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# Ground Vehicle Tracking Using Context-Based Sojourn Time Dependent Markov Model and Pseudo-Measurement

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**ABSTRACT** Tracking maneuvering vehicles in complex dynamic environment is a challenging problem for advanced driver assistance system and autonomous driving systems. Most conventional vehicle tracking algorithms can not model the vehicle dynamic exactly because of the uncertain moving behavior. However, due to the on-road capability, vehicles have to subject to various constraints imposed by traffic rules and roads. Taking advantage of those context information can refine and improve the performance of tracking as it provides additional prior information for vehicles' dynamic behavior. To achieve this goal, this paper presents a novel context-enhanced tracking approach that exploits the context information to reduce the uncertainty of dynamic estimation. A new context-based sojourn-time dependent semi-Markov (STDM) model, called sojourn-time dependent semi-Markov variable structure interacting multiple model (STDM-VSIMM), is proposed to describe the vehicle's longitudinal acceleration process. In order to cope with the context information into STDM model, a context-based Bayesian network is presented to replace the fixed model transition probabilities with sojourn-time dependent transition probabilities. Compared with traditional interacting multiple model tracking with fixed transition probability, this adaptation switching strategy makes the vehicle motion sequence closer to the natural behavior and improve the tracking performance. Furthermore, a novel pseudo-measurement is constructed to formulate the road-map constraint in tracking process for reducing uncertain on mobility constraints. Simulation results shows that the proposed STDM-VSIMM can achieve better performance after considering the context.

**INDEX TERMS** Vehicle tracking, context-enhanced, sojourn-time dependent Markov model, pseudo-measurements.

## I. INTRODUCTION

Tracking ground vehicle targets is important for many applications in traffic surveillance [1], advanced driver assistance systems (ADAS) [2] and autonomous driving systems [3]. The major issue in vehicle tracking research is to find exact dynamic model to estimate target's kinematics state, such as position, velocity and acceleration. Many dynamic models for target tracking have been proposed in recent years [4]. To handle the highly maneuvering target, the interacting multiple

model (IMM) [5], [6] estimator has proven to be a state-of-the-art framework, which operates different dynamic models in parallel all the time. However, the distinguishing feature of vehicle tracking is that the vehicle motions subjected to various constraints imposed by traffic rules and roads. These external information can significantly help to improve tracking performance, since they provide additional prior knowledge to reduce target motions uncertainty. Therefore, the goal of this paper is to track maneuvering vehicles with the prior information.

In the field of information fusion, the prior knowledge that "surrounds" a situation of interest in the world could be

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considered as contextual information [7]. The significance of contextual information have been gradually demonstrated in an increasing number of works [8], [9]. Many existing works on contextual information that enhance target tracking can be classified in terms of the contextual sources and divided into the following two categories.

The first category of context is the static context, which includes physical and logical structures, such as road maps, geographic information system (GIS) data, urban environment, etc. The static context is generally considered as a constraining parameter of motion model which can refine state estimators directly. Many works have been proposed to model this context into tracking process. In [1], a road map constrained approach was presented where the directional process noise (DPN) of dynamics model was proposed and handled by variable structure interacting multiple model (VSIMM). The road constraint assume the DPN in longitudinal direction is more uncertainty than lateral noise, which the standard motion models assume the noise in both directions are equal. The different of IMM and VSIMM is the motion model set, where the VSIMM is an adaptive version of the fixed IMM estimator, and the variable model set is adjusted according to the current road information. This VSIMM has proven to be one of the most popular estimator for its simplicity and intuitiveness, and it has become a benchmark framework in vehicle target [10]–[12]. Unlike the DPN method into the dynamics modeling, it is usual having the on-road constrained represented as a set of waypoints and junctions to describe the equality constrained dynamics [13]–[17]. The linear equality constraint method was proposed in [13], [14] and the optimal solution is the projection of Kalman filter estimates onto the road. A unified framework for constrained dynamic modeling is proposed in [16] by giving the constraints and an approximate dynamic equation. The static context constraints also be extended to the non-linear filters. Making use of the particle filter, an attractor-directed particle filter [18] was presented where the attractive potential field [19] was used to guide the particles in inference propagation. In this approach, an attractor point is assigned according to the prior distribution to form an attractive force on the particles. In an analogous way, incorporating the static context knowledge with the particle filter also can be found in visual tracking applications [20], [21].

In other cases, the context is dynamic context, which includes contextual variables, such environment conditions, traffic conditions, road vehicles, etc. The dynamic context can come from knowledge, indirect inference processes and learned data and can not update the tracking processes directly. Therefore, the dynamic context is usual in the examples at high-level fusion processes (JDL Levels 2-4), such as situation understanding [22] and intent assessment [23]. Since situations and intent rarely happen independently, incorporation of dynamic context not only provide the situation understanding, but also improve accuracy and robustness of a tracking process.

While the utilization of static context (road maps) has been explored in detail in the literature, using dynamic context in tracking process has not yet been well-addressed. Therefore, using dynamic context for vehicle tracking is a topic that received lots of attention in the recent years. The main challenge is how to develop a representation of dynamic context and utilize the context in the tracking process. One way of incorporating the dynamic context into the filtering process is described with the help of dynamic Bayesian networks (DBNs) [24], in which a composite Bayesian approach is proposed for wildlife protection application. The inferred dynamic context from DBNs allows a better discrimination and analysis of complex maneuvering behavior of the targets. However, the proposed works combining the context information with particle-filter, which may lead to high computational cost. Combining context reasoning with the spatio-temporal relationships of the events to support tracking process is proposed in [25], in which target actions were viewed as a hidden Markov model with a relevant spatial and event context associated with each node. However, the context in [25] is only selected based on goal driven sets of actions, such as going right, going left and going straight.

Thus the motivation for this paper is to propose a new vehicle tracking algorithm that comprehensively integrates various types of context with sojourn-time-dependent Markov model. A new context-based sojourn-time dependent semi-Markov (STDM) model, called sojourn-time dependent semi-Markov variable structure interacting multiple model (STDM-VSIMM), is proposed to integrate the longitudinal acceleration process into the tracking process. The transition probability matrix of STDM-VSIMM is calculated by context-based Bayesian network and the sojourn time probability mass function (pmf). Two important issues are addressed in this paper. First, in order to cope with the dynamic context, the context BN is presented to inference heterogeneity context variables. Unlike [9], [24] use the BN for describing the mobility of the targets, the proposed method incorporation the BN with STDM-VSIMM, which the fixed model transition probabilities matrix (TPM) are replaced by a set of transition probability functions of sojourn time. Using the dynamic context in TPM is more suitable for describing driving behavior than using context directly. Second, the effects of the model parameter on the estimation accuracy are analyzed and evaluated by the posterior Cramer-Rao lower bound (PCRLB). What's more, considering the static context with road map, a novel context pseudo-measurement is also proposed to constraint the motion.

The main contribution of the paper can be described as four aspects. First, the longitudinal acceleration is modeled as a semi-Markov jump process (semi-Markov chain) to describe the vehicles' dynamic behavior. Second, a new context-based sojourn-time dependent semi-Markov model, the STDM-VSIMM, in which replaced the fixed model transition probabilities with sojourn-time dependent transition probabilities, is presented to integrate the context-based

motion behavior into the tracking process. Third, a Bayesian approach is proposed to sound incorporation of different types of uncertain context information on vehicle’s mobility. Fourth, a new kind of pseudo-measurement is constructed to model road constraints into estimation process.

The rest of this paper is organized as follows. Section II describes the vehicle tracking in detail, which includes tracking scenario, dynamics model, directional process noise and measurement model. In Section III, the STDm-VSIMM is proposed, which incorporated the context BNs with sojourn time. Then the pseudo-measurement is also proposed to describe the road-map constraint. Section IV introduces the context-enhanced multi-vehicle tracking algorithm. Section V shows the experimental results of the proposed context-enhanced algorithm. The comparisons are made among different tracking algorithm. The conclusions are given in Section VI.

**II. PROBLEM FORMULATION**

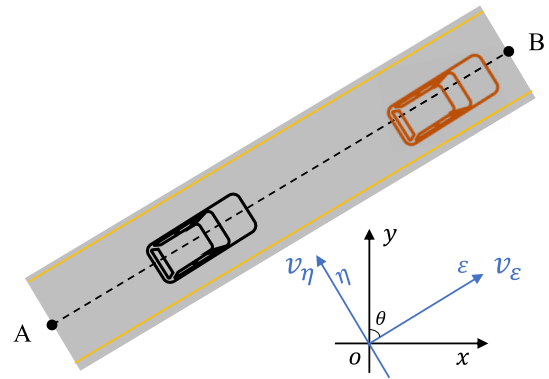
**A. CONTEXT VEHICLE TRACKING SCENARIO**

This work considers the problem of tracking the highly maneuvering ground vehicle with the application to ADAS and autonomous driving system. The ground vehicles are moving on the road with diverse motion states (speed up, slow down, stop, etc.) based on the traffic situation. Two major challenges for vehicle tracking arises from the motion uncertainty and measurement uncertainty. Those uncertainty refer to the fact that the true dynamic behaviors of the vehicle are unknown for tracker and the sensors always have clutter and false alarms, respectively. Thus, the task of vehicle tracking is aim at handling those uncertainty.

Since the road network and vehicle targets in ADAS and autonomous driving system are usually known or detected, the motion of targets are constrained by some external factors. Those external factors are considered as the context information, which affects or constrains the dynamic behavior of targets and the measurement process. The context might be partially or completely known during the estimation and should be used to obtain better estimates.

In this paper, the context information addressed with two kinds of information: road maps and interaction information which are divided into static context and dynamic context, respectively. The road map [1], [11] is defined using a sequence of connected linear segments with way points, which is belong to static context. The interaction information [12], [26], [27] describes the interaction effect with surrounding vehicles. For example, the vehicle usually keep a safe distance from other vehicles and avoid potential collision risks. This interactions between vehicles have been considered as dynamic context.

