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Models for Multiple Attribute Decision Making With Fuzzy Number Intuitionistic Fuzzy Hamy Mean Operators and Their Application

ZHENG LIANG
Business School, Sichuan University, Chengdu 610064, China
e-mail: zhengliang1986@yeah.net

ABSTRACT In this paper, we expand the Hamy mean (HM) operator and weighted Hamy mean (WHM) with fuzzy number intuitionistic fuzzy numbers (FNIFNs) to propose fuzzy number intuitionistic fuzzy Hamy mean (FNIFHM) operator and fuzzy number intuitionistic fuzzy weighted Hamy mean (FNIFWHM) operator. Then the MADM methods are proposed with these operators. In the end, we utilize an applicable example for comprehensive evaluation of agricultural economic development quality to prove the proposed methods.

INDEX TERMS Multiple attribute decision making (MADM), fuzzy number intuitionistic fuzzy Hamy mean (FNIFHM) operator, fuzzy number intuitionistic fuzzy weighted Hamy operator (FNIFWHM), comprehensive evaluation, agricultural economic development quality.

I. INTRODUCTION

Intuitionistic fuzzy sets (IFSs) designed by Atanassov [1], a generalization of fuzzy set (FS) theory [2], is one of the most successful extensions of FS and employed for depicting the uncertainties in the data. In this set each element is expressed in terms of membership, as well as non-membership, degrees, such that their sum is less than unity [3]. Since its appearance, IFSs and its extension have been successfully applied by the more and more researchers into the various domains for solving the decision-making problems [4]–[10]. Xu [11] investigated the MADM problems, in which attribute weights is incomplete and the attribute values are depicted in intuitionistic fuzzy numbers (IFNs). Li and Wang [12] investigated MADM problems through using IFSs, which the interval fractional programming model is used based on the TOPSIS methodology for solving such problems. Based on the normalized score matrix, Xu and Hu [13] proposed an entropy-based procedure to derive attribute weights. Liu *et al.* [14] presented some novel intuitionistic fuzzy operators by extending the BM operator on the basis of the Dombi operations [15] and designed some MAGDM methods. Gou *et al.* [16] pointed out a novel exponential operational law about IFNs and offered a method which was utilized to aggregate intuitionistic fuzzy information He *et al.* [17] integrated the power averaging operators with IFSs and defined several intuitionistic fuzzy power interaction aggregation operators. Gupta *et al.* [18] extended the fuzzy entropy [19] to IFSs with axiomatic justification and proposed importance of parameter alpha. Bao *et al.* [20] put forward a decision method depending on the prospect theory and the evidential reasoning method under IFSs Krishankumar *et al.* [21] developed IFSP (intuitionistic fuzzy set based PROMETHEE) which was a novel ranking method Lu and Wei [3] designed the TODIM method for performance appraisal on social-integration-based rural reconstruction under IVIFSs. Wu *et al.* [22] designed the algorithms for competiveness evaluation of tourist destination with some interval-valued intuitionistic fuzzy Hamy mean operators Wu *et al.* [23] proposed some interval-valued intuitionistic fuzzy Dombi Heronian mean operators for evaluating the ecological value of forest ecological tourism demonstration areas Zeng *et al.* [24] defined the new MADM method based on the nonlinear programming (NLP) algorithms and the TOPSIS method and interval-valued intuitionistic fuzzy values (IVIFVs) Zeng *et al.* [25] built a new MADM method on the basis of the proposed new score function of IFNs and the proposed modified VIKOR method. In order to make aggregate use of the advantages of both Schweizer-Sklar T-norm and T-conorm (SSTT) and Maclaurin symmetric mean (MSM), Wang and Liu [26] extended SSTT to IFNs and defined Schweizer-Sklar operational rules of IFNs.

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To combine the intuitionistic fuzzy sets (IFSs) and the triangular fuzzy sets (TFSs), Liu and Yuan [27] proposed the fuzzy number intuitionistic fuzzy sets (FNIFSs) which fundamental feature of the FNIFSs is that the membership degree and non-membership degree are represented as triangular fuzzy numbers (TFNs) rather than exact values. Wang [28] proposed some operational laws of FNIFSs and developed some new arithmetic aggregation operators. Li *et al.* [29] defined some similarity measure and fuzzy entropy of FNIFSs Wei *et al.* [30] proposed some MADM issues based on the induced choquet integral with FNIFSs and defined some aggregation operators. Wang [31] defined some geometric aggregation operators under FNIFSs. Wei *et al.* [32] defined the induced OWG operator and induced OWA operator under FNIFSs. Verma [33] developed some generalized Bonferroni mean operator called generalized fuzzy number intuitionistic fuzzy weighted Bonferroni mean (GFNIFWBM) operator for aggregating the FNIFSs. Wang and Wang [34] designed the fuzzy number intuitionistic fuzzy Hamacher correlated geometric (FNIFHCG) operator for performance evaluation of communication network. Wang and Yu [35] defined the fuzzy number intuitionistic fuzzy Hamacher correlated average (FNIFHCA) operator for estimating the rural landscape design schemes. Zhao *et al.* [36] proposed the fuzzy number intuitionistic fuzzy Hamacher power weighted average (FNIFHPWA) operator for evaluating the software quality. Chen and Wang [37] developed the induced fuzzy number intuitionistic fuzzy Hamacher OWA (IFNIFHOWA) operator for performance evaluation of projects loaned by international financial organizations. Fan [38] defined the fuzzy number intuitionistic fuzzy Hamacher power weighted geometric (FNIFHPWG) operator for assessing the knowledge innovation ability of new ventures based on knowledge management. Lu [39] developed the induced fuzzy number intuitionistic fuzzy Hamacher correlated geometric (IFNIFHCG) operator for assessing the international competitiveness of financial system.

However, all above the aggregation operators can not consider the relationship between arguments being aggregated. To overcome this shortcoming, the main goal of this paper is to combine the FNIFSs with the Hamy operators [40]–[42] to propose some new aggregation operators. The innovativeness of the paper can be summarized as follows: (1) we expand the Hamy mean (HM) operator and weighted Hamy mean (WHM) with fuzzy number intuitionistic fuzzy numbers (FNIFNs) to propose fuzzy number intuitionistic fuzzy Hamy mean (FNIFHM) operator and fuzzy number intuitionistic fuzzy weighted Hamy mean (FNIFWHM) operator; (2) Then the MADM methods are proposed with these operators; (3) a case study for comprehensive evaluation of agricultural economic development quality is supplied to show the developed approach; and (4) some comparative studies are provided with the existing methods.

