IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received May 8, 2020, accepted May 30, 2020, date of publication June 2, 2020, date of current version June 12, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2999459

Restricted Stabilization of Markovian Jump Systems Based on a Period and Random Switching Controller

GUOLIANG WANG[1], YADONG CHEN[1], AND XIJIE LI[2]
[1]School of Information and Control Engineering, Liaoning Shihua University, Fushun 113001, China
[2]Fushun Experimental Primary School, Fushun 113008, China

Corresponding author: Guoliang Wang (glwang@lnpu.edu.cn)

This work was supported in part by the Liaoning Revitalization Talents Program under Grant XLYC1807030, in part by the National Natural Science Foundation of China under Grant 61473140, and in part by the Program for Liaoning Innovative Talents in University under Grant LR2017029.

ABSTRACT This paper considers the stabilization of continuous-time Markovian jump systems (MJSs) via a restricted controller. It is actually a period and random switching controller. It also contains some existing controllers as special ones. Sufficient conditions for existence of such a controller are established by studying a discrete-time MJS, which are presented in terms of LMIs and depend on its period and probability. Moreover, an extension about a similar but aperiodic controller is considered. Finally, a numerical example is used to demonstrate the effectiveness and superiority of the proposed methods.

INDEX TERMS Markovian jump systems, stabilization, period and random switching, semi-Markov process, linear matrix inequalities (LMIs).

I. INTRODUCTION

It is known that Markovian jump system (MJS) [1], [2] is a particular kind of hybrid systems. There are two kinds of mechanisms simultaneously involved. One is time-evolving and closed to system state over time. The other one is event-driven mechanism and named as operation mode driven by a Markov chain. During the past decades, a lot of topics on all kinds of MJSs have been studied such as stability [3]–[6], stabilization [7]–[13], H_∞ control [14]–[16] and filtering [17]–[21], fault detection [22]–[24], estimation [25], [26], adaptive control [27], [28], synchronization [29], and so on.

In the above problems, stabilization is one of most important problems and could get better performance. By investigating the references about MJSs, it is found that most of them are mainly classified as three cases. The first kind of controller is a usual one and always referred to be mode-dependent. Because of operation mode available online and synchronous, it is the least conservative. This is also its drawback since the above assumption about operation mode is hard to be satisfied in applications. In order to remove this assumption, another mode-independent methods [30], [31] were proposed and has nothing to do with mode. Because it ignored operation mode totally even it is available sometimes, it is said to be an absolute approach. Recently, a kind of partially mode-dependent method was presented in [32] and bridged the above two cases, where a Bernoulli variable was introduced. By applying the polytopic uncertainty method to a controller, the fault-tolerant control of MJSs was considered in [34]. Though the above methods can be applied to non-mode-dependent cases, it is seen that the switchings of Bernoulli variable and polytopic uncertainty are fast even instantaneous. It is said that such a fast switching will lead to a higher cost even a damage to an equipment. In this case, it is natural to design a controller for an MJS which could sustain a period. A typical example is semi-Markov jump systems. Because of its sojourn time being any distribution, the corresponding switching will be slower than one of traditional MJSs. Very recently, the stability and stabilization of discrete-time semi-Markov jump linear systems subject to exponentially modulated periodic probability density function of sojourn time was considered in [35] and very important to make further research about semi-Markov jump systems. Particularly, necessary and sufficient

The associate editor coordinating the review of this manuscript and approving it for publication was Usama Mir.

criterion about mean square stability was first developed. However, there are still many problems to be further studied which of them will be different in essence. For example, when the considered system is continuous-time, the stability criterion of discrete-time case based on the Lyapunov function approach will be disabled because of the switching signal right-continuous and belonging to any distribution instead of only exponential distribution. Moreover, when some general kinds of controllers such as ones mentioned above are considered, how to obtain the easily solvable conditions will be not easy but necessary and important. Thus, in order to overcome such as problems and difficulties, some new techniques for analyzing its stability and giving LMI conditions of such a generally stabilizing controller are necessary to be developed. By summarizing a large number of literatures, it is found that very few of them were considered about such a controller. All the aforementioned facts and observations motivate the current research.

In this paper, the stabilization problem of continuous-time MJSs is studied via a restricted controller. The main contributions of this paper are summarized as follows: 1) A kind of restricted controller in terms of period and random switching is proposed, which contains some existing controllers as special ones; 2) By studying a discrete-time MJS indirectly, sufficient linear matrix inequality conditions for the controller are presented, in which both period and conditional switching probabilities are included. It is shown that our results are less conservative in terms of having larger application scope; 3) Compared with some traditional controllers, the modes of original system and proposed controller are not necessary synchronous. Moreover, its switching is not so fast and has less damage to equipments; 4) More extension about an aperiodic controller is further considered to remove the assumption of constant period.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{q\times n}$ is the set of all $q \times n$ real matrices. $(\Omega, \mathcal{F}, \mathbb{P})$ is a complete probability space equipped with a filtration $\{\mathcal{F}_t : t \in \mathbb{R}^+\}$ satisfying the usual hypotheses, that is a right-continuous filtration augmented by all null sets in the $\mathbb{P}$-completion of $\mathcal{F}$. Here, Ω is the sample space, $\mathcal{F}$ is the σ-algebras of subsets of the sample space and $\mathbb{P}$ is the probability measure on $\mathcal{F}$. $\mathscr{E}[\cdot]$ denotes the expectation operator. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. $\mathbb{N}$ represents the set of natural number. $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote the smallest and largest eigenvalues of a symmetric matrix M. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag $\{\cdots\}$ for a block-diagonal matrix, and $(M)^\star \triangleq M + M^T$.

II. PROBLEM FORMULATION

Consider a class of continuous-time MJSs defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, it is described as follows

$$\dot{x}(t) = A_{r_t}x(t) + B_{r_t}u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, and $u(t) \in \mathbb{R}^m$ is the control input vector. Here, $A(r_t)$ and $B(r_t)$ are known matrices of compatible dimensions. Process $\{r_t, t \geq 0\}$ is a homogeneous Markov process taking values in a finite set $\mathbb{S} \triangleq \{1, 2, \ldots, N\}$. It is adapted to the filtration $\{\mathcal{F}_t : t \in \mathbb{R}^+\}$ and is $\mathcal{F}_t$-measurable. It is a right-continuous trajectory and represents the switching among different modes. The evolution of Markov process $\{r_t, t \geq 0\}$ with transition rate matrix $\Lambda = (\lambda_{ij}) \in \mathbb{R}^{N\times N}$ is governed by

$$\Pr\{r_{t+\Delta t} = j | r_t = i\} = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t), & i \neq j \\ 1 + \lambda_{ii}\Delta t + o(\Delta t), & i = j \end{cases} \tag{2}$$

where λ_{ij} denotes the transition rate from state i to state j, and $\lambda_{ij} \geq 0$, if $i \neq j$, and $\lambda_{ii} = -\sum_{j\neq i}\lambda_{ij}$, for all $i, j \in \mathbb{S}$. As for system synthesis problems such as stabilization, the common controllers designed for system (1) could be summarized as follows:

Mode-dependent controller [7], [8]:

$$u(t) = K_{r_t}x(t) \tag{3}$$

where K_{r_t} is the control gain and depends on operation mode r_t all the time;

Mode-independent controller [30], [31]:

$$u(t) = Kx(t) \tag{4}$$

where K is the control gain and totally ignores r_t;

Partially mode-dependent controller [32]:

$$u(t) = K_{r_t}x(t) + (1 - \alpha(t))Kx(t) \tag{5}$$

where both K_{r_t} and K are control gains and similar to the above ones, and $\alpha(t)$ is the Bernoulli variable and denotes the current operation mode available or not.

