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3-D RSS-AOA Based Target Localization Method in Wireless Sensor Networks Using Convex Relaxation

SHENGMING CHANG, YOU ZHENG, PENG AN, JIANYU BAO, AND JUN LI
School of Electronic and Information Engineering, Ningbo University of Technology, Ningbo 315211, China

Corresponding author: Shengming Chang (csm20130504@163.com)

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ABSTRACT This paper addresses a target localization problem in 3-D wireless sensor networks using a hybrid system that fuses received signal strength and angle of arrival measurements. First, we formulate the received signal strength and angle of arrival measurement models as the pseudo-linear equations. Then, the bias is derived from the 3-D angle of arrival measurements that take the measurement noise into account to improve the localization performance. Furthermore, a non-convex estimator is derived based on the Least Squares criterion. Finally, semi-definite relaxation and second-order cone relaxation are applied to transform the derived non-convex estimator into a convex one. We propose a semi-definite relaxation and second-order cone relaxation-based estimator which yields the best performance under a large measurement noise or a small measurement noise. The generalization of the proposed method for known transmit power can also be applied to the case when transmit power is not know. Theoretical analysis and computer simulations corroborate the superior performance of the proposed localization methods over the existing ones.

INDEX TERMS Target localization, received signal strength, angle of arrival, convex relaxation.

I. INTRODUCTION

Wireless sensor networks (WSNs) have been used in a wide range of applications, such as target tracking, navigation, emergency services, friends finding and intelligent transportation [1], [2]. Knowledge of the sensor node location is an indispensable part for most WSNs applications, especially when it comes to the rapid development of network and information technology in the modern society, definite information of the specific location of each object becomes quite necessary for making decisions and taking actions [3]. In general, only a number of sensor nodes, called anchor nodes, have their location information known, whereas the others, called target nodes, have their location information unknown, which remains to be determined through localization methods. Most localization methods make use of the noise measurements, such as time of arrival (TOA) [4]–[6], time difference of arrival (TDOA) [7], angle of arrival (AOA) [8], [9], and received signal strength (RSS) [10]–[15] or combinations of them [16]–[26]. Among those, TOA-based and TDOA-based localization requires clock synchronization, AOA-based localization must have an antenna array which increases the cost of hardware, RSS-based localization gradually becomes the primary concern owing to it implementation, but it will fluctuate greatly with the increase of distance. Compared with TOA and TDOA, the latter two measurements do not require time synchronization between the target and the anchor node, this makes them widely used in different situations. Furthermore, hybrid system that combining two measurements of wireless signals has also been studied [16]–[21]. The advantage of hybrid system is that more information of hybrid measurement is used to complete the localization problem. On the other hand, it will increase the cost of network and the complexity [22]–[24]. In this paper, we are focused on a combination of RSS and AOA measurements, which is to further improve the localization accuracy. The biggest challenge for the RSS and AOA measurements is its non-linearity, which makes the localization problem quite complex. The measurement errors in WSNs are usually assumed to be Gaussian noise. Thus, RSS-AOA based localization problem can be solved using Maximum Likelihood (ML) estimator. However, the

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localization accuracy of this method highly depends on the selection of initial point, which means that if the initial solution is not close to the global solution, the final solution will lead to a poor localization accuracy. To avoid this disadvantage of ML estimator, convex optimization method is proposed to solve the RSS-AOA based localization problem, where the semi-definite programming (SDP) method and the second order cone programming (SOCP) method give the best performance with a higher computational complexity [27], [28]. In addition, AOA measurement in 3D is much more challenging than AOA measurement in 2D. The main reason is that it contains azimuth angle and elevation angle measurements, which are both highly non-linear, and this makes the localization problem becomes a highly non-linear one. Moreover, there is a big deviation in the localization problem based on RSS and AOA, especially when the signal-to-noise ratio is low and the localization deployment is bad. Mostly, the most literature only consider the impact of the noise variance on the localization accuracy and ignore the deviation. But under the condition of low signal-to-noise ratio and poor localization deployment, deviation is a major factor that should not be ignored. There are a lot of literatures at present discuss the RSS-AOA based localization problem. [22]–[24] studied the RSS-AOA based localization problem by using convex relaxation technique. In [23], based on LS criterion, the authors derived a novel objective function for solving the hybrid localization problem. Then, the derived non-convex localization problem was approximated into a convex problem by applying second order cone programming relaxation, which could be solved by using the convex hull of MATLAB package CVX [29]–[31]. In [22], for the 3D RSS-AOA localization problem, the authors proposed a generalized trust region subproblem (GTRS) method that could quickly solve the target localization problem with a lower computational complexity. In [24], a new relationship between the measurement information and the unknown target location was established by using the spherical coordinate conversion, and then a closed-form solution of the original problem was obtained by LS method. Compared with the convex relaxation method, the computational complexity of these two methods was greatly reduced. However, these methods did not consider the influence of AOA measurement deviation on positioning. In [9], Yue Wang *et al.* fully considered the influence of measurement deviation in 3D AOA localization problem.

In this paper, we consider the localization problem in 3D wireless sensor networks based on RSS and AOA measurements. First of all, we make pseudo-linearization of the RSS and AOA measurement models. Also, considering the influence of the measurement deviation, we reconstruct the pseudo-linearization of AOA. Then, we establish the minimum optimization problem for solving the target location by using the LS criterion. Finally, the derived cost function and constraint are transformed into a convex optimization problem by applying convex optimization relaxation technique. At the same time, we also provide the computational complexity of the proposed methods. Simulations confirmed the performance of the proposed methods which demonstrated an improvement over the previous works. Finally, we also show that the proposed estimators for known transmit power is straightforward for the case when transmit power is not know.

