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Compatibility-Based Approach for Routing and Scheduling the Demand Responsive Connector

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ABSTRACT This study concentrates on the routing and scheduling problem of Demand Responsive Connector to build feeder plans for people traveling from and to transit hub. An in-depth analysis on the characteristics of feeder services was implemented to inspire the compatibility-based algorithm design. With the goal of reducing operating cost and passenger inconvenience, the proposed algorithm took several factors critical to the real-word operation into consideration, such as double time window assurance (the time constraints at the beginning and end of passenger travels), the flexibility of feeder plans, and the number of vehicles. Our method was validated on numerical instances of 400, 600, 800 and 1000 passengers. Simulation results show that the compatibility-based algorithm can effectively reduce the number of vehicles with acceptable increase of passengers' inconvenience, and can improve the algorithm efficiency considerably. In addition, the setting of flexible time window of shutter plan can hold some elasticity for feeder services. Sensitivity analysis was conducted to help service providers evaluate the trade-off between the operation cost and level of service.

INDEX TERMS Compatibility-based approach, demand responsive connector, routing and scheduling.

I. INTRODUCTION

The transfer problem along metro lines has been studied extensively in the context of multi-model transit networks. A well-balanced job-housing layout is a good method to reduce the commuting trips predictably [1]–[3], and also helps to mitigate the stress of transfer. But for a mature city where the job-housing pattern is already shaped, regular bus, public bicycle, and bicycle-sharing, are common ways to facilitate transfers [4]–[6], and have restrained applications in urban periphery for high operation cost or limited travel distance [7], [8]. Besides, the travel characteristic of elderly passengers needs extra attentions to construct an elderly friendly transportation system [9], [10]. Demand Responsive Connector (DRC) also provides connections for people to and from transit hub, commercial center, or some other gathering places. The shuttle bus operates in a demand responsive way, and passengers are required to reserve in advance. A properly scheduled feeder system can transfer commuters who live in the periphery of the city to transit hubs or send customers to shopping centers, with less walking distance and shorter waiting time. The transit hub and shopping center can also benefit from the DRC with expanded service area and more attracted customers.

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Vehicle Routing Problem (VRP), Pickup and Delivery Problem (PDP) and Dial-a-ride Problem (DARP) are three similar problems to the routing and scheduling problem of DRC, and can provide good references for our research.

The vehicle routing problem is to search the shortest route to deliver goods from a certain depot to some scattered demanders [11]. Different variants of VRP were developed in different application scenarios, e.g., Vehicle Routing Problem with Time Windows (VRPTW) [12], Vehicle Routing Problem with Multiple Depots (MDVRP) [13], Multi Depot Multi Period Vehicle Routing Problem with a Heterogeneous Fleet (MDMPVRPHF) [14], and Stochastic Vehicle Routing Problem (SVRP) [15]. Most variants focused on the time constraints of visiting distributed points, the number of available depots, and the fleet size restriction. Common algorithms for solving VRP include Tabu Search Algorithms, Genetic Algorithms and so on. The pickup and delivery problem, also named as Pickup and Delivery Vehicle Routing Problem (PDVRP), transports goods between pickup and delivery locations [16]. Unlike VRP with only one depot, PDP provides many-to-many cargo service, which features more than one depots. Compared with PDP where the demand points are often unpaired, DARP serves passengers with paired origins and destinations. Unlike in VRP and PDP, the passenger convenience must be considered in DARP. Various versions of DARP were also proposed, such as the Integrated Dial-a-Ride Problem (IDARP) [17], [18], Fleet Size and Mix DARP with Reconfigurable vehicle Capacity (FSM-DARP-RC) [19]. Usually, Algorithms designed for DARP are more complex than those for VRP and PDP [17], [20], [21], due to their extra constraints imposed by passenger transport, e.g., the paired visiting places and the time window constraints of serving passengers.

The routing and scheduling problem of DRC can be seen as a variant of VRP, PDP or DARP. On the one hand, the vehicles of both DRC and VRP operate with one fixed station; on the other hand, DRC shares some characteristics with DARP, such as requiring advanced reservations and taking care of passengers' convenience. Some researches contributed to the pre-evaluation in the planning level of DRC. Quadrifoglio *et al.* and Li *et al.* concentrated on the optimal zone design of DRC operation [22]–[24]. The service area with several transit stations was divided into zones and each zone was served by dedicated feeder buses. Wang *et al.* focused on the service zone optimization around single transit station, and different zones are connected to the transit station by a line-haul distance [25]. For the situation of one-vehicle operation and two-vehicle operation, [26] studied the critical demand of DRC to justify the switch from traditional bus services to DRC. Chandra *et al.* discussed the service area and passenger demands of DRC to explore the optimal cycle length of feeder trips [27]. They also predicted the performance of DRC using the index of street connectivity in their subsequent research [28]. A number of papers focused on feeder service organization and algorithm design. Kim *et al.* integrated the conventional bus and DRC into a feeder system with line haul distance [29]. It combined the analytic optimization and genetic algorithm to derive the optimal headways, fleet size and other parameters related to operation. Pickup and Delivery Problem with Shuttle routes (PDPS) is special case of PDP, and it can be seen as the combination of two DRC systems and a line haul distance. Aiming to optimize the service design of the PDPS, [30] proposed a branch-and-cut-and-price approach to improve the quality and speed of solution. Customized bus, operating with predetermined bus routes which is consist with local passenger travel pattern, also shares some operating features with DRC. Liu *et al.* analyzed the operation planning process of customized bus [31]. Focusing on the customized bus service network design, [32] jointly optimized the passenger-to-vehicle assignment problem and vehicle routing problem, and the passenger convenience was considered in constraints instead of the objective function in their model. Presented with full spatial-temporal constraints, [33] optimized vehicle routes and passenger assignment procedure to cut down the operation cost. Yu *et al.* tailored the DRC service for people traveling from a fixed rail station to their final work destinations, and a bi-level nonlinear mixed integer programming model is constructed to tackle the feeder bus network design problem (FBNDP) [34]. Focus on FBNDP, [35] optimized the collection points and vehicle routes to minimize the access cost of passengers and the operation cost. Guo *et al.* designed an exact ϵ-constraint method to solve the FBNDP, and discussed the influence of maximum walk time of passengers and route length constraint [36]. Lee *et al.* extended the regional DRC to allow alternative transit stations for passengers [37]. When transit services are frequent, or transit hubs are close together, the extended system would have little impact on passenger conveniences and bring operation cost advantages. Sun *et al.* studied the routing problem of vehicles dispatched from several depots [38]. Passengers with multiple alternative time windows were collected to rail station, and their satisfaction were measured by the deviation of the expected travel time. Jaw *et al.* constructed vehicle routes and schedules with predetermined demand stations and time tables, which impose great restrictions on feeder plan [39].

Compared with VRP and PDP, studies on DRC are insufficient in both amount and depth. For example, most researches ignored the time window constraints at the transit hub where good transfers can ensure passengers to proceed their journeys as expected. In [40], passengers' personalized subway schedules were taken as input to realize good transfers between DRC and rail transit. However, successful transfers could not be guaranteed for the soft constraint of passengers' arrival time [40]. Besides, to authors' knowledge, no research has discussed the elasticity of DRC operation plan, where unforeseen delay often occurs because of some stochastic factors. In the area of traditional bus operation, the robustness of operation plan has received more and more attention to increase service regularity [41], [42] however. In addition, researchers usually prefer not to optimize the number of vehicles considering the low demand level of DRC. However, with more and more people fleeing from big city actively or committing to suburban passively, particularly in China, the ridership of DRC will greatly increased, and the optimization of vehicle number also needs to be considered in the feeder plan.