Different from the above controllers, the state feedback controller in this paper is proposed to be

$$u(t) = K^{[\hat{r}_t]}x(t) \tag{6}$$

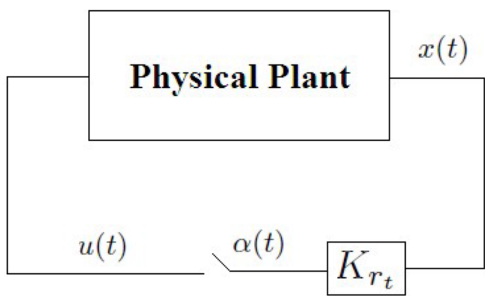

FIGURE 1. The diagram of controller (6).

where $K^{[\hat{r}_t]}$ is the control gain to be determined but different for the above ones. The detailed construction of controller (6) is clearly stated. Fig. 1. Here, operation mode $\hat{r}_t$ is another switching signal and defined as

$$\hat{r}_t = \hat{r}_{k\tau}, \forall t \in [k\tau, (k+1)\tau), \quad \forall k \in \mathbb{N} \tag{7}$$

where τ is a constant and denotes the periodic dwell time of $\hat{r}_t$. Its random switching on jump points is related to r_t and defined as

$$\xi_{h\ell} = \Pr\{\hat{r}_{k\tau} = \ell | r_{k\tau} = h\}, \ \forall \ell \in \mathbb{T} \triangleq \{1, 2, \ldots, M\}, \forall h \in \mathbb{S} \tag{8}$$

where operation mode $r_{k\tau}$ is the value of r_t at instant $k\tau$ such as $r_{k\tau} = r_{t=k\tau}$. Then it is said that the implementation of controller (6) has some restrictions such as (7) and (8). Firstly, switching signal $\hat{r}_t$ is restricted to be a piecewise constant function, whose dwell time is not very small or instantaneous but τ. In this case, the fast switching among controllers could be avoided and lead to fewer damages to equipment of a controller. However, it is also mentioned that such a dwell time is constant and will be with some limitation. A more general assumption about τ is time-varying, which will be our further work. Secondly, it is defined that the sojourn time and switching number of operation mode $r_t = i$ on interval $[k\tau, (k+1)\tau)$ are τ_i and n_i respectively. Because n_i and τ_i are very closed to $r_t = i, \forall t \in [k\tau, (k+1)\tau)$, it is reasonable that they are stochastic variables and with finite expectations. In this case, it is naturally assumed that $\sum_{j=1}^{N} n_j \leq n_{\max}$ and $\tau_i \in [\tau_{\min}, \tau_{\max}], \forall i \in \mathbb{S}$, hold for any interval $[k\tau, (k+1)\tau)$. Here, parameters $n_{\max}$, $\tau_{\min}$ and $\tau_{\max}$ are given in advance, but the preassumption are without loss of generality. Particularly, $n_{\max}$ is a natural number, while $\tau_{\min}$ and $\tau_{\max}$ are positive real constants. This assumption is also reasonable in practice, since the switching of any equipment or system among modes should be finite. So, the accumulated sojourn time of each mode should also be with upper and lower bounds. For simple description, the variables such as $x(k\tau)$ and $r_{k\tau}$ are simply denoted as $x(k)$ and $r(k)$ respectively. Then, system (1) is equal to

$$\begin{cases} \dot{x}(t) = (A_{r_t} + B_{r_t} K^{[\hat{r}_t]}) x(t), \forall t \in [k\tau, (k+1)\tau) \\ x(k) = x(t)|_{t=k\tau} \end{cases} \tag{9}$$

Remark 1: Compared with the above existing controllers, controller (6) is better in terms of having less constriction on current mode r_t but doesn't neglect it at all. In other words, when the operation mode of controller (3) such as [7], [8], [12] is not accessible on time, it will be disabled while controller (6) is an effective choice. Moreover, more information about the correlation between modes r_t and $\hat{r}_t$ are further considered and will be less conservative than controller (4) referred in [30], [31]. Thirdly, but not the last, in contrast to controllers (3) and (5) having fast switchings even instantaneously in [32], [33], the switching of (6) is more slower though r_t and $\alpha(t)$ are fast switchings. Such a slower switching will lead to less damage to equipment or system and have a wider application scope. More importantly, controller (6) could be specialized to be (3) and (4) respectively. However, it is worth mentioning that there are still some disadvantages of controller (6). One of them is that the jump points are periodic, and the dwell times are equal or constant. This assumption is ideal and will make the application with some limitations. Thus, more effort will be applied to deal with this problem.

Definition 1: System (9) or system (1) closed by controller (6)is said to be asymptotically mean square stable, if for initial conditions $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathbb{S}$, there is

$$\lim_{t\to\infty} \mathscr{E}\left[\|x(t)\|^2 | x_0, r_0\right] = 0$$

Lemma 1: [36] For any real matrix $A \in \mathbb{R}^{n\times n}$ and positive-definite matrix $P \in \mathbb{R}^{n\times n}$, if an arbitrary scalar ς is selected to be $\varsigma \geq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}$, it is always obtained that

$$A^T P A \leq \varsigma \|A\|^2 P \tag{10}$$

Particularly, by defining two pairs of conditions such as

(a) $P \geq I, P \leq \varsigma I$
(b) $P \leq I, \varsigma P \geq I$

it is known that either of them could imply inequality (10), while $\varsigma \geq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}$ could be removed.

III. MAIN RESULTS

Theorem 1: Suppose that there exists a scalar $r_i^{[\ell]}$. There is a controller (6) such that the closed-loop system (9) is asymptotically mean square stable, if for given scalars, $\overline{\rho}_i^{[\ell]} > \underline{\rho}_i^{[\ell]} > 0$ and $\varsigma > 0$, there exist matrices P_i, $X_i^{[\ell]}$, $S^{[\ell]}$ and $Y^{[\ell]}$, satisfying either of the following conditions

$$P_j \geq I, P_j \leq \varsigma I \tag{11}$$
$$P_j \leq I, \varsigma P_j \geq I \tag{12}$$

and

$$(\overline{\rho}_i^{[\ell]})^{-1} I < X_i^{[\ell]} < (\underline{\rho}_i^{[\ell]})^{-1} I \tag{13}$$

$$\begin{bmatrix} \Theta_{i1}^{[\ell]} & \Theta_{i2}^{[\ell]} \\ * & (-S^{[\ell]})^\star \end{bmatrix} < 0 \tag{14}$$

$$\varsigma \sum_{\ell=1}^{M} \sum_{j=1}^{N} \xi_{i\ell} \pi_{ij} (\max_{i\in\mathbb{S}} \{\frac{\overline{\rho}_i^{[\ell]}}{\underline{\rho}_i^{[\ell]}}\})^{2n_{\max}} e^{2(\sum_{j\in\bar{\mathbb{N}}_\ell^+} r_j^{[\ell]} \tau_{\min} + \sum_{j\in\bar{\mathbb{N}}_\ell^-} r_j^{[\ell]} \tau_{\max})} P_j) - P_i < 0 \tag{15}$$

where

$$\Theta_{i1}^{[\ell]} = (A_i S^{[\ell]} + B_i Y^{[\ell]} - r_i^{[\ell]} S^{[\ell]})^\star$$
$$\Theta_{i2}^{[\ell]} = A_i S^{[\ell]} + B_i Y^{[\ell]} - r_i^{[\ell]} S^{[\ell]} + X_i^{[\ell]} - (S^{[\ell]})^T$$
$$\bar{\mathbb{N}}_\ell^+ = \{i \in \mathbb{S} | r_i^{[\ell]} \leq 0\}, \bar{\mathbb{N}}_\ell^- = \{i \in \mathbb{S} | r_i^{[\ell]} > 0\}$$
$$\Pi \triangleq (\pi_{hj} == e^{\Lambda\tau}$$

The feedback gain of controller (3) is computed by

$$K^{[\ell]} = Y^\ell (S^{[\ell]})^{-1} \tag{16}$$

Proof: Based on system (9) with condition (8), for any $t \in [k\tau, (k+1)\tau)$ with any given $r(k) = h, \forall h \in \mathbb{S}$, a discrete-time MJS is constructed to be

$$\begin{cases} x(k+1) = e^{\int_{k\tau}^{(k+1)\tau} \bar{A}_{r_s}^{[\hat{r}_{k\tau}]} ds} x(k) \\ x_0 = x(0) \end{cases} \tag{17}$$

where $\bar{A}_{r_s}^{[\hat{r}_{k\tau}]} \triangleq A_{r_s}+B_{r_s}K^{[\hat{r}_{k\tau}]}$. According to the Kolmogorov differential equation, the transition probability matrix $\Pi = (\pi_{hj}) \in \mathbb{R}^{N\times N}$ could be obtained that

$$\Pi = e^{\Lambda\tau} = \sum_{j=1}^{\infty} \frac{(\Lambda\tau)^j}{j!} \tag{18}$$

Its detail is described to be

$$\pi_{hj} \triangleq \Pr\{r_{(k+1)\tau} = j | r_{k\tau} = h\} = \Pr\{r(k+1) = j | r(k) = h\} \tag{19}$$