A sample example are shown in Fig.1, two vehicle are moving from way points A to B in a 2-D plane and in single lane road. The vehicles are constrained by the road and surrounding vehicles, which is different from the general off-road tracking. Restrictions on whether vehicles will be influenced by other vehicles are typically determined by the



**FIGURE 1.** An example of context tracking scenario: vehicles travel along the road center line with the segment connecting A to B. Vehicles motion are constrained by the road and front vehicle.  $xoy$  is the Cartesian coordinate;  $\xi o\eta$  is the coordinate constructed by along and orthogonal road directions.

surrounding circumstances. For example, the safe distance, the velocity, the weather conditions or the traffic.

**B. DYNAMICS MODEL**

The vehicle can be dynamically model in various ways for different kind of vehicle i.e., rail, grounded, air, etc [28]–[34]. For the application of ADAS and autonomous driving system, the vehicle is usually treated as a point object without a shape, especially in using the radar sensors. In 2-D physical world, the point motion can be described by its 2-D position and velocity vectors. Thus, a general evolution of its state can be described by the discrete time nonlinear model [5] in the Cartesian coordinate system as

$$\mathbf{x}(k + 1) = f(\mathbf{x}(k), u(k), v(k)) \tag{1}$$

where  $\mathbf{x}(k) \triangleq [x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}]'$ ,  $x$  and  $y$  are the positions of vehicle in the X and Y directions, respectively. The corresponding velocities and accelerations are  $\dot{x}$  and  $\dot{y}$ , and,  $\ddot{x}$  and  $\ddot{y}$ , respectively.  $f$  is a nonlinear (prediction) function,  $u(k)$  is the control input vector.  $v(k)$  is the model process noise, which has some assumed distribution.  $k$  is the current time instant. The corresponding discrete time linear motion model is given by

$$\mathbf{x}(k + 1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G}u(k) + \mathbf{\Gamma}v(k). \tag{2}$$

As pointed out in review paper [4], the major issue for vehicle tracking is motion uncertainty. The accurate dynamic model of the target is not available to the tracker. Specifically, in the linear motion (2), the actual control input  $u$  of the target, and possibly the actual form of  $\mathbf{F}$  are most often unknown to the tracker. Due to a lack of knowledge of its dynamics, target motion modeling is thus one of the first tasks for maneuvering target tracking. In the target motion modeling, the most popular approach is to model the input  $u$  as a random process. The reader is referred to [4], [5] for a comprehensive survey on the available techniques. With a 1-D state vector  $\mathbf{x} \triangleq [x, \dot{x}, \ddot{x}]$ , we summarize the models used in this paper.

1) Nearly Constant Velocity (NCV) Model: The NCV model is a nonmaneuvering model with control input  $u$  is zero. The white noise process  $v(k)$  is used to model the effect of the control input  $u$ . Note that the “nearly constant-velocity model” emphasizes that these accelerations are small. The corresponding state model representation is given by (2) with  $u(k) \equiv 0$ , process noise variance  $\sigma_{NCV}^2$ , and

$$\mathbf{F}_{NCV} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{\Gamma}_{NCV} = \begin{bmatrix} T^2/2 \\ T \\ 0 \end{bmatrix}. \quad (3)$$

where  $T$  is the sampling interval.

2) Nearly Constant Acceleration (NCA) Model: The NCA model is a maneuvering model and the acceleration is a Wiener process. Specifically, the acceleration increment is an independent (white noise) process [5]. The corresponding state model representation is given by (2) with  $u(k) \equiv 0$ , process noise variance  $\sigma_{NCA}^2$ , and

$$\mathbf{F}_{NCA} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{\Gamma}_{NCA} = \begin{bmatrix} T^2/2 \\ T \\ 1 \end{bmatrix}. \quad (4)$$

3) Mean-Adaptive Acceleration (MAA) Model: The NCA model assumes the acceleration is an independent increment, which is uncoupled of any other time. Make use of the Markov process, the acceleration at one time is related with its values at other times. In other words, the acceleration (control input) can model as time-correlated stochastic process by considering the Markov process. The Singer model [35] model the acceleration is zero-mean first-order stationary Markov process. Further, the extension of Singer model with an adaptive mean, is called mean-adaptive acceleration (MAA) model [36], which have to a non-zero mean of the acceleration. Specifically the acceleration  $a(k) = \tilde{a}(k) + \bar{a}(k)$ , where  $\tilde{a}(k)$  is the zero-mean Singer acceleration process and  $\bar{a}(k)$  is the mean of the acceleration. The corresponding discrete-time equivalent is given by (2) with  $u(k) \equiv \bar{a}(k)$ , and

$$\mathbf{F}_{MAA} = \begin{bmatrix} 1 & T & (\alpha T - 1 + e^{-\alpha T})/\alpha^2 \\ 0 & 1 & (1 - e^{-\alpha T})/\alpha \\ 0 & 0 & e^{-\alpha T} \end{bmatrix}, \quad \mathbf{\Gamma}_{MAA} = \mathbf{I},$$

$$\mathbf{G}_{MAA} = \begin{bmatrix} T^2/2 \\ T \\ 1 \end{bmatrix} - \begin{bmatrix} (\alpha T - 1 + e^{-\alpha T})/\alpha^2 \\ (1 - e^{-\alpha T})/\alpha \\ e^{-\alpha T} \end{bmatrix}, \quad (5)$$

where  $\alpha = 1/\tau$ , where  $\tau$  is a maneuver-specific time constant. The discrete-time vector process noise covariance is  $2\alpha\sigma^2(k)^2\mathbf{Q}$ , where  $\mathbf{Q} = [q_{ij}(\alpha, T)]_{i,j=1}^3$  is a symmetric matrix [35].

$$\sigma^2(k) = \begin{cases} \frac{4 - \pi}{\pi}(a_{\max} - \hat{a}(k))^2 & \text{if } \hat{a}(k) > 0, \\ \frac{4 - \pi}{\pi}(a_{-\max} + \hat{a}(k))^2 & \text{if } \hat{a}(k) < 0. \end{cases} \quad (6)$$

where  $\hat{a}(k)$  is the filter estimate acceleration,  $a_{-\max}$  and  $a_{\max}$  is the lower and upper bounds of the estimate acceleration

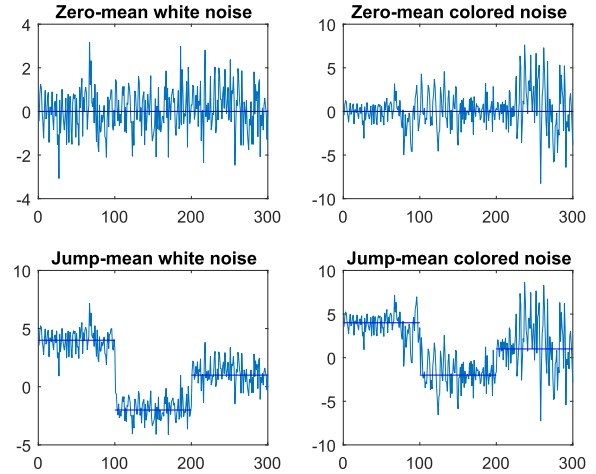


FIGURE 2. Representative random acceleration process models.

$\hat{a}(k)$ . The mean acceleration term  $\bar{a}(k + 1)$  is given by following recursion as

$$\bar{a}(k + 1) = e^{-\alpha T}\hat{a}(k) + (1 - e^{-\alpha T})\hat{a}(k). \quad (7)$$

4) Interacting Multiple Model (IMM) Model: The multiple model (MM) filter captures different dynamic models in parallel all the time, and the transition between those models is described by a first-order Markov chain. As the MM considered the hypotheses with all possible model sequences, the computational complexity of the optimal solution is increased exponentially. The most popular suboptimal solution the is IMM filter [6], where the model hypotheses are merged all branches that belong to the same model at each time instant. The IMM is extended to variable structure IMM (VS-IMM) [37] with variable mode sets. More detail about IMM and VSIMM the reader is referred to [6]. The IMM is used in paper as a benchmark model and detailed steps are introduced in Section IV-B.

5) Semi-Markov Jump Process Models: The Singer model approximates the target acceleration as a continuous-time zero-mean Markov process. However, in some case the acceleration process is non-zero mean, which are assumed piecewise constant, such as the pilot maneuver behavior with the commands [38]. The representative random noise process to model the acceleration process are shown in Fig.2. This piecewise-constant random processes is a semi-Markov jump process (the engineering-oriented description was given in [38]). The different of Markov process and semi-Markov process is that the semi-Markov process has the time interval stays in a state, which is called sojourn time.

Classic work about on the semi-Markov problems are proposed in [39], which the semi-Markov chain is used for estimating state transition. Reference [39] assumed the known jump acceleration levels  $\tilde{a}_1, \dots, \tilde{a}_n$ , and known transition probability  $P\{u(t_k) = \tilde{a}_j \mid u(t_{k-1}) = \tilde{a}_i\} = \pi_{ij}, i, j = 1, \dots, n$ , the sojourn time has an exponential distribution, the unknown acceleration mean  $u(t)$  is estimated by a weighted sum of the levels:  $u(t) = \sum \tilde{a}_i P\{u(t) = \tilde{a}_i \mid c(t)\}$  where as in a



multiple-model formulation, the weight is the posterior probability of each context level. The survey paper [4] pointed out that semi-Markov jump acceleration process with exponentially distributed sojourn time in each state is referred to as a Markovian jump-mean acceleration model. The unknown jump acceleration mean  $u(t)$  amounts to multiple models of the input in a degenerated form.

In [40] a sojourn-time dependent semi-Markov IMM (STDM-IMM) algorithm is introduced, and this semi-Markov process is actually a sojourn-time dependent Markov chain. The solution is to replace Markov chain to semi-Markov chain by a sojourn-time dependent transition probabilities. The advantage of STDM-IMM algorithm is it can handle general sojourn time distributions. Three important issues associated with this model are the design of acceleration model levels, the sojourn time and the probability mass functions of sojourn time. However, the disadvantage of STDM-IMM is to estimate the unknown sojourn time in each model, which have been pointed out by [41]. Since the vehicle acceleration behavior rarely happen independently and subject to various environmental influence, making use of those context information can provide a new way for estimating the unknown sojourn time. To achieve this goal, we integrates the context information with STDM-IMM algorithm, in which the sojourn time of each acceleration noise models can be updated by corresponding context information.

