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Leader-Following Consensus of Stochastic Perturbed Multi-Agent Systems via Variable Impulsive Control and Comparison System Method

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ABSTRACT In this paper, we explore the consensus control scheme of stochastic perturbed nonlinear multi-agent systems with impulsive protocol and comparison system method. The effective variable impulsive consensus method is used to remove the restriction of fixed impulsive instants, which is more reliable and flexible in practical applications. From the theory of impulsive differential system, comparison system and stochastic differential system, some sufficient consensus conditions are derived and the relation between the system parameters and impulsive time window is analyzed extensively. The effectiveness of the proposed control method is confirmed by the numerical simulations finally.

INDEX TERMS Multi-agent systems, consensus, impulsive control, stochastic perturbation, comparison system.

I. INTRODUCTION

The consensus (or synchronization) topic of multi-agent systems (MAS) has obtained wide attention due to the potential and meaningful application in past few years, especially in the fields of mathematics, physics, engineering, biology, and so forth [1]–[5]. In general, the consensus scheme of MAS consists of leaderless [6]–[8] and leader-following schemes [9]–[11]. For the leader-following consensus, it requires other nodes to follow the leader asymptotically. In many practical systems, it is of great significance to design distributed controller for leader-following consensus of MAS.

So far, lots of effective protocols are proposed to achieve the consensus of MAS, such as sliding mode control [12], [13], event-triggered control [14], [15], fault-tolerant control [16], [17], adaptive control [18], [19], etc. Impulsive control is a representative discontinuous control scheme, which can relieve the information transmission burden greatly [20]–[23]. The system states of the followers can achieve the instantaneous change at impulsive instants [24]–[28]. In recent years, many interesting and meaningful results on impulsive consensus of MAS have been reported. For instance, the odd variable impulsive consensus of MAS via adaptive control and comparison system method was studied in [29]. Ref. [30] investigated the asymptotic synchronization of MAS via adaptive control and variable impulsive protocol. Ref. [31] studied exponential synchronization of delayed perturbed complex networks with adaptive and impulsive protocol. Moreover, in the real world, the perturbations such as source, quantization and channel noise frequently limit the evaluations or information exchange [32]. In many cases, external perturbations correspond to some great uncertainties, and can be taken as random effects to the real systems [33], [34]. From the papers mentioned above, the stochastic disturbances are important and realistic in the consensus process of MAS.

As we know, the impulsive instants are often predesigned and independent in existing application, which means the impulsive instants were usually fixed. However, due to the internal and external constraints, it is hard to ensure the exact impulsive instants imposed at expected time exactly, and

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it often occurs fluctuation between the actual instants and expected one. This kind of impulse constraint wildly exists in real system and it is very significant to study the consensus of nonlinear MAS with impulsive time windows [35]. There are some research works about impulsive time windows, such as hybrid neural networks [36], Hopfield-type neural networks [37], stabilization of linear systems [38], coupled delayed switched neural networks [39], delayed impulsive functional differential systems [40], general nonlinear systems [41], sandwich control systems [42], and linear delayed impulsive differential systems [43]. It should be noticed that all of the above literatures are not related with comparison systems directly. However, for impulsive control systems, comparison system method shows an excellent role in the stability and stabilization analysis [44]–[46]. The best advantage of the comparison system method is the simplified system model, which is better than the existing Lyapunov stability-based methods to some extent. There are some research works to address control schemes with variable impulsive control and comparison system method [47]–[49]. However, owing to the analytical complexity of stochastic impulsive control system, there are few existing results to investigate the consensus of stochastic perturbed MAS with variable impulsive control.

The main motivation of this works is to explore the variable impulsive consensus of stochastic perturbed MAS via comparison system method. By using stochastic analytical approach and comparison system approach, some sufficient consensus conditions are derived for achieving the consensus of stochastic perturbed MAS via variable impulsive control. As far as the authors know, there is few results combining comparison system approach with the variable impulsive consensus of stochastic perturbed MAS. The proposed comparison system approach can change the dynamical analysis of high-order system into the scalar one, which simplifies the analysis complexity of the consensus scheme. Compared to the existing common impulsive control approach, the variable impulsive control techniques allow a certain imposing error of impulsive input, which is more robust, credible, practical and flexible in real system.

The organization of this literature is given as follows. Section 2 and Section 3 introduce the basic theory of impulsive control system and problem formulation respectively. In Section 4, main results for impulsive consensus of stochastic perturbed MAS are discussed. The numerical simulation examples are presented to verify the feasibility of the main results in Section 5. Finally, the conclusion of this paper is drawn in Section 6.

II. BASIC THEORY OF IMPULSIVE CONTROL SYSTEM

Consider the following stochastic impulsive control system,

$$\begin{cases} dx(t)=f_1(t,x)dt+f_2(t,x)dw(t), & t\neq t_k, \\ \Delta x(t_k)=x(t_k^+)-x(t_k^-)=U(k,x), & k\in\mathbb{N}=1,2,\ldots \\ x(t_0^+)=x(t_0), & t_0\geq 0. \end{cases} \tag{1}$$