In order to do so, the remainder of this paper is set out as follows. In the next section, we shall propose the concept of FNIFSs. In Section 3, we shall propose some Hamy mean (HM) operators with FNIFSs: the fuzzy number intuitionistic fuzzy Hamy mean (FNIFHM) operator and the fuzzy number intuitionistic fuzzy weighted Hamy mean (FNIFWHM) operator. In Section 4, we shall present a numerical example for comprehensive evaluation of agricultural economic development quality in order to illustrate the method proposed in this paper. Section 5 concludes the paper with some remarks.

II. PRELIMINARIES

A. FUZZY NUMBER INTUITIONISTIC FUZZY SET

Liu and Yuan [27] introduced the concept of FNIFS which fundamental characteristic of the FNIFS is that the values of its membership function and non-membership function are TFNs.

Definition 1 [27]: Given a fixed set $X = \{x_1, x_2, \cdots, x_n\}$, An FNIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A} = \left\{ \left\langle x_i, \tilde{t}_A(x_i), \tilde{f}_A(x_i) \right\rangle | x_i \in X \right\} \tag{1}$$

where $\tilde{t}_A(x_i) \subset [0,1]$ and $\tilde{f}_A(x_i) \subset [0,1]$ are triangular fuzzy numbers, and $\tilde{t}_A(x_i) = (a(x_i), b(x_i), c(x_i)), X \to [0,1]$, $\tilde{f}_A(x) = (l(x_i), m(x_i), p(x_i)), X \to [0,1]$, $0 \le c(x_i) + p(x_i) \le 1, \forall x \in X$.

For convenience, let $\tilde{t}_A(x_i) = (a(x_i), b(x_i), c(x_i)), \tilde{f}_A(x) = (l(x_i), m(x_i), p(x_i))$, so $\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle$ and we call $\tilde{a}(x_i)$ a fuzzy number intuitionistic fuzzy value (FNIFV).

Definition 2 [28], [31]: Let $\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle$ and $\tilde{a}(x_j) = \langle (a(x_j), b(x_j), c(x_j)), (l(x_j), m(x_j), p(x_j)) \rangle$ be two FNIFVs, then (1)–(4), as shown at the bottom of next page.

Definition 3 [28], [31]: Let $\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle$ be a FNIFV, a score function S of a FNIFV $\tilde{a}(x_i)$ can be represented (2), as shown at the bottom of next page.

Definition 4 [28], [31]: Let $\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle$ be a FNIFV, an accuracy function H of a FNIFV $\tilde{a}(x_i)$ can be represented as follows:

$$H(\tilde{a}(x_i)) = \frac{(a(x_i) + 2b(x_i) + c(x_i)) + (l(x_i) + 2m(x_i) + p(x_i))}{4}, \quad H(\tilde{a}(x_i)) \in [0,1]. \tag{3}$$

to evaluate the degree of accuracy of the FNIFV $\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle$, where $H(\tilde{a}(x_i)) \in [0,1]$. The larger the value of $H(\tilde{a}(x_i))$, the more the degree of accuracy of the FNIFV $\tilde{a}(x_i)$ is.Based on the score function S and the accuracy function H, an order relation between two FNIFV is introduced as follows:

Definition 5 [28], [31]: Let $\tilde{a}(x_i)$ and $\tilde{a}(x_j)$ be two FNIFVs, $s(\tilde{a}(x_i))$ and $s(\tilde{a}(x_j))$ be the scores of $\tilde{a}(x_i)$ and $\tilde{a}(x_j)$, respectively, and let $H(\tilde{a}(x_i))$ and $H(\tilde{a}(x_j))$ be the accuracy degrees of $\tilde{a}(x_i)$ and $\tilde{a}(x_j)$, respectively, then if $s(\tilde{a}(x_i)) < s(\tilde{a}(x_j))$, then $\tilde{a}(x_i)$ is smaller than $\tilde{a}(x_j)$, denoted by $\tilde{a}(x_i) < \tilde{a}(x_j)$; if $s(\tilde{a}(x_i)) = s(\tilde{a}(x_j))$, then

(1) if $H(\tilde{a}(x_i)) = H(\tilde{a}(x_j))$, then $\tilde{a}(x_i)$ and $\tilde{a}(x_j)$ represent the same information, denoted by $\tilde{a}(x_i) = \tilde{a}(x_j)$;
(2) if $H(\tilde{a}(x_i)) < H(\tilde{a}(x_j))$, $\tilde{a}(x_i)$ is smaller than $\tilde{a}(x_j)$, denoted by $\tilde{a}(x_i) < \tilde{a}(x_j)$.

B. HM OPERATOR

Definition 6 [42]: The HM operator is defined as follows:

$$\mathrm{HM}^{(k)}(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n)) = \frac{\sum_{1\le i_1<\ldots<i_k\le n}\left(\prod_{j=1}^{k}\tilde{a}(x_{i_j})\right)^{\frac{1}{k}}}{C_n^k} \quad (4)$$

where k is a parameter and $k = 1, 2, \ldots, n$, $i_1, i_2, \ldots, i_k$ are k integer values taken from the set $\{1, 2, \ldots, n\}$ of k integer values, C_n^k denotes the binomial coefficient and $C_n^k = \frac{n!}{k!(n-k)!}$.

III. SOME FNIFHM OPERATORS

A. FNIFHM OPERATOR

In this section, we will combine HM and FNIFNs, and propose the fuzzy number intuitionistic fuzzy Hamy mean (FNIFHM) operator.

Definition 7: Let $\tilde{a}(x_i) = \left\langle \begin{matrix}(a(x_i), b(x_i), c(x_i)),\\ (l(x_i), m(x_i), p(x_i))\end{matrix}\right\rangle$ $(i = 1, 2, \ldots, n)$ be a set of FNIFNs. The FNIFHM operator is:

$$\mathrm{FNIFHM}^{(k)}(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n)) = \frac{\bigoplus_{1\le i_1<\ldots<i_k\le n}\left(\bigotimes_{j=1}^{k}\tilde{a}(x_{i_j})\right)^{\frac{1}{k}}}{C_n^k} \quad (5)$$

Theorem 1: Let $\tilde{a}(x_i) = \left\langle \begin{matrix}(a(x_i), b(x_i), c(x_i)),\\ (l(x_i), m(x_i), p(x_i))\end{matrix}\right\rangle$ $(i = 1, 2, \ldots, n)$ be a set of FNIFNs. The aggregated value by using FNIFHM operators is also a FNIFN where (6), as shown at the bottom of next page.