The main contributions of this paper are summarized as follows:

1) The proposed methods begin with the RSS and 3D AOA measurements and derive the pseudo-linear equations. Then the bias is considered to recast the derived pseudo-linear equations in augmented form.

2) The RSS and AOA models are transformed into unified form, then a novel non-convex objective function for solving the target location is derived based on LS criterion, which tightly approximates the ML one for small noise.

3) Semi-definite programming and second order programming relaxation techniques are used to transform the derived non-convex objective function into a convex one.

The following notations are adopted throughout the paper. Bold face lower case letters and bold face upper case letters denote the vectors and matrices, respectively. $\mathbb{R}^n$ denotes the set of n-dimensional real column vectors. r_i denotes the ith entry of the vector $\boldsymbol{r}$. In addition, $\|\cdot\|$ denotes the ℓ_2-norm.

The rest of the paper is organized as follows. The RSS and AOA models and the localization scenario are given in Section II, in which we also propose the localization problem. In Section III, the proposed localization method is derived. In Section IV, the complexity of the proposed method is analyzed. Section V provides computer simulation results and analyzes the performances of the proposed methods. Finally, the main conclusions are made in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The localization problem in three dimensional (3-D) space is considered. In this section, the RSS and AOA measurement models are presented in the first place. Then the formulation for RSS-AOA localization problem is presented.

A. SYSTEM MODEL

In this localization system, only one target is to be estimated based on the corresponding target to anchors RSS measurements and AOA measurements. Under a centralized processing mode, all sensors convey their RSS and AOA measurements with respect to the target node location to the central processor, during which the locations of all the sensor nodes are supposed to be unchanged. We consider a 3D WSNs with N anchor nodes with locations $\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_N$ ($\boldsymbol{s}_i = (s_{i1}, s_{i2}, s_{i3})^T \in \mathbb{R}^3$) and one target at $\boldsymbol{x}$ ($\boldsymbol{x} = (x_1, x_2, x_3)^T \in \mathbb{R}^3$) which is unknown and is to be estimated. Fig. 1 shows the localization scenario. With log-normal shadowing, the true values of received signal path loss L_i (in dB) between the target node and the ith anchor node can be shown as

$$L_i = L_0 + 10\gamma \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{s}_i\|}{d_0}, \quad i = 1, 2, \cdots N, \tag{1}$$

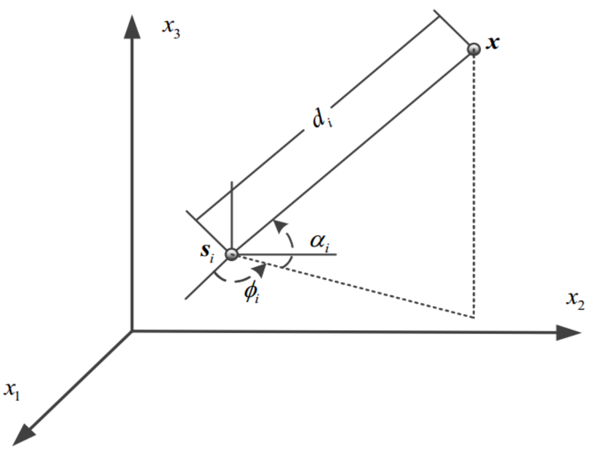

FIGURE 1. Illustration the link between the target node and the ith anchor node in 3D WSNs.

where L_0 denotes the reference path loss value at the reference distance d_0 ($\|\boldsymbol{x} - \boldsymbol{s}_i\| \geq d_0$), γ is the path loss exponent (PLE), ϕ_i, α_i respectively represents the true value of azimuth angle and elevation angle between the target node and the ith anchor node. It can be shown in a nonlinear manner as

$$\phi_i = \arctan\left(\frac{x_2 - s_{i2}}{x_1 - s_{i1}}\right), \tag{2}$$

and

$$\alpha_i = \arctan\left(\frac{x_3 - s_{i3}}{(x_1 - s_{i1})cos\phi_i + (x_2 - s_{i2})sin\phi_i}\right), \tag{3}$$

where $\phi_i \in (-\pi, \pi)$ and $\alpha_i \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $i = 1, 2, \cdots N$.

In the presence of measurement errors in practice, the observed measurement path loss, azimuth, and elevation equations are

$$\hat{\boldsymbol{L}} = \boldsymbol{L} + \boldsymbol{n}, \tag{4}$$
$$\hat{\boldsymbol{\phi}} = \boldsymbol{\phi} + \boldsymbol{m}, \tag{5}$$
$$\hat{\boldsymbol{\alpha}} = \boldsymbol{\alpha} + \boldsymbol{v}, \tag{6}$$

where $\hat{\boldsymbol{L}} = [\hat{L}_1, \hat{L}_2, \cdots, \hat{L}_N]^T$, $\hat{\boldsymbol{\phi}} = [\hat{\phi}_1, \hat{\phi}_2, \cdots, \hat{\phi}_N]^T$, and $\hat{\boldsymbol{\alpha}} = [\hat{\alpha}_1, \hat{\alpha}_2, \cdots, \hat{\alpha}_N]^T$ are the observation vectors of path loss, azimuth, and elevation measurements respectively and $\boldsymbol{L} = [L_1, L_2, \cdots, L_N]^T$, $\boldsymbol{\phi} = [\phi_1, \phi_2, \cdots, \phi_N]^T$, and $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_N]^T$ are the true values. $\boldsymbol{n} = [n_1, n_2, \cdots, n_N]^T$, $\boldsymbol{m} = [m_1, m_2, \cdots, m_N]^T$, and $\boldsymbol{v} = [v_1, v_2, \cdots, v_N]^T$ are the zero mean Gaussian measurement noises of path loss, azimuth and elevation respectively, and are modeled as $n_i \sim \mathcal{N}(0, \sigma_{n_i}^2)$, $m_i \sim \mathcal{N}(0, \sigma_{m_i}^2)$ and $v_i \sim \mathcal{N}(0, \sigma_{v_i}^2)$. For the sake of simplicity, in the rest of this paper, we assume $\sigma_{n_i}^2 = \sigma_n^2, \sigma_{m_i}^2 = \sigma_m^2, \sigma_{v_i}^2 = \sigma_v^2$ for $i = 1, \cdots, N$. Given the RSS and AOA measurements $\boldsymbol{\theta} = [\hat{\boldsymbol{L}}^T, \hat{\boldsymbol{\phi}}^T, \hat{\boldsymbol{\alpha}}^T]^T$ ($\boldsymbol{\theta} \in \mathbb{R}^{3N}$), We would like to estimate the location of the target node as accurately as possible.