This paper focused on the routing and scheduling problem of DRC. An operation model was build to give consideration simultaneously to the double time window assurance, the flexibility of feeder plans, and the number of vehicles. Based on the in-depth analysis of the characteristics of commuting travel, the notion of "compatibility" was proposed to reflect the chance of riding the same bus for two passengers. Resort to the notion of "compatibility", we developed a compatibility-based algorithm which can allocate a better initial solution and effectively speed up the solving process. All in all, following aspects were specially considered in our research:

1) Analyse that how individual appointment time affects the bus schedule;

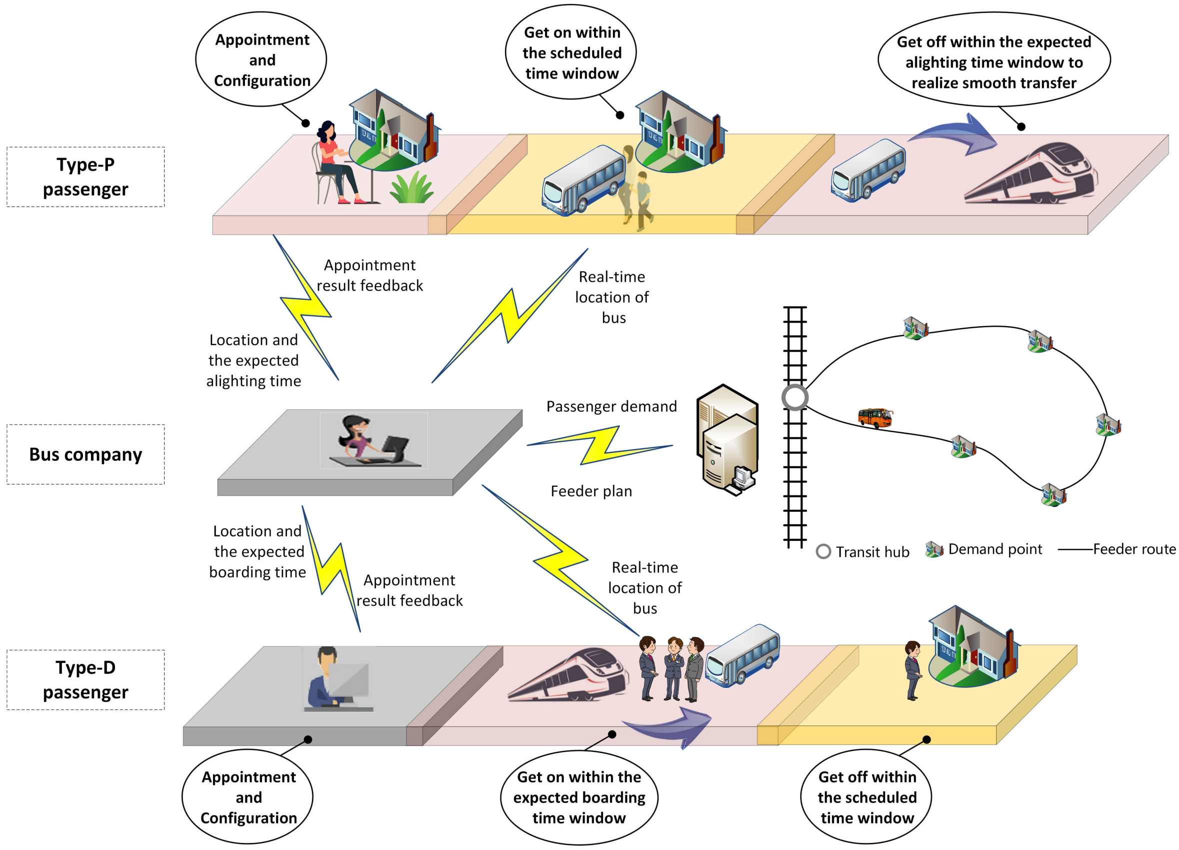

FIGURE 1. The DRC system.

2) Propose a compatibility-based algorithm to build feeder plans;
3) Adopt hard-time windows at the beginning and end of trips to ensure passenger smooth transfer;
4) Optimize the number of vehicles and retain some elasticity of feeder plan for unforeseen delay;
5) Set the maximum and minimum passengers of each feeder trip to avoid uneconomic short routes and overload problem.

II. PROBLEM STATEMENT

In this paper, we narrow the DRC to provide feeder service for commuters from and to the transit hub. Two types of passengers are defined: type P passengers who need to be picked up to the transfer point, and type D passengers who need to be delivered from the transfer point. As shown in Fig. 1, at the reservation stage, type P passengers should specify origins and expected arrival time at transit hub; while type D passengers need to provide destinations and ideal boarding time at transit hub. Feeder buses always set out from the same transit hub (transfer point) and return back after service. Considering the travel characteristics of most commuters who head to transit hub at morning and return after work, two types of passengers will be served separately.

Other assumptions and explanations of our research are stated as follows:

1) Type P passengers care more about their arrival time at the transit hub to transfer to the fixed transit lines; whereas type D passengers are more concerned with the boarding time so they can catch up the feeder bus to return home.
2) The travel time between visiting points are static, and can be estimated by distance divided by speed, or some other ways, such as Baidu GIS.
3) The Feeder bus must pick up and drop off passengers within certain time windows, and is not allowed to idle to wait the visiting time of passengers.
4) Not all passengers can reserve feeder service successfully because of the limited number of vehicles and the minimum ridership requirement.

III. OPERATION ANALYSIS

With the reservation information, a fixed time window (w) is specified by bus company to allow appropriate deviations from the expected boarding and alighting time of passengers. The Maximum Ride Time (*MRT*) is defined to enable some detours relative to the Direct Ride Time (*DRT*). The setting of w and *MRT* ensure the feasibility and flexibility of feeder plans. For each passenger, four time points are generated: the

Earliest Pick-up Time (*EPT*), the Latest Pick-up Time (*LPT*), the Earliest Drop-off Time (*EDT*) and the Latest Drop-off Time (*LDT*), and they can be calculated as in [43].

For a type P passenger i_k, we have (1)-(3), where the appointment time is used as the latest drop-off time (LDT_{i_k}) at transit hub to ensure the expected transfers to fixed transit lines.

$$EDT_{i_k} = LDT_{i_k} - w \tag{1}$$
$$LPT_{i_k} = LDT_{i_k} - DRT_{i_k} \tag{2}$$
$$EPT_{i_k} = EDT_{i_k} - MRT_{i_k} \tag{3}$$

For a type D passenger j_r, the appointment time is regarded as the earliest pick-up time (EPT_{j_r}) at transit hub in case passenger misses the feeder bus, and we have (4)-(6).

$$LPT_{j_r} = EPT_{j_r} + w \tag{4}$$
$$EDT_{j_r} = EPT_{j_r} + DRT_{j_r} \tag{5}$$
$$LDT_{j_r} = LPT_{j_r} + MRT_{j_r} \tag{6}$$

The relationships between *MRT* and *DRT* are showed as (7)-(8).

$$MRT_{i_k} = f(DRT_{i_k}) = q \times DRT_{i_k} + z \tag{7}$$
$$MRT_{j_r} = f(DRT_{j_r}) = q \times DRT_{j_r} + z \tag{8}$$

where, q and z are parameters controlling the maximum ride time deviation of passengers. Further knowledge of equations above can refer to [43]. Other variables besides above ones are declared in Table. 1.

