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Computational Intelligence Approach to Dynamic Blood Allocation With ABO-Rhesus Factor Compatibility Under Real-World Scenario

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ABSTRACT The supply chain management of blood products specifically deals with the aspect of efficient planning, implementing, and controlling of the in and out flow processes of blood unity in the blood bank system. Therefore, any improvement in the management of these chains would directly influence the manner in which blood and blood products are distributed to all the various sectors requiring such scarce and precious resources. Generally, the management of blood products is difficult due to the four ABO blood groups, which is further complicated by positive/negative rhesus factors. In this paper, a more simplified and robust dynamic mathematical model is presented for the efficient management of the blood bank. The corresponding sets of governing equations from an existing model are extended to cover the rhesus factors and the solution methods of the newly derived equations are subsequently investigated. In addition, a mathematical representation of the decision making process is presented as a function of the blood bank stockpile. Furthermore, in order to demonstrate the robustness of the developed model and to provide managerial insights, a new global hybrid symbiotic organisms search genetic particle swarm optimization algorithm is developed. Several numerical computations are performed using real-world datasets from the Enugu National Blood Transfusion Centre, in Nigeria, which fall within the monthly initial blood volume bounds of 300 over a period of eight years (2010 - 2018). Finally, the experimental results show that the mathematical model and metaheuristic optimization method proposed in this paper offer a better solution approach for blood allocation in dynamic environments. More so, the impact of some essential control parameters on the results are analysed to help the blood bank managers or decision makers to select accurately the desired parameters for optimal results yield.

INDEX TERMS Blood assignment, blood group, rhesus factors, blood compatibility, symbiotic organisms search, genetic algorithm, particle swarm optimization, symbiotic organisms search genetic particle swarm optimization.

I. INTRODUCTION

The human blood inventory management is characterized by a string of factors, which can be complicated over time [1]. Blood products are usually received from voluntary donations; these products are then stored under ideal conditions for future use by the blood banks. According to Karl Landsteiner in 1901, human blood can be classified into four main groups known as the ABO system. Then, in 1940 another system of blood grouping, known as the rhesus (Rh) blood group system was discovered, giving a total of eight major blood groups for red blood cells, namely, A^+, A^-, B^+, B^-, AB^+, AB^-, O^+, and O^-. These blood groups are significant with regards to storage and distribution, as compromising or mixing incompatible blood types can lead to blood clumping (also known as agglutination), which can be life-threatening for most patients [1]. The distribution of blood groups within a population may differ from country to country. For example, in South Africa, the O and A blood groups are known to be dominant, with 46% and 37% blood type proportions, respectively. In contrast, the B and AB blood groups are known to be less assertive, with 14% and 4% blood type proportions, respectively [2].

Blood preservation for immediate supply in the case of unexpected demand can be a daunting task. One reason is blood products are perishable commodities so have limited

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shelf life [3]–[6], which is coupled with the complexity of blood compatibility between different red blood cell types and the stochastic nature of daily blood demand and supply [7]–[9]. Therefore, it is necessary to devise more efficient and effective ways in which blood products can be stored and assigned to individuals in need. For example, the shelf-lives of blood products in South Africa are considered to be thirty-five days for whole blood, forty-two days for red blood cells, five days for platelets, and one year for plasma. Therefore, any of the blood products not used by the expiry date at a hospital blood bank are said to have ''expired''. This perishability and the continuous rise in the demand for blood products has stimulated numerous researchers to give considerable attention to the management of the scarce blood resources across the world.

The blood assignment problem (BAP) is considered in this paper as a real-world optimization problem, which tries to efficiently assign blood units to various centers (or hospitals) while trying to minimize the amount of importation and expiry within the blood bank. In general, the BAP has many components, so it becomes difficult to model mathematically. Such elements include having to deal with an adequate supply to meet daily demand or importing additional units in the face of shortages. Others include cross-matching of blood according to the blood grouping system, and expiry of blood units once a unit has exceeded its shelf-life [2], [11]. It must be remembered that blood is donated voluntarily to blood banks; therefore, there are no constant levels of supply, and while there is a daily demand for blood, it may vary. The BAP is thus said to be NP-hard, simply the supply and demand of blood involve some complexities, which are associated with blood compatibility issues and the stochastic nature of the daily needs for such a product [10]. The BAP is, therefore, studied to develop an adequate model that utilizes whole blood units more efficiently and achieves the desired objective function of minimizing unnecessary blood product importation from an external source.

An enormous amount of research has involved models relating to the policies of reducing blood wastages [2], [10]–[14]. These efforts have considered instances of double-cross matching of blood and alternative compatibility decision making in using Rh-negative blood for compatible Rh-positive patients in the event of scarcity. Several researchers have examined fixing blood bank inventory levels through the use of efficient planning of blood supply chain management schemes [15]–[19]. Some of the existing models examined close matching of demand and supply for platelets, which have a very short shelf-life, and require matching demand and supply closely. Modeling of the blood bank as an inventory system in which both the arrival of items and the demand for them are stochastic and items stored have finite lifetime was investigated in [32]. Furthermore, in [20], a new optimization framework was proposed for blood supply chain inventory management with the ABO blood group compatibility being considered. The inventory objective was to minimize the expected system out-date rate under a predetermined maximally allowable shortage level.

Similarly, research has frequently focused on integer programming-driven models for reducing wastages and shortages of blood products at hospitals. In [21], an integer programming model to minimize the total cost, shortage, and wastage levels of blood products in a hospital within a planning horizon was proposed. The two models developed by these authors, namely stochastic and deterministic, considered uncertain demand rates, demand for two types of patients, and cross-match to transfusion ratio. The results showed wastage rates decreasing from 19.9% to 2.57% on average. In [22], two mixed-integer programming models were developed, based on the FIFO and LIFO issuing policies, to maximize both the freshness of the platelet units delivered and their total cost. More so, [8] developed an integer programming model, which accounts for all the special aspects of blood supply chains involving uncertain demand and irregular supply of blood products, the perishability of blood products, and shortage avoidance. In [23], a model was formulated using approximate dynamic programming, and was investigated in terms of a blood platelet bank with eight blood types, stochastic demand, stochastic supply, and deterministic lead time.

Despite the achievements of the classical methods in minimizing blood product wastage in blood banks, researchers have sort other more efficient solution methods to tackle the blood assignment problem. The emergence of new and novel evolutionary approaches in the last few decades has created interest and introduced a great number of new metaheuristics inspired by evolutionary processes. In many instances, this new wave of metaheuristic approaches has produced the best solutions for some of the unsolved benchmark problem sets [44]. For example, the genetic algorithm (GA), particle swarm optimization (PSO) algorithm and the more recent symbiotic organisms search algorithm (SOS) are among some of the metaheuristic techniques that have shown success in solving the blood assignment problem [10], [25], [26], [31], [49], [50]. Therefore, the current study considers the possibility of combining these three algorithms to solve a similar problem with the intent to obtain even more superior results. Notably, the motivation behind selecting the three aforementioned metaheuristics for the complex optimization tasks ahead is based on their four properties. They have: (1) operators for exploration and exploitation techniques; GA, PSO, and SOS possess these functionalities; (2) parameter tuning or reduction concepts, SOS has only one main parameter which is the swarm population size; (3) adaptability to the solution of a wide range of problems, which pertains to the three chosen algorithms; and (4) stagnation prevention techniques [44]. An additional criterion was that the selected metaheuristic should also have been reported to show superior performance over the traditional heuristic approaches [41], [45], for example, the GA and PSO have good performance track records [34], [41]. The primary reason the SOS algorithm was selected was due to

its novelty, and nevertheless, its improved functionality and suitability are worth investigating for the problem at hand.

This study is concerned with a method of optimal assignment of blood in a blood bank system taking into account the proportion of each blood type in the population and the rates of blood use and donation to the stockpile available in the blood bank. The proposed approach has the advantage of minimizing wastage and workload at the blood bank. Therefore, following the steps of the mathematical model presented in [2], this paper presents a simplified and robust extended model, designed to effectively manage the demand and supply of blood in the blood bank. In the formulation of the current blood assignment model, the Rh blood grouping system is introduced into the already formulated ABO model in [2]. Note that the initial ABO model produced only four sets of governing equations, while the new model with the addition of the Rh factor would give rise to eight sets of governing equations and twenty-three constants. This effectively means that there would be fifteen degree of freedom in the new system of equations presented here. Also in this study, a hybrid nature-inspired symbiotic organisms search genetic particle swarm optimization (SOSGAPSO) algorithm is developed and used to implement and validate the applicability of the proposed mathematical model. Therefore, the main goal of the hybrid SOSGAPSO algorithm in this context is to demonstrate the practicality of the proposed model. In summary, the technical contributions of this paper are outlined as follows:

- Presentation of a novel and generalized mathematical model formulation, the objective function, and constraints representation for the optimal assignment of whole blood units in a typical real-life blood banking system,
- The formulation of the ABO-Rh compatibility matching rules as one of the factors that strongly affect management decisions,
- The proposal of hybrid symbiotic organisms search genetic particle swarm optimization algorithm for optimal routing of blood assignment. *Specifically, it is notable that the SOSGAPSO is proposed for the first time to solve the blood assignment problem.*
- The main goal of the advanced hybrid metaheuristic implementation is to demonstrate the practicality and applicability of the novel mathematical model formulated to solve a real-world problem.
- The proposed SOSGAPSO algorithm has been tested on a real-life blood bank dataset with different parameter settings and characteristics so as to demonstrate its effectiveness.

In addition to the earlier stated motivating factors for choosing the three metaheuristic algorithms that form the core of the proposed hybrid methods, it is, however, noteworthy to mention that the main motivation behind the current study is the proposal to extend the existing mathematical model presented in [2] by incorporating the rhesus factors compatibility, which introduces an additional level of complexity to the already existing model. Further, since the SOS and its hybrid variants, specifically the proposed triple hybrid methods, have not gained wide recognition in solving real-world blood assignment problems, we believe that their applications in this case to solve the complex problem at hand could also be considered as an additional significant motivation for the current study.

The remainder of this paper is organized as follows: Section 2 presents related work on the application of metaheuristic techniques for the blood assignment problem. Section 3 briefly reviews some of the preliminary concepts about blood, its storage and compatibility, and dynamic mathematical model formulation for the blood assignment problem. Section 4 presents the design of a hybrid nature-inspired metaheuristic global optimization algorithm to demonstrate the practicality of the derived mathematical model. Section 5 presents the model implementation with several sample numerical results. Finally, concluding remarks and future directions are presented in Section 6.

II. METAHEURISTIC OPTIMIZATION APPROACHES

This study builds upon the previous metaheuristic applications proposed in [10], [29]. Each of these studies was aimed at minimizing the importation and/or expiration of products experienced by a real-world blood bank system within a finite time period. However, the current study is aimed at establishing a superior mathematical model and metaheuristic algorithm that is capable of handling the BAP in a more efficient manner based on real-world blood banking scenarios. Other studies presented in [10], [25], [26], [31], [46], [48] have implemented different metaheuristic algorithms, although each of the implementations employed a similar model of randomly generated data set for their validation. These existing studies, also analyzed the BAP from a South African perspective, utilizing blood related statistics such as blood type proportions. For example, in Olusanya *et al.* [26], the PSO algorithm was implemented, and its results showed tremendous capabilities in minimizing the importations of blood units, and there was no form of wastage. The tabu search (TS), simulated annealing (SA) and a hybrid implementation were used in Olusanya and Adewumi [47], where the hybrid implementation combined both the TS and SA, and the results showed some significant improvement in solution accuracy with the hybrid as compared to using the individual algorithms. The success of the hybrid implementation on the BAP based on previous studies further motivates the exploration of more advance hybrid methods in the current study, with the aim of finding a good quality solution in terms of optimal management of the blood bank.

The genetic algorithm (GA) and its hybrid implementation with other algorithms, as well as the hill-climbing (HC) algorithm, were explored in Adewumi *et al.* [25]. While all the algorithms studied produced satisfactory results when exposed to a small blood bank, the HC algorithm provided more optimal results when applied to a much larger blood bank. The dynamic programming (DP) and

greedy randomized adaptive search procedure (GRASP) were explored by Igwe *et al.* [48]. They found that DP incurs heavy importation of blood group O for the first 50 days, while GRASP imports gradually as the number of days increased. These authors concluded that the GRASP outperformed the DP due to GRASP being a more suitable algorithm for handling situations where demand exceeds supply. A robust and effective algorithm to be incorporated in the current study is the SOS algorithm. This algorithm was previously implemented by Govender and Ezugwu [10], [31], who constructed a bi-objective function that included both importation and expiration of blood units for each blood type. Notably, these studies indicated that no form of expiration occurs due to the lifespan of the whole blood (WB) unit (assumed to be 30 days) coupled with the first-in-first-out (FIFO) issuing system. Furthermore, the SOS significantly minimized overall importation experienced within a finite period. Another implementation utilized in this study is the PSO algorithm, which was previously implemented for the BAP in Olusanya *et al.* [26].

All of the above-mentioned studies conducted their research on whole blood (WB) units and implemented randomized datasets because real-world datasets were unattainable. A concept introduced by Govender and Ezugwu [10], [31] in attempting to minimize randomness when generating such datasets, was to incorporate specific South African dates, such as public holidays and school terms. The multiple knapsack problem was used as a base for modeling the BAP [25], [26], [47], and involved using knapsacks of various capacities to maximize overall profits. Lastly, another common link among most of the metaheuristic studies includes the use of the ''bottom-up'' technique, which enables compatible blood types to be used in order to satisfy the demand for a particular type. It is important to mention in certain terms that while all the reviewed existing related study used artificial or randomly generated data set for their study, the current study used real-life blood data set to validate the new mathematical and metaheuristic techniques discussed in this paper.

III. PROBLEM STATEMENT AND MATHEMATICAL MODEL FORMULATION

Blood products comprise essential components; namely, whole blood units, red blood cells, blood plasma, and platelets. Blood products are considered to be a scarce resource but are an unavoidably daily essential, in hospitals. Also, the management of each component of these limited products is complicated by their differing life-spans and consequent storage durations. The storage duration of these products varies from a few days to a few months or even years. Ideally, the storage and management of blood products should be carried out in the blood banks. These banks, however, encounter numerous difficulties associated with the complexity of the blood economy, fragility of blood products, and compatibility. Therefore, to adequately model the management of blood supply and demand in the blood bank, some basic conceptions about this dynamic system with a very complex environment must be clarified [2].

TABLE 1. Proportion of ABO blood group in the Nigeria population [6].

AOB grouping system	O	A	B	AB
Proportion (%)	52.93	22.77	20.64	3.66

TABLE 2. Proportion of Rh blood type distribution found in the Nigeria population.

Rhesus factor	O^-	O^+	A^-	A^+	B^-	B^+	AB^-	AB^+
Proportion (%)	1.6	51.3	0.7	22.4	0.6	20.7	0.1	2.6

1. The volume of blood demand from the blood bank varies each day considerably. In particular, the advent of emergencies such as a mass accident, natural disaster, or unanticipated trauma, may result in vast amounts of blood to be requested within a minimal period. Then, the blood bank would be subjected to high pressure.
2. The scarcity of blood is often caused by the sporadic nature of its supply from generous donors. In essence, the availability of blood products depends almost solely on the willingness of individual donors to donate their blood at the various blood collections centers.
3. As mentioned earlier, there are four main blood groups, namely, O, A, B, and AB, further complicated by each being Rh positive or negative, making eight blood groups. Taking Nigeria as a case study, the population is 97% rhesus positive and 3% rhesus negative. The blood group proportions are summarized in Table 1 and 2.
4. Blood compatibility also introduces additional complexity to the system. With the classification of blood into eight major groups, the harmony of the different blood types must be considered before the transfusion process to ensure each recipient receives only blood that is compatible with their own. Considering the rhesus grouping system, the members of group O^- are considered as universal donors; their blood is compatible with blood from all other groups. Members of the AB^+ blood group can receive from all blood types, so it is often referred to as a universal receiver. The connected graph in Figure 1 illustrates the grouping compatibility for the red blood cells with the Rhesus factor. It is important to highlight the general assumption that in the event of scarcity or emergencies, with O^- blood type being a universal donor, such blood may be given in the absence or shortage of the compatible blood type. However, in a normal situation, an individual recipient should be allocated their own specific compatible blood type to avoid any unforeseen circumstances that might be life-threatening. Furthermore, as is shown in Table 2, this blood type is among the least common, so such stock should not be unnecessarily depleted by non-emergencies.

