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Dynamic Maintenance, Production and Inspection Policies, for a Single-Stage, Multi-State Production System

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ABSTRACT Over the past 20 years, integrated decision making for production systems has gained the interest of researchers and practitioners. Many studies have shown that integrated decision making can lead to substantial amount of savings. Yet, a few research work has been conducted on the areas of integrated maintenance, production and quality in dynamic environments. This paper provides an integrated multi-period, maintenance, production and quality-inspection scheduling model, which is formulated as a Markov decision process. The model minimizes the total expected maintenance, production and quality inspection costs. The structural properties of the proposed model are mathematically investigated and with using sensitivity analysis, practical insights are also provided. We mathematically provide conditions to guarantee that the optimal inspection policy is monotone non-decreasing in the state of the machine. Furthermore, we show that the optimal production policy decreases by one unit as the state of inventory increases by one unit. Sensitivity analysis demonstrates that the production parameters affect both, maintenance and inspection decisions. In addition, the maintenance parameters affect inspection decisions. Finally, it is found that among the inspection parameters (i.e., cost-of-inspection and inspection-errors), type-II error mainly affects maintenance decisions.

INDEX TERMS Decision making under uncertainty, integrated production, maintenance and quality, inspection errors, Markov decision process.

I. INTRODUCTION

In today's highly competitive business environment, driven by globalization and advances in production technology, companies adapt by increasing the efficiency of their production systems. Integrated decision making is one of the key tools to achieve production efficiency [1]. Advanced production systems are complex and accordingly they are usually modelled as multi-state ones [2], [3]. This paper addresses a multi-state production system, and aims at finding the optimal maintenance, production and inspection decisions in a dynamic environment.

Consider a multi-state deteriorating machine that produces items with imperfect quality to satisfy a random demand, the machine's deterioration will be assumed to follow a discrete-time Markov chain. Also, the quality of produced items will be dependent on the state of the machine; the worse the state of the machine the more likely it is to produce items with defects. Different maintenance actions are assumed to be available to the decision maker prior to a production run. Maintenance actions can range from doing nothing, to overhauling or replacing the machine [2]. Maintenance, followed by production, and quality inspection decisions are assumed to be taken at discrete points in time. The objective of the decision maker is to determine the optimal combination of maintenance, production and inspection policies, to minimize their corresponding total costs over a finite planning horizon. The main scientific contribution here is the development of an integrated Markov decision process (MDP) model, which addresses the maintenance, production and inspection for multi-state systems. The structural properties of the proposed model are investigated mathematically and using sensitivity analysis.

The remainder of this paper is organized as follows; Section II summarizes previous research works that are of relevance to the problem at hand. Model assumptions and

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notations are given in Section III and the developed MDP model is presented in Section IV. In Section V, the structural properties of the proposed model are provided, along with illustrative examples in Section VI. Finally, the conclusions and future research work are provided in Section VII.

II. LITERATURE REVIEW

The operations management literature spans a plethora of topics, which include integration of one or more of the following decision areas: scheduling, maintenance, production, and quality control. For more details on these topics, the reader may consult the following survey papers [4]–[6].

This section discusses two streams of research that are highly relevant to the work at hand, namely I) multi-state machine maintenance models and II) works that investigate dynamic economic lot-sizing in a multi-state setting, both using MDP.

A. MULTI-STATE MACHINE MAINTENANCE MODELS

Maintenance is usually performed correctively and preventively to maximize the availability of production systems, [7]–[9]. Over the past few decades, the growth of maintenance practices has been related to the growth of technology [10], which in turn, leads to increased complexity of industrial systems. Complex industrial systems are usually represented as multi-state systems [2], [3].

One of the earliest multi-state machine maintenance models is the one by Derman [11]. Assuming that multiple states of a machine represent different levels of the machine deterioration and the deterioration process is modeled as a discrete time Markov chain, the authors proved that the optimal replacement policy is control limit policy in one of the machine states. Later, Kolesar [12] included state occupancy cost and obtained similar control limit policy. Based on the observability of the considered system, MDP maintenance models are classified into three main categories, namely: I) partially observable (POMDP), in this type the decision maker receives noise-corrupted information about the system state II) non-observable, this type is a POMDP with just one 'null' observation and III) fully observable, in this type the decision maker knows true state of the system at the different stages of the planning horizon [13], [14]. Yet, the fully observable systems can be considered as a special case of the partially observable systems given that each state of the system has a unique signal.

One of the earliest POMDP models is the one by Ross [15]. Ross proved that the profit-maximizing objective is piecewise-linear concave function, assuming three possible maintenance actions (i.e., do nothing, inspect, and replace). Later on, many scholars extended Ross's model and investigated the existence of threshold-type structured optimal maintenance polices, which involved different assumptions on the parameters of the problem. For instance, White [16] reported that the optimal maintenance policy is an at most four-region policy (AM4R)-over the state space of the system −sorted by the first order stochastic dominance. Maillart [14] addressed the case of a system with obvious failures and consequently obtained an AM4R policy.

POMDP models are defined over multi-dimensional belief spaces. Therefore, the corresponding optimal policies are characterized over partially ordered belief spaces. The base of this research direction was established by the pioneer research work of White [13], [16]–[18]. Some authors made extensions to White's work by using other partial orders, such as the monotone likelihood ration [19] and the marginal monotonicity partial orders [2].

Recently, Papakonstantinou *et al.* [20] demonstrated the possibility of using MDP to compute optimal inspection and maintenance policies based on uncertain data in real time. Their research work addressed a non-stationary, 332-state, infinite horizon POMDP model.

B. MULTI-STATE ECONOMIC LOT-SIZING MODELS

One of the earliest dynamic optimal lot-sizing models was presented in White [21]. It addressed optimal lot sizing of items with imperfect quality and deterministic demand. This was later extended to consider: price fluctuations with deterministic demand [22], Markovian price fluctuations with random demand [23], time-inhomogeneous random prices with deterministic demand [24] and pricing decisions [25].

Parlar [26] addressed perishable items that expire after two time periods. In this case, one time-period old items were sold at less price. This work was based on stochastic analysis of demand, two classes of customers and the determination of the optimal number of new items to be processed/ordered in each time period. Haijema *et al.* [27] extended Parlar's model [26] to a larger state space (i.e., longer self-life with different item categories). Further, they used MDP to determine the optimal production amount of blood platelets. Recently, Haijema [28] optimized ordering, issuing and disposal policies of perishable goods, this was dependent on the usage policy (i.e., last in first out and first in first out).

Some authors studied different classes of customers or products. For instance, Wu *et al.* [29] presented a multi-item dynamic lot-sizing problem with downward substitution, the authors assumed that different items represented different quality grades. The objective was to find an optimal policy for selling quality-graded inventory. Iravani *et al.* [30] extended Wu *et al.* [29] by accounting for economic production decision and lost sales. Benjafar and Elhafsi's work [31] addressed the problem of one supplier and two customers to determine the joint optimal production and allocation policies.

C. INTEGRATED MODELS

So far, the presented literature showed that production lot-sizing, maintenance and inspection have infrequently been studied within the context of multi-state dynamic systems. In this section we provide a brief review of some recent integrated multi-state systems and highlights the contributions of our work.

The work by Abboud [32] was focused on multi-state maintenance and inventory system with fixed production and demand rates. Their objective was to estimate the overall system cost.

Recently, some researchers proposed MDP and other dynamic decision making models. Xiang *et al.* [33] developed an MDP production and maintenance model, and showed that the optimal production and the maintenance policies have a control limit structure. Bajestani *et al.* [34] considered joint determination of the optimal maintenance and production policies for a multi-period, multi-machine system. They formulated a multi-period model and developed sufficient conditions to guarantee monotonicity of the maintenance plan, this was based on assuming deterministic demand and Markovian deterioration. Recent applications included inspection and maintenance scheduling in infrastructural systems using POMDP [35], the provision of managerial insights about complex production systems using MDP [36] and Reinforcement learning, which also involves MDP [37].