B. PROBLEM FORMULATION

Based on (4), (5), and (6), we can formulate the ML estimation of target location $\boldsymbol{x}$ as

$$\min_{\boldsymbol{x}} \sum_{i=1}^{3N} \frac{(\theta_i - f_i(\boldsymbol{x}))^2}{\sigma_i^2}, \tag{7}$$

where, $\sigma_i = [\sigma_{n_i}, \sigma_{m_i}, \sigma_{v_i}]^T$ and

$$f_i(\boldsymbol{x}) = \Bigg[L_0 + 10\gamma \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{s}_i\|}{d_0}, \arctan(\frac{x_2 - s_{i2}}{x_1 - s_{i1}}), \arctan\left(\frac{x_3 - s_{i3}}{(x_1 - s_{i1})cos\phi_i + (x_2 - s_{i2})sin\phi_i}\right)\Bigg]^T.$$

Typically, it is seen that (7) is non-convex and difficult to solve.

III. LOCALIZATION VIA SDP AND SOCP RELAXATION

In this section, a convex relaxation method is proposed to approximately solve the ML problem (7), where the derived solution bias is minimum.

In the following part of this section, we develop a suboptimal method to solve the localization problem in (7), where the exact solution is obtained by convex optimization. Then, we show that the estimation procedure can also be applied to the case where transmit power P_T is not know.

A. LOCALIZATION METHOD WITH KNOWN P_T

We will first dispose of the RSS expression. Dividing by 10γ and taking the power on both side of (4), we obtain

$$10^{\frac{L_i - L_0}{10\gamma}} = \frac{\|\boldsymbol{x} - \boldsymbol{s}_i\|}{d_0} 10^{\frac{n_i}{10\gamma}}. \tag{8}$$

Let $\hat{d}_i = d_0 10^{\frac{L_i - L_0}{10\gamma}}$, (8) is transformed to

$$\hat{d}_i = \|\boldsymbol{x} - \boldsymbol{s}_i\| \epsilon_i, \tag{9}$$

where $\epsilon_i = 10^{\frac{n_i}{10\gamma}}$.

Then, for the AOA expressions, let us derive the pseudo-linear equations. Taking the tangent on both side of (2), we have

$$\tan\phi_i = \frac{x_2 - s_{i2}}{x_1 - s_{i1}}. \tag{10}$$

Using the relation of the trigonometric function of the same angle, i.e., $\tan\phi_i = \frac{\sin\phi_i}{\cos\phi_i}$, (10) can be equivalently written as

$$\sin\phi_i(x_1 - s_{i1}) - \cos\phi_i(x_2 - s_{i2}) = 0. \tag{11}$$

Note that ϕ_i is the true value which is not known in equation (11), therefore we need to further consider the noise value of ϕ_i. If consider the noise value, (11) should have the following form

$$\sin\hat{\phi}_i(x_1 - s_{i1}) - \cos\hat{\phi}_i(x_2 - s_{i2}) = \eta_{\hat{\phi}_i}, \tag{12}$$

where $\eta_{\hat{\phi}_i}$ is the residual caused by the noise. Next, we use the undetermined coefficient method to solve $\eta_{\hat{\phi}_i}$.

By using (5), when the measurement noise is sufficiently small, we have $\cos m_i \approx 1$, $\sin m_i \approx m_i$ and

$$\sin\hat{\phi}_i = \sin(\phi_i + m_i) \approx \sin\phi_i + m_i\cos\phi_i,$$
$$\cos\hat{\phi}_i = \cos(\phi_i + m_i) \approx \cos\phi_i - m_i\sin\phi_i. \quad (13)$$

Putting (13) into (12), we arrive at

$$\eta_{\hat{\phi}_i} = d_i\cos\alpha_i m_i, \quad (14)$$

where $d_i = \|\boldsymbol{x} - \boldsymbol{s}_i\|$ is the true distance between target and anchor.

Then, (12) can be rewritten as

$$\boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x} - \boldsymbol{s}_i) = d_i\cos\alpha_i m_i, \quad (15)$$

where $\boldsymbol{g}_{\hat{\phi}_i} = [\sin\hat{\phi}_i, -\cos\hat{\phi}_i, 0]^T$.

Similar to the derivation way of (15), according to (3) and (6), we can obtain the following form [9]

$$\boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x} - \boldsymbol{s}_i) = d_i v_i, \quad (16)$$

where $\boldsymbol{g}_{\hat{\alpha}_i} = [\sin\hat{\alpha}_i\cos\hat{\phi}_i, \sin\hat{\alpha}_i\sin\hat{\phi}_i, -\cos\hat{\alpha}_i]^T$.