Then, a stochastic Lyapunov function of system (17) is constructed as follow

$$V(x(k), r(k), k) = x^T(k)P(r(k))x(k) \tag{20}$$

Then, for any given $r(k) = h \in \mathbb{S}$, it is computed based on Lemma 1 that

$$\begin{aligned}
&\Delta V(x(k), r(k), k)\\
&= \mathscr{E}\Big[V(x(k+1), r(k+1), k+1)\Big] - V(x(k), r(k), k)\\
&= \mathscr{E}\Big[x^T(k+1)P(r(k+1))x(k+1)|x(k), r(k) = h, k\Big] - x^T(k)P_hx(k)\\
&\leq \mathscr{E}\Big[\varsigma x^T(k)||e^{\bar{A}_{r_{t_q}}^{[\hat{r}_{k\tau}]}(k\tau+\tau-t_q)} e^{\bar{A}_{r_{t_{q-1}}}^{[\hat{r}_{k\tau}]}(t_q-t_{q-1})} \ldots e^{\bar{A}_{r_{k\tau}}^{[\hat{r}_{k\tau}]}(t_1-k\tau)}||^2 P(r(k+1))x(k)|x(k), r(k) = h, k\Big]\\
&\quad - x^T(k)P_hx(k)\\
&\leq \mathscr{E}\Big[\varsigma x^T(k)||e^{\bar{A}_{r_{t_q}}^{[\hat{r}_{k\tau}]}(k\tau+\tau-t_q)}||^2||e^{\bar{A}_{r_{t_{q-1}}}^{[\hat{r}_{k\tau}]}(t_q-t_{q-1})}||^2 \ldots ||e^{\bar{A}_{r_{k\tau}}^{[\hat{r}_{k\tau}]}(t_1-k\tau)}||^2 P(r(k+1))x(k)|x(k), r(k) = h, k\Big]\\
&\quad - x^T(k)P_hx(k)
\end{aligned} \tag{21}$$

where $t_1, t_2, \ldots, t_q$, are switching instants of r_t on interval $[k\tau, (k+1)\tau)$, and q is the switching number of r_t on the same interval. For any real matrix $\bar{A}_i^{[\ell]} = A_i + B_iK^{[\ell]}$ and $\forall t \in [0, \infty)$, it is known from [37] that

$$||e^{\bar{A}_i^{[\ell]}t}|| \leq \alpha_i^{[\ell]}e^{-\beta_i^{[\ell]}t} \tag{22}$$

There, parameters $\alpha_i^{[\ell]}$ and $\beta_i^{[\ell]}$ are computed by

$$\alpha_i^{[\ell]} = \sqrt{\frac{\lambda_{\max}(H_i^{[\ell]})}{\lambda_{\min}(H_i^{[\ell]})}} \tag{23}$$

$$\beta_i^{[\ell]} = -r_i^{[\ell]} \tag{24}$$

where $r_i^{[\ell]} > \max_j \mathrm{Re}(\lambda_j(\bar{A}_i^{[\ell]}))$, and matrix $H_i^{[\ell]} > 0$ is the solution of Lyapunov equation

$$(\bar{A}_i^{[\ell]} - r_i^{[\ell]}I)^T H_i^{[\ell]} + H_i^{[\ell]}(\bar{A}_i^{[\ell]} - r_i^{[\ell]}I) < 0 \tag{25}$$

Then, formula (21) is further obtained that

$$\begin{aligned}
&\mathscr{E}\Big[\varsigma x^T(k)||e^{\bar{A}_{r_{t_q}}^{[\hat{r}_{k\tau}]}(k\tau+\tau-t_q)}||^2 \ldots ||e^{\bar{A}_{r_{k\tau}}^{[\hat{r}_{k\tau}]}(t_1-k\tau)}||^2 P(r(k+1))x(k)|x(k), r(k) = h, k\Big] - x^T(k)P_hx(k)\\
&\quad \leq \mathscr{E}\Big[\varsigma x^T(k)(\alpha_{r_{t_q}}^{[\hat{r}_{k\tau}]})^2\, e^{-2\beta_{r_{t_q}}^{[\hat{r}_{k\tau}]}(k\tau+\tau-t_q)} \ldots (\alpha_{r_{k\tau}}^{[\hat{r}_{k\tau}]})^2 e^{-2\beta_{r_{k\tau}}^{[\hat{r}_{k\tau}]}(t_1-k\tau)} \times P(r(k+1))x(k)|x(k), r(k) = h, k\Big]\\
&\quad - x^T(k)P_hx(k)\\
&= \mathscr{E}\Big[\varsigma x^T(k)(\alpha_1^{[\hat{r}_{k\tau}]})^{2n_1}e^{-2\beta_1^{[\hat{r}_{k\tau}]}\tau_1} \ldots (\alpha_N^{[\hat{r}_{k\tau}]})^{2n_N} e^{-2\beta_N^{[\hat{r}_{k\tau}]}\tau_N} P(r(k+1))x(k)|x(k), r(k) = h, k\Big]\\
&\quad - x^T(k)P_hx(k)\\
&\leq x^T(k)\Big[\varsigma \sum_{\ell=1}^{M}\sum_{j=1}^{N} \xi_{h\ell}\pi_{hj}((\alpha^{[\ell]})^{2\sum_{j=1}^{N} n_j} e^{-2\beta_1^{[\ell]}\tau_1} \ldots e^{-2\beta_N^{[\ell]}\tau_N} P_j) - P_h\Big]x(k)\\
&\quad \leq x^T(k)\Big[\varsigma \sum_{\ell=1}^{M}\sum_{j=1}^{N} \xi_{h\ell}\pi_{hj}((\alpha^{[\ell]})^{2n_{\max}} e^{-2\sum_{j\in\mathbb{N}_\ell^+}\beta_j^{[\ell]}\tau_{\min} -2\sum_{j\in\mathbb{N}_\ell^-}\beta_j^{[\ell]}\tau_{\max}} P_j) - P_h\Big]x(k)\\
&= x^T(k)\Big[\varsigma \sum_{\ell=1}^{M}\sum_{j=1}^{N} \xi_{h\ell}\pi_{hj}((\alpha^{[\ell]})^{2n_{\max}} e^{2(\sum_{j\in\bar{\mathbb{N}}_\ell^+} r_j^{[\ell]}\tau_{\min} + \sum_{j\in\bar{\mathbb{N}}_\ell^-} r_j^{[\ell]}\tau_{\max})} P_j) - P_h\Big]x(k) < 0
\end{aligned} \tag{26}$$

where $\alpha^{[\ell]} = \max_{i\in\mathbb{S}}\{\alpha_i^{[\ell]}\}$, $\mathbb{N}_\ell^+ = \{i \in \mathbb{S}|\beta_i^{[\ell]} \geq 0\}$ and $\mathbb{N}_\ell^- = \{i \in \mathbb{S}|\beta_i^{[\ell]} < 0\}$. By multiplying both sides of formula (25) with $X_i^{[\ell]} = (H_i^{[\ell]})^{-1}$ and its transpose respectively, it is equivalent to

$$\Big[(A_i + B_iK^{[\ell]} - r_i^{[\ell]}I)X_i^{[\ell]}\Big]^\star < 0 \tag{27}$$

By condition (14), it is known that $S^{[\ell]}$ is nonsingular. Then, it is concluded from formula (16) that inequality (27) could be guaranteed by condition (14) by multiplying its both sides with $\Big[\, I \;\; \bar{A}_i^{[\ell]} - r_i^{[\ell]}I \,\Big]$ and its transpose respectively. Based on inequality (13), it is easy to get that

$$\underline{\rho}_i^{[\ell]}I < H_i^{[\ell]} < \overline{\rho}_i^{[\ell]}I \tag{28}$$

which is further obtained that

$$\alpha_i^{[\ell]} < \frac{\overline{\rho}_i^{[\ell]}}{\underline{\rho}_i^{[\ell]}} \tag{29}$$

Then, inequality (26) could be implied by condition (15). As a result, one concludes that $\lim_{k\to\infty}\mathscr{E}\Big[\|x(k)\|^2|x_0, r_0\Big] = 0.$