Let $\mathbb{R}^n$ be n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ be an $n\times m$ matrix, $\mathbb{R}^+=\{x\in\mathbb{R}|x\geq 0\}$, $\mathcal{C}^{1,2}(\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^+)$ be the family of all nonnegative functions from $\mathbb{R}^+\times\mathbb{R}^n$ to $\mathbb{R}_+$ which are once continuously differentiable in the first variable and twice in the second one. The time sequence $\{t_k\}$ satisfies $0\leq t_0<t_1<\cdots<t_k<\cdots$, $\lim_{k\to\infty} t_k=\infty$, $k\in\mathbb{N}$. $x\in\mathbb{R}^n$ is the state vector, $f_1:\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^n$, $f_2:\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^{n\times m}$. $w(t)=[w_1(t),\cdots,w_m(t)]^T\in\mathbb{R}^m$ is the Brownian motion. For each $V(t,x)\in\mathcal{C}^{1,2}(\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^+)$, and an operator $\mathcal{L}$ is defined from $\mathbb{R}^+\times\mathbb{R}^n$ to $\mathbb{R}$ by

$$\mathcal{L}V(t,x)=V_t(t,x)+V_x(t,x)f_1(t,x) +\frac{1}{2}\text{trace}[f_2^T(t,x)V_{xx}(t,x)f_2(t,x)],$$

where

$$V_t(t,x)V_t(t)\doteq\partial V(t,x)/\partial t,$$
$$V_x(t,x)V_x(t)\doteq(\partial V(t,x)/\partial x_1,\cdots,\partial V(t,x)/\partial x_n),$$
$$V_{xx}(t,x)V_{xx}(t)\doteq(\partial V(t,x)/\partial x_i\partial x_j)_{n\times n}.$$

Assumption 1: $U(k,0)=0, f_1(t,0)=0, f_2(t,0)=0$ for all $k\in\mathbb{N}$.

Definition 1 [50]: Let $V(t,x)\in\mathcal{C}^{1,2}(\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^+)$, the following inequalities

$$\begin{cases} V_t(t)\leq g(t,V(t,x)), & t\neq t_k, \\ \mathcal{L}V(t,x)\leq g(t,V(t,x)), & t\neq t_k, \\ V(t,x+U(t,x))\leq\varphi_k(V(t,x)), & t=t_k, \end{cases} \tag{2}$$

hold, where $g:\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}$ is continuous for $t\in(t_{k-1},t_k]$, $k\in\mathbb{N}$, and $\varphi_k:\mathbb{R}^+\to\mathbb{R}^+$ is nondecreasing. Then the system

$$\begin{cases} \dot\omega=g(t,\omega), t\neq t_k, \\ \omega(t_k^+)=\varphi_k(\omega(t_k)), k\in\mathbb{N}, \\ \omega(t_0^+)=\omega_0\geq 0 \end{cases} \tag{3}$$

is called the comparison system of (1).

Definition 2: A function $\alpha(\nu)$ belongs to class $\mathcal{K}$ if $\alpha\in\mathcal{C}(\mathbb{R}^+\to\mathbb{R}^+)$ with $\alpha(0)=0$ is strictly increasing in ν.

Lemma 1 [50]: Assume that Assumption 1 holds. For (1) and it is comparison system (3), if there exists a $V(t,x)\in\mathcal{C}^{1,2}(\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^+)$ such that

- (H1) $\beta(\|x\|)\leq V(t,x)\leq\alpha(\|x\|)$ on $\mathbb{R}_+\times S(\rho)$ where $\alpha(\cdot),\beta(\cdot)\in\mathcal{K}$. $S(\rho)=\{x\in\mathbb{R}^n|\,\|x\|<\rho\}$.
- (H2) $\mathcal{L}V(t,x)\leq g(t,V(t,x)), t\neq t_k$.
- (H3) There exists a $\rho_0>0$ such that $x\in S(\rho_0)$ implies that $x+U(k,x)\in S(\rho_0)$ for all k and $V(t,x+U(t,x))\leq\varphi_k(V(t,x)), t=t_k, x\in S(\rho_0)$.

Then the stability of comparison system (3) implies the stochastic stability of system (1) correspondingly.

Lemma 2 [50]: Assume that Assumption 1 holds. Let $g(t,\omega)=\dot\lambda(t)\omega, \lambda\in\mathcal{C}^1(\mathbb{R}^+\to\mathbb{R}^+)$, $\varphi_k(\omega)=d_k\omega$, $d_k\geq 0$ for all k, then system (1) is asymptotically stable if the following conditions hold,

$$\lambda(t_{k+1})+\ln(\gamma d_k)\leq\lambda(t_k), \tag{4}$$

where $k \in \mathbb{N}$, $\gamma > 1$, and

$$\dot{\lambda}(t) \geq 0. \tag{5}$$

Remark 1: Lemma 1 builds a theoretical linkage between the general control system (1) and the scalar system (3). In order to decrease the complexity of theoretical analysis, the analysis of high-order system (1) is replaced by (3). If the impulsive control parameters (impulsive instants $\{t_k\}$ and control gain coefficients $d_k \geq 0$) satisfy conditions (4) and (5), the asymptotical stability of (1) can be realized.

Note that the impulsive instants $\{t_k\}$ in Lemma 2 should be preassigned ahead of time. However, due to the internal and external constraints, the actual system hardly impose impulsive input at predetermined instant correctly. This paper use the variable impulsive control method to get larger consensus region compared with the common impulsive one. The description for the impulsive time windows is shown in Fig. 1.

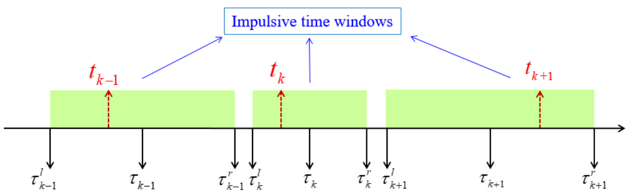

FIGURE 1. **The diagram of variable impulsive control instants and impulsive time window.**

Assumption 2: Consider the following inequality,

$$\tau_{k-1}^l < t_{k-1} < \tau_{k-1}^r < \tau_k^l < t_k < \tau_k^r < \tau_{k+1}^l < t_{k+1} < \tau_{k+1}^r, \quad k \in \mathbb{N},$$

where $\tau_k^l = \tau_k - r_k$ ($\tau_k^r = \tau_k + r_k$) and r_k denotes the left (right) endpoints and the radius of the k-th time window. $\{\tau_k\}$ indicate the centers of the impulsive time window.