Following the LS criterion, from (9), (15), and (16), we can estimate the target location $\boldsymbol{x}$ by the following LS formulation

$$\min_{\boldsymbol{x}} \sum_{i=1}^{N}\left(\frac{\hat{d}_i}{\|\boldsymbol{x}-\boldsymbol{s}_i\|}\right)^2 + \sum_{i=1}^{N}\left(\frac{\boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)}{\|\boldsymbol{x}-\boldsymbol{s}_i\|\cos\alpha_i}\right)^2 + \sum_{i=1}^{N}\left(\frac{\boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)}{\|\boldsymbol{x}-\boldsymbol{s}_i\|}\right)^2. \quad (17)$$

The Problem (17) is obviously non-convex. We shall apply SOCP and SDP relaxation technique to obtain a convex problem. For this purpose, introduce auxiliary variables

$$z_i = \left(\frac{\hat{d}_i}{\|\boldsymbol{x}-\boldsymbol{s}_i\|}\right)^2 = \frac{\hat{d}_i^{\,2}}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2}, \quad h_i = \left(\frac{\boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)}{\|\boldsymbol{x}-\boldsymbol{s}_i\|\cos\alpha_i}\right)^2 = \frac{\left(\boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)\right)^2}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2\cos^2\alpha_i}, k_i = \left(\frac{\boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)}{\|\boldsymbol{x}-\boldsymbol{s}_i\|}\right)^2 = \frac{\left(\boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)\right)^2}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2}$$

for $i = 1, 2, \cdots, N$. Then, Problem (17) can be equivalently written as

$$\min_{\boldsymbol{x},z_i,h_i,k_i} \sum_{i=1}^{N} z_i + \sum_{i=1}^{N} h_i + \sum_{i=1}^{N} k_i,$$
$$\text{s.t. } z_i = \frac{\hat{d}_i^{\,2}}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2}, \quad (18a)$$
$$h_i = \frac{\left(\boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)\right)^2}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2\cos^2\alpha_i}, \quad (18b)$$
$$k_i = \frac{\left(\boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)\right)^2}{\|\boldsymbol{x}-\boldsymbol{s}_i\|^2}. \quad (18c)$$

However, Problem (18) is still non-convex. The non-convexity of this comes from the non-linear equality constraints (18a), (18b), and (18c). In order to deal with this non-convexity, we also introduce several auxiliary variables $u_i = \|\boldsymbol{x} - \boldsymbol{s}_i\|^2$, $b_i = \boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)$, $c_i = \boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i)$, and we utilize the measurement value $\hat{\alpha}_i$ to replace α_i, we have

$$\min_{\substack{\boldsymbol{x},z_i,h_i,k_i\\u_i,b_i,c_i}} \sum_{i=1}^{N} z_i + \sum_{i=1}^{N} h_i + \sum_{i=1}^{N} k_i, \quad (19a)$$
$$\text{s.t. } u_i = \|\boldsymbol{x}-\boldsymbol{s}_i\|^2, \quad (19b)$$
$$b_i = \boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i), \quad (19c)$$
$$c_i = \boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i), \quad (19d)$$
$$z_i = \frac{\hat{d}_i^{\,2}}{u_i}, \quad (19e)$$
$$h_i = \frac{b_i^2}{u_i\cos^2\hat{\alpha}_i}, \quad (19f)$$
$$k_i = \frac{c_i^2}{u_i}. \quad (19g)$$

Let $\boldsymbol{I}_3$ as the 3×3 identity matrix and defining $\boldsymbol{X} = \begin{bmatrix}\boldsymbol{I}_3 & \boldsymbol{x}\\ \boldsymbol{x}^T & \boldsymbol{x}^T\boldsymbol{x}\end{bmatrix}$, Problem (19) can be reformulated as

$$\min_{\substack{\boldsymbol{x},z_i,h_i,k_i\\u_i,b_i,c_i,\boldsymbol{X}}} \sum_{i=1}^{N} z_i + \sum_{i=1}^{N} h_i + \sum_{i=1}^{N} k_i, \quad (20a)$$
$$\text{s.t. } u_i = \begin{bmatrix}\boldsymbol{s}_i\\-1\end{bmatrix}^T \boldsymbol{X} \begin{bmatrix}\boldsymbol{s}_i\\-1\end{bmatrix}, \quad (20b)$$
$$b_i = \boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i), \quad (20c)$$
$$c_i = \boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i), \quad (20d)$$
$$z_i = \frac{\hat{d}_i^{\,2}}{u_i}, \quad (20e)$$
$$h_i = \frac{b_i^2}{u_i\cos^2\hat{\alpha}_i}, \quad (20f)$$
$$k_i = \frac{c_i^2}{u_i}, \quad (20g)$$
$$\boldsymbol{X}_{1:3,1:3} = \boldsymbol{I}_3, \quad (20h)$$
$$\boldsymbol{X} \succeq \boldsymbol{0}_4, \quad (20i)$$
$$rank(\boldsymbol{X}) = 3. \quad (20j)$$

Dropping the non-convex constraint (20j) and applying SDP and SOCP technique, we have the mixed SD/SOCP estimator defined below as

$$\min_{\substack{\boldsymbol{x},z_i,h_i,k_i\\u_i,b_i,c_i,\boldsymbol{X}}} \sum_{i=1}^{N} z_i + \sum_{i=1}^{N} h_i + \sum_{i=1}^{N} k_i, \quad (21a)$$
$$\text{s.t. } u_i = \begin{bmatrix}\boldsymbol{s}_i\\-1\end{bmatrix}^T \boldsymbol{X} \begin{bmatrix}\boldsymbol{s}_i\\-1\end{bmatrix}, \quad (21b)$$
$$b_i = \boldsymbol{g}_{\hat{\phi}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i), \quad (21c)$$
$$c_i = \boldsymbol{g}_{\hat{\alpha}_i}^T(\boldsymbol{x}-\boldsymbol{s}_i), \quad (21d)$$
$$\begin{bmatrix}u_i & \hat{d}_i\\ \hat{d}_i & z_i\end{bmatrix} \succeq 0, \quad (21e)$$