Proof: See (7), as shown at the bottom of next page.

Thus, We obtain (8), as shown at the bottom of next page.

Thereafter, We obtain (9), as shown at the bottom of next page.

Therefore, We obtain (10), as shown at the bottom of page 5.

Hence, (6) is kept.

Example 1: Let

$$\begin{aligned}\tilde{a}(x_1) &= \{(0.2, 0.3, 0.4), (0.4, 0.5, 0.5)\},\\ \tilde{a}(x_2) &= \{(0.1, 0.1, 0.3), (0.2, 0.4, 0.6)\},\\ \tilde{a}(x_3) &= \{(0.2, 0.2, 0.6), (0.4, 0.4, 0.6)\}\\ \tilde{a}(x_4) &= \{(0.4, 0.5, 0.6), (0.5, 0.6, 0.7)\}\end{aligned}$$

be four FNIFNs, and suppose $k = 2$, then according to (6), we have $\mathrm{FNIFHM}^{(2)}(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n))$, as shown at the bottom of page 6. Then we will give some properties of FNIFHM operator.

Property 1 (Idempotency): If $\tilde{a}(x_i) = \langle(a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i))\rangle (i = 1, 2, \ldots, n)$ are equal, then

$$\mathrm{FNIFHM}^{(k)}(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n)) = \tilde{a}(x) \quad (11)$$

Property 2 (Monotonicity): Let $\tilde{a}(x_i) = \langle(a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i))\rangle (i = 1, 2, \ldots, n)$ and $\tilde{a}'(x_i) = \langle(a'(x_i), b'(x_i), c'(x_i)), (l'(x_i), m'(x_i), p'(x_i))\rangle (i = 1, 2, \ldots, n)$ be two sets of FNIFNs. If $\tilde{a}(x_i) \le \tilde{a}'(x_i)$ hold for all i, then

$$\begin{aligned}&\mathrm{FNIFHM}^{(k)}(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n))\\ &\quad\le \mathrm{FNIFHM}^{(k)}\left(\tilde{a}'(x_1), \tilde{a}'(x_2), \cdots, \tilde{a}'(x_n)\right) \quad (12)\end{aligned}$$

Property 3 (Boundedness): Let $\tilde{a}(x_i) = \langle(a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i))\rangle (i = 1, 2, \ldots, n)$ be a set of FNIFNs. If $\tilde{a}(x_i)^- = \min_i \tilde{a}(x_i)$, $\tilde{a}(x_i)^+ = \max_i \tilde{a}(x_i)$, then

$$\begin{aligned}\tilde{a}(x)^- &\le \mathrm{FNIFHM}^{(k)}(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n))\\ &\le \tilde{a}(x)^+ \quad (13)\end{aligned}$$

$$(1)\ \tilde{a}(x_i) \oplus \tilde{a}(x_j) = \left\{\left(\begin{matrix}a(x_i) + a(x_j) - a(x_i)a(x_j),\\ b(x_i) + b(x_j) - b(x_i)b(x_j),\\ c(x_i) + c(x_j) - c(x_i)c(x_j)\end{matrix}\right), \left(\begin{matrix}l(x_i)l(x_j),\\ m(x_i)m(x_j),\\ p(x_i)p(x_j)\end{matrix}\right)\right\};$$

$$(2)\ \tilde{a}(x_i) \otimes \tilde{a}(x_j) = \left\{\left(\left(\begin{matrix}a(x_i)a(x_j),\\ b(x_i)b(x_j),\\ c(x_i)c(x_j)\end{matrix}\right)\right), \left(\begin{matrix}l(x_i) + l(x_j) - l(x_i)l(x_j),\\ m(x_i) + m(x_j) - m(x_i)m(x_j),\\ p(x_i) + p(x_j) - p(x_i)p(x_j)\end{matrix}\right)\right\};$$

$$(3)\ \lambda\tilde{a}(x_i) = \left\{\left(\begin{matrix}1 - (1 - a(x_i))^{\lambda},\\ 1 - (1 - b(x_i))^{\lambda},\\ 1 - (1 - c(x_i))^{\lambda}\end{matrix}\right), \left(\begin{matrix}(l(x_i))^{\lambda},\\ (m(x_i))^{\lambda},\\ (p(x_i))^{\lambda}\end{matrix}\right)\right\}, \quad \lambda \ge 0;$$

$$(4)\ (\tilde{a}(x_i))^{\lambda} = \left\{\left(\begin{matrix}(a(x_i))^{\lambda},\\ (b(x_i))^{\lambda},\\ (c(x_i))^{\lambda}\end{matrix}\right), \left(\begin{matrix}1 - (1 - l(x_i))^{\lambda},\\ 1 - (1 - m(x_i))^{\lambda},\\ 1 - (1 - p(x_i))^{\lambda}\end{matrix}\right)\right\}, \quad \lambda \ge 0.$$

$$S(\tilde{a}(x_i)) = \frac{a(x_i) + 2b(x_i) + c(x_i)}{4} - \frac{l(x_i) + 2m(x_i) + p(x_i)}{4}, \quad S(\tilde{a}(x_i)) \in [-1, 1]. \quad (2)$$

Property 4 (Commutativity): Let $\tilde{a}(x_i) = \langle(a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i))\rangle (i = 1, 2, \ldots, n)$ and $\tilde{a}'(x_i) = \langle(a'(x_i), b'(x_i), c'(x_i)), (l'(x_i), m'(x_i), p'(x_i))\rangle (i = 1, 2, \ldots, n)$ be two sets of FNIFNs. Then

$$\mathrm{FNIFHM}^{(k)}\left(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n)\right) = \mathrm{FNIFHM}^{(k)}\left(\tilde{a}'(x_1), \tilde{a}'(x_2), \cdots, \tilde{a}'(x_n)\right) \tag{14}$$

where $\tilde{a}'(x_j)\,(j = 1, 2, \cdots, n)$ is any permutation of $\tilde{a}(x_j)\,(j = 1, 2, \cdots, n)$.