Many researchers considered static multi-state deteriorating production systems. Recent research included multi-state machine maintenance and production planning with imperfect repairs using mixed integer linear programming [38], joint production and maintenance planning under quality constraints using non-linear programming [39], Integrated production maintenance and quality using integer nonlinear programming [40] and recently, production scheduling using stochastic mixed integer programming [41].

The most relevant researches to ours are the ones by Ivy and Pollock [2], Xiang *et al.* [33] and Kuo [1]. The work by Kuo [1] neither addressed production lot-sizing nor the possibility of multiple maintenance actions and multiple machine states (Kuo assumed that if production was performed, M units would be produced by a two-state machine). Ivy and Pollock [2] considered multi-state machine with multiple maintenance actions, they did not address production lot-sizing. In this research work, we focus on a multi-state machine, with multiple maintenance actions as in [2]; however, we further extend this by accounting for production lot-sizing and inspection. In contrast to the work by [1] and [2], we address the case of fully observed system. It is worth mentioning that Xiang *et al.* [33] did not consider inspection and assumed no production setup cost [42]. In addition, they assumed only two maintenance actions, namely; do nothing and replace.

Our work contributes to the literature through providing a novel integrated stochastic dynamic, maintenance, lot-sizing and inspection model using MDP. Furthermore, The model addresses the cases of imperfect repairs and inspection errors. The structural properties of the model are investigated mathematically and using sensitivity analysis. Technical and practical insights on the model performance are provided as well.

III. MODEL ASSUMPTIONS AND NOTATIONS

This section presents the main assumptions and notations that are used to develop our proposed model.

A. MODEL ASSUMPTIONS

- The system under consideration consists of a machine and inventory of finished items that are produced by the machine.
- The system state variables are: the state of the machine and the state of inventory; both may change at discrete points of time.
- The machine has multiple operating states that represent different levels of deterioration.
- The machine deterioration is modeled as a discrete time Markov chain [2], [3].
- The more deteriorated the system, the more likely it is to deteriorate further and/or fail [14], [16].
- The state of inventory represents the number of units in stock.
- Customers' demand is IID, time-homogeneous random variable following a known probability mass function [43].
- A decision maker observes the system state at the beginning of each time period, then a maintenance action can be taken followed by production and quality inspection actions for that period.
- Maintenance actions improve the state of the machine prior to production.
- Production may lead to machine deterioration [3].
- The effect of maintenance actions is assumed to be independent of the current state, and maintenance actions are assumed to be deterministic. [2], [44].
- Quality inspection actions are error-free [1].
- At each time period, the costs incurred by the system depend on the system state, the action(s) taken by the decision maker, and the customers' demand [45].
- Backorders are not allowed [43].
- As in [1] and for simplicity, we do not keep track of defective units; they are either repaired after inspection or shipped to the market with a penalty.
- The objective function is to minimize the overall expected system maintenance, production and inspection costs, over a finite planning horizon.

B. NOTATIONS

The main notations used in this paper are given in Table 1.

IV. MODEL FORMULATION

This section starts by providing a basic MDP model for the problem at hand. Then, we derive an extended version of the model by relaxing the assumptions of error free inspection and deterministic effect of maintenance (Subsection IV-F). For simplicity of presentation, we will refer to these models by I) the basic model and II) the extended model.

An MDP model consists of system states, action space, transition probabilities between system states, and reward/cost criteria that depend on system state and the actions taken [45]. The definitions of these elements are provided successively as follows:

TABLE 1. Notations.

Index	Description
l	A subscript used with the maintenance actions, to indicate the number of state improvements due to taking a maintenance action a_l
t	Time index
u	A subscript used with the production actions to indicate the number of units that will be produced
Symbol	**Description**
a_l	The maintenance action that improves a given machine state, j_t, by at most l states. For simplicity, we refer to the maintenance action, a_l, taken at time t by a_t, (e.g. if $a_t = a_2$ then, the new machine state will be 0 for any $j_t <= 2$)
b_u	The action of producing a batch of u units, $b_u \in \{0, 1, \ldots, i_{max} - i_t\}$, where i_t is the on-hand inventory. For simplicity, we refer to the production action taken at time t by b_t
c_{al}	The fixed cost of taking maintenance action a_l
c_b	The fixed setup cost of taking production action b
c_d	The cost of a defective unit delivered to the market
c_h	Holding cost per unit of inventory per time period
c_q	Quality inspection cost per unit inspected
c_r	Cost to repair an item with defects
c_{rf}	The cost of misclassifying a good item as rework
c_u	Unit production cost
c_π	Shortage cost per unit short per time period
i_t	The state of inventory at the beginning of time period t, which represents the number of units in stock, $i_t \in \{0, 1 \ldots, i_{max}\}$
j_t	The state of the machine at the beginning of time period t, j_t increases as the machine deteriorates, $j_t \in \{0, 1, ..., j_{max}\}$
$j_t^{'}$	An intermediate state of the machine during time period t. This state results from taking a_t, when the machine is in state j_t. Once b_t is taken, the machine goes from $j_t^{'}$ to a new state $j_{t+1} \geq j_t^{'}$ (i.e., deterioration of the machine due to production)
k_t	Quality inspection action, which represents the number of items to be inspected
$p(y\|j_t, a_t)$	The conditional probability of producing a defective unit given that the machine is in state j_t, and maintenance action a_t was taken. y is a Bernoulli random variable. Since, each pair of j_t and a_t leads to a unique $j_t{'} \in \{0, 1, ..., j_{max}\}$, $p(y\|j_t, a_t)$ can be expressed as $p(y\|j_t{'})$
S_t	System state at the beginning of time period t. This represents the state of the machine and on-hand inventory of finished items (i_t, j_t)
t	A time period of the planning horizon T
X	An IID discrete random variable, which represents customers' demand, $X \sim f(x)$
α	The probability of classifying a good item as defective (type-I error)
β	The probability of classifying a defective item as good (type-II error)
$*$	A superscript to indicate optimal values

A. PLANNING HORIZON

We assume a finite planning horizon composed of T time periods. A decision maker takes decisions at time periods $t = 1, 2, \ldots, T$. At each time period, the decision maker performs machine maintenance by selecting a maintenance action, a_t, and decides on the optimal production quantity and sample size for quality inspection (i.e., b_t and k_t, respectively). These actions are taken after observing the system state, which is defined next.

B. SYSTEM STATE AT TIME t

At the beginning of time period t, the system state consists of the level of on-hand inventory, $i_t \in \{0, \ldots, i_{max}\}$, and the machine state, $j_t \in \{0, \ldots, j_{max}\}$, which represents the deterioration level of the machine. As examples: $j_t = 0$ represents a new machine; whereas $j_t = j_{max}$ represents a failed machine (i.e. nonfunctional machine, or a machine that produces 100% defectives). Given the system state at time t, $S_t = (i_t, j_t)$, the decision maker takes maintenance, production and quality inspection actions during $(t, t+1)$.

C. ACTION SPACE DURING TIME PERIOD t

At the beginning of time period t, the decision maker selects a maintenance action, followed by a production and quality inspection actions, which are described next.

a) Maintenance actions
   a_l: the action of improving the machine state by l states [2], $l \in \{0, 1, \ldots, j_{max}\}$. For instance, if the machine is in state j_t, a_l brings the machine back, instantaneously, to state $max\{0, j_t - l\}$; whereas, a_0 is the do nothing action, which keeps the current machine state unchanged. For simplicity of notation, we refer to a_l taken at time t by a_t.
b) Production actions
   b_u: the action of producing a batch of u units, $b_u \in \{0, 1, \ldots, i_{max} - i_t\}$, where i_t is the on hand inventory of finished items ($i_t \leqslant i_{max}$), and i_{max} is the maximum level of inventory; due to space constraint. This also helps keep a finite state and action spaces in our model.
c) Quality inspection actions
   k_t: the action of inspecting the quality of k_t produced items. $k_t \in \{0, 1, \ldots, b_t\}$. For simplicity of notation, we refer to the action of not inspecting any item by k_0.

