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An Improved Phenomenological Model of the Planetary Gearbox Based on Meshing Vibration Characteristics

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ABSTRACT With the construction of periodic functions relative to the meshing frequency, rotating frequency, and the faulty frequency, etc., the phenomenological model provides a simple and effective way of representing the vibration signal of the planetary gearbox. However, due to that the meshing vibration is simplified, the traditional model can only reflect the modulation characteristic of the signal, but not the impact characteristic. Therefore, an improved phenomenological model is proposed, which can satisfy these two characteristics of the vibration signal at the same time. To consider the meshing vibration correctly, two key points should be determined: occurring moment of impact signals, and their relative amplitudes. In order to calculate the occurring time, this paper proposes modifying the reference point of the meshing phase and using the modified phases to represent it. Then two experiments are designed to verify the correctness of the modified phases and the occurring moment. Fortunately, these experimental results also provide a factual basis for determining the relative amplitudes of these impact signals. Subsequently, signal model of the gearbox in the healthy state is established based on the meshing vibration characteristics. The simulation results show that the new model can satisfy the modulation and impact characteristics. In addition, combined with the time-varying meshing stiffness of the gear pair, the paper theoretically analyzes the influence of fault type, location, and size on the impact frequency in a fault meshing period. Finally, the correctness of the new phenomenological model is verified by experiments.

INDEX TERMS Planetary gearbox, phenomenological model, meshing phase, meshing vibration, vibration impact.

I. INTRODUCTION

Due to the advantages of the large speed ratio, strong carrying capacity, and compact structure, planetary gearboxes have been widely used in the fields of aviation and automobiles [1]–[3]. Commonly, the ring gear is fixed to the base, and the sun gear and the carrier are used as input and output elements, respectively. The planet gears are equally spaced in the gearbox and meshing with the sun gear and the ring gear at the same time, and consequently forming the external gear pairs and internal gear pairs, respectively. Compared with the fixed-axis gearbox, the vibration response of a planetary gearbox is more complicated due to the special operation mode.

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In order to study the vibration mechanism of planetary gearboxes, various acceleration signal models of gearboxes have been established. The phenomenological model, lumped parameter model (LPM), and finite element model (FEM) are three commonly used models. Based on LPM and FEM, nonlinear factors in the gearbox, such as tooth cracks [4] and spalling [5], pressure angle and contact ratio [6], manufacturing errors [7] and backlash [8] have been investigated indepth. However, these two methods require plenty of research work in the early stage or later stage, which results in generating the gearbox acceleration signal less efficiently than the phenomenological model. The phenomenological model directly describes the meshing vibration signal as a function related to the meshing frequency, which greatly reduces the difficulty of establishing the signal model and improves efficiency. Based on the phenomenological model, He et al. [9]

verified the correctness of the order tracking method based on the discrete spectrum correction technology; Feng et al. [10], [11] discussed the iterative generalized demodulation method and local mean decomposition method. Besides, the phenomenological models can also study the effects of non-linear factors on vibration signals. For example, Lei et al. [12] established a phenomenological model which takes into account the angular shift of a planet gear. Subsequently, their staff derived the spectrum structure of the vibration signals under non-equal load distribution conditions [13]. Some scholars have compared the results of phenomenological models with LPM, such as Parra and Vicuña [14], Koch and Vicuña [5], and Feki et al. [15]. In terms of the efficiency of generating the vibration signal of the gearbox, the phenomenological model is the highest, but it also has obvious defects. For rotating machinery, such as bearings [16]-[21], fixed-axis gearboxes [22], 2[3], and planetary gearboxes [9], [24]–[26], periodic or quasi-periodic impacts in the vibration signal are the main basis for extracting characteristic frequencies. Besides, the experimental vibration signal of the planetary gearbox shows that the impact component in the signal is extremely rich. However, since the phenomenological model is constructed based on the trigonometric function, there is no impact component in the synthesized vibration signal. Therefore, it is a disadvantage for the phenomenological model to reflect the meshing impact characteristic.

For a gearbox in the perfect condition, the impact signal is mainly caused by the sudden change in the stiffness of the gear pair. The reason for this phenomenon is the gear tooth changing. During the meshing process, the meshing point passes through the double-tooth entering point and the double-tooth exiting point in order. Therefore, many scholars have conducted a lot of research on the meshing stiffness and established various stiffness models. Because of its advantage of simplicity, the analytical method (AM) is widely used [6], [27]–[29]. However, this method needs to estimate the average value of stiffness and ignores the time-varying characteristics of the meshing stiffness. The potential energy method and the finite element method (FEM) can effectively overcome these shortcomings. Based on the potential energy method, Liang et al. [30] calculated the stiffness of gear pairs of a planetary gearbox in the cracked condition. Chen and Shao [4] carried out a deep study on the crack width and depth of the influence on the stiffness. Cui et al. [31], Chaari et al. [32], and Lei [33] compared the potential energy method with FEM and found that the stiffness calculation results of the two methods are similar. It is worth noting that the reference points of the three stiffness models are the double-tooth entering points.

Since the planetary gearbox contains multiple gear pairs, the meshing phase of the gearbox is an unavoidable research point. Parker and Lin [34] derived the meshing phases of the gearbox through the geometric relationship of the internal and external gear pairs. Based on Parker's research, Walha *et al.* [27] discussed the phases of the gearbox under different operating modes. Later, they also calculated the phases when the sun gear is cracked [35]. Guo and Parker [29] and Li et al. [36] calculated the meshing phases of a compound planetary gearbox. Besides, according to the difference of the phases, Inalpolat and Kahraman [25] divided the gearboxes into five categories and compared the sideband and the amplitude characteristics of the vibration signals of them. He et al. [26] studied the mechanism of sideband modulation of the vibration signal when the planet gears are equally spaced in the gearbox. According to the theoretical research and experimental results of Vicuña [37], Hong et al. [38] and He et al. [26], when the teeth number of the ring gear (Zr) is a multiple of the number of the planet gears (N), the amplitude of meshing frequency is high; on the contrary, it is 0. The reason for this phenomenon is also associated with the meshing phase of the gearbox. Besides, the meshing phase has important application value. In [24], [39] it was used to locate and identify the faulty planetary gear. Thus, research on the meshing phase is helpful to reveal the vibration mechanism and fault diagnosis of gearboxes. Different from the stiffness model, the reference point of the meshing phase is the pitch point of the gear pair. More importantly, the meshing phase has not been experimentally verified so far.