B. THE FNIFWHM OPERATOR

In actual MADM, it's important to consider attribute weights. This section will propose the fuzzy number intuitionistic fuzzy weighted Hamy mean (FNIFWHM) operator as follows.

Definition 8: Let $\tilde{a}(x_i) = \left\langle \begin{array}{l} (a(x_i), b(x_i), c(x_i)), \\ (l(x_i), m(x_i), p(x_i)) \end{array} \right\rangle$ $(i = 1, 2, \ldots, n)$ be a set of FNIFNs with their weight vector be $w_i = (w_1, w_2, \ldots, w_n)^T$, thereby satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Then we can define the FNIFWHM

$$\mathrm{FNIFHM}^{(k)}\left(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n)\right) = \frac{\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}} \tilde{a}(x_{i_j})\right)^{\frac{1}{k}}}{C_n^k}$$
$$= \left\{ \begin{array}{l} \left(\begin{array}{l} 1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} a(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}, \\ 1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} b(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}, \\ 1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} c(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}} \end{array} \right), \left(\begin{array}{l} \left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} (1-l(x_{i_j}))\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}, \\ \left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} (1-m(x_{i_j}))\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}, \\ \left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} (1-p(x_{i_j}))\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}} \end{array} \right) \end{array} \right\} \tag{6}$$

$$\overset{k}{\underset{j=1}{\otimes}} \tilde{a}(x_{i_j}) = \left\{ \begin{array}{l} \left(\prod_{j=1}^{k} a(x_{i_j}), \prod_{j=1}^{k} b(x_{i_j}), \prod_{j=1}^{k} c(x_{i_j})\right), \\ \left(1-\prod_{j=1}^{k}(1-l(x_{i_j})), 1-\prod_{j=1}^{k}(1-m(x_{i_j})), 1-\prod_{j=1}^{k}(1-p(x_{i_j}))\right) \end{array} \right\} \tag{7}$$

$$\left(\overset{k}{\underset{j=1}{\otimes}} \tilde{a}(x_{i_j})\right)^{\frac{1}{k}} = \left\{ \begin{array}{l} \left(\left(\prod_{j=1}^{k} a(x_{i_j})\right)^{\frac{1}{k}}, \left(\prod_{j=1}^{k} b(x_{i_j})\right)^{\frac{1}{k}}, \left(\prod_{j=1}^{k} c(x_{i_j})\right)^{\frac{1}{k}}\right), \\ \left(\begin{array}{l} 1-\left(\prod_{j=1}^{k}(1-l(x_{i_j}))\right)^{\frac{1}{k}}, 1-\left(\prod_{j=1}^{k}(1-m(x_{i_j}))\right)^{\frac{1}{k}}, \\ 1-\left(\prod_{j=1}^{k}(1-p(x_{i_j}))\right)^{\frac{1}{k}} \end{array} \right) \end{array} \right\} \tag{8}$$

$$\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}} \tilde{a}(x_{i_j})\right)^{\frac{1}{k}} = \left\{ \left(\begin{array}{l} 1-\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} a(x_{i_j})\right)^{\frac{1}{k}}\right), \\ 1-\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} b(x_{i_j})\right)^{\frac{1}{k}}\right), \\ 1-\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k} c(x_{i_j})\right)^{\frac{1}{k}}\right) \end{array} \right), \left(\begin{array}{l} \prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}(1-l(x_{i_j}))\right)^{\frac{1}{k}}\right), \\ \prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}(1-m(x_{i_j}))\right)^{\frac{1}{k}}\right), \\ \prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}(1-p(x_{i_j}))\right)^{\frac{1}{k}}\right) \end{array} \right) \right\} \tag{9}$$

operator as follows:

$$\mathrm{FNIFWHM}_w^{(k)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)=\frac{\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}}\left(\tilde{a}(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}}{C_n^k} \tag{15}$$

Theorem 2: Let $\tilde{a}(x_i)=\left\langle\begin{matrix}(a(x_i),b(x_i),c(x_i)),\\(l(x_i),m(x_i),p(x_i))\end{matrix}\right\rangle$ $(i=1,2,\ldots,n)$ be a set of FNIFNs. The aggregated value by using FNIFWHM operator is also a FNIFN where (16), as shown at the bottom of next page.

Proof: From definition 2, we can obtain,

$$\left(\tilde{a}(x_{i_j})\right)^{w_{i_j}}=\left\{\left(\begin{matrix}\left(a(x_{i_j})\right)^{w_{i_j}},\\\left(b(x_{i_j})\right)^{w_{i_j}},\\\left(c(x_{i_j})\right)^{w_{i_j}}\end{matrix}\right),\left(\begin{matrix}1-\left(1-l(x_{i_j})\right)^{w_{i_j}},\\1-\left(1-m(x_{i_j})\right)^{w_{i_j}},\\1-\left(1-p(x_{i_j})\right)^{w_{i_j}}\end{matrix}\right)\right\} \tag{17}$$

Thus,

$$\overset{k}{\underset{j=1}{\otimes}}\left(\tilde{a}(x_{i_j})\right)^{w_{i_j}}=\left\{\left(\begin{matrix}\prod_{j=1}^{n}\left(a(x_{i_j})\right)^{w_{i_j}},\\\prod_{j=1}^{n}\left(b(x_{i_j})\right)^{w_{i_j}},\\\prod_{j=1}^{n}\left(c(x_{i_j})\right)^{w_{i_j}}\end{matrix}\right),\left(\begin{matrix}1-\prod_{j=1}^{n}\left(1-l(x_{i_j})\right)^{w_{i_j}},\\1-\prod_{j=1}^{n}\left(1-m(x_{i_j})\right)^{w_{i_j}},\\1-\prod_{j=1}^{n}\left(1-p(x_{i_j})\right)^{w_{i_j}}\end{matrix}\right)\right\} \tag{18}$$

Therefore, We obtain (19), as shown at the bottom of page 7.

Thereafter, We obtain (20), as shown at the bottom of page 7.

Furthermore, We obtain (21), as shown at the bottom of page 7.

Hence, (16) is kept.