Having the "do nothing" actions (i.e., a_0, b_0, and k_0) in our model, means that the model will not only determine the optimal maintenance, production and inspection actions, rather, it will also determine the time between them [2]. As example, if $b_t^* > 0$ at $t = 3$, followed by b_0 at $t = 4$ and $b_t^* > 0$ at $t = 5$, then, from $t = 3$, the time between producing is two time units, and so on. The same thing applies to a_t^* and k_t^*.

D. TRANSITION PROBABILITIES BETWEEN THE SYSTEM STATES

Given a system state (i_t, j_t) at time t, the conditional state transition probability depends on (i_t, j_t), the action(s) taken and the customers' demand (X), which is assumed to be an IID discrete random variable. To visit the next system state (i_{t+1}, j_{t+1}) from (i_t, j_t), the system goes through an intermediate state $(i_t, j_t^{'})$; this is due to the maintenance action taking place prior to the production action.

Given a system state (i_t, j_t) at time t, a maintenance action is first taken. This alters the state of the machine from the initial state, j_t, to $j_t{'}$. Production and inspection actions follow the maintenance action and lead to transition in the state of

TABLE 2. List of events taking place in a typical time period *t*.

Point of time	Event	System state (i, j)
t	Beginning of the time period t	(i_t, j_t)
$t+t'$	Completion of the maintenance action	(i_t, j'_t)
$t+t'+t''$	Completion of the production and inspection actions	(i_t+b_t, j_{t+1})
$t+t'+t''+t'''$	Demand takes place	$(i_{t+1} = max\{i_t+b_t-x, 0\}, j_{t+1})$
$t+1$	Beginning of the next time period	(i_{t+1}, j_{t+1})

inventory to $i_t + b_t$, and possible deterioration of the machine (probabilistic failure due to operating the machine). Thus, j'_t becomes j_{t+1} and remains there until $t+1$. Finally, a random demand, $X = x$, takes place, which is satisfied from available inventory. Table 2, summarizes the event list in a given time period t, where t', t'' and $t''' > 0$, and $t' + t'' + t''' < 1$.

Assuming that i_t and j_t are independent; the transition probability to (i_{t+1}, j_{t+1}); given the current system state,(i_t, j_t), the actions taken, $(a_t$, b_t, and $k_t)$ and the demand, x_t, is expressed as follows

$$P(S_{t+1}|S_t, a_t, b_t, k_t, x_t) = P(i_{t+1}, j_{t+1}|i_t, j_t, a_t, b_t, x_t)$$
$$= P(i_{t+1}|i_t, b_t, x)\times P(j_{t+1}|j_t, a_t, b_t)$$

Assuming IID and time-homogeneous demand, $X \sim f_X(x)$, we have

$$P(i_{t+1}|i_t, b_t, x) = \begin{cases} p(X = x) = f(x) & \text{if } i_{t+1} = max\{i_t + b_t - x, 0\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

And, since the deterioration of the machine is Markovian, we have

$$P(j_{t+1}|j_t, a_t = a_l, b_t) = P(j_{t+1}|j'_t, b_t) \times P(j'_t|j_t, a_t = a_l),$$

such that

$$P(j'_t|j_t, a_t = a_l) = \begin{cases} 1 & \text{if } j'_t = max\{j_t - l, 0\} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Depending on b_t, $P(j_{t+1}|j'_t, b_t)$ has two expressions: The state of the machine may deteriorate due to production

$$P(j_{t+1}|j'_t, b_t \neq b_0) = \begin{cases} P(j_{t+1}|j'_t) & \text{if } j_{t+1} \geqslant j'_t \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Or, remains the same in the case of no production

$$P(j_{t+1}|j'_t, b_t = b_0) = \begin{cases} 1 & \text{if } j_{t+1} = j'_t \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Please note that the probabilities $P(j_{t+1}|j'_t)$ are assumed to be Markovian.

Having the state and action spaces of the problem, the cost criteria is defined next.

E. COST CRITERIA

Depending on the system state (i_t, j_t) at the start of t, the actions taken during t, and the demand and state transitions taking place (as demonstrated by Table 2), the system incurs the following random cost:

$$\begin{aligned} &R(S_t, a_t = a_l, b_t = b_u, k_t, x_t) \\ &= c_{a_l} + c_b + c_u b_u + c_q k_t \\ &\quad +(c_r k_t + c_d(b_u - k_t))p(y|j_t, a_l) + c_h[i_t + b_u - x_t]^+ \\ &\quad +c_\pi[x_t - i_t - b_u]^+ \quad \forall \quad a_t, b_t, k_t, t, x_t \end{aligned} \quad (5)$$

The first two terms of equation 5, $c_{a_l} + c_{b_u}$, account for the fixed costs of performing maintenance action a_l and production action b_u. The terms $c_u b_u + c_q k_t$ represent variable production and inspection costs. We assume error-free inspection and define $p(y|j_t, a_t) = p(y|j'_t)$ as the probability of producing a defective unit when the machine is in the intermediate state j'_t (Recall from Table 2 that the machine is in state j'_t prior to production). The term $(c_r k_t + c_d(b_t - k_t)) \times p(y|j_t, a_l)$, represents the cost to repair the units with defects found in inspection $(c_r k_t p(y|j_t, a_l))$, plus the cost of the defective units delivered to the market $(c_d(b_t - k_t)p(y|j_t, a_l))$. The term, $c_h[i_t + b_u - x_t]^+$ accounts for the holding cost of the units left unsold at the end of the time period t. In contrast, the last term, $c_\pi[x_t - i_t - b_u]^+$, accounts for penalty incurred when the demand exceeds the sum of the inventory at the beginning of period t and the quantity produced during t. For simplicity, and without loss of generality, it is assumed that $R(S_t, a_t, b_t, k_t, x_t) = 0$ at $t = T$. Due to the assumed random demand, the quantity $R(S_t, a_t, b_t, k_t, x_t)$ is random variable and the expected value of $R(S_t, a_t, b_t, k_t, x_t)$ is given by

$$E(R) = \sum_x R(S_t, a_t, b_t, k_t, x) \times f(x) \quad (6)$$

1) VALUE FUNCTION

For multiple time periods over a finite planning horizon (T), the expected total system costs can be expressed as

$$\begin{aligned} &V(i_t, j_t, a_t, b_t, k_t) \\ &= \sum_x R(i_t, j_t, a_t, b_t, k_t, x) \times f(x) \\ &\quad + \sum_{j_{t+1}} \sum_{i_{t+1}} V^*(i_{t+1}, j_{t+1}) p(i_{t+1}|i_t) p(j_{t+1}|j_t, a_t, b_t) \\ &= \sum_x R(i_t, j_t, a_t, b_t, k_t, x) \times f(x) \\ &\quad + \sum_{j_{t+1}} \sum_x V^*(max\{i_t + b_t - x, 0\}, j_{t+1}) f(x) p(j_{t+1}|j_t, a_t, b_t) \end{aligned} \quad (7)$$

where

$$V^*(i_t, j_t) = \min_{a_t, b_t, k_t} V(i_t, j_t, a_t, b_t, k_t) \quad \forall t \quad (8)$$

Furthermore, the state transition probability, $p(i_{t+1}|i_t, b_t)$ is given by Equation 1, and $p(j_{t+1}|j_t, a_t, b_t)$ is given by

$$p(j_{t+1}|j_t, a_t, b_t) = p(j'_t|j_t, a_t,) \times p(j_{t+1}|j'_t, b_t),$$

where $p(j'_t|j_t, a_t,)$ and $p(j_{t+1}|j'_t, b_t)$ are given by Equations 2−4.