To overcome the shortcomings of the above research status, the paper proposes to establish an improved phenomenological model that satisfies both modulation and impact characteristics. Considering that the sudden change of stiffness is the main cause of system impact [1], it is very important to accurately calculate the occurring moment of stiffness sudden change and relative amplitudes of the impacts caused by it.

Considering the periodicity of the gear pair and the phase relationship between them [34], this paper proposes to use phase and contact ratio to calculate the occurring moment. However, the reference points of the existing phase model in [34] and the stiffness model are set at the pitch point and the double-tooth entering point, respectively. This leads to the fact that phases in [34] cannot be directly used to express the occurring moment. So, this paper proposes to unify the reference points of both models to the double-tooth entering point, and designs two experiments to verify the modified phase. Fortunately, these experimental results also provide a factual basis for determining the relative amplitudes of the impact signals. It can be seen that the new model does not pay attention to the stiffness value of the gear pair.

The structure of the paper is as follows. Shortcomings of the existing phenomenological model are analyzed in Section 2. In Section 3, the initial reference points of the meshing stiffness and phase models are unified, and the modified meshing phases are verified by experiments. Based on the meshing vibration characteristics, a new phenomenological model of the vibration acceleration signal of the planetary gearbox is established in Section 4. In Section 5, the correctness of the model is verified experimentally from the perspective of the time and frequency domain. The conclusions are in Section 6.

II. THE PHENOMENOLOGICAL MODEL

The accurate establishment of the vibration signal model is of great significance for studying the vibration mechanism and fault diagnosis of gearboxes. According to the previous research, many models have been established to obtain the dynamic response of the planetary gearbox [1]–[3], such as the phenomenological model, LPM, and FEM. Different from the other two models, the phenomenological model directly establishes the meshing vibration signal as a function related to the meshing frequency [14], [37] and it has been widely used in fault diagnosis and identification.

In the perfect condition, the vibration signal x(t) of the gearbox can be expressed as the following equation or its modified form [15], [37], [40], [41]

$$\begin{aligned} x(t) &= \sum_{i=1}^{N} x_{i}^{r}(t) + \sum_{i=1}^{N} x_{i}^{s}(t) \\ &= \sum_{i=1}^{N} a_{i}^{r}(t - \gamma_{ri}/f_{c}) \cdot v_{i}^{r}(t - \gamma_{ri}/f_{c}) \\ &+ \sum_{i=1}^{N} a_{i}^{s}(t - \gamma_{ri}/f_{c}) \cdot v_{i}^{s}(t - (\gamma_{ri} + \gamma_{rs})/f_{c}) \end{aligned}$$
(1)

where $x_i^r(t)$ and $x_i^s(t)$ represent the vibration of the planet-ring gear pair and the sun-planet gear pair; $a_i^{r,s}$ (t) represents the amplitude modulation effect caused by the carrier, which is a function related to the carrier rotation frequency f_c ; $v_i^{r,s}$ (t) represents the meshing vibration signals, and it is a function of the meshing frequency f_m and its multiple $n \cdot f_m$; γ_{ri} represents the meshing phase of the *i*-th planet gear relative to the first planet gear; γ_{rs} represents the phase of the internal gear pair relative to the external gear pair.

According to the theoretical research in [26], [37], [38], it is known that whether the amplitude A_{fin} , corresponding to the meshing frequency f_m in the spectrum of the vibration signal, is 0 is related to the teeth number of ring gear Zr and the number of planet gears N.

$$\begin{cases} A_{fm} \neq 0, & \text{if } Zr/N \in Z \\ A_{fm} = 0, & otherwise \end{cases}$$
(2)

The theoretical analysis and experimental results of the planetary gearbox have proved the correctness of the above equation. For example, when N = 3, Zr is 72 in [14], 90 in [15], 96 in [38], 102 in [26], and 150 in [15]; when N = 4, Zr is 72 in [5] and 100 in [41] etc. In these cases, Zr is an integer multiple of N. Therefore, the amplitude A_{fm} of the meshing frequency is not 0. Figure 1 (a) and (b) show the simulation results of [14]. In contrast, when the Zr and N do not satisfy the relationship, the amplitude of the meshing frequency A_{fm} of the experimental signal is relatively low (disturbed by the background noise, normally the amplitude is not 0). For example, Zr = 101 and N = 3 in [26], Zr = 81 and N = 4 in [11], Zr = 99 and N = 4 in [38], and Zr = 62 and N = 3 in this paper. The simulation vibration signal and envelope order spectrum of the gearbox in this paper are



FIGURE 1. The vibration signals and frequency spectrum of gearboxes. (a) and (b) correspond to Zr = 72 and N = 3 in [14]. (c) and (d) correspond to Table 1 and n = 2. (e) and (f) correspond to Table 1 and n = 5. (g) and (h) correspond to Table 1 and n = 8.

TABLE 1. Physical parameters of the planetary gearbox.

Parameters	Sun gear	Planet gear	Ring gear
Number of Teeth	Zs=19	Zp=22	Zr=62
Module (mm)	4	4	4
Mass (Kg)	0.7	1.822	5.982
Face width(m)	0.055	0.042	0.053
Reduction ratio		4.263	

shown in Figure 1 (c) and (d), respectively. It shows that the amplitude of the meshing order of the vibration signal is 0 and it verifies the correctness of (2). The basic parameters of the planetary gearbox in this paper are shown in Table 1.

For the sidebands of the meshing order of these two types of gearbox, they are also different. In Figure 1 (b), the sidebands are the 69th order (Zr-3) and the 75th order (Zr + 3). In figure (d), the sidebands are the 60th order (Zr-2) and 66th order (Zr + 4). Both the 69th and 75th orders of the first gearbox and the 60th and 66th orders of the second gearbox are integral multiples of N (N = 3). Although expressions of the sidebands of these two types are different, the numerical law of them can be considered the same.

According to equation (1), the vibration signal x(t) is composed by meshing frequency f_m and its harmonics $n \cdot f_m$ $(n = 1, 2, 3 \cdots)$. When *n* is different, signal x(t) is also different. In Figure 1 (a) and (c), n = 2. As a result, it can be seen from Figure 1 (b) and (d) that the highest order of the envelope order spectrum of the vibration signal is $2 \cdot O_m$.

In order to study the influence of the harmonic component of the meshing frequency $n \cdot f_m$ $(n = 1, 2, 3 \cdots)$ on the vibration signal, we set n = 5 and 8 respectively to simulate the vibration signal of the gearbox in Table 1 again. Figure 1 (e) and (g) show the vibration signal curves and Figure 1 (f) and (h) are the envelope order spectra, respectively. It can be seen from Figure 1 (d), (f) and (h) that the highest order of them is $2 \cdot O_m$, $5 \cdot O_m$, and $8 \cdot O_m$ respectively. The simulated results show that the highest order *n* of the vibration signal is preset. When the parameter *n* is not set properly, it may have an inestimable influence on the subsequent signal processing methods.

