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Fault Detection for 2-D Continuous-Discrete State-Delayed Systems in Finite Frequency Domains

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ABSTRACT This paper investigates the fault detection problem for two dimensional (2-D) continuous discrete state-delay Roesser systems in finite frequency domains. Two performance indexes H_∞ and H_- are used to measure the fault sensitivity and the disturbance robustness in finite frequency. Based on this, the fault detection problem is converted into a filtering problem by designing a filter to generate a residual signal. By the generalized KYP lemma, convex design conditions are obtained, which are expressed in terms of linear matrix inequalities (LMIs). An example is provided to demonstrate the feasibility and effectiveness of the proposed method.

INDEX TERMS Finite frequency, fault detection, 2-D continuous discrete systems.

I. INTRODUCTION

During the past decades, fault detection has attracted more attention and a lot of detection approaches have been presented. Wherein, one of the main method for fault detection technology researches is the model-based fault detection method [1], [2]. The main goal of fault detection is to distinguish faults from disturbances. It is common to construct a residual signal by designing fault detection observer or filter to minimize the influence of disturbances and maximize the influence of faults simultaneously [3], [4]. Then, compare the residual signal with a predefined threshold, if the residual exceeds the threshold, an alarm is generated. Moreover, in practice, fault usually emerge in the low frequency domain [5], e.g., actuator failures in flight control systems [6]. The generalized Kalman-Yakubivich-Popov (KYP) lemma [7], which establishes an equivalence between the finite frequency condition and LMIs, allows researchers to better tailor specific frequency and solve the fault detection problems. For instance, the fault detection problem in finite frequency for one-dimension (1-D) systems was studied in [2], [8].

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On the other hand, in recent years, a large attention has been paid to two-dimension (2-D) systems, which can be continuous-continuous, discrete-discrete or continuous-discrete settings. The Roesser state-space model [9] is one of the most representative one. Based on this model, a number of methodologies and techniques have been developed for analysis and synthesis of 2-D system [10]–[13]. Recently, based on the generalized KYP lemma for 2-D Roesser system model, the fault detection problem for 2-D systems has been reported in the literature [14]–[20]. By the generalized KYP lemma for 2-D discrete Roesser systems in [21], the fault detection observer and filter design is formulated as a multi-objective optimization problem in [14]–[16], respectively. In [17], the problem of fault detection observer design for 2-D continuous Roesser systems was studied. Similar problem for 2-D continuous-discrete Roesser systems was discussed in [20]. More recently, the fault detection observer design method have been extend to 2-D continuous nonlinear systems and 2-D T-S fuzzy systems [18], [19].

As we all know, time-delay phenomenon, which usually cause system instability, is widespread in the practical engineering. Therefore, the study of time-delay systems fault detection problem has important theoretical significance. However, the time-delay phenomenon was not considered in [20]. Recentlly, a generalized KYP lemma for 2-D continuous-discrete state-delay Roesser systems was given in [22]. Based on this work, in the paper, we focus on the fault detection filtering for 2-D continuous-discrete state-delay Roesser systems. This fact motivates the present work.

This paper discuss the fault detection problem for a class of 2-D continuous-discrete state-delay systems described by the Roesser model. Different from existing results [20], the state-delay is considered. Moreover, a fault detection filter is designed to satisfy a finite-frequency H_- index and a finite-frequency H_∞ index simultaneously. The remainder of the paper is organized as follows. The problem statement and preliminaries are presented in Section 2. Section 3 presents the main results of the paper, where a finite-frequency fault detection method is proposed for 2-D continuous-discrete state-delay Roessor systems. Section 4 gives an example to illustrate the effectiveness of the proposed method. Section 5 concluded this paper.

Notation: Throughout this paper, we use $\mathbb{R}^{m\times n}$, $\mathbb{C}^{m\times n}$, $\mathbb{H}^{m\times n}$, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{Z}^+$ to represent the $m \times n$ real matrix set, complex matrix set, Hermite matrix set, real numbers set, positive real numbers set and positive integers set, respectively. The superscript T, $*$ denote the real matrix transpose and the complex matrix transpose, respectively. $\prod_{l=0}^{L}$ denotes the Cartesian product of sets A_l. $\langle \mathbb{X}_i \rangle$ denotes the diagonal matrix with diagonal entries $\mathbb{X}_1, \ldots, \mathbb{X}_n$, where $\mathbb{X}_i$ could be numbers or matrices. The symbols $\sigma_{max}(\cdot)$ and $\sigma_{min}(\cdot)$ denote the spectral norm of a matrix. In addition, $He(A)$ indicates $A + A^T$. $\mathcal{U}^\perp$ denotes the orthogonal complement of $\mathcal{U}$. I is the identity matrix with appropriate dimension. $\varrho(G, \Pi)$ is a function defined as $\varrho(G, \Pi) := [G^* \ \ I]\Pi[G^* \ \ I]^*$.

II. PRELIMINARIES AND PROBLEM FORMULATION

In this paper, we consider the 2-D continuous-discrete state-delay systems with state-space equations

$$
\begin{aligned}
(\mathcal{E}) : \begin{bmatrix} \frac{\partial}{\partial t} x_h(t,k) \\ x_v(t,k+1) \end{bmatrix} &= A \begin{bmatrix} x_h(t,k) \\ x_v(t,k) \end{bmatrix} + A_\tau \begin{bmatrix} x_h(t-\tau_h,k) \\ x_v(t,k-\tau_v) \end{bmatrix} \\
&\quad + B_f f(t,k) + B_d d(t,k), \\
y(t,k) &= C \begin{bmatrix} x_h(t,k) \\ x_v(t,k) \end{bmatrix} + C_\tau \begin{bmatrix} x_h(t-\tau_h,k) \\ x_v(t,k-\tau_v) \end{bmatrix} \\
&\quad + D_f f(t,k) + D_d d(t,k).
\end{aligned}
$$

where $x_h(t,k) \in \mathbb{R}^{n_h}$ and $x_v(t,k) \in \mathbb{R}^{n_v}$ are the horizontal state and vertical state, respectively. $y(t,k)$, $f(t,k)$ and $d(t,k)$ are the external output, fault input and disturbance input vectors. The exogenous disturbance $d(t,k)$ is assumed energy-bounded in the paper. τ_l $(l = h, v)$ are the constant state delays of the system which satisfying $0 < \tau_l \leq \bar{\tau}_l$ and $\bar{\tau}_h \in \mathbb{R}^+$, τ_v, $\bar{\tau}_v \in \mathbb{Z}^+$. $\bar{\tau}_l$ are the upper bound of state-delays. In the following discussion, unless specifically noted otherwise, the subscript "l" represents either the subscript h or v. Matrices A, $A_\tau \in \mathbb{R}^{n\times n}$ $(n = n_h + n_v)$, $B_f \in \mathbb{R}^{n\times n_f}$, $B_d \in \mathbb{R}^{n\times n_d}$, C, $C_\tau \in \mathbb{R}^{n_y\times n}$, $D_f \in \mathbb{R}^{n_y\times n_f}$ and $D_d \in \mathbb{R}^{n_y\times n_d}$ are system matrices.