In a real-world situation, the complex flow of blood products through a typical blood bank system is presented in Figure. 2. Meanwhile, the daily operational performance of

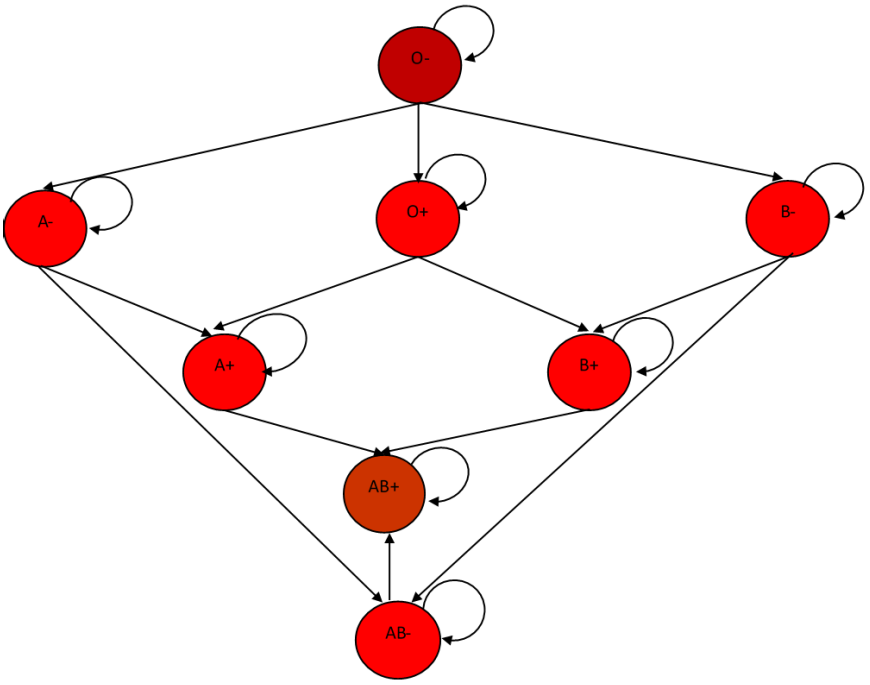

FIGURE 1. Group compatibility for red blood cells with Rhesus.

any blood bank, which is measured in terms of shortage, operational costs, demand, supply, importation, expiration, or wastage, is determined by several variables. These include the frequency of blood donation camps, collection quantity (from camps and blood bank donations), percentage of the componentizing, cross-match release period, issuing policy for blood products, blood type balance, shelf-life of the incoming blood units, etc [33].

A. MASS BALANCE FORMULATION

This section presents an extended mathematical model iformulation that was initially proposed in [2]. As aforementioned earlier, the extension here is the introduction of the rhesus factor blood grouping system, which further introduces additional complexity into the existing ABO blood type model. For the sake of simplifying notation, blood group AB will for the remainder of the article be referred to a C. We begin our formulation by denoting Δ to represent the amount of blood required by a hospital. According to [2], an approximate value of Δ should realistically be available to the management of the blood bank. More so, the amount of blood needed for each blood type is assumed to be proportional to the representation of blood types in the population denoted by $n_{O^-}, n_{O+}, n_{A^-}, n_{A^-}, n_{B^-}, n_{B^+}, n_{C^-}$ and n_{C^+}, and these values can be expressed as follows:

$$\begin{aligned} \Delta_{O^+} &= n_{O^+}\Delta; \quad \Delta_{O^-} = n_{O^-}\Delta, \\ \Delta_{A^+} &= n_{A^+}\Delta; \quad \Delta_{A^-} = n_{A^-}\Delta, \\ \Delta_{B^+} &= n_{B^+}\Delta; \quad \Delta_{B^-} = n_{B^-}\Delta, \\ \Delta_{C^+} &= n_{C^+}\Delta; \quad \Delta_{C^-} = n_{C^-}\Delta. \end{aligned}$$

Mathematically, the compatibility table can be expressed as

$$\Delta_{O^-} = D_{O^-O^-}, \tag{1}$$
$$\Delta_{O^+} = D_{O^-O^+} + D_{O^+O^+}, \tag{2}$$
$$\Delta_{A^-} = D_{O^-A^-} + D_{A^-A^-}, \tag{3}$$
$$\Delta_{A^+} = D_{O^-A^+} + D_{O^+A^+} + D_{A^-A^+} + D_{A^+A^+}, \tag{4}$$
$$\Delta_{B^-} = D_{O^-B^-} + D_{B^-B^-}, \tag{5}$$
$$\Delta_{B^+} = D_{O^-B^+} + D_{O^+B^+} + D_{B^-B^+} + D_{B^+B^+}, \tag{6}$$
$$\Delta_{C^-} = D_{O^-C^-} + D_{A^-C^-} + D_{B^-C^-} + D_{C^-C^-}, \tag{7}$$
$$\Delta_{C^+} = D_{O^-C^+} + D_{O^+C^+} + D_{A^-C^+} + D_{A^+C^+} + D_{B^-C^+} + D_{B^+C^+} + D_{C^-C^+} + D_{C^+C^+}, \tag{8}$$

where, D_{XY} represent the amount of blood donated by people with blood type X to people with blood type Y. Equations (1) - (8) display where optimization may take place. Apart from type O, all blood groups may be substituted with at least one other blood type. Optimization consists of choosing the best replacement method. To begin, all blood quantities D_{XY} should be expressed as functions of V_{O^-}, V_{O^+}, V_{A^-}, V_{A^+}, V_{B^-}, V_{B^+}, V_{C^-}, and V_{C^+}, Which represents the amount of blood of each type available in the blood bank. The simplest model is a linear relationship:

$$D_{XY} \propto V_X.$$

Note that if there is a lot of type X blood in the bank, V_X is large and this blood type may be used to supply the request in blood of type Y. Conversely, if V_X is small, D_{XY} will be low and very little blood of type X will be used to substitute blood of type Y. Therefore, using this model, equations (1-8)

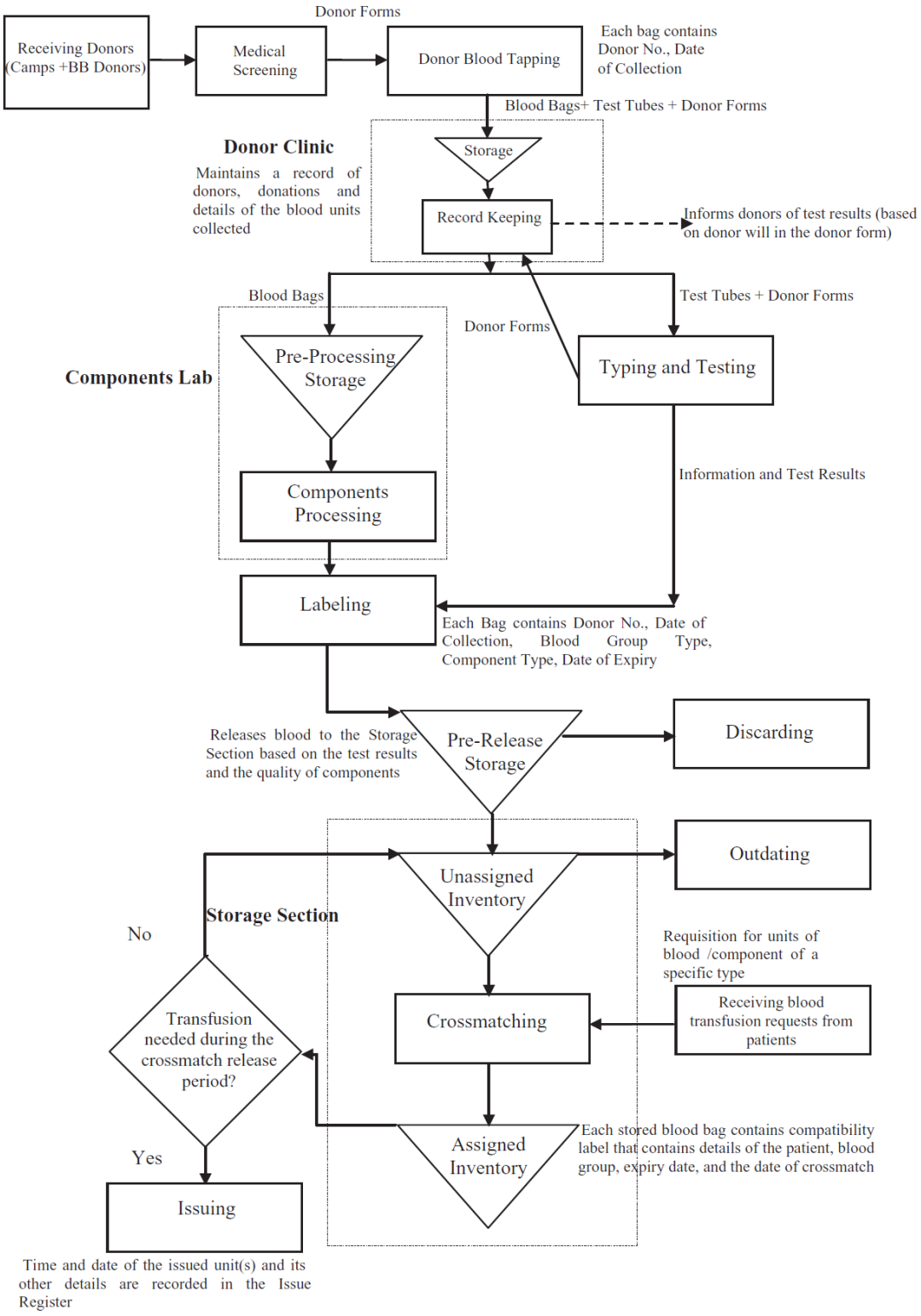

FIGURE 2. A traditional framework for the flow of blood units in a real-world blood banking system [33].

can be rewritten as follows;

$$\Delta_{O^-} = \alpha_1 V_{O^-}, \tag{9}$$

$$\Delta_{O^+} = \alpha_2 V_{O^-} + \beta_2 V_{O^+}, \tag{10}$$

$$\Delta_{A^-} = \alpha_3 V_{O^-} + \gamma_3 V_{A^-}, \tag{11}$$

$$\Delta_{A^+} = \alpha_4 V_{O^-} + \beta_4 V_{O^+} + \gamma_4 V_{A^-} + \delta_4 V_{A^+}, \tag{12}$$

$$\Delta_{B^-} = \alpha_5 V_{O^-} + \epsilon_5 V_{B^-}, \tag{13}$$

$$\Delta_{B^+} = \alpha_6 V_{O^-} + \beta_6 V_{O^+} + \epsilon_6 V_{B^-} + \varphi_6 V_{B^+}, \tag{14}$$

$$\Delta_{C^-} = \alpha_7 V_{O^-} + \gamma_7 V_{A^-} + \epsilon_7 V_{B^-} + \lambda_7 V_{C^-}, \tag{15}$$

$$\Delta_{C^+} = \alpha_8 V_{O^-} + \beta_8 V_{O^+} + \gamma_8 V_{A^-} + \delta_8 V_{A^+} + \epsilon_8 V_{B^-} + \varphi_8 V_{B^+} + \lambda_8 V_{C^-} + \sigma_8 V_{C^+}, \quad (16)$$

where α_i, β_i, γ_i, δ_i, λ_i, ϵ_i, φ_i, and σ_i are positive constants with values still to be determined. The value of α_1 is determined from

$$\alpha_1 = \frac{\Delta_{O^-}}{V_{O^-}}.$$

If $\alpha_1 < 1$, the quantity of blood needed for that given day is larger than the amount of blood of type O^- that must be imported from outside the system. Otherwise, there will be a shortage at some stage in the day. If no substitution is allowed, then the rate of self-replacement for the parameters, γ_3, δ_4, λ_5, φ_5, χ_8, and σ_8, will be as follows:

$$\beta_2^s = \frac{\Delta_{O^+}}{V_{O^+}}, \quad \gamma_3^s = \frac{\Delta_{A^-}}{V_{A^-}}, \ \delta_4^s = \frac{\Delta_{A^+}}{V_{A^+}}, \ \epsilon_5^s = \frac{\Delta_{B^-}}{V_{B^-}},$$
$$\varphi_6^s = \frac{\Delta_{B^+}}{V_{B^+}}, \ \lambda_7^s = \frac{\Delta_{C^-}}{V_{C^-}}, \ \sigma_8^s = \frac{\Delta_{C^+}}{V_{C^+}}, \quad (17)$$

where the superscript denotes self-replacements. Here again, if the parameters have a value larger than 1, this indicates a potential shortage of the respective blood types. If all eight parameters α_1, β_2, γ_3, δ_4, ϵ_5, φ_5, λ_8, and σ_8 are greater than 1, the amount of blood requested for the day will be more than the amount of blood in the bank and additional blood should be pulled into the system. In an ideal situation, these parameters should be as small as possible, which would mean that the blood bank is well stocked in all types of blood.

B. THE DYNAMIC MASS BALANCE SYSTEM

Next, the dynamical mass balance equation for this system may be described as represented in equations (18-25).

$$\frac{dV_{O^-}}{dt} = Q_{O^-} - (D_{O^-O^-} + D_{O^-O^+} + D_{A^-O^-} + D_{O^-A^+} + D_{O^-B^-} + D_{O^-B^+} + D_{O^-C^-} + D_{O^-C^+}), \quad (18)$$

$$\frac{dV_{O^+}}{dt} = Q_{O^+} - (D_{O^+O^+} + D_{O^+A^+} + D_{O^+B^+} + D_{O^+C^+}), \quad (19)$$

$$\frac{dV_{A^-}}{dt} = Q_{A^-} - (D_{A^-A^-} + D_{A^-A^+} + D_{A^-C^-} + D_{A^-C^+}), \quad (20)$$

$$\frac{dV_{A^+}}{dt} = Q_{A^+} - (D_{A^+A^+} + D_{A^+C^+}), \quad (21)$$

$$\frac{dV_{B^-}}{dt} = Q_{B^-} - (D_{B^-B^-} + D_{B^-B^+} + D_{B^-C^-} + D_{B^-C^+}), \quad (22)$$

$$\frac{dV_{B^+}}{dt} = Q_{B^+} - (D_{B^+B^+} + D_{B^+C^+}), \quad (23)$$

$$\frac{dV_{C^-}}{dt} = Q_{C^-} - (D_{C^-C^-} + D_{C^-C^+}), \quad (24)$$

$$\frac{dV_{C^+}}{dt} = Q_{C^-} - D_{C^+C^+}. \quad (25)$$

For each time unit, say at time $t = 1$, which corresponds to one day, blood enters and leaves the system. Similarly, the variations of the blood volume in the blood bank will correspond to the amount of blood donated for each blood group, denoted by $Q_{O^-}, Q_{O^+}, Q_{A^-}, Q_{A^+}, Q_{B^-}, Q_{B^+}, Q_{C^-}$ and Q_{C^+} minus the amount of blood from that group used during the day. The following equations denote variations in the volume of bloodstock in the blood bank, in terms of the actual quantity of blood leaving the bank, which may be expressed as a function of D_{XY}. Similarly, using the above-derived model, the governing equations may, therefore, be written as follows:

$$\frac{dV_{O^-}}{dt} = Q_{O^-} - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8)V_{O^-}, \quad (26)$$

$$\frac{dV_{O^+}}{dt} = Q_{O^+} - (\beta_2 + \beta_4 + \beta_6 + \beta_8)V_{O^+}, \quad (27)$$

$$\frac{dV_{A^-}}{dt} = Q_{A^-} - (\gamma_3 + \gamma_4 + \gamma_7 + \gamma_8)V_{A^-}, \quad (28)$$

$$\frac{dV_{A^+}}{dt} = Q_{A^+} - (\delta_4 + \delta_8)V_{A^+}, \quad (29)$$

$$\frac{dV_{B^-}}{dt} = Q_{B^-} - (\epsilon_5 + \epsilon_6 + \epsilon_7 + \epsilon_8)V_{B^-}, \quad (30)$$

