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Optimal Transmission Line Assignment Considering Reliabilities and Assignment Cost in the Multi-State Network

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ABSTRACT As the improvement of the network reliability can be achieved by the components assignments, the optimal transmission line assignment with maximal reliabilities and minimal cost (OTLAMRMC) problem under the transmission time constraints is investigated. The OTLAMRMC problem contains two sub-problems: the reliabilities and assignment cost evaluation under the transmission time constraint (RACETTC) and the multi-objective transmission line assignment optimization problem. First, the RACETTC algorithm is proposed to evaluate reliabilities and assignment cost under the transmission time constraints for a certain transmission line configuration. Then, the Non-dominated Sorting Genetic Algorithm III (NSGA-III) is adopted to search the optimal transmission line assignment based on the results obtained by the RACETTC algorithm. Therefore, combining the RACETTC algorithm and the NSGA-III together, the RACETTC-NSGA-III algorithm is proposed to solve the OTLAMRMC problem. Finally, example simulations are given to illustrate the proposed algorithm. The example results show that the RACETTC-NSGA-III algorithm can provide efficient solutions in a reasonable time.

INDEX TERMS Multi-state network, network reliability, NSGA-III, transmission line assignment, transmission time.

I. INTRODUCTION

With the rapid development and wildly usage of the computer networks in real life, it is very important to guarantee network reliability in order to provide satisfied services to users. The network reliability is the probability of the event that a given amount of data can be transmitted through the network successfully under the given time.

In real life, the network system is to perform the given tasks under specific conditions. Due to the influence of its own or external objective conditions, the network and its components (arcs and nodes) will generally show a variety of different performance levels in the operation process. A network with only two performance levels is called a binary-state network, and a multi-state network (MSN) is a network with multiple performance levels. Generally, the components of a multi-state network have multiple performance levels, so they are usually multi-state. In practical work, the performance level of the components generally shows a gradual decline, that is to say, components usually experience many intermediate states from normal working state to complete failure state.

The network reliability related problems have attracted lots of attentions [1]–[4]. The networks have first been treated as the binary-state networks [1], [5], [6]. However, the arcs and nodes of the network may have multiple states due to full failure, partial failure, maintenance, and other conditions with different performance [7]. In this case, the network is modeled as the MSN. For example, in real-life system, a computer network is usually modeled as a MSN due to communication congestion, physical facilities damage, etc.

In recent years, the MSN has attracted lots of attentions [8]–[18]. Garia *et al.* [10] focused on the evaluation of the reliability indices such as reliability, mean time to failure and sensitivity analysis of the multi-state complex system. Alamoudy [11] presented a comparison between some methods for evaluation the reliability of a multi-state communication flow networks systems. Lin *et al.* [12] constructed a time-constrained multi-state network to investigate the capacity of a computer network. They proposed an approach to evaluate the probability that d units of data can

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be sent from source node to sink node in no more than T units of time. Huang [14] proposed an algorithm to obtain minimal capacity vectors for evaluating the system reliability that the multi-state distribution network can meet all retailers' demand under stocks. As the MSN reliability can be computed in terms of all minimal capacity vectors meeting the demand requirements, Huang *et al.* [16] proposed a group approach with both the concepts of minimal cut and minimal path (MP) in order to narrow the search range of feasible capacity vectors.

As network reliability plays an important role in ensuring the normal output of data, many researches focus on reliability assurance or reliability improvement. For a given network, in order to guarantee the network reliability, lots of studies focus on the network backup paths or the spare paths [19]–[24], the network tolerance [25], [26], the network resilience [27], [28], etc.

For the network under construction, as the improvement of the network reliability can be achieved by the component assignments, the network reliability related assignment problems have attracted lots of attentions [29]–[35]. The components can be resources, commodities, transmission lines, etc.

Hsieh and Lin [29] considered the multi-resource allocation problem to maximize the network reliability in the multi-source multi-sink multi-state network (MMMSN). Lin and Yeh [34] focused on the optimal component assignment with maximal network reliability. Based on the research of [34], Lin and Yeh [7] extended the problem to consider the transmission budget.

The researches [7], [29], [34], [35] mainly focused on the problems of finding the optimal multi-state resources or transmission lines to maximize the network reliability. However, these researches [7], [34]–[36] only considered the single-objective resources or transmission line assignment problem, they do not consider the multi-objective components assignments.

In the practical environment, decision makers usually need to consider several objectives comprehensively. For the network reliability related component assignments, decision makers often need to consider to maximize the network reliability, to minimize the transmission time, to minimize the transmission cost comprehensively.

Meanwhile, the researches [7], [34]–[36] only consider the single-source single-sink multi-state network, they do not consider the multi-source multi-sink multi-state network. The single-sink means that there is only one sink node or terminal node in the network. As the MMMSN is very common in practical application, the MMMSN reliability related component assignments problems are worth studying. Zhang *et al.* [37] investigated the optimal transmission line assignment with maximal reliabilities with the cost constraint in the MMMSN. However, they do not consider the transmission time.

Therefore, this paper considers the optimal transmission line assignment with maximal $s-t$ pair reliabilities and minimal cost (OTLAMRMC) problem under the transmission time constraints in the MMMSN with fixed network topology. The $s-t$ pair reliability is the probability of the event that the data between source node s and sink node t can be transmitted through the paths which connect s and t under the given time. In this paper, there are some multi-state transmission lines can be selected to be assigned to the arcs, where a transmission line can be assigned to at most one arc.

As the OTLAMRMC problem is a multi-objective optimization problem, the objective function needs to be calculated before optimization. So the OTLAMRMC problem can be regarded as having two sub-problems: the $s-t$ pair reliabilities and assignment cost evaluation under the transmission time constraint (RACETTC) for a certain transmission line configuration and the multi-objective transmission line assignment optimization problem.

For the RACETTC problem, a RACETTC algorithm is proposed to calculate the $s-t$ pair reliabilities and assignment cost under the transmission time constraint for a certain transmission line configuration.

For the multi-objective transmission line assignment optimization problem, as the Non-dominated Sorting Genetic Algorithm III (NSGA-III) proposed by Deb and Jain [38] is an effective way to find the multi-objective optimization solution, the NSGA-III is adopted to find the optimal transmission line assignment based on the multi-objective values obtained by the RACETTC algorithm.

Combining the RACETTC algorithm and the NSGA-III together, the RACETTC-NSGA-III algorithm is proposed to solve the OTLAMRMC problem. The simulation results show that the RACETTC-NSGA-III algorithm can obtain the efficient solutions in a reasonable time. For the multi-objective transmission line assignment optimization problem, there may exists more than one solution that can get the maximal $s-t$ pair reliabilities and minimal assignment cost. After obtaining the optimal solutions, the decision makers can determine the final transmission line assignment according to their preferences.

The remainder of this paper is organized as follows. Section II provides the network model and the formulation of the OTLAMRMC problem. Section III introduces the RACETTC algorithm in details. Section IV gives the illustrations of the NSGA-III and the RACETTC-NSGA-III algorithm. Section V illustrates the RACETTC-NSGA-III algorithm with numerical examples. In the end, the conclusions are presented in Section VI. Table 1 gives the Acronyms.

II. PROBLEM FORMULATION

In this section, the multi-source multi-sink multi-state network model is provided first, then the OTLAMRMC problem formulation is given subsequently.

A. NETWORK MODEL

Let $(\mathbf{N}, \mathbf{A}, \mathbf{S}, \mathbf{T})$ be the MMMSN, where $\mathbf{N}$ denotes the set of nodes, $\mathbf{A} = \{a_i \mid i = 1, 2, \ldots, n\}$ represents the set

TABLE 1. Acronyms.

Acronyms	Decsription
MSN	multi-state network
MMMSN	multi-source multi-sink multi-state network
OTLAMRMC	optimal transmission line assignment with maximal $s-t$ pair reliabilities and minimal cost
RACETTC	the $s-t$ pair reliabilities and assignment cost evaluation under the transmission time constraint
MP	minimal path
NSGA-III	Non-dominated Sorting Genetic Algorithm III
NSGA-II	Non-dominated Sorting Genetic Algorithm II
LBP	lower boundary point
RSDP	the recursive sum of disjoint products

of n number of arcs connecting the nodes, $\mathbf{S} = \{s_u \mid u = 1, 2, \ldots, \alpha\} \subset \mathbf{N}$ denotes the set of the source nodes and $\mathbf{T} = \{t_v \mid v = 1, 2, \ldots, \beta\} \subset \mathbf{N}$ denotes the set of the sink nodes. Let $\mathbf{LEN} = \{len_i \mid i = 1, 2, \ldots, n\}$ be the set of the lengths of the arcs, $d_{uv}, u = 1, 2, \ldots, \alpha, v = 1, 2, \ldots, \beta$ represents the demand which needs to be transmitted from s_u to t_v, $\mathbf{P}_{uvj}$ denotes the jth, $j = 1, 2, \ldots, m_{uv}$ minimal path connecting s_u and t_v, where a minimal path is a path set such that if any arc is removed from this set, then the remaining set is no longer a path set [37]. In this paper, s_u are supposed to transmit d_{uv} units of data through $\mathbf{P}_{uvj}$ to t_v under the transmission time constraints.

