Sub-Region Warranty Differential Pricing Optimization Strategy Based on Regional Granularity of Use Reliability

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ABSTRACT Despite the various advantages of existing warranty designs, these designs are still limited to inherent reliability and ignore the interaction between use reliability and regional difference. A new sub-region warranty differential pricing strategy based on the regional granularity of use reliability is proposed to solve these issues. A use reliability prediction model based on regional granularity partition results and after-sales failure data is established to evaluate product use reliability accurately in different sub-regions. Then, a novel high-dimensional optimization model that considers the regional difference in use reliability, warranty, and price is developed to optimize the regional warranty differential pricing strategy. Two scenarios for pricing and warranty, namely, unified and partition warranty schemes, are considered, and the necessary optimality conditions for each scenario are determined. Afterward, a practical case study is conducted in the air-conditioning industry to verify the performance of the proposed model. The sensitivity of the model is also analyzed. Numerical experiments show that the sub-region warranty differential pricing strategy allows for a suitable trade-off among use reliability, warranty, and selling price. Moderately reducing the selling price of sub-regions with high use reliability and increasing the selling price of sub-regions with low use reliability can enhance profitability. This work provides manufacturers with guidelines on designing sub-region warranty differential pricing strategies.

INDEX TERMS Sub-region warranty, differential pricing, use reliability, high-dimensional optimization model, air-conditioner.

I. INTRODUCTION

Warranty, as a competitive marketing tool, is crucial in enhancing perceived customer value and stimulating market demand [1]. With the rapid development of production technologies and increase in market competitiveness, product quality and reliability have been continuously improved. This improvement has led to high customer expectation for after-sales service quality and length of the warranty period and coverage, thereby affecting the expected warranty costs and integrated profit. Warranty policy decisions should consider not only the key marketing factors,

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including selling price, warranty period, and post-warranty commitments, but also the inherent technical factors, such as product quality and reliability [2], [3]. However, the use reliability of products distributed in different geographical regions exhibits a significant difference because the working conditions and service time of products vary from region to region [4]. Therefore, developing a scientific and reasonable warranty strategy from the perspective of product use reliability is meaningful. Doing so can control and reduce warranty costs, increase manufacturers' profits and customer satisfaction with after-sales service, and enable manufacturers to achieve regional warranty management, differential extended warranty service, and personalized marketing.

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Researchers from diverse disciplines have conducted many studies on warranty decisions from different perspectives due to the importance of such decisions [5]–[7]. Reliability-based warranty decisions mainly involve product reliability evaluation and formulation of a reasonable warranty period on the basis of reliability information. A number of extensions of reliability evaluation methods and reliability-based warranty modeling have been reported in recent years.

A. RELIABILITY EVALUATION

Reliability evaluation methods in literature are divided into three basic types, namely, methods based on reliability tests, component-based numerical and probabilistic simulations, and statistical analysis of field data [8]–[11]. Although reliability tests have high popularity and extensive applications, they are unsuitable for current high-reliability or long-life products because they are time consuming and expensive [12]. Simulations can be used to support reliability tests in certain cases to solve this problem partly. However, simulations might demonstrate low accuracy when applied to complex products with various failure mechanisms [11]–[13]. Thus, reliability evaluations based on tests and simulations are limited in many application fields. By contrast, reliability evaluations based on field return data have low time consumption and cost and are accurate. Accordingly, they can overcome the limitations of the two other methods.

Various field data-based reliability evaluation approaches have been developed in recent years. For instance, Lu [14] proposed a reliability prediction method based on early field failure data to identify vehicle parts that are likely to become actionable items. Ion *et al.* [15] studied field reliability prediction by using early warranty data from repair centers to check if the reliability of consumer electronics is at the proper level. Al-Garni *et al.* [16] utilized Weibull and gamma distribution models to estimate the reliability of air-conditioners by using field data. Moreover, Yuan and Kuo [17] used the Weibull exponential distribution to simulate the L-shaped hazard rate function. A change-point-based reliability prediction model using field return data was investigated by Altun and Comert to predict the reliability performance of high-volume complex electronic products during their warranty period [18]. Kleyne and Bender [19] introduced a practical method of reliability prediction that merges a military standard approach with manufacturer's warranty data to improve the accuracy of failure rate prediction. Gurel and Cakmakci [20] used a parametric Weibull model and the linear regression method to estimate hazard rates of LCD TV on the basis of lifetime data obtained by service departments. Furthermore, Hsu *et al.* [21] proposed a hierarchical reliability model for the joint modeling of laboratory and field data. With this model, the information from multiple products of the same type can be integrated efficiently. Wang *et al.* [22] studied a reliability evaluation model of vehicles based on support vector regression by using 2D warranty data. To investigate the effect of usage rate on product degradation, Dai *et al.* [23] presented

