement conditions), environmental conditions, vehicle conditions and driver behaviors could also affect the trajectories of turning vehicles. Such influences may also be considered in future studies.

## VI. CONCLUSIONS

Realistic representation of turning vehicle trajectories reflecting the impact of intersection geometry is important in safety estimation tools such as driving simulators, virtual reality applications and microscopic traffic simulations to enhance reliability. Existing studies have not adequately addressed this issue. In this study, a modeling approach based on the

minimum-jerk principle was proposed and tested for estimating turning vehicle trajectories (paths, speed and acceleration profiles). The comparison of modeled and empirical real world trajectories suggests that the proposed modelling method can reproduce trajectories with high accuracy. Input variables of the model are intersection geometries and the entering speed of the vehicles, and no pre-determined trends (or shapes) were assumed to model speed and acceleration profiles. The modeled profiles in this study follow the kinematic features of turning vehicles and are more realistic than previous studies. The trajectories produced can be used as the initial or ideal pattern for representing turning vehicle maneuvers. Sensitivity analysis shows that the model realistically represents the effects of geometric features of an intersection on trajectories. The applicability of the model in different geometric settings and boundary conditions was also confirmed through the sensitivity analysis.

Only free-flow turning trajectories were modeled in this study and therefore the model cannot be applied in modeling vehicle trajectories that interact with pedestrians, cyclists, or other vehicles in an intersection. Applicability of the proposed modeling approach in such cases should be verified in future studies using additional empirical data. Nevertheless, free-flow turning trajectories can be considered as the base patterns for any simulation model used in driving simulator or virtual reality applications. The findings in this study may be useful in enhancing the reliability and accuracy of such applications.

## ACKNOWLEDGMENT

The authors would like to thank Professor Hideki Nakamura of Nagoya University, Japan for providing the trajectory data used in this study.

## REFERENCES

- [1] K. M. Bauer and D. W. Harwood, “Statistical models of at-grade intersection accidents,” Federal Highway Administration, Washington, DC, USA, Tech. Rep. FHWA-RD-96-125, 1996.
- [2] S. angenehpour, C. Chung, and S. Saneinejad, “Impact of curb radius reduction on pedestrian safety a before-after surrogate safety study in Toronto,” in *Proc. ITS World Congr.*, Detroit MI, USA, 2017, pp. 1–7.
- [3] W. K. M. Alhajyaseen, M. Asano, and H. Nakamura, “Left-turn gap acceptance models considering pedestrian movement characteristics,” *Accident Anal. Prevention*, vol. 50, pp. 175–185, Jan. 2013.
- [4] D. M. Tan, W. K. M. Alhajyaseen, M. Asano, and H. Nakamura, “Development of microscopic traffic simulation model for safety assessment at signalized intersections,” *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2316, no. 1, pp. 122–131, Jan. 2012.
- [5] S. T. Chrysler, O. Ahmad, and C. W. Schwarz, “Creating pedestrian crash scenarios in a driving simulator environment,” *Traffic Injury Prevention*, vol. 16, no. sup1, pp. S12–S17, Jun. 2015.
- [6] J. Wu, *Analysis of Pedestrian Safety Using Micro-simulation and Driving Simulator. Doctoral Dissertation, Department of Civil*. Orlando, FL, USA: University of Central Florida, 2017.
- [7] C.-G.-J. Gerardo, A.-L. Jesús, and O.-M. Ricardo, “Modeling the turning speed and car following behaviors of autonomous vehicles in a virtual world,” *Ingeniería, Investigacl. Tecnol.*, vol. 16, no. 3, pp. 391–405, Jul. 2015.
- [8] Y. Gu, Y. Hashimoto, L.-T. Hsu, M. Iryo-Asano, and S. Kamijo, “Human-like motion planning model for driving in signalized intersections,” *IATSS Res.*, vol. 41, no. 3, pp. 129–139, Oct. 2017.