$$\left\|\begin{bmatrix} 2b_i \\ u_i \cos^2 \hat{\alpha}_i - h_i \end{bmatrix}\right\| \leq u_i \cos^2 \hat{\alpha}_i + h_i, \tag{21f}$$

$$\left\|\begin{bmatrix} 2c_i \\ u_i - k_i \end{bmatrix}\right\| \leq u_i + k_i, \tag{21g}$$

$$\boldsymbol{X}_{1:3,1:3} = \boldsymbol{I}_3, \tag{21h}$$

$$\boldsymbol{X} \succeq \boldsymbol{0}_4. \tag{21i}$$

This is the proposed convex estimator for known transmit power P_T based on hybrid RSS-AOA measurements. We label the estimator (21) as "SD/SOCP1-new" method in this paper.

B. LOCALIZATION METHOD WITH UNKNOWN P_T

In order to reduce the cost of implementation, testing and calibration are not necessary. Therefore, the transmit power of the sensor is often unknown, i.e. P_T is not know. Not knowing P_T in RSS measurement model corresponds to not knowing L_0 in model (1). The generalization of the proposed method for known L_0 is straightforward for the case where L_0 is not known. Then, the equation (8) can be rewritten as

$$\beta_i \eta = \|\boldsymbol{x} - \boldsymbol{s}_i\| \epsilon_i, \tag{22}$$

where $\beta_i = d_0 10^{\frac{L_i}{10\gamma}}$, $\epsilon_i = 10^{\frac{n_i}{10\gamma}}$ and $\eta = 10^{\frac{-L_0}{10\gamma}}$ is an unknown parameter that need to be estimated.

For the sufficiently small noise, according to (22), (15), and (16), we can get the following LS problem as

$$\min_{\boldsymbol{x},\eta} \sum_{i=1}^{N} \left(\frac{\beta_i \eta}{\|\boldsymbol{x} - \boldsymbol{s}_i\|}\right)^2 + \sum_{i=1}^{N} \left(\frac{\boldsymbol{g}_{\hat{\phi}_i}^T (\boldsymbol{x} - \boldsymbol{s}_i)}{\|\boldsymbol{x} - \boldsymbol{s}_i\| \cos \alpha_i}\right)^2 + \sum_{i=1}^{N} \left(\frac{\boldsymbol{g}_{\hat{\alpha}_i}^T (\boldsymbol{x} - \boldsymbol{s}_i)}{\|\boldsymbol{x} - \boldsymbol{s}_i\|}\right)^2. \tag{23}$$

Introduce auxiliary variable $\hat{z}_i = \left(\frac{\beta_i \eta}{\|\boldsymbol{x} - \boldsymbol{s}_i\|}\right)^2$. Using the similar steps as described in Section III-A, we obtain the following convex estimator

$$\min_{\substack{\boldsymbol{x},\hat{z}_i,h_i,k_i \\ u_i,b_i,c_i,\boldsymbol{X},\eta}} \sum_{i=1}^{N} \hat{z}_i + \sum_{i=1}^{N} h_i + \sum_{i=1}^{N} k_i, \tag{24a}$$

$$\text{s.t. } u_i = \begin{bmatrix} \boldsymbol{s}_i \\ -1 \end{bmatrix}^T \boldsymbol{X} \begin{bmatrix} \boldsymbol{s}_i \\ -1 \end{bmatrix}, \tag{24b}$$

$$b_i = \boldsymbol{g}_{\hat{\phi}_i}^T (\boldsymbol{x} - \boldsymbol{s}_i), \tag{24c}$$

$$c_i = \boldsymbol{g}_{\hat{\alpha}_i}^T (\boldsymbol{x} - \boldsymbol{s}_i), \tag{24d}$$

$$\begin{bmatrix} u_i & \beta_i \eta \\ \beta_i \eta & \hat{z}_i \end{bmatrix} \succeq 0, \tag{24e}$$

$$\left\|\begin{bmatrix} 2b_i \\ u_i \cos^2 \hat{\alpha}_i - h_i \end{bmatrix}\right\| \leq u_i \cos^2 \hat{\alpha}_i + h_i, \tag{24f}$$

$$\left\|\begin{bmatrix} 2c_i \\ u_i - k_i \end{bmatrix}\right\| \leq u_i + k_i, \tag{24g}$$

$$\boldsymbol{X}_{1:3,1:3} = \boldsymbol{I}_3, \tag{24h}$$

$$\boldsymbol{X} \succeq \boldsymbol{0}_4. \tag{24i}$$

This is the proposed convex estimator for unknown transmit power P_T based on hybrid RSS-AOA measurements. We label the estimator (24) as "SD/SOCP2-new" method in this paper.

IV. COMPLEXITY ANALYSIS

The computational complexity is analyzed in this section. The *worst-case* complexity of the considered method is used, which is based on the following formula [32]:

$$\mathcal{O}\left(\sqrt{L}\left(m \sum_{i=1}^{N_{sd}} n_i^{sd^3} + m^2 \sum_{i=1}^{N_{sd}} n_i^{sd^2} + m^2 \sum_{i=1}^{N_{soc}} n_i^{soc} + \sum_{i=1}^{N_{soc}} n_i^{soc^2} + m^3\right)\right), \tag{25}$$

where L is the number of iterations, m is the number of equality constraints, N_{soc} is the number of the second order (SOC) constraints, and n_i^{soc} is dimension of the ith SOC. Assume that $K_{max} = 30$ is the maximum number of steps in the bisection procedure used in [22] and [24]. In the next section, simulation results will indicate that the proposed methods show better performance.