an accelerated failure time model in which the parameters are estimated by the stochastic expectation–maximization algorithm using censored and field data. To improve prediction accuracy, Ghasemieh *et al.* [24] proposed a new hybrid model based on heuristic optimization methodologies and artificial neural network. On the basis of 2D warranty data, He *et al.* [25] developed a log-linear regression model of the product failure rate by setting the usage rate and manufacturing day as covariates and utilized the maximum likelihood approach to estimate the parameters. Meanwhile, Prakash and Mukhopadhyay[26] performed a complete failure mode (CFM) analysis based on warranty data to determine the reliability of individual components.

Although these field data-based methods achieve desirable results, they still have the limitations. First, they do not fully utilize after-sale fault data to evaluate the use reliability of products in different regions quantitatively. Second, they cannot establish a mathematical model that reflects the quantitative relationship between use reliability and different geographical regions nor can they reveal the quantitative relationship between inherent reliability and use reliability. Third, these models were developed only for specific problems and thus lack generalizability. Therefore, establishing a generalized prediction model to evaluate the use reliability of products in different regions is essential.

B. RELIABILITY-BASED WARRANTY MODELING

Many studies have focused on optimal strategies that link an engineering design variable (product reliability) with warranty to either maximize manufacturers' profit or minimize the total cost. Several of these works, such as [2], [3], [27], and [28], considered only qualitative aspects. Meanwhile, previous quantitative studies have investigated the interactions among decisions on reliability, warranty, price, and production. In particular, quantitative studies have provided a series of warranty decision models from the perspective of cost minimization or profit maximization.

Warranties based on cost minimization have been examined in prior research. Chien [29] proposed an analytical model to determine the optimal warranty period and out-of-warranty replacement age for a general repairable product sold under a failure-free renewing warranty agreement by minimizing the corresponding warranty cost functions from the perspective of the seller and buyer. To set the optimal reliability, price, warranty length, and warranty service quality, Wang and Liu [30] reconstructed the product demand function by analyzing the effect of warranty service quality on customer satisfaction and repeated purchase and further introduced a modified cost function and warranty decision model. Ambad and Kulkarni [31] presented an objective optimization function weighted by the mean time between failures (MTBF) and warranty cost to increase product reliability and minimize warranty costs. The authors developed a solution based on the genetic algorithm and analyzed the relationship among MTBF, reliability, and warranty costs. Chen *et al.* [32] studied a comprehensive warranty cost

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model for repairable products presenting two types of failure under burn-in and free replacement warranty/pro-rata warranty policies. They obtained the failure rate distribution and optimal warranty period length from the warranty cost model and after-sales service data.

Warranties based on profit maximization have also been investigated. Murthy [33] introduced an analytical model that comprehensively considers the reliability, warranty, and price of a new product with the aim of maximizing the generated profit. To determine the optimal quality and price strategy for a new product, Teng and Thompson [34] developed a two-parameter decision model of quality and price and used the maximum principle method to solve this model. Wu *et al.* [35] developed a new price–warranty decision model based on [34] by replacing quality level with warranty period length to maximize the total profit. DeCroix [36] introduced a new model based on game theory to represent companies that are compelled to design warranties, reliability parameters, and selling prices. Another optimal strategy was proposed by Fang and Huang [37], who utilized a Bayesian analysis approach to set the optimal price and warranty period when the manufacturer does not have sufficient historical sales data. Huang *et al.* [38] investigated a mathematical model of selecting the optimal reliability, warranty period, and price to maximize the discounted expected profit for a general repairable product sold under an FRW policy. The study developed solutions for stable and dynamic markets by using the maximum principle method. On the basis of the work of [38], Darghouth *et al.* [39] proposed an analytical model for the joint optimization of the design, warranty, and price of a new product sold with a maintenance service contract considering one unique service provider. The goal was to maximize the discounted expected profit over the product life cycle. Two specific types of markets and four maintenance service contracts were considered. Shang *et al.* [40] investigated the condition-based renewable replacement warranty policy by combining the inverse Gaussian degradation model, which maximizes profit by optimizing the replacement threshold, sale price, and warranty period. Zhu *et al.* [41] recommended an integrated model that makes simultaneous optimal decisions on product reliability, warranty policy, regular price, promotion price, and lengths of regular sales and promotions to maximize the total profit over the product lifecycle. Other relevant multi-objective models have also been developed [42]–[44] to solve multi-objective warranty optimization issues due to research and development (R&D) expenditures, production cost, market share, warranty attractiveness index, warranty costs, spare parts cost, MTBF, and net profit.