The following Lemmas 3 and 4 are helpful to realize the consensus scheme via variable impulsive control.

Lemma 3: Assume that Assumptions 1 and 2 hold. Let $g(t,\omega) = \dot{\lambda}(t)\omega, \lambda \in \mathcal{C}^1(\mathbb{R}^+ \to \mathbb{R}^+)$, $\varphi_k(\omega) = d_k\omega$, $d_k \geq 0$ for all k, then system (1) is asymptotically stable if the following conditions hold

$$\lambda(\tau_{k+1}^l) + \ln(\gamma d_k) \leq \lambda(\tau_k^l), \tag{6}$$

where $k \in \mathbb{N}$, $\gamma > 1$, and

$$\dot{\lambda}(t) \geq 0. \tag{7}$$

Proof: From $g(t,\omega) = \dot{\lambda}(t)\omega$ and $\varphi_k(\omega) = d_k\omega$, the comparison system is changed into the following form:

$$\begin{cases} \dot{\omega} = \dot{\lambda}(t)\omega, t \neq t_k, \\ \omega(t_k^+) = d_k\omega(t_k), \\ \omega(t_0^+) = \omega_0 \geq 0. \end{cases} \tag{8}$$

Next, we discuss the solution $\omega(t, t_0, u_0)$ of system (8)

For $t \in (t_0, t_1]$, one has

$$\omega(t) = \omega_0 \exp(\lambda(t) - \lambda(t_0)),$$

which leads to

$$\omega(t_1) = \omega_0 \exp(\lambda(t_1) - \lambda(t_0)).$$

For $t \in (t_1, t_2]$,

$$\begin{aligned} \omega(t) &= \omega(t_1^+) \exp(\lambda(t) - \lambda(t_1)) \\ &= d_1\omega(t_1) \exp(\lambda(t) - \lambda(t_1)) \\ &= \omega_0 d_1 \exp(\lambda(t_1) - \lambda(t_0)) \exp(\lambda(t) - \lambda(t_1)) \\ &= \omega_0 d_1 \exp(\lambda(t) - \lambda(t_0)). \end{aligned}$$

In general, for $t \in (t_k, t_{k+1}]$,

$$\omega(t, t_0, u_0) = \omega_0 \prod_{t_0 < t_k < t} d_k \exp(\lambda(t) - \lambda(t_0)). \tag{9}$$

Since $\dot{\lambda}(t) \geq 0$, it follows from (6) and (9) that

$$\begin{aligned} &\omega(t, t_0, u_0) \\ &= \omega_0 \prod_{t_0 < t_k < t} d_k \exp(\lambda(t) - \lambda(t_0)) \\ &\leq \omega_0 \prod_{t_0 < t_k < t} \left(\frac{1}{\gamma} \exp(\lambda(\tau_k^l) - \lambda(\tau_{k+1}^l)) \right) \exp(\lambda(t) - \lambda(t_0)) \\ &\leq \omega_0 (\frac{1}{\gamma^k} \exp(\lambda(\tau_1^l) - \lambda(\tau_2^l)) \exp(\lambda(\tau_2^l) - \lambda(\tau_3^l)) \\ &\quad \cdots \exp(\lambda(\tau_k^l) - \lambda(\tau_{k+1}^l))) \exp(\lambda(t_{k+1}) - \lambda(t_0)) \\ &= \frac{\omega_0}{\gamma^k} \exp(\lambda(\tau_1^l) - \lambda(\tau_{k+1}^l)) \exp(\lambda(t_{k+1}) - \lambda(t_0)) \\ &= \frac{\omega_0}{\gamma^k} \exp(\lambda(\tau_1^l) - \lambda(t_0)) \exp(\lambda(t_{k+1}) - \lambda(\tau_{k+1}^l)). \end{aligned}$$

Note that $\omega_0 \exp(\lambda(\tau_1^l) - \lambda(t_0)) \exp(\lambda(t_{k+1}) - \lambda(\tau_{k+1}^l))$ is finite, and $1/\gamma^k \to 0$ as $k \to \infty$, it has $\lim_{t\to\infty} \omega(t, t_0, u_0) = 0$. It is easy to conclude from Lemma 1 that system (1) is asymptotically stable. □

In the proof of Lemma 3, the consensus condition is described by the left endpoints of the adjacent impulsive time windows $t \in (\tau_k - r_k, \tau_{k+1} - r_{k+1}] = (\tau_k^l, \tau_{k+1}^l]$, $k \in \mathbb{N}$. The following Lemma 4 will discuss the centers of the adjacent impulsive time windows $t \in (\tau_{k-1}, \tau_k]$.

Lemma 4: Assume that Assumptions 1 and 2 hold. Let $g(t,\omega) = \dot{\lambda}(t)\omega, \lambda \in \mathcal{C}^1(\mathbb{R}^+ \to \mathbb{R}^+)$, $\varphi_k(\omega) = d_k\omega$, $d_k \geq 0$ for all k, then system (1) is asymptotically stable if the following conditions hold,

$$\lambda(\tau_{k+1}) + \ln(\gamma d_k) \leq \lambda(\tau_k), \tag{10}$$

where $k \in \mathbb{N}$, $\gamma > 1$, and

$$\dot{\lambda}(t) \geq 0. \tag{11}$$

Proof: From $g(t,\omega) = \dot{\lambda}(t)\omega$ and $\varphi_k(\omega) = d_k\omega$, the comparison system is changed into the following form:

$$\begin{cases} \dot{\omega} = \dot{\lambda}(t)\omega, t \neq t_k, \\ \omega(t_k^+) = d_k\omega(t_k), \\ \omega(t_0^+) = \omega_0 \geq 0. \end{cases} \tag{12}$$