In the paper, we assume the frequency variables ω_{f_l} of the fault input $f(t,k)$ and the frequency variables ω_{d_l} of disturbance input $d(t,k)$ satisfy $\omega_{f_l} \in \mathbb{U}_{f_l}$ and $\omega_{d_l} \in \mathbb{U}_{d_l}$, respectively. The frequency ranges $\mathbb{U}_{f_l}$ and $\mathbb{U}_{d_l}$ have the following low frequency range:

$$
\mathbb{U}_{f_l} := \{\omega_{f_l} : |\omega_{f_l}| \leq \varpi_{f_l}\}, \quad \mathbb{U}_{d_l} := \{\omega_{d_l} : |\omega_{d_l}| \leq \varpi_{d_l}\}, \quad (1)
$$

where ϖ_{f_h}, $\varpi_{d_h} \in \mathbb{R}^+$, ϖ_{f_v}, $\varpi_{d_v} \in [0,\ \pi]$.

In this paper, we are interested in designing a fault detection filter in the following form:

$$
\begin{aligned}
(\hat{\mathcal{E}}) : \begin{bmatrix} \frac{\partial}{\partial t} \hat{x}_h(t,k) \\ \hat{x}_v(t,k+1) \end{bmatrix} &= \hat{A} \begin{bmatrix} \hat{x}_h(t,k) \\ \hat{x}_v(t,k) \end{bmatrix} + \hat{B} y(t,k), \\
\hat{y}(t,k) &= \hat{C} \begin{bmatrix} \hat{x}_h(t,k) \\ \hat{x}_v(t,k) \end{bmatrix},
\end{aligned}
$$

where $\hat{x}_h(t,k)$ and $\hat{x}_v(t,k)$ are the system state estimations. $\hat{y}(t,\mathbf{k})$ is the output estimation. Matrices $\hat{A}$, $\hat{B}$ and $\hat{C}$ are the filter matrices to be determined.

Let $r(t,k) = \hat{y}(t,k) - y(t,k)$ represents the residual signal and

$$
\begin{aligned}
\tilde{x}_l(t,k) &= x_l(t,k) - \hat{x}_l(t,k), \\
\tilde{x}_h(t-\tau_h,k) &= x_h(t-\tau_h,k) - \hat{x}_h(t-\tau_h,k), \\
\tilde{x}_v(t,k-\tau_v) &= x_v(t,k-\tau_v) - \hat{x}_v(t,k-\tau_v).
\end{aligned}
$$

Then, we obtain the filtering error dynamic system:

$$
\begin{aligned}
(\tilde{\mathcal{E}}) : \begin{bmatrix} \frac{\partial}{\partial t} \tilde{x}_h(t,k) \\ \tilde{x}_v(t,k+1) \end{bmatrix} &= \tilde{A} \begin{bmatrix} \tilde{x}_h(t,k) \\ \tilde{x}_v(t,k) \end{bmatrix} + \tilde{A}_\tau \begin{bmatrix} \tilde{x}_h(t-\tau_h,k) \\ \tilde{x}_v(t,k-\tau_v) \end{bmatrix} \\
&\quad + \tilde{B}_f f(t,k) + \tilde{B}_d d(t,k), \\
r(t,k) &= \tilde{C} \begin{bmatrix} \tilde{x}_h(t,k) \\ \tilde{x}_v(t,k) \end{bmatrix} + \tilde{C}_\tau \begin{bmatrix} \tilde{x}_h(t-\tau_h,k) \\ \tilde{x}_v(t,k-\tau_v) \end{bmatrix} \\
&\quad + \tilde{D}_f f(t,k) + \tilde{D}_d d(t,k),
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{A} &= \mathcal{T}^T \begin{bmatrix} A & 0 \\ \hat{B}C & \hat{A} \end{bmatrix} \mathcal{T}_n,\ \tilde{A}_\tau = \mathcal{T}^T \begin{bmatrix} A_\tau & 0 \\ \hat{B}C_\tau & 0 \end{bmatrix} \mathcal{T}, \\
\tilde{B}_f &= \mathcal{T}^T \begin{bmatrix} B_f \\ \hat{B}D_f \end{bmatrix},\ \tilde{B}_d = \mathcal{T}^T \begin{bmatrix} B_d \\ \hat{B}D_d \end{bmatrix},\ \tilde{D}_d = D_d, \\
\tilde{C} &= \begin{bmatrix} C & -\hat{C} \end{bmatrix} \mathcal{T},\ \tilde{C}_\tau = \begin{bmatrix} C_\tau & \mathbf{0} \end{bmatrix} \mathcal{T},\ \tilde{D}_f = D_f, \\
\mathcal{T} &:= \begin{bmatrix} I_{n_h} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{n_v} & \mathbf{0} \\ \mathbf{0} & I_{n_h} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I_{n_v} \end{bmatrix}. \qquad (2)
\end{aligned}
$$

Let $x(t,k) = [\tilde{x}_h^*(t,k) \ \ \tilde{x}_v^*(t,k)]^*$ and $\mathbf{X}$, $\mathbf{F}$, $\mathbf{D}$ are the *Laplace* $- \mathcal{Z}$ transform of $x(t,k)$, $f(t,k)$ and $d(t,k)$, respectively. Then, suppose $d(t,k) = 0$, by the *Laplace* $- \mathcal{Z}$ transform [12], the transfer function from the fault $f(t,k)$ to residual output $r(t,k)$ of the error dynamic system $(\tilde{\mathcal{E}})$ is given by

$$
G_{rf} = \left(\tilde{C} + \tilde{C}_\tau \Lambda_f\right)\left(\Omega_f - \tilde{A} - \tilde{A}_\tau \Lambda_f\right)^{-1} \tilde{B}_f + \tilde{D}_f, \quad (3)
$$

where, for $\omega_{f_h} \in \mathbb{R}$, $\omega_{f_v} \in [0,\ \pi]$,

$$
\Lambda_f := \langle e^{-\tau_l j\omega_{f_l}} I_{2n_l} \rangle,\ \Omega_f := diag\{j\omega_{f_h} I_{2n_h},\ e^{j\omega_{f_v}} I_{2n_v}\}.
$$

Similarly, let $f(t,k)=0$, the transfer function from the disturbance $d(t,k)$ to residual output $r(t,k)$ of the error dynamic system $(\tilde{\mathcal{E}})$ can be written as

$$G_{rd}=\left(\tilde{C}+\tilde{C}_\tau\Lambda_d\right)\left(\Omega_d-\tilde{A}-\tilde{A}_\tau\Lambda_d\right)^{-1}\tilde{B}_d+\tilde{D}_d, \quad (4)$$

where, for $\omega_{d_h}\in\mathbb{R}$, $\omega_{d_v}\in[0,\ \pi]$,

$$\Lambda_d:=\langle e^{-\tau_l j\omega_{d_l}}I_{2n_l}\rangle,\ \Omega_d:=diag\{j\omega_{d_h}I_{2n_h},\ e^{j\omega_{d_v}}I_{2n_v}\}.$$

To formulate the fault detection problem, the following two definitions are needed, which are similar to the definition of H_∞ index in [22] and H_- index in [14].

Definition 1: The H_∞ index of the transfer function G_{rd} is defined as

$$\|G_{rd}\|_\infty^{\mathbb{U}_d}:=\sup_{\omega_{d_l}\in\mathbb{U}_{d_l}}\sigma_{max}\left(G_{rd}\right), \quad (5)$$

where $\mathbb{U}_d:=\mathbb{U}_{d_h}\times\mathbb{U}_{d_v}$. $\mathbb{U}_{d_l}$ is defined in (1).

Definition 2: The H_- index of the transfer function G_{rf} is defined as

$$\|G_{rf}\|_-^{\mathbb{U}_f}:=\inf_{\omega_{f_l}\in\mathbb{U}_{f_l}}\sigma_{min}\left(G_{rf}\right), \quad (6)$$

where $\mathbb{U}_f:=\mathbb{U}_{f_h}\times\mathbb{U}_{f_v}$. $\mathbb{U}_{f_l}$ is defined in (1).