Besides, the experimental results of planetary gearboxes show that the impact component in the acceleration response signal is extremely rich. However, the results based on the traditional phenomenological model can only reflect the modulation characteristic and cannot reflect the meshing impact characteristic. Therefore, the inability to simulate the impact characteristic of the vibration signal is a limitation of the traditional phenomenological model.

Although LPM and FEM can overcome the above shortcoming, their computational efficiency is reduced sequentially, and both need to prepare a large amount of preliminary or later basic research work. Therefore, this paper proposes to construct a novel phenomenological model, which considers the meshing impact characteristic without reducing the efficiency of generating the acceleration signal.

To establish the phenomenological model with meshing impact characteristic accurately, the occurring moment of the meshing impact and its amplitude are 2 key points.

However, the planetary gearbox contains multiple gear pairs, so it is difficult to directly express these moments. Due to the periodicity of the stiffness of the gear pair and the fact that multiple-planet gears are usually equally spaced in the gearbox, it provides the possibility to express these moments. Therefore, before calculating moments when the stiffness of multiple gears changes suddenly, it is necessary to study the stiffness and phase model of gear pairs first.

III. MESHING STIFFNESS AND PHASE

A. MODEL BEFORE UNIFYING REFERENCE POINT

Since a planetary gearbox contains multiple gear pairs, three meshing phases are employed to represent the relationship between them [34]–[36], [39]. Definitions of them are as follows:

1) γ_{si} , the phase of the *i*-th external gear pair relative to the first external gear pair;

2) γ_{ri} , the phase of the *i*-th internal gear pair relative to the first internal gear pair;

3) γ_{rs} , the phase between the *i*-th internal gear pair and the external gear pair.

According to the periodicity of the gear pair and the distribution of the planet gears, the first two phases can be easily obtained [34].

$$\gamma_{si} = (i-1) \cdot Zs / N \gamma_{ri} = -(i-1) \cdot Zr / N, \quad i = 1 \cdots N$$
(3)

The latter phase needs to be calculated based on the basic parameters of the entire gearbox. The meshing relationship



FIGURE 2. Meshing diagram of the planet gear, sun gear, and ring gear [34].

between the planet gear and the sun gear and the internal gear is shown in Figure 2.

Lines N_1N_2 and N_3N_4 represent the theoretical meshing lines of the external and internal gear pairs, and lines B_1E_1 and B_2E_2 are the actual meshing lines of them. Point $N_i(i =$ 1, 2, 3, 4) represents the tangent point of the meshing line with the base circle. Points B_i (i = 1,2) and $D_i(i = 1,2)$ represent the double-tooth entering points. Points C_i (i = 1, 2) and $E_i(i = 1, 2)$ indicate the double-tooth exiting points. Although they both are the entering points or the exiting points, they have different meanings. Take the external gear pair as an example. At the initial time, one sun gear tooth enters the meshing process at point B_1 and then exits at point E_1 . During this time, other gear teeth enter and exit the meshing process. Point C_1 indicates that the former gear tooth is out of meshing, and point D_1 indicates that the latter gear tooth is in meshing. So, two double-tooth meshing areas $(B_1C_1 \text{ and } D_1E_1)$ and one single tooth meshing area (C_1D_1) are formed. When meshing points are at B1, C_1 , D_1 , and E_1 , the stiffness changes abruptly, causing system impact. Point $P_i(i = 1, 2)$ represents the pitch point. Point Q_1 represents the projection point of the pitch point P_1 on the base circle of the planet gear. Point Q_2 represents a point that is separated from Q_1 by the base round tooth thickness t_p of the planet gear. Point Q_3 indicates the position on the meshing line of the internal gear pair and arc length Q_2Q_3 is an integer multiple of the base pitch.

Therefore, in order to calculate phase γ_{rs} , it is only necessary to calculate the length of P_2Q_3 . According to the

TABLE 2. Phases of the planetary gearbox.

geometric relationship in Figure 2

$$P_2 Q_3 = B_2 Q_3 - B_2 P_2 \tag{4}$$

$$B_2 P_2 = N_4 P_2 - N_4 B_2 = r_{br} \tan \alpha_2 - \sqrt{r_{ar}^2 - r_{br}^2}$$
(5)

$$B_2Q_3 = p - \text{mod}(Q_2B_2, p)$$
(6)
$$Q_2B_2 = P_1P_2 - B_2P_2 - t_p$$

$$= r_{bp}(\tan \alpha_1 + \tan \alpha_2 + (\pi - \alpha_1 - \alpha_2)) - (r_{br} \tan \alpha_2 - \sqrt{r_{ar}^2 - r_{br}^2}) - t_p$$
(7)

where α_1 and α_2 are pressure angles, and normally $\alpha_1 = \alpha_2$. r_{br} and r_{ar} represent the radius of the base circle and the addendum circle of the ring gear, respectively; r_{bp} represents the radius of the base circle of the planet gear; p is the base pitch and $p = \pi m \cdot \cos \alpha$; m is the gear modulus.

Moreover, the planet tooth thickness at the base circle is

$$t_p = m \cdot \cos \alpha \cdot (\pi/2 + zp \cdot inv(\alpha))$$

Then phase γ_{rs} can be derived as [34]

$$\gamma_{rs} = \frac{P_2 Q_3}{p} \tag{8}$$

Normally, N planet gears are equally spaced in the gearbox to evenly distribute the load. Since the planet gear meshes with the sun gear and the ring gear at the same time, three planet gears form 6 gear pairs. Phases of the planetary gearbox in Table 1 between the pitch point are shown in Table 2.

From the definitions above, the initial reference point is set at the pitch point $P_i(i = 1, 2)$ when establishing the meshing phase model. However, the reference point of the stiffness model is different.

Normally, the contact ratio ε of the gear pair is not an integer. So as the gear rotates, the number of teeth in contact changes periodically. The more the number of teeth in contact, the greater the stiffness of the gear pair. When the double-tooth entering point B_1 is used as the reference point of the stiffness model, the equation of the time-varying meshing stiffness k can be expressed as

$$k = \begin{cases} k_d & t \in ((i-1) \cdot T_m, (\varepsilon - 2 + i) \cdot T_m] \\ k_s & t \in ((\varepsilon - 2 + i) \cdot T_m, i \cdot T_m] \end{cases}$$
(9)

where T_m represents the meshing period of the gear pair, k_d and k_s are the stiffness of the double-tooth and signal-tooth meshing area, respectively.