Equation 7 states that the expected cost at time t equals the immediate expected cost at t when actions a_t, b_t, and k_t are taken, plus the expected total costs incurred in the future $(t+1, \ldots, T)$, where the decision maker is assumed to follow an optimal policy. Finally, given any system state (i_t, j_t) at time t, the optimal actions a_t^*, b_t^*, k_t^* are expressed as

$$a_t^*, b_t^*, k_t^* = \underset{a_t,b_t,k_t}{argmin}\, V^*(i_t, j_t) \tag{9}$$

F. AN EXTENSION TO INCLUDE INSPECTION ERRORS AND PROBABILISTIC EFFECT OF MAINTENANCE

So far, we have provided the mathematical details on the basic model. In this section, we present the details on the extended model. The extended model is a general form of the basic one and it primarily addresses the systems that have inspection errors, and when maintenance of a system has a probabilistic effect (i.e., the after-maintenance posterior state is attained with a probability of less than one). The extended model is derived through relaxing two of the basic model assumptions, namely: I) error-free inspection and II) deterministic effect of maintenance actions. The main advantage of relaxing these assumptions is to make the model closer to reality. The development of the extended model starts by relaxing these parameters separately, and afterwards, combined.

There are two types of quality inspection errors [46]:

- Type-I inspection error (α): this represents the producer's risk due to classifying a "good item" as defective.
- Type-II inspection error (β): this represents the consumer's risk due to classifying a defective item as "good".

Both error types can be included in the basic model by extending Equation 5, which will provide the following form:

$$\begin{aligned}
&R(S_t, a_t = a_l, b_t = b_u, k_t, x_t)\\
&= c_{a_l} + c_b + c_u b_u + c_q k_t\\
&\quad + k_t p(y|j_t, a_l)(1-\beta)c_r + k_t p(y|j_t, a_l)\beta c_d\\
&\quad + k_t(1 - p(y|j_t, a_l))\alpha c_{rf} + (b_t - k_t)p(y|j_t, a_l)c_d\\
&\quad + c_h[i_t + b_u - x_t]^+ + c_\pi[x_t - i_t - b_u]^+ \quad \forall\, a_t, b_t, k_t, t, x_t
\end{aligned} \tag{10}$$

The first four terms in Equation 10 are identical to those in Equation 5. The term, $k_t p(y|j_t, a_l)(1-\beta)c_r$, represents the repair cost of defective items that are correctly identified in inspection. The cost of committing type-II error and shipping defective items to the market is given by the term $k_t p(y|j_t, a_l)\beta c_d$. The term, $k_t(1 - p(y|j_t, a_l))\alpha c_{rf}$, represents the cost of misclassifying a "good" component as rework [46]. The term, $(b_t - k_t)p(y|j_t, a_l)c_d$, accounts for the cost of defective units in the non-inspected portion of the lot. Finally, the holding and shortage cost terms are identical to those in Equation 5. Note that we will obtain Equation 5, if we substitute 0s, for α and β in Equation 10.

The basic mathematical model can also be extended to include imperfect repairs by amending Equation 2 to the following form:

$$P(j'_t|j_t, a_t = a_l) = \begin{cases} p(j'_t|j_t, a_l) & \text{if } j'_t \leqslant j_t \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Equation 11 states that the repair actions, will either improve the state of the machine probabilistically, or keep the state of the machine unchanged. Therefore, the new state of the machine after repair, j'_t, is now a random variable. The probability of producing defective units (i.e., $p(y|j_t, a_l) = p(y|j'_t)$) can be obtained by conditioning on j' [47], as follows

$$E(p(y|j'_t)) = \sum_{j'_t} p(y|j'_t) \times p(j'_t|j_t, a_t) \tag{12}$$

Since the objective is to minimize the expected total cost, the term, $p(y|j'_t)$, in Equation 5, should also be replaced by $E(p(y|j'_t))$, when the repair actions are imperfect.

To relax both assumptions of the basic model, Equation 2 should be replaced by Equation 11, and Equation 5 should be replaced by Equation 10, which also has to incorporate $E(p(y|j'_t))$ in place of $p(y|j'_t)$.

V. STRUCTURAL PROPERTIES OF THE MODEL

For simplicity of presentation, this section provides some structural properties of the basic model that was presented in Section IV(A-E).

First, we mathematically state our assumption on the machine deterioration mechanism, that is, the more deteriorated the machine, the more likely it will deteriorate further.

Definition 1: A probability transition matrix $p(j_{t+1}|j'_t, b_t)$ has an increasing failure rate if $\sum_{j_{t+1}\geq q} p(j_{t+1}|j'_t, b_t) \geq \sum_{j_{t+1}\geq q} p(j_{t+1}|j'_t - 1, b_t), \forall j'_t, q \in \{0, 1, \ldots, j_{max}\}$. Equivalently, this implies that $p(j_{t+1}|j'_t, b_t)$ first-order stochastically dominates $p(j_{t+1}|j'_t - 1, b_t)$ [14], [16], [45].

Next, Lemma 1, shows that the optimal inspection action, k_t^*, is always to inspect all, or none of the units that are produced during time t, b_t^*.

Lemma 1: At any system state (i_t, j_t), the optimal inspection action will be always to inspect all units produced, or to inspect 0 units. Therefore, $k_t^* = b_t^*$ or $k_t^* = 0, \forall (i_t, j_t)$.

Proof: The second term of Equation 7, $\sum_{j_{t+1}}\sum_x V^*(i_t + b_t - x, j_{t+1})f(x)p(j_{t+1}|j_t, a_t, b_t)$, is independent of k_t. Additionally, the first term of Equation 7, given by Equations 5 and 6, shows that, the corresponding quality cost function is linear function in k_t ($c_q k_t + (c_r k_t + c_d(b_u - k_t))p(y|j_t, a_l)$). This implies that, if $c_q + (c_r - c_d)\, p(y|j_t, a_t) < 0$, then $k_t^* = b_t^*$; and in case $c_q + (c_r - c_d)p(y|j_t, a_t) > 0$, then $k_t^* = 0$. ∎

Next, Lemma 2 states that if the optimal maintenance policy results in a non-decreasing values of the after-maintenance intermediate machines state, j'_t, in j_t; and if b_t^* is monotone non-increasing in j_t, then, for given an initial level of inventory, i_t, the optimal inspection action, k^*, is either to: I) inspect 0 units $\forall j_t$, II) inspect all the produced units $\forall j_t$, or III) inspect 0 units starting from the new machine state ($j_t = 0$) to some state $j_t'' \in \{0, 1, 2, \ldots, j_{max}\}$, then inspect all of the produced units for $j_t > j_t''$.

Lemma 2: If $p(y|j_t, a_t^*)$ is non-decreasing in j_t and b_t^* is non-increasing in j_t, then for a given level of inventory, i_t, the optimal inspection action, k_t^* will be either:

- $k_t^* = b_t^* \,\forall j_t$, or
- $k_t^* = 0 \,\forall j_t$, or
- $k_t^* = 0 \,\forall j_t, j_t < j''$ and $k_t^* = b_t^* \,\forall j_t, j_t \geq j''$, where $j'' \in \{0, 1, \ldots, j_{max}\}$.

Proof: Based on Lemma 1, if $c_q + (c_r - c_d)p(y|j_t, a_t^*) > 0$ then $k_t^* = 0$. This condition can be re-written as: if $c_d p(y|j_t, a_t^*) < c_q + c_r p(y|j_t, a_t^*)$, then $k_t^* = 0$. Since the cost elements are positive constants, and given our assumption that $p(y|j_t, a_t^*)$ is non-decreasing in j_t, then it is impossible that this condition does not hold true for $j_t < j''$ then holds true for $j_t \geq j''$, where j'' is some intermediate machine state. Yet, the three remaining cases stated by Lemma 2 are the only possible ones. ■

The following proposition states that for a given optimal maintenance action, a_t^*, and a machine state, j_t, the optimal production action, b_t^*, will decrease by 1 unit as i_t increases by 1 unit.