Let $\mathbf{L} = \{l_k \mid k = 1, 2, \ldots, n_L\}$ be the set of n_L number of transmission lines, c_k denotes the assignment cost of unit length of l_k, $h_k(w)$ represents the wth, $w = 1, 2, \ldots, M_k$ capacity of l_k, where M_k is the total number of states that l_k owns, $h_k(M_k)$ represents the maximal capacity of l_k. A transmission line assignment is denoted as $\mathbf{B} = (b_1, \ldots, b_i, \ldots, b_n)$ where $b_i = k$ if transmission line l_k is selected to be assigned to arc a_i for $i = 1, 2, \ldots, n$. Therefore, the maximal capacity of a_i under $\mathbf{B}$ is $h_{b_i}(M_{b_i})$. Table 2 gives the term definitions.

Before modeling the OTLAMRMC problem, some assumptions are made as follows.

1) The transmission line assignments do not consider the nodes.

2) The capacities of different transmission lines are statistically independent.

3) The network flow must satisfy the flow-conservation law.

B. PROBLEM FORMULATION

The aim of this paper is to find the optimal transmission line assignment with minimal assignment cost and maximal s_u-t_v reliabilities under the transmission time constraints.

Let $C(\mathbf{B})$ be the total transmission line assignment cost under $\mathbf{B}$, $Re_{uv}(\mathbf{B})$ denotes the $s_u - t_v$ pair reliability under $\mathbf{B}$, $t_{uv}(\mathbf{B})$ represents the transmission time to transmit d_{uv} from s_u to t_v under $\mathbf{B}$, T_{uv} denotes the transmission time constraints between the $s_u - t_v$ pair.

TABLE 2. Term definitions.

Notation	Decsription
$\mathbf{N}$	the set of nodes
n	the total number of the arcs
a_i	the i th, $i=1,2,\ldots,n$ arc
$\mathbf{A}$	$\mathbf{A}=\{a_i \mid i=1,2,\ldots,n\}$, the set of n number of arcs connecting the nodes
α	the total number of the source nodes
s_u	the u th, $u=1,2,\ldots,\alpha$ source node
$\mathbf{S}$	$\mathbf{S}=\{s_u \mid u=1,2,\ldots,\alpha\}\subset\mathbf{N}$, the set of the source nodes
β	the total number of the sink nodes
t_v	the v th, $v=1,2,\ldots,\beta$ sink node
$\mathbf{T}$	$\mathbf{T}=\{t_v \mid v=1,2,\ldots,\beta\}\subset\mathbf{N}$, the set of the sink nodes
len_i	the length of a_i
$\mathbf{LEN}$	$\mathbf{LEN}=\{len_i \mid i=1,2,\ldots,n\}$, the set of the lengths of the arcs
d_{uv}	the demand which needs to be transmitted from s_u to t_v
m_{uv}	the total number of the minimal path connecting s_u and t_v
$\mathbf{P}_{uvj}$	the j th, $j=1,2,\ldots,m_{uv}$ minimal path connecting s_u and t_v
n_L	the total number of the transmission lines
l_k	the k th, $k=1,2,\ldots,n_L$ transmission line
$\mathbf{L}$	$\mathbf{L}=\{l_k \mid k=1,2,\ldots,n_L\}$, the set of n_L number of transmission lines
c_k	the assignment cost of unit length of l_k
M_k	the total number of states that l_k owns
$h_k(w)$	the w th, $w=1,2,\ldots,M_k$ capacity of l_k
$h_k(M_k)$	the maximal capacity of l_k
$\mathbf{B}$	$\mathbf{B}=(b_1,\ldots,b_i,\ldots,b_n)$, a transmission line assignment
$h_{b_i}(M_{b_i})$	the maximal capacity of a_i under $\mathbf{B}$
$C(\mathbf{B})$	the total assignment cost under $\mathbf{B}$
$Re_{uv}(\mathbf{B})$	the s_u-t_v pair reliability under $\mathbf{B}$
$t_{uv}(\mathbf{B})$	the transmission time to transmit d_{uv} from s_u to t_v under $\mathbf{B}$
T_{uv}	the transmission time constraints between s_u-t_v pair
x_i	the current capacity of a_i
$\mathbf{X}$	$\mathbf{X}=(x_1,x_2,\ldots,x_n)$, the network capacity vector
ff_i	the total flow through a_i
f_{uvj}	the flow through $\mathbf{P}_{uvj}$
t_{uv}	the maximal transmission time of the paths connecting s_u to t_v
t_{uvj}	the transmission time through $\mathbf{P}_{uvj}$
$\mathbf{X}_{uv}$	$\mathbf{X}_{uv}=(x_{uv1},x_{uv2},\ldots,x_{uvn})$, the state capacity vector of the arcs belonging to $\mathbf{P}_{uv}$

As the optimization targets contain $Re_{uv}(\mathbf{B})$ maximization and $C(\mathbf{B})$ minimization, in order to unity the

TABLE 2. ***(Continued.)*** **Term definitions.**

$\mathbf{U}_{uv\mathbf{B}}$	the set of all the satisfied $\mathbf{X}_{uv}$ under assignment $\mathbf{B}$
$\mathbf{X}_{uvLBP}$	the lower boundary points of $\mathbf{X}_{uv}$
h_{uv}	the number of $\mathbf{X}_{uvLBP}$ s
$n_{\mathbf{\Psi}}$	the total number of the qualified $\mathbf{X}$ s
$\mathbf{\Psi}_{min}$	the set to store the $\mathbf{X}_{LBP}$ s
$\mathbf{K}$	the set to store the index of the qualified $\mathbf{X}$ s
$\mathbf{I}$	the set to store the index of non- $\mathbf{X}_{LBP}$ s
$\mathbf{J}$	The set to store the index of the $\mathbf{X}_{LBP}$ s

optimization trend, the problem formulation converts $C(\mathbf{B})$ minimization to $-C(\mathbf{B})$ maximization. The function maximization means to maximize every element of the vector function. The mathematical model of the OTLAMRMC problem is given as follows.

$$\max \text{function } (\mathbf{B}) = (Re_{11}(\mathbf{B}), \ldots, Re_{uv}(\mathbf{B}), \ldots, Re_{\alpha\beta}(\mathbf{B}), -C(\mathbf{B}))$$
$$\text{s.t.} \begin{cases} b_i = k, k \in \{1, 2, \ldots, n_L\} & \text{for } i = 1, 2, \ldots n \\ b_i \neq b_j, & \text{for } i \neq j \\ t_{uv}(\mathbf{B}) \leq \mathrm{T}_{uv}, u = 1, 2, \ldots, \alpha, & v = 1, 2, \ldots, \beta, \end{cases} \quad (1)$$

where $\mathbf{B} = (b_1, \ldots, b_i, \ldots, b_n)$ is the decision variable with n dimensions. The evaluations of $C(\mathbf{B})$, $t_{uv}(\mathbf{B})$ and $Re_{uv}(\mathbf{B})$ are given in Section III.

III. RACETTC ALGORITHM

In this section, the RACETTC algorithm is proposed to evaluate the $s_u - t_v$ pair reliabilities and assignment cost under transmission time constraints for a transmission line configuration.

For a certain transmission line configuration, the RACETTC algorithm first determines the flow and capacity of the arcs, then calculate the transmission time, at last evaluate the assignment cost and the $s_u - t_v$ pair reliabilities.

A. FLOW AND CAPACITY

In order to evaluate the transmission time and the $s_u - t_v$ pair reliabilities, we first need to know the flow through the paths and arcs.

As there are many kinds of path capacity combinations in MSN, it is time-consuming to enumerate all path capacity combinations to calculate reliability. If we can find the minimum capacity of the path that meets the transmission requirements, we can obtain the network reliability by calculating the probability of the minimal path. The maximum capacity of the MP is the minimum capacity of all the paths that meet the transmission requirements.

We first calculate the flow through the minimal path $\mathbf{P}_{uvj}$, then the smallest capacity of the arcs which satisfy the transmission flow can be obtained.