Then, the fault detection filter design problem to be addressed in this paper can be expressed as follows.

Given a system $(\mathcal{E})$ and $\gamma_d>0$, $\gamma_f>0$, the fault detection observer described by $(\hat{\mathcal{E}})$ is defined such that the error dynamic system $(\tilde{\mathcal{E}})$ satisfies the following conditions:

(i) the system $(\tilde{\mathcal{E}})$ is asymptotically stable;
(ii) $\|G_{rf}\|_-^{\mathbb{U}_f}>\gamma_f$, $\forall\,(\omega_{f_h},\omega_{f_v})\in\mathbb{U}_f$;
(iii) $\|G_{rd}\|_\infty^{\mathbb{U}_d}<\gamma_d$, $\forall\,(\omega_{d_h},\omega_{d_v})\in\mathbb{U}_d$.

The following lemmas will be used in the paper.

Lemma 1: (Finsler Lemma) Let $x\in\mathbb{R}^n$, $\mathcal{Q}\in\mathbb{R}^{n\times n}$, $\mathcal{U}\in\mathbb{R}^{n\times m}$. For all $\mathcal{U}^\perp$ such that $\mathcal{U}^\perp\mathcal{U}=0$, the following statements are equivalent.

1) $x^T\mathcal{Q}x<0, \forall\mathcal{U}^\perp x=0, x\neq 0$;
2) $\mathcal{U}^\perp\mathcal{Q}\mathcal{U}^{\perp T}\prec 0$;
3) $\exists\mu\in\mathbb{R}:\mathcal{Q}-\mu\mathcal{U}^T\mathcal{U}\prec 0$;
4) $\exists\mathcal{Y}\in\mathbb{R}^{m\times n}:\mathcal{Q}+\mathcal{U}\mathcal{Y}+\mathcal{Y}^T\mathcal{U}^T\prec 0$.

III. MAIN RESULTS

Before presenting the main results of this paper, we first present the following conclusions in this section. According to Theorem 1 and Theorem 2 in [22], for system $(\mathcal{E})$, let $f(t,k)=0$, we have the following Corollaries:

Corollary 1: Consider the system $(\mathcal{E})$ with $f(t,\mathbf{s})=0$. Given finite frequency ranges $\mathbb{U}_{f_d}$ and scalars $\bar{\tau}_h\in\mathbb{R}^+$, $\bar{\tau}_v\in\mathbb{Z}^+$, for any delays τ_l satisfying $0<\tau_l\leq\bar{\tau}_l$, if there exist matrixes $S=\langle S_l\rangle$, $Z=\langle Z_l\rangle$, $P=\langle P_l\rangle$, $Q=\langle Q_l\rangle\in\mathbb{H}^n$ and $Z\succ 0$, $Q\succ 0$, such that

$$F_1^T\Pi_1F_1+diag\{\Pi_2,\ \mathbf{0}\}+F_2^T\Theta F_2\prec 0, \quad (7)$$

where Θ is a given symmetric matrix with appropriate dimension. $\Pi_1:=\mathcal{T}\left\langle\Phi_l\otimes P_l+\Psi_l\otimes Q_l+\Phi_{\tau_l}\otimes Z_l\right\rangle\mathcal{T}^T$, $\Pi_2:=\Phi_s\otimes S+\mathcal{T}\left\langle\Phi_{zl}\otimes Z_l\right\rangle\mathcal{T}^T$ with $\mathcal{T}$ in (2) and

$$F_1:=\begin{bmatrix}A & A_\tau & B_d\\ I & \mathbf{0} & \mathbf{0}\end{bmatrix},\ F_2:=\begin{bmatrix}C & C_\tau & D_d\\ \mathbf{0} & \mathbf{0} & I\end{bmatrix},$$
$$\Phi_h:=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ \Phi_v:=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\ \Psi_h:=\begin{bmatrix}-1 & j\varpi_h^+\\ -j\varpi_h^+ & -\varpi_h\end{bmatrix},$$
$$\Psi_v:=\begin{bmatrix}0 & e^{j\varpi_v^+}\\ e^{-j\varpi_v^+} & -2\cos\varpi_v^-\end{bmatrix},\ \varpi_h^+:=\frac{\varpi_{h_2}+\varpi_{h_1}}{2},$$
$$\varpi_h=\varpi_{h_1}\varpi_{h_2},\ \varpi_v^+:=\frac{\varpi_{v_2}+\varpi_{v_1}}{2},\ \varpi_v^-:=\frac{\varpi_{v_2}-\varpi_{v_1}}{2},$$
$$0<\varpi_{v_2}-\varpi_{v_1}<2\pi,\ \Phi_{\tau_h}:=diag\{\bar{\tau}_h,0\},\ \Phi_s:=\Phi_v,$$
$$\Phi_{\tau_v}:=\begin{bmatrix}\bar{\tau}_v & -\bar{\tau}_v\\ -\bar{\tau}_v & \bar{\tau}_v\end{bmatrix},\ \Phi_{zl}:=\begin{bmatrix}-\bar{\tau}_l^{-1} & \bar{\tau}_l^{-1}\\ \bar{\tau}_l^{-1} & -\bar{\tau}_l^{-1}\end{bmatrix}.$$

Then the following finite frequency condition holds:

$$\varrho(G_{yd},\ \Theta)<0,\ \forall\,\omega_l\in\mathbb{U}_l, \quad (8)$$

where G_{yd} is the transfer function from $d(t,k)$ to $y(t,k)$.

Corollary 2: The system $(\mathcal{E})$ is asymptotically stable, if there exist matrices $X=\langle X_l\rangle$, $S_\tau=\langle S_{\tau_l}\rangle$, $Z_\tau=\langle Z_{\tau_l}\rangle\in\mathbb{H}^n$ with $X\succ 0$, $S_\tau\succ 0$ and $Z_\tau\succ 0$, such that

$$\begin{bmatrix}A & A_\tau\\ I & \mathbf{0}\end{bmatrix}^T\mathcal{X}_1\begin{bmatrix}A & A_\tau\\ I & \mathbf{0}\end{bmatrix}+\mathcal{X}_2\prec 0, \quad (9)$$

where $\mathcal{X}_1:=\mathcal{T}\langle\Phi_l\otimes X_l+\Phi_{\tau_l}\otimes Z_{\tau_l}\rangle\mathcal{T}^T$, $\mathcal{X}_2:=\Phi_s\otimes S_\tau+\mathcal{T}\langle\Phi_{zl}\otimes Z_{\tau_l}\rangle\mathcal{T}^T$ with $\mathcal{T}$, Φ_l, Φ_{τ_l}, Φ_s, Φ_{zl} are defined in (2) and (7).

A. DISTURBANCE ATTENUATION CONDITION

For system $(\tilde{\mathcal{E}})$, let $f(t,k)=0$, we have

$$\tilde{\mathcal{E}}_d:\begin{bmatrix}\frac{\partial}{\partial t}\tilde{x}_h(t,k)\\ \tilde{x}_v(t,k+1)\end{bmatrix}=\tilde{A}\begin{bmatrix}\tilde{x}_h(t,k)\\ \tilde{x}_v(t,k)\end{bmatrix}+\tilde{A}_\tau\begin{bmatrix}\tilde{x}_h(t-\tau_h,k)\\ \tilde{x}_v(t,k-\tau_v)\end{bmatrix}+\tilde{B}_dd(t,k),$$
$$r(t,k)=\tilde{C}\begin{bmatrix}\tilde{x}_h(t,k)\\ \tilde{x}_v(t,k)\end{bmatrix}+\tilde{C}_\tau\begin{bmatrix}\tilde{x}_h(t-\tau_h,k)\\ \tilde{x}_v(t,k-\tau_v)\end{bmatrix}+\tilde{D}_dd(t,k),$$

where $\tilde{A}$, $\tilde{A}_\tau$, $\tilde{B}_d$, $\tilde{C}$, $\tilde{C}_\tau$ and $\tilde{D}_d$ were given by (2).