A. THE COMMON TIME WINDOW OF PASSENGERS SHARING ONE FEEDER BUS

This article considers both the boarding time and alighting time of all passengers. It is logical that passengers taking one feeder bus must have a common time window at the transit hub. For type P passengers, people taking the trip m must have a common alighting time window $(TW_1^{P,m}, TW_2^{P,m})$ (see (9)-(10)). While, a common boarding time window $(TW_1^{D,g}, TW_2^{D,g})$ should exist for type D passengers taking trip g (see (11)-(12)).

$$TW_1^{P_m} = max\{EDT_{i_k} | i_k \in P_m\} \tag{9}$$
$$TW_2^{P_m} = min\{LDT_{i_k} | i_k \in P_m\} \tag{10}$$
$$TW_1^{D_g} = max\{EPT_{j_r} | j_r \in D_g\} \tag{11}$$
$$TW_2^{D_g} = min\{LPT_{j_r} | j_r \in D_g\} \tag{12}$$

After passengers joining specific feeder routes, their initial four time points need to be renewed because of the contracted time window at transit hub.

For passenger i_k taking the feeder trip m, the four time points are renewed as Equations (13)-(16)

$$EDT_{i_k l}^{P,m} = TW_1^{P,m}(i_k \in P_m) \tag{13}$$
$$LDT_{i_k l}^{P,m} = TW_2^{P,m}(i_k \in P_m) \tag{14}$$
$$EPT_{i_k l}^{P,m} = TW_1^{P,m} - MRT_{i_k}(i_k \in P_m) \tag{15}$$
$$LPT_{i_k l}^{P,m} = TW_2^{P,m} - DRT_{i_k}(i_k \in P_m) \tag{16}$$

TABLE 1. Variable declaration.

Variables	Declaration
EPT_{i_k}, LPT_{i_k}	The earliest pick-up time, the latest pick-up time of the i_k
EDT_{i_k}, LDT_{i_k}	The earliest drop-off time and the latest drop-off time of the i_k
EPT_{j_r}, LPT_{j_r}	The earliest pick-up time, the latest pick-up time of j_r
EDT_{j_r}, LDT_{j_r}	The earliest drop-off time and the latest drop-off time of j_r
$(TW_1^{P,m}, TW_2^{P,m})$	The common time window of passengers taking feeder trip *m*
$(TW_1^{D,g}, TW_2^{D,g})$	The common time window of passengers taking feeder trip *g*
$RTW^{P,m}, RTW^{D,g}$	The flexible time window of the route *m* and route *g*
$EPT_{i_k}^{P,m}, LPT_{i_k}^{P,m}$	The renewed earliest pick-up time and the renewed latest pick-up time of i_k after joining in trip *m*
$EDT_{j_r}^{D,g}, LDT_{j_r}^{D,g}$	The renewed earliest drop-off time and the renewed latest drop-off time of j_r after joining the trip *g*
$EST^{P,m}, LST^{P,m}$	The earliest and the latest start time of trip *m* at transit hub
$EAT_{i_k}^{P,m}, LAT_{i_k}^{P,m}$	The earliest and the latest time to visit the boarding place of i_k during trip *m*
$ERT^{P,m}, LRT^{P,m}$	The earliest and the latest return time of vehicle running trip *m*
$EST^{D,g}, LST^{D,g}$	The earliest and the latest start time of trip *g* at transit hub
$EAT_{j_r}^{D,g}, LAT_{j_r}^{D,g}$	The earliest and the latest time to visit the alighting place of j_r during trip *g*
$ERT^{D,g}, LRT^{D,g}$	The earliest and the latest return time of vehicle running the trip *g*
$AST^{P,m}$,$APT_{i_k}^{P,m}$, $ART^{P,m}$	(For trip *m*), The actual setting out time, the actual time picking up i_k, and the actual time returning to the transit hub
$AST^{D,g}$,$ADT_{j_r}^{D,g}$, $ART^{D,g}$	(For trip *g*), The actual setting out time, the actual time dropping off j_r, and the actual time returning to the transit hub
$TB_{i_k}^{P,m}, TA_{i_k}^{P,m}$	The travel time Before and After the vehicle reaching the boarding place of i_k during feeder trip *m*
$TB_{j_r}^{D,g}, TA_{j_r}^{D,g}$	The travel time Before and After the vehicle reaching the alighting place of j_r during feeder trip *g*
$T^{P,m}, T^{D,g}$	The whole travel time of trip *m* and trip *g*
P_m, D_g	The set of type P passengers served by trip *m*; the set of type D passengers served by trip *g*
Q_{min}, Q_{max}	The minimum and maximum number of passengers on the bus
V_{max}	The maximum number of vehicles
M; G	The total number of trips serving type P passengers and The total number of trips serving type D
N; V; H_v	The collection of all trips, $N = M \cup G$;The collection of all vehicles;Trips served by vehicle v
$x_n^{v,h}$	Whether the trip n is the h-th trip of vehicle v
$t_{(i_k,i_{k+1})}$, $t_{(j_r,j_{r+1})}$	The minimum and maximum number of passengers on the bus

For passenger j_r taking the trip g, we have (17)-(20).

$$EPT_{j_r}^{D,g} = TW_1^{D,g}(j_r \in D_g) \tag{17}$$
$$LPT_{j_r}^{D,g} = TW_2^{D,g}(j_r \in D_g) \tag{18}$$
$$EDT_{j_r}^{D,g} = TW_1^{D,g} + DRT_{j_r}(j_r \in D_g) \tag{19}$$
$$LDT_{j_r}^{D,g} = TW_2^{D,g} + MRT_{j_r}(j_r \in D_g) \tag{20}$$

B. THE RELATIONSHIP BETWEEN INDIVIDUAL PASSENGERS AND FEEDER ROUTES

Equations (13)-(20) constrain the feeder plan form the perspective of individual passengers. However, a feeder trip always serves more than one customer, so we need to clarify that how passengers influence the feeder plan. Fig. 2 illustrates the visiting time windows of feeder trips. Each rectangle represents a time window of single visiting point (corresponding to one passenger), and the first one and last one denote time window at the transit hub. The start time and end time of rectangles are the renewed access time of passengers (see (13)-(20)). The length of the black part is the actual visiting time window because of the interaction of passengers. Note that the horizontal distance between two visiting points represents the travel time between them, and the horizontal order corresponds with the sequence of access time of passengers. The vertical distance is not the actual space distance, so the slope of the line is inconsistent.

Fig. 2 (a) details the visiting time windows of a trip m serving type P passengers. The latest return time of the feeder trip m ($LRT^{P,m}$) is set as the upper bound of the common time window (see (21)). By doing this, all passengers can reach the transfer station before their expected arrival time. Then, we calculate the latest visiting time of each passenger in a backward way (see (22)), and do the same for the latest start time ($LST^{P,m}$) of feeder bus (see (23)). Equations (24)-(26) ensure that the feeder bus to collect passengers after their earliest pick-up time.

$$LRT^{P,m} = TW_2^{P,m} \tag{21}$$
$$LAT_{i_k}^{P,m} = LRT^{P,m} - TA_{i_k}^{P,m} \tag{22}$$
$$LST^{P,m} = LRT^{P,m} - T^{P,m} \tag{23}$$
$$EST^{P,m} = max\{EPT_{i_k}^{P,m} - TB_{i_k}^{P,m} | i_k \in P_m\} \tag{24}$$
$$EAT_{i_k}^{P,m} = EST^{P,m} + TB_{i_k}^{P,m} \tag{25}$$
$$ERT^{P,m} = EST^{P,m} + T^{P,m} \tag{26}$$