Let x_i be the current capacity of a_i, $\mathbf{X} = (x_1, x_2, \ldots, x_n)$ denotes the network capacity vector, $f\!f_i$ represents the total flow through a_i, f_{uvj} denotes the flow through $\mathbf{P}_{uvj}$. For a given transmission assignment $\mathbf{B}$, $f\!f_i$ and f_{uvj} should not exceed the maximal capacity of a_i and the maximal capacity of $\mathbf{P}_{uvj}$ under $\mathbf{B}$, respectively, that are

$$f\!f_i = \sum_{u=1}^{\alpha}\sum_{v=1}^{\beta}\sum_{j=1}^{m_{uv}} \{f_{uvj} \mid a_i \in \mathbf{P}_{uvj}\} \leq h_{b_i}(M_{b_i}), \quad i = 1, 2, \ldots, n. \quad (2)$$

$$f_{uvj} \leq \min\{h_{b_i}(M_{b_i}) \mid a_i \in \mathbf{P}_{uvj}\}, \quad u = 1, 2, \ldots, \alpha, \quad v = 1, 2, \ldots, \beta, \quad j = 1, 2, \ldots, m_{uv}, \quad (3)$$

where $h_{b_i}(M_{b_i})$ is the maximal capacity of a_i under $\mathbf{B}$. As the maximal capacity of the path should not exceed the maximal capacity of the arcs belonging to this path, the minimum value of the maximum capacity of all arcs is the maximum capacity of the path. That is, $\min\{h_{b_i}(M_{b_i}) \mid a_i \in \mathbf{P}_{uvj}\}$ is the maximal capacity of $\mathbf{P}_{uvj}$ under $\mathbf{B}$.

In order to transmit d_{uv} units of data through $s_u - t_v$ pair, the total flow through $\mathbf{P}_{uvj}$ should equal to d_{uv}, that is

$$\sum_{j=1}^{m_{uv}} f_{uvj} = d_{uv}, \quad u = 1, 2, \ldots, \alpha, \; v = 1, 2, \ldots, \beta. \quad (4)$$

According to (2)-(4), $f\!f_i$, $i = 1, 2, \ldots, n$ can be obtained. Then, we can derive x_i from $f\!f_i$ through (5).

$$x_i = h_{b_i}(w) \text{ if there exist a } w \in 1, 2, \ldots M_{b_i} \text{ such that } h_{b_i}(w-1) < f\!f_i \leq h_{b_i}(w), \quad (5)$$

where $h_{b_i}(w)$ is the smallest capacity of l_{b_i} to satisfy the flow loading. Using (5), we can get all the qualified network capacity vectors $\mathbf{X}$s under the transmission assignment $\mathbf{B}$.

B. ASSIGNMENT COST AND TRANSMISSION TIME EVALUATION

The transmission line assignment cost is related with the length of the arc to which this transmission line is configured. Let $C(\mathbf{B})$ be the total transmission line assignment cost under $\mathbf{B}$, we have

$$C(\mathbf{B}) = \sum_{i=1}^{n} (c_{b_i} \cdot len_i), \quad (6)$$

where $c_{b_i} \cdot len_i$ is the assignment cost of l_{b_i} based on the length of a_i.

The transmission time of the $s_u - t_v$ pair, which is represented by t_{uv}, is the maximal transmission time of the paths connecting s_u to t_v, that is

$$t_{uv} = \max(t_{uvj}), \quad j = 1, \ldots, m_{uv}, \quad (7)$$

where t_{uvj} is the transmission time through $\mathbf{P}_{uvj}$. t_{uvj} is the sum of the transmission time that the flow pass through the arcs which belonging to $\mathbf{P}_{uvj}$, that is

$$t_{uvj} = \sum \frac{f\!f_i}{x_i}, \quad a_i \subset \mathbf{P}_{uvj}. \quad (8)$$

C. RELIABILITES EVALUATION

Let $\mathbf{X}_{uv} = (x_{uv1}, x_{uv2}, \ldots, x_{uvn})$ be the state capacity vector of the arcs belonging to $\mathbf{P}_{uv}$, $\mathbf{U}_{uv\mathbf{B}}$ denotes the set of all the satisfied $\mathbf{X}_{uv}$ under assignment $\mathbf{B}$. The $s_u - t_v$ pair reliability $Re_{uv}(\mathbf{B})$ is

$$Re_{uv}(\mathbf{B}) = \sum Pr\{\mathbf{X}_{uv} | \mathbf{X}_{uv} \in \mathbf{U}_{uv\mathbf{B}}\}. \tag{9}$$

According to (9), we need to found all the qualified network capacity vectors first. However, Lin *et al.* [39] proved that it is an inefficient way to enumerate all the qualified $\mathbf{X}$s and summing up their probabilities to obtain the network reliability. Instead, we can use the lower boundary point (LBP) of $\mathbf{X}_{uv}$s, which are represented by $\mathbf{X}_{uvLBP}$s, to calculate $Re_{uv}(\mathbf{B})$. The $\mathbf{X}_{uvLBP}$s can be obtained by the $\mathbf{X}_{LBP}$s generation algorithm proposed by Lin and Yeh [36].

Let $n_{\boldsymbol{\Psi}}$ be the total number of the qualified $\mathbf{X}$s, $\boldsymbol{\Psi}_{min}$ denote the set to store the $\mathbf{X}_{LBP}$s, $\mathbf{K} = \{1, 2, \ldots, n_{\boldsymbol{\Psi}}\}$ and $\mathbf{I}$ represent the sets to store the index of the qualified $\mathbf{X}$s and non-$\mathbf{X}_{LBP}$s, respectively, $\mathbf{J}$ denote the set to store the index of the $\mathbf{X}_{LBP}$s. The pseudo codes to choose $\mathbf{X}_{LBP}$s from the qualified $\mathbf{X}$s are given in Algorithm 1 [36].

Algorithm 1 The $\mathbf{X}_{LBP}$s Generation Algorithm

```
function Ψ_min = LBP(X_1, ..., X_nΨ)
[Initialization] I = ∅, J = ∅.
for i = 1 to nΨ with i ∉ I
    for j = i + 1 to nΨ with j ∉ I
        if X_i ≥ X_j
            I = I ∪ {i}
        else
            I = I ∪ {j}
        end
        j = j + 1
    end
    i = i + 1
end
J = K − I, Ψ_min = {X_J(1), X_J(2), ..., X_J(length(J))}.
```

As the capacity states of the arcs which do not belonging to $\mathbf{P}_{uvj}$ have no influence on $Re_{uv}(\mathbf{B})$, $\mathbf{X}_{uv}$ can be derived from $\mathbf{X}$ as follows:

$$x_{uvi} = \begin{cases} \mathrm{x_i} & \text{if } \mathrm{a_i} \in \mathbf{P}_{\mathrm{uvj}}, \\ 0 & \text{if others.} \end{cases} \tag{10}$$

According to algorithm 1, we can obtain $\mathbf{X}_{LBP}$s, then we can derive $\mathbf{X}_{uvLBP}$s by (10).

Suppose that there are h_{uv} number of $\mathbf{X}_{uvLBP}$s, $Re_{uv}(\mathbf{B})$ can be calculated as follows.

$$Re_{uv}(\mathbf{B}) = Pr\{\bigcup_{k=1}^{h_{uv}} \{\mathbf{X}_{uv} | \mathbf{X}_{uv} \geq \mathbf{X}_{uvLBPk}, \mathbf{X}_{uv} \in \mathbf{U}_{uv\mathbf{B}}\}\}, \tag{11}$$

where $Pr\{\mathbf{X}_{uv} \geq \mathbf{X}_{uvLBPk}\}$ can be calculated by (12).

$$\begin{aligned} &Pr\{\mathbf{X}_{uv} \geq \mathbf{X}_{uvLBPk}\} \\ &\quad = Pr\{x_{uv1} \geq x_{uvLBPk1}\} \\ &\qquad \times Pr\{x_{uv2} \geq x_{uvLBPk2}\} \times \ldots \times Pr\{x_{uvn} \geq x_{uvLBPkn}\}. \end{aligned} \tag{12}$$

In detail, $Re_{uv}(\mathbf{B})$ can be calculated by the recursive sum of disjoint products (RSDP) algorithm proposed by Zuo *et al.* [40]. The basic idea of the RSDP algorithm is that the probability of a union with h_v vectors can be calculated by evaluating the probabilities of several unions with $h_v - 1$ vectors or less [37], [40]. The pseudo codes of the RSDP algorithm are given in Algorithm 2 [37].