- [9] S. Mahmud Khan and M. Chowdhury, "Situation-aware left-turning connected and automated vehicle operation at signalized intersections," 2019, *arXiv:1908.00981*. [Online]. Available: <http://arxiv.org/abs/1908.00981>
- [10] D. Gettman, L. Pu, T. Sayed, and S. G. Shelby, "Surrogate safety assessment model and validation," FHWA, Washington, DC, USA, Tech. Rep., FHWA-HRT-08-051, 2008.
- [11] W. K. M. Alhajyaseen, M. Asano, H. Nakamura, and D. M. Tan, "Stochastic approach for modeling the effects of intersection geometry on turning vehicle paths," *Transp. Res. C, Emerg. Technol.*, vol. 32, pp. 179–192, Jul. 2013.
- [12] A. Wolferrmann, W. K. Alhajyaseen, and H. Nakamura, "Modeling speed profiles of turning vehicles at signalized intersections," in *Proc. 3rd Int. Conf. Road Saf. Simulation*, 2011, pp. 1–7.
- [13] F. Wei, W. Guo, X. Liu, C. Liang, and T. Feng, "Left-turning vehicle trajectory modeling and guide line setting at the intersection," *Discrete Dyn. Nature Soc.*, vol. 2014, pp. 1–7, 2014.
- [14] K. Long and Y. Ni, "Modeling and simulation of right-turn vehicle behaviors under influences of conflicting bicycles at intersection," *Transp. Res. Board Annu. Meeting*, Jan. 2012.
- [15] W. Huang, M. Fellendorf, and R. Schönauer, "Social force-based vehicle model for two-dimensional spaces," in *Proc. TRB Annu. Meeting*, Washington, DC, USA, 2012, pp. 1–16.
- [16] Y. Xu, Z. Ma, and J. Sun, "Simulation of turning vehicles' behaviors at mixed-flow intersections based on potential field theory," *Transportmetrica B, Transp. Dyn.*, vol. 7, no. 1, pp. 498–518, Dec. 2019.
- [17] M. Shokrolah Shirazi and B. T. Morris, "Trajectory prediction of vehicles turning at intersections using deep neural networks," *Mach. Vis. Appl.*, vol. 30, no. 6, pp. 1097–1109, Sep. 2019.
- [18] J. Sun, X. Qi, Y. Xu, and Y. Tian, "Vehicle turning behavior modeling at conflicting areas of mixed-flow intersections based on deep learning," *IEEE Trans. Intell. Transp. Syst.*, pp. 1–12, 2019.
- [19] P. Wang, L. Zhang, and S. Fang, "Trajectory prediction for left-turn vehicles at T-shaped intersections based on location based service," in *Proc. Int. Conf. Transp. Inf. Saf. (ICTIS)*, Jun. 2015, pp. 29–33.
- [20] H. Jiang, S. An, J. Wang, and J. Cui, "Eco-approach departure system for left-turn vehicles at fixed-time signalized intersection," *Sustainability*, vol. 10, no. 1, p. 273.
- [21] J. Zhao, V. L. Knoop, and M. Wang, "Two-dimensional vehicular movement modelling at intersections based on optimal control," *Transp. Res. B, Methodol.*, vol. 138, pp. 1–22, Aug. 2020.
- [22] Z. Ma, J. Sun, and Y. Wang, "A two-dimensional simulation model for modelling turning vehicles at mixed-flow intersections," *Transp. Res. C, Emerg. Technol.*, vol. 75, pp. 103–119, Feb. 2017.