Similarly, Fig. 2 (b) details the visiting time window of the trip g serving type D passengers. The earliest time for feeder bus to visit each unloading place is calculated in a forward way (see (27)-(29)), while the latest deliver time is derived in a backward way like (30)-(32).

$$EST^{D,g} = TW_1^{D,g} \tag{27}$$
$$EAT_{j_r}^{D,g} = EST^{D,g} + TB_{j_r}^{D,g} \tag{28}$$
$$ERT^{D,g} = EST^{D,g} + T^{D,g} \tag{29}$$
$$LRT^{D,g} = min\{LDT_{j_r}^{D,g} + TA_{j_r}^{D,g} | j_r \in D_g\} \tag{30}$$
$$LAT_{j_r}^{D,g} = LRT^{D,g} - TA_{j_r}^{D,g} \tag{31}$$
$$LST^{D,g} = LRT^{D,g} - T^{D,g} \tag{32}$$

C. THE FLEXIBLE TIME WINDOW OF FEEDER ROUTE

As shown in Fig. 2, the black part of rectangles represent the actual visiting time windows, and they are of equal length actually. We take trip m as an example:

When feeder bus picks up passenger i_k, we have:

$$\begin{aligned} &LAT_{i_k}^{P,m} - EAT_{i_k}^{P,m} \\ &= (LRT^{P,m} - TA_{i_k}^{P,m}) - (EST^{P,m} + TB_{i_k}^{P,m}) \\ &= LRT^{P,m} - EST^{P,m} - (TA_{i_k}^{P,m} + TB_{i_k}^{P,m}) \\ &= LRT^{P,m} - EST^{P,m} - T^{P,m} \end{aligned} \tag{33}$$

Equation (33) indicates that the length of visiting time window is not related to k, so the actual visiting time window of all passengers in trip m are of equal length. In fact, these access time windows shape a elastic time band for the feeder plan, and the band width of route m ($RTW^{P,m}$) is calculated as (34).

$$RTW^{P,m} = LRT^{P,m} - EST^{P,m} - T^{P,m} \tag{34}$$

Similarly, for feeder trip g serving type D passengers, we have:

$$RTW^{D,g} = LRT^{D,g} - EST^{D,g} - T^{D,g} \tag{35}$$

The time band gives the feeder plan some elasticity allowing some unforeseen delay along the way, and leaves the chance to further optimize the schedule of feeder bus so as to reduce the number of vehicles.

IV. MODEL FORMULATION

A. OBJECTIVE FUNCTION

The optimization goal is to minimize the disutility of passengers and the operation cost. For type P passengers, the disutility is measured by the weighted sum of passengers' arrival time deviations and their ride time deviations (see (36)). Similarly, the disutility of type D passengers is calculated as the weighted sum of passenger's boarding time deviations and their ride time deviations (see (37)). The operation cost is measured by the total running time((38)) and vehicle configuration cost which directly related to the number of vehicles.

$$C_1^P = \sum_{m=1}^{M} \sum_{i_k \in P_m} [a_1(LDT_{i_k} - ART^{P,m}) + a_2(TA_{i_k}^{P,m} - DRT_{i_k})] \tag{36}$$

$$C_1^D = \sum_{g=1}^{G} \sum_{j_r \in D_g} [a_1(AST^{D_g} - EPT_{j_r}) + a_2(TB_{j_r}^{D,g} - DRT_{j_r})] \tag{37}$$

$$C_2 = \sum_{m=1}^{M} T^{P,m} + \sum_{g=1}^{G} T^{D,g} \tag{38}$$

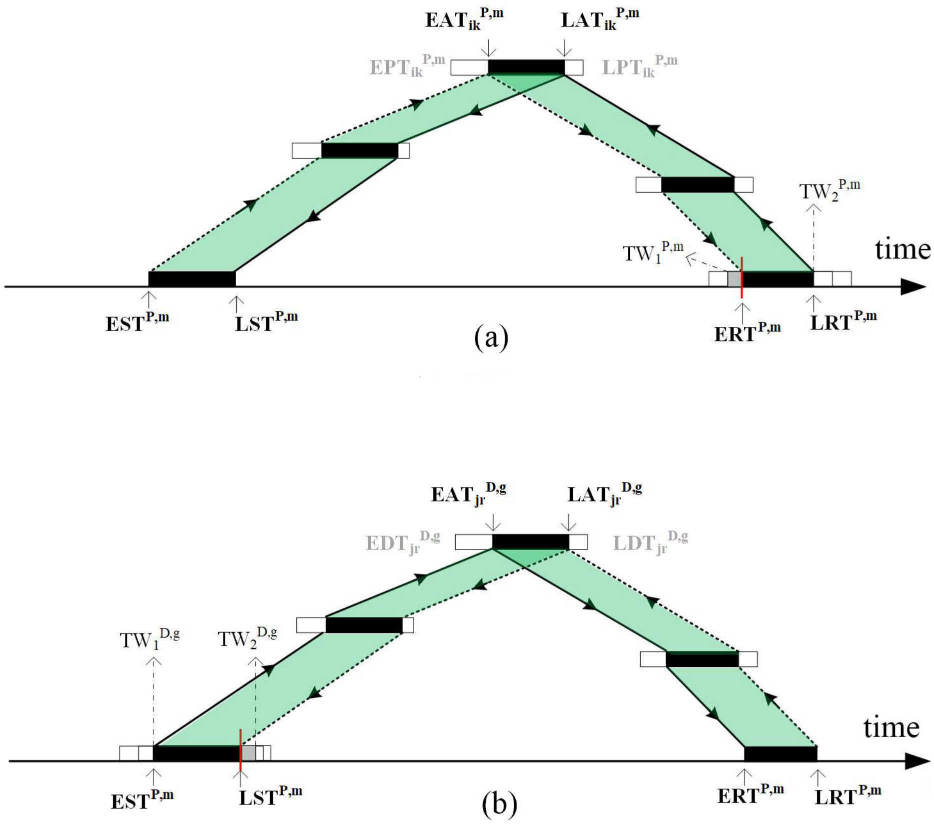

FIGURE 2. **The constraints faced by feeder trips.**

Thus the objective function can be expressed as (39).

$$Minimize\ C = b_1(C_1^P + C_1^D) + b_2 \times C_2 + b_3 \times V \quad (39)$$

where a_1 and a_2 are coefficients to weigh the travel deviation of passengers; b_1, $b_2$and b_3 are to value the passengers' disutility, total running time and the number of vehicle configuration.

B. CONSTRAINTS

From the perspective of passengers, trips and vehicles respectively, we formulate following constraints for the routing and scheduling problem of DRC operation.