$$\frac{dV_{B^+}}{dt} = Q_{B^+} - (\varphi_6 + \varphi_8)V_{B^+}, \quad (31)$$

$$\frac{dV_{C^-}}{dt} = Q_{C^-} - (\lambda_7 + \lambda_8)V_{C^-}, \quad (32)$$

$$\frac{dV_{C^+}}{dt} = Q_{C^+} - \sigma_8 V_{C^+}. \quad (33)$$

The solutions to the differential equation (26-31) above can also be written as follows:

$$V_{O^-}(t) = \frac{Q_{O^-}}{\sum_{n=1}^{8}\alpha_n} + \left(V_{O^-}^0 - \frac{Q_{O^-}}{\sum_{n=1}^{8}\alpha_n}\right) e^{-\left(\sum_{n=1}^{8}\alpha_n\right)t}, \quad (34)$$

$$V_{O^+}(t) = \frac{Q_{O^+}}{\sum_{n=1}^{4}\beta_{2n}} + \left(V_{O^+}^0 - \frac{Q_{O^+}}{\sum_{n=1}^{4}\beta_{2n}}\right) e^{-\left(\sum_{n=1}^{4}\beta_n\right)t}, \quad (35)$$

$$V_{A^-}(t) = \frac{Q_{A^-}}{\gamma_3 + \gamma_4 + \gamma_7 + \gamma_8} + \left(V_{A^-}^0 - \frac{Q_{A^-}}{\gamma_3 + \gamma_4 + \gamma_7 + \gamma_8}\right) \times e^{-(\gamma_3+\gamma_4+\gamma_7+\gamma_8)t}, \quad (36)$$

$$V_{A^+}(t) = \frac{Q_{A^+}}{\delta_4 + \delta_8} + \left(V_{A^+}^0 - \frac{Q_{A^+}}{\delta_4 + \delta_8}\right) e^{-(\delta_4+\delta_8)t}, \quad (37)$$

$$V_{B^-}(t) = \frac{Q_{B^-}}{\epsilon_5 + \epsilon_6 + \epsilon_7 + \epsilon_8} + \left(V_{B^-}^0 - \frac{Q_{B^-}}{\epsilon_5 + \epsilon_6 + \epsilon_7 + \epsilon_8}\right) \times e^{-(\epsilon_5+\epsilon_6+\epsilon_7+\epsilon_8)t}, \quad (38)$$

$$V_{B^+}(t) = \frac{Q_{B^+}}{\varphi_6 + \varphi_8} + \left(V_{B^+}^0 - \frac{Q_{B^+}}{\varphi_6 + \varphi_8}\right) e^{-(\varphi_6+\varphi_8)t}, \quad (39)$$

$$V_{C^-}(t) = \frac{Q_{C^-}}{\lambda_7 + \lambda_8} + \left(V_{C^-}^0 - \frac{Q_{C^-}}{\lambda_7 + \lambda_8}\right) e^{-(\lambda_7+\lambda_8)t}, \tag{40}$$

$$V_{C^+}(t) = \frac{Q_{C^+}}{\sigma_8} + \left(V_{C^+}^0 - \frac{Q_{C^+}}{\sigma_8}\right) e^{-(\sigma_8)t}, \tag{41}$$

where V_X^0 denotes the amount of blood available in the blood bank at the beginning of the day. If the parameters V_X^0 and Q_X remain the same for some time, the volume of blood at the blood bank will reach its asymptotic level, resulting in the following equations:

$$V_{O^-}^l = \frac{Q_{O^-}}{\sum_{n=1}^{8} \alpha_n}, \tag{42}$$

$$V_{O^+}^l = \frac{Q_{O^+}}{\sum_{n=1}^{4} \beta_{2n}}, \tag{43}$$

$$V_{A^-}^l = \frac{Q_{A^-}}{\gamma_3 + \gamma_4 + \gamma_7 + \gamma_8}, \tag{44}$$

$$V_{A^+}^l = \frac{Q_{A^+}}{\delta_4 + \delta_8}, \tag{45}$$

$$V_{B^-}^l = \frac{Q_{B^-}}{\epsilon_5 + \epsilon_6 + \epsilon_7 + \epsilon_8}, \tag{46}$$

$$V_{B^+}^l = \frac{Q_{B^+}}{\varphi_6 + \varphi_8}, \tag{47}$$

$$V_{C^-}^l = \frac{Q_{C^-}}{\lambda_7 + \lambda_8}, \tag{48}$$

$$V_{C^+}^l = \frac{Q_{C^+}}{\sigma_8}, \tag{49}$$

where the superscript l represents the long term value, and this should reflect the proportions of each blood type in the population (n_{O^-}, n_{O^+}, n_{A^-}, n_{A^+}, n_{B^-}, n_{B^+}, n_{C^-}, and n_{C^+}). More so, these proportions describe what should be the state of blood stocked in the bank in an ideal situation. Relating the population of the donor types to the amount of each blood type available expressed by equations (39-46), the following equations are obtained:

$$n_{O^-} = \frac{V_{O^-}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}, \tag{50}$$

$$n_{O^+} = \frac{V_{O^+}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}, \tag{51}$$

$$n_{A^-} = \frac{V_{A^-}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}, \tag{52}$$

$$n_{A^+} = \frac{V_{A^+}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}, \tag{53}$$

$$n_{B^-} = \frac{V_{B^-}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}, \tag{54}$$

$$n_{B^+} = \frac{V_{B^+}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}, \tag{55}$$

$$n_{C^-} = \frac{V_{C^-}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}, \tag{56}$$

$$n_{C^+} = \frac{V_{C^+}^l}{V_{O^-}^l + V_{O^+}^l + V_{A^-}^l + V_{A^+}^l + V_{B^-}^l + V_{B^+}^l + V_{C^-}^l + V_{C^+}^l}. \tag{57}$$

Using the assumption that the donated blood quantity Q_x, is proportional to the amount of blood in the population n_x, equations (47-54) can be rewritten as

$$\frac{n_{O^-}-1}{\alpha} + \frac{n_{O^+}}{\beta} + \frac{n_{A^-}}{\gamma} + \frac{n_{A^+}}{\delta} + \frac{n_{B^-}}{\epsilon} + \frac{n_{B^+}}{\varphi} + \frac{n_{C^-}}{\lambda} + \frac{n_{C^+}}{\sigma} = 0, \tag{58}$$

$$\frac{n_{O^-}}{\alpha} + \frac{n_{O^+}-1}{\beta} + \frac{n_{A^-}}{\gamma} + \frac{n_{A^+}}{\delta} + \frac{n_{B^-}}{\epsilon} + \frac{n_{B^+}}{\varphi} + \frac{n_{C^-}}{\lambda} + \frac{n_{C^+}}{\sigma} = 0, \tag{59}$$

$$\frac{n_{O^-}}{\alpha} + \frac{n_{O^+}}{\beta} + \frac{n_{A^-}-1}{\gamma} + \frac{n_{A^+}}{\delta} + \frac{n_{B^-}}{\epsilon} + \frac{n_{B^+}}{\varphi} + \frac{n_{C^-}}{\lambda} + \frac{n_{C^+}}{\sigma} = 0, \tag{60}$$

$$\frac{n_{O^-}}{\alpha} + \frac{n_{O^+}}{\beta} + \frac{n_{A^-}}{\gamma} + \frac{n_{A^+}-1}{\delta} + \frac{n_{B^-}}{\epsilon} + \frac{n_{B^+}}{\varphi} + \frac{n_{C^-}}{\lambda} + \frac{n_{C^+}}{\sigma} = 0, \tag{61}$$

$$\frac{n_{O^-}}{\alpha} + \frac{n_{O^+}}{\beta} + \frac{n_{A^-}}{\gamma} + \frac{n_{A^+}}{\delta} + \frac{n_{B^-}-1}{\epsilon} + \frac{n_{B^+}}{\varphi} + \frac{n_{C^-}}{\lambda} + \frac{n_{C^+}}{\sigma} = 0, \tag{62}$$

$$\frac{n_{O^-}}{\alpha} + \frac{n_{O^+}}{\beta} + \frac{n_{A^-}}{\gamma} + \frac{n_{A^+}}{\delta} + \frac{n_{B^-}}{\epsilon} + \frac{n_{B^+}-1}{\varphi} + \frac{n_{C^-}}{\lambda} + \frac{n_{C^+}}{\sigma} = 0, \tag{63}$$

$$\frac{n_{O^-}}{\alpha} + \frac{n_{O^+}}{\beta} + \frac{n_{A^-}}{\gamma} + \frac{n_{A^+}}{\delta} + \frac{n_{B^-}}{\epsilon} + \frac{n_{B^+}}{\varphi} + \frac{n_{C^-}-1}{\lambda} + \frac{n_{C^+}}{\sigma} = 0, \tag{64}$$

$$\frac{n_{O^-}}{\alpha} + \frac{n_{O^+}}{\beta} + \frac{n_{A^-}}{\gamma} + \frac{n_{A^+}}{\delta} + \frac{n_{B^-}}{\epsilon} + \frac{n_{B^+}}{\varphi} + \frac{n_{C^-}}{\lambda} + \frac{n_{C^+}-1}{\sigma} = 0, \tag{65}$$

where

$$\alpha = \sum_{n=1}^{8} \alpha_n,$$
$$\beta = \sum_{n=1}^{4} \beta_n,$$
$$\gamma = \gamma_3 + \gamma_4 + \gamma_7 + \gamma_8,$$
$$\delta = \delta_4 + \delta_8,$$

$$\epsilon = \epsilon_5 + \epsilon_6 + \epsilon_7 + \epsilon_8,$$
$$\varphi = \varphi_6 + \varphi_8,$$
$$\lambda = \lambda_7 + \lambda_8,$$
$$\sigma = \sigma_8,$$

and

$$n_{O^-} + n_{O^+} + n_{A^-} + n_{A^+} + n_{B^-} + n_{B^+} + n_{C^-} + n_{C^+} = 1.$$

Equations (55-61) is rewritten to have seven independent parameters, and solving further will yield the following equations:

$$\alpha = \beta = \gamma = \delta = \epsilon = \varphi = \lambda = \sigma. \tag{66}$$

In the matrix system given below, we express $\alpha, \beta, \gamma, \delta, \epsilon, \varphi,$ and λ as a function of σ_8. Expressing the system of equation (9) - (16) using (66) results in the matrix representation

$$\mathbb{QC} = \mathbb{R}, \tag{67}$$

where From the matrix system given in equation (68), there are 14 equations and 25 unknowns. In general, the equations above gives 14 independent relationships between the parameters $\alpha_i, \beta_i\gamma_i\delta_i, \epsilon_i, \varphi_i, \lambda_i$ and σ_8, which are expressed as follows:

The parameters have to be positive, which, therefore, results in the following inequalities:

$$\beta_8 \le \frac{\Delta_{O^+}}{V_{O^+}}, \quad \gamma_3 \le \frac{\Delta_{A^-}}{V_{A^-}}, \ \delta_4 \le \frac{\Delta_{A^+}}{V_{A^+}}, \ \epsilon_5 \le \frac{\Delta_{B^-}}{V_{B^-}},$$
$$\varphi_6 \le \frac{\Delta_{B^+}}{V_{B^+}}, \ \lambda_7 \le \frac{\Delta_{C^-}}{V_{C^-}}$$
$$\beta_2 + \beta_4 + \beta_6 \le \sigma_8$$
$$\gamma_3 + \gamma_4 + \gamma_7 \le \sigma_8$$
$$\delta_4 \le \sigma_8$$
$$\epsilon_5 + \epsilon_6 + \epsilon_7 \le \sigma_8$$
$$\varphi_6 \le \sigma_8$$
$$\lambda_7 \le \sigma_8.$$

Here, the equality holds only when $\Delta_x = D_{xx}$.

Note that by combining the above equations with equations (9) - (16), fourteen of the required parameters have been defined. However, the remaining parameters will be determined by optimizing the decision-making process, for which insight is provided in the subsequent discussion in Section 3.3.

C. DECISION MAKING

At the end of the first day ($t = 1$), equations (32-38) give the stocks of blood available in the bank as:

$$V_{O^-}(1) = \frac{Q_{O^-}}{\sum_{n=1}^{8}\alpha_n} + \left(V_{O^-}^0 - \frac{Q_{O^-}}{\sum_{n=1}^{8}\alpha_n}\right) e^{-\left(\sum_{n=1}^{8}\alpha_n\right)}, \tag{68}$$

$$V_{O^+}(1) = \frac{Q_{O^+}}{\sum_{n=1}^{4}\beta_n} + \left(V_{O^+}^0 - \frac{Q_{O^+}}{\sum_{n=1}^{4}\beta_n}\right) e^{-\left(\sum_{n=1}^{4}\beta_{2n}\right)}, \tag{69}$$

$$V_{A^-}(1) = \frac{Q_{A^-}}{\gamma_3+\gamma_4+\gamma_7+\gamma_8} + \left(V_{A^-}^0 - \frac{Q_{A^-}}{\gamma_3+\gamma_4+\gamma_7+\gamma_8}\right) e^{-(\gamma_3+\gamma_4+\gamma_7+\gamma_8)}, \tag{70}$$

$$V_{A^+}(1) = \frac{Q_{A^+}}{\delta_4+\delta_8} + \left(V_{A^+}^0 - \frac{Q_{A^+}}{\delta_4+\delta_8}\right) e^{-(\delta_4+\delta_8)}, \tag{71}$$

$$V_{B^-}(1) = \frac{Q_{B^-}}{\epsilon_5+\epsilon_6+\epsilon_7+\epsilon_8} + \left(V_{B^-}^0 - \frac{Q_{B^-}}{\epsilon_5+\epsilon_6+\epsilon_7+\epsilon_8}\right) e^{-(\epsilon_5+\epsilon_6+\epsilon_7+\epsilon_8)}, \tag{72}$$

$$V_{B^+}(1) = \frac{Q_{B^+}}{\varphi_6+\varphi_8} + \left(V_{B^+}^0 - \frac{Q_{B^+}}{\varphi_6+\varphi_8}\right) e^{-(\varphi_6+\varphi_8)}, \tag{73}$$

$$V_{C^-}(1) = \frac{Q_{C^-}}{\lambda_7+\lambda_8} + \left(V_{C^-}^0 - \frac{Q_{C^-}}{\lambda_7+\lambda_8}\right) e^{-(\lambda_7+\lambda_8)}, \tag{74}$$

$$V_{C^+}(1) = \frac{Q_{C^+}}{\sigma_8} + \left(V_{C^+}^0 - \frac{Q_{C^+}}{\sigma_8}\right) e^{-\sigma_8}. \tag{75}$$

Combined with equation (66), the volumes become

$$V_{O^-}(1) = \frac{Q_{O^-}}{\sigma_8} + \left(V_{O^-}^0 - \frac{Q_{O^-}}{\sigma_8}\right) e^{-\sigma_8}, \tag{76}$$

$$V_{O^+}(1) = \frac{Q_{O^+}}{\sigma_8} + \left(V_{O^+}^0 - \frac{Q_{O^+}}{\sigma_8}\right) e^{-\sigma_8}, \tag{77}$$

$$V_{A^-}(1) = \frac{Q_{A^-}}{\sigma_8} + \left(V_{A^-}^0 - \frac{Q_{A^-}}{\sigma_8}\right) e^{-\sigma_8}, \tag{78}$$

$$V_{A^+}(1) = \frac{Q_{A^+}}{\sigma_8} + \left(V_{A^+}^0 - \frac{Q_{A^+}}{\sigma_8}\right) e^{-\sigma_8}, \tag{79}$$

$$V_{B^-}(1) = \frac{Q_{B^-}}{\sigma_8} + \left(V_{B^-}^0 - \frac{Q_{B^-}}{\sigma_8}\right) e^{-\sigma_8}, \tag{80}$$

$$V_{B^+}(1) = \frac{Q_{B^+}}{\sigma_8} + \left(V_{B^+}^0 - \frac{Q_{B^+}}{\sigma_8}\right) e^{-\sigma_8}, \tag{81}$$

$$V_{C^-}(1) = \frac{Q_{C^-}}{\sigma_8} + \left(V_{C^-}^0 - \frac{Q_{C^-}}{\sigma_8}\right) e^{-\sigma_8}, \tag{82}$$

$$V_{C^+}(1) = \frac{Q_{C^+}}{\sigma_8} + \left(V_{C^+}^0 - \frac{Q_{C^+}}{\sigma_8}\right) e^{-\sigma_8}. \tag{83}$$

And the corresponding blood proportion in the bank may be expressed in (84)–(91), as shown at the bottom of the 11th page.