Example 2: Let

$$\begin{aligned}\tilde{a}(x_1)&=\{(0.2,0.3,0.4),(0.4,0.5,0.5)\}\\\tilde{a}(x_2)&=\{(0.1,0.1,0.3),(0.2,0.4,0.6)\},\\\tilde{a}(x_3)&=\{(0.2,0.2,0.6),(0.4,0.4,0.6)\},\\\tilde{a}(x_4)&=\{(0.4,0.5,0.6),(0.5,0.6,0.7)\}\end{aligned}$$

be four FNIFNs, and suppose $k=2$, $w=(0.2,0.1,0.5,0.2)^T$, then according to (16), we have $\mathrm{FNIFWHM}_w^{(2)}(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n))$, as shown at the bottom of page 8. Then we will give some properties of FNIFWHM operator.

Property 5 (Idempotency): If $\tilde{a}(x_i)=\langle(a(x_i),b(x_i),c(x_i)),(l(x_i),m(x_i),p(x_i))\rangle(i=1,2,\ldots,n)$ are equal, then

$$\mathrm{FNIFWHM}_w^{(k)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)=\tilde{a}(x) \tag{22}$$

Property 6 (Monotonicity): Let $\tilde{a}(x_i)=\langle(a(x_i),b(x_i),c(x_i)),(l(x_i),m(x_i),p(x_i))\rangle(i=1,2,\ldots,n)$ and $\tilde{a}'(x_i)=\langle(a'(x_i),b'(x_i),c'(x_i)),(l'(x_i),m'(x_i),p'(x_i))\rangle(i=1,2,\ldots,n)$ be two sets of FNIFNs. If $\tilde{a}(x_i)\le\tilde{a}'(x_i)$ hold for all i, then

$$\begin{aligned}&\mathrm{FNIFWHM}_w^{(k)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)\\&\quad\le\mathrm{FNIFWHM}_w^{(k)}\left(\tilde{a}'(x_1),\tilde{a}'(x_2),\cdots,\tilde{a}'(x_n)\right)\end{aligned} \tag{23}$$

Property 7 (Boundedness): Let $\tilde{a}(x_i)=\langle(a(x_i),b(x_i),c(x_i)),(l(x_i),m(x_i),p(x_i))\rangle(i=1,2,\ldots,n)$ be a set of FNIFNs. If $\tilde{a}(x_i)^-=\min_i\tilde{a}(x_i)$, $\tilde{a}(x_i)^+=\max_i\tilde{a}(x_i)$, then

$$\begin{aligned}\tilde{a}(x)^-&\le\mathrm{FNIFWHM}_w^{(k)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)\\&\le\tilde{a}(x)^+\end{aligned} \tag{24}$$

Property 8 (Commutativity): Let $\tilde{a}(x_i)=\langle(a(x_i),b(x_i),c(x_i)),(l(x_i),m(x_i),p(x_i))\rangle(i=1,2,\ldots,n)$ and $\tilde{a}'(x_i)=\langle(a'(x_i),b'(x_i),c'(x_i)),(l'(x_i),m'(x_i),p'(x_i))\rangle(i=1,2,\ldots,n)$ be two sets of FNIFNs. Then

$$\begin{aligned}&\mathrm{FNIFWHM}_w^{(k)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)\\&\quad=\mathrm{FNIFWHM}_w^{(k)}\left(\tilde{a}'(x_1),\tilde{a}'(x_2),\cdots,\tilde{a}'(x_n)\right)\end{aligned} \tag{25}$$

where $\tilde{a}'(x_j)(j=1,2,\cdots,n)$ is any permutation of $\tilde{a}(x_j)$ $(j=1,2,\cdots,n)$.

IV. NUMERICAL EXAMPLE AND COMPARATIVE ANALYSIS

A. NUMERICAL EXAMPLE

China's agriculture is changing from traditional agriculture to modern agriculture. Due to the restriction of resource

$$\mathrm{FNIFHM}^{(k)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)=\frac{\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}}\tilde{a}(x_{i_j})\right)^{\frac{1}{k}}}{C_n^k}$$

$$=\left\{\left(\begin{matrix}1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}a(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}b(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}c(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}\end{matrix}\right),\left(\begin{matrix}\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}\left(1-l(x_{i_j})\right)\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}\left(1-m(x_{i_j})\right)\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}\left(1-p(x_{i_j})\right)\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}\end{matrix}\right)\right\} \tag{10}$$

endowment and the influence of production mode, agricultural economy has been in a state of low quality development for a long time [43], [44]. Under the background of "new normal" of economic development, the main purpose of the

$$\text{FNIFHM}^{(2)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)=\frac{\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}}\tilde{a}(x_{i_j})\right)^{\frac{1}{k}}}{C_n^k}$$

$$=\left\{\left(\begin{array}{l}1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}a(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}b(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\1-\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}c(x_{i_j})\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}\end{array}\right),\left(\begin{array}{l}\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}(1-l(x_{i_j}))\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}(1-m(x_{i_j}))\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\\left(\prod_{1\le i_1<\ldots<i_k\le n}\left(1-\left(\prod_{j=1}^{k}(1-p(x_{i_j}))\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}\end{array}\right)\right\}$$