Obviously, any solution $\omega(t, t_0, u_0)$ of system (12) is given by

$$\omega(t, t_0, u_0) = \omega_0 \prod_{t_0<t_k<t} d_k \exp(\lambda(t) - \lambda(t_0)),$$
$$for\ t \in (t_k, t_{k+1}], \quad (13)$$

Since $\dot{\lambda}(t) \geq 0$, it follows from (10) and (13) that

$$\begin{aligned}
&\omega(t, t_0, u_0)\\
&= \omega_0 \prod_{t_0<t_k<t} d_k \exp(\lambda(t) - \lambda(t_0))\\
&\leq \omega_0 \prod_{t_0<t_k<t} \left(\frac{1}{\gamma} \exp(\lambda(\tau_k) - \lambda(\tau_{k+1}))\right) \exp(\lambda(t) - \lambda(t_0))\\
&\leq \omega_0 (\frac{1}{\gamma^k} \exp(\lambda(\tau_1) - \lambda(\tau_2)) \exp(\lambda(\tau_2) - \lambda(\tau_3))\\
&\quad \cdots \exp(\lambda(\tau_k) - \lambda(\tau_{k+1}))) \exp(\lambda(t_{k+1}) - \lambda(t_0))\\
&= \frac{\omega_0}{\gamma^k} \exp(\lambda(\tau_1) - \lambda(\tau_{k+1})) \exp(\lambda(t_{k+1}) - \lambda(t_0))\\
&= \frac{\omega_0}{\gamma^k} \exp(\lambda(\tau_1) - \lambda(t_0)) \exp(\lambda(t_{k+1}) - \lambda(\tau_{k+1})).
\end{aligned}$$

Note that $\omega_0 \exp(\lambda(\tau_1) - \lambda(t_0)) \exp(\lambda(t_{k+1}) - \lambda(\tau_{k+1}))$ is finite, and $1/\gamma^k \to 0$ as $k \to \infty$, it has $\lim_{t\to\infty} \omega(t, t_0, u_0) = 0$. It is easy to conclude from Lemma 1 that system (1) is asymptotically stable. □

Remark 2: As shown in Lemma 2, the impulsive instants $\{t_k\}$ should be predetermined ahead of time to realize the asymptotical stability of impulsive control system (1). By comparison, Lemmas 3 and 4 just need the impulsive instants $\{t_k\}$ exert within a time interval (impulsive time window).

III. PROBLEM FORMULATION

In this paper, directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ denotes the communication graph among the agents. $\mathcal{V} = \{v_1, \ldots, v_N\}$ and $\mathcal{E} \in (\mathcal{V} \times \mathcal{V})$ denote the finite set of N nodes and the set of edges or arcs. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes the associate adjacency matrix. $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $a_{ij} = 0$, where (v_j, v_i) means an edge rooted at node j and ended at node i. $a_{ii} = 0 (i \in \mathbb{N})$ means that there are no self-loops. $N_i = \{j | (v_j, v_i) \in \mathcal{E}\}$ denotes the set of neighbors of node i. $d_i = \sum_{j=1}^{N} a_{ij}$ is the in-degree of node i and $D = \text{diag}\{d_i\} \in \mathbb{R}^{N\times N}$ is the corresponding in-degree matrix. The Laplacian matrix is defined as $L = D - \mathcal{A}$. For a directed graph, a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$ is a direct path from node i to node j. A directed graph is called strongly connected iff there exists a directed path from i to j for any nodes v_i and v_j.

Consider the leader node as

$$\dot{x}_0(t) = Ax_0(t) + \psi(x_0(t)), \quad (14)$$

where $x_0 = [x_{01}, x_{02}, \ldots, x_{0n}]^T \in \mathbb{R}^n$ is the state vector. $A \in \mathbb{R}^{n\times n}$ is a known matrix, $\psi : \mathbb{R}^n \to \mathbb{R}^n$ is the nonlinear function.

Consider the stochastic perturbed MAS with N agents via impulsive control method is described by

$$\begin{cases} dx_i(t) = (Ax_i(t) + \psi(x_i(t)))dt \\ \qquad +\vartheta_i(t, \delta_i(t))dw(t), \quad t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = u_i(t_k), \quad k \in \mathbb{N}, \end{cases} \quad (15)$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T \in \mathbb{R}^n$ is the state vector. $\Delta x_i(t_k)$ is the state jump of the follower node i at impulsive instant t_k. $\{t_k\}$ is the impulsive sequence. Let the state be the left continuous (i.e., $x_i(t_k) = x_i(t_k^-)$). $\delta_i = x_i(t) - x_0(t)$ is the consensus error, and the matrix $\vartheta_i : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ satisfies the subsequent Assumption.

Assumption 3: $\vartheta_i(t, \delta_i(t))$ satisfies the linear growth condition with the initial condition $\vartheta(t, 0) = 0$. Moreover, $\vartheta_i(t, \delta_i(t))$ satisfies the following inequality condition with known constant matrix Σ,

$$\text{trace}[\vartheta_i^T(t, \delta_i(t))\vartheta_i(t, \delta_i(t))] \leq \|\Sigma\delta_i(t)\|^2. \quad (16)$$

Assumption 4: The function $\psi : \mathbb{R}^n \to \mathbb{R}^n$ satisfies the following condition

$$\|\psi(x_1) - \psi(x_2)\| \leq l \|x_1 - x_2\|. \quad (17)$$

where $l > 0$ is the known positive constant.