D. GENERALIZED OBJECTIVE FUNCTION

Managing the blood bank requires getting these proportions at the end of the day to be as close as possible to the ideal proportions of the bank. Achieving optimum management involves minimizing the objective function, which is given in equation (92).

$$E = (p_{O^-} - n_{O^-})^2 + (p_{O^+} - n_{O^+})^2 + (p_{A^-} - n_{A^-})^2 + (p_{A^+} - n_{A^+})^2 + (p_{B^-} - n_{B^-})^2 + (p_{B^+} - n_{B^+})^2 + (p_{C^-} - n_{C^-})^2 + (p_{C^+} - n_{C^+})^2. \tag{92}$$

This function depends only on the parameter σ_8. The rest of the parameter values, namely $\beta_2, \gamma_3, \delta_4, \epsilon_5, \varphi_6,$ and λ_7, should

be provided by the optimization process such that their values are close as possible to those of the self-replacement values β_2^s, γ_3^s, δ_4^s, ϵ_5^s, φ_6^s, and λ_7^s. Thus, E can be modified and given as:

$$\begin{aligned} E = {} & (p_{O^-} - n_{O^-})^2 + (p_{O^+} - n_{O^+})^2 \\ & + (p_{A^-} - n_{A^-})^2 + (p_{A^+} - n_{A^+})^2 + (p_{B^-} - n_{B^-})^2 \\ & + (p_{B^+} - n_{B^+})^2 + (p_{C^-} - n_{C^-})^2 + (p_{C^+} - n_{C^+})^2 \\ & + \left(1 - \frac{\beta_2}{\beta_2^s}\right)^2 + \left(1 - \frac{\gamma_3}{\gamma_3^s}\right)^2 + \left(1 - \frac{\delta_4}{\delta_4^s}\right)^2 \\ & + \left(1 - \frac{\epsilon_5}{\epsilon_5^s}\right)^2 + \left(1 - \frac{\varphi_6}{\varphi_6^s}\right)^2 + \left(1 - \frac{\lambda_7}{\lambda_7^s}\right)^2. \end{aligned}$$

However, it is noteworthy to mention here that the objective function is only a possibility, and since solving E is rather complicated, numerical methods would be preferred over analytic methods as an alternative method for computing its minimum value. As a preference, later in this paper, we will explore the possibility of using standard global optimization methods to calculate the minimum value of E, which probably satisfies some of the typical constraints easily associated with real-world experiences in the blood banking system. Therefore, once the minimum value of E is computed, and the values of the parameters $\beta_2\gamma_3\delta_4$, ϵ_5, φ_6, λ_7 and σ_8 may be determined alongside all other outstanding constants. Afterward, based on these findings, reasonable suggestions would be available for the blood bank manager to make feasible and accurate decisions on the proportions of blood volume needed

$$\mathbb{Q} = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
V_{O^-} & 0 & 0 & 0 & 0 & 0 & 0 & V_{O^+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & V_{O^-} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{A^-} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & V_{O^-} & 0 & 0 & 0 & 0 & 0 & V_{O^+} & 0 & 0 & 0 & V_{A^-} & 0 & 0 & V_{A^+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & V_{O^-} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{B^-} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & V_{O^-} & 0 & 0 & 0 & 0 & V_{O^+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{B^-} & 0 & 0 & V_{B^+} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & V_{O^-} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{A^-} & 0 & 0 & 0 & 0 & 0 & V_{B^-} & 0 & 0 & 0 & V_{C^-} \\
0 & 0 & 0 & 0 & 0 & 0 & V_{O^-} & 0 & 0 & 0 & V_{O^+} & 0 & 0 & 0 & V_{A^-} & 0 & V_{A^+} & 0 & 0 & 0 & V_{B^-} & 0 & V_{B^+} & 0 & V_{C^-}
\end{pmatrix}$$

$$\mathbb{C} = \begin{pmatrix}
\alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \\ \beta_2 \\ \beta_4 \\ \beta_6 \\ \beta_8 \\ \gamma_3 \\ \gamma_4 \\ \gamma_7 \\ \gamma_8 \\ \delta_4 \\ \delta_8 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \varphi_6 \\ \varphi_8 \\ \lambda_7 \\ \lambda_8
\end{pmatrix}, \quad \mathbb{R} = \begin{pmatrix}
\sigma_8 - \alpha_1 \\ \sigma_8 \\ \sigma_8 \\ \sigma_8 \\ \sigma_8 \\ \sigma_8 \\ \sigma_8 \\ \Delta_{O^+} \\ \Delta_{A^-} \\ \Delta_{A^+} \\ \Delta_{B^-} \\ \Delta_{B^+} \\ \Delta_{C^-} \\ \Delta_{C^+} - \sigma_8 V_{C^+}.
\end{pmatrix}.$$

in the replacement strategy, which can be expressed using the follows:

$$\mu_{O^-A^-}=\frac{D_{O^-A^-}}{\Delta_{A^-}},\quad \mu_{O^-A^-}=\frac{D_{A^-A^-}}{\Delta_{A^-}},\ \mu_{O^-A^+}=\frac{D_{O^-A^+}}{\Delta_{A^+}},$$
$$\mu_{O^+A^+}=\frac{D_{O^+A^+}}{\Delta_{A^+}},\quad \mu_{A^-A^+}=\frac{D_{A^-A^+}}{\Delta_{A^+}},\ \mu_{A^+A^+}=\frac{D_{A^+A^+}}{\Delta_{A^+}},$$
$$\mu_{O^-B^-}=\frac{D_{O^-B^-}}{\Delta_{B^-}},\quad \mu_{B^-B^-}=\frac{D_{B^-B^-}}{\Delta_{B^-}},\ \mu_{O^-B^+}=\frac{D_{O^-B^+}}{\Delta_{B^+}},$$
$$\mu_{O^+B^+}=\frac{D_{O^+B^+}}{\Delta_{B^+}},\quad \mu_{B^-B^+}=\frac{D_{B^-B^+}}{\Delta_{B^+}},\ \mu_{B^+B^+}=\frac{D_{B^+B^+}}{\Delta_{B^+}},$$
$$\mu_{O^-C^{--}}=\frac{D_{O^-C^-}}{\Delta_{C^-}},\quad \mu_{A^-C^-}=\frac{D_{A^-C^-}}{\Delta_{C^-}},$$

$$\alpha_1=\frac{\Delta_{O^-}}{V_{O^-}}$$
$$\alpha_2=\frac{\Delta_{O^-}-\beta_2 V_{O^+}}{V_{O^-}}$$
$$\alpha_3=\frac{\Delta_{A^-}-\gamma_3 V_{A^-}}{V_{O^-}}$$
$$\alpha_4=\frac{\Delta_{A^+}-\beta_4 V_{O^+}-\gamma_4 V_{A^-}-\delta_4 V_{A^+}}{V_{O^-}}$$
$$\alpha_5=\frac{\Delta_{B^-}-\epsilon_5 V_{B^-}}{V_{O^-}}$$
$$\alpha_6=\frac{\Delta_{B^+}-\beta_6 V_{O^+}-\epsilon_6 V_{B^-}-\varphi_6 V_{B^+}}{V_{O^-}}$$
$$\alpha_7=\frac{\Delta_{C^-}-\lambda_7 V_{A^-}-\epsilon_7 V_{B^-}-\lambda_7 V_{C^-}}{V_{O^-}}$$
$$\alpha_8=\sigma_8-\left(\frac{\Delta_{O^-}+V_{O^+}+V_{A^-}+V_{A^+}+V_{B^-}+V_{B^+}+V_{C^-}}{V_{O^-}}\right)+\left(\frac{\beta_2 V_{O^+}+\gamma_3 V_{A^-}+\beta_4 V_{O^+}+\gamma_4 V_{A^-}+\delta_4 V_{A^+}+\epsilon_5 V_{B^-}+\beta_6 V_{O^+}+\epsilon_6 V_{B^-}+\varphi_6 V_{B^+}+\lambda_4 V_{A^-}+\epsilon_7 V_{B^-}+\lambda_7 V_{C^-}}{V_{O^-}}\right)$$
$$\beta_8=\sigma_8-(\beta_2+\beta_4+\beta_6)$$
$$\gamma_8=\sigma_8-(\gamma_3+\gamma_4+\gamma_7)$$
$$\delta_8=\sigma_8-\delta_4$$
$$\epsilon_8=\sigma_8-(\epsilon_5+\epsilon_6+\epsilon_7)$$
$$\varphi_8=\sigma_8-\varphi_6$$
$$\lambda_8=\sigma_8-\lambda_7.$$

$$p_{O^-}=\frac{V^l_{O^-}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)},\tag{84}$$
$$p_{O^+}=\frac{V^l_{O^+}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)},\tag{85}$$
$$p_{A^-}=\frac{V^l_{A^-}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)},\tag{86}$$
$$p_{A^+}=\frac{V^l_{A^+}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)},\tag{87}$$
$$p_{B^-}=\frac{V^l_{B^-}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)},\tag{88}$$
$$p_{B^+}=\frac{V^l_{B^+}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)},\tag{89}$$
$$p_{C^-}=\frac{V^l_{C^-}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)},\tag{90}$$
$$p_{C^+}=\frac{V^l_{C^+}(1)}{V^l_{O^-}(1)+V^l_{O^+}(1)+V^l_{A^-}(1)+V^l_{A^+}(1)+V^l_{B^-}(1)+V^l_{B^+}(1)+V^l_{C^-}(1)+V^l_{C^+}(1)}.\tag{91}$$

$$\mu_{B^-C^-} = \frac{D_{B^-C^-}}{\Delta_{C^-}}, \quad \mu_{C^-C^-} = \frac{D_{C^-C^-}}{\Delta_{C^-}}, \quad \mu_{O^-C^+} = \frac{D_{O^-C^+}}{\Delta_{C^+}},$$
$$\mu_{O^+C^+} = \frac{D_{O^+C^+}}{\Delta_{C^+}}, \quad \mu_{A^-C^+} = \frac{D_{A^-C^+}}{\Delta_{C^+}}, \quad \mu_{A^+C^+} = \frac{D_{A^+C^+}}{\Delta_{C^+}},$$
$$\mu_{B^-C^+} = \frac{D_{B^-C^+}}{\Delta_{C^+}}, \quad \mu_{B^+C^+} = \frac{D_{B^+C^+}}{\Delta_{C^+}}, \quad \mu_{C^-C^+} = \frac{D_{C^-C^+}}{\Delta_{C^+}},$$
$$\mu_{C^+C^+} = \frac{D_{C^+C^+}}{\Delta_{C^+}},$$

where μ_{XY} represents the percentage proportion of blood of type X used to replace the blood of type Y in the blood bank. μ_{XY}, in essence, is the critical piece of information that the blood bank manager needs.

As previously mentioned, solving the blood assignment problem will involve applying three standard global optimization algorithms, the symbiotic organisms search, genetic, and particle swarm optimization algorithms to find an optimum solution for the problem at hand. Moreover, it should be noted that these global optimization techniques are preferred to their traditional counterparts because they have been proven to be more efficient and robust in finding reasonable or near-optimal solutions to nearly all complex optimization tasks [27], [28]. Therefore, the next step involves solving the given objective function by implementing the hybridization of the three metaheuristic algorithms to efficiently identify the optimal routing for each blood type described above. *The aim here is to determine the replacement proportion of all blood groups based on the volume of importation and supply of the different blood groups considering the rhesus factor compatibility effect as well*. The solution methodology and implementation process of the triple hybrid algorithms is discussed subsequently.

IV. SOLUTION METHODOLOGY

In blood supply chains, the daily management of the blood bank with respect to blood demand and supply units has frequently been observed to be threatened by several risks that arise from uncertainty in available parameter values, such as the accurate demand forecast and possible unforeseen disruptions [30]. This highlights the need for an effective technique that can determine the optimal assignment of blood units in the dynamic blood bank environment. Note that the case where highly limited importation of blood from external sources to the blood bank is needed, then the process is considered to represent optimal assignment. However, because the optimal assignment of blood in most cases is not usually a possibility, an optimization method is preferred. Such a method should be able to assign blood units commensurate to the available blood volume, and the volume requested, with minimal or no form of importation from an external source. Therefore, because good quality solutions to the same blood assignment problem studied in this paper have resulted from some global metaheuristic algorithms, namely, SOS [10], PSO [26], and GA [25], [31], these three efficient algorithms are combined in this study to determine an even better, near-optimal, assignment of blood units relative to blood demands.

Implementing the proposed hybrid optimization techniques involves multiple heuristic methods. For example, the SOS algorithm is first used as the global optimization technique, which maintains the required optimal routing for assigning available blood units relative to demands. Then the GA, with its crossover, mutation, and selection operator effects, is used to handle the aspect of rhesus factor blood compatibility concerns. We also use the PSO, which is similar in operation to the SOS and is used to enhance the overall performance stability of the optimal routing process. The PSO, with its global best variable characteristic, serves the main purpose of filtering out the best cross-matching among the different blood groups from any inappropriate cross-matches. The search strategy of the PSO differs from the SOS in the sense that it does not implement survival of the fittest like the SOS search mechanism. Finally, a more traditional heuristic technique called the bottom-up technique is implemented as part of the proposed hybrid algorithm, with the aim of pulling in required blood units from other compatible blood types. The reason for introducing the bottom-up technique is to maximize the replacement options of the available compatible blood types. For example, there are situations or instances when the blood units at hand cannot meet the demand for a day, so additional units from other compatible blood types can be used to augment the temporary shortage. However, it is necessary that each blood type must first fulfill its own corresponding requests before being redirected to satisfy other compatible blood types. Specifically, the bottom-up technique is used to minimize unnecessary importation of blood units from an external source; such minimization should reduce the running cost of the blood bank. Next, we briefly discuss the three stages of the hybrid SOSGAPSO implementation steps.

A. SYMBIOTIC ORGANISMS SEARCH OPTIMIZATION TECHNIQUE

The SOS algorithmic design concept is inspired by the three main interactive relationships that occur between paired organisms within ecosystems [34]. These evolving relationships are mutualism, commensalism, and parasitism. Each of these interactions presents a unique behavioral pattern between the organisms, which they can be mutually beneficial, one-sided, or even destructive to one organism while benefiting the other. The integration of each of the three aforementioned interactions into the SOS computational, algorithmic design model is briefly presented below, as three phases.

1) MUTUALISM INTERACTION PHASE

The mutualism phase of the SOS algorithm involves the interaction between two different organisms, for which the overall relationship goal is that both organisms receive mutual benefits during their interaction. In other words, both organisms will attempt to provide some form of positive exchange that would assist in the sustainability of both individual organisms. Given below are two updating equations (90 and 91), which exemplify the mutualism phase. Let X_i and X_j

represent randomly selected individuals from an ecosystem, whilst *rand* represents a random number generator with a set bound range between 0 and 1. The term BF1 and BF2 are stochastic values between 1 and 2 that depict each individual organism's benefit factor. The term X_{best} corresponds to an individual organism with the highest degree of adaptation to the habitat or ecosystem. Finally, the term MV, which is calculated in equation 92, represents a mutual vector between X_i and X_j.

$$X_{inew} = X_i + rand(0, 1) \cdot (X_{best} - MV \cdot BF1), \quad (93)$$

$$X_{jnew} = X_j + rand(0, 1) \cdot (X_{best} - MV \cdot BF2), \quad (94)$$

where

$$MV = (X_i + X_j)/2. \quad (95)$$

2) COMMENSALISM INTERACTION PHASE

The commensalism phase involves one organism benefiting from the interaction, whilst the other organism does not receive any benefit, but at the same time, the interaction between the two organisms does not cause harm to the targeted organism. Equation 93 below exemplifies the commensalism phase of the SOS algorithm. Let X_{best} and X_{inew} represent the best organism and newly calculated organism, respectively.

$$X_{inew} = X_i + rand\ (-1, 1) \cdot (X_{best} - X_j), \quad (96)$$

where the organisms X_i Benefits from X_j by means of $(X_{best} - X_j)$.