$$=\left\{\begin{array}{l}\left(\begin{array}{l}1-\left(\begin{array}{l}\left(1-(0.2\times0.1)^{\frac{1}{2}}\right)\times\left(1-(0.2\times0.2)^{\frac{1}{2}}\right)\times\left(1-(0.2\times0.4)^{\frac{1}{2}}\right)\\\times\left(1-(0.1\times0.2)^{\frac{1}{2}}\right)\times\left(1-(0.1\times0.4)^{\frac{1}{2}}\right)\times\left(1-(0.2\times0.4)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\1-\left(\begin{array}{l}\left(1-(0.3\times0.1)^{\frac{1}{2}}\right)\times\left(1-(0.3\times0.2)^{\frac{1}{2}}\right)\times\left(1-(0.3\times0.5)^{\frac{1}{2}}\right)\\\times\left(1-(0.1\times0.2)^{\frac{1}{2}}\right)\times\left(1-(0.1\times0.5)^{\frac{1}{2}}\right)\times\left(1-(0.2\times0.5)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\1-\left(\begin{array}{l}\left(1-(0.4\times0.3)^{\frac{1}{2}}\right)\times\left(1-(0.4\times0.6)^{\frac{1}{2}}\right)\times\left(1-(0.4\times0.6)^{\frac{1}{2}}\right)\\\times\left(1-(0.3\times0.6)^{\frac{1}{2}}\right)\times\left(1-(0.3\times0.6)^{\frac{1}{2}}\right)\times\left(1-(0.6\times0.6)^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\end{array}\right),\\\left(\begin{array}{l}\left(\begin{array}{l}\left(1-((1-0.4)\times(1-0.2))^{\frac{1}{2}}\right)\times\left(1-((1-0.4)\times(1-0.4))^{\frac{1}{2}}\right)\times\left(1-((1-0.4)\times(1-0.5))^{\frac{1}{2}}\right)\\\times\left(1-((1-0.2)\times(1-0.4))^{\frac{1}{2}}\right)\times\left(1-((1-0.2)\times(1-0.5))^{\frac{1}{2}}\right)\times\left(1-((1-0.4)\times(1-0.5))^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\\left(\begin{array}{l}\left(1-((1-0.5)\times(1-0.4))^{\frac{1}{2}}\right)\times\left(1-((1-0.5)\times(1-0.4))^{\frac{1}{2}}\right)\times\left(1-((1-0.5)\times(1-0.6))^{\frac{1}{2}}\right)\\\times\left(1-((1-0.4)\times(1-0.4))^{\frac{1}{2}}\right)\times\left(1-((1-0.4)\times(1-0.6))^{\frac{1}{2}}\right)\times\left(1-((1-0.4)\times(1-0.6))^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\\left(\begin{array}{l}\left(1-((1-0.5)\times(1-0.6))^{\frac{1}{2}}\right)\times\left(1-((1-0.5)\times(1-0.6))^{\frac{1}{2}}\right)\times\left(1-((1-0.6)\times(1-0.7))^{\frac{1}{2}}\right)\\\times\left(1-((1-0.6)\times(1-0.6))^{\frac{1}{2}}\right)\times\left(1-((1-0.6)\times(1-0.7))^{\frac{1}{2}}\right)\times\left(1-((1-0.6)\times(1-0.7))^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\end{array}\right)\end{array}\right\}$$

$$=\left\langle(0.2102,0.2526,0.4685),(0.3763,0.4769,0.6028)\right\rangle$$

$$\text{FNIFWHM}_w^{(k)}\left(\tilde{a}(x_1),\tilde{a}(x_2),\cdots,\tilde{a}(x_n)\right)=\frac{\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}}\left(\tilde{a}(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}}{C_n^k}$$

$$=\left\{\left(\begin{array}{l}1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(a(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(b(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(c(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}\end{array}\right),\left(\begin{array}{l}\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-l(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}},\\\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-m(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}},\\\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-p(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}}\end{array}\right)\right\} \tag{16}$$

supply-side structural reform of agriculture is to improve the quality of agricultural economic development by optimizing the agricultural supply system [45], [46]. Although the high quality development of agricultural economy in developed areas has achieved fruitful results, the research on the high quality development of agricultural economy in northwest China is still relatively backward [47], [48]. Due to the differences in regional advantages and agricultural production modes of economic development models between different regions, there are corresponding differences in modes and countermeasures to improve the quality of agricultural economy between different regions. The comprehensive evaluation of agricultural economic development quality is really a decision making issue [49]–[53]. Thus, in this section we shall present a numerical example for comprehensive evaluation of agricultural economic development quality with FNIFNs in order to illustrate the method proposed in this paper. There is a panel with five possible agricultural economic zones $A_i\,(i = 1, 2, 3, 4, 5)$ to select. The experts select four attributes to evaluate the five possible agricultural economic zones: ① G_1 is the agricultural product quality factor; ② G_2 is the agricultural environmental; ③ G_3 is the agricultural technological process; ④ G_4 is the agriculture system innovation. The five possible agricultural economic zones $A_i\,(i = 1, 2, 3, 4, 5)$ are to be evaluated using the FNIFNs under the above four attributes (whose weighting vector $\omega = (0.2, 0.1, 0.5, 0.2)$.) which is listed in Table 1.

In the following, we utilize the approach developed to select agricultural economic zones.

$$\left(\underset{j=1}{\overset{k}{\otimes}}\left(\tilde{a}\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}} = \left\{\left(\begin{array}{l}\left(\prod_{j=1}^{n}\left(a\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}},\\ \left(\prod_{j=1}^{n}\left(b\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}},\\ \left(\prod_{j=1}^{n}\left(c\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\end{array}\right), \left(\begin{array}{l}1-\left(\prod_{j=1}^{n}\left(1-l\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}},\\ 1-\left(\prod_{j=1}^{n}\left(1-m\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}},\\ 1-\left(\prod_{j=1}^{n}\left(1-p\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\end{array}\right)\right\} \tag{19}$$

$$\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\underset{j=1}{\overset{k}{\otimes}}\left(\tilde{a}\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}} = \left\{\left(\begin{array}{l}1-\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(a\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right),\\ 1-\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(b\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right),,\\ 1-\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(c\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right),\end{array}\right), \left(\begin{array}{l}\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-l\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right),\\ \prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-m\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right),\\ \prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-p\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\end{array}\right)\right\} \tag{20}$$

$$\mathrm{FNIFWHM}_{w}^{(k)}\left(\tilde{a}\left(x_1\right), \tilde{a}\left(x_2\right), \cdots, \tilde{a}\left(x_n\right)\right) = \frac{\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\underset{j=1}{\overset{k}{\otimes}}\left(\tilde{a}\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}}{C_n^k}$$

$$= \left\{\left(\begin{array}{l}1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(a\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\ 1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(b\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\ 1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(c\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}\end{array}\right), \left(\begin{array}{l}\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-l\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}},\\ \left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-m\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}},\\ \left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-p\left(x_{i_j}\right)\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}}\end{array}\right)\right\} \tag{21}$$

Step 1: According to table 1, we can aggregate all FNIFNs r_{ij} by using the FNIFWHM operator to get the overall FNIFNs A_i $(i = 1, 2, 3, 4, 5)$ of the green suppliers A_i. Suppose that $k = 2$, then the aggregating results are shown in Table 2.

Step 2: According to the aggregating results shown in Table 2 and the score functions of the agricultural economic zones are shown in Table 3.

Step 3: According to the score functions shown in Table 3 and the comparison formula of score functions, the ordering of the agricultural economic zones are shown in Table 4. Note that "$>$" means "preferred to". As we can see, depending on the aggregation operators used, the best agricultural economic zones is A_3.