V. SIMULATION RESULTS

In this section, a set of Monte Carlo simulations are conducted to evaluate the performance of the proposed methods in comparison with the existing ones for RSS-AOA hybrid localization with known and unknown P_T. N anchor nodes and one target node are randomly distributed in a box with an edge length $B = 15$ m. We assume that all sensor nodes are fully connected, i.e., all anchor nodes and target node can communicate with each other. The models (4), (5), and (6) are used to generate measurements. All sensor nodes are assumed to have the same transmit power and the received signal path loss (which corresponds to the received signal power) at the reference distance $d_0 = 1$ m is set to $L_0 = 40$ dB, the PLE is fixed as $\gamma = 4$. The performance of all discussed methods are evaluated using the root mean square error (RMSE), defined as RMSE$=\sqrt{\sum_{i=1}^{Mc} \frac{\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|^2}{Mc}}$, where $\hat{\boldsymbol{x}}$ and $\boldsymbol{x}$ is the estimated location and true location of the ith Monte Carlo (Mc) run respectively, and Mc=3000 being the number of runs. In order to demonstrate the benefit of hybrid systems versus traditional localization ones, we present also the performance results of the proposed methods when only RSS measurements are employed, called here RSS-only-K and RSS-only-U for known and unknown P_T, respectively. Also, it is worth mentioning that for the sake of fairness, the LS method in [19] is given here. We use ML-Ture-K and ML-Ture-U to denote the ML solution of (7) for known and unknown P_T by using the starting point provided by the true location. Furthermore, we also give the Cramér-Rao lower bound (CRLB) on the RMSE of any unbiased estimator as a performance benchmark, and use the CRLB-K and CRLB-U to denote the CRLB of the two cases when known and unknown P_T, respectively. All of the presented methods are solved by using the MATLAB package CVX [31], where the solver is SDPT3 [30]. The corresponding ML problems are

TABLE 1. **Summary of the Considered Methods In Section III-A and Section V-A.**

Method	Description	Complexity
SD/SOCP1	The SD/SOCP method in [25]	$\mathcal{O}(N^{3.5})$
GTRS-SRWLS1	The GTRS method in [22]	$\mathcal{O}(K_{max}N)$
SC-WLS1	The WLS method in [24]	$\mathcal{O}(K_{max}N)$
SOCP	The SOCP method in [23]	$\mathcal{O}(N^{3.5})$
LS	The LS method in [19]	$\mathcal{O}(N)$
SD/SOCP1-new	The proposed method in III-A	$\mathcal{O}(N^{3.5})$

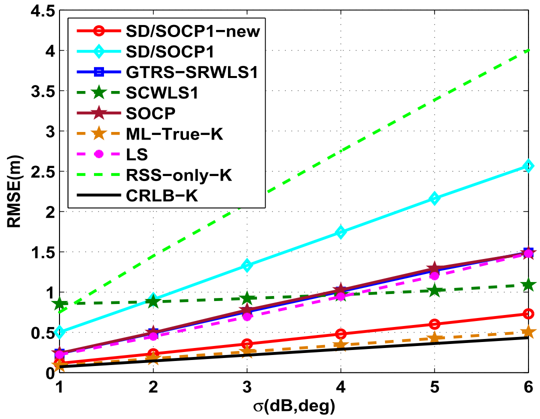

FIGURE 2. **RMSE versus the σ (dB, deg) comparison, when $N = 9$ and $\gamma = 4$, $B = 15$ m, $L_0 = 40$ dB, $d_0 = 1$ m, $Mc = 3000$.**

solved by MATLAB function "Isqnonlin". In the following part of this section, we will verify the performance of the proposed methods through different scenarios.

A. P_T KNOWN

Table 1 gives an overview of the compared methods in this case, together with their complexities. From Table 1, we can see that the complexity of each algorithm mainly depends on the network size, i.e., the number of sensors in WSNs. As shown in this table, the proposed method SD/SOCP1-new has the same computational complexity as SOCP and SD/SOCP1 methods, but slightly higher than GTRS-SRWLS1, SC-WLS1, and LS methods. However, it shows the superior performance in the sense of localization accuracy than the other considered methods, as we will see in the following simulation results.

1) Effect of the noise standard deviation. The RMSE of all discussed methods versus the noise standard deviation is given in Fig.2. From this figure, the RMSE of all the discussed methods increases as the noise standard deviation σ increases, i.e., the performance of all considered methods deteriorates as σ grows. It also shows that the effectiveness of combined measurements of RSS and AOA versus using only a single measurement for our proposed method. Moreover, with the increase of σ, the gap between the methods is increasing. Finally, the proposed method performs better than other discussed methods and is closest to the ML-True-K and CRLB-K for all choices of σ.

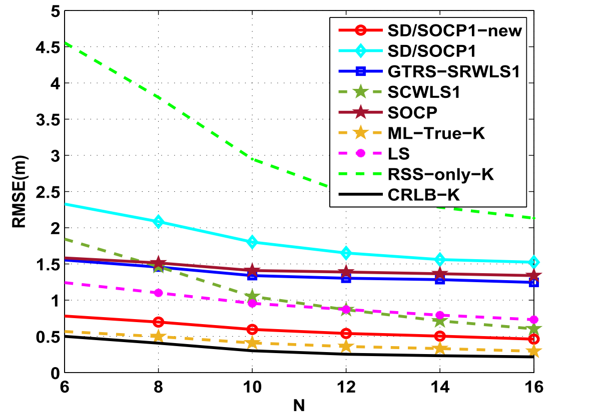

FIGURE 3. **RMSE versus of sensor number N comparison, when $\sigma_n = 4$ dB, $\sigma_m = 5$ deg, $\sigma_v = 5$ deg, $\gamma = 4$, $B = 15$ m, $L_0 = 40$ dB, $d_0 = 1$ m, $Mc = 3000$.**