At the same time, for any $t \in [k\tau, (k+1)\tau)$, it is known that

$$\begin{aligned}
\mathscr{E}\left[||x(t)||^2\right] &\le \mathscr{E}\left[x^T(k)||\Phi(t,k\tau)||^2 x(k)\right] \\
&= \mathscr{E}\left[x^T(k)||e^{\bar{A}^{[\hat{r}_{k\tau}]}_{r_{t_m}}(t-t_m)} e^{\bar{A}^{[\hat{r}_{k\tau}]}_{r_{t_{m-1}}}(t_m-t_{m-1})} \right. \\
&\quad \left. \ldots e^{\bar{A}^{[\hat{r}_{k\tau}]}_{r_{k\tau}}(t_1-k\tau)}||^2 x(k)\right] \\
&\le \mathscr{E}\left[x^T(k)||e^{\bar{A}^{[\hat{r}_{k\tau}]}_{r_{t_m}}(t-t_m)}||^2 \right. \\
&\quad \left. \ldots ||e^{\bar{A}^{[\hat{r}_{k\tau}]}_{r_{k\tau}}(t_1-k\tau)}||^2 x(k)\right] \\
&\le \mathscr{E}\left[x^T(k)(\alpha^{[\hat{r}_{k\tau}]}_{r_{t_m}})^2 e^{-2\beta^{[\hat{r}_{k\tau}]}_{r_{t_m}}(t-t_m)} \right. \\
&\quad \left. \ldots (\alpha^{[\hat{r}_{k\tau}]}_{r_{k\tau}})^2 e^{-2\beta^{[\hat{r}_{k\tau}]}_{r_{k\tau}}(t_1-k\tau)} x(k)\right] \\
&\le \mathscr{E}\left[(\alpha^{[\hat{r}_{k\tau}]}_{r_{t_m}})^2 e^{2r^{[\hat{r}_{k\tau}]}_{r_{t_m}}(t-t_m)} \right. \\
&\quad \left. \ldots (\alpha^{[\hat{r}_{k\tau}]}_{r_{k\tau}})^2 e^{2r^{[\hat{r}_{k\tau}]}_{r_{k\tau}}(t_1-k\tau)}\right] ||x(k)||^2 \\
&< \sum_{\ell=1}^{M} \xi_{h\ell}((\alpha^{[\ell]})^{2n_{\max}} \\
&\quad e^{2(\sum_{j\in\bar{\mathbb{N}}^+_\ell} r^{[\ell]}_j \tau_{\min} + \sum_{j\in\bar{\mathbb{N}}^-_\ell} r^{[\ell]}_j \tau_{\max})} ||x(k)||^2 \\
&= \gamma \mathscr{E}\left[||x(k)||^2\right]
\end{aligned} \tag{30}$$

where

$$\gamma = \max_{h\in\mathbb{S}}\{\sum_{\ell=1}^{M} \xi_{h\ell}((\alpha^{[\ell]})^{2n_{\max}} e^{2(\sum_{j\in\bar{\mathbb{N}}^+_\ell} r^{[\ell]}_j \tau_{\min} + \sum_{j\in\bar{\mathbb{N}}^-_\ell} r^{[\ell]}_j \tau_{\max})}\}$$

is a value with limited bound. It is further obtained that

$$\begin{aligned}
0 &\le \lim_{t\to\infty} \mathscr{E}\left[||x(t)||^2|x_0, r_0\right] \\
&\le \gamma \lim_{k\to\infty} \mathscr{E}\left[||x(k)||^2|x_0, r_0\right] = 0
\end{aligned} \tag{31}$$

Then, the closed-loop system (1) is asymptotically mean square stable. This completes the proof.

Remark 2: For continuous-time MJSs, it is said that the time-scheduled Lyapunov function method proposed for LTI control systems [38] is not suitable. The main reason is that nonlinear term $e^{\bar{A}^{[\ell]}_i t}$ has NM modes and is very closed to stabilizing controller, which cannot be handled by the above method. In other words, when stabilization problems are considered, solvable conditions with easy computation forms such as LMI conditions are not easily obtained. All these facts will make the analysis and synthesis of system (9) difficult, and novel methods should be developed. To the contrary, based on the proposed methods, convex conditions for the existence of controller (6) are obtained and computed easily. Moreover, our results will include the deterministic case as a special one. Thus, the obtained results can be viewed as extension results on stabilization by a controller with failures from deterministic systems to stochastic systems.

Remark 3: As for the conditions in this theorem, some additional explanations are necessary given in the following. Firstly, all the conditions are given in terms of LMIs and could be solved directly and easily. However, the complexity of computation will be larger, especially M and N becomes very large. There will be $(N+1)M+N$ variables to be computed, while $N^2+(3M+1)N$ inequalities are needed to solved; Secondly, more information about probability (8), parameters $n_{\max}$, $\tau_{\min}$ and $\tau_{\max}$ are taken into account and could further demonstrate theirs effect on system analysis and synthesis. On the other hand, there is also an unavoidable problem that parameter $n_{\max}$ related to $\max_{i\in\mathbb{S}}\{\frac{\overline{\rho}^{[\ell]}_i}{\underline{\rho}^{[\ell]}_i}\}$ plays a large negative effect in terms of making condition (15) having smaller region of solvable solution. This phenomenon results from the switching property of r_t on interval $[k\tau, (k+1)\tau)$ and is inevitable. Fortunately, one could reduce this effect by selecting suitable values of $r^{[\ell]}_j$, $\tau_{\min}$ and $\tau_{\max}$. However, how to further reduce this negative effect and obtain less conservative results with smaller computation complexity are not easy and will be our further work.

Since the jump points of controller (6) are periodic, another type of jump system is considered and described as

$$\dot{x}(t) = A_{\eta_t}x(t) + B_{\eta_t}u(t) \tag{32}$$

where $\{\eta_t, t \ge 0\}$ is a jump process and takes values in a finite set $\mathbb{S} \triangleq \{1, 2, \ldots, N\}$, $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, and $A(\eta_t)$ and $B(\eta_t)$ are known matrices of compatible dimensions. Different from Markovian switching r_t, η_t jumps randomly. As a result, the instant of switching is random, whose dwell time of a given mode is also random and may be not an exponential distribution. Without loss of generality, the jump points are denoted as $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots, k \in \mathbb{N}$. Here, $\tau(k)$ denotes the dwell time of a given switching η_t and is defined as $\tau(k) \triangleq t_{k+1} - t_k$. It is stated above that η_t is different from r_t and not a Markov signal. In detail, dwell time $\tau(k)$ is time-varying and random but without any statistical property. The switching among modes is different from (2) and described as

$$\Pr\{r_{t_{k+1}} = j | r_{t_k} = h\} = \begin{cases} \theta_{hj}, & h \ne j \\ 0, & h = j \end{cases} \tag{33}$$

where θ_{hj} denotes the switching probability from state h to state j, and $\theta_{hj} \in [0, 1]$, if $h \ne j$, and $\theta_{hh} \equiv 0$. Moreover, it should also be satisfied that $\sum_{j=1}^{N}\theta_{hj} = 1$, for all $h, j \in \mathbb{S}$. In other words, the jump happening in the next time should be changed to another different mode. Without loss of generality, for any given interval $[t_k, t_{k+1})$, its operation mode is assumed to be $\forall\eta_t = h \in \mathbb{S}$, $\forall t \in [t_k, t_{k+1})$. Thus, system (32) with above description $\forall\eta_t = h \in \mathbb{S}$, $\forall t \in [t_k, t_{k+1})$ becomes to

$$\begin{cases} \dot{x}(t) = A_h x(t) + B_h u(t), \forall\eta_t = h \in \mathbb{S}, \forall t \in [t_k, t_{k+1}) \\ x(k) \triangleq x(t)|_{t=t_k} \end{cases} \tag{34}$$

Similarly, the state feedback controller in this section is also proposed to be

$$u(t) = K^{[\hat{\eta}_t]}x(t) \tag{35}$$

where $K^{[\hat{\eta}_t]}$ is the control gain to be determined. And $\hat{\eta}_t$ is another switching signal and defined as

$$\hat{\eta}_t = \hat{\eta}_{t_k}, \quad \forall t \in [t_k, t_{k+1}), \forall k \in \mathbb{N} \tag{36}$$

Its value is related to η_t and defined as

$$\omega_{h\ell} \triangleq \Pr\{\hat{\eta}_{t_k} = \ell | \eta_{t_k} = h\}, \quad \forall \ell \in \mathbb{T} \triangleq \{1, 2, \ldots, M\}, \quad \forall h \in \mathbb{S} \tag{37}$$

Since $\tau(k)$ is time-varying and random, it is naturally assumed that $\tau(k) \in [\tau_{\min}, \tau_{\max}]$, where $0 < \tau_{\min} < \tau_{\max}$. Similarly, variables such as $x(k\tau)$ and $\eta_{k\tau}$ are simply denoted as $x(k)$ and $\eta(k)$ respectively.