Proposition 1: Given an arbitrary system state (i_t^1, j_t), such that a_1^* and $b_1^* > 0$ are the optimal maintenance and production actions respectively at (i_t^1, j_t); then for any system state (i_t^2, j_t), such that $i_t^2 > i_t^1, i_t^2 - i_t^1 < b_1^*$, and $a_2^* = a_1^*$, then $b_2^* = b_1^* - (i_t^2 - i_t^1)$ will be optimal production action at (i_t^2, j_t). Thus, if b_1^*, right before customers' demand, leads to inventory state $i_{t+1} = d$, then b_2^* will lead to $i_{t+1} = d$ as well.

Proof: The proof is given in the appendix. ■

The following proposition states that if two machine states (j_t^1 and j_t^2), have their a_t^* values such that their after-maintenance state, j_t' is the same, then their b_t^* values will be the same as well.

Proposition 2: If $a_1^* > 0$ and $b_1^* > 0$ are the optimal maintenance and production actions at state (i_t, j_t^1), then for any system state (i_t, j_t^2), such that $j_t^2 = j_t^1 + l, l > 0$ and $a_2^* = a_1^* + l$, is the optimal maintenance action at (i_t, j_t^2), then the optimal production action at (i_t, j_t^2) is $b_2^* = b_1^*$.

Proof: The proof is given in the appendix. ■

VI. NUMERICAL EXAMPLES

This section demonstrates the applicability of the proposed models through a number of numerical examples. Subsection VI-A provides a basic example to demonstrate the structural properties that were presented in Section V. And Subsection VI-B provides the generalized model use to study the interactions between the optimal polices of the model. It is focused on providing practical insights and further clarifications on how the parameters of one decision area (e.g., production, maintenance or quality) affect the entire system.

TABLE 3. Parameters of the demonstrative example.

c_{a0}	0	c_r	1
c_{a1}	2	c_u	2
c_{a2}	4	c_π	6
c_b	3	i_{max}	6
c_d	7	j_{max}	2
c_h	0.5	T	3
c_q	0.5		

TABLE 4. Optimal decision rules (a^* and b^*) at $t = 1$.

a^* at $t = 1$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	1	2
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 1$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	0	0	0
5	0	0	0
6	0	0	0

A. DEMONSTRATIVE EXAMPLE

The following example of a joint optimal maintenance, production and inspection policies illustrates the structural properties of the proposed model. The solutions were obtained by solving the mathematical model presented in Section IV using the value iteration algorithm [45], it was implemented using MATLAB. The parameters of this example are given in Table 3.

Furthermore, the following probabilities are assumed: $f(x) \sim Binomial\ (n = 13, p = 0.4)$

$$p(j_t'|j_t, a_t = a_0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p(j_t'|j_t, a_t = a_1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$p(j_t'|j_t, a_t = a_2) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$p(j_{t+1}|j_t', b_t = b_0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p(j_{t+1}|j_t', b_t \neq b_0) = \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p(y|j_t) = \begin{bmatrix} 0.1 & 0.3 & 1 \end{bmatrix}$$

Tables 4–6 provide the optimal maintenance and production decision rules (i.e., a^* and b^*) at $t = 1, 2$ and 3 respectively. Figures 1 and 2 demonstrate the optimal maintenance and production policies a^* and b^*, given in Table 4.

Figure 1 demonstrates that when the machine is in state $j = 1$ and there are 0–3 units inventory, the optimal maintenance decision rule is to repair the machine ($a^* = 1$).

TABLE 5. Optimal decision rules (a^* and b^*) at $t = 2$.

a^* at $t = 2$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	0	1
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 2$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	2	0	0
5	0	0	0
6	0	0	0

TABLE 6. Optimal decision rules (a^* and b^*) at $t = 3$.

a^* at $t = 3$

i \ j	0	1	2
0	0	0	1
1	0	0	1
2	0	0	1
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 3$

i \ j	0	1	2
0	5	5	5
1	4	4	4
2	3	3	3
3	2	2	0
4	0	0	0
5	0	0	0
6	0	0	0

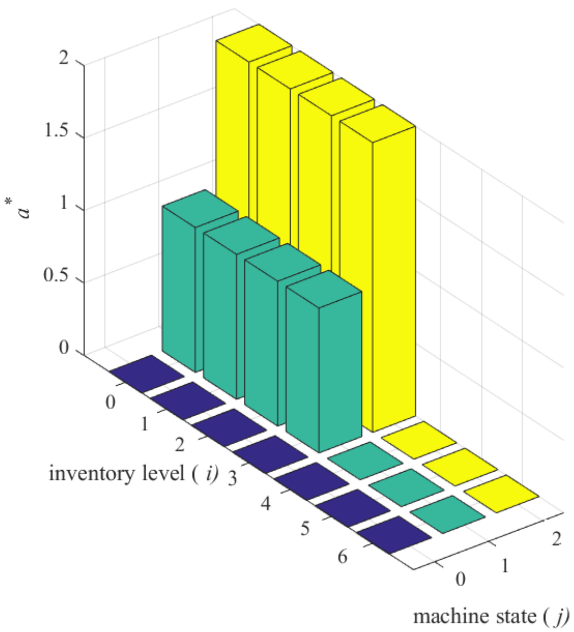

FIGURE 1. Optimal maintenance decision rules, a^*, at states (i, j), $t = 1$.

In addition, Figure 2 demonstrates how the corresponding optimal production decision rules decrease by 1 unit when i increases for $i = 0 - 3$. After this, it was experimentally observed that, in many cases, no production will take place. The reason for this structure is primarily the fixed production and maintenance cost elements of the objective function. For instance, if we set $c_a = c_b = 0$, then production would have kept decreasing by 1 unit to reach 1 then 0, giving the well-known base stock policy [42]. Furthermore, the results demonstrate that if a^* brings the machine to the same

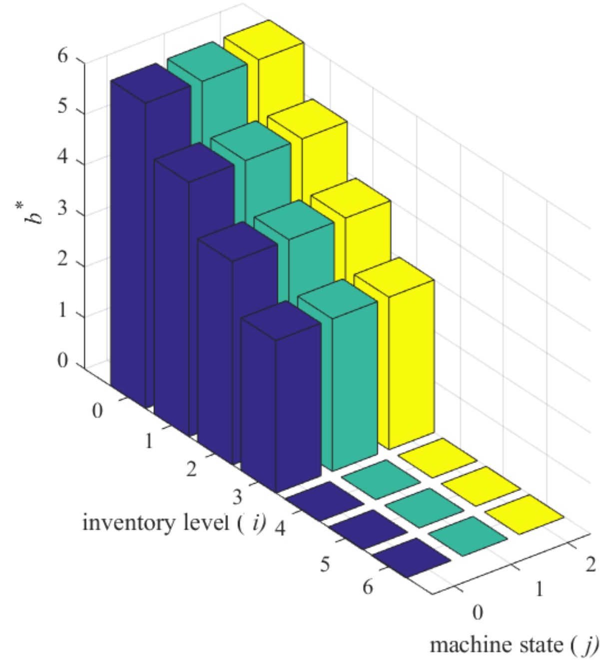

FIGURE 2. Optimal production decision rules, b^*, at states (i, j), $t = 1$.

TABLE 7. Parameters of the baseline example.

c_{a0}	0	c_{rf}	1
c_{a1}	1.8	c_u	2
c_{a2}	3	c_π	6
c_b	3	i_{max}	6
c_d	7	j_{max}	2
c_h	0.5	T	4
c_q	0.5	α	0
c_r	1	β	0

intermediate state, j'_t, then the equal value of b^* will be produced $\forall\, j$. (e.g., row 1 of Table 5 gives $a^* = 0$ 1, and 2 at $j = 0$, 1 and 2, respectively; this gives the same value of $j'_t = 0$ and therefore the same value of $b^* = 6$).

As for inspection, the optimal inspection actions, k^*_t, were are equal to b^*_t, $t \in \{1, 2, 3\}$.

B. NUMERICAL SENSITIVITY ANALYSIS

Sensitivity analysis is conducted on the extended model by varying its parameters one at a time, and combined for some cases. Next, we define a baseline example that is similar to the previous one, albeit with slight changes to the parameters used (Table 7), this is to emphasize on the model characteristics in terms of sensitivity and performance.