The time-varying meshing stiffness of the external and internal gear pairs k_{spi} and k_{rpi} $(i = 1, 2, \dots, N)$ can be obtained by AM, the energy method, and the FEM. Considering the periodicity of them, only stiffness k_{sp1} and k_{rp1} formed by the first planet gear need to be calculated. Combined with the meshing phase, the time-varying stiffness of multiple gear pairs in the planetary gearbox can be obtained [34].

$$k_{spi} = k_{sp1}(t - \gamma_{si} \cdot T_m) \quad i = 1, 2, \dots, N$$
(10)



FIGURE 3. Meshing phases and curves of meshing stiffness. (a) to (c) The external gear pairs. (d) to (f) The internal gear pairs.

$$k_{rpi} = k_{rp1}(t - \gamma_{ri} \cdot T_m - \gamma_{rs} \cdot T_m) \quad i = 1, 2, \dots, N$$
(11)

Finally, stiffness curves of gear pairs of the planetary gearbox are shown in Figure 3, and all of them exhibit significant periodicity. The abscissa is two complete meshing periods. Two mutations in each stiffness curve are observed in one period, one rising edge and one falling edge. Green dots on the curves indicate the stiffness corresponding to the pitch point. However, the amount of stiffness changing is not obvious when the contact position is at the pitch point. Therefore, the phase with the pitch point as the reference point cannot be used to express the occurring moment of system impacts.

B. MODEL AFTER UNIFYING REFERENCE POINT

Based on the previous research, the reference points of the phase model and the stiffness model are the pitch point and the double-tooth entering point, respectively. Which makes it impossible to use the phase in [34] to represent the moment of stiffness sudden change.

In order to study the vibration mechanism of the planetary gearbox, the paper proposes to unify the reference points of these two models. Considering that the sudden change of the stiffness is the main factor causing the impact signals, and the double tooth entering point is the reference point of the stiffness model, the reference point of the phase model is changed from the pitch point $P_i(i = 1, 2)$ to the double-tooth entering point $B_i(i = 1, 2)$. Figure 4 shows the schematic diagram of meshing phases before and after.

Definitions of the new meshing phases are modified to

1) γ'_{si} , the phase of the *i*-th external gear pair relative to the first external gear pair between the double-tooth entering point;



FIGURE 4. Schematic diagram before and after phase modification.

2) γ'_{ri} , the phase of the *i*-th internal gear pair relative to the first internal gear pair between the double-tooth entering point;

3) γ'_{rs} , the phase of the *i*-th internal gear pair and the external gear pair between the double-tooth entering point.

Although the definitions have changed, it is a coincidence that the values of the first two phases have not changed. The modified phase γ'_{rs} can be obtained according to Figure 4 and derived from the following equation.

$$\gamma_{rs}' = \frac{B_2 B_1}{p} = \frac{B_2 P_2 - B_1 P_1 + P_2 P_1}{p} = \frac{B_2 P_2 - B_1 P_1}{p} + \gamma_{rs}$$

$$B_1 P_1 = B_1 N_2 - P_1 N_2 = \sqrt{r_{ap}^2 - r_{bp}^2 - r_{bp}} \tan \alpha_1$$
(12)

Phases of the planetary gearbox in this paper between the double-tooth entering points are shown in Table 3.

Then meshing stiffness of the gearbox after modifying the reference point of the phase model can be obtained by the following equations.

$$k_{spi} = k_{sp1}(t - \gamma'_{si} \cdot T_m)i = 1, 2, 3$$
(13)

$$k_{rpi} = k_{rp1}(t - \gamma'_{rs}T_m - \gamma'_{ri} \cdot T_m) \quad i = 1, 2, 3 \quad (14)$$

Figure 5 shows the modified meshing stiffness curves k_{spi} and $k_{rpi}(i = 1, 2, 3)$. Phase differences of gear pairs between their double-tooth entering points are represented by the modified phases $\gamma'_{si}, \gamma'_{ri}$, and γ'_{rs} . For the convenience of later study, in addition to phase γ'_{sr} , phase T_{rs} is also used to indicate the modified phase between the internal and external gear pairs, where

$$T_{rs} = 1 - \gamma_{rs}' = 0.1439 \tag{15}$$

Total stiffness of the gearbox changes 12 times in one meshing period due to the phase factor, and 12 impacts will be generated in one period. Due to the periodic characteristic of the stiffness curves, the vibration signal in the later periods can also be established based on these 12 impacts. These impacts do not proceed synchronously and they occur sequentially in time order. Taking the double-tooth entering point of the first external gear pair as time 0, time of the rising and falling edges of the time-varying stiffness in the *j*-th meshing period is shown in equation (16).

$$T_{up} = \begin{cases} j - \gamma'_{s1} - 1 \\ j - \gamma'_{s2} \\ j - \gamma'_{s3} \\ j - \gamma'_{rs} - \gamma'_{r1} \\ j - \gamma'_{rs} - \gamma'_{r2} \\ j - \gamma'_{rs} - \gamma'_{r3} \end{cases} \quad T_{down} = \begin{cases} j + \varepsilon_{sp} - \gamma'_{s1} - 1 \\ j + \varepsilon_{sp} - \gamma'_{s2} \\ j + \varepsilon_{sp} - \gamma'_{s3} \\ j + \varepsilon_{pr} - \gamma'_{rs} - \gamma'_{r1} \\ j + \varepsilon_{pr} - \gamma'_{rs} - \gamma'_{r2} \\ j + \varepsilon_{pr} - \gamma'_{rs} - \gamma'_{r3} \end{cases}$$
(16)



FIGURE 5. Modified meshing phases and curves of the meshing stiffness. (a) to (c) The external gear pairs, (d) to (f) The internal gear pairs.

 TABLE 3. Modified phases of the planetary gearbox.

γ'_{s1}	γ'_{s2}	γ′s3	γ'_{r1}	γ'_{r2}	γ′ _{r3}	γ'_{rs}
0	2/3	1/3	0	-1/3	-2/3	0.8561

where, ε_{sp} and ε_{pr} represent the contact ratios of the external and the internal gear pairs, respectively.

Moreover, Figure 5 also shows curves of the time-varying meshing stiffness when the external gear pair is in root crack or broken tooth condition, as shown by the red dashed line and the solid line in Figure 5 (a). Crack is a common failure and has a significant effect on the meshing stiffness [1]–[3]. As the fault level increases, the changing amount in stiffness increases. When the faulty system continues running, the cracked tooth may change to a broken one. However, curves in Figure 5 (a) show that the sudden change moment of the stiffness is consistent with that in the perfect condition. This indicates that some certain health conditions of the gear pair do not affect the modified phase, such as the root crack or broken fault. A similar phenomenon occurs to the planet gear and the ring gear.