Many integrated models have been proposed to address warranty optimization problems with single and multiple objectives. However, these analytical models still have deficiencies. First, they consider only the product quality levels or inherent reliability aspects and ignore the regional differences of use reliability. Second, studies on warranty decisions that simultaneously consider use reliability, warranty period,

FIGURE 1. Regional granularity partition of the use reliability of AC.

and sale price are unavailable. Research on sub-region warranty differential pricing strategies is also lacking.

To deal with the problems that cannot be addressed by these models, the present study considers the significant regional difference in the use reliability of products in different regions. To the best of our knowledge, very few studies have focused on the correlation between use reliability and regional difference. In our previous work [45], [46], we constructed a multivariable high-dimensional clustering model of use reliability and utilized the improved clustering algorithm to solve the model, which is aimed at the minimum difference of use reliability in similar regions. We obtained optimal regional granularity partition results on use reliability. As shown in Fig. 1, the use reliability of air-conditioner (AC) in 31 provincial administrative areas of Mainland China is divided into 10 groups of geographical regions. Given that product reliability, warranty costs, warranty period, and sale price are closely related [27], [47], [48], the regional difference in use reliability and length of the warranty period directly affects the actual warranty cost per unit and selling price in different sub-regions. This difference should be considered when evaluating optimal warranty policy decision-making options. On the basis of these considerations, we design a sub-region warranty differential pricing strategy based on the regional granularity of use reliability.

Accurate prediction of use reliability is an important prerequisite for achieving a sub-region warranty differential pricing strategy. With this concept in mind, we establish a mathematical model that reflects the quantitative relationship between use reliability and different geographical regions based on regional granularity partition results. We introduce a general formula of the expected warranty cost and modify the sales model for each sub-region. Furthermore, a high-dimensional optimization model that considers the regional difference in the use reliability, warranty period, and selling price of each sub-region is developed to optimize the sub-region warranty differential pricing strategy. Two scenarios for pricing and warranty, namely, unified and partition warranty schemes, are considered, and the necessary

optimality conditions for each scenario are derived. The proposed model is then applied to a practical case study in the air-conditioning industry for illustration. Analyses of the two scenarios and model sensitivity are also conducted.

Our study differs from prior studies in the following aspects. First, previous studies determined optimal warranty decisions on the basis of product inherent reliability. The present study determines optimal sub-region warranty decisions by considering the product use reliability, warranty period, and selling price of each sub-region. Second, unlike prior work, this study investigates the quantitative relationship between use reliability and different geographical regions and evaluates the use reliability in different sub-regions on the basis of regional granularity partition results and after-sales failure data. Third, this work quantitatively examines the effect of the regional difference in use reliability on product warranty and selling price. Moreover, the proposed high-dimensional optimization model uses a differential pricing strategy instead of a unified pricing strategy in different sub-regions.

The remainder of this paper is structured as follows. Section II describes the sub-region warranty differential pricing strategy with a modeling framework. Section III introduces the proposed mathematical model, and Section IV presents the model optimization analysis. Section V provides a practical case study that illustrates the proposed models and a discussion of the results. The conclusions and future work directions are given in Section VI.

II. ANALYSIS OF THE SUB-REGION WARRANTY DIFFERENTIAL PRICING STRATEGY

Warranties are closely related to many complex factors, including product design decision, manufacturing and production process control, selling, after-sales service, and environmental conditions [27]. Hence, key factors should be determined and analyzed prior to developing a mathematical model for the sub-region warranty differential pricing strategy.