$$max\{EDT_{i_k} | i_k \in P_m\} < \{LDT_{i_k} | i_k \in P_m\}, \quad \forall m \in M \quad (40a)$$

$$max\{EPT_{j_r} | j_r \in D_g\} < \{LPT_{j_r} | j_r \in D_g\}, \quad \forall g \in G \quad (40b)$$

$$EPT_{i_k}^{P,m} \leq APT_{i_k}^{P,m} \leq LPT_{i_k}^{P,m}, \quad \forall i_k \in P_m,\ m \in M \quad (41a)$$

$$EDT_{j_r}^{D,g} \leq ADT_{j_r}^{D,g} \leq LDT_{j_r}^{D,g}, \quad \forall j_r \in D_g,\ g \in G \quad (41b)$$

$$TA_{i_k}^{P,m} \leq MRT_{i_k}, \quad \forall i_k \in P_m,\ m \in M \quad (42a)$$

$$TB_{j_r}^{D,g} \leq MRT_{j_r}, \quad \forall j_r \in D_g,\ g \in G \quad (42b)$$

$$1 - (\sum_{m=1}^{M} |P_m| + \sum_{g=1}^{G} |D_g|)/(|P| + |D|) < \delta \quad (43)$$

$$EST_n \leq AST_n \leq LST_n, \quad \forall n \in N \quad (44a)$$

$$ERT_n \leq ART_n \leq LRT_n, \quad \forall n \in N \quad (44b)$$

$$Q_min \leq P_n \leq Q_max, \quad \forall n \in N \quad (45)$$

$$RTW_n \geq \theta, \quad \forall n \in N \quad (46)$$

For $\forall n, n' \in N, v \in V, h \in H_v$:

$$AST_n(1 - x_n^{v,h}) < ART_{n'}(1 - x_{n'}^{v,h-1}) \quad (47)$$

$$\sum_{v=1}^{V} \sum_{h=1}^{H_v} x_n^{v,h} = 1, \quad \forall n \in N \quad (48)$$

$$V \leq V_{max} \quad (49)$$

Equation (40) is to ensure that passengers in the same bus have common time window at the transit hub. Equation (41) constrains that every type P passenger(type D passenger) should be collected (distributed) within their own time windows. Equation (42) guarantees that the in-vehicle time of passengers no longer than their maximum ride time. Equation (43) limits the reservation failure rate of passengers. Constraint of (44) restricts the actual start and end time of trips. Equation (45) avoids the over load of trips, where P_n is the number of passengers of trip n. The constraint described by (46) is to give each feeder trip elasticity of θ minutes at least. Equation (47) requires the starting time of next running must later than the return time of previous running for the same feeder bus. Equation (48) guarantees that every trip will be covered by a vehicle. The maximum number of vehicles is limited in (49).

V. THE COMPATIBILITY-BASED ALGORITHM

In this section, we define the compatibility of passengers first, which reflects the characteristic of commuter travels. Different with other heuristic algorithm, we cooperated the compatibility analysis result into the algorithm design. This trick would build a more closely band between the algorithm design and the operation problem of DRC. A compatibility-based heuristic algorithm is thus proposed to build feeder routes and schedules for type P passengers and type D passengers separately in the **PROBLEM STATEMENT**. At last, the schedule will be jointly adjusted to reduce the number of vehicles in **Vehicle Deficit**.

A. COMPATIBILITY ANALYSIS

Compatibility analysis is to determine whether two passengers have the chance to ride the same bus. The compatibility of two passengers depends on two conditions: sharing common time window at the transit hub (**Condition 1**), and the time window of both passengers can be satisfied if a vehicle is dispatched to serve them specially (**Condition 2**). The compatibility of type P passengers and type D passengers will be discussed separately.

Let i_k and i_r denote two passengers of type P. The common time window at the transit hub means that their expected arrival time are overlapped, and can be calculated by (9)-(10). Then, their time points are renewed according to (13)-(16). The compatibility of two type P passengers can be judged by following conditions:

- **Condition 1** is met **if** $TW_1^{P,new} < TW_2^{P,new}$;
- **Condition 2** is met **if** $max\{EPT_{i_k}^{P,new} + t_{kr}^P, EPT_{i_r}^{P,new}\} < min\{LPT_{i_k}^{P,new} + t_{kr}^P, LPT_{i_r}^{P,new}\}$ (when serving i_k first) **OR** $max\{EPT_{i_r}^{P,new} + t_{rk}^P, EPT_{i_k}^{P,new}\} < min\{LPT_{i_r}^{P,new} + t_{rk}^P, LPT_{i_k}^{P,new}\}$ (when serving i_r first).

The superscript *new* represents the new route including only passenger i_k and i_r.

We can judge the compatibility of two type D passengers similarly. It is easy to conclude that two "incompatible" passengers can never be served by the same vehicle. The compatibility of passengers can suggest the initial number of vehicles and direct the shortcut of routing and scheduling feeder service.

B. ROUTES AND SCHEDULES

In this part, we will give the objective function of routing and scheduling first, then a three-step compatibility-based algorithm will be developed, including **Initialization**, **Compatibility-based Insertions** and **End-effect Handling**.

1) INITIALIZATION

Based on the result of compatibility analysis, the *Minimum InCompatible Set* (*MICS*) of passengers is generated to determine the initial number of feeder routes and seed passengers. *MICS* is the minimum subset of passengers where any two passengers are incompatible, and passengers outside the set are compatible with at least one passenger of the *MICS*. We take the size of *MICS* as the initial number of routes, and each passenger in *MICS* is regarded as a seed passenger corresponding to one feeder route.

Step(1): Construct the compatibility matrix as follow:

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{bmatrix}$$

Elements in the matrix above represent the compatibility of passengers (1,2 . . . n). x_{ij} values 1 if passenger i is compatible with passenger j, otherwise x_{ij} values 0.

Step (2): Determine the *MICS*

The *MICS* of type P passengers and type D passengers should be determined separately in the same way: firstly, mark the column and row having the most "1"s with "→"; then cross off ("×") all columns and rows compatible with the passenger we marked before;Repeat the above process until all rows and columns are marked with "→" or crossed off with "×". The incompatible set is the set of passengers corresponding to the columns or rows marked with "→". We take eight passengers as an example, and the process of determining the incompatible set of them is as follows:

$$\begin{pmatrix} 0&0&0&1&1&0&1&0 \\ 0&0&1&0&0&0&1&1 \\ 0&1&0&0&1&0&0&1 \\ 1&0&0&0&1&0&0&0 \\ 1&0&1&1&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 1&1&0&0&1&0&0&0 \\ 0&1&1&0&0&1&0&0 \end{pmatrix}$$

(a)

$$\begin{array}{c|cccccccc} & \times & \times & \times & \times & \downarrow & \times & \times & \downarrow \\ \hline \times & 0&0&0&1&1&0&1&0 \\ \times & 0&0&1&0&0&0&1&1 \\ \times & 0&1&0&0&1&0&0&1 \\ \times & 1&0&0&0&1&0&0&0 \\ \rightarrow & 1&0&1&1&0&0&1&0 \\ \times & 0&0&0&0&0&0&0&1 \\ \times & 1&1&0&0&1&0&0&0 \\ \rightarrow & 0&1&1&0&0&1&0&0 \end{array}$$

(b)

Where (b) is the last step of searching for the incompatible set. So one of the incompatible set of the eight passengers is {5,8}. We can repeat the process above to find the incompatible set with the least number of passengers as the *MICS*.

Step (3): Initialize routes and define compatible route set for non-seed customers.

Each passenger in *MICS* is a seed passenger corresponding to one feeder route. We assume that {5,8} is the *MICS* in the previous example, so the initial number of feeder trips is 2, and passenger 5 and 8 are seed passengers. Using "0" to denote the transfer point, we have ROUTE1 = {0,5,0} and ROUTE2 = {0,8,0}.

We define a Compatible Route Set (*CRS*) for each non-seed passenger. The seed passengers of routes in the *CRS* are compatible with the non-seed passenger. The *CRS* of passenger 1 in the previous example is ROUTE1, because passenger 1 is compatible with passenger 5 who is the seed passenger of ROUTE1. Any unrouted passenger can only be served by feeder routes in his *CRS*, so the search range of feasible solutions is reduced effectively.

2) COMPATIBILITY-BASED INSERTIONS

In this part, a parallel insertion heuristic algorithm is used to incorporate unrouted customers into current feeder plan. Considering the length and concise of this paper, we only discuss the process of routing type P passengers. Type D passengers can also be routed following the steps below, and relevant formulas will be given without details.