From (14) and (15), the error system is obtained as

$$\begin{cases} d\delta_i(t) = (A\delta_i(t) + \psi(x_i(t)) - \psi(x_0(t)))dt \\ \qquad +\vartheta_i(t, \delta_i(t))dw(t), \quad t \neq t_k, \\ \Delta\delta_i(t_k) = \delta_i(t_k^+) - \delta_i(t_k) = u_i(t_k), \quad k \in \mathbb{N}. \end{cases} \quad (18)$$

In order to achieve the consensus goal, the following impulsive controller is designed,

$$u_i(t_k) = b_k(\sum_{j\in N_i} a_{ij}(x_i(t_k) - x_j(t_k))$$
$$+c_i(x_i(t_k) - x_0(t_k))), \quad k \in \mathbb{N}, \quad (19)$$

where b_k is the control gain coefficient, $c_i \geq 0$ is the edge weight from the leader to the follower. If there is an edge from the leader to the follower, it has $c_i > 0$ and $C = \text{diag}\{c_i\} \in \mathbb{R}^{N\times N}$.

Assumption 5: The graph contains a spanning tree and the leader node 0 is the root node.

By (19) and the properties of Kronecker product, system (18) is rewritten as

$$\begin{cases} d\delta(t) = ((I_N \otimes A)\delta(t) + \bar{\psi}(x(t), \bar{x}_0(t)))dt \\ \qquad +\vartheta(t, \delta(t))d\bar{w}(t), \quad t \neq t_k, \\ \Delta\delta(t_k) = b_k((L + C) \otimes I_n)\delta(t_k), \quad k \in \mathbb{N}, \end{cases} \quad (20)$$

where $\delta(t) = [\delta_1^T(t), \ldots, \delta_N^T(t)]^T$, $\bar{w}(t) = 1_N \otimes w(t)$, $x(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$, $\bar{x}_0(t) = 1_N \otimes x_0(t)$, $\vartheta(t, \delta(t)) = \text{diag}\{\vartheta_i(t, \delta_i(t))\}$, $\bar{\psi}(x(t), \bar{x}_0(t)) = [\psi^T(x_1(t)) - \psi^T(x_0(t)), \ldots, \psi^T(x_N(t)) - \psi^T(x_0(t))]^T$.

The goal of this paper is to design the controller for the stochastic perturbed MAS, which can assure all follower nodes synchronize asymptotically in mean square case to the leader for any initial condition, i.e., $\lim_{t\to\infty} E(\|\delta(t)\|^2) = 0$.

IV. VARIABLE IMPULSIVE CONSENSUS OF STOCHASTIC PERTURBED MULTI-AGENT SYSTEMS

Before starting the impulsive consensus problem of stochastic perturbed MAS, we first give the consensus conditions with fixed impulsive instants (correspond to Lemma 2).

Theorem 1: If Assumptions 1-5 hold, and there exists $\xi > 1$ such that

$$(\lambda_A + 2l + \lambda_\Sigma)(t_{k+1} - t_k) + \ln(\lambda_k \xi) < 0, \tag{21}$$

where λ_A and λ_k are the maximum eigenvalue of $A + A^T$ and $(b_k(L + C) + I_N)^T(b_k(L + C) + I_N)$, λ_Σ is the maximum eigenvalue of $\Sigma^T\Sigma$ respectively. Then the consensus of multi-agent system (15) can be achieved.

Proof: Firstly, we will find out whether system (20) satisfies the conditions of Lemma 1. Let the Lyapunov function be $V(t) = \delta^T\delta$, it is easy to find that, (H1) of Lemma 1 holds with $\beta(\delta) = \lambda_1 \|\delta\|^2$ and $\alpha(\delta) = \lambda_2 \|\delta\|^2$, where $\lambda_1 \in (0, 1)$ and $\lambda_2 > 0$.

$$\begin{aligned}\mathcal{L}V(t) &= V_t(t) + V_\delta(t)((I_N \otimes A)\delta + \bar{\psi}(x(t), \bar{x}_0(t)))\\ &\quad + \frac{1}{2}\text{trace}[\vartheta^T(t, \delta)V_{\delta\delta}(t, \delta)\vartheta(t, \delta)]\\ &= \delta^T(I_N \otimes (A + A^T))\delta + 2\delta^T\bar{\psi}(x, \bar{x}_0)\\ &\quad + \text{trace}[\vartheta^T(t, \delta)\vartheta(t, \delta)]. \end{aligned}\tag{22}$$

From Assumption 3, it can yield

$$\text{trace}[\vartheta^T(t, \delta)\vartheta(t, \delta)] \le \delta^T\Sigma^T\Sigma\delta \le \lambda_\Sigma V(t).$$

Then, it can get

$$\begin{aligned}\mathcal{L}V(t) &= \delta^T(I_N \otimes (A + A^T))\delta + 2\delta^T\bar{\psi}(x, \bar{x}_0)\\ &\quad + \text{trace}[\vartheta^T(t, \delta)\vartheta(t, \delta)]\\ &\le (\lambda_A + 2l + \lambda_\Sigma)V(t).\end{aligned}$$

Hence, (H2) of Lemma 1 is satisfied with $g(t, \omega) = (\lambda_A + 2l + \lambda_\Sigma)\omega$.

Given any $\rho_0 > 0$ and $\delta \in S(\rho_0)$, one gets

$$\begin{aligned}&\|\delta + U(k, \delta)\|\\ &\le \|\delta + b_k((L + C) \otimes I_n)\delta\|\\ &= \|(b_k((L + C) \otimes I_n) + I_{nN})\delta\|\\ &= \sqrt{\delta^T((b_k(L + C) + I_N)^T(b_k(L + C) + I_N) \otimes I_n)\delta}\\ &\le \sqrt{\lambda_k}\,\|\delta\| \le \|\delta\|. \end{aligned}\tag{23}$$

From $\|\delta + U(k, \delta)\| \le \|\delta\|$, it has $\delta + U(k, \delta) \in S(\rho_0)$.