3) PARASITISM INTERACTION PHASE

The final step within the SOS algorithm optimization search process is known as the parasitism relationship, and generally relates to improving the structure of one organism at the cost of harming the second organism involved. Typically, the organism that feeds (i.e., becomes stronger) is known as the parasite, while the organism being fed off is regarded as the host. The parasitic phase offers powerful capabilities in improving the chances of obtaining a possible solution for a given problem. The current study utilizes the form of parasitic mechanism between two individuals, as depicted in Figure 3 below.

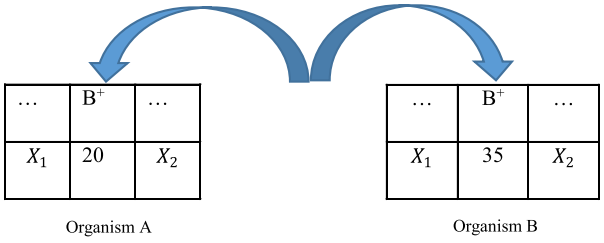

FIGURE 3. Illustration of the SOS algorithm parasitic phase considered in this study.

With reference to Figure 3, the illustration provides an example with specific reference to blood type B^+. Assume that organism A is the parasite, and organism B is the host. Furthermore, assume that the demand for blood type B^+ is 30 for the given day. Since the host contains a value that is much closer to the daily demand, the parasite will then scan the host and, if the present value within the parasite is not enough, swap values. This mechanism is implemented across all blood types, performing swap-overs when necessary. If the parasite contains a value that is better than the host, then the swap-over is not performed. The enhanced SOS algorithm for the blood assignment problem is illustrated in Figure 4.

B. PARTICLE SWARM OPTIMIZATION TECHNIQUE

Originally the construction of the PSO algorithm was motivated by the behavior of birds and schools of fish [35]. The general description of the PSO algorithm comprises various particles within the solution search space of a problem. The PSO algorithm has, over the years, gained wide popularity and acceptance among the optimization research community due to its flexibility, simplicity of implementation and adaptation, and versatility in solving a wide range of optimization problems [36]. The algorithm has each candidate solution as a 'particle'. It starts its optimization search process by enabling individual particles to move randomly with different velocities, which they use to update their individual positions. While moving within the search domain, a particle tries to attain its best velocity according to its own local best (represented here as P_{best}) value and its neighbor's global best (represented here as G_{best}) value. The change in position of a particle in the search domain is dependent on the current position of the particle, current velocity of the particle, the distance between P_{best} and current position, and distance between G_{best} and current position [37], [38]. The velocity guides the movement of particles within the search space. The control parameters of PSO are inertia weight, acceleration coefficients, and velocity of the particle. Given an n-dimensional space, every particle is made up of a position vector $X_i = (X_{i1}, X_{i2}, \ldots, X_{in})$ and a velocity vector $V_i = (V_{i1} V_{i2}, \ldots, V_{in})$, whilst using the particle and velocity updating equations (94 and 95) to iteratively update the best source of direction within the solution search space.

$$V_i\,(t+1) = \omega V_i\,(t) + c_1 r_1\,(P_{best} - X_i) + c_2 r_2\,(G_{best} - X_i) \quad (97)$$

$$X_i\,(t+1) = X\,(t) + V_i(t+1) \quad (98)$$

where:

P_{best} :	personal best position
G_{best} :	global best position
r_1, r_2 :	random values between [0, 1]
c_1, c_2 :	scaling parameters.
ω :	inertia weight
t :	iteration index.

C. GENETIC ALGORITHM OPERATORS

A genetic algorithm (GA) can be described as a global stochastic search method that duplicates the processes

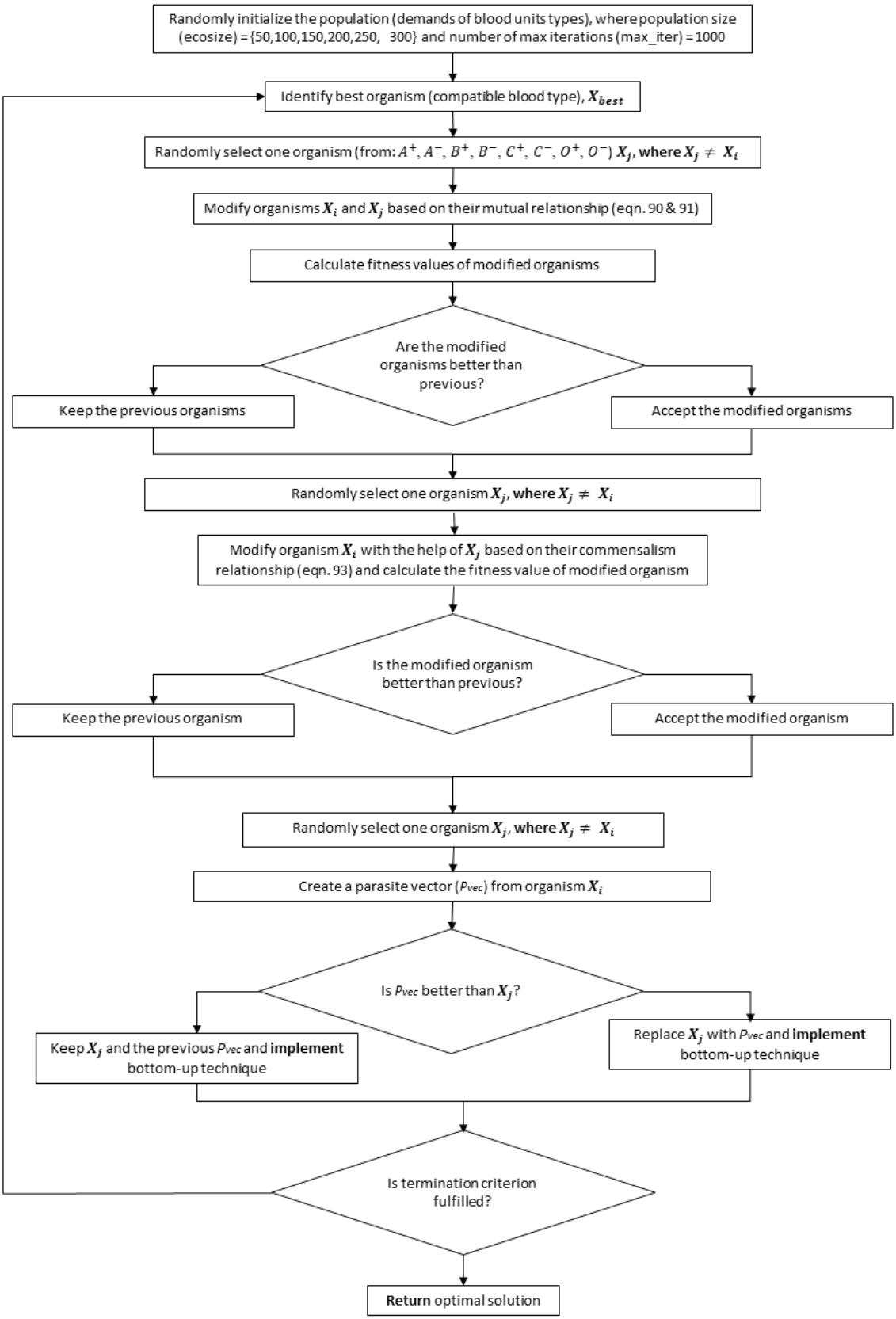

FIGURE 4. Enhanced SOS algorithm flowchart for optimal routing of blood assignment.

associated with the evolutionary processing of biology [39], [40]. The GA comprises three stages, which are summarized below: selection, crossover, and mutation.

1) SELECTION

this genetic operator emphasizes the need to locate better solutions within a population. This generally encompasses some form of selection mechanism, such as the roulette wheel, tournament selection, etc.

2) CROSSOVER

the selection operator chooses two individuals from the population and selects segments within the individuals to crossover. As with the selection stage, there are several types of crossover possible, such as uniform, multi-point, etc. In the current study of the blood allocation problem, the hybrid algorithm is modeled in such a way that it is possible to randomly select positions between two organisms and perform uniform crossovers to exchange segments between corresponding blood types. Figure 5, below, provides an example of the crossover mechanism implemented for the proposed SOSGAPSO considered in this study.

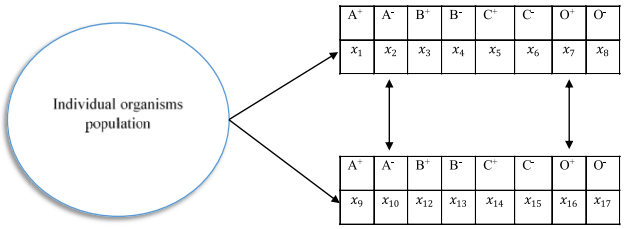

FIGURE 5. Representation of a crossover operator for the SOSGAPSO algorithm.

Figure 5 provides an illustration of the crossover mechanism adopted within the GA segment of the proposed SOSGAPSO hybrid algorithm. In figure illustrates two individuals within the population being selected and then being subjected to the crossover mechanism. Afterward, random segments are selected (in this example segments 2 and 7 are selected), and the values (denoted by x) are then swapped between individuals (segments 10 and 16).

3) MUTATION

The process of mutation encompasses changing the original genetic make-up of an individual, which in turn creates a new opportunity for a potential solution to be located. The mutation process used in this study involves the process of recalculation and multi-randomized selection. If a segment within an individual is randomly selected and the value does not match the daily demand for the day (per blood type), then the segment is subjected to recalculation, which leads to replacement if the new value is an improvement over the pre-existing value.

D. BOTTOM-UP TECHNIQUE

Ensuring adequate blood units supply relative to demand is a vital managerial decision making practice that must be maintained, if the blood bank is to function well. However, this is only feasible if there is sufficient information regarding blood units supply, demand and importation. Such accounting information is also required to avoided unnecessary blood importation from external sources. Therefore, in the proposed blood assignment optimization task, the bottom-up technique is used to model the blood units compatibility matching pattern based on the compatible replacement probability of each of the available blood units in the bank. The probability pattern is derived from similar daily historical replacement options that had been previously implemented in the bank. For N similar days, the replacement probability pattern P_{in} of the blood type i is computed as follows.

$$P_i = \frac{\sum_{n=1}^{N-1} X_{ij}^n}{N-1}$$
$$X_{ij} = \begin{cases} 1 & D_{ij} > P_i \\ 0 & D_{ij} > P_i \end{cases} \tag{99}$$

where $X_{ij}^n = 1$ denotes compatible blood type i being considered as a suitable replacement for blood type j on a similar day n. While $X_{ij}^n = 0$ implies that the blood type i cannot be used to replace blood type j. The variable D_{ij} represent the amount of blood donated by people with blood type i to people with blood type j. Therefore, the replacement probability pattern for all compatibility blood types can be written as follows.

$$\boldsymbol{P_i} = \left\{ \boldsymbol{p_{ij}^1}, \quad \boldsymbol{p_{ij}^2}, \quad \ldots, \quad \boldsymbol{p_{ij}^m} \right\} \tag{100}$$

where m represents the number of blood types, which in this case is eight (8).

Ideally, the bottom-up technique favors replacement with the blood of the same type. However, with the complexity of blood compatibilities issues introduced by the addition of the rhesus factors, replacement options become even more complex. The bottom-up techniques is modelled to handle such complexity by exploring different replacement possibilities or alternatives based on historical behavior probability. Note that the bottom-up techniques are considered and used, in general, only when the primary blood type replacement option is not possible because of a shortage in that blood type. In this case, a secondary compatible replacement blood type option is invoked by implementing the aforementioned replacement technique.

E. SYMBIOTIC ORGANISM SEARCH GENETIC PARTICLE SWARM OPTIMIZATION

The hybrid optimization algorithm proposed in this paper consists of three well-known global metaheuristics, namely SOS, GA, and PSO. Further, these three algorithms have been proven to be highly effective, efficient, and robust in solving very complex global optimization problems [41]. Of note is the aforementioned individual algorithms, each having been previously used separately to solve the blood assignment problem, with their respective performance studies having been reported in [10], [26], [31]. Therefore, it is assumed that

a hybrid of these algorithms should prove capable of routing the assignment and minimizing overall levels of whole blood units importation over a finite period of time in the blood bank. Further, our combination of the algorithms entails using the SOS algorithm as the control algorithmic structure, while embedding the essential components of the GA and PSO to improve the overall performance further and refine the quality of the partial results obtained by the SOS. In other words, the resulting hybrid SOSGAPSO algorithm generally leverages and incorporates the strength of the individual algorithms to form a more robust and complex optimization algorithm. However, due to the fusion of multiple sub-processes such as the mutation, crossover, particle position and velocity equations of the remaining algorithms, it is most likely that the time complexity of the new hybrid algorithm would increase to some extent. Nevertheless, the anticipated slight increase in the amount of computational time needed for the new method is likely to be compensated by the hybrid algorithm's improved performance.

For the SOSGAPSO swarm solution representation, each individual organism and particle is encoded using strings of characters, which in this case represent the eight blood types, namely, A^+, A^-, B^+, B^-, C^+, C^-, O^+, and O^-. However, because the values of the sample blood unit are represented as strings, they cannot be directly used to compute the updating formulas for the SOS and PSO algorithms, which form the major part of the hybrid SOSGAPSO. Therefore, the strings of the blood types are transformed or converted into numeric values of type double, so as to take into consideration significant values of supply, demand, and importation. Note that while there may be other approaches for transforming the given string of blood type into equivalent numeric values, the conversion methods used in the current study are with respect to the blood type compatibility equations given in (1) to (8) above and the proportion of Rh blood type distribution found in the Nigerian Population [2]. Note that by storing each value in a separate location, as shown in Table 3, we can manipulate the individual swarm in the calculations used later in the algorithms updating formulas, which are represented in equations (90) – (95) above.

TABLE 3. Blood types values and proportions.

Blood type	O^-	O^+	A^-	A^+	B^-	B^+	C^-	C^+
Value	0.04	0.07	0.07	0.15	0.07	0.15	0.30	0.15
Proportion (%)	1.6	51.3	0.7	22.4	0.6	20.7	0.1	2.6

The standard SOS and PSO were initially designed to handle continuous optimization problems. Since the blood assignment problem considered in this paper is a discrete optimization problem, a discretization transformation method is required to map a continuous position of each organism to a discrete position. The conversion method used here for the SOS algorithm is a simple random number permutation defined in equation (101) and also discussed in [29].

$$X_i = \lfloor X_i + k \rfloor \, mod \;\; m \tag{101}$$

where k and $m > 0$ are integers.