Based on Corollary 1, filter design conditions satisfying disturbance attenuation performance (iii) are derived as follows.

Theorem 1: Consider the error dynamic system $(\tilde{\mathcal{E}}_d)$. Given finite frequency ranges $\mathbb{U}_{d_l}$, scalars $\alpha_i>0$ $(i=1,\ldots,6)$, $\gamma_d>0$ and $\bar{\tau}_h\in\mathbb{R}^+$, $\bar{\tau}_v\in\mathbb{Z}^+$, for any delays τ_l satisfying $0<\tau_l\leq\bar{\tau}_l$, the error dynamic system $(\tilde{\mathcal{E}}_d)$ satisfies specification (ii), if there exist matrices $\mathcal{A}$, $\mathcal{V}$, $Y_{d_i}\in\mathbb{R}^{n\times n}$, $W_{d_1}\in\mathbb{R}^{n\times n_r}$, $W_{d_2}\in\mathbb{R}^{n\times n_f}$, $\mathcal{B}\in\mathbb{R}^{n_y\times n}$, $\mathcal{C}\in\mathbb{R}^{n_r\times n}$, Z_{dl_2}, S_{dl_2}, P_{dl_2}, $Q_{dl_2}\in\mathbb{R}^{n\times n}$, and symmetric matrices Z_{dl_1}, Z_{dl_3}, S_{dl_1}, S_{dl_3}, P_{dl_1}, P_{dl_3}, Q_{dl_1}, $Q_{dl_3}\in\mathbb{R}^{n\times n}$ satisfying

$$\bar{Q}_d\succ 0,\ \bar{Z}_d\succ 0, \quad (10)$$
$$diag\{\bar{\Xi}_d,\Theta_d\}+He\left(\Sigma^d\right)\prec 0, \quad (11)$$

where $\Theta_d = diag\{I_{n_r}, -\gamma_d^2 I_{n_d}\}$ and

$$\begin{aligned}
\bar{\Xi}_d &:= diag\{\bar{\Pi}_d,\ \mathbf{0}\} + diag\{\mathbf{0},\ \bar{\Upsilon}_d\},\\
\bar{\Upsilon}_d &:= \sum_{l=h,v} \left(\Phi_s \otimes (\bar{S}_f J_l) + \Phi_{z_l} \otimes (\bar{Z}_f J_l)\right),\\
\bar{\Pi}_d &:= \sum_{l=h,v} \left(\Phi_l \otimes (\bar{P}_f J_l) + \Psi_l \otimes (\bar{Q}_f J_l) + \Phi_{\tau_l} \otimes (\bar{Z}_d J_l)\right),\\
J_h &:= diag\{I_{n_h},\ \mathbf{0}_{n_v},\ I_{n_h},\ \mathbf{0}_{n_v}\},\\
J_v &:= diag\{\mathbf{0}_{n_h},\ I_{n_v},\ \mathbf{0}_{n_h},\ I_{n_v}\},\\
\bar{P}_d &:= \begin{bmatrix} \langle P_{d_{l_1}}\rangle & \langle P_{d_{l_2}}\rangle \\ \star & \langle P_{d_{l_3}}\rangle \end{bmatrix},\ \bar{S}_d := \begin{bmatrix} \langle S_{d_{l_1}}\rangle & \langle S_{d_{l_2}}\rangle \\ \star & \langle S_{d_{l_3}}\rangle \end{bmatrix},\\
\bar{Q}_d &:= \begin{bmatrix} \langle Q_{d_{l_1}}\rangle & \langle Q_{d_{l_2}}\rangle \\ \star & \langle Q_{d_{l_3}}\rangle \end{bmatrix},\ \bar{Z}_d := \begin{bmatrix} \langle Z_{d_{l_1}}\rangle & \langle Z_{d_{l_2}}\rangle \\ \star & \langle Z_{d_{l_3}}\rangle \end{bmatrix},\\
\Sigma^d &= \begin{bmatrix}
-Y_{d_1} & \cdots & -Y_{d_6} & -W_{d_1} & -W_{d_2}\\
-\alpha_1\mathcal{V} & \cdots & -\alpha_6\mathcal{V} & \mathbf{0} & \mathbf{0}\\
\Sigma_{31}^d & \cdots & \Sigma_{36}^d & A^T W_{d_1} - C^T & A^T W_{d_2}\\
a_{d_1}\mathcal{A} & \cdots & a_{d_6}\mathcal{A} & -\mathcal{C}^T & \mathbf{0}\\
\Sigma_{51}^d & \cdots & \Sigma_{56}^d & A_\tau^T W_{d_1} - C_\tau^T & A_\tau^T W_{d_2}\\
\mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \cdots & \mathbf{0} & -I_{n_r} & \mathbf{0}\\
\Sigma_{81}^d & \cdots & \Sigma_{86}^d & B_d^T W_{d_1} - D_d^T & B^T W_{d_2}
\end{bmatrix},\\
\Sigma_{3i}^d &= A^T Y_{d_i} + \alpha_i C^T \mathcal{B},\ \Sigma_{5i}^d = A_\tau^T Y_{d_i} + \alpha_i C_\tau^T \mathcal{B},\\
\Sigma_{7i}^d &= B_d^T Y_{d_i} + \alpha_i D_d^T \mathcal{B},\ i = 1, \ldots, 6.
\end{aligned}$$

Proof: For the matrix variables $P_d = \langle P_{d_l}\rangle$, $0 \prec Q_d = \langle Q_{d_l}\rangle$, $S_d = \langle S_{d_l}\rangle$ and $0 \prec Z_d = \langle Z_{d_l}\rangle$ with appropriate dimensions, it is easy to verify

$$P_d = \mathcal{T}^T \bar{P}_d \mathcal{T},\ Q_d = \mathcal{T}^T \bar{Q}_d \mathcal{T},$$
$$S_d = \mathcal{T}^T \bar{S}_d \mathcal{T},\ Z_d = \mathcal{T}^T \bar{Z}_d \mathcal{T}.$$

Let $\Xi_d := diag\{\Pi_{d_1}, \mathbf{0}\} + diag\{\mathbf{0}, \Pi_{d_2}\}$, where

$$\Pi_{d_1} := \mathcal{T}\langle \Phi_l \otimes P_{d_l} + \Psi_l \otimes Q_{d_l} + \Phi_{\tau_l} \otimes Z_{f_l}\rangle \mathcal{T}^T,$$
$$\Pi_{d_2} := \Phi_s \otimes S_d + \mathcal{T}\langle \Phi_{z_l} \otimes Z_{d_l}\rangle \mathcal{T}^T.$$