A. FACTORS FROM THE TECHNOLOGY PERSPECTIVE

The warranties offered by manufacturers are generally associated with product reliability [41], [49]. Specifically, high product inherent reliability equates to a long and improved warranty term from manufacturers. Product inherent reliability is calculated through a reliability test under standard conditions. Product design, control of raw materials, and quality control in the production phase determine product inherent reliability, which gradually increases with investments in R&D and production. Thus, product inherent reliability can be enhanced by structural improvements, strict quality control activities, and process management measures during product development and production.

The reliability of products placed into the actual use environment is called use reliability. Inherent reliability is the basis of use reliability. That is, inherent reliability positively affects use reliability, which implies that use reliability increases as inherent reliability increases. Furthermore, the service quality level in different sub-regions can meet the same quality standard requirements through continuous quality improvement and strict quality control activities because the service quality factors are controllable. Thus, its effect on use reliability is ignored in this study. In addition, the use reliability of products distributed in different sub-regions exhibits a significant difference due to the regional difference of the factors that influence use reliability. Furthermore, the regional difference of use reliability inevitably affects the length of the warranty period and the expected warranty cost. Therefore, product use reliability is an important basis for manufacturers to formulate effective warranty strategies.

B. FACTORS FROM THE MARKETING PERSPECTIVE

Warranty ensures product reliability and can be used as a valuable marketing tool [50]. Generally, product warranty cost, warranty period, selling price, and sales volume vary under different warranty policies. Designing an effective warranty strategy can enhance customers' perceived value and stimulate market demand, thus affecting the sales volume and sale price. Sale price is the only factor that can generate profit in the market decision, but it is also subject to market environment factors, such as competitive position and competitive situation of the firm. In addition, sale price reversely affects sales volume, that is, sales volume is sensitive to sale price [51]–[53]. Sales volume is constrained by the market environment and regional difference in consumer demand in different regions. Accordingly, the length of the warranty period, the warranty cost for a unit product, the market environment, and the regional difference in consumer demand inevitably lead to fluctuations in the sale price and sales volume of products in different sub-regions.

In accordance with this analysis, a modeling framework of the sub-region warranty differential pricing strategy is established based on the regional granularity of product use reliability. As presented in Fig. 2, the overall idea of this model is to comprehensively consider the effects of key technical features, marketing variables, and regional difference in use reliability on warranty decisions. The multiple and complex interactions among product warranty period, warranty cost of each sub-region, R&D and production costs, sale price of each sub-region, sales volume of each sub-region, sales revenue of each sub-region, and integrated profit are analyzed to determine the optimal combination of product use reliability, warranty period, and sale price of each sub-region, thereby maximizing the manufacturer's profit.

Overall, the sub-region warranty differential pricing strategy effectively improves market competitiveness and warranty benefits. It can also overcome the deficiencies brought about by inherent reliability.

III. MATHEMATICAL MODEL

This section describes product warranty policy and failure. A general formula of expected warranty cost is developed, and the sales model for each sub-region is modified.
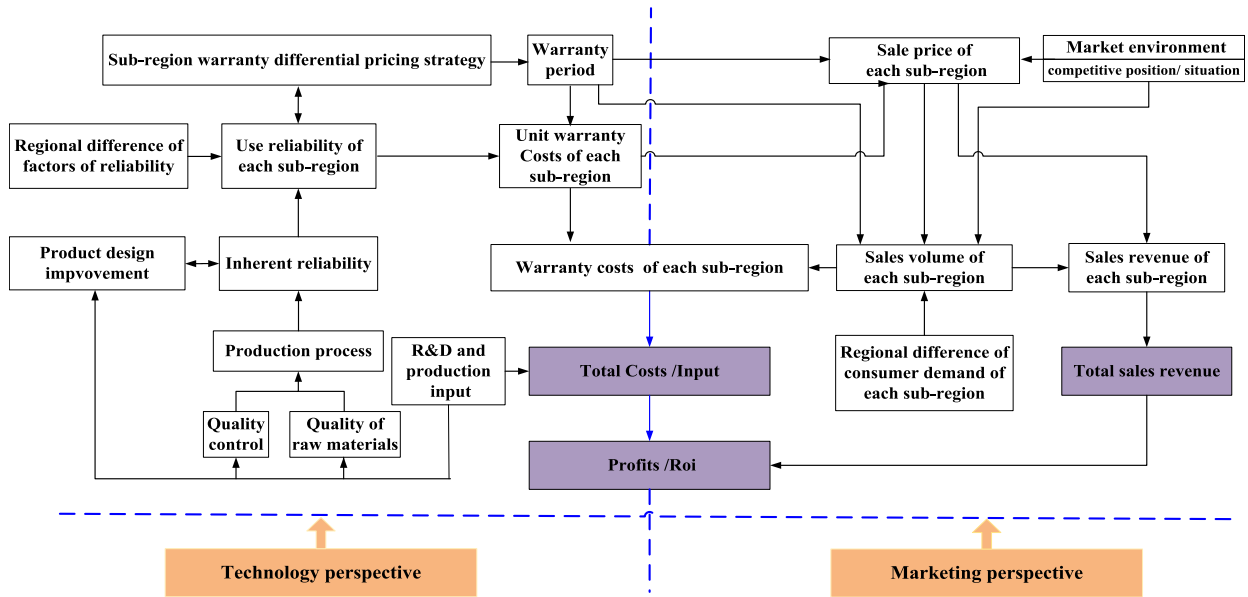
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FIGURE 2. Modeling framework for the sub-region warranty differential pricing strategy based on use reliability.