**C. DIRECTIONAL PROCESS NOISE**

The traditional dynamic models in Cartesian coordinates assume that the target can move in any direction. Therefore, use equal process noise variances in both the X and Y directions. This means that for off-road targets the motion uncertainties in both directions are identical. Due to the on-road attribute, the directional process noise (DPN) were used in [1] for modeling the process noise with corresponding road network. Furthermore, by modeling vehicle dynamic directly in road coordinates, the motion uncertainty of vehicle is along and across the direction of the road, which is called longitudinal and lateral process noise in 2-d road coordinate system [11]. As shown in Fig. 1, the process noise components along X and Y directions are given by  $v_x$  and  $v_y$ , respectively. Similarly,  $v_\xi$  and  $v_\eta$  are the longitudinal and lateral process noise, respectively.

Taking advantage of DPN, we can describe the vehicle longitudinal and lateral maneuvering behavior using different dynamic models. In this paper, we consider the high maneuvering behavior in the longitudinal direction, which means the longitudinal uncertainty is higher than lateral direction. Thus, the longitudinal noise  $v_\xi$  is assume as semi-Markov jump process (semi-Markov chain). Likewise to the traditional modeling, the lateral noise is assumed as a small white noise. Thus, this problem is how to use the context information to describe the noise  $v_\xi$  with the semi-Markov jump process.

It is noted that the if the state estimation is carried out in the  $xoy$  coordinate system, the variances of the longitudinal

and lateral process noise components in the on-road motion models need to be converted into unified Cartesian  $oxy$  coordinates. Given the angle between Cartesian coordinates and 2-D road coordinates, the transformation covariance matrix  $\mathbf{Q}$  can be written as

$$\mathbf{Q} = \begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix} \begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \quad (8)$$

**D. MEASUREMENT MODEL**

Many sensors can be used to track vehicles, such as camera, lidar, millimeter-wave radar, etc. In connected environment, the cooperative tracking sensors such as global positioning system (GPS) and V2X communication are generally used for vehicle tracking. In this paper, we assumed all the measurement are converted to 2-D road coordinate [11], and the measurement vector  $\mathbf{z}(k)$  is given by

$$\mathbf{z}(k) = \begin{bmatrix} \xi(k) \\ \eta(k) \end{bmatrix} + \mathbf{w}(k) = \begin{bmatrix} \xi(k) \\ \eta(k) \end{bmatrix} + \begin{bmatrix} w_\xi(k) \\ w_\eta(k) \end{bmatrix} \quad (9)$$

where  $\mathbf{w}(t)$  is the  $2 \times 1$  Gaussian measurement noise vector.  $w_\xi(k)$  and  $w_\eta(t)$  are corresponding independent Gaussian with measurement noises with  $w_\xi(k) \sim \mathcal{N}(0, \sigma_\xi^2)$  and  $w_\eta(t) \sim \mathcal{N}(0, \sigma_\eta^2)$ . The covariance matrix  $\mathbf{R}(t)$  of measurement is given by

$$\mathbf{R}(t_k) = \begin{bmatrix} R_{\xi\xi}(t) & R_{\eta\xi}(t) \\ R_{\xi\eta}(t) & R_{\eta\eta}(t) \end{bmatrix} = \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}. \quad (10)$$

We denote the set of all measurements received at time step  $t_k$  as

$$\mathbf{z}(k) = \{\mathbf{z}_m(k), m = 1, 2, \dots, M(k)\} \quad (11)$$

where  $M(k)$  is the number of measurements in the a time stamp  $k$ .

The cumulative set of measurements available up to time step  $k$  as

$$\mathbf{z}^k = \{\mathbf{z}(i), i = 1, 2, \dots, k\} \quad (12)$$

In the vehicle tracking scenario, the vehicle is moving along the road and the trajectory of a target along the road-map can be considered as a context information. For example, the vehicle is assumed to travel on a straight road in Fig.1. Using the road line constraint from A to B in the filtering process can significant improve the tracking performance. This is also called destination constraint in [42], [43], which is used for tracking an anti-radiation missile to protect a radar array from its threat. Unlike the constraint method for target motion model [14], [17], [44], the idea reformulated the measurement model with the road constraint. In this paper, the pseudo-measurement is constructed to model road constraints, which is discussed in Section III-D.

### III. CONTEXT-BASED SOJOURN-TIME DEPENDENT MARKOV MODEL AND PSEUDO-MEASUREMENTS

This section introduces the context-enhanced tracking algorithm that incorporating the context with sojourn-time dependent Markov Model and pseudo-measurements. We first model the longitudinal acceleration process as a semi-Markov chain, then we discuss type of dynamic context and how to incorporate it into the semi-Markov chain with the Bayesian network. Lastly, the pseudo-measurement is proposed to describe the road map constraint.

#### A. SOJOURN-TIME DEPENDENT MARKOV MODEL

In intelligent transportation system, the driver behavior under the different context cases, will have different maneuvering behavior with a random sojourn time. For example, the driver will make a heavy deceleration in a short time to avoid accident under the crisis situation. While in normal situation, the driver will have a slightly deceleration with a longer time to keep a safe distance. To incorporate the sojourn-time dependence vehicle motion, the semi-Markov jump process is considered for longitudinal state estimation in this paper.

One optimal solution to the semi-Markov jump problem was proposed with STDM-IMM algorithm in [40], which extends the IMM filter from the normal Markov system to the semi-Markov system with sojourn-time-dependent transition probability. This method can handle arbitrary sojourn-time distributions, and it is more effective to model the acceleration than the a white noise model with a mean of random jumps [39]. In STDM-IMM, the Markov chain of the model transitions is replaced with sojourn-time dependent transition probabilities.

Let us consider a first-order semi-Markov chain, the transition probability of the STDM chain is a function of sojourn time  $\tau$  and is defined as [45]

$$\pi'_{ij}(\tau) = P\{m_j(k) | m_i(k-1), \tau_i(k-1) = \tau\} \quad (13)$$

where  $\tau_i(k-1)$  is the sojourn time of model  $i$  at time  $k-1$ . If the  $k=0$ , the sojourn time is assumed as  $\tau=1$ . Thus the  $\tau$  can take are from 1 to  $k$ .  $m(k)$  is the currently active model (model state) at time  $k$ . The event mode  $m_j(k)$  is denoted as  $m_j(k) \triangleq \{m(k) = m_j\}$ .

In order to get the sojourn-time-dependent transition probability, the prior knowledge of the sojourn-time pmf is defined as

$$g_i(\tau) \triangleq P\{\tau_i(k) = \tau | m_i(k), \mathbf{z}^k\}, \quad (14)$$

and the cumulative distribution function of the sojourn time pmf is written as

$$G_i(\tau) = \sum_{\tau=1}^k g_i(\tau). \quad (15)$$

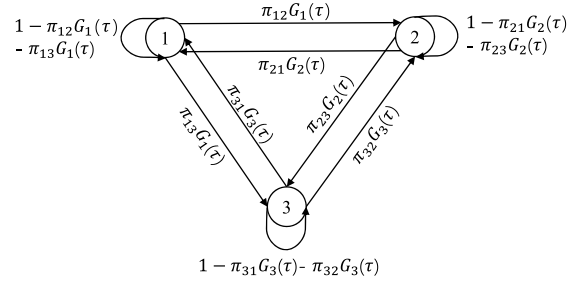


FIGURE 3. A sojourn-time-dependent semi-Markov system with three models.

The conditional probability of transition from  $i$  to  $j$  at time  $k-1$  using the (13) and (15) as follow:

$$\pi'_{ij}(\tau) = \begin{cases} \pi_{ij} G_i(\tau) & \text{if } i \neq j \\ 1 + \pi_{ii} G_i(\tau) - \sum_{j'=1}^m \pi_{ij'} G_i(\tau) & \text{if } i = j \end{cases} \quad (16)$$

where  $\pi'_{ij}(\tau)$  is the conditional probability of transition with sojourn time.  $\pi_{ij}$  is the original transition probability.  $m$  is the number of models in the STDM system. A three models STDM system is shown in Fig.3. It can be seen that the STDM system becomes a Markov system when  $G_i(\tau)$  is set as 1, which can be solved by the traditional IMM.

However, due to the sojourn time  $\tau$  at each model is unknown, the conditional probability mass function (pmf) of the sojourn time  $\pi_{ij}(\tau)$  is difficult to obtain, this disadvantage is also pointed out in [41].

To address the limitations of sojourn time, we propose the a context-based sojourn-time-dependent semi-Markov model, called STDM-VSIMM. The main improvement is to propose adaptive sojourn time estimator, in which the sojourn time of each acceleration noise models can be estimated by corresponding dynamic context. Moreover, the STDM-IMM can be extend to the variable structure framework with context.

In proposed STDM-VSIMM method, the sojourn time  $\tau$  is calculated in each context hypothesis as the model history. The following assumption is made

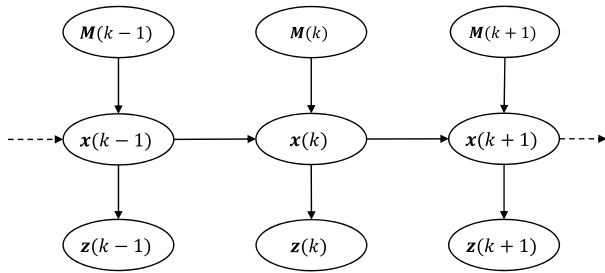
$$P\{\tau_i(k) | \mathbf{c}^k\} = P\{\tau_i(k) | \mathbf{z}^k\} \quad (17)$$

where  $\mathbf{c}^k$  denotes the entire interaction information about the context influence collected at this location up to time  $k$ .

In order to model and infer the context influence for sojourn-time in the STDM process, the probability distributions  $P\{m_i(k) | \mathbf{c}^k\}$  is defined, where  $m_i(k) = true$  means that the vehicle target is moving with the specific acceleration noise  $m_i$  at time  $k$ . Then, we can rewrite sojourn-time probability distribution as

$$P\{\tau_i(k) | \mathbf{c}^k\} = \sum_{m_i} P\{\tau_i(k) | m_i(k)\} P\{m_i(k) | \mathbf{c}^k\} \quad (18)$$

where the simplest case  $m_i(k)$  is a binary variable:  $m_i(k) = true$  represents the model is selected in the context hypotheses, while  $m_i(k) = false$  represents the context model is



**FIGURE 4.** Bayesian network describing correlations between the states  $x(k)$  of a dynamic process, measurement  $z(k)$  and context interaction influences on the process.