Step (1): the feasibility of insertion

Whether a passenger can be inserted into a certain route depends on three constraints: (a) the capacity constraint of feeder bus, (b) the maximum ride time constraint of passengers, and (c) the visiting time constraints.

For a type P passenger i_{new} attempting to join the feeder route m between i_v and i_{v+1}, the feasibility of insertion is judged as follows in the order of complexity.

(a) The capacity constraint (see (50)).

$$P_m + 1 < Q_{max} \tag{50}$$

(b) The maximum ride time constraint

The actual ride time should be shorter than the maximum ride time for passenger i_{new}, as well as passengers already included in route m (see (51)-(53)).

$$TA_{i_{new}}^{P,m} \le MRT_{i_{new}} \tag{51}$$

$$\Delta T^{P,m} = t_{i_v,i_{new}} + t_{i_{new},i_{v+1}} - t_{i_v,i_{v+1}} \tag{52}$$

$$TA_{i_k}^{P,m} + \Delta T^{P,m} \le MRT_{i_k}, \quad (k = 1, 2, \ldots, v) \tag{53}$$

where $\Delta T^{P,m}$ is the increased travel time of route m caused by i_{new}. Equation (53) is to validate the maximum ride time constraints of passengers boarding before i_{new}, because the ride time of passengers getting on bus later than i_{new} will not be influenced.

Similarly, for a type D passenger j_{new} attempting to join the route g between j_z and j_{z+1}, we have (54)-(56):

$$TB_{j_{new}}^{D,g} \le MRT_{j_{new}} \tag{54}$$

$$\Delta T^{D,g} = t_{j_z,j_{new}} + t_{j_{new},j_{z+1}} - t_{j_z,j_{z+1}} \tag{55}$$

$$TB_{j_r}^{D,g} + \Delta T^{D,g} \le MRT_{j_r}, \quad (r = z+1, \ldots, D_g) \tag{56}$$

(c) The visiting time constraint

According to the discussion in the section of **OPERATION ANALYSIS**, we can verify the visiting time constraints of all passengers on the bus by (57).

$$LRT^{P,m_{new}} - EST^{P,m_{new}} - T^{P,m_{new}} \ge \theta \tag{57}$$

where the m_{new} in the superscript means the renewed route m including i_{new}, and the θ is the minimum time retained for flexibility of feeder plan. The method of calculating $LRT^{P,m_{new}}$,$EST^{P,m_{new}}$ and $T^{P,m_{new}}$ can look back to the section of **OPERATION ANALYSIS**. The visiting time constraint of type D passengers can be validated in a similar way.

Step (2): The marginal cost caused by new passenger

The marginal cost (ΔC) is the added value of the objective function caused by the new inserted passenger. The new passenger will increase the travel disutility of passengers, including arrival time deviations and detours of some passengers. Besides, the travel time of feeder route will also be extended. The ΔC caused by i_{new} is:

$$\begin{aligned}\Delta C^{P,m} = b_1 \Big\{ & a_1 \left[\Delta TW_2^{P,m} \times |P_m|\right] \\ & + a_1 \left[LDT_{i_{new}} - TW_2^{P,m_{new}}\right] \\ & + a_2 \left[v \times \Delta T^{P,m}\right] + a_2 \left[(TA_{i_new}^{P,m} - DRT_{i_{new}})\right]\Big\} \\ & + b_2 \Delta T^{P,m}\end{aligned}$$

where $\Delta TW_2^{P,m}$ is the reduction of common time window caused by passenger i_{new}. v is the number of passengers boarding before i_{new}. For a type D passenger j_{new}, the marginal cost of new insertion is:

$$\begin{aligned}\Delta C^{D,g} = b_1 \Big\{ & a_1 \left[\Delta TW_1^{D,g} \times |D_g|\right] \\ & + a_1 \left[TW_1^{D,g_{new}} - EPT_{j_{new}}\right] \\ & + a_2 \left[(|D_g| - z) \times \Delta T^{D,g}\right] \\ & + a_2 \left[(TB_{j_new}^{D,g} - DRT_{j_{new}})\right]\Big\} \\ & + b_2 \Delta T^{D,g}\end{aligned}$$

where $|D_g| - z$ is the number of passengers getting off the bus later than j_{new}.

Step (3): determine the next passenger joining in current feeder plan

Step (1) - (2) help to find the best route and insertion location for each unrouted customer. However, only the passenger with the least marginal cost can be selected as the next one joining in current feeder plan.

Procedure **Routes Construction**
While(the proportion of unrouted customers exceeds δ) **Do:**
  Find the MICS for remaining customers;
  Initialize new routes and determine the CRS for each non-seed passenger;
  Repeat
    Step (1) Judge the feasibility of every possible insertion for each unrouted customer;
    Step (2) Calculate the marginal cost of every feasible insertion;
    Step (3) Determine the priority customer to join in current feeder plan;
  Until (none of the remaining customers can join in current routes)
End Do
Perform *end-effect handling*

FIGURE 3. The procedure of routes optimization.

With all details described above, the process of building feeder routes can be organized as Fig.3, where δ is the threshold to execute **End-effect handling** procedure.

3) END-EFFECT HANDLING

We run the routes and schedules construction procedure until only "a few" unrouted passengers left. to avoid generating new short routes, *End-effect handling* try removing a passenger already in feeder plan to find a feasibility insertion location for an unrouted passenger. In this case, a new route must be generated successfully from the removed passenger and the remaining unrouted passengers. In fact, the *CRS* proposed in this paper can limit the search scope of feasible insertions, and speed up the **End-effect handling** process. In this paper, we only introduce the idea of **End-effect handling**, and more details can refer to [44].

C. VEHICLE DEFICIT

After routing and scheduling type P passengers and type D passengers separately, we will adjust the feeder plan slightly to cut down the number of vehicles needed. The logic is to adjust the departure time of certain pairs of feeder routes which are critical to the vehicle configuration. Passengers will not be removed from the feeder plan unless the the number of vehicles exceed the limit. This section contains two parts: **Critical routes detection** and **Schedule adjustment**.

1) CRITICAL ROUTES DETECTION

Deficit Function (DF) is commonly used to determine the number of vehicles and critical routes in traditional bus planning [45]. DF is a step function, counting +1 for a bus departing from the transit hub and counting −1 for a return. In addition, DF can be illustrated intuitively by graph. Fig. 4 is a small example to build the DF when scheduling the DRC feeder services. Each segment in the upper part of Fig. 4 represents a feeder route, and the start and end of the segment represent the departure and return time of a feeder bus respectively.

In the figure of DF, the maximum value of DF is the total number of vehicles needed, and the paired routes corresponding to the peak number of vehicles are critical routes, e.g., the route ⑦ and ⑧ in Fig. 4. Of particular note is that the departure time of routes serving type P passengers are initialized as their latest departure time, while those of routes serving type D passengers are set as their earliest departure time when constructing the DF. This setting can minimize the passenger inconvenience before adjusting the shuttle schedule.

2) SCHEDULE ADJUSTMENT

The number of vehicles will be reduced by 1 if we can eliminate the time gap within every pair of critical routes. For example, we can advance the return time of route ⑦ or postpone the departure time of ⑧, or do both together to flatten the peak of vehicle use. If we want to level the deficit function further, more critical routes need be detected and adjusted. The flexible time window retained before will matter to this procedure.

We divide pairs of critical routes into four types: P+D, P+P, D+D and D+P. Type "P+D" means a pair of critical routes where the route serving type P passengers departs from the transit hub earlier than the return time of the route serving type D passengers. Other three types of critical routes can be similarly defined. There are four adjustment strategies corresponding to four types of critical route pairs. However, because of the departure time setting when building the deficit function, all routes serving type P passengers cannot be postponed and all routes serving type D passengers cannot be advanced further.