C. PHASE VERIFICATION

No matter where the reference point of the phase model is, it is necessary to verify it experimentally. However, there is no literature so far to verify it experimentally. It can be seen from Figure 3 and Figure 5 that the stiffness at the pitch point is not special. So, it is difficult to prove the correctness of the phase by the experimental method when the pitch point is referenced. However, when the double-tooth entering point is used as the reference point, the modified phases between gear pairs can be experimentally verified.

In order to verify the correctness of the modified phase, two experiments are designed in this paper. The experimental



FIGURE 6. (a) Test of the planetary gearbox. (b) Sun gear in the perfect condition. (c) Sun gear in the fault condition.



FIGURE 7. Vibration acceleration curve of gearbox without fault.

platform and two sun gears with different health states are shown in Figure 6. The experimental test mainly includes a driving motor, a torque sensor, a planetary gearbox, a magnetic powder brake, and several acceleration sensors. Parameters of the gearbox are listed in Table 1. Sensors are mounted outside the gearbox to obtain the vibration signal in the radial direction. During the experiment, the driving speed of the motor is 60 r/min and the sampling rate is 81920 Hz. The purpose of setting low rotational speed and high sampling rate is to obtain a sufficiently detailed vibration impact information. Then the meshing period T_m of the gearbox is 0.0687s, and the fault period T_s is 0.4355s when the sun gear is broken.

The vibration acceleration signal of the planetary gearbox in the perfect condition is shown in Figure 7, and it shows an obvious impact characteristic. It is observed that three equally spaced impact signals appear in one meshing period in the partially enlarged view. This result verifies that the phase between the internal (or external) gear pairs is 1/3.

However, since the meshing periods of the internal and external pairs are the same, the source of these impacts cannot be determined. Considering that the sensor is directly mounted on the ring gear, the transient impact signals generated by the external gear pair are weakened a lot, so it is speculated that these impacts in the figure are generated by the internal pairs. In order to verify this conjecture, the paper carried out another experiment. During this situation, the perfect sun gear is replaced by a faulty one with a missing tooth. And the faulty sun-planet gear pair can generate fault impact signals with sufficient energy, as shown in Figure 8.

Faulty impact signals with T_s as the period and meshing impact signals with T_m as the period are clearly shown in Figure 8. There is no doubt that impacts with high amplitude are caused by the broken sun gear tooth. It is observed that there is a time interval T_t between the fault impact and a normal one, where $T_t = 0.0103s$. After converting it to



FIGURE 8. Vibration acceleration curve of the gearbox with sun gear fault.

phase, the interval $T'_{rs} = 0.145$. In Figure 5, the theoretical result of phase T_{rs} between the internal and the external gear pair is 0.144. The relative error between the theoretical and experimental results is 4.17%. Considering factors such as the speed fluctuation of the gear system, the tooth deformation, and the installation error, phase T'_{rs} is considered to be the same with phase T_{rs} . Therefore, it can be confirmed that the meshing impact in Figure 7 and Figure 8 is caused by the rising edge of the internal gear pair.

In summary, except for the modified phase γ'_{si} , the experimental results above have proved the correctness of the modified phases γ'_{ri} and γ'_{rs} . According to our current survey results, this is the first time that the meshing phase of the planetary gearbox is verified experimentally. Besides, the experimental results also prove that the impact amplitude of the internal gear pair is higher than that of the external pair; the impact amplitude caused by the rising edge is higher than the falling edge. The correctness of the modified phase provides strong experimental support for establishing a new phenomenological model.

IV. IMPROVED PHENOMENOLOGICAL MODEL

A. SINGLE PLANET GEAR

Sudden change in the stiffness of the gear pair (the rising and falling edges) is the main cause of the impact signal [1]. The vibration frequency and the attenuation coefficient of the impact are both related to the inherent property of the gearbox, and the transient impacts can be expressed by the following equation

$$x(t) = e^{(-C_r \cdot \mod(t, T_m))} \cdot \cos(2\pi \cdot \omega_0 \cdot \mod(t, T_m)) \quad (17)$$

where Cr and ω_0 represent the attenuation coefficient and the natural frequency of the gearbox, respectively. Which are available through the experimental and analytical methods. The fluctuation of these two parameters are ignored in this paper and takes Cr = 1000 and $\omega_0 = 1600$ Hz; mod (,) represents the remainder operation.

To facilitate understanding, the paper first establishes the vibration signal model considering only one planet gear firstly. The planet gear meshes with the sun gear and the ring gear simultaneously, forming two gear pairs, including an external gear pair and an internal gear pair. One gear pair will change gears twice in a meshing period. So, when only one planet gear is considered, the gearbox generates a



FIGURE 9. Gearbox diagram. (a) Gearbox with one planet gear. (b) Gearbox with three planet gears.

total of 4 transient impact signals in one meshing period. As the gear system rotates, the meshing stiffness of each gear pair periodically changes suddenly, and these 4 impact signals are periodically generated. Therefore, the vibration signal of the gearbox contains 4 types of transient signals: $x_{sp1_u}(t), x_{sp1_d}(t), x_{pr1_u}(t)$, and $x_{pr1_d}(t)$, which are respectively caused by the rising edges and falling edges of the external and internal gear pairs. Positions of the planet gear and the sensor at the initial moment are shown in Figure 9 (a). The rising edge of the external gear pair is set as the initial time 0 and at this time the modified phases $\gamma'_{s1} = 0$ and $\gamma'_{r1} = 0$. Then the 4 transient impact signals can be expressed as

$$x_{sp1_u}(t) = A \cdot (e^{(-C_r \cdot \mod(t,T_m))} \cdot \cos(2\pi \cdot \omega_0 \cdot \mod(t,T_m)))$$
(18)
$$x_{sp1_d}(t)$$

$$=B \cdot \begin{pmatrix} e^{(-C_r \cdot \mod(t - (\varepsilon_{sp} - 1) \cdot T_m, T_m))} \\ \cos(2\pi \cdot \omega_0 \cdot \mod(t - (\varepsilon_{sp} - 1) \cdot T_m, T_m)) \end{pmatrix}$$
(19)
$$X_{nr1} \quad u(t)$$

$$= C \cdot \begin{pmatrix} e^{(-C_r \cdot \operatorname{mod}(t - (1 - \gamma'_{rs}) \cdot T_m, T_m))} \\ \cos(2\pi \cdot \omega_0 \cdot \operatorname{mod}(t - (1 - \gamma'_{rs}) \cdot T_m, T_m)) \end{pmatrix}$$
(20)

$$= D \cdot \begin{pmatrix} e^{(-C_r \cdot \mod(t - (\varepsilon_{pr} - \gamma'_{rs}) \cdot T_m, T_m))} \\ \cos(2\pi \cdot \omega_0 \cdot \mod(t - (\varepsilon_{pr} - \gamma'_{rs}) \cdot T_m, T_m)) \end{pmatrix}$$
(21)

where A, B, C, and D represent the contributions or the amplitude coefficients of the impacts, respectively.