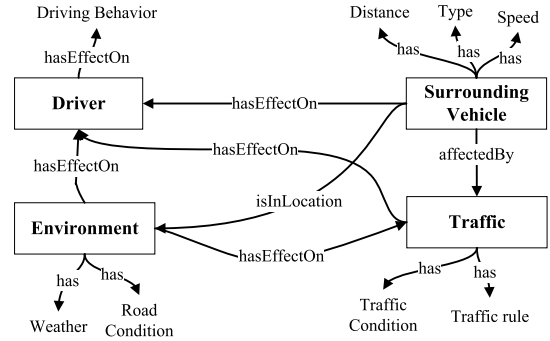
not selected.  $P\{\tau_i(k) | m_i(k)\}$  is the probability about the  $\tau_i(k)$  when  $m_i(k)$  is selected, which can be set from the prior knowledge.  $P\{m_i(k) | \mathbf{c}^k\}$  is the model probability under the contextual information, this probability is inferred in following Section III-B and Section III-C.

For each model, the highest probability  $P\{\tau_i(k) | \mathbf{c}^k\}$  is selected, then sojourn-time  $\tau_i(k)$  updated by the probability history of each models. Accordingly, the  $P(m_i(k) | \mathbf{c}^k)$  meet  $\sum_{i=1}^m P(m_i(k) | \mathbf{c}^k) = 1$ . An overall of the discrete causal model describing the context STDm process is shown in Fig.4. Note that the  $\mathbf{m}(k) = \{m_i(k), i = 1, 2, \dots, M\}$  is the set of the motion models.

**B. DYNAMIC CONTEXT INTERACTION FACTORS**

This section discusses correlated context information which can influence the longitudinal acceleration noise of the motion model. The contextual information [7] is a prior knowledge in which aids understanding the situation. The significance of context information for tracking problem has been demonstrated in [24], [25]. However, many works considers the road network as the constraining factor of motion model, few of relevant context research provides the context interaction in understanding the driving maneuvering behavior. In this paper, we develop a representation of the interaction relationships between vehicles and utilizes context information in the filtering process. This preliminary works about context-aided tracking have been presented in the conference paper [27].

Considering the major factors of road incidents [46], the context information about the interaction influence is generalized for representing the uncertain driving behavior. As shown in Fig. 5, Surrounding vehicles (pedestrian), traffic, environment and driver can assumed as four types of context interaction. As the data of driver (drowsiness, distraction) are hardly to obtain, this impact factor is not considered in this paper. The impact factor of surrounding vehicles consider the interaction between vehicles. Vehicles usually keep a safe distance to avoid potential collision risks. The motion of vehicles will changing when interaction happens. The context of traffic information is seen as the constraint in motion estimators. For example, the vehicle will slow down and stop when traffic light is red or the stop sign in front.



**FIGURE 5.** Semantic network of main factors involved in driving maneuvering behavior.

Environment context include the weather, road condition and even the noises of tracking sensor. For example, the detection probability  $P_d$  is influenced by the foggy or snowy weather.

**C. DYNAMIC CONTEXT FACTORS IN STDm PROCESSES**

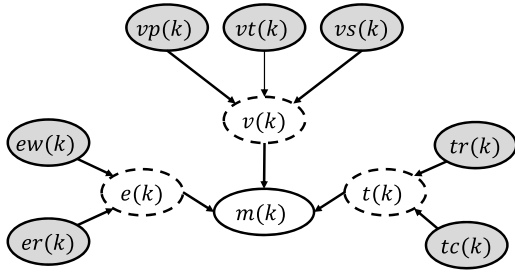
In this section, the Bayesian network is used to efficient handle the dynamic context between such diverse interaction information and the maneuvering model probability  $P\{m_i(k)|\mathbf{c}^k\}$ . The contextual information in Bayesian network are definition in the Fig.6 and by the following parents nodes:

- $v(k)$  is a multi-state variable that represents the vehicle nodes for the maneuvering probability of each acceleration noise.  $vs(k)$  and  $vp(k)$  are the position and speed represent the distance and speed between the tracking target and the interactive objects, respectively.  $vt(k)$  is the type of interaction target, such as car, truck, etc.
- $t(k)$  is a multi-state variable that represents the congestion level of the traffic condition. It denotes the maneuvering probabilistic of the vehicle.  $tr(k)$  is the traffic rule states, such as stop and keep. It represents the red light or stop sign for driving behavior.  $tc(k)$  is the traffic condition state, such as heavy(congestion), medium(normal), light(smooth).
- $e(k)$  is a multi-state variable, which represent environmental factors effecting driving behavior.  $er(k)$  is road nodes such as snow road, rain road, dry road, etc.  $ew(k)$  represents the weather visibility, such as far, medium, near, etc.
- $m_i(k)$  is a multi-state variable, which represent interaction model probability.

As shown in Fig.6, the joint probability distribution of the context factors representation is shown in (19).

$$\begin{aligned}
 &P\{\mathbf{v}(k)\} \\
 &= P\{vp(k)\}P\{vt(k)\}P\{vs(k)\} \\
 &\quad * P\{ew(k)\}P\{er(k)\}P\{t(k)\}P\{tc(k)\} \\
 &\quad * P\{v(k) | vp(k), vt(k), vs(k)\}P\{e(k) | ew(k), er(k)\} \\
 &\quad * P\{e(k) | ew(k), er(k)\}P\{m_i(k) | v(k), e(k), t(k)\} \quad (19)
 \end{aligned}$$

where the  $P\{\mathbf{v}(k)\}$  is the set of all variables involved in the factorization. This factorization is key to cost efficient



**FIGURE 6.** The Bayesian network describing contextual interaction. Variables  $v$ ,  $t$  and  $e$  represent vehicle, traffic, environment of the context. The grey variables are real context data. The variable  $m_i$  represent derived interaction model probability for each model  $i$ .

construction of the context interaction models and it supports sound and efficient computation of  $p(m_i(k)|\mathbf{c}^k)$  with a suitable inference algorithm, such as message-passing algorithm, junction-tree propagation. The essential of the Bayesian network based context interaction model is calculating the posterior probability distribution  $p(m_i(k)|\mathbf{c}^k)$  of the model probability index node  $m_i(k)$  according context evidence set.

Considering that the dynamic context interaction model is a single connected network, and the path is short generally, the message-passing algorithm [47] is suitable for the threat model reasoning to meet the real time requirement. As shown in Fig.7, each node in the message-passing algorithm calculates own posterior probability according to the message from evidence nodes and internal conditional probability of the node, and propagates to adjacent nodes, until influence of the evidence spreads to all nodes of the network. Assume the graph into two parts: an upper subgraph  $G_{BA}^+$ , and a lower subgraph  $G_{BA}^-$ , then the context data is denoted as  $D_{BA}^+$  and  $D_{BA}^-$ , respectively. Thus, using Bayes' rule and Bayes Network conditional independence assumption, the posterior probability distribution of  $A_i$  can be written as

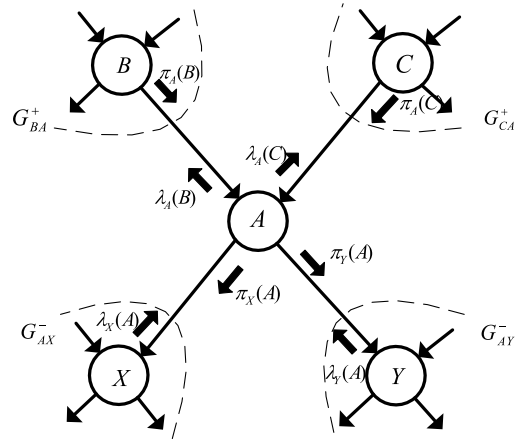
$$\begin{aligned}
 &P(A_i|D_{BA}^+, D_{CA}^+, D_{AX}^-, D_{AY}^-) \\
 &= \alpha P(A_i|D_{BA}^+, D_{CA}^+) P(D_{AX}^-|A_i) P(D_{AY}^-|A_i) \\
 &= \alpha P(D_{AX}^-|A_i) P(D_{AY}^-|A_i) \\
 &\quad \cdot \left[ \sum_{jk} P(A_i|B_j, C_k) P(B_j|D_{BA}^+) P(C_k|D_{CA}^+) \right] \quad (20)
 \end{aligned}$$

where  $\alpha$  is a normalizing constant.

Starting from the definition of  $\pi_A(B_j) = P(B_j|D_{BA}^+)$ , then it becomes:

$$\begin{aligned}
 &P(A_i|D_{BA}^+, D_{CA}^+, D_{AX}^-, D_{AY}^-) \\
 &= \alpha \lambda_X(A_i) \lambda_Y(A_i) \cdot \left[ \sum_{jk} P(A_i|B_j, C_k) \pi_A(B_j) \pi_X(C_k) \right]
 \end{aligned}$$

For more detail about probabilistic reasoning of BN, the reader is referred to [47]. As soon as  $P\{m_i(k)|\mathbf{c}^k\}$  is updated, it is integrated in (18) to calculate sojourn time probability.



**FIGURE 7.** Message-passing algorithm in a part of Bayesian network.

**D. PSEUDO-MEASUREMENTS**

In this section, pseudo-measurements are constructed to describe the road map constraint. Inspired by [42], the pseudo-measurement constraints are constructed for anti-radiation missile system. We proposed a pseudo-measurement method for vehicle tracking problem. As the vehicle move on the road, the vehicle always follows the road line. If the positions of two road map points on the straight line are available at  $(\xi(r-1), \eta(r-1))$  and  $(\xi(r), \eta(r))$ , the constraint trajectory can be constructed by

$$\frac{\eta(r-1) - \eta(r)}{\xi(r-1) - \xi(r)} = \frac{\eta(k) - \eta(r)}{\xi(k) - \xi(r)} = \frac{\dot{\eta}(k)}{\dot{\xi}(k)} \quad (21)$$

where the  $(\xi(k), \eta(k))$  and  $(\dot{\xi}(k), \dot{\eta}(k))$  are vehicle position and velocity at time k, respectively. If  $(\xi(r-1), \eta(r-1))$  and  $(\xi(r), \eta(r))$  are both known, the constraints is in (21).