(1) Type P+D: The time gap within this type of paired critical routes cannot be eliminated by only adjusting the departure time of routes.

(2) Type P+P: Try to advance the departure time of the later returning feeder route under the limit of *FTW* to eliminate the time gap.

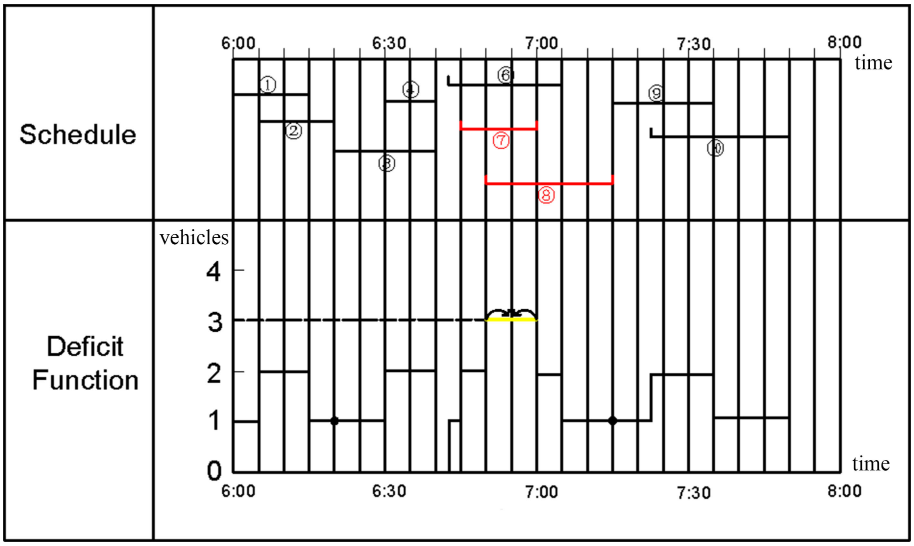

FIGURE 4. Build the deficit function for a feeder schedule.

(3) Type D+D: Try to postpone the departure time of the first leaving feeder route under the limit of *FTW* to eliminate the time gap.

(4) Type D+P: the departure time of both routes can be adjusted. Delaying the departure time of the first leaving feeder route or (and) advancing the departure time of the later returning route to level the deficit function.

Like we said before, the schedule optimization can cut down the number of vehicles without removing any routed passengers. However, revoking some passengers' feeder services will be considered if the maximum number of vehicles (V_{max}) is breached.

VI. COMPUTATIONAL ANALYSIS

As stated in the **INTRODUCTION**, existing researches on the routing and scheduling problem of DRC are limited, and they were developed for different scenarios and focused on different practical problem during DRC operation. It is unfair and cursory to compare our model and algorithm with other studies of different problems during DRC operation. So, three variants of our method are generated to validate the proposed compatibility-based algorithm. **Algorithm 1**: ignore the compatibility analysis and schedule optimization. In this case, the initial number of routes is computed from the total number of unrouted passengers divided by average passenger load, and seed passengers are chosen randomly; **Algorithm 2**: ignore the compatibility of passengers but the schedule optimization; **Algorithm 3**: ignore the schedule optimization but the compatibility analysis. **Algorithm 4** is compatibility-based algorithm proposed in this paper, including both compatibility analysis and schedule optimization. The demand data and parameter setting are stated as follows.

(1) Generated instances: requests distribute uniformly over a square area of 2km wide and 4km long, and demand within 1km of the transit hub will be discarded. The travel time between two demand points is calculated by their Euclidean distance. The appointment time (the expected alighting time) of type P passengers are simulated using a normal distribution with a mean of 9:00 and a standard deviation of 2 hours, and preliminary boarding and alighting time window will be determined by (1)-(3). The appointment time (the expected boarding time) of type D passengers follow a normal distribution with a mean of 17:00 and a standard deviation of 2 hours. Equation (4)-(6)are used to calculate other time points. Fig.5 shows the spatial and temporal distribution of generated requests, where (a) is the overall distribution and (b) is the projection of the demand points onto the spatial plane. Appointment time outside of 8:00 to 18:00 will be discarded. The number of instances are 400, 600, 800 and 1000. Type P passengers and type D passengers are of the same amount.

(2) Parameter setting: a_1=2, a_2=b_1=b_2=1; q=2, z=0.25; RTW=0.1; w=0.25h; δ=0.1; Q_{min}=3, Q_{max}=25; V_{max}=15.

We choose evaluation indicators from the interest of both the service provider and passengers. The number of vehicles and the total travel time are two indicators related to operation cost. Depot inconvenience and trip inconvenience are of concern to passengers. The depot inconvenience is the arrival time deviation for type P passengers, and the departure time deviation for type D passengers. The travel time inconvenience is the time exceeding passengers' desired ride time.

A. SIMULATION RESULT

Our algorithm was coded in MATLAB 2018b and performed on an i7-4790CPU (16RAM) computer. Each simulation was repeated 50 times, and Table.2 shows the simulation result.

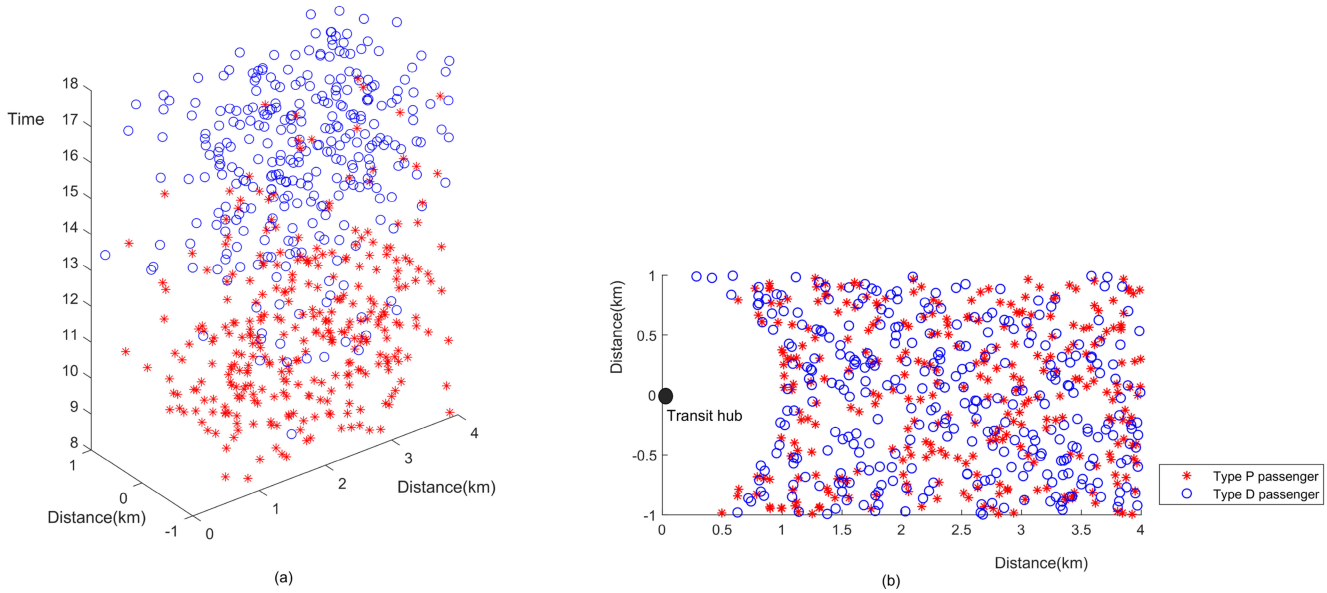

FIGURE 5. The spatial and temporal distribution of requests.