For the PSO aspect of the hybrid algorithm, the conversion techniques proposed by Veeramachaneni *et al.* [42], appear to be more appropriate for the problem at hand. The Veeramachaneni PSO is an extension of the well-known binary PSO, in which each variable is allowed to assume any of θ discrete values. The technique uses the generalized Sigmund function given in equation (102) to map the updated velocity into $[0, \theta - 1]$ interval.

$$S_{ij} = \frac{\theta - 1}{1 + \exp\left(-V_{ij}\right)} \tag{102}$$

While the velocity update remains unchanged from the binary case, the particle position update is modified to allow for more than two states [43]. Consequently, each particle's position is updated according to the normal distribution $X_{ij} = N\left(S_{ij}, \tau \times (\theta - 1)\right)$ random number generator. Then the piecewise function given in equation (100) is applied to ensure that all values fall within the specified interval $[0, \theta - 1]$, as described in [43].

$$X_{ij} = \begin{cases} \theta - 1 & X_{ij} > \theta - 1 \\ 0 & X_{ij} < 0 \\ X_{ij} & otherwise \end{cases} \tag{103}$$

The incorporation of the GA entails using its three special operators, namely selection, crossover, and mutation, which have been explained above. Finally, the bottom-up technique is used to assign compatible blood types to those special replacement cases where shortages might arise. For example, in the special cases arising when, as a result of scarcity or shortages, the request for some units of a specific blood type cannot be met by assigning the same blood type. In that case, any available compatible blood units from other blood types are pulled to satisfy the immediate demand. What this simply means is that, whenever blood is to be assigned, the requested blood type must first be considered. Only when there is a shortage of the requested blood type in the blood bank then aside from the universal donor (O^-), the other next available compatible blood type should be considered for a possible assignment. However, in the case where the O^- and the other compatible blood types are not available, and then blood will be imported into the bank from an external source(s) in accordance with the request at hand [26]. One main advantage of implementing the bottom-up technique is that it has every tendency to minimize the importation of blood units into the blood bank, and by so doing, increases the effective utilization of available resources. The Algorithm listing one below provides a brief summary of the sequence of steps performed in the implementation of the SOSGAPSO algorithm. The process starts by setting the size of the initial swarm population, which does not exceed the initial blood volume. The first optimization task is initiated by the control algorithm, which is the SOS, where the three interaction phases of the algorithm are executed and updated iteratively. The mutualism phase of the SOS algorithm, which consists of equations (90) - (91), is implemented in conjunction with

Algorithm 1 Pseudocode of SOSGAPSO

Begin:
Initialize organism population $X_i(i = 1, 2, \ldots, n)$
Initialize *ecosize, MaxDay, Day: nday*
While (*itr <Maximum number of iterations*) **do**
  for *nday = 1: MaxDay*
    Let $i = 1$
      for $i = 1$: *ecosize*
        Calculate fitness of all organisms
        X_{best} = Organism with lowest fitness value
      end for
  //Begin SOS phases
  Mutualism Phase:
  Select organism X_j randomly where $X_j \neq X_i$
    Apply SOS updating formula on new X_i and X_j using equations 90 and 91
      Implement GA operator: Selection
      Implement GA operator: Crossover
      Implement GA operator: Mutation
  Commensalism Phase:
      Whilst in Commensalism Phase: Apply SOS updating formula on new X_i using equations 93
      Apply PSO updating formula on new X_i using equations 94 and 95
  Parasitic Phase:
Select organism X_j randomly where $X_j \neq X_i$
  Create a parasite vector P_{vec}
      Update the organism X_j
      Calculate the fitness value of X_j
      if (*fitness* (P_{vec}) *<fitness* (X_j)) **then**
        $X_j = P_{vec}$
      end if
    Implement the bottom-up technique: $\mathbf{P_i} = \left\{\mathbf{p_{ij}^1}, \mathbf{p_{ij}^2}, \ldots, \mathbf{p_{ij}^m}\right\}$
   Identify the best individual X_{best}
  end for
$Itr = itr + 1$
end while
return X_{best}
end

the GA operators. Afterward, the commensalism phase follows with the SOS updating formula given in equation (95): it causes one individual within an ecosystem to benefit from another individual without causing the secondary individual any harm. Further, in parallel, the commensalism phase is implemented in conjunction with the updating formula of PSO given in equations (94) – (95). The parasitic phase of the SOS algorithm is similarly implemented in conjunction with the bottom-up technique. Lastly, the SOSGAPSO algorithm exits the search optimization process and returns the best individual as the solution for the day; that is, after the algorithm set termination condition is satisfied.

The main advantage of the above algorithm listing is its leverage of the enhanced performance of the three individual algorithms, namely SOS, GA, and PSO, and most importantly, it strikes a good balance between exploration and exploitation during the hybrid algorithm implementation. The parameter *nday* represents the number of days, *MaxDay* represents the maximum number of days the algorithm will execute before termination, *ecosize* represents the size of the ecosystem, X is the organism within the ecosystem, X_{best} represents the best organism that will be returned as a solution for the day, and *fitness ()* represents the fitness function used to evaluate organism X.

F. DATASET SUMMARY

The following study implements a hybrid Symbiotic Organism Search – Genetic Algorithm – Particle Swarm Optimization (SOSGAPSO) algorithm in conjunction with real-world blood dataset. The dataset was adapted from the Nigerian Enugu blood bank over a period of eight years (2010 - 2018). The datasets provide the issued levels of blood units for each blood type per month. To gauge the effectiveness of SOSGAPSO algorithm, the algorithm was tested with various population sizes. For each population size, the number of

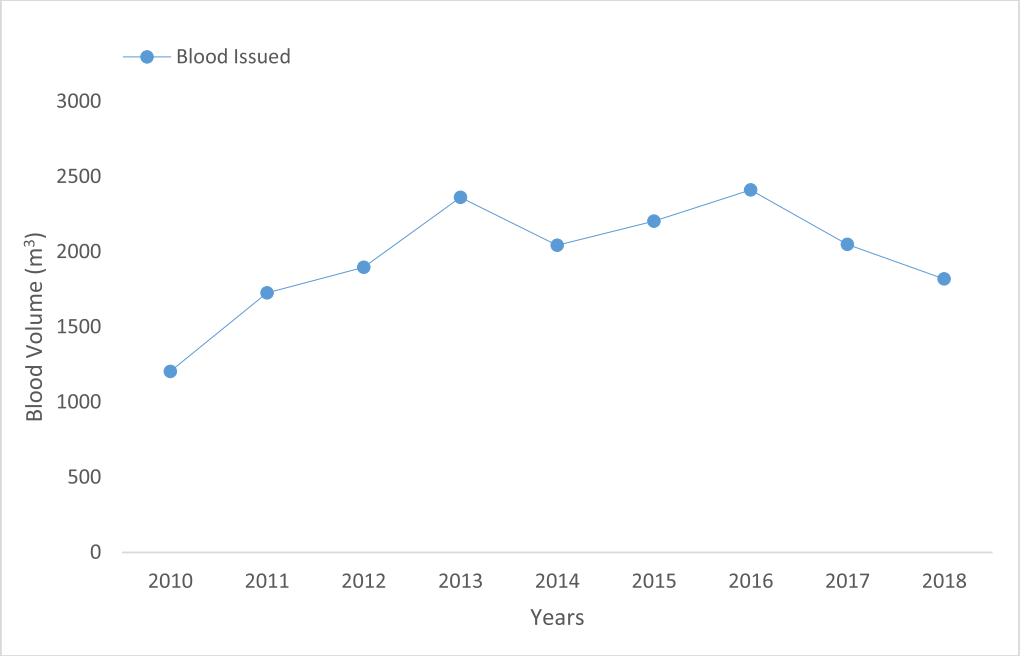

FIGURE 6. Line graph showing the summed blood levels across all blood types.

solutions, levels of importation, and the time complexity were compared. The following further elaborates on each point:

a) Population size: Relates to the number of organisms generated within each population. An organism in this study is represented by a basic array of size eight, with each segment relating to each blood type. Additionally, the organism representation relates to both the supply generated for a month and the organism with the closest make-up to the demand for the month.

b) Number of Solutions: When the demand for a month matches the randomly generated supply, then this is classified as a solution. A solution implies that no form of importation occurred within that month, nor were any blood units carried over to the next month. Due to the nature of the algorithm, finding solutions would occur mainly at the start of the time frame, because the algorithm carries over any blood units from the previous month into the current month. Due to this behavior, blood units would be expected to stockpile as time progresses, thus making the aspect of finding a solution much rarer.

c) Importation: This variable relates to importing additional blood units from external sources when the blood bank cannot meet the demand for a given month. This is deemed to have a negative connotation, as the act of importing additional blood units incurs more expenses for the blood bank.

In total, five different population sizes were tested, ranging from a population size of 50 to 300, in increments of 50. Due to the issued amount of blood units being documented (use of real-world dataset) and not randomly generated, the average levels will be held constant across the five simulations. Table 4 below illustrates the averages attained for each blood type, over a test period of 108 months.

TABLE 4. Average per blood type for demand levels in accordance with the Nigerian Enugu Blood dataset used within this study across a period of 108 Months.

Rhesus factor	O^+	O^-	A^+	A^-	B^+	B^-	C^+	C^-
Demand	85.99	5.21	36.23	1.81	29.36	0.89	4.08	0.31

As previously stated, this study used real-world blood demand values from Enugu, Nigeria. According to the dataset, in none of the months did demand values exceed 300 units. Figure 6 is given below, which illustrates the annual demand for blood units from 2009 until 2018. The supply is generated using a random generator with an upper and lower percentage bound of 100 and 0, respectively. Furthermore, the initial blood unit volume is set to 300.

Figure 6 indicates the summed amounts of blood units across all blood types for the period 2010 to 2018. It is noticeable that an increasing trend occurs from 2010 until 2013 when the trend shows a somewhat steady-state for the next three years. The trend then starts to decrease from 2016 onwards. It is unclear if the issued blood supply would continue this decreasing trend, as the datasets only provided values between these time ranges. Furthermore, since we are currently in 2019 for this study, we opted to ignore the few months of data provided for 2019, as it would give an unfair comparison with data for complete years.

V. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, extensive experiments are described, which were conducted in order to investigate the practicality of the mathematical model formulation and the efficacy of the proposed hybrid SOSGAPSO, in comparison to the performance of other existing algorithms, in solving the blood assignment problem. For this purpose, a real-life blood use

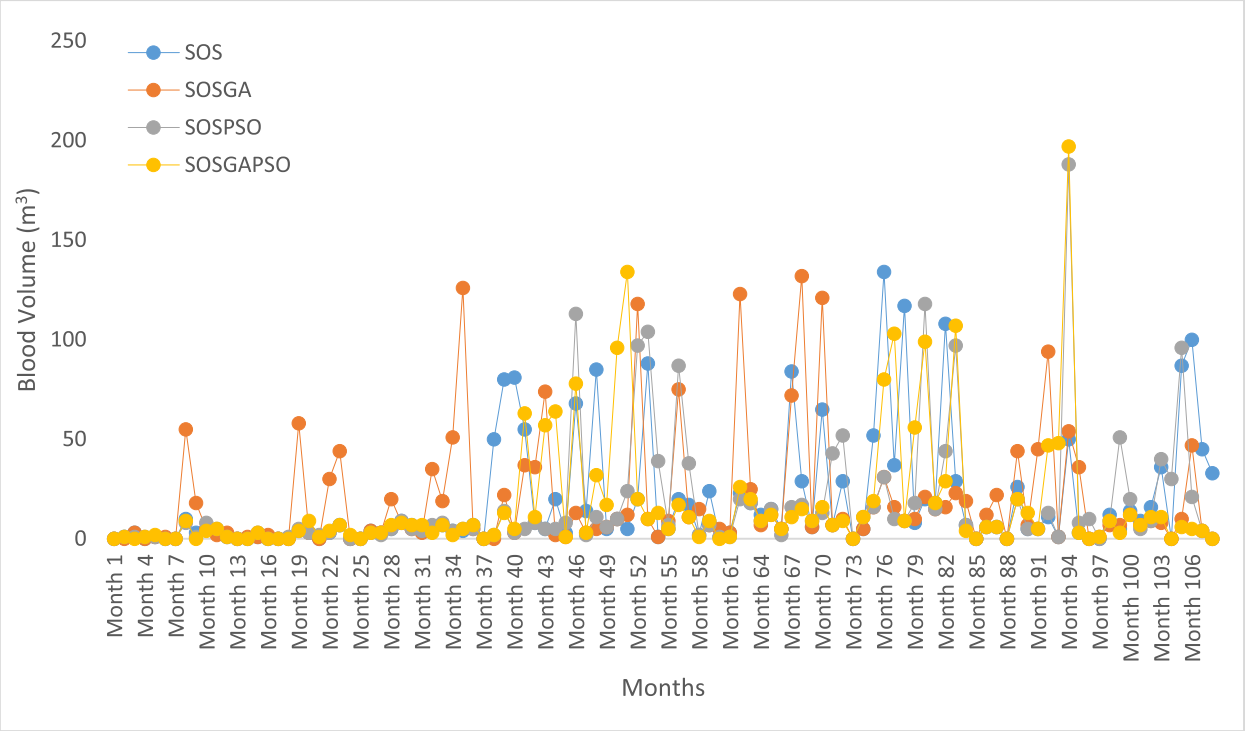

FIGURE 7. Levels of blood unit importation for the population size of 50 over a period of 108 months (2010 - 2018) for the SOS, SOSGA, SOSPSO and SOSGAPSO algorithms.

dataset is utilized. The experimental testing platform for the proposed algorithms was conducted on an Intel Core i5 CPU with 2.5 GHz and 4 GB RAM and Windows 10.0 operating system. The implementation software for the three algorithmic methods was Java.

This study utilizes the following parameter settings for the proposed SOSGAPSO algorithm: population size N or $ecosize = 50$, crossover rate Cr $= 0.07$, mutation rate m $= 0.03$, personal and global learning coefficients $c_1 = c_2 = 1.7$, inertia weight $\omega = 0.715$, the inertia weight damping ratio $wdamp = 0.99$, $r_1 = r_2 =$ are randomly generated values between [0, 1]. Each algorithm was subjected to a maximum of 1000 iterations, where each iteration number stands for each day. Different population sizes of 50, 100, 150, 200, 250, and 300 were used to conduct the experiment. Whilst the supply values were set to constant percentage bounds between 0 and 100%, with the initial blood volume not exceeding 300 units. Specifically, the representative algorithms namely, the SOS, PSO and GA choice of parameter selection is similar to the implementation by Govender and Ezugwu [10], [31], [46] and Ezugwu *et al.* [29], in terms of parameter configuration, however, they differ with regards to the real-life data set used for the current study. Table 5 shows the parameter setting for the three representative algorithms.

TABLE 5. Parameter configuration for SOS, PSO, and GA algorithms.

SOS Parameters	PSO Parameters	GA Parameters
N = 50	N = 50	N = 50
MaxIt= 1000	MaxIt = 1000	MaxIt = 1000
	$c_1 = c_2 = 1.7$	m = 0.03
	$\omega = 0.715$	Cr = 0.07
	$wdamp = 0.99$	

VI. COMPARISON WITH COMMONLY USED BLOOD ASSIGNED ALGORITHM

To examine the effectiveness of SOSGAPSO algorithm, the results obtained from the algorithm would be contrasted with the values obtained in relation to the SOSGA [29], SOSPSO, and standard SOS algorithm [46]. Furthermore, the aspect that would be evaluated is the average level of importation and computational time. Note that no solutions were found by any of the algorithms. A solution is achieved when the supply meets every value across all the eight blood types, thus resulting in no form of importation or carry-over of stock. More so, the algorithms could not find any potential solutions due to the varying issuing percentages exhibited by the real-life datasets. Additionally, the act of carrying over remaining blood units to the next month increases the likelihood of stockpiling and diminishes the potential for the algorithms to locate solutions.

The monthly volumes of importation with the initial population size, according to all the algorithms, are shown in Figure 7 and Table 6. As can be seen from the figure, all the algorithms (except the SOSGA, algorithm) experienced low importation levels for the initial 37 months. Thereafter, sporadic spikes in importation occurred for the SOS, SOSGA, and SOSPSO algorithms across the finite time period. The importation spikes occurred much earlier (after month 7) for

TABLE 6. Supply and importation averages attained per algorithm across eight blood types in accordance with population size 50.