Algorithm 2 The RSDP Algorithm

```
function Re = RSDP(X_LBP1, X_LBP2, ..., X_LBPh_v)
for i = 1 to h_v
    if i == 1
        Re = Pr{X | X ≥ X_LBPi}
    else
        re_1 = Pr{X | X ≥ X_LBPi}
        if i == 2
            re_2 = Pr{X | X ≥ (X_LBP1 ⊕ X_LBPi)}
        else
            for j = 1 to i − 1
                X_LBPji = X_LBPj ⊕ X_LBPi
            end
            Ψ_minji = LBP(X_LBP1i, X_LBP2i, ..., X_LBP(i−1)i)
            re_2 = RSDP(Ψ_minji)
        end
    end
    Re = Re + re_1 − re_2
end
```

As there may exist some $\mathbf{X}_{uvLBP}$s larger than more than one of the remaining $\mathbf{X}_{uvLBP}$s, the true $\mathbf{X}_{uvLBP}$s, which are represented by $\mathbf{X}'_{uvLBP}$s, can be obtained by algorithm 1. After the determination of $\mathbf{X}'_{uvLBP}$s, $Re_{uv}(\mathbf{B})$ can be obtained by the RSDP algorithm

D. RACETTC ALGORITHM

After the calculation method of the transmission time, the assignment cost and the $s_u - t_v$ pair reliabilities are given, the RACETTC algorithm can be formed as follows.

IV. RACETTC-NSGA-III ALGORITHM

This section first gives the processes of the NSGA-III, then provides the procedures of the RACETTC-NSGA-III algorithm.

A. NSGA-III

After the calculation of the $s_u - t_v$ pair reliabilities and the assignment cost for one transmission line configuration, we need to find the optimal transmission line assignments. As the implicit enumeration method to solve the OTLAMRMC problem is time consuming, the NSGA-III proposed by Deb and Jain [38], which has been proved to be an effective way to find the multi-objective optimization solution, is adopted in this paper.

The NSGA-III [38] is an effective upgrade of the Non-dominated Sorting Genetic Algorithm II (NSGA-II) proposed by Deb *et al.* [41]. The NSGA-III and the NSGA-II seek

Algorithm 3 The RACETTC Algorithm

[Remark] For a transmission line assignment $\mathbf{B}$, run the following steps.

Step 1. Find all $\mathbf{F} = (ff_1, \ldots, ff_i, \ldots, ff_n)$ through (13). If no feasible $\mathbf{F}$ exist, then jump out of the following steps. Find another assignment, run Step 1 again.

$$\sum_{j=1}^{m_{uv}} f_{uvj} = d_{uv}, \quad u = 1, 2, \ldots, \alpha, \; v = 1, 2, \ldots, \beta,$$
$$ff_i = \sum_{u=1}^{\alpha}\sum_{v=1}^{\beta}\sum_{j=1}^{m_{uv}} \{f_{uvj} \mid a_i \in \mathbf{P}_{uvj}\} \le h_{b_i}(M_{b_i}),$$
$$i = 1, 2, \ldots, n,$$
$$f_{uvj} \le \min\{h_{b_i}(M_{b_i}) \mid a_i \in \mathbf{P}_{uvj}\}, \quad u = 1, 2, \ldots, \alpha,$$
$$v = 1, 2, \ldots, \beta, \quad j = 1, 2, \ldots, m_{uv}, ,$$
$$i = 1, 2, \ldots, n. \tag{13}$$

Step 2. Find the minimum value of the arc capacity satisfying the flow conditions by (14).

$$x_i(\min) = h_{b_i}(w) \text{ if there exist a } w \in 1, 2, \ldots M_{b_i}$$
$$\text{such that } h_{b_i}(w-1) < ff_i \le h_{b_i}(w). \tag{14}$$

Step 3. Find all the state vectors $\mathbf{X} = (x_1, \ldots, x_i, \ldots, x_n)$, where $x_i(\min) \le x_i \le h_{b_i}(M_{b_i})$. As the total number of the states that one transmission line owns is not large, we can obtain $\mathbf{X}$ by going through all the qualified states of the arcs. The estimated computational complexity is $O(M_{\mathrm{i}}) \times O((MQ_{\mathrm{i}})^{\mathrm{n}})$, where $O(M_{\mathrm{i}})$ is the computational complexity to find qualified states of x_i, M_{i} is the number of all the states that a_i owns, MQ_{i} is the number of the qualified states that a_i owns.

Step 4. Calculate the transmission time of the $s_u - t_v$ pair by (15).

$$t_{uv} = \max(t_{uvj}), \quad j = 1, \ldots, m_{uv},$$
$$t_{uv\mathrm{j}} = \sum \frac{ff_i}{x_i}, \quad a_i \subset \mathbf{P}_{uvj}. \tag{15}$$

Step 5. Select the state vectors that satisfy the transmission time constraints $\mathbf{X}'$. Compare t_{uv} with T_{uv}, if $t_{uv} \le T_{uv}$, $u = 1, 2, \ldots, \alpha$, $v = 1, 2, \ldots, \beta$, then $\mathbf{X}' = \mathbf{X}$; else, jump out of the following steps. Find another assignment, run Step 1.

Step 6. Use algorithm 1 to obtain $\mathbf{X}_{LBP}$s form $\mathbf{X}'$.

Step 7. Derive $\mathbf{X}_{uvLBP} = (x_{uvLBP1}, \ldots, x_{uvLBPi}, \ldots, x_{uvLBPn})$ from $\mathbf{X}_{LBP}$s by (16).

$$x_{uvLBPi} = \begin{cases} \mathrm{x_{LBPi}} & \text{if } \mathrm{a_i} \in \mathbf{P}_{\mathrm{uvj}}, \\ 0 & \text{if others.} \end{cases} \tag{16}$$

Step 8. Use algorithm 1 to obtain $\mathbf{X'}_{uvLBP}$s.

Step 9. Calculate the total transmission line assignment cost $C(\mathbf{B})$ using (17).

$$C(\mathbf{B}) = \sum_{i=1}^{n} (c_{b_i} \cdot len_i). \tag{17}$$

Step 10. Use the RSDP algorithm to calculate $Re_{uv}(\mathbf{B})$ for all $u = 1, 2, \ldots, \alpha$, $v = 1, 2, \ldots, \beta$.

Step 11. According to the above steps, we can get the objective function under $\mathbf{B}$, that is

$$\text{function}(\mathbf{B}) = (Re_{11}(\mathbf{B}), \ldots, Re_{uv}(\mathbf{B}),$$
$$\ldots, Re_{\alpha\beta}(\mathbf{B}), -C(\mathbf{B})). \tag{18}$$

the optimal solution by imitating the selection and genetic mechanism of nature. They have the following advantages

1) They do not have too many mathematical requirements for optimization problems. They can deal with any form of objective functions and constraints, whether linear or nonlinear, discrete or continuous.

2) They can search all the solutions in the solution space quickly without falling into the fast descent trap of local optimal solution.

3) They can compare multiple individuals at the same time, so as to improve the solution speed.

4) They have scalability and are easy to combine with other algorithms.

The difference between the NSGA-II and the NSGA-III is mainly due to the change of selection mechanism. The operation of selecting the superior individual from the population and eliminating the inferior individual is called selection. The purpose of selection is to pass the optimized individuals (or solutions) directly to the next generation or to generate new individuals to the next generation through crossover and mutation. The selection operation is based on the fitness evaluation of individuals in the population.

The NSGA-II mainly relies on the crowding degree for ranking, which obviously plays a less obvious role in high-dimensional target space. Instead of using crowding degree ranking, the NSGA-III introduces the reference point mechanism to retain the population individuals that are not dominated and close to the reference point.

Compared with the NSGA-II, the NSGA-III has better convergence and diversity, especially for the optimization problems with three or more objectives. As the NSGA-III can obtain the optimal solutions in a reasonable time, this paper adopts the NSGA-III to find the optimal set of the transmission line assignment with maximal $s_u - t_v$ pair reliabilities and the minimal assignment cost.

Let N_{pop} be the population size, N_{fun} denotes the number of the optimization objectives, the complexity of the NSGA-III is $O(N_{fun}N_{pop}^2)$.

Let T_{cycle} be the maximal number of the generation, t represents the current iteration number, p_c and p_m denote the crossover and mutation rate, respectively,. The basic processes of the NSGA-III are given in Algorithm 4.

B. RACETTC-NSGA-III ALGORITHM

Combining the RACETTC algorithm and the NSGA-III together, the RACETTC-NSGA-III algorithm is proposed to solve the OTLAMRMC problem in the MMMSN without changing the network topology.

In the RACETTC-NSGA-III algorithm, a chromosome represents a transmission line assignment and the fitness function is $(Re_{11}(\mathbf{B}), \ldots, Re_{uv}(\mathbf{B}), \ldots, Re_{\alpha\beta}(\mathbf{B}), -C(\mathbf{B}))$ under the transmission line assignment $\mathbf{B}$. The procedures of the RACETTC-NSGA-III algorithm are given in Fig. 1.