Remark 4: By investigating models (1) and (32), it is found that the main difference is about the property of switching signal. In the former one, r_t is a traditional Markov process, and its dwell time belongs to an exponential distribution. The latter η_t is only a switching one, whose dwell time is arbitrary. Moreover, the current operation mode must change to another different one at jump points. It may be seen as a semi-Markovian process. The reason considering such a system is to remove periodic interval $[k\tau, (k+1)\tau)$, $\forall k \in \mathbb{N}$. In this case, the switching of controller (35) is naturally not instantaneous, and only conditional probability (37) is needed considered. However, it is not said that model (1) could be removed. The reason is either of them cannot be totally included or specialized as another one, though some aspects in one model are more general than ones in the other one.

Theorem 2: Suppose that there exists a scalar $r_i^{[\ell]}$. There is a controller (35) such that the closed-loop system (34) is asymptotically mean square stable, if for given scalars, $\overline{\rho}_i^{[\ell]} > \underline{\rho}_i^{[\ell]} > 0$ and $\varsigma > 0$, there exist matrices P_i, $X_i^{[\ell]}$, $S^{[\ell]}$ and $Y^{[\ell]}$, satisfying conditions (11) or (12), (13), (14), and

$$\varsigma \sum_{j=1}^{N} \left[\sum_{\ell \in \bar{\mathbb{N}}_{+}^{[h]}} \omega_{h\ell}\theta_{hj} (\max_{i\in\mathbb{S}}\{\frac{\overline{\rho}_i^{[\ell]}}{\underline{\rho}_i^{[\ell]}}\})^2 e^{2r_h^{[\ell]}\tau_{\min}} + \sum_{\ell \in \bar{\mathbb{N}}_{-}^{[h]}} \omega_{h\ell}\theta_{hj} (\max_{i\in\mathbb{S}}\{\frac{\overline{\rho}_i^{[\ell]}}{\underline{\rho}_i^{[\ell]}}\})^2 e^{2r_h^{[\ell]}\tau_{\max}} \right] P_j - P_h < 0 \tag{38}$$

where $\bar{\mathbb{N}}_{+}^{[h]} = \{\ell \in \mathbb{T} | r_h^{[\ell]} \geq 0\}$, $\bar{\mathbb{N}}_{-}^{[h]} = \{\ell \in \mathbb{T} | r_h^{[\ell]} < 0\}$. Then, the gain of controller (35) could be obtained by (16).

Proof: Based on system (34) closed by controller (35), for any $t \in [t_k, t_{k+1})$ with any given $\eta(k) = h$, $\forall h \in \mathbb{S}$, a discrete-time jump system is constructed to be

$$\begin{cases} x(k+1) = e^{\int_{t_k}^{t_{k+1}} \bar{A}_h^{[\hat{\eta}_{t_k}]} ds} x(k) \\ x_0 = x(0) \end{cases} \tag{39}$$

where $\bar{A}_h^{[\hat{\eta}_{t_k}]} \triangleq A_h + B_h K^{[\hat{r}_{t_k}]}$. A stochastic Lyapunov function of system (39) is constructed to be

$$V(x(k), \eta(k), k) = x^T(k) P(\eta(k)) x(k) \tag{40}$$

Then, it is computed that

$$\begin{aligned} &\Delta V(x(k), \eta(k), k) \\ &= \mathscr{E}\Big[V(x(k+1), \eta(k+1), k+1)\Big] - V(x(k), \eta(k), k) \\ &= \mathscr{E}\Big[x^T(k+1)P(\eta(k+1))x(k+1)|x(k), \eta(k) = h, k\Big] - x^T(k)P_h x(k) \\ &\leq \mathscr{E}\Big[\varsigma x^T(k)||e^{\bar{A}_h^{[\hat{\eta}_{t_k}]}(t_{k+1}-t_k)}||^2 P(\eta(k+1))x(k) \,|x(k), \eta(k) = h, k\Big] - x^T(k)P_h x(k) < 0 \end{aligned} \tag{41}$$

Based on inequality (22) implied by conditions (13) and (14), it is further computed that

$$\begin{aligned} &\mathscr{E}\Big[\varsigma x^T(k)||e^{\bar{A}_h^{[\hat{\eta}_{t_k}]}(t_{k+1}-t_k)}||^2 P(\eta(k+1))x(k)|x(k), \eta(k) = h, k\Big] - x^T(k)P_h x(k) \\ &\leq \mathscr{E}\Big[\varsigma x^T(k)(\alpha_h^{[\hat{\eta}_{t_k}]})^2 e^{-2\beta_h^{[\hat{\eta}_{t_k}]}\tau(k)} P(\eta(k+1))x(k)|x(k), \eta(k) = h, k\Big] - x^T(k)P_h x(k) \\ &\leq x^T(k)\Bigg[\varsigma \sum_{j=1}^{N}\Big(\sum_{\ell \in \mathbb{N}_{+}^{[h]}} \omega_{h\ell}\theta_{hj}(\alpha_h^{[\ell]})^2 e^{-2\beta_h^{[\ell]}\tau_{\min}} + \sum_{\ell \in \mathbb{N}_{-}^{[h]}} \omega_{h\ell}\theta_{hj}(\alpha_h^{[\ell]})^2 e^{-2\beta_h^{[\ell]}\tau_{\max}}\Big)P_j - P_h\Bigg] x(k) \\ &= x^T(k)\Bigg[\varsigma \sum_{j=1}^{N}\Big(\sum_{\ell \in \bar{\mathbb{N}}_{+}^{[h]}} \omega_{h\ell}\theta_{hj}(\alpha_h^{[\ell]})^2 e^{2r_h^{[\ell]}\tau_{\min}} + \sum_{\ell \in \bar{\mathbb{N}}_{-}^{[h]}} \omega_{h\ell}\theta_{hj}(\alpha_h^{[\ell]})^2 e^{2r_h^{[\ell]}\tau_{\max}}\Big)P_j - P_h\Bigg] x(k) < 0 \end{aligned} \tag{42}$$

where $\mathbb{N}_{+}^{[h]} = \{\ell \in \mathbb{T} | \beta_h^{[\ell]} \geq 0\}$ and $\mathbb{N}_{-}^{[h]} = \{\ell \in \mathbb{T} | \beta_h^{[\ell]} < 0\}$. It is further obtained by

$$\varsigma \sum_{j=1}^{N}\Big(\sum_{\ell \in \bar{\mathbb{N}}_{+}^{[h]}} \omega_{h\ell}\theta_{hj}(\alpha_h^{[\ell]})^2 e^{2r_h^{[\ell]}\tau_{\min}} + \sum_{\ell \in \bar{\mathbb{N}}_{-}^{[h]}} \omega_{h\ell}\theta_{hj}(\alpha_h^{[\ell]})^2 e^{2r_h^{[\ell]}\tau_{\max}}\Big)P_j - P_h < 0 \tag{43}$$

which is guaranteed by (38). The next steps are similar to the ones in Theorem 1, which are omitted here. This completes the proof.

Remark 5: Compared with conditions in Theorems 1 and 2, only conditions (15) and (38) are different. Such a difference is totally determined by the considered different systems. It seems that the latter one could lead to a higher probability about solvable solutions. This doesn't say that Theorem 2 is better than Theorem 1. The reason is that the considered problems between them are different, and no any additional jumps in interval $[t_k, t_{k+1})$ happen in the latter. Thus, condition (38) could easily obtain solvable solutions. How to obtain more general conditions containing them both is not easy and will be our future research topics.

Based on the above analysis, an algorithm for Markovian jump systems with a period and random switching controller is presented to solve this problem.

Computation Algorithm:

Step 1: For system (9) with given r_i^{ℓ}, $i \in \mathbb{S}$, $\ell \in \mathbb{T}$, and determine sets $\bar{\mathbb{N}}_{\ell}^{+}$, $\bar{\mathbb{N}}_{\ell}^{-}$ and maximum iteration time $k_{\max}$.

Step 2: Select suitable values of $\overline{\rho}_i^{[\ell]}$, $\underline{\rho}_i^{[\ell]}$ and ς such as $\overline{\rho}_i^{[\ell]} > \underline{\rho}_i^{[\ell]} > 0$ and $\varsigma > 0$, and set $k = 0$.

Step 3: Find solvable solutions $(P_i, X_i^{[\ell]}, S^{[\ell]}, Y^{[\ell]})$ satisfying (11) or (12), and (13)-(15).

Step 4: If there are solvable solutions, compute control gains by (16), and exit; Otherwise go to Step 5, and set $k = k + 1$;

Step 5: If $k \leq k_{\max}$, increasing $\overline{\rho}_i^{[\ell]}$ or decreasing $\underline{\rho}_i^{[\ell]}$, while increasing ς or not depends on which one of conditions (11) and (12) is used, and go to Step 3; Otherwise exit. It means that there is no solvable solution to controller (6) for system (9) with given r_i^{ℓ}, $i \in \mathbb{S}$, $\ell \in \mathbb{T}$. In order to obtain solutions, one could select much more smaller values of r_i^{ℓ} such as $r_i^{\ell} < 0$, $i \in \mathbb{S}$, $\ell \in \mathbb{T}$. Then, repeat this process from Step 1.