In this paper, we consider use the measurement point  $(\xi_m(k), \eta_m(k))$  to replace the road point  $(\xi(r-1), \eta(r-1))$ , then a pseudo-measurement can be written as:

$$\gamma(k) = \dot{\eta}(k)(\xi_m(k) - \xi(r)) - \dot{\xi}(k)(\eta_m(k) - \eta(r)) = 0 \quad (22)$$

where  $\gamma(k)$  is the constant and error-free. This error-free pseudo-measurement can also be augmented into the measurement vector to convert a context constrained problem into the regular filtering problem. The pseudo-measurement with the context road-map constraint can be written as:

$$\begin{aligned}
 \mathbf{z}(t_k) &= \begin{bmatrix} \xi(k) \\ \eta(k) \\ \gamma(k) \end{bmatrix} \\
 &= \begin{bmatrix} \xi(k) \\ \eta(k) \\ \dot{\eta}(k)(\xi_m(k) - \xi(r)) - \dot{\xi}(k)(\eta_m(k) - \eta(r)) \end{bmatrix} + \begin{bmatrix} w_\xi(k) \\ w_\eta(k) \\ w_\gamma(k) \end{bmatrix} \quad (24)
 \end{aligned}$$

with the covariances

$$\mathbf{R}(k) = \begin{bmatrix} R_{\xi\xi}(k) & R_{\xi\eta}(k) & R_{\xi\gamma}(k) \\ R_{\xi\eta}(k) & R_{\eta\eta}(k) & R_{\eta\gamma}(k) \\ R_{\xi\gamma}(k) & R_{\eta\gamma}(k) & R_{\gamma\gamma}(k) \end{bmatrix} \quad (25)$$



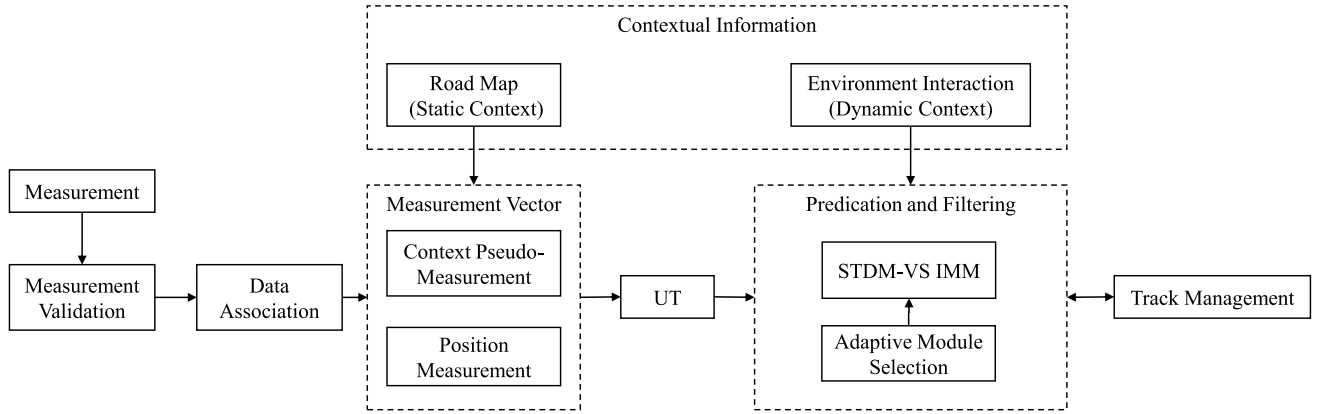


FIGURE 8. Block diagram of the proposed context-based multi-vehicle tracking algorithm.

$$= \begin{bmatrix} \sigma_{\xi}^2 & 0 & 0 \\ 0 & \sigma_{\eta}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (26)$$

where  $R_{\xi\eta}(k)$  and  $R_{\eta\xi}(k)$  are the cross-covariances of the of measurement  $\xi$  and  $\eta$ , respectively.  $\sigma_{\gamma}^2$  is the variance of pseudo-measurement  $\gamma$ , which is assumed as error-free.

#### IV. CONTEXT-BASED VEHICLE TRACKING ALGORITHM

In this section, the context-enhanced multi-vehicle tracking algorithm was proposed, which considers the context information by integrating the context-based STDM system and pseudo-measurement into tracking process. Based on the methods described in section III, the block diagram of proposed tracking algorithm is shown in Fig.8.

##### A. UNSCENTED TRANSFORMATION

Considering the high nonlinearity of (24), the unscented transformation (UT) is to deal with the nonlinear measurements. 1) Generate Sigma Points: Sigma points are generated as the following:

$$\begin{aligned} \chi_0(k) &= \hat{\mathbf{x}}(k|k-1) \\ \chi_i(k) &= \hat{\mathbf{x}}(k|k-1) + \left[ \sqrt{(N_z + \lambda)\mathbf{P}(k|k-1)} \right]_i, \\ & \quad i = 1, \dots, N_x^s \\ \chi_i(k) &= \hat{\mathbf{x}}(k|k-1) + \left[ \sqrt{(N_z + \lambda)\mathbf{P}(k|k-1)} \right]_{i-N_z}, \\ & \quad i = N_x^s + 1, \dots, 2N_z \end{aligned} \quad (27)$$

where  $\left[ \sqrt{(N_z + \lambda)\mathbf{P}(k|k-1)} \right]_i$  represents the  $i$ th column of matrix  $\left[ \sqrt{(N_z + \lambda)\mathbf{P}(k|k-1)} \right]$ .  $N_z$  denotes the dimension of the augmented measurement vector  $\mathbf{z}(k)$ ,  $\lambda$  can be any number except for  $\lambda + N_z = 0$ .  $\mathbf{P}(k|k-1)$  is one step predict covariance. The weights  $W_i^m$  and  $W_i^c$  are calculated as

$$\begin{aligned} W_0^m &= \frac{\lambda}{N_z + \lambda} \\ W_0^c &= \frac{\lambda}{N_z + \lambda} + (1 - \alpha^2 + \beta) \end{aligned}$$

$$W_i^m = W_i^c = \frac{1}{2(N_z + \lambda)}, \quad i = 1, \dots, 2N_z. \quad (28)$$

where  $\alpha$  and  $\beta$  are empirical parameters of the sigma points, and  $\alpha$  is used to determine the spread of the sigma points around  $\bar{\mathbf{z}}(k)$ ,  $\beta$  incorporates prior knowledge of the distribution of  $\mathbf{z}(k)$ .

Then, instantiating each sigma point using the nonlinear functions in (24) to obtain the posterior sigma points as

$$\hat{\chi}_i(k|k-1) = h(\chi_i(k)) = \begin{bmatrix} \xi_i(k) \\ \eta_i(k) \\ \gamma_i(k) \end{bmatrix}, \quad i = 0, 1, \dots, 2N_z. \quad (29)$$

Finally, the mean  $\hat{\mathbf{z}}(k)$ , variance and cross covariance are given by

$$\hat{\mathbf{z}}(k) = \sum_{i=0}^{2N_z} W_i^{mean} \hat{\chi}_i(k) \quad (30)$$

$$\mathbf{P}_z(k) = \sum_{i=0}^{2N_z} W_i^c (\hat{\chi}_i(k|k-1) - \hat{\mathbf{z}}(k)) (\hat{\chi}_i(k|k-1) - \hat{\mathbf{z}}(k))' \quad (31)$$

$$\mathbf{P}_{zx}(k) = \sum_{i=0}^{2N_z} W_i^c (\chi_0(k) - \hat{\mathbf{x}}(k|k-1)) (\hat{\chi}_i(k|k-1) - \hat{\mathbf{z}}(k))' \quad (32)$$

##### B. STDM-VSIMM FILTER

The proposed STDM-VSIMM filter is modified by the basis of the STDM-IMM filter, where vehicle dynamics are captured by multiple motion models, and where the transition between those models is described by a sojourn-time dependent Markov chain.

The main improvement is to propose an adaptive sojourn-time dependent transition probability based on dynamic context, which is closer to the natural behavior of the maneuvering vehicle. The adaptive sojourn-time dependent transition probability of STDM system is introduced in Section III. Another modification is that the IMM algorithm is extended to VSIMM algorithm with the current context. This context-based variable structure eliminates the need for carrying all

the possible models throughout the entire tracking period, significantly reducing computational complexity and improving tracking accuracy.

Let us consider the linear discrete-time kinematic models given in (2) with multiple models by

$$\mathbf{x}(k) = \mathbf{F}\mathbf{x}(k-1) + \mathbf{\Gamma}v(k, m(k)) \quad (33)$$

where  $m(k)$  denotes the model at time  $k$ . The acceleration noise is semi-Markov jump Gaussian noise. For each model  $i$ ,  $v(k, m_i(k)) \sim \mathcal{N}(u_i, \mathbf{Q}_i)$  with mean  $u_i$  and covariance  $\mathbf{Q}_i$ . One cycle of STDMM-VSIMM at time  $k$  can be summarized as following steps.

1) Calculation of the mixing probabilities:

$$\mu_{ij}(k-1|k-1) = \frac{1}{\bar{c}_j} \pi_{ij} \mu_i(k-1|k-1), \quad i, j = 1, \dots, N \quad (34)$$

where  $\bar{c}_j = \sum_{i=1}^N \pi_{ij} \mu_i(k-1|k-1)$ ,  $j = 1, \dots, N$ .  $\pi_{ij}$  is the sojourn-time-dependent transition probability from model  $i$  to model  $j$ , which is proposed in Section III.