Due to the rotation of the carrier, the vibration signal collected by the sensor shows an obvious amplitude modulation phenomenon. Many scholars used different window functions to simulate it [1] and in this paper $W_{p1}(t) = (0.54 - 0.46 \cdot \cos(2\pi \cdot f_c \cdot t - \pi))$. Then the vibration signal of a gearbox containing only one planet gear can be expressed as

$$x_{p1}(t) = (x_{sp1_u}(t) + x_{sp1_d}(t) + x_{pr1_u}(t) + x_{pr1_d}(t))W_{p1}(t)$$
(22)

In order to accurately model the gearbox vibration signal, the amplitude coefficients A-D need to be further determined. According to the experimental result in Figure 8, it shows that: 1) the impact amplitude caused by the internal gear pair is greater than that of the external gear pair; 2) the impact



FIGURE 10. (a) Curves of meshing stiffness of external and internal gear pairs. (b) Impact signal in one meshing period. (c) Window function. (d) Vibration curve with one planet gear.

amplitude caused by the rising edge is greater than the falling edge. Then, coefficients in (18)-(21) can be taken as A = 0.8, B = 0.4, C = 1, and D = 0.6, respectively. Figure 10 (a) shows the stiffness curves of gear pairs. The stiffness of the system changes 4 times during one meshing period, and the corresponding impacts appeared in Figure 10 (b). Figure 10 (c) shows the window function curve of one revolution of the carrier. Figure 10 (d) shows the vibration response curve of the system. Obvious amplitude modulation appears in the vibration signal.

B. MULTIPLE PLANET GEARS

When the gearbox contains multiple planet gears, each planet gear will pass through the sensor in turn as the carrier rotates, as shown in Figure 9 (b). Since the planet gears are equally spaced in the gearbox, the vibration impact signals generated by the *i*-th planet gear can be obtained by the following equations.

$$x_{spi_u}(t) = x_{sp1_u}(t - (i - 1) \cdot Zs / N \cdot T_m)$$
(23)

$$x_{spi_d}(t) = x_{sp1_d}(t - (i - 1) \cdot Zs / N \cdot T_m)$$

$$(24)$$

$$x_{pri_u}(t) = x_{pr1_u}(t - (N + 1 - i) \cdot Zr / N \cdot T_m) \quad (25)$$

$$x_{pri_d}(t) = x_{pr1_d}(t - (N + 1 - i) \cdot Zr / N \cdot T_m) \quad (26)$$

Window functions are expressed as

$$W_{pi} = (0.54 - 0.46 \cdot \cos\left(2\pi f_c(t - (i - 1) \cdot Zr/N) - \pi\right))$$
(27)

The overall simulated vibration signal is

$$x(t) = \sum_{i=1}^{N} \left(x_{spi_u}(t) + x_{spi_d}(t) + x_{pri_u}(t) + x_{pri_d}(t) \right) \cdot W_{pi}(t)$$
(28)

Figure 11 (a) shows the vibration signals generated by the three planet gears in three colors. It can be found that



FIGURE 11. (a) Vibration signals generated by three planet gears in a meshing period. (b) Overall signal of the gearbox. (c) The frequency spectrum of the signal. (d) Local enlargement.

12 impact signals appear in sequence and corresponding to the sudden change moments in stiffness in Figure 5. In addition, at the initial time, since the distance between the first planet gear and the sensor is the closest, the vibration amplitude of the signal x_{p1} is the largest. Therefore, the vibration signal of the planetary gearbox in perfect condition in one meshing period can always be simulated using these 12 transient impact signals. For a planetary gearbox with *N* planet gears, the number of transient impact signals is 4N.

The overall signal of the gearbox is shown in Figure 11 (b) and it exhibits 3 amplitude fluctuations in one revolution of the carrier.

Figure 11 (c) shows the frequency spectrum of the simulated signal. The frequency is mainly concentrated in the vicinity of the multiplication of the meshing frequency $n \cdot f_m$ ($n = 1, 2, \dots$) and the natural frequency ω_0 of the system. Generally, the highest order in the frequency spectrum based on the traditional phenomenological model is preset, which is shown in Figure 1. However, the results based on the method proposed in the paper are not limited to this.

Figure 11 (d) shows that the amplitude at the meshing order O_m is 0, and the amplitudes of the 60-*th* and 63-*rd* orders are not 0. This result is consistent with the theoretical analysis in [26], [37], [38].

Thus, compared with the signal in Figure 1 base on the traditional method, the results based on the improved phenomenological model are closer to the real signal, which can reflect the modulation characteristic and the impact characteristics.

C. LOCAL FAULT CONDITION

When a gear tooth fails, normal gear teeth and the faulty tooth alternately participate in the meshing process. Then, two kinds of transient impact signals will appear in the vibration signal: normal impacts and faulty impacts.

1) TOOTH ROOT FAILURE

When a fault occurs at the root of a gear tooth, such as the crack and missing tooth conditions, it can be known from Figure 5 that the moment of the sudden change in the stiffness is consistent with the perfect condition. So the vibration signal model under these fault conditions can still be constituted by 12 transient impact signals. However, the amplitudes A to D of the impacts in (18) to (21) will change according to the faulty gear.