IV. NUMERICAL EXAMPLES

Example 1: Consider an VTOL helicopter model partly cited from [39]. Its form is described as (1), whose parameters are given to be

$$A_1 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.4200 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.0664 & -0.707 & 0.1198 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.4422 & 0.1761 \\ 0.9775 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.5047 & -0.707 & 2.5460 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0.4422 & 0.1761 \\ 5.1120 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}$$

Here, the state variables x_1, x_2, x_3 and x_4 are denoted as the horizontal velocity, the vertical velocity, the pitch rate and the pitch angle respectively. Jump parameter $r(t) \in \mathbb{S} \triangleq \{1, 2, 3\}$ indicates the airspeed and corresponds to the three airspeeds during helicopter flight: 135 knots, 60 knots and 170 knots. Without loss of generality, the transition rate matrix is given as

$$\Lambda = \begin{bmatrix} -0.0450 & 0.0002 & 0.0448 \\ 0.0171 & -0.0171 & 0 \\ 0.0894 & 0 & -0.0894 \end{bmatrix}$$

By the traditional methods for designing a mode-dependent controller such as [7], [8], [12], one could get the corresponding gains as

$$K^{[1]} = \begin{bmatrix} 2.6919 & 25.9312 & -16.6173 & -5.7369 \\ 5.8135 & 12.3155 & -20.5112 & -7.8846 \end{bmatrix}$$

$$K^{[2]} = \begin{bmatrix} -137.6000 & -1061.5 & 705.6 & 275.6 \\ -280.4000 & -147.6 & 873.5 & 356.3 \end{bmatrix}$$

$$K^{[3]} = \begin{bmatrix} -69.6 & -1141.7 & 708.0 & 205.4 \\ -188.5 & -164.1 & 855.9 & 234.7 \end{bmatrix}$$

For this kind of controller, it is said that it is ideal sometimes, since its operation mode should be available online. In other word, when its mode experiences general case such as (7), the desired controller may be disabled. Without loss of generality, it is assumed here that stochastic process $\hat{r}(t)$ has three modes such as $\hat{r}(t) \in \mathbb{M} \triangleq \{1, 2, 3\}$. Meanwhile, the other parameters are given as $\tau = 20$, $\tau_{\min} = 10$ and $n_{\max} = 2$. The Π is computed to

$$\Pi = \begin{bmatrix} 0.6692 & 0.0258 & 0.3050 \\ 0.2206 & 0.7148 & 0.0646 \\ 0.6086 & 0.0151 & 0.3764 \end{bmatrix}$$

and $\Xi \triangleq (\xi_{h\ell}) \in \mathbb{R}^{3\times 3}$ is given to be

$$\Xi = \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.7 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}$$

Under initial condition $x_0 = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T$ and after applying the above controller experiencing condition (7), one could get the corresponding simulation of the closed-loop system given in Fig. 2, while Fig. 3 is the simulation

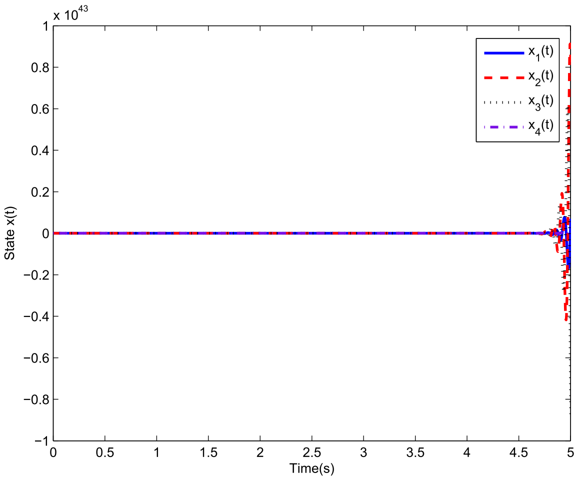

FIGURE 2. The state responses of closed-loop system.

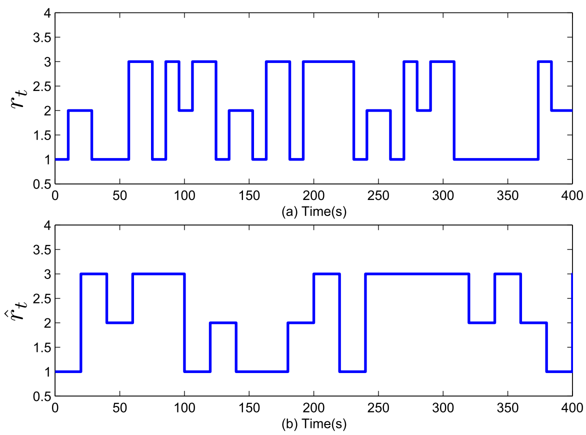

FIGURE 3. The simulations of operation modes r_t and $\hat{r}_t$.

of operation modes r_t and $\hat{r}_t$. Based on these simulations, it is seen that the closed-loop system becomes unstable when controller experiences condition (7). In other words, general condition (7) plays a negative effect in terms of reducing system performance even making the stable system unstable. At the same time, one could design controller (6) based on Theorem 1. Without loss of generality, when given $\varsigma = 1.1$, $\underline{\rho}_i^{[\ell]} = 1.1$, $\overline{\rho}_i^{[\ell]} = 100$, $\forall i \in \mathbb{S}$, $\forall \ell \in \mathbb{M}$, $r_1^{[1]} = -1.2$, $r_1^{[2]} = -0.9$, $r_1^{[3]} = -0.9$, $r_2^{[1]} = -1$, $r_2^{[2]} = -0.8$, $r_2^{[3]} = -0.9$, $r_3^{[1]} = -0.9$, $r_3^{[2]} = -0.8$ and $r_3^{[3]} = -0.9$, it is obtained by Theorem 1 that

$$S^{[1]} = \begin{bmatrix} 0.0381 & 0.0231 & -0.0491 & 0.0623 \\ 0.0070 & 0.2446 & 0.0619 & 0.0263 \\ -0.0984 & -0.0801 & 0.2293 & -0.2200 \\ 0.0610 & 0.0843 & -0.0356 & 0.1133 \end{bmatrix}$$

$$Y^{[1]} = \begin{bmatrix} -0.0357 & 0.0787 & 0.1839 & -0.0358 \\ -0.0436 & 0.0547 & 0.0727 & -0.0483 \end{bmatrix}$$

$$S^{[2]} = \begin{bmatrix} 0.1329 & -0.2797 & -0.1555 & 0.1539 \\ -0.1117 & 2.2169 & -0.6605 & 0.2462 \\ -0.2980 & -0.5563 & 1.1295 & -0.6872 \\ 0.2029 & 0.0458 & -0.4292 & 0.3499 \end{bmatrix}$$

$$Y^{[2]} = \begin{bmatrix} -0.1372 & 0.2731 & 0.4195 & -0.1949 \\ -0.1707 & 0.3986 & 0.2920 & -0.2195 \end{bmatrix}$$

$$S^{[3]} = \begin{bmatrix} 0.1121 & -0.1729 & -0.1612 & 0.1462 \\ -0.0591 & 1.2735 & -0.4781 & 0.1715 \\ -0.2868 & -0.4337 & 1.0633 & -0.6444 \\ 0.1850 & 0.0228 & -0.4032 & 0.3239 \end{bmatrix}$$

$$Y^{[3]} = \begin{bmatrix} -0.1219 & 0.0978 & 0.4426 & -0.1955 \\ -0.1492 & 0.2398 & 0.3046 & -0.2133 \end{bmatrix}$$

Then, the control gains are computed as

$$K^{[1]} = \begin{bmatrix} -4.8002 & -0.1186 & 0.2451 & 2.8248 \\ -2.5711 & 0.2144 & -0.2082 & 0.5327 \end{bmatrix}$$

$$K^{[2]} = \begin{bmatrix} -3.2539 & -0.1294 & 0.8457 & 2.6263 \\ -1.1426 & 0.1147 & 0.3554 & 0.4925 \end{bmatrix}$$

$$K^{[3]} = \begin{bmatrix} -3.7277 & -0.0206 & 0.9530 & 2.9876 \\ -1.4473 & 0.2551 & 0.4451 & 0.7258 \end{bmatrix}$$

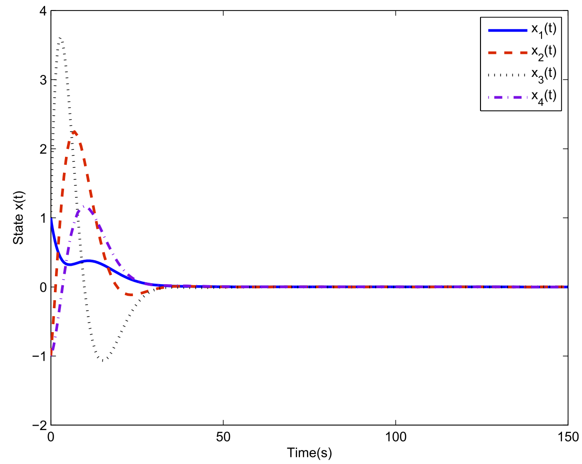

FIGURE 4. The state responses of closed-loop system (1).