Denote $\bar{\mathcal{T}} := diag\{\mathcal{T}, \mathcal{T}, \mathcal{T}\}$, it is easy to prove that $\Xi_d = \bar{\mathcal{T}}^T \bar{\Xi}_d \bar{\mathcal{T}}$. According to inequation (7), we have

$$F_{d_1}^T \bar{\mathcal{T}}^T \bar{\Xi}_f \bar{\mathcal{T}} F_{d_1} + F_{d_2}^T \Theta_d F_{d_2} \prec 0, \tag{12}$$

where

$$F_{d_1} = \begin{bmatrix} \tilde{A} & \tilde{A}_\tau & \tilde{B}_d \\ I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} \end{bmatrix},\ F_{d_2} = \begin{bmatrix} \tilde{C} & \tilde{C}_\tau & \tilde{D}_d \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix}.$$

Then, inequality (12) can be written as

$$\begin{bmatrix} \mathcal{M}_d \\ I_{4n+n_d} \end{bmatrix}^T \bar{\mathcal{T}}_1 diag\{\bar{\Xi}, \Theta_d\} \bar{\mathcal{T}}_1^T \begin{bmatrix} \mathcal{M}_d \\ I_{4n+n_d} \end{bmatrix} \prec 0, \tag{13}$$

where $\bar{\mathcal{T}}_1 = \mathcal{T}_d diag\{\bar{\mathcal{T}}^T,\ I_{n_r+n_d}\}$ and

$$\mathcal{M}_d = \begin{bmatrix} \tilde{A} & \tilde{A}_\tau & \tilde{B}_d \\ \tilde{C} & \tilde{C}_\tau & \tilde{D}_d \end{bmatrix},\ \mathcal{T}_d = \begin{bmatrix} I_{2n} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{n_r} & \mathbf{0} \\ \mathbf{0} & I_{4n} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I_{n_d} \end{bmatrix}.$$

Using Lemma 1, condition (13) is equivalent to the existence of a matrix $\mathcal{Y}$ such that

$$\bar{\mathcal{T}}_1 diag\{\bar{\Xi}, \Theta_d\} \bar{\mathcal{T}}_1^T + He\left(\begin{bmatrix} -I_{2n+n_r} \\ \mathcal{M}_d^T \end{bmatrix} \mathcal{Y}\right) \prec 0. \tag{14}$$

Let $\mathcal{Y} \in \mathbb{R}^{(2n+n_r)\times(6n+n_r+n_d)}$ be the following specific block form:

$$\mathcal{Y} = \bar{\mathcal{T}}_2^T \begin{bmatrix} Y_{d_1} & Y_{d_2} & W_1 & Y_{d_3} & \cdots & Y_{d_6} & W_2 \\ a_1\mathcal{V} & a_2\mathcal{V} & 0 & a_3\mathcal{V} & \cdots & a_6\mathcal{V} & 0 \\ 0 & 0 & I_{n_r} & 0 & \cdots & 0 & 0 \end{bmatrix} \bar{\mathcal{T}}_3, \tag{15}$$

where $\bar{\mathcal{T}}_2 = diag\{\mathcal{T}, I_{n_r}\}$, $\bar{\mathcal{T}}_3 = diag\{\mathcal{T}, I_{n_r}, \mathcal{T}, \mathcal{T}, I_{n_d}\}$ and $a_i \in \mathbb{R}^+$, $i = 1, \ldots, 6$.

Note that, $\bar{\mathcal{T}}_1^T \bar{\mathcal{T}}_1 = I_{6n+n_r+n_d}$, pre-and post-multiplying (14) by $\bar{\mathcal{T}}_1^T$ and $\bar{\mathcal{T}}_1$ and substituting (2) and (15) into (14) given (11). According to Corollary 1, specification (ii) holds if inequations (10) and (11) holds. The proof is completed.

B. FAULT SENSITIVITY CONDITION

For system $(\tilde{\mathcal{E}})$, let $d(t,k) = 0$, we have

$$\begin{aligned}
\tilde{\mathcal{E}}_f : \begin{bmatrix} \frac{\partial}{\partial t}\tilde{x}_h(t,k) \\ \tilde{x}_v(t,k+1) \end{bmatrix} &= \tilde{A} \begin{bmatrix} \tilde{x}_h(t,k) \\ \tilde{x}_v(t,k) \end{bmatrix} + \tilde{A}_\tau \begin{bmatrix} \tilde{x}_h(t-\tau_h,k) \\ \tilde{x}_v(t,k-\tau_v) \end{bmatrix}\\
&\quad + \tilde{B}_f f(t,k),\\
r(t,k) &= \tilde{C} \begin{bmatrix} \tilde{x}_h(t,k) \\ \tilde{x}_v(t,k) \end{bmatrix} + \tilde{C}_\tau \begin{bmatrix} \tilde{x}_h(t-\tau_h,k) \\ \tilde{x}_v(t,k-\tau_v) \end{bmatrix}\\
&\quad + \tilde{D}_f f(t,k),
\end{aligned}$$

where $\tilde{A}$, $\tilde{A}_\tau$, $\tilde{B}_f$, $\tilde{C}$, $\tilde{C}_\tau$ and $\tilde{D}_f$ were given by (2).

Based on Corollary 1, filter design conditions satisfying fault sensitivity performance (ii) are derived as follows.

Theorem 2: Consider the error dynamic system $(\tilde{\mathcal{E}})$ with $d(t,k) = 0$. Given finite frequency ranges U_{f_l}, scalars $\alpha_i > 0$ $(i = 1, \ldots, 6)$, $\gamma_f > 0$, $\beta > 0$, the vector η with $\|\eta\|_2 = \beta$ and $\bar{\tau}_h \in \mathbb{R}^+$, $\bar{\tau}_v \in \mathbb{Z}^+$, for any delays τ_l satisfying $0 < \tau_l \leq \bar{\tau}_l$, the error dynamic system $(\tilde{\mathcal{E}})$ satisfies specification (ii), if there exist matrices $\mathcal{A}$, $\mathcal{V}$, Y_{f_1}, Y_{f_2}, Y_{f_3}, Y_{f_4}, Y_{f_5}, $Y_{f_6} \in \mathbb{R}^{n\times n}$, $W_f \in \mathbb{R}^{n\times n_f}$, $\mathcal{B} \in \mathbb{R}^{n_y\times n}$, $\mathcal{C} \in \mathbb{R}^{n_r\times n}$, $Z_{f_{l_2}}$, $S_{f_{l_2}}$, $P_{f_{l_2}}$, $Q_{f_{l_2}}$ $\in \mathbb{R}^{n\times n}$, and symmetric matrices $Z_{f_{l_1}}$, $Z_{f_{l_3}}$, $S_{f_{l_1}}$, $S_{f_{l_3}}$, $P_{f_{l_1}}$, $P_{f_{l_3}}$, $Q_{f_{l_1}}$, $Q_{f_{l_3}} \in \mathbb{R}^{n\times n}$ $(l = h, v)$ satisfying

$$\bar{Q}_f \succ 0,\ \bar{Z}_f \succ 0, \tag{16}$$
$$\beta^2 - \mathcal{C}\eta < 0, \tag{17}$$
$$diag\{\bar{\Xi}_f, \mathbf{0}\} + diag\{\mathbf{0}, \bar{\Sigma}_f\} + He\left(\Sigma^f\right) \prec 0, \tag{18}$$