The proposed use reliability prediction model and the high-dimensional optimization model are also illustrated.

A. PRODUCT WARRANTY POLICY

Warranty assures the purchaser that failed items can be replaced or repaired by the manufacturer during the warranty period at no cost or at a reduced cost depending on the terms of the warranty contract. Manufacturers offer multiple types of warranty policies for various products. FRW, PRW, and renewing FRW (RFRW) are the most common warranty policies [39]. Among them, FRW has been widely applied in various products, including consumer, industrial, and commercial ones.

On the basis of the former analysis, we consider that the product in this report is sold with the FRW policy. The policy states the following: "under the policy, the manufacturer agrees to replace or repair failed items free of charge up for a limited period from the time of the initial purchase" [54].

B. PRODUCT FAILURE

The assumption that $f(t, \lambda) = \{dF(t, \lambda)/dt\}$ denotes the failure density function, where $F(t, \lambda)$ is the cumulative failure distribution function for the first time to failure. The failure rate function is given as follows:

$$r(t, \lambda) = \frac{f(t, \lambda)}{1 - F(t, \lambda)}, \qquad (1)$$

where λ is an indicator of product reliability, i.e., a small value of λ indicates good product quality. Parameter λ can be used to characterize the failure distribution. Specifically, λ is the failure rate for an exponential failure distribution, and it is the scale parameter for Weibull and gamma distributions.

The number of failures under a warranty is a random variable that depends on the type of repair actions implemented

to rectify the problem. The most common repair actions are as follows [5]: (i) replace the failed item with a new one; (ii) repaired items have a distribution that is different from $F(t, \lambda)$; and (iii) the failed item is rectified through minimal repair. The third case is suitable for multi-component products wherein the item failure is due to a single or only a few components [39]. Thus, we suppose that the product under consideration is repairable, and any failure can be rectified. We also pay attention to the action of minimal repair, which brings the system to a condition similar to that prior to item failure.

We assume that repair times are negligible, and failures occur over time according to a non-homogeneous Poisson process with an intensity function given by the failure rate function $r(t, \lambda)$. Thus, the expected number of failures over $[0, t)$, $N(t, \lambda)$ is obtained as follows:

$$N(t, \lambda) = \int_0^t r(t, \lambda) \, dt. \qquad (2)$$

C. EXPECTED WARRANTY COST FOR EACH SUB-REGION

When an item is repaired under a warranty, the manufacturer incurs high costs, including material, transportation, labor, management, and intangible costs. We combine these costs into a single cost called the repair cost, which is a random variable due to the uncertainty of several costs [6]. We let c_m be the expected value of the repair cost and W be the warranty period. The expected number of failures for the ith sub-region in consideration of minimal repairs over the warranty period, $N(W, \lambda_i)$, is calculated as

$$N(W, \lambda_i) = \int_0^W r_0(t, \lambda_i) \, dt \qquad (3)$$

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with

$$r_0(t, \lambda_i) = \frac{f_0(t, \lambda_i)}{\int_t^\infty f_0(x, \lambda_i)\, dx}. \quad (4)$$

Thus, the expected warranty cost for a unit product in the ith sub-region, $\omega(W, \lambda_i)$, is given by

$$\omega(W, \lambda_i) = c_m \int_0^W r_0(t, \lambda_i)\, dt, \quad i = 1, 2, \cdots, I. \quad (5)$$

where c_m is the expected cost of each repair, λ_i is an indicator of the use reliability of products in the ith sub-region, and the parameter I is the number of regional granularity partitions of product use reliability.