	Variable	O^+	O^-	A^+	A^-	B^+	B^-	C^+	C^-
SOS	Supply	182.85	1.79	95.24	1.79	325.14	14.99	1.79	1.79
	Importation	9.09	3.63	2.90	0.80	0.00	0.01	2.76	0.00
SOSGA	Supply	130.21	1.88	59.06	1.88	70.75	2.79	1.88	1.88
	Importation	8.30	3.55	2.90	0.69	1.29	0.12	2.70	0.01
SOSPSO	Supply	250.35	1.83	129.84	1.83	401.19	16.05	1.83	1.83
	Importation	7.83	3.58	2.20	0.72	0.00	0.01	2.71	0.01
SOSGAPSO	Supply	302.52	1.81	158.75	1.81	359.41	13.14	1.81	1.81
	Importation	7.50	3.62	2.62	0.78	0.00	0.00	2.71	0.02

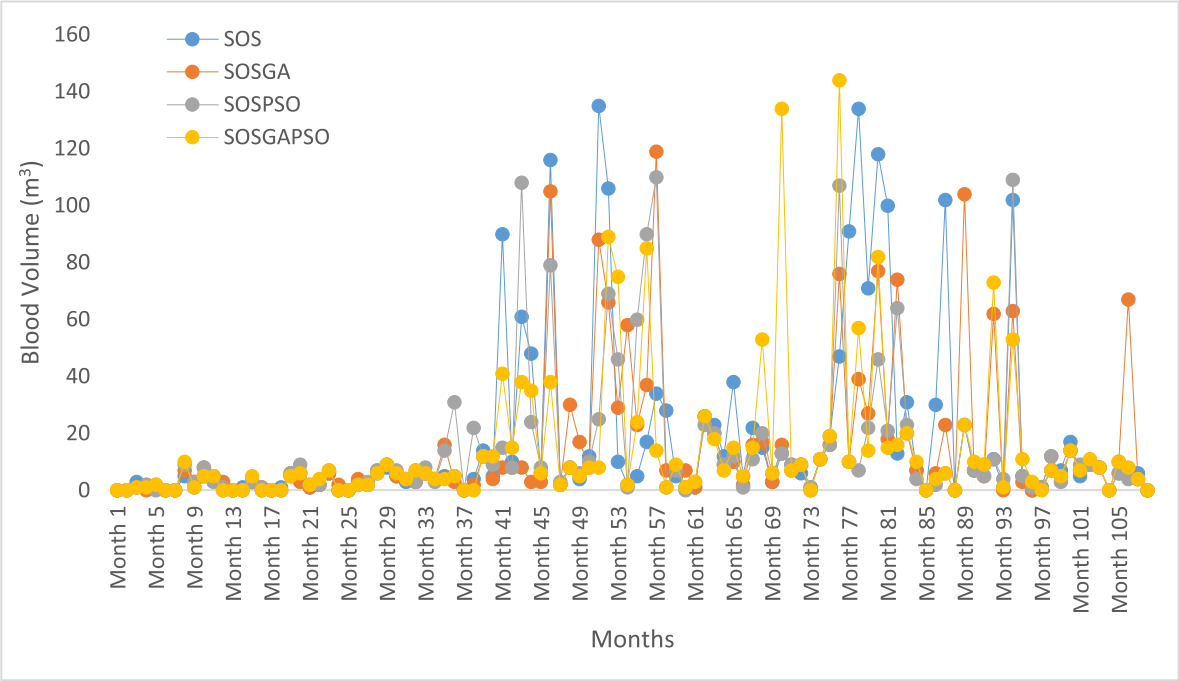

FIGURE 8. Levels of blood unit importation for the population size of 100 over a period of 108 months (2010 - 2018) for the SOS, SOSGA, SOSPSO and SOSGAPSO algorithms.

the SOSGA algorithm. Once importation was necessary, similar levels of importation were predicted by all the algorithms, except for the proposed hybrid SOSGAPSO algorithm, which seemed to incur lower peaks in the importation, as illustrated in Figure 7. It is expected that an increase in population size will have a positive effect on importation levels.

Table 6 shows the capabilities of the competing algorithms in handling the assignment of certain blood types more efficiently. For instance, the proposed SOSGAPSO indicates lower importation levels for O^+ as compared to the SOSGA and SOS, with the SOSPSO showing similar capabilities. In contrast, the SOSPSO incurs lower importation rates for blood type A^+. Likewise, the SOSGA low importation rates are linked to blood type A^- and SOS is linked to type C^- blood. It is unclear at this point whether this behavior would continue as the population sizes increase.

Results for a population size of 100 are shown in Figure 8 and Table 7. According to Figure 8, the four algorithms incurred minimal levels of importation over the first 30 months. It is then noticeable that the three algorithms (SOS, SOSGA, SOSPSO) start to increase importation rates around months 35-37, but the SOSGAPSO algorithm starts

TABLE 7. Supply and importation averages attained per algorithm across eight blood types in accordance with population size 100.

	Variable	O^+	O^-	A^+	A^-	B^+	B^-	C^+	C^-
SOS	Supply	250.53	1.78	123.41	1.78	331.29	9.73	1.78	1.78
	Importation	8.58	3.61	2.78	0.80	0.00	0.00	2.76	0.02
SOSGA	Supply	212.05	1.83	109.79	1.83	277.44	10.48	1.83	1.83
	Importation	6.52	3.57	2.03	0.74	0.00	0.00	2.75	0.02
SOSPSO	Supply	232.24	1.85	120.87	1.85	382.12	12.65	1.85	1.85
	Importation	6.45	3.55	1.14	0.77	0.00	0.00	2.71	0.01
SOSGAPSO	Supply	206.76	1.79	124.95	1.79	400.44	16.99	1.79	1.79
	Importation	6.03	3.60	1.66	0.74	0.00	0.00	2.76	0.01

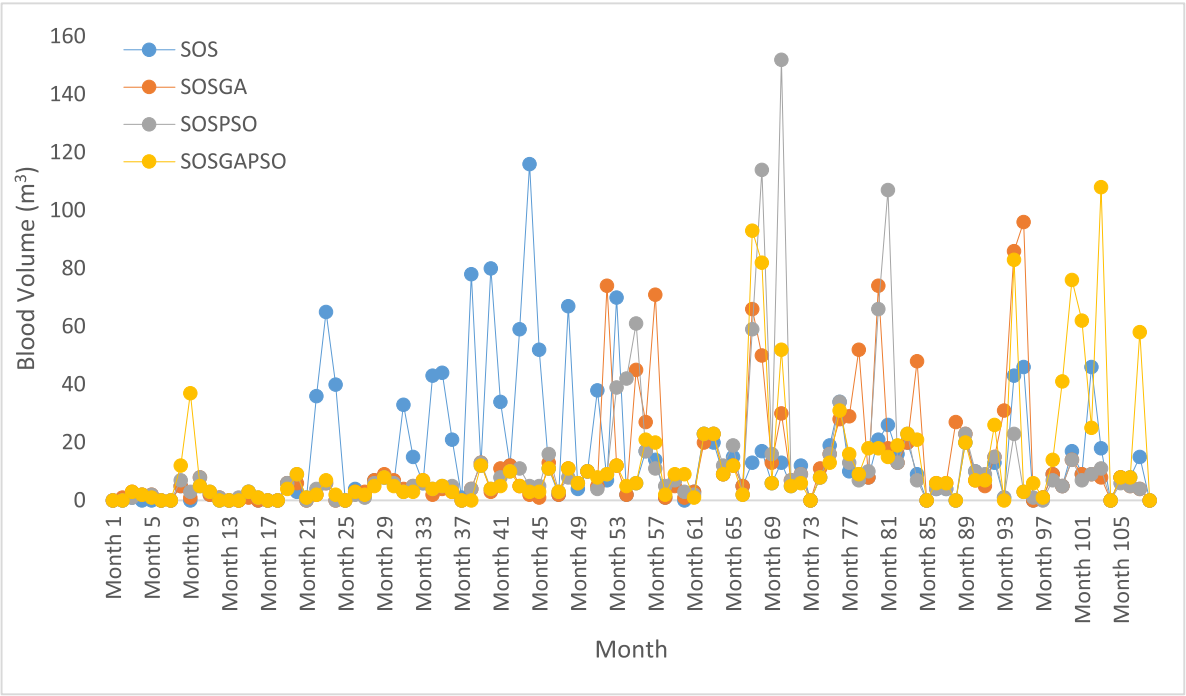

FIGURE 9. Levels of blood unit importation for population size of 150 over a period of 108 months (2010 - 2018) for the SOS, SOSGA, SOSPSO and SOSGAPSO algorithms.

to increase importation rates a few months later, around month 40. The SOSGA and SOSPSO algorithms incur very similar peaks, with the SOS algorithm incurring more frequent and higher peaks, whilst the proposed SOSGAPSO incurs noticeably lower levels of blood units importation. Months 41-95 incurred the majority of blood unit importation across all the four algorithms, with the levels lowering back to minimal values from month 95 onwards, whilst the SOSGA algorithm experiencing one large spike in month 106.

From Table 7, it can be seen that no form of importation occurred for blood group B, which can, therefore, be excluded from comparisons. It is evident that the SOSGAPSO algorithm incurs lower importation values for blood types O^+, A^-, and C^-. This is important because O^+ is one of the most common blood types within Nigeria, so any algorithm that incurs lower importation for O^+ would reduce expenses for the blood bank. The SOSPSO algorithm marginally outperforms the SOSGA algorithm with lower importations for blood types O^+, O^-, A^+, C^+, and C^-. Finally, the standard SOS algorithm did not succeed in producing lower importation levels for any of the blood types.

For an initial population of 150, the need for importation is delayed. As seen in Figure 9, the overall levels of blood unit importation begun to elevate only around month 43,

TABLE 8. Supply and importation averages attained per algorithm across eight blood types in accordance with population size: 150.

	Variable	O^+	O^-	A^+	A^-	B^+	B^-	C^+	C^-
SOS	Supply	175.32	1.91	80.72	1.91	111.57	5.40	1.91	1.91
	Importation	6.03	3.51	1.89	0.72	0.51	0.04	2.68	0.01
SOSGA	Supply	274.32	1.87	179.06	1.87	461.86	13.79	1.87	1.87
	Importation	4.95	3.53	0.81	0.71	0.00	0.00	2.71	0.01
SOSPSO	Supply	295.98	1.78	187.02	1.78	418.71	19.23	1.78	1.78
	Importation	4.61	3.62	0.71	0.80	0.00	0.00	2.79	0.03
SOSGAPSO	Supply	250.81	1.85	158.60	1.85	438.93	17.66	1.85	1.85
	Importation	4.60	3.56	0.80	0.71	0.00	0.00	2.11	0.00

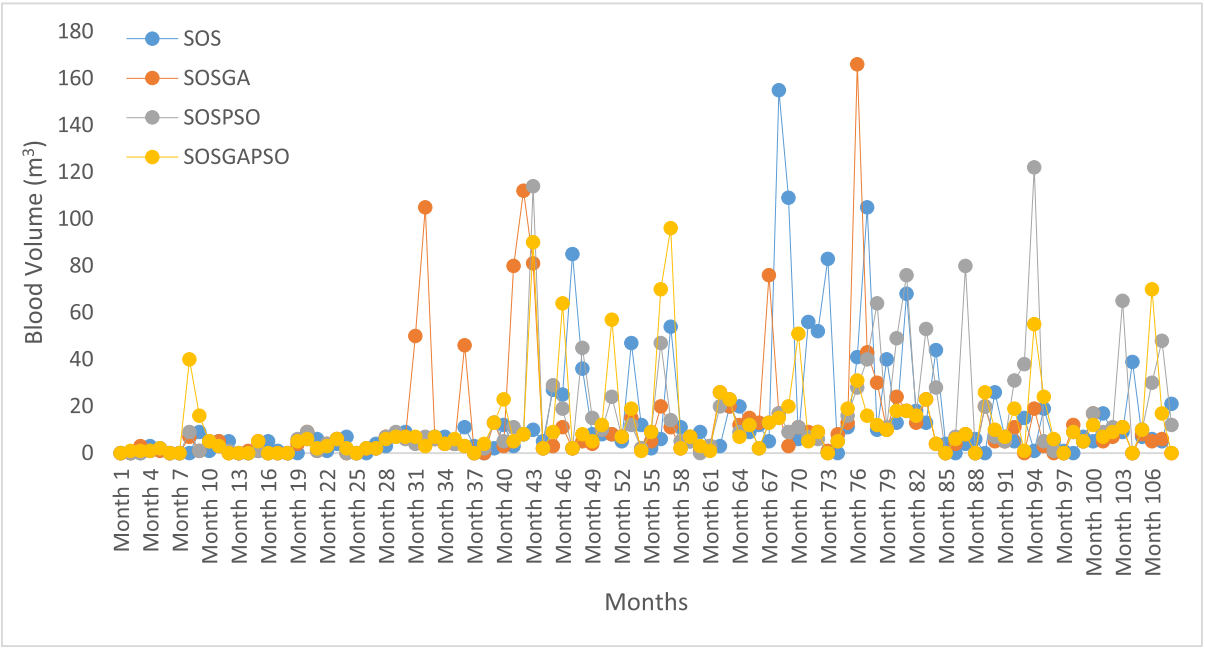

FIGURE 10. Levels of blood unit importation for the population size of 200 over a period of 108 months (2010 - 2018) for the SOS, SOSGA, SOSPSO and SOSGAPSO algorithms.

which is 13 months later than the effects seen in Figure 7 (population size of 50). Once again, it is evident that the proposed SOSGAPSO algorithm exhibited lower peaks in as compared to the remaining algorithms, which is a good sign that the SOSGAPSO had the least importation level compared to the other algorithms.

Results illustrated in Table 8 are also in favor of the SOSGAPSO algorithm, which required minimal importation levels for four out of the eight blood types, with an equal finish for blood type B across the three hybrid implementations, whilst the standard SOS incurred minimal importation averages for blood type B. Furthermore, the SOSGAPSO algorithm seemed to incur no importation for blood type C^- as compared to the other algorithms.

Figure 10 gives results for an initial population of 200. It shows an increased importation rate amongst most of the algorithms around month 40, which is similar to the effects seen in Figure 9. The importation rates do not level out as the time frame continues, with sporadic importation spikes occurring across the four algorithms. For the SOSGAPSO and SOSPSO algorithms, there were numerous low peaks, whilst the standard SOS, and SOSGA algorithm experienced the two highest peaks within the time frame.

TABLE 9. Supply and importation averages attained per algorithm across eight blood types in accordance with population size: 200.

	Variable	O^+	O^-	A^+	A^-	B^+	B^-	C^+	C^-
SOS	Supply	209.80	1.76	106.00	1.76	316.55	14.98	1.76	1.76
	Importation	5.84	3.62	2.09	0.70	0.00	0.00	2.76	0.00
SOSGA	Supply	237.46	1.94	113.40	1.94	425.35	16.33	1.94	1.94
	Importation	5.22	3.46	1.09	0.70	0.00	0.00	2.63	0.00
SOSPSO	Supply	259.85	1.82	132.40	1.82	390.39	16.51	1.82	1.82
	Importation	5.63	3.57	1.72	0.69	0.00	0.00	2.75	0.01
SOSGAPSO	Supply	207.37	1.82	106.14	1.82	341.89	20.18	1.82	1.82
	Importation	4.30	3.54	1.03	0.74	0.00	0.03	2.73	0.01

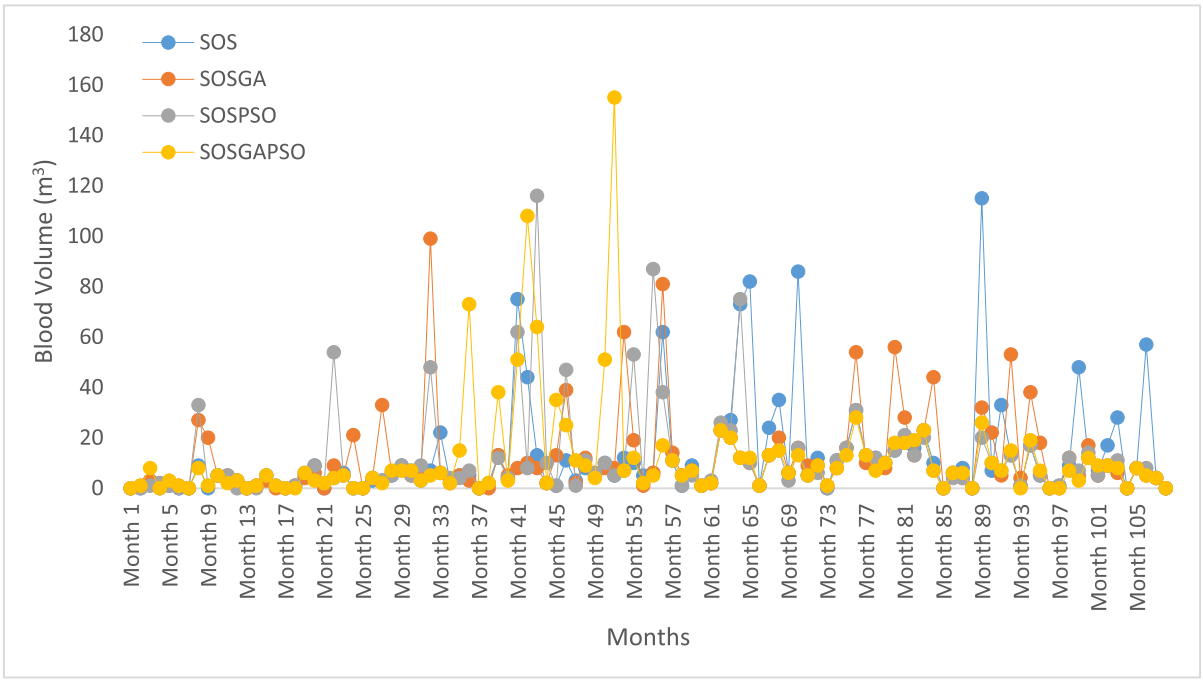

FIGURE 11. Levels of blood unit importation for the population size of 250 over a period of 108 months (2010 - 2018) for the SOS, SOSGA, SOSPSO and SOSGAPSO algorithms.