When $t = t_k$, one can get

$$\begin{aligned}E(V(t_k^+)) &= E(\delta^T(t_k^+)\delta(t_k^+))\\ &= E(((b_k((L + C) \otimes I_n) + I_{nN})\delta(t_k))^T\\ &\quad ((b_k((L + C) \otimes I_n) + I_{nN})\delta(t_k)))\\ &= E(\delta^T(t_k)((b_k(L + C) + I_N)^T\\ &\quad (b_k(L + C) + I_N) \otimes I_n)\delta(t_k))\\ &\le \lambda_k E(V(t_k)).\end{aligned}$$

Note that (H3) in Lemma 1 holds with $\varphi_k(\omega) = \lambda_k\omega$, and $\lambda_1 \|\delta\|^2 \le V(\delta) \le \lambda_2 \|\delta\|^2$. From Lemma 1, it yields that the asymptotic stability of system (15) is proven by the asymptotic stability of the following comparison system:

$$\begin{cases}\dot{\omega} = (\lambda_A + 2l + \lambda_\Sigma)\omega, t \neq t_k,\\ \omega(t_k^+) = \lambda_k\omega(t_k),\\ \omega(t_0^+) = \omega_0 \ge 0.\end{cases}$$

From $\dot{\lambda}(t) = \lambda_A + 2l + \lambda_\Sigma \ge 0$, it has $\lambda(t_{k+1}) = (\lambda_A + 2l + \lambda_\Sigma)t_{k+1}$ and $\lambda(t_k) = (\lambda_A + 2l + \lambda_\Sigma)t_k$, which shows (21) is equivalent to (4) in Lemma 2. Then system (20) is asymptotically stable, which implies that the multi-agent systems (15) can realize consensus. □

In the following, it takes the impulsive time window into the consideration, and the consensus of stochastic perturbed MAS with left endpoints and the centers of the time windows (correspond to Lemmas 3 and 4) will be given respectively.

Theorem 2: If Assumptions 1-5 hold, and there exists $\xi > 1$ such that

$$(\lambda_A + 2l + \lambda_\Sigma)(\tau_{k+1}^l - \tau_k^l) + \ln(\lambda_k\xi) < 0, \tag{24}$$

where λ_A, λ_k and λ_Σ have the same meanings in Theorem 1. Then the consensus of stochastic perturbed multi-agent systems (15) can be achieved.

Proof: Let the Lyapunov function be $V(t) = \delta^T\delta$, and similar to (22) in Theorem 1, for $t \in (t_{k-1}, t_k]$, one get

$$\mathcal{L}V(t) \le (\lambda_A + 2l + \lambda_\Sigma)V(t).$$

Thus, (H2) in Lemma 1 holds with $g(t, \omega) = (\lambda_A+2l+\lambda_\Sigma)\omega$.

Similar to (23) in Theorem 1, for any $\rho_0 > 0$ and $\delta \in S(\rho_0)$, one gets

$$\|\delta + U(k, \delta)\| \le \|\delta\|.$$

From $\|\delta + U(k, \delta)\| \le \|\delta\|$, it has $\delta + U(k, \delta) \in S(\rho_0)$

When $t = t_k$, one can get

$$\begin{aligned}E(V(t_k^+)) &= E(\delta^T(t_k^+)\delta(t_k^+))\\ &= E(((b_k((L + C) \otimes I_n) + I_{nN})\delta(t_k))^T\\ &\quad \times((b_k((L + C) \otimes I_n) + I_{nN})\delta(t_k)))\\ &= E(\delta^T(t_k)((b_k(L + C) + I_N)^T\\ &\quad \times(b_k(L + C) + I_N) \otimes I_n)\delta(t_k))\\ &\le \lambda_k E(V(t_k)).\end{aligned}$$

Thus, (H3) in Lemma 1 holds with $\varphi_k(\omega) = \lambda_k\omega$, and (H1) in Lemma 1 also holds with $\beta(\delta) = \lambda_1 \|\delta\|^2$ and $\alpha(\delta) = \lambda_2 \|\delta\|^2$, where $\lambda_1 \in (0, 1)$ and $\lambda_2 > 0$. From Lemma 1, it yields that the asymptotic stability of system (20) is proven by the asymptotic stability of the following comparison system:

$$\begin{cases}\dot{\omega} = (\lambda_A + 2l + \lambda_\Sigma)\omega, t \neq t_k,\\ \omega(t_k^+) = \lambda_k\omega(t_k),\\ \omega(t_0^+) = \omega_0 \ge 0.\end{cases}$$

From $\dot{\lambda}(t) = \lambda_A + 2l + \lambda_\Sigma \ge 0$, $\lambda(\tau_{k+1}^l) = (\lambda_A + 2l + \lambda_\Sigma)\tau_{k+1}^l$ and $\lambda(\tau_k^l) = (\lambda_A + 2l + \lambda_\Sigma)\tau_k^l$ are obtained easily, which shows (24) is equivalent to (6) in Lemma 3. Then the origin of (20) is asymptotically stable, which implies that the consensus of stochastic perturbed MAS (15) can be achieved. □

Theorem 3: If Assumptions 1-5 hold and there exists $\xi > 1$ such that

$$(\lambda_A + 2l + \lambda_\Sigma)(\tau_{k+1} - \tau_k) + \ln(\lambda_k \xi) < 0, \tag{25}$$

where λ_A, λ_k, and λ_Σ, have the same meanings in Theorem 1. Then the consensus of multi-agent systems (15) can be achieved.