When the sun gear is faulty

В

$$A = \begin{cases} A_{fault} & 0 < \operatorname{mod}(t, Zs/3 \cdot T_m) < (\varepsilon_{pr} - \gamma_{rs}' - 1) \cdot T_m \\ A & others \end{cases}$$
(29)

$$=\begin{cases} B_{fault} & 0 < \operatorname{mod}(t - (\varepsilon_{sp} - 1)T_m, Zs/3 \cdot T_m) \\ & < (\varepsilon_{pr} - \gamma'_{rs} - 1)T_m \\ B & others \end{cases}$$
(30)

$$C = C_{fault} \tag{31}$$

$$D = D_{fault} \tag{32}$$

When the planet gear is faulty

$$A = \begin{cases} A_{fault} & 0 < \operatorname{mod}(t, Z_p \cdot T_m) < (\varepsilon_{pr} - \gamma'_{rs} - 1) \cdot T_m \\ A & others \end{cases}$$

$$B = \begin{cases} B_{fault} & 0 < \operatorname{mod}(t - (\varepsilon_{sp} - 1)T_m, Z_p \cdot T_m) \\ & < (\varepsilon_{pr} - \gamma'_{rs} - 1)T_m \\ B & others \end{cases}$$
(34)

$$C = \begin{cases} C_{fault} & 0 < \operatorname{mod}(t - (1 - \gamma'_{rs})T_m, Z_p \cdot T_m) \\ & < (\varepsilon_{pr} - \gamma'_{rs} - 1)T_m \\ C & others \end{cases}$$
(35)

$$D = \begin{cases} D_{fault} & 0 < \operatorname{mod}(t - (\varepsilon_{pr} - \gamma'_{rs}) \cdot T_m, Z_p T_m) \\ & < (\varepsilon_{pr} - \gamma'_{rs} - 1) \cdot T_m \\ D & others \end{cases}$$
(36)

When the ring gear is faulty

$$A = A_{fault} \tag{37}$$

$$B = B_{fault} \tag{38}$$

$$C = \begin{cases} C_{fault} & 0 < \operatorname{mod}(t - (1 - \gamma'_{rs})T_m, Zr/3 \cdot T_m) \\ & < (\varepsilon_{pr} - \gamma'_{rs} - 1)T_m \\ C & others \end{cases}$$
(39)

$$D = \begin{cases} D_{fault} & 0 < \operatorname{mod}(t - (\varepsilon_{pr} - \gamma'_{rs})T_m, Zr/3 \cdot T_m) \\ & < (\varepsilon_{pr} - \gamma'_{rs} - 1)T_m \\ D & others \end{cases}$$
(40)

where coefficient j_{fault} depends on the fault level, and $j_{fault} > j, j = A, B, C, D$.

To verify the correctness of the signal model of the gearbox in faulty conditions, this paper takes the sun gear with a broken tooth as an example. At this time, the coefficients Cand D of the transient impact signals generated by the planetring gear pair stay unchanged. The amplitude coefficients of



FIGURE 12. (a) Vibration signals generated by three planet gears. (b) The vibration signal of the gearbox. (c) The envelope order spectrum. (d) Local enlargement when the sun gear is faulty.

the transient impact signal generated by the faulty external gear pairs become large, and $A_{fault} = 5$ and $B_{fault} = 2$ are selected during the simulation.

Figure 12 (a) shows the vibration signals of the 3 planet gears when one sun gear tooth is missing. Figure 12 (b) shows the overall signal of the gearbox. Consistent with the perfect condition, the fault vibration curve also shows an amplitude modulation phenomenon. Transient impact signals are clearly shown in the figures, including both the fault impacts with large amplitude and the normal meshing impacts. Theoretically, the fault period $T_s = Zs \cdot T_m/N$ when the sun gear is faulty. However, due to the amplitude modulation of the carrier, some faulty impacts do not appear clearly. This results in the interval between fault impacts not being equal to T_s . Figure 12 (c) shows the envelope order spectrum of the vibration signal. Due to the existence of the fault, the frequency components become rich, but they are mainly concentrated at the meshing order and its harmonic frequency. Besides, fault frequency orders are shown in the low-frequency zone. Figure 12 (d) is a partially zoom view of Figure 12 (c) around the meshing order. It shows that the frequency components consistent with that of a healthy gearbox appear, such as $O_m + O_c$, O_m -2· O_c , etc., but also contain frequencies related to the failure, such as $O_m - O_s$, $O_m - 2 \cdot O_s + 2 \cdot O_c$, $O_m + O_s - 2 \cdot O_s + 2 \cdot O_s$ O_c , $O_m + O_s + 2 \cdot O_c$, etc. Where O_c is the rotation order of the carrier, and Os is the sun gear fault order. Moreover, the frequency with the largest amplitude around the meshing order O_m is still the 63-*rd*, shown in order $(O_m + O_c)$.

Therefore, for a planetary gearbox with N planet gears, the vibration signal in one meshing period can also be simulated by 4N transient impact signals when the fault occurs at the gear tooth root. However, amplitude coefficients of



FIGURE 13. Stiffness curves of gear pairs when the fault occurs near the pitch point. (a) Broken tooth condition. (b) Spalling tooth condition.

1

Meshing periods (Tm)

1.2

1.4

1.6

1.8

2

0.8

the transient impacts are related to the health of the gear pair.

2) NONE TOOTH ROOT FAILURE

0

0

0.2

0.4

0.6

When a gear fault does not occur at the root, such as a position near the pitch point, the sudden change situation in stiffness will become complicated, and more transient impact signals may occur. However, the idea of establishing the phenomenological model of the gearbox is still applicable. The following is a study of the broken or spalling failure near the pitch point of the sun gear tooth.

The stiffness curves of the faulty gear pairs of these two conditions are shown by the red dotted lines in Figure 13 (a) and (b), respectively. The green dashed lines in the figures indicate the locations of the sudden change in stiffness during a meshing period. Due to the gear fault, the mutation frequency in the stiffness curves is 1 or 2 more respectively compared with that in the perfect condition in Figure 5. As a result, the mutation frequency of the gear system in a faulty meshing period will be changed from 4N to 4N + 1 (Figure 5 (a)) or 4N + 2 (Figure 5 (b)) times.

$$x(t) = \sum_{i=1}^{N} \left(\underbrace{\underbrace{x_{spi_u}(t) + x_{spi_d}(t) + x_{pri_u}(t) + x_{pri_d}(t)}_{Meshing impacts}}_{fault impacts} \right) \cdot W_{pi}(t)$$

$$(41)$$

Although the number of transient impact signals becomes more, the phenomenological model proposed in the paper is still applicable. According to the location and size of the fault, the moment when the mutation occurs can be calculated. Subsequently, faulty impact signals $x_{spi_fault}(t)$ and $x_{pri_fault}(t)$ can be established. Finally, the overall vibration signal x(t)of the gearbox will change from equation (28) to (41).



FIGURE 14. Comparison of two kinds of tooth break faults. (a) In the time domain. (b) Envelope order spectrum. (c) Local enlargement at low frequency band. (d) Local enlargement around the meshing order. (e) Local enlargement in the time domain.