where J_l were given by (11) and

$$\begin{aligned}
\bar{\Xi}_f &:= diag\{\bar{\Pi}_f, \mathbf{0}\} + diag\{\mathbf{0}, \bar{\Upsilon}_f\},\\
\bar{\Upsilon}_f &:= \sum_{l=h,v} \left(\Phi_s \otimes (\bar{S}_f J_l) + \Phi_{z_l} \otimes (\bar{Z}_f J_l)\right),\\
\bar{\Pi}_f &:= \sum_{l=h,v} \left(\Phi_l \otimes (\bar{P}_f J_l) + \Psi_l \otimes (\bar{Q}_f J_l) + \Phi_{\tau_l} \otimes (\bar{Z}_f J_l)\right),\\
\bar{P}_f &:= \begin{bmatrix} \langle P_{f_{l_1}}\rangle & \langle P_{f_{l_2}}\rangle \\ \star & \langle P_{f_{l_3}}\rangle \end{bmatrix},\ \bar{S}_f := \begin{bmatrix} \langle S_{f_{l_1}}\rangle & \langle S_{f_{l_2}}\rangle \\ \star & \langle S_{f_{l_3}}\rangle \end{bmatrix},
\end{aligned}$$

$$\bar{Q}_f := \begin{bmatrix} \langle Q_{fi_1}\rangle & \langle Q_{fi_2}\rangle \\ \star & \langle Q_{fi_3}\rangle \end{bmatrix}, \ \bar{Z}_f := \begin{bmatrix} \langle Z_{fi_1}\rangle & \langle Z_{fi_2}\rangle \\ \star & \langle Z_{fi_3}\rangle \end{bmatrix},$$

$$\tilde{\Sigma}_f = \begin{bmatrix} -C^TC & C^T\mathcal{C} & -C^TC_\tau & 0 & -C^TD_f \\ \mathcal{C}^TC & -\beta^2 I & \mathcal{C}^TC_\tau & 0 & \mathcal{C}^TD_f \\ -C_\tau^TC & C_\tau^T\mathcal{C} & -C_\tau^TC_\tau & 0 & -C_\tau^TD_f \\ 0 & 0 & 0 & 0 & 0 \\ -D_f^TC & D_f^T\mathcal{C} & -D_f^TC_\tau & 0 & -D_f^TD_f+\gamma_f^2 I \end{bmatrix},$$

$$\Sigma^f = \begin{bmatrix} -Y_{f_1} & \cdots & -Y_{f_6} & -W_f \\ -\alpha_1\mathcal{V} & \cdots & -\alpha_6\mathcal{V} & \mathbf{0} \\ \Sigma_{31}^f & \cdots & \Sigma_{36}^f & A^TW_f \\ \alpha_1\mathcal{A} & \cdots & \alpha_6\mathcal{A} & \mathbf{0} \\ \Sigma_{51}^f & \cdots & \Sigma_{56}^f & A_\tau^TW_f \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \Sigma_{71}^f & \cdots & \Sigma_{76}^f & B_f^TW_f \end{bmatrix},$$

$$\Sigma_{3i}^f = A^TY_{f_i} + \alpha_iC^T\mathcal{B}, \ \Sigma_{5i}^f = A_\tau^TY_{f_i} + \alpha_iC_\tau^T\mathcal{B},$$
$$\Sigma_{7i}^f = B_f^TY_{f_i} + \alpha_iD_f^T\mathcal{B}, \ i = 1, \ldots, 6.$$

Proof: For the matrix variables $P_f = \langle P_{fl}\rangle$, $0 \prec Q_f = \langle Q_{fl}\rangle$, $S_f = \langle S_{fl}\rangle$ and $0 \prec Z_f = \langle Z_{fl}\rangle$ with appropriate dimensions, it is easy to verify

$$P_f = \mathcal{T}^T\bar{P}_f\mathcal{T}, \ Q_f = \mathcal{T}^T\bar{Q}_f\mathcal{T},$$
$$S_f = \mathcal{T}^T\bar{S}_f\mathcal{T}, \ Z_f = \mathcal{T}^T\bar{Z}_f\mathcal{T}.$$

Let $\Xi_f := diag\{\Pi_{f_1}, \mathbf{0}\} + diag\{\mathbf{0}, \Pi_{f_2}\}$, where

$$\Pi_{f_1} := \mathcal{T}\langle\Phi_l\otimes P_{fl} + \Psi_l\otimes Q_{fl} + \Phi_{\tau_l}\otimes Z_{fl}\rangle\mathcal{T}^T,$$
$$\Pi_{f_2} := \Phi_s\otimes S_f + \mathcal{T}\langle\Phi_{zl}\otimes Z_{fl}\rangle\mathcal{T}^T.$$

It is easy to prove that $\Xi_f = \bar{\mathcal{T}}^T\bar{\Xi}_f\bar{\mathcal{T}}$. According to inequation (7), we have

$$F_{f_1}^T\Big(diag\{\bar{\mathcal{T}}^T\bar{\Xi}_f\bar{\mathcal{T}}, \mathbf{0}\} + F_{f_2}^T\Theta_fF_{f_2}\Big)F_{f_1} \prec 0, \quad (19)$$

where $\Theta_f = diag\{-I_{n_r}, \ \gamma_f^2 I_{n_f}\}$ and

$$F_{f_1} = \begin{bmatrix} \tilde{A} & \tilde{A}_\tau & \tilde{B}_f \\ I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix}, \ F_{f_2} = \begin{bmatrix} \mathbf{0} & \tilde{C} & \tilde{C}_\tau & \tilde{D}_f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix}.$$

Define $\mathcal{M}_f := [\tilde{A} \ \tilde{A}_\tau \ \tilde{B}_f]$, $\mathcal{U}^\perp := \begin{bmatrix} \mathcal{M}_f^T & I \end{bmatrix}$ and

$$\mathcal{Q} := diag\{\bar{\mathcal{T}}^T\bar{\Xi}_f\bar{\mathcal{T}}, \mathbf{0}\} + F_{f_2}^T\Theta_fF_{f_2}$$
$$= \tilde{\mathcal{T}}^T\big(diag\{\bar{\Xi}_f, \mathbf{0}\} + diag\{\mathbf{0}, \bar{\Sigma}_f\}\big)\tilde{\mathcal{T}}$$

where $\tilde{\mathcal{T}} = diag\{\bar{\mathcal{T}}, I_{n_f}\}$ and

$$\bar{\Sigma}_f = \begin{bmatrix} -C^TC & C^T\hat{C} & -C^TC_\tau & 0 & -C^TD_f \\ \hat{C}^TC & -\hat{C}^T\hat{C} & \hat{C}^TC_\tau & 0 & \hat{C}^TD_f \\ -C_\tau^TC & C_\tau^T\hat{C} & -C_\tau^TC_\tau & 0 & -C_\tau^TD_f \\ 0 & 0 & 0 & 0 & 0 \\ -D_f^TC & D_f^T\hat{C} & -D_f^TC_\tau & 0 & -D_f^TD_f+\gamma_f^2 I_{n_f} \end{bmatrix}.$$

Using Lemma 1, condition (19) is equivalent to the existence of a matrix $\mathcal{Y}$ such that

$$\mathcal{Q} + He\left(\begin{bmatrix} -I_{2n} \\ \mathcal{M}_f^T \end{bmatrix}\mathcal{Y}\right) \prec 0. \quad (20)$$

Let $\mathcal{Y} \in \mathbb{R}^{(2n)\times(6n+n_f)}$ be the following specific block form:

$$\mathcal{Y} = \mathcal{T}^T\begin{bmatrix} Y_{f_1} & \cdots & Y_{f_6} & W_f \\ \alpha_1\mathcal{V} & \cdots & \alpha_6\mathcal{V} & 0 \end{bmatrix}\tilde{\mathcal{T}}. \quad (21)$$

Let $\mathcal{A} := \hat{A}^T\mathcal{V}$, $\mathcal{B} := \hat{B}^T\mathcal{V}$, by substituting (2) and (21) into (20), we have

$$\mathcal{Q} + \tilde{\mathcal{T}}^THe(\Sigma^f)\tilde{\mathcal{T}} \prec 0. \quad (22)$$

Note that $\tilde{\mathcal{T}}\tilde{\mathcal{T}}^T = I_{6n+n_f}$. Pre-and post-multiplying (29) by $\tilde{\mathcal{T}}$ gives

$$diag\{\bar{\Xi}_f, \mathbf{0}\} + diag\{\mathbf{0}, \bar{\Sigma}_f\} + He(\Sigma^f) \prec 0. \quad (23)$$

Let $\mathcal{C} := \hat{C}^T$, for given $\beta > 0$ and vector η with $\|\eta\|_2 = \beta$, by inequation (17), we have $\mathcal{C}^T\mathcal{C} < \beta^2 I$. Thus, inequation (23) holds if inequation (18) holds. According to Corollary 1, specification (ii) holds. The proof is completed.