2) Mixing: the mean and the covariance matrix for the  $j$ th mode-matched filter are given by

$$\begin{aligned} \hat{\mathbf{x}}^{0j}(k-1|k-1) &= \sum_{i=1}^N \mu_{ij}(k-1|k-1) \hat{\mathbf{x}}^i(k-1|k-1) \quad (35) \\ \mathbf{P}^{0j}(k-1|k-1) &= \sum_{i=1}^N \mu_{ij}(k-1|k-1) \{ \mathbf{P}^i(k-1|k-1) \\ &\quad + [\hat{\mathbf{x}}^i(k-1|k-1) - \hat{\mathbf{x}}^{0j}(k-1|k-1)] \\ &\quad \cdot [\hat{\mathbf{x}}^i(k-1|k-1) - \hat{\mathbf{x}}^{0j}(k-1|k-1)]' \} \quad (36) \end{aligned}$$

3) Mode matched filtering: for  $j = 1, 2, \dots, N$ , use the estimate (35), covariance (36) and observation  $\mathbf{z}(k)$  as input to match  $m_j(k)$ . The Kalman filter with unscented transformation (27)-(32) are used to yield  $x_j(k|k)$  and  $P_j(k|k)$ . The Gaussian likelihood for the measurement can be calculated from the innovation vector  $v_j(k)$  and its covariance  $\hat{\mathbf{S}}_j(k)$  following

$$\Lambda_j(k) = \frac{\exp \left[ -(1/2) (v_j(k))' (\hat{\mathbf{S}}_j(k))^{-1} v_j(k) \right]}{\sqrt{|2\pi \hat{\mathbf{S}}_j(k)|}} \quad (37)$$

where  $v_j(k) = \mathbf{z}(k) - \hat{\mathbf{z}}(k|k-1; \hat{\mathbf{x}}^{0j}(k-1|k-1))$ ,  $z_k$  is the measurement and  $\hat{\mathbf{z}}(k|k-1; \hat{\mathbf{x}}^{0j}(k-1|k-1))$  is the predicted measurement for mode filter  $m_j(k)$ , respectively.  $\hat{\mathbf{S}}_j(k)$  is the innovation covariance,  $\hat{\mathbf{S}}_j(k) \triangleq \hat{\mathbf{S}}_j(k, \mathbf{P}^{0j}(k-1|k-1))$ .

4) Mode probability update:

$$\mu_j(k|k) = \frac{\mu_j(k|k-1) \Lambda_j(k)}{\sum_j \mu_j(k|k-1) \Lambda_j(k)} \quad (38)$$

5) Estimate: the combined state and the corresponding covariance matrix are calculated from the weighted state

estimates and its covariances

$$\hat{\mathbf{x}}(k|k) = \sum_{i=1}^N \mu_j(k|k) \hat{\mathbf{x}}^i(k|k) \quad (39)$$

$$\begin{aligned} \mathbf{P}(k|k) &= \sum_{i=1}^N \mu_j(k|k) \{ \mathbf{P}^i(k|k) + [\hat{\mathbf{x}}^i(k|k) - \hat{\mathbf{x}}(k|k)] \\ &\quad \cdot [\hat{\mathbf{x}}^i(k|k) - \hat{\mathbf{x}}(k|k)]' \} \quad (40) \end{aligned}$$

6) Variable Filter Module Selection: Since the vehicle may have non-maneuvering state, the variable model is considered to handle this possibility, where the NCV model is used. The basic idea to make multiple motion models adaptive is to use the dynamic context in Section III.

### C. MEASUREMENT VALIDATION

All the measurements should be validated before data association, which can effectively reduce false alarms and avoid erroneous assignments. Validation is performed in two stages.

1) Road constraint validation: As the vehicle target is moving with road constraint, the valid measurement confidence regions will intersect with segment of the road. Based on the pseudo-measurement described in Section III-D, the validation problem is equivalent to testing whether the pseudo-measurement  $\gamma(k)$  is in the confidence regions. To be specific, the confidence region of measurement is given by

$$\mathcal{V}_1(k, \gamma_1) = \gamma(k) \leq \gamma_1 \quad (41)$$

where  $\gamma_1$  is the confidence threshold.

2) Tracking gate: the tracking gate is commonly used for measurement validation, which is formed for each track based on its predicted measurement. The measurement  $\mathbf{z}(k)$  is valid for associating with the  $n$ th track only if the measurement will be in the following region

$$\mathcal{V}_2(k, \gamma_1) = v(k)' \mathbf{S}(k) v(k) \leq \gamma_2 \quad (42)$$

where  $v(k) = \mathbf{z}(k) - \hat{\mathbf{z}}(k|k-1)$ ,  $\gamma_2$  is the validation threshold,  $\hat{\mathbf{z}}(k+1|k)$  is the predicted measurement and  $\mathbf{S}(k)$  is the innovation covariance. Because of the multiple models,  $\hat{\mathbf{z}}(k|k-1)$  and  $\mathbf{S}(k)$  correspond to the IMM module in (37) with the largest  $|\mathbf{S}(k)|$  so that the measurement is validated or rejected by all modules [1].

After the on-road constraint validation and gating validation, the measurements can be considered for data association and track initialization.

### D. DATA ASSOCIATION

In order to track multiple targets with imperfect sensors, it is necessary to handle the measurement-to-track association problem. Measurement provided by sensors are always affected by some clutter, while the nearest neighbor (NN) [6], multiple hypothesis tracking (MHT) [48], joint probabilistic data association (JPDA) [49] and 2-D assignment data association are commonly used for deal with the measurement origin uncertainty. In this paper, the 2-D assignment data

association is used for vehicle tracking. The details on 2-D assignment algorithm can be found in [50].

**E. TRACK MANAGEMENT**

Based on the data association results, track management is used to improve the true track detection probability and to reduce the false track acceptance probability. The track state includes three states: tentative, confirmed and dead. In this paper, the sliding-window based  $M/N$  logic management method is applied to manage the track. All measurements that after measurement validation and data association were not associated with any existing track and are initialized as tentative tracks. Then, a  $M/N$  test (a track is confirmed if at least  $M$  measurement is associated together over  $N$  frames; otherwise, the track needs to be deleted) is used for tentative track. For confirmed track, if the  $M/N$  test is not satisfied, then delete the track. The performance of  $M/N$  logic management method is affected by the variable probability of false alarm (PFA)  $P_d$ . Considering the occlusion environment in vehicle tracking, the 2/2 & 2/3 logic can be used to form tentative tracks.

**V. EXPERIMENTAL RESULTS**

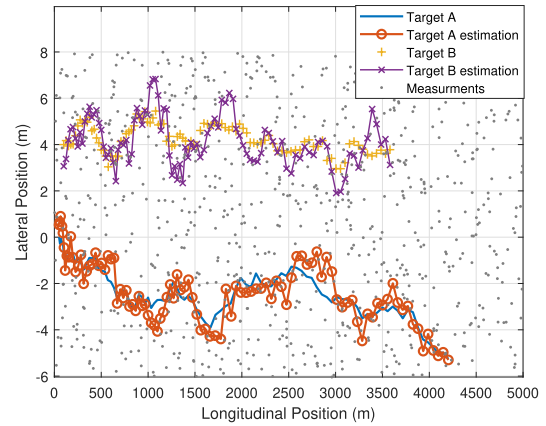
In this section, we demonstrate the performance of the STDM-VSIMM multi-vehicle tracking algorithm through simulation experiments. To analyze the performance, the proposed algorithm is compared with the KF and IMM filters. In order to test the road constraint, the proposed are compared with pseudo-measurement.

**A. SIMULATION SCENARIO**

Two tracking vehicles are simulated to move along the road. They initial positions at 2D road coordinate system are  $(50m, 0m)$  and  $(100m, 4m)$  with corresponding initial longitudinal speed of 10m/s and 25m/s, respectively. Since the lane changing is not the considered in this paper, for simplicity, the lateral speed is assumed as zero and a Gaussian white process noise with variance  $\sigma_{la} = 0.2m/s^2$  is added to model the lateral motion. The longitudinal maneuvers are generated with the non-zeros jump-mean acceleration input, which are shown in detail in Table. 1 and Table. 2. The measurements are assumed reported only position measurement in the 2D road coordinate every  $T = 2s$  with the standard deviations of error being  $\sigma_{\xi} = 2m$  and  $\sigma_{\eta} = 1m$  along the longitudinal and lateral directions, respectively. The target detection probability  $P_D = 0.95$ , and the false alarms are assumed to be uniformly distributed, and with the spatial density  $\lambda = 2.0 \times 10^{-5}/m^2$ . The ground truth trajectories and estimation trajectories are illustrated in Fig.9.

**B. CONFIGURATION OF THE LONGITUDINAL TRACKING ALGORITHMS**

1) KF-based tracking algorithm: the KF tracking algorithm is configured with the nearly constant-acceleration (NCA) model for longitudinal direction. The longitudinal process noise variance of the NCA model is  $\sigma_{lo} = 0.3m/s^2$ .



**FIGURE 9. The ground truth and estimation trajectories.**

**TABLE 1. The context maneuver parameter of target A.**

Times	Acceleration input	Position context	Velocity context
5s-7s	$2m/s^2$	{0.1; 0.3;0.6}	{0.2; 0.3;0.5}
8s-11s	$1m/s^2$	{0.2; 0.5;0.3}	{0.3; 0.4;0.3}
12s-17s	$0.5m/s^2$	{0.3; 0.4;0.3}	{0.6; 0.3;0.2}
40s-46s	$0.5m/s^2$	{0.7; 0.2;0.1}	{0.5; 0.3;0.2}
48s-51s	$1m/s^2$	{0.1; 0.7;0.3}	{0.3; 0.5;0.2}
53s-56s	$2m/s^2$	{0.2; 0.2;0.6}	{0.3; 0.3;0.4}

**TABLE 2. The context maneuver parameter of target B.**

Times	Acceleration input	Position context	Velocity context
20s-22s	$-2m/s^2$	{0.1; 0.3;0.6}	{0.1; 0.5;0.4}
30s-34s	$-0.5m/s^2$	{0.25; 0.5;0.25}	{0.2; 0.5;0.3}
51s-53s	$2m/s^2$	{0.1; 0.2;0.7}	{0.25; 0.25;0.5}
70s-74s	$1m/s^2$	{0.15; 0.6;0.25}	{0.3; 0.3;0.4}

2) IMM-based tracking algorithm: the IMM-based tracking algorithm is configured as nearly constant velocity (NCV) and mean-adaptive acceleration model (MAA) to model the longitudinal motion. The process noise variance of NCV model is  $\sigma_{lo}^{NCA} = 0.1m/s^2$ . The parameters for MAA model is given with  $\alpha = 1/\tau = 1/5$ ,  $a_{max} = 4m/s^2$ , and  $a_{max} = -4m/s^2$ . The process noise variance of the MAA is  $\sigma_{lo} = 0.3m/s^2$ . After the configurations to model the longitudinal motion with NCV and MAA, the IMM is used for estimating the vehicle motion. The initial model probability  $\mu_0 = [0.5, 0.5]$  and the fixed TPM of IMM is giving by (43)

$$\mathbf{\Pi} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}. \tag{43}$$

3) STDM-VSIMM based tracking algorithm: Three jump acceleration process model and a NCV model are deployed

TABLE 3. PCT of  $p(c|v)$ .