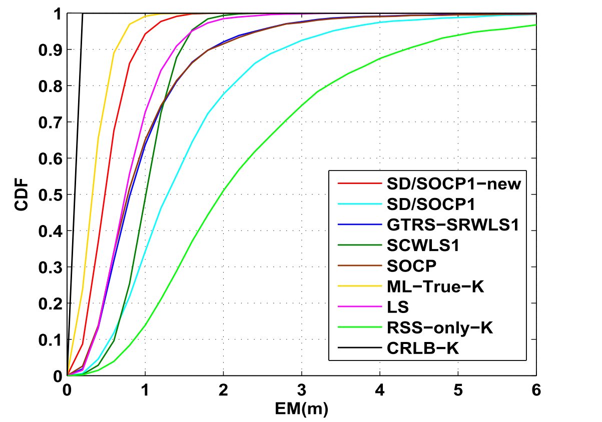

FIGURE 4. **CDF versus the mean error comparison, when $N = 9$ and $\sigma_n = 4$ dB, $\sigma_m = 5$ deg, $\sigma_v = 5$ deg, $\gamma = 4$, $B = 15$ m, $L_0 = 40$ dB, $d_0 = 1$ m, $Mc = 3000$.**

2) Effect of the number of the anchor nodes. Fig. 3 compares the RMSE versus the number of anchor nodes N when $\sigma_n = 4$ dB, $\sigma_m = 5$ deg, and $\sigma_v = 5$ deg. Fig. 3 shows that the RMSE of all considered methods decreases when the number of anchor nodes increases, and the proposed SD/SOCP1-new method results to the smallest RMSE among the discussed methods. Moreover, it is shown that a smaller N leads to a larger performance margin, and the performance margin slightly decreases with the increase of N. From the figure, it is clear that the proposed method outperforms the considered methods for all choices of N. Furthermore, the proposed method is very close to ML-Ture-K and CRLB-K for all choices of N. Finally, even though we derive our method under the assumption that the noise is small enough, Fig. 3 shows that the proposed method has high positioning performance in high noise environment. This behavior is since the considering the influence of the measurement deviation of AOA.

3) Effect of the cumulative distribution function (CDF) of the mean error (ME). CDF can be used to evaluate the accuracy of localization method. From the CDF trend figure with localization error, we can clearly see the deviation range

TABLE 2. Summary of the Considered Methods In Section III-B and Section V-B.

Method	Description	Complexity
SD/SOCP2	The SD/SOCP method in [25]	$2 \cdot \mathcal{O}(N^{3.5})$
GTRS-SRWLS2	The GTRS method in [22]	$2 \cdot \mathcal{O}(K_{max}N)$
SC-WLS2	The WLS method in [24]	$2 \cdot \mathcal{O}(K_{max}N)$
SD/SOCP2-new	The proposed method in III-B	$\mathcal{O}(N^{3.5})$

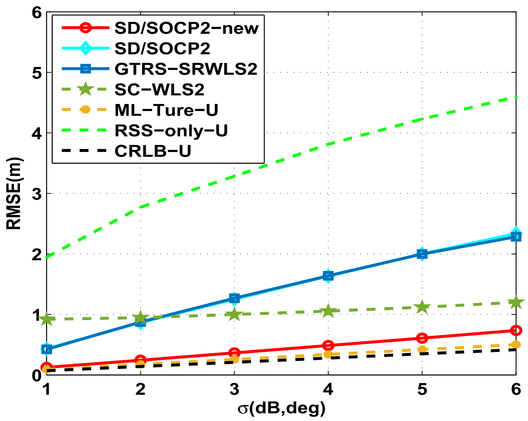

FIGURE 5. RMSE versus the σ (dB, deg) comparison, when $N = 9$ and $\gamma = 4$, $B = 15$ m, $d_0 = 1$ m, $Mc = 3000$.

of the localization method. Generally, the closer the CDF is to 100%, the better localization accuracy and the faster convergence speed there will be. Fig. 4 shows the CDF comparison of the error in target location estimation of the considered methods. From Fig. 4, it can be seen that the proposed method outperforms the other considered methods for all range of ME. Furthermore, we can see that the proposed method achieves ME$\leq$ 1 m in 95% of the cases, whereas the other considered methods attain the same ME in less than 75% of the cases.

B. P_T UNKNOWN

Table 2 also gives an overview of the considered methods in this case when P_T is not know, together with their complexities. From table 2, the proposed method SD/SOCP2-new is slightly more complex than other considered methods, due to the convex optimization process. However, the higher computational complexity of the proposed method than the other considered methods is justified by its higher estimation accuracy as we will show in the following part in this section.

1) Effect of the noise standard deviation. Fig. 5 illustrates the RMSE of all discussed methods versus the noise standard deviation. As anticipated, Fig. 5 reveals that the RMSE of all the discussed methods increases as the noise standard deviation σ increases. The proposed method performs better than other discussed methods in the entire range of the noise standard deviation. This is mainly because the proposed method SD/SOCP2-new can get the target location directly through joint estimation of target location and P_T, while the other methods need to iterative bisection procedure,

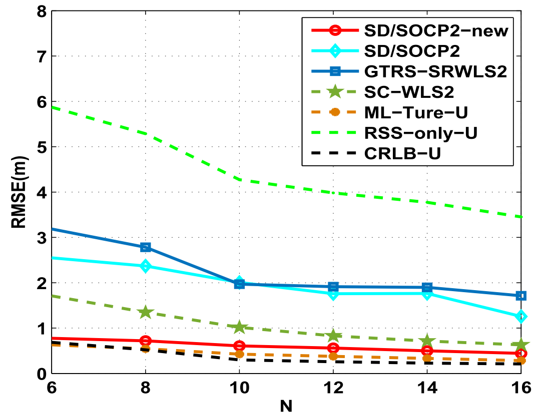

FIGURE 6. RMSE versus of sensor number N comparison, when $\sigma_n = 4$ dB, $\sigma_m = 5$ deg, $\sigma_v = 5$ deg, $\gamma = 4$, $B = 15$ m, $d_0 = 1$ m, $Mc = 3000$.