C. STABILITY CONDITION

Based on Corollary 2, we have the following asymptotically stable conditions.

Theorem 3: Consider the error dynamic system $(\tilde{\mathcal{E}})$. Given scalars $\alpha_i > 0$ $(i = 1, \ldots, 6)$, $\bar{\tau}_h \in \mathbb{R}^+$, $\bar{\tau}_v \in \mathbb{Z}^+$, for any delays τ_l satisfying $0 < \tau_l \leq \bar{\tau}_l$, the error dynamic system $(\tilde{\mathcal{E}})$ is asymptotically stable, if there exist matrices $\mathcal{A}$, $\mathcal{V}$, $Y_{\tau_i} \in \mathbb{R}^{n\times n}$, $Z_{\tau_{l_2}}$, $S_{\tau_{l_2}}$, $X_{l_2} \in \mathbb{R}^{n\times n}$, $\mathcal{B} \in \mathbb{R}^{n_y\times n}$ and symmetric matrices $Z_{\tau_{l_1}}$, $Z_{\tau_{l_3}}$, $S_{\tau_{l_1}}$, $S_{\tau_{l_3}}$, X_{l_1}, $X_{l_3} \in \mathbb{R}^{n\times n}$ satisfying

$$\bar{X} \succ 0, \ \bar{S}_\tau \succ 0, \ \bar{Z}_\tau \succ 0, \quad (24)$$
$$\bar{\Xi}_\tau + He(\Sigma^\tau) \prec 0, \quad (25)$$

where J_l were given by (11) and

$$\bar{\Xi}_\tau := diag\{\bar{\Pi}_\tau, \mathbf{0}\} + diag\{\mathbf{0}, \bar{\Upsilon}_\tau\},$$
$$\bar{\Upsilon}_\tau := \sum_{l=h,v}\left(\Phi_s\otimes(\bar{S}_\tau J_l) + \Phi_{zl}\otimes(\bar{Z}_\tau J_l)\right),$$
$$\bar{\Pi}_\tau := \sum_{l=h,v}\left(\Phi_l\otimes(\bar{X}J_l) + \Phi_{\tau_l}\otimes(\bar{Z}_\tau J_l)\right),$$
$$\bar{X} := \begin{bmatrix} \langle X_{l_1}\rangle & \langle X_{l_2}\rangle \\ \star & \langle X_{l_3}\rangle \end{bmatrix}, \ \bar{S}_\tau := \begin{bmatrix} \langle S_{\tau_{l_1}}\rangle & \langle S_{\tau_{l_2}}\rangle \\ \star & \langle S_{\tau_{l_3}}\rangle \end{bmatrix},$$

$$\Sigma^\tau = \begin{bmatrix} -Y_{\tau_1} & \cdots & -Y_{\tau_6} \\ -\alpha_1\mathcal{V} & \cdots & -\alpha_6\mathcal{V} \\ \Sigma_{31}^\tau & \cdots & \Sigma_{36}^\tau \\ \alpha_1\mathcal{A} & \cdots & \alpha_6\mathcal{A} \\ \Sigma_{51}^\tau & \cdots & \Sigma_{56}^\tau \\ \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}, \quad \bar{Z}_\tau := \begin{bmatrix} \langle Z_{\tau_{l_1}}\rangle & \langle Z_{\tau_{l_2}}\rangle \\ \star & \langle Z_{\tau_{l_3}}\rangle \end{bmatrix},$$
$$\Sigma_{3i}^\tau = A^TY_{\tau_i} + \alpha_iC^T\mathcal{B}, \quad \Sigma_{5i}^d = A_\tau^TY_{d_i} + \alpha_iC_\tau^T\mathcal{B}, \quad i = 1, \ldots, 6.$$

Proof: For the matrix variables $0 \prec X = \langle X_l\rangle$, $S_\tau = \langle S_{\tau_l}\rangle$ and $0 \prec Z_\tau = \langle Z_{\tau_l}\rangle$ with appropriate dimensions, it is easy to verify

$$X = \mathcal{T}^T\bar{X}\mathcal{T}, \ S_\tau = \mathcal{T}^T\bar{S}_\tau\mathcal{T}, \ Z_\tau = \mathcal{T}^T\bar{Z}_\tau\mathcal{T}.$$

Let $\Xi_\tau := diag\{\Pi_{\tau_1}, \mathbf{0}\} + diag\{\mathbf{0}, \Pi_{\tau_2}\}$, where

$$\Pi_{\tau_1} := \mathcal{T}\langle \Phi_l \otimes X_l + \Phi_{\tau_l} \otimes Z_{\tau_l}\rangle \mathcal{T}^T,$$
$$\Pi_{\tau_2} := \Phi_s \otimes S_\tau + \mathcal{T}\langle \Phi_{z_l} \otimes Z_{\tau_l}\rangle \mathcal{T}^T.$$

It is easy to prove that $\Xi_\tau = \bar{\mathcal{T}}^T \bar{\Xi}_\tau \bar{\mathcal{T}}$. According to inequation (9), we have

$$\begin{bmatrix} \tilde{A} & \tilde{A}_\tau \\ I & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix}^T \bar{\mathcal{T}}^T \bar{\Xi}_\tau \bar{\mathcal{T}} \begin{bmatrix} \tilde{A} & \tilde{A}_\tau \\ I & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix} \prec 0. \tag{26}$$

Define $\mathcal{M}_\tau := \begin{bmatrix} \tilde{A} \ \tilde{A}_\tau \end{bmatrix}$, $\mathcal{U}^\perp := \begin{bmatrix} \mathcal{M}_\tau^T \ I \end{bmatrix}$ and $\mathcal{Q} := \bar{\mathcal{T}}^T \bar{\Xi}_\tau \bar{\mathcal{T}}$. Using Lemma 1, condition (26) is equivalent to the existence of a matrix $\mathcal{Y}$ such that

$$\mathcal{Q} + He\left(\begin{bmatrix} -I_{2n} \\ \mathcal{M}_\tau^T \end{bmatrix} \mathcal{Y}\right) \prec 0. \tag{27}$$

Let $\mathcal{Y} \in \mathbb{R}^{(2n)\times(6n)}$ be the following specific block form:

$$\mathcal{Y} = \mathcal{T}^T \begin{bmatrix} Y_{f_1} & \cdots & Y_{f_6} \\ \alpha_1 \mathcal{V} & \cdots & \alpha_6 \mathcal{V} \end{bmatrix} \bar{\mathcal{T}}. \tag{28}$$

Let $\mathcal{A} := \hat{A}^T \mathcal{V}$, $\mathcal{B} := \hat{B}^T \mathcal{V}$, by substituting (2) and (28) into (27), we have

$$\mathcal{Q} + \bar{\mathcal{T}}^T He(\Sigma^\tau) \bar{\mathcal{T}} \prec 0. \tag{29}$$

Note that $\bar{\mathcal{T}}\bar{\mathcal{T}}^T = I_{6n}$. Pre-and post-multiplying (29) by $\bar{\mathcal{T}}$ gives (25). Thus, inequation (26) holds if inequations (24) and (25) holds. According to Corollary 2, the error dynamic system $(\tilde{\mathcal{E}})$ is asymptotically stable. The proof is completed.