B. INFLUENCE OF THE PARAMETER ON THE FINAL RESULT

In order to show the effects on the ranking results by changing parameters of k in the FNIFWHM (FNIFWDHM) operators, all the results are shown in Tables 5.

$$\mathrm{FNIFWHM}_w^{(2)}\left(\tilde{a}(x_1), \tilde{a}(x_2), \cdots, \tilde{a}(x_n)\right) = \frac{\underset{1\le i_1<\ldots<i_k\le n}{\oplus}\left(\overset{k}{\underset{j=1}{\otimes}}\left(\tilde{a}(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}}{C_n^k}$$

$$= \left\{\begin{array}{l}\left(\begin{array}{l}1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(a(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\ 1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(b(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}},\\ 1-\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(c(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^k}}\end{array}\right), \left(\begin{array}{l}\left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-l(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}},\\ \left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-m(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}},\\ \left(\prod_{1\le i_1<\ldots<i_x\le n}\left(1-\left(\prod_{j=1}^{n}\left(1-p(x_{i_j})\right)^{w_{i_j}}\right)^{\frac{1}{k}}\right)\right)^{\frac{1}{C_n^x}}\end{array}\right)\end{array}\right\}$$

$$= \left\{\begin{array}{l}\left(\begin{array}{l}1-\left(\begin{array}{l}\left(1-(0.2^{0.2}\times 0.1^{0.1})^{\frac{1}{2}}\right)\times\left(1-(0.2^{0.2}\times 0.2^{0.5})^{\frac{1}{2}}\right)\times\left(1-(0.2^{0.2}\times 0.4^{0.2})^{\frac{1}{2}}\right)\\ \times\left(1-(0.1^{0.1}\times 0.2^{0.5})^{\frac{1}{2}}\right)\times\left(1-(0.1^{0.1}\times 0.4^{0.2})^{\frac{1}{2}}\right)\times\left(1-(0.2^{0.5}\times 0.4^{0.2})^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\ 1-\left(\begin{array}{l}\left(1-(0.3^{0.2}\times 0.1^{0.1})^{\frac{1}{2}}\right)\times\left(1-(0.3^{0.2}\times 0.2^{0.5})^{\frac{1}{2}}\right)\times\left(1-(0.3^{0.2}\times 0.5^{0.2})^{\frac{1}{2}}\right)\\ \times\left(1-(0.1^{0.1}\times 0.2^{0.5})^{\frac{1}{2}}\right)\times\left(1-(0.1^{0.1}\times 0.5^{0.2})^{\frac{1}{2}}\right)\times\left(1-(0.2^{0.5}\times 0.5^{0.2})^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\ 1-\left(\begin{array}{l}\left(1-(0.4^{0.2}\times 0.3^{0.1})^{\frac{1}{2}}\right)\times\left(1-(0.4^{0.2}\times 0.6^{0.5})^{\frac{1}{2}}\right)\times\left(1-(0.4^{0.2}\times 0.6^{0.2})^{\frac{1}{2}}\right)\\ \times\left(1-(0.3^{0.1}\times 0.6^{0.5})^{\frac{1}{2}}\right)\times\left(1-(0.3^{0.1}\times 0.6^{0.2})^{\frac{1}{2}}\right)\times\left(1-(0.6^{0.5}\times 0.6^{0.2})^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\end{array}\right),\\ \left(\begin{array}{l}\left(\begin{array}{l}\left(1-((1-0.4)^{0.2}\times(1-0.2)^{0.1})^{\frac{1}{2}}\right)\times\left(1-((1-0.4)^{0.2}\times(1-0.4)^{0.5})^{\frac{1}{2}}\right)\times\left(1-((1-0.4)^{0.2}\times(1-0.5)^{0.2})^{\frac{1}{2}}\right)\\ \times\left(1-((1-0.2)^{0.1}\times(1-0.4)^{0.5})^{\frac{1}{2}}\right)\times\left(1-((1-0.2)^{0.1}\times(1-0.5)^{0.2})^{\frac{1}{2}}\right)\times\left(1-((1-0.4)^{0.5}\times(1-0.5)^{0.2})^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\ \left(\begin{array}{l}\left(1-((1-0.5)^{0.2}\times(1-0.4)^{0.1})^{\frac{1}{2}}\right)\times\left(1-((1-0.5)^{0.2}\times(1-0.4)^{0.5})^{\frac{1}{2}}\right)\times\left(1-((1-0.5)^{0.2}\times(1-0.6)^{0.2})^{\frac{1}{2}}\right)\\ \times\left(1-((1-0.4)^{0.1}\times(1-0.4)^{0.5})^{\frac{1}{2}}\right)\times\left(1-((1-0.4)^{0.1}\times(1-0.6)^{0.2})^{\frac{1}{2}}\right)\times\left(1-((1-0.4)^{0.5}\times(1-0.6)^{0.2})^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}},\\ \left(\begin{array}{l}\left(1-((1-0.5)^{0.2}\times(1-0.6)^{0.1})^{\frac{1}{2}}\right)\times\left(1-((1-0.5)^{0.2}\times(1-0.6)^{0.5})^{\frac{1}{2}}\right)\times\left(1-((1-0.6)^{0.2}\times(1-0.7)^{0.2})^{\frac{1}{2}}\right)\\ \times\left(1-((1-0.6)^{0.1}\times(1-0.6)^{0.5})^{\frac{1}{2}}\right)\times\left(1-((1-0.6)^{0.1}\times(1-0.7)^{0.2})^{\frac{1}{2}}\right)\times\left(1-((1-0.6)^{0.5}\times(1-0.7)^{0.2})^{\frac{1}{2}}\right)\end{array}\right)^{\frac{1}{C_4^2}}\end{array}\right)\end{array}\right\}$$

$$= \langle(0.7033, 0.7311, 0.8511), (0.1123, 0.1397, 0.1936)\rangle$$

TABLE 1. FNIFN decision matrix (*R*).