We suppose that product failure distribution is an exponential distribution with parameter λ_i. Thus, $F(t, \lambda_i) = 1 - e^{-\lambda_i t}$. The expected warranty cost for a unit product in the ith sub-region, $\omega(W, \lambda_i)$, can be rewritten as follows:

$$\omega(W, \lambda_i) = c_m \lambda_i W. \quad i = 1, 2, \cdots, I. \quad (6)$$

Eq. (6) shows that the expected warranty cost for a unit product in different sub-regions exhibits dynamic fluctuation due to the regional difference in product use reliability.

D. USE RELIABILITY PREDICTION

On the basis of the discussion above, the first task is to predict the use reliability of products in different sub-regions to achieve warranty decision optimization. The upgrading of mass-produced products shows continuity and gradual improvement, that is, different generations of the same type of products can maintain a stable structure and function (i.e., the factors of reliability have the same effect on product use reliability). Thus, we can make full use of the historical data of the same type of products to quantitatively compute the effect degree and predict the actual use reliability of new products in different regions.

As previously discussed, the regional granularity partition of use reliability accurately reveals that the use reliability of products is as similar as possible in the same sub-region and as different as possible in different sub-regions. Essentially, the difference degree of product use reliability depends entirely on the integrated distance between different sub-regions. Therefore, the integrated distance between different sub-regions is introduced in this study to characterize the difference degree of actual use reliability in different sub-regions.

The least squares method is used to fit the polynomial in advance to clarify the mathematical relationship between the difference degree of use reliability and the integrated distance between different sub-regions. The polynomial fitting function is as follows:

$$y(x) = \sum_{k=0}^K a_k x^k, \quad (7)$$

where $y(x)$ represents the difference degree of use reliability in two sub-regions, x represents the integrated distance between two sub-regions, a_k is an undetermined coefficient,

and K is the highest power of the polynomial. In consideration of the sensitivity of the difference degree to the choice of K, preliminary experiments are conducted by estimating different K values. The best value is selected for K through a comparative analysis of the calculation errors and objective actual conditions.

On this basis, a mathematical model that reflects the quantitative relationship between use reliability and the integrated distance for different sub-regions is established as follows:

$$\begin{cases} |\Delta\lambda_{ij}| = |\lambda_i - \lambda_j| = a_K D_{ij}^K + \cdots + a_k D_{ij}^k + \cdots + a_1 D_{ij} + a_0 \\ D_{ij} = \omega_1 \cdot d_{ij}(AQED) + \omega_2 \cdot d_{ij}(ISED) + \omega_3 \cdot d_{ij}(VCED), \end{cases}$$
$$0 \le k \le K. \quad (8)$$

where λ_i and λ_j are the use failure rates of the same type of products in the ith and jth sub-regions, respectively, and they are used to characterize the actual use reliability level. $\Delta\lambda_{ij}$ represents the difference degree of the use failure rate for the ith and jth sub-regions, and a_k ($0 \le k \le K$) is an undetermined coefficient. D_{ij} is the integrated distance between the ith and jth sub-regions, and $d_{ij}(AQED)$, $d_{ij}(ISED)$, and $d_{ij}(VCED)$ are the absolute quantity Euclidean distance, the increment speed Euclidean distance, and the variation coefficient Euclidean distance, respectively. ω_1, ω_2, and ω_3 are the weight coefficients of three types of distance and satisfy the constrained equation $\omega_1 + \omega_2 + \omega_3 = 1$. The three types of distance and the weight coefficients are similar to those used in [4].

The corresponding evaluation criteria for selecting K are as follows.

In the first case, i.e., when $K = 0$ and $y(x) = a_0$, regardless of the changes in the integrated distance between two different sub-regions, the difference degree of use failure rate in the two sub-regions is assumed to be constant, which is inconsistent with [4] and [45]. Consequently, $K = 0$ is not consistent with objective actual conditions.