Proof: Note that $\lambda(\tau_{k+1}) = (\lambda_A + 2l + \lambda_\Sigma)\tau_{k+1}$, and $\lambda(\tau_k) = (\lambda_A + 2l + \lambda_\Sigma)\tau_k$, which shows (25) is equivalent to (10) in Lemma 4. The analysis process is similar to the proof of Theorem 2, which is omitted here for simplicity.

V. NUMERICAL EXAMPLES

In this section, we consider the Chua's system [51] as the example, which is a very typical nonlinear one with Lipschitz condition and usually used for the numerical simulation in some existing consensus or synchronization cases. The stochastic perturbed MAS is described as

$$\begin{cases} dx_i(t) = (Ax_i(t) + \psi(x_i(t)))dt \\ \qquad\qquad + \vartheta_i(t, \delta_i(t))dw(t), \quad t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = u_i(t_k), \quad k \in \mathbb{N}, \end{cases}$$

where

$$A = \begin{bmatrix} -p_1(1+m_2) & p_1 & 0 \\ 1 & -1 & 1 \\ 0 & -p_2 & 0 \end{bmatrix},$$

$$\psi(x_i) = \begin{bmatrix} -0.5p_1(m_1 - m_2)(|x_{i1}+1| - |x_{i1}-1|) \\ 0 \\ 0 \end{bmatrix}.$$

Let the system parameters be $m_1 = -1.25$, $m_2 = -0.758$, $p_1 = 9.21$, $p_2 = 15.995$, $\vartheta_i(t, \delta_i) = \text{diag}\{1/\sqrt{2}\delta_{i1}, 1/\sqrt{2}\delta_{i2}, 1/\sqrt{2}\delta_{i3}\}$, then it yields $\lambda_A = 16.5492$, $l = |m_1 p_1| = 11.5125$, and $\Sigma = I_3$.

In this section, consider the MASs with a communication topology shown in Fig. 2 consisting of four follower agents and one leader.

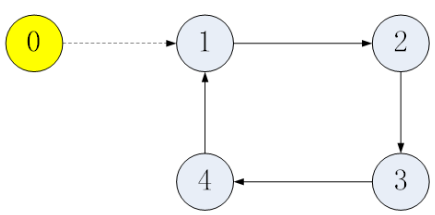

FIGURE 2. Diagram of communication topology.

One has the following matrices from topology $\mathcal{G}$:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$L = D - \mathcal{A} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Let $b_k = -0.6$, $\xi = 1.01$, then it can get $\lambda_k = 0.8721$.

From condition (21) in Theorem 1, it yields

$$t_{k+1} - t_k < -\frac{\ln(\lambda_k \xi)}{\lambda_A + 2l + \lambda_\Sigma} = -\frac{\ln(1.01 \times 0.8721)}{40.5742}.$$

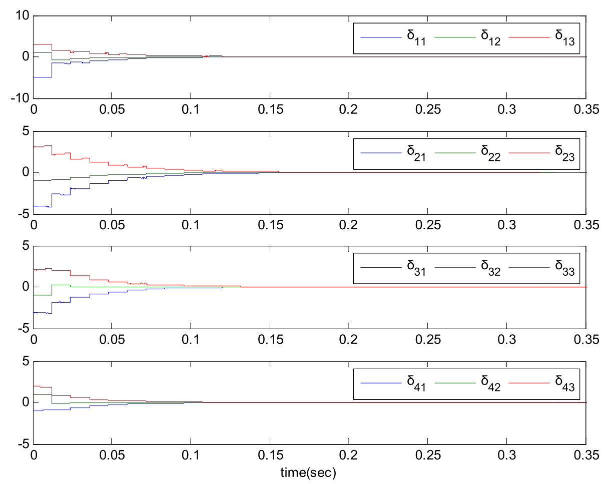

FIGURE 3. Consensus error trajectory for Theorem 1.

From Fig. 3, it is observed that the consensus error can converge to zero asymptotically, which verifies the correctness of the proposed comparison system method. Fig. 4 shows the impulsive sequence t_k in the simulations.

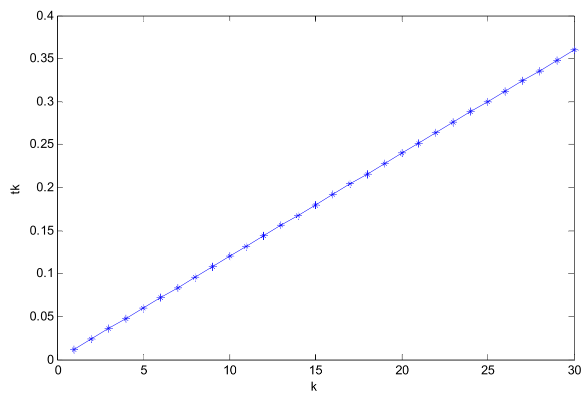

FIGURE 4. The impulsive instant t_k vs k.