	G_1	G_2
A_1	$\langle(0.2,0.3,0.4),(0.4,0.5,0.5)\rangle$	$\langle(0.1,0.1,0.3),(0.2,0.4,0.6)\rangle$
A_2	$\langle(0.2,0.5,0.5),(0.3,0.4,0.6)\rangle$	$\langle(0.5,0.7,0.7),(0.2,0.5,0.8)\rangle$
A_3	$\langle(0.7,0.7,0.8),(0.2,0.2,0.4)\rangle$	$\langle(0.4,0.7,0.7),(0.2,0.3,0.4)\rangle$
A_4	$\langle(0.1,0.4,0.6),(0.2,0.3,0.7)\rangle$	$\langle(0.2,0.2,0.4),(0.3,0.3,0.4)\rangle$
A_5	$\langle(0.5,0.5,0.6),(0.3,0.3,0.5)\rangle$	$\langle(0.1,0.1,0.4),(0.3,0.3,0.4)\rangle$
	G_3	G_4
A_1	$\langle(0.2,0.3,0.6),(0.4,0.4,0.6)\rangle$	$\langle(0.4,0.5,0.6),(0.5,0.6,0.7)\rangle$
A_2	$\langle(0.5,0.6,0.7),(0.4,0.6,0.6)\rangle$	$\langle(0.4,0.4,0.7),(0.2,0.5,0.8)\rangle$
A_3	$\langle(0.7,0.7,0.9),(0.2,0.4,0.5)\rangle$	$\langle(0.5,0.6,0.6),(0.1,0.2,0.2)\rangle$
A_4	$\langle(0.1,0.4,0.7),(0.2,0.5,0.8)\rangle$	$\langle(0.4,0.6,0.7),(0.5,0.6,0.6)\rangle$
A_5	$\langle(0.2,0.2,0.4),(0.1,0.2,0.2)\rangle$	$\langle(0.3,0.4,0.5),(0.4,0.5,0.5)\rangle$

TABLE 2. The aggregating results of the agricultural economic zones by the FNIFWHM operator.

	FNIFWHM
A_1	<(0.7033,0.7311,0.8511), (0.1123,0.1397,0.1936)>
A_2	<(0.8030,0.8647,0.9045), (0.0783,0.1542,0.2373)>
A_3	<(0.8893,0.9125,0.9448), (0.0439,0.0750,0.1078)>
A_4	<(0.6614,0.8105,0.8974), (0.0752,0.1305,0.2326)>
A_5	<(0.7311,0.7447,0.8392), (0.0618,0.0825,0.1023)>

TABLE 3. The score functions of the green suppliers.

	FNIFWHM
A_1	0.6078
A_2	0.7032
A_3	0.8393
A_4	0.6528
A_5	0.6826

C. COMPARATIVE ANALYSIS

Then, we compare our proposed method with other existing methods including FNIFWA [33] operator and FNIFWG [35] operator, FNIFPWA operator [54], FNIFPWG operator [54], the generalized fuzzy number intuitionistic fuzzy weighted Bonferroni mean (GFNIFWBM) operator [33], the fuzzy number intuitionistic fuzzy Hamacher power weighted geometric (FNIFHPWG) operator [38], induced fuzzy number intuitionistic fuzzy choquet ordered averaging (IFNIFCOA) operator [30], induced fuzzy number intuitionistic fuzzy choquet ordered geometric (IFNIFCOG) operator [30]. The comparative results are shown in Table 6.

TABLE 4. Ordering of the agricultural economic zones.

	Ordering
FNIFWHM	$A_3 > A_2 > A_5 > A_4 > A_1$

From above, Compare the values of our proposed FNIFWHM operator with FNIFWA operator, FNIFWG operator, FNIFPWA operator, FNIFPWG operator, GFNIFWBM operator, FNIFHPWG operator, IFNIFCOA operator, IFNIFCOG operator, the results in ranking of alternatives are slightly different and the best alternatives are same. However, the existing aggregation operators, such as FNIFWA and FNIFWG operators, do not consider the information about the relationship between arguments being aggregated, and thus cannot eliminate the influence of unfair arguments on decision result. The FNIFPWA operator and FNIFPWG operator are effective to deal with MADM problems in which the attributes are in different priority level. The prominent characteristic of GFNIFWBM is its capability to capture the interrelationship between two input arguments. Our proposed FNIFWHM operator consider the information about the relationship between arguments being aggregated.

TABLE 5. Ranking results for different operational parameters of the FNIFWHM operator.

	$s(A_1)$	$s(A_2)$	$s(A_3)$	$s(A_4)$	$s(A_5)$	Ordering
$k=1$	0.6507	0.7405	0.8581	0.7052	0.7171	$A_3>A_2>A_5>A_4>A_1$
$k=2$	0.6078	0.7032	0.8393	0.6528	0.6826	$A_3>A_2>A_5>A_4>A_1$
$k=3$	0.5892	0.6887	0.8289	0.6281	0.6643	$A_3>A_2>A_5>A_4>A_1$
$k=4$	0.5782	0.6803	0.8220	0.6140	0.6523	$A_3>A_2>A_5>A_4>A_1$

TABLE 6. Ordering of the green suppliers.

	Ordering
FNIFWA [33]	$A_3>A_5>A_2>A_4>A_1$
FNIFWG [35]	$A_3>A_2>A_5>A_4>A_1$
FNIFPWA[54]	$A_3>A_2>A_5>A_4>A_1$
FNIFPWG[54]	$A_3>A_2>A_5>A_4>A_1$
GFNIFWBM[33]	$A_3>A_2>A_5>A_4>A_1$
FNIFHPWG[38]	$A_3>A_2>A_5>A_4>A_1$
FNIFCOA[30]	$A_3>A_2>A_5>A_4>A_1$
FNIFCOG[30]	$A_3>A_2>A_5>A_4>A_1$
FNIFWHM	$A_3>A_2>A_5>A_4>A_1$
FNIFWDHM	$A_3>A_5>A_2>A_4>A_1$

V. CONCLUSION

In this paper, we investigate the MADM problems with FNIFNs. Then, we utilize the Hamy mean (HM) operator and weighted Hamy mean (WHM) operator to develop some Hamy mean aggregation operators with FNIFNs: fuzzy number intuitionistic fuzzy Hamy mean (FNIFHM) operator and fuzzy number intuitionistic fuzzy weighted Hamy mean (FNIFWHM) operator. The prominent characteristic of these proposed operators are studied. Then, we have utilized these operators to develop some approaches to solve the MADM problems with FNIFNs. Finally, a practical example for comprehensive evaluation of agricultural economic development quality is given to verify the developed approach and to demonstrate its practicality and effectiveness. In the future, the application of the proposed aggregating operators of FNIFNs needs to be explored in the decision making, risk analysis and many other fields under uncertain environments [55]–[64].

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