The demand is assumed to follow a Binomial distribution given by $f(x) \sim Binomial(n = 20, p = 0.3)$. The fraction of defective units at the different machine states are:

$$p(y|j_t) = \begin{bmatrix} 0.05 & 0.3 & 1 \end{bmatrix}$$

Furthermore, we assume that the state transition matrices of the baseline example are identical to those given in the previous example. For the case of imperfect repair, in place

TABLE 8. Optimal decision rules (a_t^* and b_t^*), $t \in \{1, 2, 3\}$, $c_b = 4$.

$a_t^*, t \in \{1,2,3\}$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	1	2
4	0	1	2
5	0	0	0
6	0	0	0

$b_t^*, t \in \{1,2,3\}$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	2	2	2
5	0	0	0
6	0	0	0

of $p(j_t'|j_t, a_t = a_1)$, which is given in the previous example, we use the following state transition matrix:

$$p(j_t'|j_t, a_t = a_{1(imperfect)}) = \begin{bmatrix} 1 & 0 & 0 \\ 0.7 & 0.3 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

This matrix implies that if the machine is in states 2 or 3, and the action $a_{1(imperfect)}$ is taken, then the machine will go to a better state in a probabilistic way.

Starting from this baseline example, we sequentially varied the model parameters and investigated the effects on the optimal policies. Unless otherwise stated, the parameters in each of the following experiments are equivalent to the ones given in Table 7. Furthermore, the effect(s) of the given parameters on the optimal production, maintenance and inspection policies are discussed. Remarks on the resulting structural properties are also given when necessary.

1) THE EFFECT OF c_b

If we take the production apart from the maintenance and inspection decisions, then it can be shown that the optimal inventory policy is (s, S) policy [42], i.e., to produce when the level of inventory drops below s and bring it back to S. In this case, s represents a production threshold and S is the maximum inventory level. In most of our experiments we obtained (s, S) optimal inventory polices. Furthermore, if the production setup cost (i.e., c_b) is increased, a higher production threshold will be obtained. This was observed by varying c_b from 3 to 10. Sample representative results, for $c_b = 4$ and 10 are demonstrated by Tables $8 - 11$.

Compared with Tables 8 and 9, Tables 10 and 11 demonstrate the decrease in some a^* values when c_b of the baseline example is increased to 10.

At $c_b = 4$, the optimal inspection policy is $k_t^* = b_t^*$, for $(i = 3, j = 1)$, $t = 4$ and $k_t^* = k_0$ elsewhere. In contrast, the optimal inspection policy at $c_b = 10$ is $k_t^* = k_0$, $\forall(i, j)$, $\forall t$.

Remark: Note that Table 9 gives $a^*(i = 3, j = 0, 1$ and $2) = 0, 0, 2$, which implies that the corresponding values of j' are 0, 1, 0 respectively. Hence, the condition of Lemma 2 is not satisfied and the optimal inspection decision rule at $i = 3$, $t = 4$ can be non-monotone in j.

TABLE 9. Optimal decision rules (a^* and b^*) at $t = 4$, $c_b = 4$.

a^* at $t = 4$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	0	2
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 4$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	2	0	0
5	0	0	0
6	0	0	0

TABLE 10. Optimal decision rules (a_t^* and b_t^*), $t \in \{1, 2, 3\}$, $c_b = 10$.

$a_t^*, t \in \{1,2,3\}$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

$b_t^*, t \in \{1,2,3\}$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

TABLE 11. Optimal decision rules (a^* and b^*) at $t = 4$, $c_b = 10$.

a^* at $t = 4$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 4$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

TABLE 12. Optimal decision rules (a^* and b^*) at $t = 1$, $c_\pi = 4$.

a^* at $t = 1$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 1$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

2) THE EFFECT OF c_π

In this part, c_π of the baseline case is incremented between $4 - 7$ in a step of 1. As a result, there was an increase in some of the production and maintenance actions. The optimal inspection policy, k^*, for $c_\pi = 4$ and 7, was $k^* = 0\ \forall(i, j)$ (according to Lemma 1, k^*, is not directly affected by π).

Furthermore, by simultaneously varying the values of c_b and c_π over the given ranges, it was observed that a^* increases or remains the same when b^* increases and vice versa.

Sample representative results for $c_\pi = 4$ and 7, $t = 1$ are provided in Tables 12 and 13 respectively.

TABLE 13. Optimal decision rules (a^* and b^*) at $t = 1$, $c_\pi = 7$.

a^* at $t = 1$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	1	2
4	0	1	2
5	0	0	0
6	0	0	0

b^* at $t = 1$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	2	2	2
5	0	0	0
6	0	0	0

TABLE 14. Optimal decision rules (a^* and b^*) at $t = 1$, $c_u = 3$.

a^* at $t = 1$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	1	2
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 1$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	0	0	0
5	0	0	0
6	0	0	0

TABLE 15. Optimal decision rules (a^* and b^*) at $t = 1$, $c_u = 4.5$.

a^* at $t = 2$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

b^* at $t = 2$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

Note that the effect of c_h is opposite to that of c_π and therefore omitted.

3) THE EFFECT OF c_u

The value of c_u was increased from 2−6 in a step of 2. As a result, some values of a^* and b^* changed. Tables 14 and 15 provide sample representative decision rules (at $t = 1$), for $c_u = 3$ and 4.5, respectively.

Regarding the effect on inspection, k^* was not affected. However, if we assume that the repair cost in our model, c_r, is function of c_u, then k^* would have been possibly affected as per Lemma 1.

4) THE EFFECT OF c_q

Increasing the quality inspection cost c_q from 0 to an arbitrary large number (i.e., 100) in a step of 10, did not show any effect on the production and maintenance policies. However, smaller values of c_q resulted in $k^* = b^*$ and larger values gave $k^* = 0$. This is consistent with the results of Lemma 1. Yet, according to equations 8 and 9, $k_t^* > 0$ affects $V^*(i_t, j_t)$ and therefore, based on Equation 7, this can affect a_{t-1}^* and b_{t-1}^*. However, this was found to be very insignificant effect.

TABLE 16. Optimal decision rules (a^* and b^*) at $t = 1$, $c_d = 14$, $\beta = 0$ and 0.3.

a^* at $t = 1$

i \ j	0	1	2
0	0	1	2
1	0	1	2
2	0	1	2
3	0	1	2
4	0	1	2
5	0	0	0
6	0	0	0

b^* at $t = 1$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	2	2	2
5	0	0	0
6	0	0	0

Overall, the parameter c_q had very negligible effect on the structure of the optimal production and maintenance policies.

5) THE EFFECT OF c_d

Increasing c_d from 7 to 14 in a step of 1 did not show a significant effect on the optimal production and maintenance polices. However, in consistence with Lemma 1, k^* values increased. As example, if $c_d = 7$ then $k^* = 0$, and if $c_d = 14$ then $k^* = b^*$ $\forall (i, j)$, $\forall t$. Note that if c_d increases, then the cost of defectives shipped to the market will increase. In this case, the system tends to do more inspection and incur the inspection cost, rather than not performing inspection and allowing items with defects be sold in the market.

6) THE EFFECT OF α

In this part, we incremented α between $0 - 0.3$ in a step of 0.1 and did not observe any effect on a^*, b^* or k^* values. As shown by Equation 10, α appears in the term $k_t(1 - p(y|j_t, a_l))\alpha c_{rf}$, which means that α will have an effect if $k^* \neq k_0$ (i.e., should be equal to b^* as per Lemma 1). This can happen in multiple ways. As example, c_a values were increased so that $a^* = a_0$ and $k^* = b^*$ $\forall (i, j)$ and $\forall t$. Then, α was incremented again and few k^* values became equal to to 0. On the other hand, c_{rf} can have an effect, but in practice the value of c_{rf} should be much less than c_d (i.e., a relatively small number around the value assumed in Table 7). In conclusion, the optimal maintenance, production and inspection polices are not sensitive to changes in α.