Algorithm 4 The NSGA-III

Step 1. Define N_{pop}, p_c, p_m, and T_{cycle}.
Step 2. Generate the initial parent population randomly.
Step 3. Generate the offspring population Q_t by using the roulette wheel selection, crossover and mutation operations.
Step 4. Form the intermediate population $R_t = PP_t \cup Q_t$.
Step 5. Evaluate the fitness function of each individual in R_t. Normalize the target value of each individual in R_t.
Step 6. Sort the individuals belong to R_t according to the fast non-dominated sorting method.
Step 7. Calculate the reference point of the individuals in R_t.
Step 8. Choose the best N_{pop} individuals from R_t to form the parent population P_{t+1} of the next generation.
Step 9. If $t < T_{cycle}$, set $t = t + 1$ and return to Step 3, otherwise, end the NSGA-III and output the results.

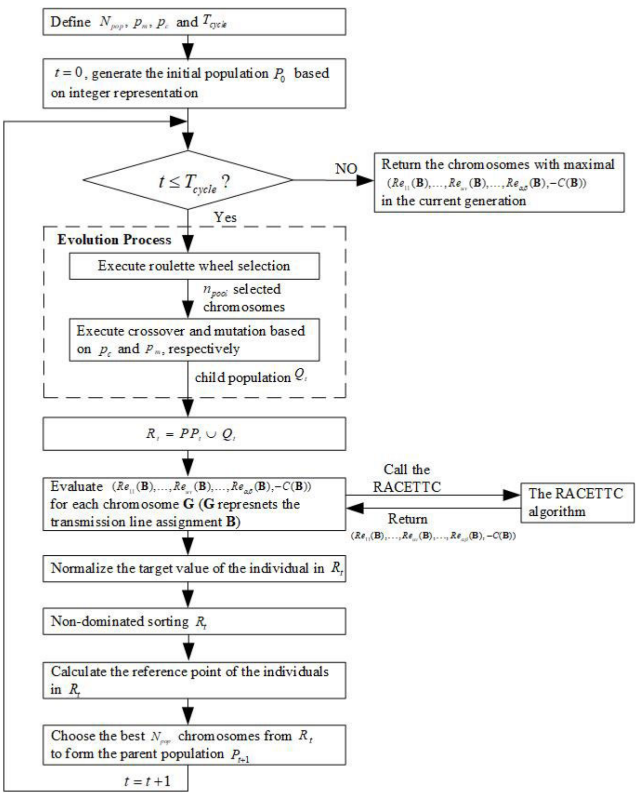

FIGURE 1. The procedures of the RACETTC-NSGA-III algorithm.

As shown in Fig. 1, the roulette wheel selection, which is also called the proportion selection, is used in the RACETTC-NSGA-III algorithm. The basic idea of the roulette wheel selection is that the probability of individual selection is directly proportional to their fitness. That is, the individuals with high fitness are more likely to be selected to the next generation.

V. NUMERICAL EXAMPLE

The following examples are programmed with the MATLAB programming language and executed on the personal computer with Intel Core i7-7700HQ, CPU 2.8GHz and 8GB RAM.

A. EXAMPLE 1

In this example, the RACETTC-NSGA-III algorithm is compared with the implicit enumeration method based on the results and running time in terms of a two-source two-sink multi-state computer network. Fig. 2 gives the topology of the network. Table 3 gives the length of the arcs.

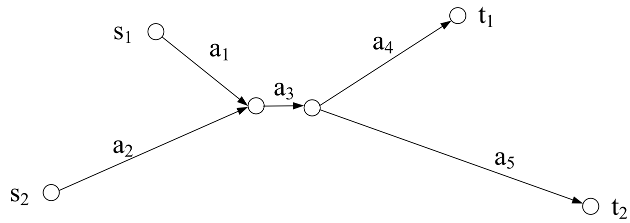

FIGURE 2. A two-source two-sink multi-state computer network.

TABLE 3. The length of the arcs of example 1.

i	1	2	3	4	5
len_i (km)	30	60	10	45	80

The paths are $\mathbf{P}_{111} = \{a_1, a_3, a_4\}$, $\mathbf{P}_{121} = \{a_1, a_3, a_5\}$, $\mathbf{P}_{211} = \{a_2, a_3, a_4\}$, and $\mathbf{P}_{221} = \{a_2, a_3, a_5\}$. The transmission demands are $d_{11} = 1$ Gb, $d_{12} = 1$ Gb, $d_{21} = 1$ Gb, and $d_{22} = 1$ Gb. The transmission time constraint are $T_{11} = 2$ s, $T_{12} = 2$ s, $T_{21} = 2$ s, and $T_{22} = 2$ s.

There are 6 transmission lines ready to be assigned to the arcs. The assignment cost of unit kilometer of the transmission line is counted in Ren Min Bi (RMB). Table 4 gives the unit assignment cost and the probability distribution of the transmission lines which can be selected to be assigned to the arcs.

TABLE 4. Probability distribution and cost of the transmission lines of example 1.

l_k	Cost (RMB/km)	Capacity (Gbps)				
		0	1	2	3	4
1	4200	0.03	0	0.02	0	0.95
2	3800	0.03	0	0.03	0	0.94
3	3800	0.02	0	0.05	0	0.93
4	3700	0.04	0	0.03	0	0.93
5	3200	0.02	0	0.02	0.96	0
6	3000	0.03	0	0.06	0	0.91

The parameters of the RACETTC-NSGA-III are $N_{pop} = 100$ and $T_{cycle} = 1000$. Examples are implemented by the implicit enumeration method and the RACETTC-NSGA-III algorithm to explore the results of $(Re_{11}(\mathbf{B}), Re_{12}(\mathbf{B}), Re_{21}(\mathbf{B}), Re_{22}(\mathbf{B}), -C(\mathbf{B}))$. Table 5 and Table 6 give the results obtained by the implicit enumeration method and the

TABLE 5. The results obtained by the implicit enumeration method of example 1.

No.	Satisfied transmission lines assignment	$(Re_{11}(\mathbf{B}), Re_{12}(\mathbf{B}), Re_{21}(\mathbf{B}), Re_{22}(\mathbf{B}), -C(\mathbf{B}))$
1	(2 3 1 6 5)	(0.893855 0.90307 0.90307 0.91238 -775000)
2	(2 5 1 3 6)	(0.90307 0.893855 0.91238 0.90307 -759000)
3	(3 2 1 5 6)	(0.91238 0.90307 0.90307 0.893855 -768000)
4	(3 4 1 5 6)	(0.91238 0.90307 0.89376 0.88464 -762000)
5	(3 4 2 5 6)	(0.902776 0.893564 0.884352 0.875328 -758000)
6	(3 5 1 2 6)	(0.90307 0.90307 0.90307 0.90307 -759000)
7	(3 5 1 4 6)	(0.89376 0.90307 0.89376 0.90307 -754500)
8	(3 5 2 4 6)	(0.884352 0.893564 0.884352 0.893564 -750500)
9	(3 6 1 2 5)	(0.90307 0.91238 0.893855 0.90307 -763000)
10	(3 6 1 4 5)	(0.89376 0.91238 0.88464 0.90307 -758500)
11	(4 3 1 6 5)	(0.88464 0.89376 0.90307 0.91238 -772000)
12	(4 5 1 3 6)	(0.89376 0.88464 0.91238 0.90307 -756000)
13	(4 5 2 3 6)	(0.884352 0.875328 0.902776 0.893564 -752000)

TABLE 6. The results obtained by the RACETTC-NSGA-III algorithm ($p_c = 0.9$, $p_m = 0.01$) of example 1.

No.	Satisfied transmission lines assignment	$(Re_{11}(\mathbf{B}), Re_{12}(\mathbf{B}), Re_{2,}(\mathbf{B}), Re_{22}(\mathbf{B}), -C(\mathbf{B}))$
1	(2 3 1 6 5)	(0.893855 0.90307 0.90307 0.91238 -775000)
2	(2 5 1 3 6)	(0.90307 0.893855 0.91238 0.90307 -759000)
3	(3 2 1 5 6)	(0.91238 0.90307 0.90307 0.893855 -768000)
4	(3 4 2 5 6)	(0.902776 0.893564 0.884352 0.875328 -758000)
5	(3 5 1 2 6)	(0.90307 0.90307 0.90307 0.90307 -759000)
6	(3 5 1 4 6)	(0.89376 0.90307 0.89376 0.90307 -754500)
7	(3 5 2 4 6)	(0.884352 0.893564 0.884352 0.893564 -750500)
8	(3 6 1 2 5)	(0.90307 0.91238 0.893855 0.90307 -763000)
9	(3 6 1 4 5)	(0.89376 0.91238 0.88464 0.90307 -758500)
10	(4 3 1 6 5)	(0.88464 0.89376 0.90307 0.91238 -772000)
11	(4 5 1 3 6)	(0.89376 0.88464 0.91238 0.90307 -756000)
12	(4 5 2 3 6)	(0.884352 0.875328 0.902776 0.893564 -752000)

RACETTC-NSGA-III algorithm ($p_c = 0.9$, $p_m = 0.01$) respectively.