D. DESIGN OF FAULT DETECTION FILTERS

Theorems 1-3 present a group of LMI conditions for the fault detection filter design. By combining Theorems 1-3, an algorithm is proposed to obtain the parameters of a desired filter.

Algorithm 1: Given adjustable parameters α_i $(i = 1, \ldots, 6)$, weighting factors $p, q \in \mathbb{R}^+$, which satisfying $p + q = 1$, the upper bound of state delay $\tau_l \in \mathbb{Z}^+$, finite frequency ranges $\mathbb{U}_{f_l}$ and $\mathbb{U}_{d_l}$, β and vector η, solve the following convex optimization problem

$$\begin{aligned} &\min\ \gamma = p\gamma_d + q\gamma_f \\ &s.t.\ (10), (11), (16), (17), (18), (24), (25). \end{aligned} \tag{30}$$

Then, the filter parameter matrixes satisfying

$$\hat{A} = (\mathcal{A}\mathcal{V}^{-1})^T,\ \hat{B} = (\mathcal{B}\mathcal{V}^{-1})^T,\ \hat{C} = \mathcal{C}.$$

Remark 1: In Algorithm 1, a much better fault detection filter can be designed by choosing different adjustable parameters α_i (i $= 1, \ldots, 6$) for inequations in 1-3. However, up to now, there is no feasible method for optimizing these parameters [23]. Thus, we just choose same adjustable parameters to simply show the efficacy of α_i in the paper.

E. RESIDUAL EVALUATION FUNCTION AND THRESHOLD

Motivated by [15] and [17], we choose the following residual evaluation function $J_r(t, k)$ and the threshold J_{th} :

$$J_r(t,k) := \sqrt{\frac{1}{\hat{t}}\frac{1}{\hat{k}}\sum_{k=0}^{\hat{k}} \int_0^{\hat{t}} r^T(t,k) r(t,k) d\tilde{t}}, \tag{31}$$

$$J_{th} := \sup_{d \neq 0, f=0} J_r(t,k), \tag{32}$$

where $J_r(t, k)$ is the residual evaluation function, $\hat{t}$, $\hat{k}$ are the horizontal range and the vertical range of evaluation window, respectively. J_{th} is the threshold.The occurrence of faults can be detected using the following logic rules:

$$\begin{aligned} &J_{th} < J_r(t,k) \implies \textit{with faults} \implies \textit{alarm}; \\ &J_{th} \geq J_r(t,k) \implies \textit{no faults} \implies \textit{no alarm}. \end{aligned} \tag{33}$$

IV. SIMULAIONS

Considering the 2-D continuous-discrete state-delay Roesser system in the form of $\mathcal{E}$ with

$$A = \begin{bmatrix} -0.5 & 0.2 \\ 0.3 & -0.1 \end{bmatrix},\ A_\tau = \begin{bmatrix} -0.1 & -0.1 \\ 0 & 0.1 \end{bmatrix},\ B_d = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$B_f = \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix},\quad C = \begin{bmatrix} 1.3 & 0.7 \end{bmatrix},\ C_\tau = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$
$$D_d = 0.7,\ D_f = 0.3.$$

Let $p = 0.6$, $q = 0.4$, finite frequency ranges $\mathbb{U}_{f_l} = \mathbb{U}_{f_l} = [-\frac{\pi}{3}, \frac{\pi}{3}]$, $\alpha_1 = 2$, $\alpha_2 = 2$, $\alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 1$, $\beta = 2$, $\eta = [1\ \ 1]^T$, state delays $\bar{\tau}_h = 1$ and $\bar{\tau}_v = 3$. By solving the optimization problem in Algorithm 1 with the Matlab LMI Toolbox, we can obtain $\gamma = 0.0483$ and the following fault detection filter gain

$$\hat{A} = \begin{bmatrix} -1.5148 & -0.6655 \\ -0.0180 & -0.1011 \end{bmatrix},\quad \hat{B} = \begin{bmatrix} -0.1307 \\ 0.0429 \end{bmatrix},$$
$$\hat{C} = \begin{bmatrix} 1.3454 & 0.6566 \end{bmatrix}.$$

As an example, let the fault input and the disturbance input be

$$f(t,k) = \begin{cases} 3.8 & (t \geq 10), \\ 0 & otherwise, \end{cases}$$
$$d(t,k) = 0.2\cos(0.3t) + 0.2\sin(0.3k).$$

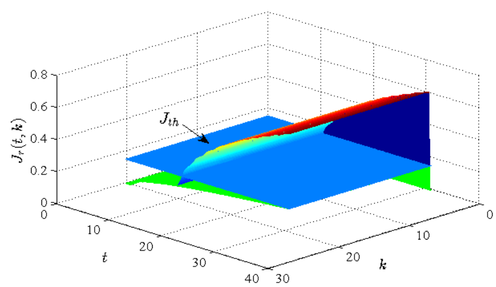

FIGURE 1. The residual evaluation function $J_r(t, k)$ and the threshold J_{th} in three-dimensional space.

By (32), we can obtain the threshold $J_{th} = 0.1538$. The simulation results are shown in Figs. 1-2. Fig. 1 depicts the residual evaluation function $J_r(t, k)$ and the threshold J_{th} in three-dimensional space. Figs. 2 depicts the residual evaluation function $J_r(t, k)$ and the threshold J_{th} in two-dimensional space. It can be seen from Fig. 2 that the fault can be detected at 10.8.

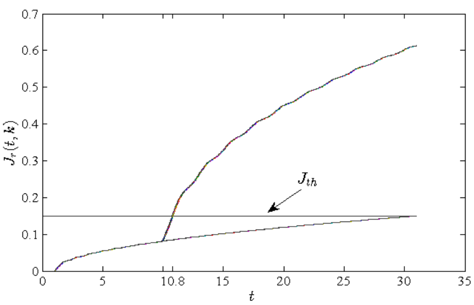

FIGURE 2. The residual evaluation function $J_r(t, k)$ and the threshold J_{th} in two-dimensional space.

V. CONCLUSION

In the paper, by the generalized KYP lemma, the finite frequency H_- and H_∞ indexes have been used to design the fault detection filters for 2-D continuous-discrete state-delay Roesser systems. Finite-frequency performance analysis conditions are firstly obtained. Convex filter design conditions are derived by constructing a hyperplane tangent combined with matrix inequality techniques. Then, an algorithm is proposed to construct a desired fault detection filter. The effectiveness of the proposed fault detection method is illustrated by an example. Furthermore, system parametric uncertainties are frequently encountered in many practical systems and often a primary source of instability and performance degradation of a control system. Thus, it is a worthiness subject to study system parametric uncertainties in the future.

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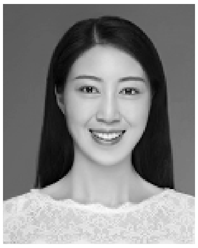

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