- [23] Brisk Synergies Tech Corp. (2017). *BriskLUMINA Service. Obtenido de Brisk Synergies*. [Online]. Available: <https://brisksynergies.com/brisklumina/>
- [24] L. Scholl, M. Elagaty, B. Ledezma-Navarro, E. Zamora, and L. Miranda-Moreno, "A surrogate video-based safety methodology for diagnosis and evaluation of low-cost pedestrian-safety countermeasures: The case of cochabamba, bolivia," *Sustainability*, vol. 11, no. 17, p. 4737, Aug. 2019.
- [25] T. Fu, W. Hu, L. Miranda-Moreno, and N. Saunier, "Investigating secondary pedestrian-vehicle interactions at non-signalized intersections using vision-based trajectory data," *Transp. Res. C, Emerg. Technol.*, vol. 105, pp. 222–240, Aug. 2019.
- [26] K. Suzuki and H. Nakamura, "TrafficAnalyzer-the integrated video image processing system for traffic flow analysis," in *Proc. 13th ITS World Congr.*, London, U.K., Oct. 2006, pp. 8–12.
- [27] T. Flash and N. Hogan, "The coordination of arm movements: An experimentally confirmed mathematical model," *J. Neurosci.*, vol. 5, no. 7, pp. 1688–1703, Jul. 1985.
- [28] Q. C. Pham, "The formation of trajectories during goal-oriented locomotion in humans II maximum smoothness model," *Eur. J. Neurosci.*, vol. 26, no. 8, 2007, pp. 2391–2403.
- [29] C. Dias, M. Abdullah, M. Sarvi, R. Lovreglio, and W. Alhajyaseen, "Modeling and simulation of pedestrian movement planning around corners," *Sustainability*, vol. 11, no. 19, p. 5501, 2019.
- [30] Y. Wada and M. A. Kawato, "Via-point time optimization algorithm for complex sequential trajectory formation," *Neural Netw.*, vol. 17, no. 3, 2004, pp. 353–364.
- [31] A. Aloulou and O. O. Boubaker, "Minimum jerk-based control for a three dimensional bipedal robot," in *Intelligent Robotics and Applications (Lecture Notes in Computer Science)*, vol. 7102, S. Jeschke, H. Liu, and D. chilberg, Eds. Berlin, Germany: Springer, 2011, pp. 251–262.
- [32] C. Guarino Lo Bianco and M. Romano, "Bounded velocity planning for autonomous vehicles," in *Proc. IEEE/RJS Int. Conf. Intell. Robots Syst.*, Oct. 2005, pp. 685–690.
- [33] T. Shamir, "How should an autonomous vehicle overtake a slower moving vehicle: Design and analysis of an optimal trajectory," *IEEE Trans. Autom. Control*, vol. 49, no. 4, pp. 607–610, Apr. 2004.
- [34] T. Hiraoka, T. Kunimatsu, O. Nishihara, and H. Kumamoto, "Modeling of driver following behavior based on minimum-jerk theory," in *Proc. 12th World Congr. ITS*, San Francisco, CA, USA, 2005, pp. 1–7.
- [35] C. Dias, T. Oguchi, and K. Wimalasena, "Drivers' speeding behavior on expressway curves: Exploring the effect of curve radius and desired speed," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2672, no. 17, pp. 48–60, Dec. 2018.
- [36] C. Dias, M. Iryo-Asano, and T. Oguchi, "Predicting optimal trajectory of left-turning vehicle at signalized intersection," *Transp. Res. Procedia*, vol. 21, pp. 240–250, 2017.