Context Interaction $c$	Vehicle interaction $v$		
	HVI	MVI	LVI
HI	0.92	0.12	0
MI	0.2	0.98	0.01
LI	0	0.02	0.98

TABLE 4. PCT of  $p(v|vp)$ .

Vehicle interaction $v$	Relative position $vp$		
	HRP	MTP	LTP
HVI	0.70	0.28	0.02
MVI	0.25	0.55	0.20
LVI	0.02	0.10	0.88

as the model set of STDMM. The jump acceleration noises, namely, low maneuvering, medium maneuvering, high maneuvering, respectively. For each model, the directional process noises are  $w_h(k) \sim \mathcal{N}(2, 1)$ ,  $w_m(k) \sim \mathcal{N}(1, 0.75)$ ,  $w_k(k) \sim \mathcal{N}(0.5, 0.5)$ . In the STDMM-VSIMM filter, the probability mass functions of sojourn time of the jump acceleration noises are

$$\begin{aligned} g_h(\tau) &= a_1 e^{-|\tau-3|} \\ g_m(\tau) &= a_2 e^{-|\tau-5|} \\ g_l(\tau) &= a_3 e^{-|\tau-7|} \end{aligned} \quad (44)$$

where  $a_i, i = 1, 2, 3$  are set to the value which meet  $\sum_{\tau}^{\infty} a_i(\tau) = 1$ .

The STDMM-VSIMM estimator uses initial model probability  $\mu_0 = [0.25, 0.25, 0.25, 0.25]$  and the initial TPM is giving by (16)

$$\mathbf{\Pi} = \begin{bmatrix} 0.1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.1 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.1 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.1 \end{bmatrix}. \quad (45)$$

The STDMM-VSIMM based tracking algorithm estimates the sojourn time according to the pdf of sojourn time using (44) and the context-based Bayesian network. In order to make the BN work, parameterizing the conditional probability table (CPT) for each node is the critical step. For simplicity, the context interaction information in Fig. 6, only the the vehicle interaction node  $v(k)$  is considered in our simulation. According the our previous work about the bayesian network threat assessment in [51], the CPT table is given in Table.3, Table.4, Table.5 to calculate the conditional probability for each node over time after receiving the contextual information.

4) Posterior Cramer-Rao lower bound (PCRLB): The posterior Cramer-Rao lower bound is applied to evaluate the performance of proposed estimation method. According [52]

TABLE 5. PCT of  $p(v|vs)$ .

Vehicle interaction $v$	Relative Speed $vs$		
	HRS	MRS	LRS
HVI	0.72	0.25	0.03
MVI	0.20	0.50	0.30
LVI	0.10	0.20	0.70

and [53], the PCRLB on the augmented state estimation error covariance matrix  $\mathbf{P}_{k+1}$  is the inverse of the Fisher information matrix (FIM)  $J_{k+1}$ , thus

$$\mathbf{P}_{k+1} \triangleq E \left[ \left( \hat{Y}_{k+1} - Y_{k+1} \right) \left( \hat{Y}_{k+1} - Y_{k+1} \right)^T \right] \geq J_{k+1}^{-1}. \quad (46)$$

where  $\hat{Y}_{k+1} \triangleq [\hat{X}_{k+1}^T, \hat{X}_{t,k+1}^T]^T$  be the unbiased estimate of  $Y_{k+1} \triangleq [X_{k+1}^T, X_{t,k+1}^T]^T$  conditioned on the measurement set  $Z_{k+1}$ .

The posterior FIM  $J_{k+1}$  is given with a recursive equation in [54]:

$$J_{k+1} = \underbrace{D_k^{22} - D_k^{21}(J_k + D_k^{11})^{-1}D_k^{12}}_{J_{X,k+1}} + J_{Z,k+1} \quad (47)$$

where

$$D_k^{11} = \mathbb{E}\{-\Delta_{X_k}^{X_k} \ln p(X_{k+1}|X_k)\} \quad (48)$$

$$D_k^{12} = \mathbb{E}\{-\Delta_{X_k}^{X_{k+1}} \ln p(X_{k+1}|X_k)\} \quad (49)$$

$$D_k^{21} = (D_k^{12})^T \quad (50)$$

$$D_k^{22} = \mathbb{E}\{-\Delta_{X_{k+1}}^{X_{k+1}} \ln p(X_{k+1}|X_k)\} \quad (51)$$

$$J_{Z,k+1} = \mathbb{E}\{-\Delta_{X_{k+1}}^{Z_{k+1}} \ln p(Z_{k+1}|X_{k+1})\}. \quad (52)$$

For the linear dynamic system, the FIM  $J_{k+1}$  is given by

$$J_{k+1} = \left( \mathbf{F}_k J_k^{-1} \mathbf{F}_k^T + \mathbf{Q}_k \right)^{-1} + J_{Z,k+1} \quad (53)$$

where  $\mathbf{F}_k \triangleq \text{diag}\{F_k, F_k\}$ ,  $\mathbf{Q}_k \triangleq \text{diag}\{Q_k, Q_{t,k}\}$ , and the FIM is initialized as  $J_0 = \mathbf{P}_0^{-1}$ . Matrix  $J_{Z,k+1}$  gives the measurement contributions to the PCRLB and is given by

$$J_{Z,k+1} = \left[ \frac{\partial h_{k+1}}{\partial \hat{Y}_{k+1}} \right]^T (R_{k+1})^{-1} \left[ \frac{\partial h_{k+1}}{\partial \hat{Y}_{k+1}} \right]. \quad (54)$$

### C. CONFIGURATION OF THE LATERAL TRACKING ALGORITHMS

The lateral tracking algorithms are compared with the method of using the pseudo-measurements. The without pseudo-measurements method is a normal KF filter with the NCV model to tracking vehicles. The without one is the same NCV model as the normal one. The true position of the target A at 100s in road coordinates  $(x_{100}, y_{100}) = (2903m, 0m)$ , is assumed as the prior known road center for testing the proposed pseudo constraint.



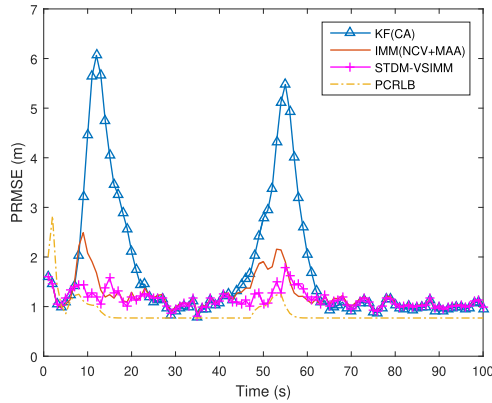


FIGURE 10. Longitudinal PRMSE of vehicle A.

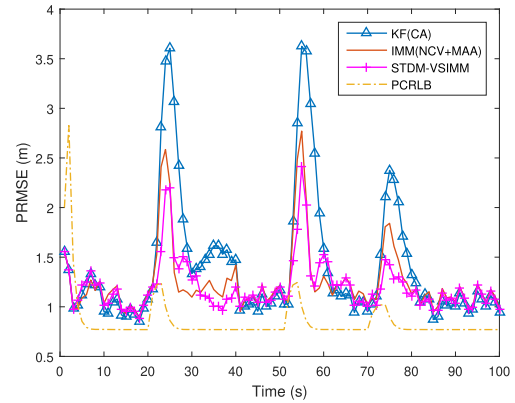


FIGURE 12. Longitudinal PRMSE of vehicle B.

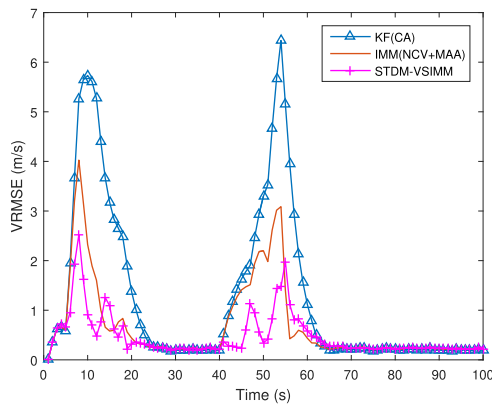


FIGURE 11. Longitudinal VRMSE of vehicle A.

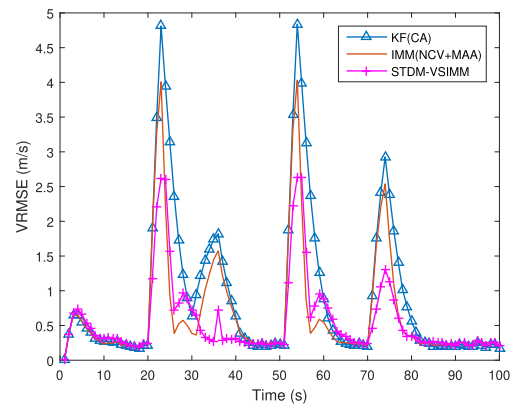


FIGURE 13. Longitudinal VRMSE of vehicle B.

**D. SIMULATION RESULT OF LONGITUDINAL**

In this experiment, the four filters are applied on the test scenario for 200 Monte Carlo runs. The two benchmark trajectories regarding longitudinal and lateral position are shown in Fig.9. The position root mean square error (PRMSE) and velocity root mean square error (VRMSE) are used for measuring the accuracy of the longitudinal motion. Furthermore, the optimal sub-pattern assignment (OSPA) [55], and the proposed PCRLB are also used to evaluate performance.