D. APPLICATION

The improved phenomenological model proposed in the paper can also be used to simulate the vibration signals of a gearbox with different gear tooth faulty level. To prove this advantage, this paper takes two sizes of broken tooth fault as examples. Tooth breaking at the pitch point and the root are case Broken 1 and case Broken 2, respectively. The difference between the vibration signals of them is compared in the time domain and the frequency domain, respectively.

Figure 14 shows the vibration curves in the time domain and envelope order spectra in the frequency domain. Except for the amplitude, the frequency difference between the two types is very small, shown in Figure 14 (b) - Figure (d). Thus, it is difficult to distinguish the two faults from the frequency domain. However, due to the different fault level of the broken tooth, the engagement time of the fault gear pair is also different. This can be seen from the partially enlarged view of the time domain curve in Figure 14 (e). So, the phase between the faulty impact and the normal meshing impact is determined by the fault level. In turn, this feature can be used to determine the fault level of the gear tooth. For example, when the actual vibration signal of the gearbox is collected, the fault position and fault level of the gear pair can be obtained according to the time difference (or phase) between the fault impact and the meshing impact.

Therefore, for a gearbox containing N planet gears, when a fault occurs, the transient impact signal generated by the faulty gear pair in one meshing period is related to the type and position of the fault. When the fault occurs at the root, the vibration signals in a meshing period can always be simulated by 4N transient impact signals. When the fault occurs at



FIGURE 15. Flow chart of the paper.

other positions, it can be simulated by 4N + 1 transient impact signals for the crack and broken tooth fault conditions and 4N + 2 transient impact signals for the spalling fault condition.

In summary, the improved phenomenological model proposed in the paper is not only applicable to the perfect condition but also suitable for single and compound fault conditions. This model has the advantages of high computational efficiency and the ability to highlight the impact characteristic. However, according to Figure 7, we know that only the meshing impact signals generated by the rising edge of the internal gear pair can be collected by the sensor when the gearbox is perfect, and the other shock signals are drowned by strong noise. Therefore, in order to effectively extract these 4*N* transient signals, it is necessary to use advanced signal processing methods to reduce the noise. But this is not the focus of the paper, so the content about signal processing will not be expanded in this paper.

E. FLOW CHART

The purpose of this paper is to establish an improved phenomenological model which satisfies the modulation and impact characteristics of the vibration signal of the planetary gearbox. Figure 15 shows the flow chart of this paper.

There are two key points to establish the model, A): the moment when the transient impact occurs; B): the relative amplitude of different impacts. Among them, the occurring moment of the transient impact is theoretically derived by the geometry relationship of the gear pair and verified by experiments. The relative amplitude of impact signals is also derived from these experimental results. Then, the vibration signal of the gearbox with one planet gear is generated first. Considering the periodicity and the phase difference of gear pairs, vibration signal with one planet gear is extended to



FIGURE 16. Flow chart for generating vibration signals of the planetary gearbox. (a) In perfect condition. (b) In unhealthy condition.



FIGURE 17. (a) Experimental acceleration signal. (b) Envelope order spectrum. (c) Local enlargement around the meshing order.

multiple planet gears and the signal model of the gearbox is established. Then, signal models when the gearbox is faulty are established and discussed. Finally, the correctness of the new model is verified by experiments.

Figure 16 shows the flow charts for generating vibration signals of the planetary gearbox in different healthy conditions.

V. EXPERIMENTAL VERIFICATION

The existing research results [1]–[3] show that the vibration signal of the planetary gearbox has the characteristics of impact and modulation. Generally, the impact characteristic is generated by meshing vibration and the modulation characteristic is caused by the rotation of the carrier. Therefore, in order to ensure the correctness of the new model, the paper needs to verify these two key points by experiments. Among them, the impact characteristic of the vibration signal has been proved by the phase experiments in the III Section of the paper, as shown in Figures 7 and 8. In order to verify the modulation characteristic, another experiment is carried out on the experimental platform when the sun gear tooth is broken. At this time, the input speed of the gearbox is set at 300 rpm and the fault period Ts of the sun gear is 0.087s. Then, the vibration signal of the gearbox is shown in Figure 17.

When the impacts caused by other interfering factors are not considered, impacts in Figure 17 (a) can be classified into the fault impacts with high amplitude and normal meshing impacts with low amplitude, respectively. However, some fault impacts do not appear or show, as indicated by the red dotted line in the figure, resulting in the fault impulses not appearing at equal intervals. The reason is due to the amplitude modulation caused by the rotation of the carrier and this is consistent with the results of the theoretical analysis in Figure 12.

The envelope order spectrum of the vibration signal is shown in Figure 17 (b). The main orders of the signal are the meshing order and its harmonics, the carrier rotation order, and fault frequency order. Figure 17 (c) is a partially enlarged view around the meshing order. The amplitude of the meshing order O_m is close to 0, and amplitudes of the 63rd order $(O_m + O_c)$ and the 60-th order $(O_m - 2O_c)$ are large. Besides, fault-related orders such as $O_m - O_s$, $O_m + O_s - O_c$, $O_m + O_s + 2 \cdot O_c$ are also clearly shown. The frequency components of the experimental results are consistent with the theoretical analysis results in Figure 12, which prove that the improved phenomenological model based on the meshing vibration characteristics is correct.

VI. CONCLUSION

The paper unifies the reference points of the stiffness model and the phase model, and obtains the modified meshing phases of gear pairs. Based on the meshing vibration of the gearbox, an improved phenomenological model of the vibration signal of the planetary gearbox is established. It overcomes the shortcomings of the traditional phenomenological model and makes the simulation results closer to the real situation. The paper has the following conclusions:

(1) The paper derives the modified meshing phase of the gear pair and proves the correctness of the modified meshing phase experimentally. This makes up for the lack of experimental proof of the meshing phase;

(2) In the phase experiment results shown in Figure 7, only the meshing impact signals caused by the rising edges of the internal gear pairs are picked up by the sensor. This shows 2 facts. A): the amplitude of the vibration signal generated by the internal gear pair is greater than that generated by the external gear pair; B): the amplitude caused by the rising edge of the gear pair is greater than that caused by the falling edge. These experimental phenomena lay the experimental and theoretical basis for revealing the complex vibration mechanism of the planetary gearbox; (3) The vibration signal of the gearbox can be regarded as a superposition of 4 kinds of transient impact signals generated in sequence by the meshing process. And the occurring moments of the transient impact signals can be expressed using the modified phase;

(4) Based on the meshing vibration characteristics, an improved phenomenological model of the planetary gearbox is proposed and established. This model can reflect not only the modulation characteristic of the vibration signal, but also its impact characteristic. Compared with the traditional phenomenological model, the novel one has both efficiency and accuracy.

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