Under the same conditions and after applying the above controller, one could get the simulation of closed-loop system given in Fig. 4. It is obvious that it is stable and demonstrates the utility of the proposed method. On the other hand, one could also design controller (35) with conditions (36) and (37) for a jump system described by (33) and (34). Jump parameters η_t and $\hat{\eta}_t$ take values in sets $\mathbb{S}$ and $\mathbb{M}$ respectively. The transition probability of $\Theta \triangleq (\theta_{hj}) \in \mathbb{R}^{3\times 3}$ is given as

$$\Theta = \begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.7 & 0 & 0.3 \\ 0.2 & 0.8 & 0 \end{bmatrix}$$

And $\Upsilon \triangleq (\omega_{h\ell}) \in \mathbb{R}^{3\times 3}$ is given to be

$$\Upsilon = \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.7 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}$$

When the related parameters of Theorem 2 are given to be same, one has

$$S^{[1]} = \begin{bmatrix} 0.0376 & 0.0265 & -0.0488 & 0.0619 \\ 0.0061 & 0.2421 & 0.0674 & 0.0241 \\ -0.0962 & -0.0789 & 0.2211 & -0.2160 \\ 0.0598 & 0.0865 & -0.0325 & 0.1117 \end{bmatrix}$$

$$Y^{[1]} = \begin{bmatrix} -0.0351 & 0.0781 & 0.1820 & -0.0347 \\ -0.0429 & 0.0516 & 0.0719 & -0.0475 \end{bmatrix}$$

$$S^{[2]} = \begin{bmatrix} 0.1333 & -0.2821 & -0.1557 & 0.1541 \\ -0.1120 & 2.2524 & -0.6768 & 0.2531 \\ -0.2991 & -0.5717 & 1.1410 & -0.6924 \\ 0.2034 & 0.0498 & -0.4332 & 0.3518 \end{bmatrix}$$

$$Y^{[2]} = \begin{bmatrix} -0.1378 & 0.2739 & 0.4211 & -0.1958 \\ -0.1712 & 0.4014 & 0.2926 & -0.2200 \end{bmatrix}$$

$$S^{[3]} = \begin{bmatrix} 0.1124 & -0.1725 & -0.1624 & 0.1468 \\ -0.0584 & 1.2822 & -0.4857 & 0.1747 \\ -0.2884 & -0.4417 & 1.0728 & -0.6489 \\ 0.1857 & 0.0255 & -0.4071 & 0.3258 \end{bmatrix}$$

$$Y^{[3]} = \begin{bmatrix} -0.1223 & 0.0956 & 0.4453 & -0.1968 \\ -0.1496 & 0.2388 & 0.3065 & -0.2143 \end{bmatrix}$$

Then, the control gains are computed as

$$K^{[1]} = \begin{bmatrix} -4.7226 & -0.0622 & 0.1975 & 2.7033 \\ -2.5191 & 0.2484 & -0.2398 & 0.4536 \end{bmatrix}$$

$$K^{[2]} = \begin{bmatrix} -3.2537 & -0.1293 & 0.8456 & 2.6261 \\ -1.1411 & 0.1147 & 0.3556 & 0.4919 \end{bmatrix}$$

$$K^{[3]} = \begin{bmatrix} -3.7258 & -0.0183 & 0.9543 & 2.9892 \\ -1.4482 & 0.2578 & 0.4471 & 0.7298 \end{bmatrix}$$

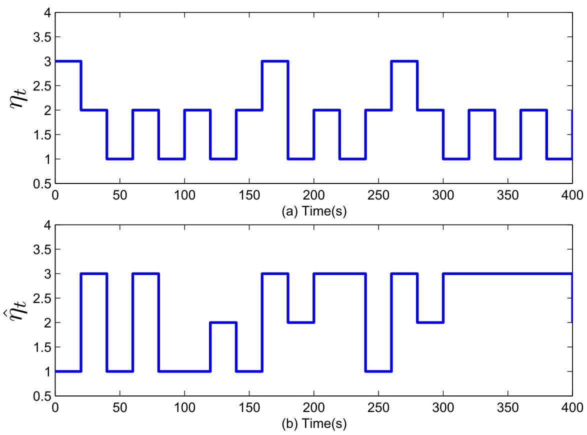

FIGURE 5. The simulations of operation modes η_t and $\hat{\eta}_t$.

Under the same initial condition, one could also obtain the simulations of the resulting closed-loop system. Particularly, Fig. 5 is the simulation of operations modes satisfying (33), (36) and (37). From Fig. 6, it is seen that the designed controller is still effective since the states of closed-loop system are stable. Based on such simulations and comparisons, it is said that our methods are less conservative that they can be used to more cases in terms of general operation mode of controller.

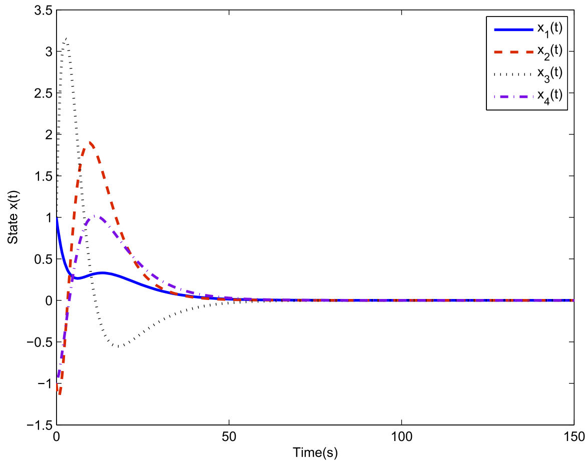

FIGURE 6. The state curves of closed-loop system (34).

V. CONCLUSIONS

In this paper, the stabilization problem of continuous-time Markovian jump systems has been investigated by a restrict controller. Different from the existing ones, the main restriction about the controller is that the dwell time of each controller is period, whose switching signal is a piece-wise continuous function. Moreover, the switching of controllers at jump points is random and conditionally dependent on the original mode at such jump points. By studying a discrete-time MJS indirectly, the existence conditions have been with LMI forms and related to period and conditional probability. Then, the proposed model and method have been applied to propose an aperiodic controller. The utility and advantage of the established results have been proved by a numerical example. Finally, it is said that there are still many problems to be considered. Firstly, because of lots of LMIs and variables to be solved, how to further reduce the complexity is another important problem; Secondly, it is seen that in order to obtain solvable conditions, some enlarged inequalities have been introduced, which also bring some conservatism. How to further reduce the conservatism is necessary to be considered; Thirdly, some extensions about such models could be applied to describe other problems directly, such as filtering, observer design, and fault detection. Fourthly, but not the last, it is worth mentioning that there are still some disadvantages of the proposed controller. One of them is that the jump points are periodic, and the dwell times are equal or constant. This assumption is ideal and will make the application with some limitations. Thus, more effort will be applied to deal with this problem. In a word, all the observations are necessary studied, and some of them may be not easy.

REFERENCES

[1] O. L. V. Costa, M. D. Fragoso, and M. G. Todorov, *Continuous-Time Markov Jump Linear Systems*. Berlin, Germany: Springer, 2012.

[2] G. L. Wang, Q. L. Zhang, and X. G. Yan, *Analysis and Design of Singular Markovian Jump Systems*. Cham, Switzerland: Springer, 2014.