The running time of the implicit enumeration method and the RACETTC-NSGA-III algorithm ($p_c = 0.9$, $p_m = 0.1$) with 20 times are 8194 s and 273 s, respectively.

As show by Table 5, the implicit enumeration method finds 13 kinds of transmission lines assignments with maximal $s_u - t_v$ pair reliabilities and minimal assignment cost under the transmission time constraints. Table 6 reveals that the RACETTC-NSGA-III algorithm obtains 12 kinds of transmission lines assignments with the optimal function values.

There are more than one kinds of transmission lines assignments for the decision makers to choose based on their preferences or experiences. If the decision makers take Re_{11} in the first place, assignment (3 2 1 5 6) or assignment (3 4 1 5 6) will be the best choices. If the decision makers pay more attentions on assignment cost, assignment (4 5 2 3 6) will be the best one.

From Table 5 and Table 6, we know that the results obtained by the RACETTC-NSGA-III algorithm is only one less than that of the implicit enumeration method. The missing transmission lines assignment is (3 4 1 5 6). The results can prove that the RACETTC-NSGA-III algorithm can find the approximate optimal solutions. As the solutions obtained by the RACETTC-NSGA-III algorithm are related to the crossover rate and the mutation rate, we can set different parameter values to run the RACETTC-NSGA-III algorithm. Table 7 gives the results obtained by the RACETTC-NSGA-III algorithm with $p_c = 0.9$ and $p_m = 0.01$.

TABLE 7. The results obtained by the RACETTC-NSGA-III algorithm ($p_c = 0.9$, $p_m = 0.1$) of example 1.

No.	Satisfied transmission lines assignment	$(Re_{11}(\mathbf{B}), Re_{12}(\mathbf{B}), Re_{21}(\mathbf{B}), Re_{22}(\mathbf{B}), -C(\mathbf{B}))$
1	(2 3 1 6 5)	(0.893855 0.90307 0.90307 0.91238 -775000)
2	(2 5 1 3 6)	(0.90307 0.893855 0.91238 0.90307 -759000)
3	(3 2 1 5 6)	(0.91238 0.90307 0.90307 0.893855 -768000)
4	(3 4 1 5 6)	(0.91238 0.90307 0.89376 0.88464 -762000)
5	(3 4 2 5 6)	(0.902776 0.893564 0.884352 0.875328 -758000)
6	(3 5 1 2 6)	(0.90307 0.90307 0.90307 0.90307 -759000)
7	(3 5 1 4 6)	(0.89376 0.90307 0.89376 0.90307 -754500)
8	(3 5 2 4 6)	(0.884352 0.893564 0.884352 0.893564 -750500)
9	(3 6 1 2 5)	(0.90307 0.91238 0.893855 0.90307 -763000)
10	(3 6 1 4 5)	(0.89376 0.91238 0.88464 0.90307 -758500)
11	(4 5 1 3 6)	(0.89376 0.88464 0.91238 0.90307 -756000)
12	(4 5 2 3 6)	(0.884352 0.875328 0.902776 0.893564 -752000)

The running time of the RACETTC-NSGA-III algorithm ($p_c = 0.9$, $p_m = 0.01$) with 20 times is 268 s. Table 7 reveals that the RACETTC-NSGA-III algorithm ($p_c = 0.9$, $p_m = 0.01$) obtains 12 kinds of transmission lines assignments with the optimal function values. Compared to the implicit enumeration method, the missing transmission lines assignment is (4 3 1 6 5).

Combing the results obtained by the RACETTC-NSGA-III algorithm with two different p_m, there are 13 kinds of transmission lines assignments which are the same as the results obtained by the implicit enumeration method. Meanwhile, the running time of the RACETTC-NSGA-III algorithm is much less than that of the implicit enumeration method.

Considering the optimization objective results and algorithm running time of example 1, we can prove that the RACETTC-NSGA-III algorithm can provide efficient solutions in a reasonable time.

B. EXAMPLE 2

Example 1 is to select 5 out of 6 transmission lines to be assigned to the arcs. The total number of the transmission line assignment choice is $6 \times 5 \times 4 \times 3 \times 2 = 720$. As the number of transmission line assignment in the implicit enumeration method is not very large, the computing time advantage of the RACETTC-NSGA-III algorithm is not so obvious.

TABLE 8. Probability distribution and cost of the transmission lines of example 2.

l_k	Cost (RMB/ km)	Capacity (Gbps)				
		0	1	2	3	4
1	4200	0.03	0	0.02	0	0.95
2	3800	0.03	0	0.03	0	0.94
3	3800	0.02	0	0.05	0	0.93
4	3700	0.04	0	0.03	0	0.93
5	3200	0.02	0	0.02	0.96	0
6	3000	0.03	0	0.06	0	0.91
7	2200	0.03	0.03	0.94	0	0
8	2800	0.02	0.01	0.09	0.88	0
9	3000	0.01	0.02	0.02	0.95	0
10	2300	0.03	0.04	0.12	0.81	0
11	2500	0.06	0	0.12	0	0.82
12	2500	0.03	0	0.97	0	0
13	2900	0.05	0	0.08	0	0.87
14	2700	0.02	0	0.14	0	0.84
15	4300	0.03	0.01	0	0	0.96
16	1900	0.05	0.14	0.81	0	0
17	2900	0.03	0.09	0	0	0.88
18	2500	0.02	0.02	0.13	0.83	0
19	2500	0.01	0.02	0.14	0.83	0
20	3100	0.03	0.07	0	0.9	0

In example 2, the network topology is the same as that in example 1. There are 20 transmission lines can be assigned to the arcs. Table 8 gives the unit assignment cost and the probability distribution of the transmission lines.

As the total number of the transmission line assignment choice is $20 \times 19 \times 18 \times 17 \times 16 = 1860480$, the running time of the implicit enumeration method well be very long. The running time of the implicit enumeration method in example 1 is used as reference for conversion, the running time of the implicit enumeration method in example 2 will be $8194\ s \div 720 \times 1860480 = 21173296\ s = 5881.47\ h = 245.06\ day$. This means that the implicit enumeration method needs to run more than 245 days to obtain the results, which is not realistic in the real world.

We have run the implicit enumeration method in example 2 for more than two days, the program is still running. Then, we run the RACETTC-NSGA-III algorithm twice for $N_{pop} = 100$ and $T_{cycle} = 1000$ with different mutation rate. The running time of the RACETTC-NSGA-III algorithm ($p_c = 0.9$, $p_m = 0.01$) and ($p_c = 0.9$, $p_m = 0.1$) with 20 times are 1129 s and 1153 s, respectively. Table 9 gives the merging results of the two runs of the RACETTC-NSGA-III algorithm. There are 75 kinds of transmission lines assignments with the optimal function values. The decision makers can choose the final transmission line assignment based on preferences or experiences.

The results of example 2 prove the effectiveness of the RACETTC-NSGA-III algorithm again. For the case of a large number of optimization choices, it takes a long time to run

TABLE 9. The results obtained by the RACETTC-NSGA-III algorithm of example 2.