7) THE EFFECT OF β

Similar to α, it was observed that β values (in the range of 0−0.3) did not make a significant impact on the optimal polices. However, this is not the case when c_d is made high. As example, for the baseline case, but with $c_d = 14$, if β is increased (from 0 to 0.3); then k_t^* decreases from b_t^* to 0 $\forall (i, j)$, $\forall t$. Regarding the optimal production policy, it was not affected by β (Table 16 provides sample results). Therefore, for large values of c_d, we conclude that the inspection polices are affected by β (i.e., if β increases, then less quality inspections are performed in this case).

8) THE EFFECT OF c_a

In this part, we first incremented c_{a_1} between $1.8 - 3$ (the value of c_{a_2}). As a result, production actions were not

TABLE 17. Optimal decision rules (a^* and b^*) at $t = 1$, using $a_{1(imperfect)}$.

a^* at $t = 1, 2$

i \ j	0	1	2
0	0	2	2
1	0	2	2
2	0	2	2
3	0	2	2
4	0	1	2
5	0	1	0
6	0	1	0

b^* at $t = 1, 2$

i \ j	0	1	2
0	6	6	6
1	5	5	5
2	4	4	4
3	3	3	3
4	2	2	2
5	0	0	0
6	0	0	0

affected. The only difference was that the system either took the replace action, a_2 along with k_0 at $j = 1, i \leq 3$; or a_0 along with $k^* = b^*$ at $j = 1, i > 3$. In both cases, quality is ensured by either performing maintenance or by inspection. Finally, increasing c_{a_1} and c_{a_2} simultaneously led to the trivial case that maintenance is not performed and inspecting all the produced units ($k^* = b^*$). Furthermore, it was experimentally observed that the optimal maintenance decision rule, a^*, is monotone in i. This is demonstrated by all of the optimal maintenance polices presented in this paper.

9) THE EFFECT OF IMPERFECT MAINTENANCE

In this part we solve the instance where $p(j'_t|j_t, a_1)$ of the baseline case is replaced by $p(j'_t|j_t, a_t = a_{1(imperfect)})$. This led to an increased level of maintenance at $j = 1$ (i.e. taking $a^* = 2$ at $j = 1$). In particular, this happened at the lower values of i (these have larger values of b^*). Thus, the system chose the most expensive maintenance action $a^* = 2$ as it guarantees taking the machine to state 0 (less rate of defectives), instead of the $a_{1(imperfect)}$, which has a probabilistic effect. Table 17 lists the obtained results.

Compared to the baseline case, more inspection was done at $t = 4$. This is to avoid performing expensive or imperfect maintenance at the last time epoch of the planning horizon. In summary, maintenance actions with probabilistic effect had an impact on both the maintenance and inspection policies.

Remark: Table 17 shows that $a^*(i = 5 \text{ and } 6, j = 1) = 1$ while the corresponding b^* values are 0s. The reason for this is that maintenance actions tend to decrease the value of j_{t+1} and therefore, can decrease V^*_{t+1} (the optimal value function at $t + 1$). However, such scenario can't happen at $t = T$ to have $a^* > 0$ and the corresponding $b^* = 0$.

10) THE EFFECTS OF $p(j_{t+1}|j'_t, b_t)$ AND $p(y|j'_t)$

It was experimentally observed that the effect of the failure rate matrix $p(j_{t+1}|j'_t, b_t \neq b_0)$ is similar to that of c_a. Using a matrix with higher failure rate (in the sense of first order stochastic dominance) led to a decrease in maintenance and increase in inspection. Also, when the probability of producing items with defects was increased, similar to the discussion related to c_q, almost no effect took place on the optimal maintenance and production polices.

VII. CONCLUSIONS AND FUTURE RESEARCH WORK

We proposed an integrated maintenance, production and quality inspection dynamic model, formulated as an MDP. Compared to previous research work, we developed an integrated model that considers multiple maintenance actions and quality inspection. The proposed model has the advantage of enabling integrated decision making under uncertainty. It also provides new structural properties and some practical insights on the interaction ways between production, maintenance and quality inspection polices.

The structural properties of the model were investigated mathematically and with sensitivity analysis. The mathematical approach demonstrated that, under some mathematical assumptions/conditions, the quality inspection policies are monotone in the machine state. Furthermore, it was shown that the optimal production decision rules decrease by 1 unit as the state of inventory increases by 1 unit, to a point where it becomes optimal to produce 0 units. This was evident at any time period of a finite planning horizon. Numerical examples indicated that the optimal production decision rule remains at 0. This finding yields to the well-known (s, S) inventory policy.

Sensitivity analysis was performed by changing the model parameters separately and combined when needed. The results showed that the quality related parameters such as the inspection cost and type-I inspection error had negligible effect on the optimal policies. Furthermore, if it was costly to deliver a product with defects to the market, type-II inspection error would affect the optimal maintenance policies. Maintenance and quality inspection actions were proportional to the production actions. In other words, more production leads to increase in maintenance and/or inspection. Maintenance and inspection were the tools to achieve quality of production. Imperfect maintenance actions were considered such that, if an imperfect maintenance action is taken, the system will lie somewhere between the desired machine state and its pre-maintenance condition. Based on the given examples, imperfect maintenance actions led to extra maintenance (i.e. taking the replace action instead of the imperfect maintenance) or resulted in increased inspection.

Future research will focus on the areas of partially observed systems, multiple items with correlated demand, two or more stages of production and the case of non-repairable items, where imperfect items are sold in a secondary market.

APPENDIX

A. PROOF OF PROPOSITION 1

Proposition 3: Given an arbitrary system state (i^1_t, j_t), such that a^*_1 and $b^*_1 > 0$ are the optimal maintenance and production actions respectively at (i^1_t, j_t); then for any system state (i^2_t, j_t), such that $i^2_t > i^1_t, i^2_t - i^1_t < b^*_1$, and $a^*_2 = a^*_1$, then $b^*_2 = b^*_1 - (i^2_t - i^1_t)$ will be optimal production action at (i^2_t, j_t). Thus, if b^*_1, right before customers' demand, leads to inventory state $i_{t+1} = d$, then b^*_2 will lead to $i_{t+1} = d$ as well.

Proof: Consider a system at state (i_t^1, j_t), let b_1^* the optimal production action such that $i_t^1 + b_1^* = d$. To prove this we consider two cases, namely:

Case 1: $b^- > 0$ be any action such that $i_t^1 + b^- = e < d$,
Case 2: $b^+ > 0$ be any action such that $i_t^1 + b^+ = f > d$.
Considering case 1:

$V^*(i_t^1, j_t) = V(i_t^1, j_t, a^*, b_1^*, k_t) \leq V(i_t^1, j_t, a^*, b^-, k_t)$, using Equations 5–8, this is equivalent to:

$$\sum_x (c_{a^*} + c_{b_1^*} + c_u b_1^* + c_q k_t + (c_r k_t + c_d(b_1^* - k_t)) \times p(y|j_t, a^*) + c_h[i_t + b_1^* - x_t]^+ + c_\pi[x_t - i_t - b_1^*]^+) \times f(x) + \sum_{j_{t+1}} \sum_x V^*(i_t + b_1^* - x, j_{t+1}) \times f(x) p(j_{t+1}|j_t, a^*, b_1^*) \leq \sum_x (c_{a^*} + c_{b^-} + c_u b_1^- + c_q k_t + (c_r k_t + c_d(b^- - k_t))p(y|j_t, a_l) + c_h[i_t + b^- - x_t]^+ + c_\pi[x_t - i_t - b^-]^+) \times f(x) + \sum_{j_{t+1}} \sum_x V^*(i_t + b^- - x, j_{t+1}) \times f(x) p(j_{t+1}|j_t, a^*, b^-) \tag{13}$$

Substituting $b_1^* = d - i_t^1$ and $b^- = e - i_t^1$ in Inequality 13, and noticing from Equations (2–4) that $p(j_{t+1}|j_t, a^*, b_1^*) = p(j_{t+1}|j_t, a^*, b^-) = p(j_{t+1}|j_t, a^*, b_t \neq b_0)$, using Lemma 1 and simplifying gives:

$$\sum_x (c_u(d-e) + c_q(d-e) + (c_r(d-e)) \times p(y|j_t, a_t) + c_h([d - x_t]^+ - [e - x_t]^+) + c_\pi([x_t - d]^+ - [x_t - e]^+) f(x) + \sum_{j_{t+1}} \sum_x (V^*([d-x]^+, j_{t+1}) - V^*([e-x]^+, j_{t+1})) f(x) p(j_{t+1}|j_t, a^*, b_t \neq b_0) \leq 0. \tag{14}$$

Now, consider the system state (i_t^2, j_t), such that the optimal maintenance action at this state is equal to a^* and $i_t^2 > i_t^1$, $i_t^2 - i_t^1 < b_1^*$. Let b_2, be the production action such that $i_t^2 + b_2 = d$. Next, we will prove that $b_2 = b_2^*$ is optimal production action at (i_t^2, j_t).