**CHARITHA DIAS** received the master's degree from The University of Tokyo (UTokyo), Japan, in 2007, and the Ph.D. degree from Monash University, Australia, in 2015. After completing the master's degree, he worked as a transportation modeler at two reputed consultancy firms in Tokyo for around three and a half years. He worked as a Postdoctoral Project Researcher at the Institute of Industrial Science, UTokyo, from November 2015 until he moved to Qatar. He is currently

working as an Assistant Professor with the Qatar Transportation and Traffic Safety Center, College of Engineering, Qatar University. His research interests include pedestrian crowd dynamics, modeling and simulation, mixed traffic modeling and simulation, traffic safety, driver behavior, and intelligent transportation systems. While doing the Ph.D. degree, he received several awards including the Australian Road Research Board (ARRB)–Monash Transportation Research Prize, in 2013, and the Postgraduate Publication Award from the Department of Civil Engineering, Monash University, in 2014.



**MIHO IRYO-ASANO** (Member, IEEE) is an Associate Professor with Nagoya University, Japan, and has been working on international/national research projects on traffic control and user behavior analysis. Prior to Nagoya University, she has working experiences with the Intelligent Transportation Systems (ITS) Division, National Institute for Land and Infrastructure Management, Japan, where she was involved in the development of vehicle-to-infrastructure communication systems. The developed system is currently used in whole Japan for providing transportation services with road users as well as for the road administration. She has good knowledge about traffic control and road design, especially from pedestrians' viewpoint, and conducted several research projects with Dr. W. Alhajyaseen for traffic safety and user behavior analyses. She has also developed a pedestrian traffic simulator and virtual reality simulator combined with the pedestrian simulation model. The achievements of these projects have been published in more than 80 refereed journal articles or conference papers (including 14 SCI journal articles). These knowledge and research experiences are reflected in practice as well. She is a Committee Member of technical guidelines of the Japan Society of Traffic Engineers and contributed to the latest publications of Manual of Intersection Design and Traffic Control, Roundabout Manual, and Pavement Marking Design Manual.



**MUHAMMAD ABDULLAH** was born in Mianwali, Pakistan. He received the B.S. degree in transportation engineering from the University of Engineering and Technology Lahore, Pakistan, in 2013, and the M.S. and Ph.D. degrees from The University of Tokyo, Japan, in 2016 and 2019, respectively. He was also an Exchange Researcher at the Institute for Transport Planning and Traffic Engineering, Technical University of Darmstadt, Germany, from September to October 2018. Since

October 2019, he has been an Assistant Professor with the Department of Civil Engineering, University of Management and Technology, Lahore, Pakistan. His research interests include traffic signal design and optimization, road safety, and pedestrian behavior.



**TAKASHI OGUCHI** was born in Tokyo, Japan. He received the degree from the Department of Civil Engineering, Faculty of Engineering, The University of Tokyo (UTokyo), in 1988, and the Ph.D. degree from the Graduate School of Engineering, UTokyo, in 1993. After joining Nissan Motor Corporation Ltd., he moved to Tokyo Metropolitan University and promoted to a Professor, in 2007. He moved to the Institute of Industrial Science (IIS), UTokyo, in 2011, where he was

assigned as the Director of the Advanced Mobility Research Center (ITS Center), in 2018. He is also a member of the Mobility Innovation Collaborative Research Center (UTmobI) started in the university, in July 2018, and is in charge of education with the Department of Civil Engineering, UTokyo. He is currently working as a Professor with the IIS, UTokyo. He also serves as the Leader of the Service Implementation Promotion Working Group of Japanese national project Cross-Ministerial Strategic Innovation Promotion Program in the Innovation of Automated Driving for Universal Services (SIP-adus). His major research fields are traffic management and control, including advanced traffic signal control, advanced highway network traffic management systems, and integrated transport and mobility system design, including automated vehicles, the harmonized design of infrastructure facilities, and hierarchical road transport systems.



**WAEEL ALHAJJYASEEN** received the M.S. degree in transportation engineering from The University of Jordan, in 2005, and the D.Eng. degree in civil engineering from Nagoya University, in 2010. He is currently an Associate Professor with the Qatar Transportation and Traffic Safety Center, College of Engineering, Qatar University. He is specialized in traffic engineering, traffic safety (human factors, crash data, causality, and driving simulation), road user behavior analysis (motor-

ized and non-motorized), and intelligent transportation systems. He received several regional and international awards for his contribution to the field of traffic engineering such as the Yasoshima Yoshinosuke Prize provided by the Eastern Asia Society for Transportation Studies, in 2009. He is the Chair of the Traffic Safety Analysis and Policy Special Interest Group, World Conference on Transport Research Society (WCTRS).

...