No.	Satisfied transmission lines assignment	$(Re_{11}(\mathbf{B}), Re_{12}(\mathbf{B}), Re_{2_1}(\mathbf{B}), Re_{22}(\mathbf{B}), -C(\mathbf{B}))$
1	(1 10 15 19 2)	(0.903264 0.903264 0.866016 0.866016 723500)
2	(2 3 8 5 6)	(0.912576 0.903264 0.921984 0.912576 769000)
3	(2 3 8 6 7)	(0.903264 0.903264 0.912576 0.912576 760000)
4	(2 5 8 6 3)	(0.903264 0.912576 0.912576 0.921984 788000)
5	(2 6 8 3 5)	(0.912576 0.912576 0.912576 0.912576 764000)
6	(2 6 8 3 7)	(0.912576 0.903264 0.912576 0.903264 748000)
7	(2 6 8 5 3)	(0.912576 0.912576 0.912576 0.912576 785000)
8	(2 6 8 7 3)	(0.903264 0.912576 0.903264 0.912576 776000)
9	(3 3 1 3 3)	(0.91238 0.91238 0.91238 0.91238 859000)
10	(3 3 1 3 5)	(0.91238 0.91238 0.91238 0.91238 811000)
11	(3 3 1 5 3)	(0.91238 0.91238 0.91238 0.91238 832000)
12	(3 3 1 5 5)	(0.91238 0.91238 0.91238 0.91238 784000)
13	(3 5 1 3 3)	(0.91238 0.91238 0.91238 0.91238 823000)
14	(3 5 1 3 5)	(0.91238 0.91238 0.91238 0.91238 775000)
15	(3 5 1 5 3)	(0.91238 0.91238 0.91238 0.91238 796000)
16	(3 5 1 5 5)	(0.91238 0.91238 0.91238 0.91238 748000)
17	(3 6 2 5 7)	(0.902776 0.893564 0.893564 0.884446 716000)
18	(3 6 4 5 7)	(0.893172 0.884058 0.884058 0.875037 715000)
19	(3 12 4 11 6)	(0.856716 0.884058 0.847974 0.875037 653500)
20	(4 3 8 5 6)	(0.903168 0.893952 0.921984 0.912576 766000)
21	(4 3 8 6 7)	(0.893952 0.893952 0.912576 0.912576 757000)
22	(4 11 6 14 16)	(0.856128 0.707616 0.838292 0.692874 564500)
23	(4 11 15 18 8)	(0.884736 0.893952 0.866304 0.875328 640500)
24	(4 15 2 7 19)	(0.848256 0.875328 0.848256 0.875328 706000)
25	(4 18 3 5 11)	(0.874944 0.839232 0.874944 0.839232 643000)
26	(5 3 1 3 3)	(0.91238 0.91238 0.91238 0.91238 841000)
27	(5 3 1 3 5)	(0.91238 0.91238 0.91238 0.91238 793000)
28	(5 3 1 5 3)	(0.91238 0.91238 0.91238 0.91238 814000)
29	(5 3 1 5 5)	(0.91238 0.91238 0.91238 0.91238 766000)
30	(5 5 1 3 5)	(0.91238 0.91238 0.91238 0.91238 757000)
31	(5 5 1 5 3)	(0.91238 0.91238 0.91238 0.91238 778000)
32	(5 5 1 5 5)	(0.91238 0.91238 0.91238 0.91238 730000)
33	(5 5 2 5 5)	(0.902776 0.902776 0.902776 0.902776 726000)
34	(5 5 4 5 5)	(0.893172 0.893172 0.893172 0.893172 725000)
35	(5 6 1 7 9)	(0.87514 0.90307 0.86621 0.893855 657000)
36	(5 12 17 13 7)	(0.81928 0.810656 0.81092 0.802384 581500)
37	(5 18 17 14 16)	(0.845152 0.698544 0.827904 0.684288 548500)
38	(6 5 4 8 12)	(0.875037 0.875037 0.884058 0.884058 645000)
39	(6 7 17 2 16)	(0.827992 0.691416 0.802384 0.670032 574000)
40	(6 8 15 14 1)	(0.912576 0.903264 0.912576 0.903264 758500)
41	(6 9 1 13 11)	(0.875425 0.86621 0.875425 0.86621 642500)
42	(6 13 3 5 10)	(0.884058 0.838953 0.86583 0.821655 630000)
43	(6 16 4 3 12)	(0.884058 0.875037 0.738234 0.730701 612000)
44	(6 19 1 20 14)	(0.82935 0.90307 0.82935 0.90307 637500)
45	(7 9 17 11 19)	(0.777568 0.802384 0.802384 0.827992 587500)
46	(7 16 3 20 14)	(0.78678 0.856716 0.67797 0.738234 573500)
47	(8 2 1 11 16)	(0.86621 0.746415 0.86621 0.746415 618500)
48	(8 9 13 14 18)	(0.827022 0.810144 0.827022 0.810144 614500)
49	(9 7 14 18 8)	(0.782208 0.790356 0.758016 0.765912 585500)
50	(9 20 1 3 8)	(0.90307 0.893855 0.8379 0.82935 713000)
51	(10 16 15 13 6)	(0.84816 0.866016 0.73872 0.754272 596500)
52	(10 19 4 16 13)	(0.700569 0.821655 0.730701 0.856995 573500)
53	(11 6 2 10 5)	(0.821748 0.865928 0.847974 0.893564 652500)
54	(11 10 2 19 9)	(0.857092 0.857092 0.847974 0.847974 603500)
55	(12 8 2 16 13)	(0.738558 0.86621 0.738558 0.86621 598500)
56	(13 3 4 9 19)	(0.856995 0.856995 0.884058 0.884058 687000)
57	(13 11 1 20 14)	(0.81225 0.88445 0.8037 0.87514 634500)
58	(13 16 2 18 11)	(0.85728 0.83942 0.730944 0.715716 551500)
59	(13 18 3 11 5)	(0.83049 0.86583 0.839232 0.874944 643500)
60	(14 5 2 4 16)	(0.884352 0.746172 0.884352 0.746172 629500)
61	(14 10 4 1 5)	(0.884058 0.893172 0.838953 0.847602 701000)
62	(14 11 2 19 4)	(0.893564 0.884352 0.857092 0.848256 677500)
63	(14 12 2 4 17)	(0.884352 0.810656 0.875328 0.802384 667500)
64	(14 16 1 5 13)	(0.91238 0.88445 0.75411 0.731025 613000)
65	(15 6 2 14 17)	(0.884352 0.794112 0.893564 0.802384 700500)
66	(16 11 2 9 7)	(0.738558 0.715716 0.857092 0.830584 556000)
67	(17 12 4 6 10)	(0.793848 0.761112 0.875037 0.838953 593000)
68	(17 12 4 15 14)	(0.785664 0.802032 0.866016 0.884058 683500)
69	(17 18 1 11 9)	(0.78584 0.81092 0.85728 0.88464 631500)
70	(19 3 15 17 7)	(0.819456 0.875328 0.827904 0.884352 652500)
71	(19 7 17 16 8)	(0.691416 0.827992 0.670032 0.802384 545500)
72	(19 12 4 14 3)	(0.884058 0.884058 0.884058 0.884058 687500)
73	(19 14 2 4 11)	(0.875328 0.857092 0.884352 0.865928 641500)
74	(19 16 4 6 10)	(0.875037 0.838953 0.730701 0.700569 545000)
75	(19 20 15 14 3)	(0.912576 0.912576 0.84672 0.84672 729500)

the implicit enumeration, while the RACETTC-NSGA-III algorithm can obtain most of the optimization solutions in a relatively reasonable time. So the RACETTC-NSGA-III algorithm can be used widely in the cases that the number of optimization choices is large, which is very common for the multi-objective optimization problem is real environment.

VI. CONCLUSION

As the improvement of the network reliability can be achieved by the component assignments, how to assign the component to the arcs to guarantee the network reliability is an important problem worthy of studying. This paper considers the optimal transmission line assignment problem with maximal $s_u - t_v$ pair reliabilities and minimal cost under the transmission time constraints. The OTLAMRMC problem contains two sub-problems: the reliabilities and assignment cost evaluation under the transmission time constraint problem and the multi-objective transmission line assignment optimization problem. First, the RACETTC algorithm is proposed to evaluate the $s_u - t_v$ pair reliabilities and assignment cost under the transmission time constraints for a certain transmission line configuration. Then, the NSGA-III is adopted to search the optimal transmission line assignment based on the results obtained by the RACETTC algorithm. At last, combining the RACETTC algorithm and the NSGA-III together, the RACETTC-NSGA-III algorithm is proposed to solve the OTLAMRMC problem. The example simulations are given to illustrate the proposed algorithm. The example results show that the RACETTC-NSGA-III algorithm can provide efficient solutions in a reasonable time.

The aim of this paper is to maximize the source nodes and sink nodes pair reliabilities and to minimize the transmission line assignment cost comprehensively. Through the transmission line assignment optimization, the network reliability can be guaranteed. As the backup paths or the spare paths application is also a useful way to improve the network reliability, one of the future researches is to consider the backup paths or the spare paths combine with the network reliability related transmission line assignment problem.

REFERENCES

[1] L. Xing, C. Wang, and G. Levitin, "Competing failure analysis in non-repairable binary systems subject to functional dependence," *Proc. Inst. Mech. Eng., Part O, J. Risk Rel.*, vol. 226, no. 4, pp. 406–416, Apr. 2012.

[2] J. Sun, G. Zhu, G. Sun, D. Liao, Y. Li, A. K. Sangaiah, M. Ramachandran, and V. Chang, "A reliability-aware approach for resource efficient virtual network function deployment," *IEEE Access*, vol. 6, pp. 18238–18250, Mar. 2018.

[3] Z. Zhao, B. Xiao, N. Wang, X. Yan, and L. Ma, "Selective maintenance optimization for a multi-state system with degradation interaction," *IEEE Access*, vol. 7, pp. 99191–99206, Jul. 2019.

[4] S.-Y. Hsieh and C.-C. Lai, "A novel scheme for improving the reliability in smart grid neighborhood area networks," *IEEE Access*, vol. 7, pp. 129942–129954, Aug. 2019.