Suppose there is a production action, b_2^-, such that $0 < b_2^- < b$ and $i_t^2 + b_2^- = e < d$, that results in a better objective function value than b_2, then:

$V(i_t^2, j_t, a^*, b_2^-, k_t = b_2^-) < V(i_t^2, j_t, a^*, b_2, k_t = b_2)$. Then, again, using equations 5 − 8:

$$\sum_x (c_{a^*} + c_{b_2^-} + c_u b_2^- + c_q k_t + (c_r k_t + c_d(b_2^- - k_t)) \times p(y|j_t, a_t) + c_h[i_t^2 + b_2^- - x_t]^+ + c_\pi[x_t - i_t^2 - b_2^-]^+) f(x) + \sum_{j_{t+1}} \sum_x V^*(i_t^2 + b_2^- - x, j_{t+1}) f(x) p(j_{t+1}|j_t, a^*, b_2^-) < \sum_x (c_{a^*} + c_{b_2} + c_u b_2 + c_q k_t + (c_r k_t + c_d(b_2 - k_t)) \times p(y|j_t, a_t) + c_h[i_t^2 + b_2 - x_t]^+ + c_\pi[x_t - i_t^2 - b_2]^+) f(x) + \sum_{j_{t+1}} \sum_x V^*(i_t^2 + b_2 - x, j_{t+1}) f(x) p(j_{t+1}|j_t, a^*, b_2).$$

Substituting $b_2^- = e - i_t^2$, $b_2 = d - i_t^2$, and observing that $p(j_{t+1}|j_t, a^*, b_2^-) = p(j_{t+1}|j_t, a^*, b_2) = p(j_{t+1}|j_t, a^*, b_t \neq b_0)$ and simplifying, gives Inequality 15 which contradicts with Inequality 14, hence action b_2^- can't be optimal at (i_t^2, j_t).

$$\sum_x (c_u(d-e) + c_q(d-e) + (c_r(d-e)) p(y|j_t, a_t) + c_h([d - x_t]^+ - [e - x_t]^+) + c_\pi([x_t - d]^+ - [x_t - e]^+)) \times f(x) + \sum_{j_{t+1}} \sum_x V^*([d-x]^+, j_{t+1}) - V^*([e-x]^+, j_{t+1}) f(x) \times p(j_{t+1}|j_t, a^*, b_t \neq b_0) > 0. \tag{15}$$

The proof of case 2, follows similarly and hence omitted. ∎

B. PROOF OF PROPOSITION 2

Proposition 4: If $a_1^* > 0$ and $b_1^* > 0$ are the optimal maintenance and production actions at state (i_t, j_t^1), then for any system state (i_t, j_t^2), such that $j_t^2 = j_t^1 + l$, $l > 0$ and $a_2^* = a_1^* + l$, is the optimal maintenance action at (i_t, j_t^2), then the optimal production action at (i_t, j_t^2) is $b_2^* = b_1^*$.

Proof: If a_1^* and $b_1^* > 0$ are the optimal maintenance and production actions at state (i_t, j_t^1), then for any system state (i_t, j_t^2), such that $j_t^2 = j_t^1 + l$, $l > 0$ and $a_2^* = a_1^* + l$, the optimal production action $b_2^* = b_1^*$.

Since a_1^* and b_1^* are the optimal maintenance and production actions at (i_t, j_t^1), and by considering the general case of Lemma 1, i.e. inspection of all produced units ($k_1^* = b_1^*$), $V^*(i_t, j_t)$ can be expressed using a_1^*, b_1^* and Equations 5 − 8, as follows:

$$V^*(i_t, j_t^1) = \sum_x [c_{a_1^*} + c_{b_1^*} + c_u b_1^* + c_q b_1^* + (c_r b_1^* + c_d(b_1^* - b_1^*)) p(y|j_t, a_t) + c_h[i_t + b_1^* - x_t]^+ + c_\pi[x_t - i_t - b_1^*]^+] f(x) + \sum_{j_{t+1}} \sum_x V^*(i_t + b_1^* - x, j_{t+1}) f(x) p(j_{t+1}|j_t^1, a_1^*, b_1^*)$$

Also, for any arbitrary production action b other than b_1^*:

$$V^*(i_t, j_t^1) = \sum_x [c_{a_1^*} + c_{b_1^*} + c_u b_1^* + c_q b_1^* + (c_r b_1^* + c_d(b_1^* - b_1^*)) p(y|j_t, a_t) + c_h[i_t + b_1^* - x_t]^+ + c_\pi[x_t - i_t - b_1^*]^+] f(x) + \sum_{j_{t+1}} \sum_x V^*(i_t + b_1^* - x, j_{t+1}) f(x) p(j_{t+1}|j_t^1, a_1^*, b_1^*) \leq \sum_x [c_{a_1^*} + c_b + c_u b + c_q k_t + (c_r b + c_d(b - b)) p(y|j_t, a_t) + c_h[i_t + b - x_t]^+ + c_\pi[x_t - i_t - b]^+] f(x) + \sum_{j_{t+1}} \sum_x V^*(i_t + b - x, j_{t+1}) f(x) p(j_{t+1}|j_t^1, a_1^*, b) = V'(i_t, j_t^1) \tag{16}$$

It can also be stated that b_1^* minimizes the RHS of Inequality 16, $V'(i_t, j_t^1)$.

Let's now consider a state (i_t, j_t^2), such that $j_t^2 = j_t^1 + l$, $l > 0$ with $a_2^* = a_1^* + l$ (note that a_1^* and a_2^* will take the machine, from states j_t^1 and j_t^2 respectively, to the same machine state j_t'), substituting this in Equations $5-8$ gives:

$$
\begin{aligned}
&V''(i_t, j_t^2) \\
&= \sum_x [c_{a_2^*} + c_b + c_u b + c_q b + (c_r b + c_d(b{-}b)) \\
&\quad \times p(y|j_t, a_t) + c_h[i_t + b - x_t]^+ + c_\pi[x_t - i_t - b]^+] f(x) \\
&\quad + \sum_{j_{t+1}} \sum_x V^*(i_t + b - x, j_t^2 - a_2^*) f(x) p(j_t^2 - a_2^* | j_t^2, a_2^*, b)
\end{aligned}
\tag{17}
$$

Such that b is any arbitrary production action. Since $a_2^* = a_1^* + l$, Equations $2-4$ shows that $p(j_t^2 - a_2^* | j_t^2, a_2^*, b) = p(j_{t+1} | j_t^1, a_1^*, b)$. Additionally, since $c_{a_2^*} > c_{a_1^*}$, then $V''(i_t, j_t^2) = V'(i_t, j_t^1) + (c_{a_2} - c_{a_1})$, and b_1^* minimizes $V''(i_t, j_t^2)$ too, hence, $b_2^* = b_1^*$. ■

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