TABLE 2. Simulation results.

Algorithms	1	2	3	4	1	2	3	4
Indicators of operation cost								
Number of requests	Number of vehicles (vehs)				Total travel time (h)			
400	8.52	6.60	4.50	3.30	39.11	38.90	19.72	19.87
600	9.76	8.08	5.14	4.08	52.39	49.83	24.70	24.78
800	11.32	9.48	5.64	4.52	60.71	63.17	28.98	29.20
1000	13.40	10.60	5.92	4.94	73.11	71.54	32.70	33.11
Indicators of service level								
Number of requests	Number of vehicles (vehs)				Total travel time (h)			
400	3.00	3.00	3.42	3.36	6.24	6.36	2.64	3.00
600	3.18	3.24	3.54	3.54	6.66	6.60	3.66	3.66
800	3.24	3.24	3.66	3.66	6.96	6.96	4.38	4.32
1200	3.30	3.30	3.72	3.72	7.08	7.08	4.80	5.04
Simulation time (s)								
Number of requests	Algorithm 1		Algorithm 2		Algorithm 3		Algorithm 4	
400	14		14		2		2	
600	50		49		5		5	
800	83		94		9		9	
1200	240		243		15		15	

From the data above, algorithm 3 and algorithm 4 have clear advantages over algorithm 1 and 2 in reducing the number of vehicles and total travel time. We are pleased to find that a 50 percent reduction on the vehicle number and total travel time have increased the depot inconvenience of passengers by no more than 1 minute. In addition, the trip inconvenience of passenger in algorithm 3 and 4 is even lower than that of algorithm 1 and 2. This result means that apparent decline in operation cost in algorithm 3 and 4 has little influence on the disutility of passengers. The improvement of trip inconvenience may be explained by more compact feeder trips where passengers taking the same bus are closer in space. The data of simulation time shows that algorithm 3 and 4 run almost 10 times faster than algorithm 1 and 2 do.

The simulation result affirms the advantage of compatibility analysis in determining the initial number of routes and choosing the seed passengers. Besides, the compatible route set of passengers effectively narrow the search range of feasible insertions, and contribute to the improvement of computation time. In addition, the increase of passenger demand has resulted a steady growth in vehicle number and total travel time, which validated the stability of algorithm 3 and 4. The schedule optimization also helps to reduce the number of vehicles if we compare the result of algorithm 1 and algorithm 2 (or algorithm 3 and algorithm 4).

B. SENSITIVITY ANALYSIS

The operation cost and level of service always stand on the opposite sides of the routing and scheduling problem of DRC. To figure out how the compatibility-based algorithm performs in this dilemma, our algorithm is conducted with different parameter settings to generate the service plan for 800 customers. We have a sensitivity analysis on the fixed time window (w), the elastic time window (θ), and the maximum number of vehicles (V_{max}). According to (1) and (4), the fixed time window (w) allows some deviation of passenger's appointment time. The elastic time window (θ) is the spare time retained for unforeseen delay, and affects the flexibility of feeder plan. The maximum number of vehicles (V_{max}) can be adjusted according to the vehicle configuration of bus company.

The parameter settings and simulation results are reported in Table. 3, and other parameters are valued as the preceding part.

TABLE 3. The settings and simulation result.

	Indicators of operation cost			Indicators of service level		
	No. vehicles (vehs)	Total travel time (h)	Average load (Persons)	Depot inconvenience (minutes)	Trip inconvenience (minutes)	Rejection rate
w (minutes)						
9	5.7	36.1	5.4	1.2	1.8	0.15
15	4.0	25.8	10.5	3.6	4.2	0.05
30	3.1	20.1	16.2	9.0	6.0	0.01
θ (minutes)						
0	4.0	22.5	13.5	6.2	5.7	0.02
3	3.9	24.0	12.2	4.8	5.4	0.03
6	4.0	25.8	10.5	3.6	4.2	0.05
9	4.1	29.6	8.2	2.4	3.6	0.07
12	5.1	32.8	5.4	1.2	1.8	0.22
V_{max} (vehs)						
1	1.0	10.4	8.2	2.8	3.8	0.73
2	2.0	20.1	8.3	3.0	3.6	0.36
3	3.0	25.8	10.4	3.6	4.8	0.06
4	3.9	25.8	10.6	3.6	4.8	0.05
5	3.9	25.8	10.6	3.6	4.8	0.05
15	4.0	25.8	10.5	3.6	4.2	0.05

According to the simulation results, several findings and possible explanations are presented as follows:

(1) The increase of fixed time window (w) can reduce the number of vehicles and total travel time at the cost of increased passenger inconvenience. According to (1) and (4), wider time window makes passenger spend more time at transit hub. Besides, the increase of average load gave rise to longer feeder trips, which indirectly resulted in more trip inconvenience of passengers.

(2) Larger flexible time window θ brings about lower cycle load, and accordingly the operation cost increases and the passenger inconveniences are reduced. It is somewhat contrary to our expectation that retaining more flexible time when building feeder routes may give more room for schedule optimization to cut down the number of vehicles. A possible explanation might be that the help of θ in schedule optimization cannot offset its impact on average load. There seems to be a subtle relationship between θ and the number of vehicles, and we leave it for future work.

(3) We can see that an ample number of vehicles ensures lower rejection rate of passengers. The service providers can easily make a decision on the number of vehicle configurations based on their expected rejection rate, e.g., the best number of vehicles may be 3 for 800 passengers from the data above.

Based on the above analysis, we may find that w and θ can be regarded as knobs of balancing the operation cost and the passenger inconvenience. Smaller w and larger θ correspond to lower operation cost but higher passenger inconvenience. In addition, larger θ brings more flexibility for the feeder plan. The simulation result of V_{max} in Table. 3 should be interpreted with caution, because that the same rejection rate may correspond to different vehicle numbers in scenarios with different number of passengers. The service providers of DRC should choose appropriate parameter setting according to their acceptable operation cost and expected level of service.

VII. CONCLUSIONS

This paper provided an in-depth analysis on the routing and scheduling problem of DRC. The common time window of passengers at the transit hub and the elastic time window of feeder plan were specially discussed to clarify how individual passengers influence the feasibility and elasticity of feeder plan. A compatibility analysis was implemented to help initialize routes and choose seed passengers when routing and scheduling the feeder service, and it was proved to be helpful in generating better feeder plans and improving the efficiency of the algorithm. In order to be more consistent with the actual operation, we incorporated the double time window of passengers' travel, the elasticity requirement of feeder plan, the maximum vehicle number constraint and the ridership limit into the three-step compatibility-based algorithm. The operation cost of bus company and passengers' inconvenience could be balanced well in the proposed algorithm. The simulation results showed that, compared with other three variants, the proposed algorithm can effectively reduce the number of vehicles and cut down the total travel time by up to 50 percent with acceptable increase of passenger inconvenience. The computation time of algorithm can also be reduced to one tenth. The sensitivity analysis on the fixed time window (w), the elastic time window (θ)

and the maximum number of vehicles (V_{max}) revealed the trade-off between operation cost and passenger satisfaction. The method proposed in this paper can provide the decision maker a pre-evaluation in the planning level of demand responsive connector, and generate feeder plans which balance the operation cost and the level of service well in the operation stage. Future work would concentrate on an in-depth analysis on the factors critical to the performance of feeder plans, such as the subtle relationship between the minimum flexible time window and the number of vehicles.

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