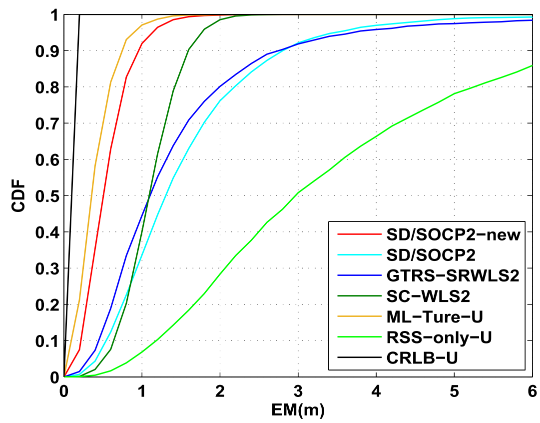

FIGURE 7. CDF versus the mean error comparison, when $N = 9$ and $\sigma_n = 4$ dB, $\sigma_m = 5$ deg, $\sigma_v = 5$ deg, $\gamma = 4$, $B = 15$ m, $d_0 = 1$ m, $Mc = 3000$.

which will lead to local minimum value, resulting in large localization error. Moreover, it can be seen that the proposed method SD/SOCP2-new has the similar properties as the SD/SOCP1-new method. Finally, the proposed method is also closest to the ML-True-U and CRLB-U for all choices of σ.

2) Effect of the number of the anchor nodes. Fig. 6 compares the RMSE versus the number of anchor nodes N when $\sigma_n = 4$ dB, $\sigma_m = 5$ deg, and $\sigma_v = 5$ deg. Similar to Fig. 3, Fig. 6 shows that the RMSE of all considered methods decreases when the number of anchor nodes increases when P_T unknown, and the proposed SD/SOCP2-new method results to the smallest RMSE among the discussed methods for all choices N. Moreover, the proposed method is also closest to the ML-True-U and CRLB-U for all choices N.

3) Effect of the CDF of the ME. we also investigate the CDF comparison of the ME in the target localization estimation of the considered methods when P_T unknown. Fig.6 shows that the proposed method outperforms the other considered methods for all range of ME, improving the localization accuracy by more than 0.4 m on average. Furthermore, we can see that the proposed method achieves ME$\leq$ 1 m in 90% of the cases, whereas the other considered methods attain the same ME in less than 50% of the cases.

VI. CONCLUSION

In this paper, the hybrid RSS-AOA localization problem has been considered. We have presented a convex SD/SOCP method for both the cases when P_T is known and unknown. According to the measurement models, we transform the models into pseudo-linear equation. Then a non-convex LS estimator is derived. By using the SDP and SOCP technique, the derived non-convex estimator is approximated into a convex one. The generalization of the proposed method for known transmit power can also be applied to the case when transmit power is not know. The simulation results verify the excellent performance of the proposed method in solving the localization problem efficiently in variety of scenarios for hybrid RSS-AOA measurements.

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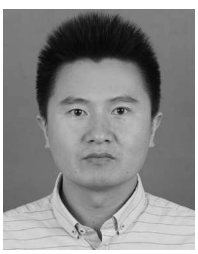

SHENGMING CHANG received the B.Sc. degree in mathematics from the Leshan Teachers College, Leshan, China, in 2008, the M.S. degree in applied mathematics from Ningbo University, Ningbo, China, in 2014, and the Ph.D. degree from the Faculty of Electrical Engineering and Computer Science, Ningbo University, in 2019. He is currently a Lecturer of electronic engineering with the Ningbo University of Technology, Ningbo. His current research interests include wireless localization and underwater acoustic wireless sensor networks localization.

YOU ZHENG received the Ph.D. degree from the University of Technology of Belfort-Montbéliard, Belfort, France, in 2007 and 2012. He is currently an Associate Professor of electronic engineering with the Ningbo University of Technology, Ningbo, China. He has authored or coauthored many of journals and conference papers in the areas of indoor positioning and navigation.

PENG AN received the B.Sc. and Ph.D. degrees from Tsinghua University, Beijing, China, in 2004 and 2009, respectively. He is currently a Professor of electronic engineering with the Ningbo University of Technology, Ningbo, China. He has authored or coauthored many of journals and conference papers in the areas of artificial intelligence and robot controlling systems. His current research interests include wireless localization, artificial intelligence, machine learning, and multimedia information processing.

JIANYU BAO received the B.Sc. degree from Yanshan University, Qinhuangdao, China, in 1998, and the M.Sc. and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 2001 and 2007, respectively, all in power electronics. Since 2010, he has been an Associate Professor with the Ningbo Institute of Technology, Zhejiang University. From 2017 to 2018, he was a Visiting Scholar with the Dresden University of Technology. He is currently a Professor of electrical engineering with the Ningbo University of Technology, Ningbo, China. He has authored or coauthored many of journal and conference papers in the areas of power electronics.

JUN LI received the B.Sc. degree in computer science and technology from Jiamusi University, Heilongjiang, China, in 2003, the M.Sc. degree in computer application technology from Xihua University, Chengdu, China, in 2008, and the Ph.D. degree in computer application technology from Shanghai University, China, in 2011. She is currently a Lecturer of electronic engineering with the Ningbo University of Technology, Ningbo, China. Her research interests include virtual reality, machine learning, and multimedia information processing.