Fig.10 and Fig.12 show the PRMSE for vehicle A and vehicle B, respectively. We see from the figures that the error gain comes mainly from the maneuvering instant (the maneuvering inputs are given in Table 1 and 2). The KF tracking algorithm gives the poor longitudinal position estimates when the maneuvering happen. The IMM algorithm shows a better result than the KF one. This is because the MAA model is more accurate to model the maneuvering state. It can be seen that the filter performance improvement with the proposed STDM-VSIMM is hence larger when the maneuvering segments are longer. This is because that the STDM-VSIMM longitudinal estimator is more accurate to predicate the sojourn time of the motion behaviors, then it gets better results than other algorithms. What’s more, the STDM-VSIMM based tracking algorithm is further towards

the PCRLB than others. Fig.11 and Fig.13 shows the VRMES for each filter. It is also seen that the proposed filter outperform the KF filter and fixed set IMM filter. This confirms that the sojourn time dependent transition probabilities represent target behavior better than the fixed transition probability.

The overall statistic of longitudinal PRMSE and VRMSE in 200 Monte Carlo runs is shown in Table. 6. As we can see, the STDM-VSIMM filter yields better performance than the other filters with respect to both position and velocity metrics. To be more specific, comparing with the IMM filter, the proposed STDM-VSIMM filter gain 7.4 % PRMSE performance and 18% VRMSE performance.

However, the performance increase is not as much as other paper [12], [41], this is because the measurement standard deviations is  $\sigma_{\xi} = 2m$ . If the we use the noise sensors with the  $\sigma_{\xi} = 5m$  or  $\sigma_{\xi} = 10m$ , the performance increase is better. In fact, it depends on the actual measurement error of the sensors. Nevertheless, it is noticed that the improvement of proposed filter is more obvious than others at the maneuvering time segments. Since the performance gain comes mainly from maneuvering segments, the proposed are shown to have a better ability to track high maneuvering target, such as aircraft, unmanned aerial vehicle, etc.

TABLE 6. The RMSE of different longitudinal tracking algorithms.

	KF	IMM	STDM-VSIMM
Position RMSE A	1.87 m	1.33 m	1.23 m
Velocity RMSE A	1.38 m/s	0.75 m/s	0.59 m/s
Position RMSE B	1.54 m	1.36 m	1.26 m
Velocity RMSE B	0.92 m/s	0.67 m/s	0.57 m/s

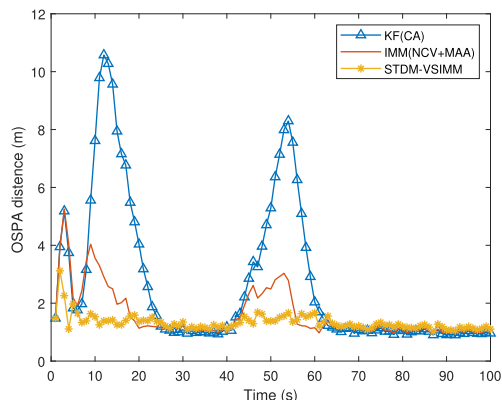


FIGURE 14. The OSPA distance of vehicle A.

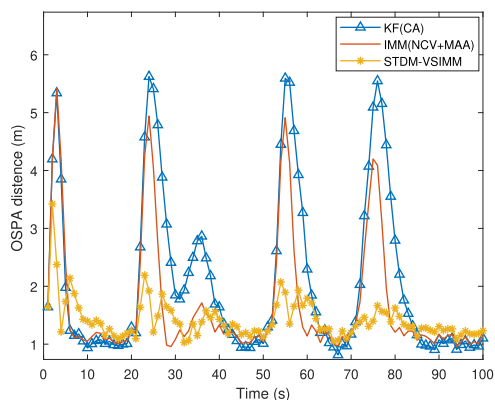


FIGURE 15. The OSPA distance of vehicle B.

The proposed method is also compared using the optimal sub-pattern assignment (OSPA) metric [55]. The OSPA metric computes the distance between two track sets by the localisation error and track label error. The track label component can effectively capture the data association performance. The results of OSPA distance can be seen in Fig.14 and Fig.15. The proposed has the smallest OSPA distance, which represents the smallest estimation error and least amount of incorrect data association. Besides, as we can see in Table. 7, the overall OSPA distance of proposed STDM-VSIMM algorithm performance also outperforms than other algorithms.

Apart from error performance, it is also of interest to study the longitudinal motion model switching probability. The modeling switching is shown with the maneuvering from 40s-56s of vehicle A. In this segment, the acceleration input is  $0.5 m/s^2$  at 40s-46s,  $1 m/s^2$  at 48s-51s,  $2 m/s^2$

TABLE 7. The OSPA distance of different longitudinal tracking algorithms.

	KF	IMM	STDM-VSIMM
Target A	2.67 m	1.59 m	1.33 m
Target B	2.08 m	1.67 m	1.41 m

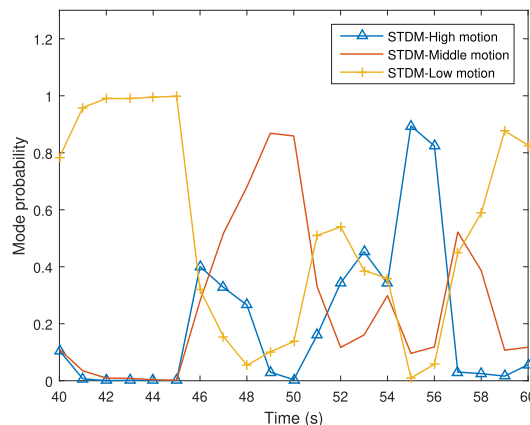


FIGURE 16. The mode probability of vehicle A from 40s to 60s.

TABLE 8. The PRMSE of different lateral tracking algorithms.

	With Pseudo-Measurements	Without Pseudo-Measurements
Position RMSE A	1.02 m	0.83 m
Position RMSE B	0.53 m	0.46 m

at 53s-56s. As we can see from Fig.16, the low motion probability is very close to 1 at 41s-45s. Then the middle motion and high motion probability increases at 46s-51s and 54s-56s, respectively. So the proposed context-based STDM works as expected and estimates the model switching probability accurately. This is because that the sojourn time in model is clearly estimated with context information, whereas the sojourn time in IMM is ambiguous.

E. SIMULATION RESULT OF LATERAL

The lateral position RMSE is shown in Fig.17. The proposed method, which incorporates the road map constraint with pseudo-measurement, produces estimates more accurate than the unconstrained one. The overall statistics of lateral mean PRMSE are shown in Table. 8. Apparently, the lateral estimation result are significantly improved when using the road-map constraint. The mean performance gain is 16.9 % of two vehicles. This demonstrates the effectiveness of incorporating the pseudo-measurement as the road constraint, which significantly improve estimate accuracy.

F. COMPUTATIONAL COST

The lowest computational cost of KF is observed than other filters. The computational complexity of KF is  $O(m)$ , where

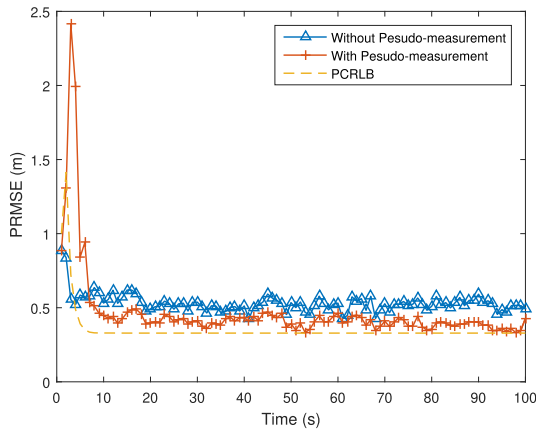


FIGURE 17. Lateral PRMSE of vehicle A.

TABLE 9. Average computation times of longitudinal algorithms.

Methods	KF	IMM	STDM-VSIMM
Times	6.33s	7.21s	7.82s

the  $m$  is the number of targets. The proposed filter has the same computational complexity ( $O(m^2)$ ) as the IMM. But the proposed filter has only slightly higher requirements than the IMM. This is because the computation of adaptive state transition probability cost some executing time. In order to test the cost of the transition probability using Bayesian network, the computation times are compared as shown in Table. 9. All algorithms are running with 2.4 GHz and 8G ram PC after 200 Monte Carlo simulations. As expected, the KF method is fastest algorithms in which the context information is considered. Comparing with the IMM method, the STDM-VSIMM approach requires a higher computation cost due to the computation of the Bayesian network. Note that the computational cost is under 200 Monte Carlo run. However, for only one run case, it still shows an acceptable computational for a real time vehicle tracking application.

## VI. CONCLUSION

This paper introduced a novel context-enhanced vehicle tracking approach, which integrated with types of context information. Based on the fact that the longitudinal acceleration of vehicles is a semi-Markov jump process, a sojourn-time dependent semi-Markov model has been proposed through sufficiently incorporating the dynamic context information. The proposed approach calculate the model switching probabilities with the sojourn time probability mass function and its likelihood, then the normal Markov switching is replaced by the semi-Markov switching. Unlike the traditional way which utilize the context in tracking process directly, the proposed approach inference and reason the context interaction based on the Bayesian networks, which can is more appropriate and accurately. Furthermore, considering the static context with road map, a novel con-

text pseudo-measurement is also proposed to constraint the motion.

The proposed filter is evaluated on numerical simulations, the results demonstrated that the proposed context-enhanced tracking algorithms outperform other algorithms, due to better prediction of target maneuvers caused by vehicle interaction and the road map constraint. It was also shown that the context can significantly reduce the estimation uncertainties and make the overall filtering process more efficient. It is worth noting that the assuming of the sojourn time probability mass function should be on the basis of target maneuver behavior. Nevertheless, since we use a Bayesian inference approach to correlate with the context, the impact of such problems is mitigated.

Furthermore, it would be interesting to apply the proposed dynamic model to other tracking scenarios including the following aspects. First, the STDM-VSIMM can be used in air traffic surveillance due to the fact that most aerial vehicles have a jump acceleration [40], and thereby the tracking performance could be further improved. Second, the context-based method is good for lack of observations (measurements) scenario, which can reduce the estimation uncertainties. Third, some turn models [4] (constant turn model, circular motion model) can also be integrated with the proposed STDM-VSIMM to track more specific targets. In the future research, we will investigate the extension of context-enhanced method for vehicle tracking. For example, using context for improving data association in missing measurement occasions. What's more, we will explore the spatio-temporal context representations for sojourn time with a hidden Markov model.

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