As can be seen in Table 9, for a population of 200, whilst the triple hybrid implementation shows minimal importation values for the majority of the blood types, it is apparent that the standard SOS and SOSGA algorithms turn over lower importation levels for blood types C^- and B^- in comparison to the other hybrid implementations.

In Figure 11, giving results for a population of 250, the SOSGAPSO algorithm incurs the highest peak of all the algorithms around month 51. Nevertheless, it makes up for this large increase by having far lower importation levels across the remaining time period. The SOSPSO algorithm seems to incur the majority of its importations between months 31 and 67, whilst the SOSGA algorithm incurs importations between month 25 and 58, followed by a brief period of low importations, after which importation levels start to increase again around month 76. Finally, the SOS algorithm started incurring larger importation rates around month 33, and sporadically incurred spikes throughout the remaining time frame.

While focusing on Table 10, which depicts the importation averages attain for each metaheuristic implementation in conjunction with a population size of 250, it can be seen that the SOSPSO hybrid significantly improves its results as compared to those for a population size of 200. The SOSGAPSO

TABLE 10. Supply and importation averages attained per algorithm across eight blood types in accordance with population size: 250.

	Variable	O^+	O^-	A^+	A^-	B^+	B^-	C^+	C^-
SOS	Supply	200.35	1.83	104.32	1.83	238.51	6.17	1.83	1.83
	Importation	4.08	3.59	1.23	0.72	0.44	0.03	2.75	0.02
SOSGA	Supply	157.47	1.91	77.04	1.91	136.93	6.05	1.91	1.91
	Importation	4.02	3.49	0.92	0.70	0.05	0.04	2.69	0.01
SOSPSO	Supply	228.64	1.92	100.40	1.92	359.81	13.45	1.92	1.92
	Importation	3.77	3.47	0.98	0.73	0.00	0.03	2.67	0.02
SOSGAPSO	Supply	216.08	1.97	122.80	1.97	336.49	10.65	1.97	1.97
	Importation	3.97	3.46	1.04	0.67	0.00	0.03	2.61	0.00

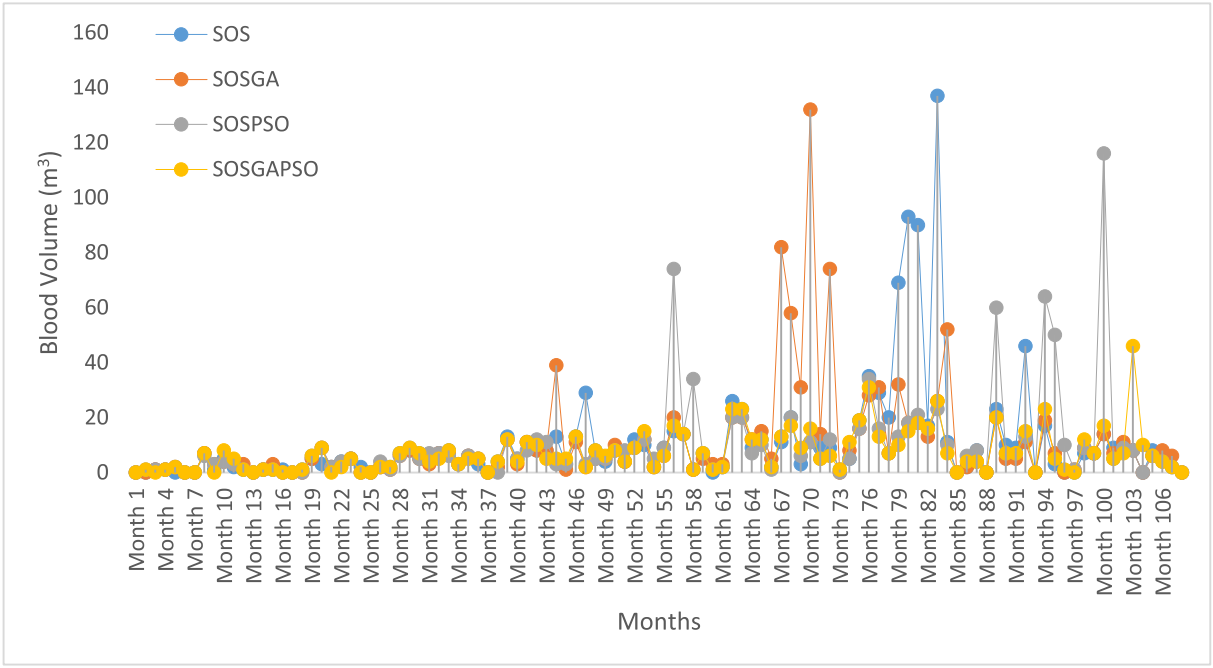

FIGURE 12. Levels of blood unit importation for the population size of 300 over a period of 108 months (2010 - 2018) for the SOS, SOSGA, SOSPSO and SOSGAPSO algorithms.

provides lower importation values for only four out of the eight blood types with the SOS algorithm incurring higher importation values.

With the population set at a maximum of 300, the SOSGAPSO hybrid implementation dramatically outperformed the remaining three algorithms, as shown in Figure 12. As depicted, the SOSGAPSO algorithm incurred many fewer high peaks than did the remaining algorithms. The SOSGA and SOSPSO algorithms seem to produce almost similar results, whilst the SOS algorithm sees some improvement with a larger population, but still incurs much higher importation peaks.

The superiority of the SOSGAPSO algorithm with a population size of 300 can also be witnessed in relation to specific blood groups. The SOSGAPSO algorithm clearly outperforms the other algorithms in all eight blood types, requiring nil imports for several blood groups. This behavior is expected to continue as the population size increases, but it can be seen from the results above that none of the algorithms were capable of minimizing importation for blood types C+.

VII. DISCUSSION

As depicted in Figures 7 – 12, and Tables 6 – 11 the SOSGAPSO offers improved results in comparison to the other

TABLE 11. Supply and importation averages attained per algorithm across eight blood types in accordance with population size: 300.

	Variable	O^+	O^-	A^+	A^-	B^+	B^-	C^+	C^-
SOS	Supply	337.03	1.94	191.98	1.94	457.40	14.90	1.94	1.94
	Importation	3.00	3.55	0.79	0.71	0.00	0.00	2.62	0.01
SOSGA	Supply	289.62	1.92	198.12	1.92	526.99	18.92	1.92	1.92
	Importation	3.82	3.46	0.08	0.72	0.00	0.00	2.67	0.01
SOSPSO	Supply	310.34	1.85	195.41	1.85	482.75	14.69	1.85	1.85
	Importation	2.53	3.54	0.39	0.74	0.00	0.01	2.69	0.00
SOSGAPSO	Supply	324.34	1.87	224.10	1.87	524.16	18.21	1.87	1.87
	Importation	0.42	3.52	0.00	0.70	0.00	0.00	2.61	0.00

TABLE 12. Computational time associated with each metaheuristic algorithm in comparison to the different population sizes.

Metaheuristic Algorithm	Population size	Computational Time
SOS	50	7315
SOSGA	50	13627
SOSPSO	50	10811
SOSGAPSO	50	19758
SOS	100	13921
SOSGA	100	38577
SOSPSO	100	16227
SOSGAPSO	100	42733
SOS	150	26247
SOSGA	150	40870
SOSPSO	150	28355
SOSGAPSO	150	48615
SOS	200	21097
SOSGA	200	46099
SOSPSO	200	36140
SOSGAPSO	200	62248
SOS	250	39189
SOSGA	250	49486
SOSPSO	250	63403
SOSGAPSO	250	77923
SOS	300	75088
SOSGA	300	63480
SOSPSO	300	35669
SOSGAPSO	300	43899

two hybrid implementations and the standard SOS algorithm. Even though none of the algorithms where able to find a solution, it is apparent that an algorithm with more operators that modify an individual within a population will have a greater chance of obtaining individuals that are a closer match to the demand for a month. In addition, the computational times associated with each metaheuristic algorithm in conjunction with the varying population sizes is shown below in Table 12 and these CPU times give a clearer picture of the amount of computing resource consumed by the individual algorithm to obtained the results presented in Tables 6 – 11.

From Table 12, it can be seen that for each metaheuristic algorithm, the computational time tends to increase as the population size increases. However, this relationship is not consistent. In comparing the results for the SOSGAPSO algorithm for population sizes of 250 and 300, it can be seen that the computational time decreases considerably between these two variables. It appears that a decrease in importation rates (as depicted in Table 11) influences the overall computational time experienced per algorithm. Another observation concerns the standard SOS algorithm, which, except for the case of SOSGAPSO with population size 300 mentioned above, shows much lower computational time values than that for the other algorithms. This can be attributed to the hybrid metaheuristic algorithms needing to execute additional functions which can be computationally intensive, but offer improved results. In order words, the SOS algorithm consumed the least computational time, but at the expense of sound quality results. The hybrid methods produced better results, but at the cost of computational time. Additionally, it can be seen that increasing the number of individuals within a population decreases the chances of obtaining large importation averages.

To further investigate the effect of population size on the level of importation, this study summed the average amounts of importation experienced per blood type in order to find a percentage increase or decrease in the amount of importation experienced. Below is Figure 13, which illustrates the trend associated with the behavior between importation and population size.

As shown in Figure 13, each algorithm starts with a 0% percentage change from a population size of 50, as this was the initial starting population size. Afterwards, there is a decrease in overall importation percentage for all algorithms, with the SOSPSO and SOSGAPSO incurring approximately 14.20 % decrease in importation for a population size of 100. Following the population size of 100, a further increase in population size to 150 imposes a smaller percentage decrease for the standard SOS, SOSGA and SOSPSO algorithm, whilst SOSGAPSO incurs a great decrease of 20.54%. Population

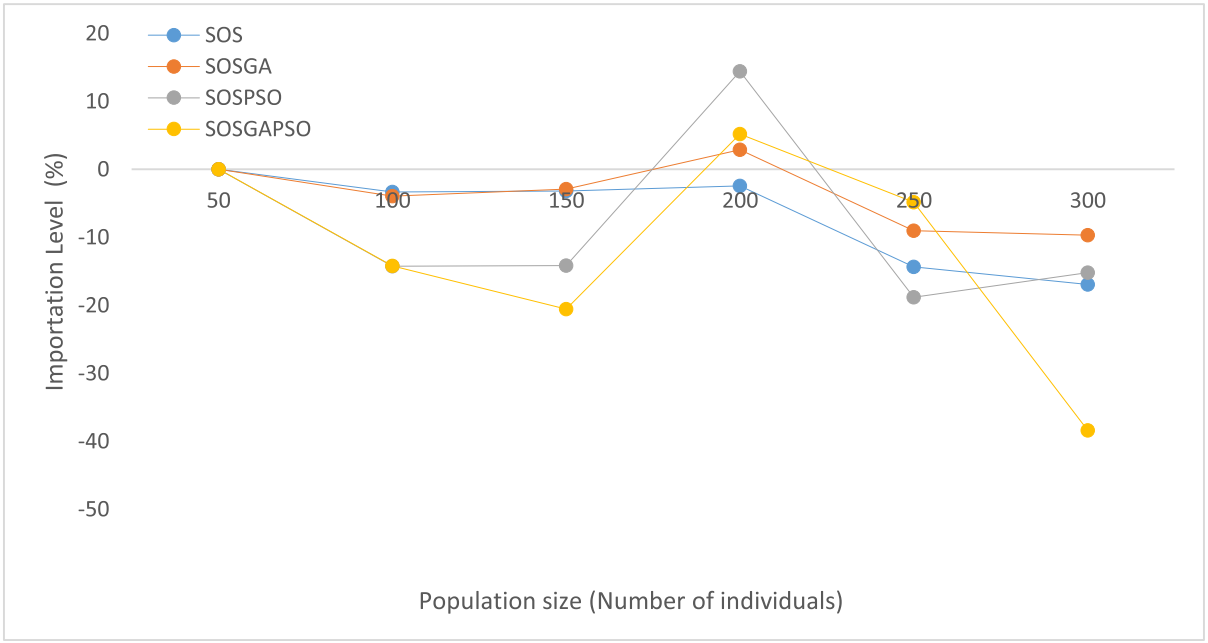

FIGURE 13. Percentage increase/decrease between importation levels and population sizes for the SOS, SOSGA, SOSPSO and SOSGAPSO algorithms.

size of 200 suddenly increases importation for all algorithms, which is largely caused by the increases in blood types A^+ and O^+ as shown in Table 9. Although a sudden increase was experienced with a population size of 200, a further increase dramatically decreases all percentage importation, with SOSPSO incurring the highest decrease of 18.79%. Lastly, the population size of 300 establishes the dominance of SOSGAPSO, indicated by a decrease importation levels by 38.40%.

In terms of the individual blood types, type O^+ incurs as one of the highest average levels of importation due to high percentage of individuals in Nigeria living with blood type O^+ as was shown earlier in Table 2. Blood group B was handled well by each metaheuristic algorithm for all population sizes, even though blood type B^+ has the third highest percentage proportion within Nigeria. Stock-piling, as the accumulation of blood stocks of different types over a prolonged finite period, can be seen with positive connotations, because it greatly reduces importation levels associated with a blood type. The relationship can be seen as directly proportional, as a higher blood type percentage proportion leads to a higher probability of stock-piling occurring at a faster rate. Taking into account the aspect of stock-piling, it appears that all algorithms struggled to reduce importation rates associated with blood types O^- and C^+, which is mainly associated with these blood types having a low population proportion. In summary, the SOSGAPSO hybrid metaheuristic can be deemed as a superior implementation in relation to the data from a real-life blood sample collection process at the National Blood Transfusion Service, Enugu Center, Nigeria.

VIII. CONCLUSION AND FUTURE DIRECTION

This paper has considered the formulation of a novel mathematical model for the task of assigning whole blood units by blood bank managers to meet requests for blood supplies from hospitals and emergency call centres. In the developed model, the relationship between the proportion of each blood group in the population and the blood use and donations to the stocks available in the blood bank was established. Unlike the existing mathematical models, the full ABO-Rhesus blood compatibility factor was considered and its associated complexities estimated using an optimization procedure that minimizes the difference between the ideal situation of constantly available stocks and the real-world blood bank scenario. Further, the proposed model accounts for the provision of an accurate managerial discernment on the proportion of blood type X that can be replaced with blood type Y.

Subsequently, efficient advanced nature-inspired SOSGAPSO metaheuristic algorithm has been proposed for the optimal routing of blood units in the bank. The metaheuristic optimization method that was proposed combined the efforts of the SOS, GA, PSO and a unique bottom-up modelling technique. The bottom-up technique was modelled according to certain observed historical behavioural probability of blood compatibility replacement trends in the blood bank. The implemented bottom-up technique reflects some additional constraints that needed to be satisfied for the blood bank to be efficiently managed, and so avoiding unnecessary importation of blood from external source(s). The multiple experiments conducted using real-life datasets proved the practicality and validity of the proposed mathematical model formulation. Further, the numerical results showed that by

using the SOSGAPSO global optimization and bottom-up techniques, the total units of blood types imported into the bank were greatly minimized.

The current model formulation only considered red blood cells. Therefore, in future work, we intend to incorporate further components of a variety of blood products by extending the current proposed mathematical model formulation. More so, it will be interesting to consider integrating special cases of mass emergencies and blood shelf-life relative to expiration into the proposed model. Although incorporating these two last components would slightly increase the model complexity, but the results of the proposed model implementation presented in this study would not be significantly affected. It would also be interesting to test the robustness of the proposed hybrid SOSGAPSO metaheuristic algorithm when applied to solve other complex real-world optimization problems.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of the paper.

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