[3] Y. Kao, L. Shi, J. Xie, and H. R. Karimi, "Global exponential stability of delayed Markovian jump fuzzy cellular neural networks with generally incomplete transition probability," *Neural Netw.*, vol. 63, pp. 18–30, Mar. 2015.
[4] T. Hou and H. Ma, "Exponential stability for discrete-time infinite Markov jump systems," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4241–4246, Dec. 2016.
[5] M. Meng, L. Liu, and G. Feng, "Stability and l_1 gain analysis of Boolean networks with Markovian jump parameters," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4222–4228, Aug. 2017.
[6] B. Wang and Q. Zhu, "Stability analysis of semi-Markov switched stochastic systems," *Automatica*, vol. 94, pp. 72–80, Aug. 2018.
[7] Y. Zhang, Y. He, M. Wu, and J. Zhang, "Stabilization for Markovian jump systems with partial information on transition probability based on free-connection weighting matrices," *Automatica*, vol. 47, no. 1, pp. 79–84, Jan. 2011.
[8] G. Wang, Q. Zhang, and C. Yang, "Stabilization of singular Markovian jump systems with time-varying switchings," *Inf. Sci.*, vol. 297, pp. 254–270, Mar. 2015.
[9] L. Zhang, Y. Leng, and P. Colaneri, "Stability and stabilization of discrete-time semi-Markov jump linear systems via semi-Markov kernel approach," *IEEE Trans. Autom. Control*, vol. 61, no. 2, pp. 503–508, Feb. 2016.
[10] W. M. Chen, S. Y. Xu, B. Y. Zhang, and Z. D. Qi, "Stability and stabilization of neutral stochastic delay Markovian jump systems," *IET Control Theory Appl.*, vol. 10, no. 15, pp. 1798–1807, Jun. 2016.
[11] M. Shen, J. H. Park, and D. Ye, "A separated approach to control of Markov jump nonlinear systems with general transition probabilities," *IEEE Trans. Cybern.*, vol. 46, no. 9, pp. 2010–2018, Sep. 2016.
[12] B. Jiang, Y. Kao, H. R. Karimi, and C. Gao, "Stability and stabilization for singular switching semi-Markovian jump systems with generally uncertain transition rates," *IEEE Trans. Autom. Control*, vol. 63, no. 11, pp. 3919–3926, Nov. 2018.
[13] Y. Wang, Y. Xia, H. Shen, and P. Zhou, "SMC design for robust stabilization of nonlinear Markovian jump singular systems," *IEEE Trans. Autom. Control*, vol. 63, no. 1, pp. 219–224, Jan. 2018.
[14] J. Cheng, H. Zhu, S. Zhong, Y. Zeng, and X. Dong, "Finite-time H_∞ control for a class of Markovian jump systems with mode-dependent time-varying delays via new Lyapunov functionals," *ISA Trans.*, vol. 52, no. 6, pp. 768–774, Nov. 2013.
[15] J. Zhu, X. Yu, T. Zhang, Z. Cao, Y. Yang, and Y. Yi, "The mean-square stability probability of H_∞ control of continuous Markovian jump systems," *IEEE Trans. Autom. Control*, vol. 61, no. 7, pp. 1918–1924, Jul. 2016.
[16] J. Cheng, J. H. Park, Y. J. Liu, Z. Liu, and L. Tang, "Finite-time H_∞ fuzzy control of nonlinear Markovian jump delayed systems with partly uncertain transition descriptions," *Fuzzy Sets Syst.*, vol. 314, pp. 99–115, May 2017.
[17] S. Xu, J. Lam, and X. Mao, "Delay-dependent H_∞ control and filtering for uncertain Markovian jump systems with time-varying delays," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 54, no. 9, pp. 2070–2077, Sep. 2007.
[18] H. Shen, Z.-G. Wu, and J. H. Park, "Reliable mixed passive and H_∞ filtering for semi-Markov jump systems with randomly occurring uncertainties and sensor failures," *Int. J. Robust Nonlinear Control*, vol. 25, no. 17, pp. 3231–3251, Nov. 2015.
[19] X. Li, J. Lam, H. Gao, and J. Xiong, "H_∞ and H_2 filtering for linear systems with uncertain Markov transitions," *Automatica*, vol. 67, pp. 252–266, May 2016.
[20] H. Shen, F. Li, Z.-G. Wu, and J. H. Park, "Finite-time asynchronous H_∞ filtering for discrete-time Markov jump systems over a lossy network," *Int. J. Robust Nonlinear Control*, vol. 26, no. 17, pp. 3831–3848, Nov. 2016.
[21] M. Shen, D. Ye, and Q.-G. Wang, "Event-triggered H_∞ filtering of Markov jump systems with general transition probabilities," *Inf. Sci.*, vols. 418–419, pp. 635–651, Dec. 2017.
[22] F. Li, P. Shi, X. Wang, and R. Agarwal, "Fault detection for networked control systems with quantization and Markovian packet dropouts," *Signal Process.*, vol. 111, pp. 106–112, Jun. 2015.
[23] X. Su, P. Shi, L. Wu, and Y.-D. Song, "Fault detection filtering for nonlinear switched stochastic systems," *IEEE Trans. Autom. Control*, vol. 61, no. 5, pp. 1310–1315, May 2016.
[24] S. He, "Fault detection filter design for a class of nonlinear Markovian jumping systems with mode-dependent time-varying delays," *Nonlinear Dyn.*, vol. 91, no. 3, pp. 1871–1884, Feb. 2018.
[25] H. Shen, Y. Zhu, L. Zhang, and J. H. Park, "Extended dissipative state estimation for Markov jump neural networks with unreliable links," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 2, pp. 346–358, Feb. 2017.
[26] H. Shen, S. Huo, J. Cao, and T. Huang, "Generalized state estimation for Markovian coupled networks under round-robin protocol and redundant channels," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1292–1301, Apr. 2019.
[27] G. Wang, Q. Zhang, and C. Su, "Adaptive control of singular Markovian jump systems with uncertain switchings," *IET Control Theory Appl.*, vol. 9, no. 12, pp. 1766–1773, Aug. 2015.
[28] H. Li, P. Shi, and D. Yao, "Adaptive sliding-mode control of Markov jump nonlinear systems with actuator faults," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 1933–1939, Apr. 2017.
[29] X. Liu, W. P. Tay, Z.-W. Liu, and G. Xiao, "Quasi-synchronization of heterogeneous networks with a generalized Markovian topology and event-triggered communication," *IEEE Trans. Cybern.*, early access, Jan. 24, 2019, doi: 10.1109/TCYB.2019.2891536.
[30] C. E. de Souza, "Robust stability and stabilization of uncertain discrete-time Markovian jump linear systems," *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 836–841, May 2006.
[31] G. L. Wang, "Mode-independent control of singular Markovian jump systems: A stochastic optimization viewpoint," *Appl. Math. Comput.*, vol. 286, pp. 527–538, Aug. 2016.
[32] G. Wang, B. Li, Q. Zhang, and C. Yang, "A partially delay-dependent and disordered controller design for discrete-time delayed systems," *Int. J. Robust Nonlinear Control*, vol. 27, no. 16, pp. 2646–2668, Nov. 2017.
[33] G. Wang and C. Yi, "Fault estimation for nonlinear systems by an intermediate estimator with stochastic failure," *Nonlinear Dyn.*, vol. 89, no. 2, pp. 1195–1204, Jul. 2017.
[34] G. Wang, Q. Zhang, and C. Yang, "Fault-tolerant control of Markovian jump systems via a partially mode-available but unmatched controller," *J. Franklin Inst.*, vol. 354, no. 17, pp. 7717–7731, Nov. 2017.
[35] L. Zhang, T. Yang, and P. Colaneri, "Stability and stabilization of semi-Markov jump linear systems with exponentially modulated periodic distributions of sojourn time," *IEEE Trans. Autom. Control*, vol. 62, no. 6, pp. 2870–2885, Jun. 2017.
[36] G. L. Wang and Y. Y. Sun, "Consensus of multi-agent systems with random switching topologies and its application," *Int. J. Robust Nonlinear Control*, vol. 30, no. 5, pp. 2079–2096, 2020.
[37] M. Tanelli, B. Picasso, P. Bolzern, and P. Colaneri, "Almost sure stabilization of uncertain continuous-time Markov jump linear systems," *IEEE Trans. Autom. Control*, vol. 55, no. 1, pp. 195–210, Jan. 2010.
[38] W. Xiang, G. Zhai, and C. Briat, "Stability analysis for LTI control systems with controller failures and its application in failure tolerant control," *IEEE Trans. Autom. Control*, vol. 61, no. 3, pp. 811–816, Mar. 2016.
[39] D. P. De Farias, J. C. Geromel, J. B. R. Do Val, and O. L. V. Costa, "Output feedback control of Markov jump linear systems in continuous-time," *IEEE Trans. Autom. Control*, vol. 45, no. 5, pp. 944–949, May 2000.