Similarly, let $b_k = -0.6$, $\xi = 1.01$, from condition (24) in Theorem 2 and condition (25) in Theorem 3, the following estimation can be obtained as

$$\tau_{k+1}^{l} - \tau_{k}^{l} < -\frac{\ln(\lambda_k \xi)}{\lambda_A + 2l + \lambda_\Sigma},$$
$$\tau_{k+1} - \tau_{k} < -\frac{\ln(\lambda_k \xi)}{\lambda_A + 2l + \lambda_\Sigma}.$$

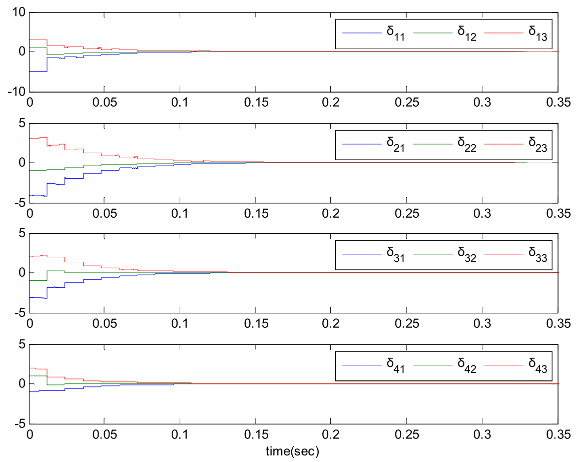

FIGURE 5. **Consensus error trajectory for Theorem 2.**

The consensus error trajectories are shown in Fig. 5. From this figure, it can obtain that the synchronization time between other nodes and the leader node is less than 0.15s. It can verify the effectiveness of Theorem 2. Furthermore, the relation between t_k and τ_k^l is shown in Fig. 6. The actual impulsive instant and impulsive left endpoint are represented by blue stars and red circles. Obviously, the actual t_k is greater than τ_k^l.

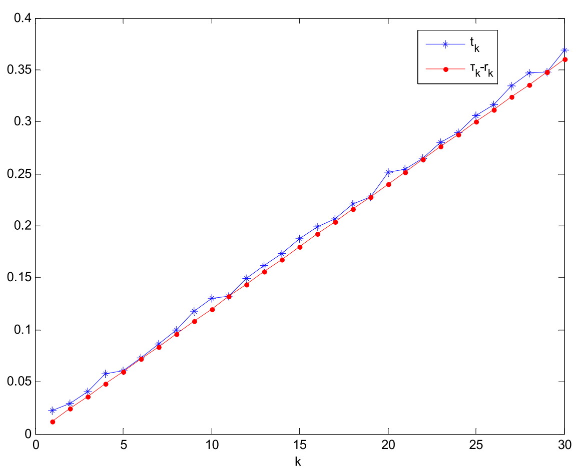

FIGURE 6. **The relation between t_k and τ_k^l.**

Under the condition in Theorem 3, consensus of the stochastic perturbed MAS with impulsive center points can be realized in Fig. 7, which is less than 0.15s. The relationship between t_k and τ_k is shown in Fig. 8. The actual impulsive instant and impulsive center point are represented by blue stars and red circles. From this figure, it can obtain impulsive instants are exerted at left and right sides of the centers.

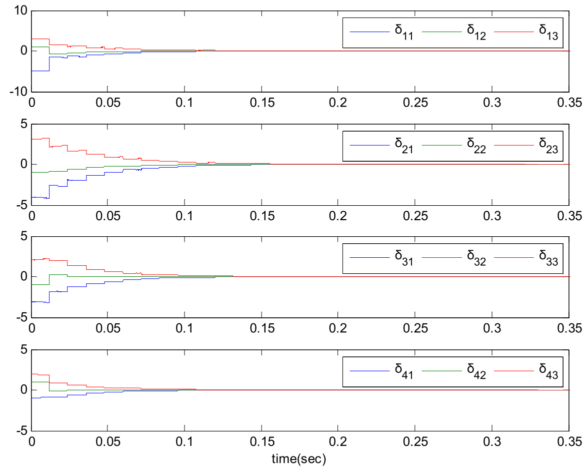

FIGURE 7. **Consensus error trajectory for Theorem 3.**

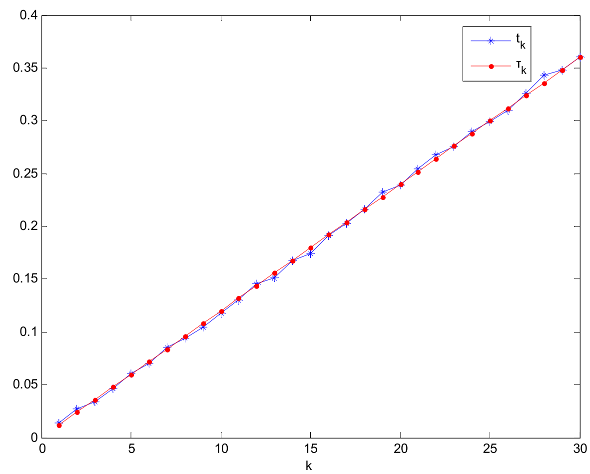

FIGURE 8. **The relationship between t_k and τ_k.**

In the simulation section, the initial states are chosen as $x_1 = [-3, 1, 2]^T$, $x_2 = [-2, -1, 2]^T$, $x_3 = [-1, -1, 1]^T$, $x_4 = [1, 1, 1]^T$, $x_0 = [2, 0, -1]^T$.

VI. CONCLUSION

In this paper, the variable impulsive consensus scheme of stochastic perturbed MAS is investigated. Based on the theory of stochastic and impulsive differential systems, some comparison system-based sufficient conditions are obtained to realize the consensus goal. Compared with the typical and existing impulsive control method, the consensus conditions of variable impulsive protocol in this works is more reasonable and flexible in real application. Finally, the numerical examples are given to illustrate the feasibility of the control method. It should be pointed out that future research topics include further promotion and improvement as well as various potential applications, mainly involving the following aspects.

(1) The impulsive consensus condition in this paper is only a sufficient condition, and it is necessary to further reduce its conservation in our future work.

(2) In practical applications, time delay is inevitable, and how to extend the results in this paper to a more general delayed system is an important issue.

(3) For the stochastic perturbed consensus problem in real system, the additive or multiplicative noise is the more realistic stochastic one [52], [53], which deserves further deep explanation.

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