[5] M. Agarwal and V. K. Sharma, "Ant colony approach to constrained redundancy optimization in binary systems," *Appl. Math. Model.*, vol. 34, no. 4, pp. 992–1003, Apr. 2010.

[6] C. Jackson and A. Mosleh, "Downwards inference: Bayesian analysis of overlapping higher-level data sets of complex binary-state on-demand systems," *Proc. Inst. Mech. Eng., Part O, J. Risk Rel.*, vol. 226, no. 2, pp. 182–193, Jul. 2011.

[7] Y.-K. Lin and C.-T. Yeh, "Evaluation of optimal network reliability under components-assignments subject to a transmission budget," *IEEE Trans. Rel.*, vol. 59, no. 3, pp. 539–550, Sep. 2010.

[8] S. Zarezadeh, M. Asadi, and S. Eftekhar, "Signature-based information measures of multi-state networks," *Probab. Eng. Informational Sci.*, vol. 33, no. 3, pp. 438–459, Jul. 2019.

[9] J. Kostolny, E. Zaitseva, P. Rusnak, and M. Kvassay, "Application of multiple-valued logic in importance analysis of k-out-of-n multi-state systems," in *Proc. ISMVL*, Linz, Austria, May 2018, pp. 19–24.

[10] M. Garia, S. B. Singh, and A. Kumar, "Reliability analysis of multi-state complex system with multi-state weighted subsystems," *Int. J. Qual. Rel. Manag.*, vol. 36, no. 4, pp. 552–568, Feb. 2019.

[11] M. A. Alamoudy, "Reliability evaluation of communication flow network considering a multi-state system," *IOSR J. Electron. Commun. Eng.*, vol. 14, pp. 17–25, Mar. 2019.

[12] Y.-K. Lin, L. Fiondella, and P.-C. Chang, "Reliability of time-constrained multi-state network susceptible to correlated component faults," *Ann. Oper. Res.*, Oct. 2019, doi: 10.1007/s10479-019-03428-3.

[13] G. Qiu and Y. Gu, "Dynamic diagnosis approach of multi-state degradation system using hidden Markov model," in *Proc. Prognostics Syst. Health Manage. Conf. (PHM-Qingdao)*, Qingdao, China, Oct. 2019, pp. 1–8.

[14] C.-F. Huang, "System reliability for a multi-state distribution network with multiple terminals under stocks," *Ann. Oper. Res.*, Feb. 2020, doi: 10.1007/s10479-020-03546-3.

[15] Z. P. Xu, J. E. Ramirez-Marquez, Y. Liu, and T. F. Xiahou, "A new resilience-based component importance measure for multi-state networks," *Reliab. Eng. Sys. Saf.*, vol. 193, Jul. 2020, Art. no. 106591.

[16] D.-H. Huang, C.-F. Huang, and Y.-K. Lin, "A novel minimal cut-based algorithm to find all minimal capacity vectors for multi-state flow networks," *Eur. J. Oper. Res.*, vol. 282, no. 3, pp. 1107–1114, Oct. 2019.

[17] X. Song, X. Jia, and N. Chen, "Sensitivity analysis of multi-state social network system based on MDD method," *IEEE Access*, vol. 7, pp. 167714–167725, 2019.

[18] A. Pietrabissa and L. Ricciardi Celsi, "Discrete-time selfish routing converging to the Wardrop equilibrium," *IEEE Trans. Autom. Control*, vol. 64, no. 3, pp. 1288–1294, Mar. 2019.

[19] N. Safaei, R. Tavakkoli-Moghaddam, and F. Sassani, "A series–parallel redundant reliability system for cellular manufacturing design," *Proc. Inst. Mech. Eng., Part O, J. Risk Rel.*, vol. 223, no. 3, pp. 233–250, Sep. 2009.

[20] Y.-K. Lin, "Spare routing reliability for a stochastic flow network through two minimal paths under budget constraint," *IEEE Trans. Rel.*, vol. 59, no. 1, pp. 2–10, Mar. 2010.

[21] Y.-K. Lin, "Network reliability of a time-based multistate network under spare routing with p minimal paths," *IEEE Trans. Rel.*, vol. 60, no. 1, pp. 61–69, Mar. 2011.

[22] B. Todd and J. Doucette, "Fast efficient design of shared backup path protected networks using a multi-flow optimization model," *IEEE Trans. Rel.*, vol. 60, no. 4, pp. 788–800, Dec. 2011.

[23] Y. Zhang, Z. Fang, and Z. Xu, "An optimal design of multi-protocol label switching networks achieving reliability requirements," *Rel. Eng. Syst. Saf.*, vol. 182, pp. 133–141, Feb. 2019.

[24] Y. Zhang, Z. Xu, X. Wang, J. Lu, and Y. Sun, "Single minimal path based backup path for multi-state network," *Proc. Inst. Mech. Eng., Part O, J. Risk Rel.*, vol. 228, no. 2, pp. 152–165, Apr. 2014.

[25] M.-L. Chiang, H.-C. Hsieh, and C.-W. Wang, "Improving the fault-tolerance under software-defined network based on new sight of agreement protocol," *IEEE Access*, vol. 6, pp. 40898–40908, 2018.

[26] G. Levitin, M. Xie, and T. L. Zhang, "Reliability of fault-tolerant systems with parallel task processing," *Eur. J. Oper. Res.*, vol. 177, no. 1, pp. 420–430, Feb. 2007.

[27] S. Hosseini, K. Barker, and J. E. Ramirez-Marquez, "A review of definitions and measures of system resilience," *Rel. Eng. Syst. Saf.*, vol. 145, pp. 47–61, Jan. 2016.

[28] M. Ouyang, "A mathematical framework to optimize resilience of interdependent critical infrastructure systems under spatially localized attacks," *Eur. J. Oper. Res.*, vol. 262, no. 3, pp. 1072–1084, Nov. 2017.

[29] C.-C. Hsieh and M.-H. Lin, "Reliability-oriented multi-resource allocation in a stochastic-flow network," *Rel. Eng. Syst. Saf.*, vol. 81, no. 2, pp. 155–161, Aug. 2003.

[30] C.-C. Hsieh and Y.-T. Chen, "Reliable and economic resource allocation in an unreliable flow network," *Comput. Oper. Res.*, vol. 32, no. 3, pp. 613–628, Mar. 2005.

[31] C.-C. Hsieh and Y.-T. Chen, "Resource allocation decisions under various demands and cost requirements in an unreliable flow network," *Comput. Oper. Res.*, vol. 32, no. 11, pp. 2771–2784, Nov. 2005.
[32] C.-C. Hsieh and M.-H. Lin, "Simple algorithms for updating multi-resource allocations in an unreliable flow network," *Comput. Ind. Eng.*, vol. 50, nos. 1–2, pp. 120–129, May 2006.
[33] W. Xu, S. He, R. Song, and J. Li, "Reliability based assignment in stochastic-flow freight network," *Appl. Math. Comput.*, vol. 211, no. 1, pp. 85–94, May 2009.
[34] Y.-K. Lin and C.-T. Yeh, "Optimal resource assignment to maximize multistate network reliability for a computer network," *Comput. Oper. Res.*, vol. 37, no. 12, pp. 2229–2238, Dec. 2010.
[35] Y.-K. Lin and C.-T. Yeh, "Maximal network reliability with optimal transmission line assignment for stochastic electric power networks via genetic algorithms," *Appl. Soft Comput.*, vol. 11, no. 2, pp. 2714–2724, Mar. 2011.
[36] Y.-K. Lin and C.-T. Yeh, "Using minimal cuts to optimize network reliability for a stochastic computer network subject to assignment budget," *Comput. Oper. Res.*, vol. 38, no. 8, pp. 1175–1187, Aug. 2011.
[37] Y. Zhang, Z.-G. Xu, W.-H. Wang, J.-G. Lu, and Y.-X. Sun, "Optimal transmission lines assignment with maximal reliabilities in multi-source multi-sink multi-state computer network," *J. Central South Univ.*, vol. 20, no. 7, pp. 1868–1877, Jul. 2013.
[38] K. Deb and H. Jain, "An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints," *IEEE Trans. Evol. Comput.*, vol. 18, no. 4, pp. 577–601, Aug. 2014.
[39] J.-S. Lin, C.-C. Jane, and J. Yuan, "On reliability evaluation of a capacitated-flow network in terms of minimal pathsets," *Networks*, vol. 25, no. 3, pp. 131–138, May 1995.
[40] M. J. Zuo, Z. Tian, and H.-Z. Huang, "An efficient method for reliability evaluation of multistate networks given all minimal path vectors," *IIE Trans.*, vol. 39, no. 8, pp. 811–817, May 2007.
[41] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

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