In the second case, i.e., when $K = 1$ and $y(x) = a_0 + a_1 x$, the difference degree of use failure rate increases monotonically with the increase in the integrated distance between the two different sub-regions. Then, the larger integrated distance indicates a larger difference degree of the use failure rate. This finding is crucial in selecting the best value for K.

In the third case, i.e., when $K = 2$ and $y(x) = a_0 + a_1 x + a_2 x^2$, $y(x)$ decreases monotonically in the interval $(0, \frac{a_1}{2a_2})$, indicating that the difference degree of use failure rate decreases as the integrated distance between two different sub-regions increases. This situation is inconsistent with objective actual conditions. Accordingly, $K = 2$ is also undesirable in this study.

On the basis of this analysis, we determine the best value $K = 1$ and obtain $y(x) = a_0 + a_1 x$. We let δ and ξ be undetermined coefficients. To make a concise expression, Eq. (2) can be rewritten as follows:

$$\begin{cases} |\Delta\lambda_{ij}| = |\lambda_i - \lambda_j| = \delta D_{ij} + \xi \\ D_{ij} = \omega_1 \cdot d_{ij}(AQED) + \omega_2 \cdot d_{ij}(ISED) + \omega_3 \cdot d_{ij}(VCED). \end{cases} \quad (9)$$

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After determining the mathematical relationship between product use reliability and integrated distance for different sub-regions, the next key problem to be resolved is the estimation of undetermined coefficients δ and ξ to accurately predict the product use reliability in different sub-regions. The specific steps are as follows:

(1) The after-sales fault data are classified and statistically analyzed on the basis of the regional granularity partition of product use reliability. Then, the actual use failure rate of products in certain regions is estimated in accordance with the statistical analysis results of existing after-sales fault data. Here, product use failure rate λ_i^* in the ith sub-region is determined by the maximum likelihood method. It is calculated as

$$\hat{\lambda}_i^* = \frac{F_i}{\sum_{i \in G} T_i + \sum_{i \in H} T_i^+}, \quad i = 1, 2, \cdots, I. \quad (10)$$

where F_i is the total number of product failures in the ith sub-region, T_i is the working time of faulty products in the ith sub-region, T_i^+ is the working time of fault-free products in the ith sub-region, G represents the faulty products in the ith sub-region, and H represents the fault-free products in the ith sub-region.

(2) Combined with the regional granularity partition of use reliability obtained previously, the integrated distance between different sub-regions can be calculated using Eq. (9).

(3) Parameters δ and ξ are computed using the least squares method, and a mathematical model reflecting the quantitative relationship between use reliability and integrated distance for different sub-regions is derived. Thus, the actual use failure rate of the same type of products in different sub-regions can be estimated.

(4) The influence coefficient is calculated to measure the difference degree of use reliability of the same type of products in each sub-region. We let λ_i^* be the use failure rate of the same type of products in the ith sub-region. Its influence coefficient is given by

$$\alpha_i \approx \lambda_i^* / \lambda_0^*, \quad i = 1, 2, \cdots, I. \quad (11)$$

where α_i is the reliability influence coefficient in the ith sub-region and λ_0^* is the inherent failure rate of the same type of products.

E. SALES MODEL FOR EACH SUB-REGION

Generally, the product sales volume depends on various factors, including product quality, sale price, advertising, and the competitive environment. On the one hand, purchasers often utilize warranty information to evaluate product quality and determine whether the sale price is appropriate. Hence, warranty and sale price are the two main factors that determine product sales. On the other hand, product reliability cannot be directly observed, so its effect on the purchase decision is ignored. On this basis, the Glickman–Berger model [55], in which demand is characterized by a displaced logarithmic linear function, is introduced in this study because the model has been validated in a number of relevant studies.

It is calculated as

$$Q(P, W) = k_1 (W + \tau)^\beta P^{-\gamma},$$
$$k_1 > 0, \ \tau > 0, \ 0 < \beta < 1, \ \gamma > 1. \quad (12)$$

where Q is the total demand quantity, k_1 is a scale factor, and τ is a constant of time displacement that allows for the possibility of nonzero demand when W is zero. Parameters β and γ denote warranty elasticity and price elasticity, respectively.

Afterward, the regional difference in use reliability presented above is used to divide the entire region into different sub-regions and achieve the sub-region warranty differential pricing optimization strategy. In accordance with this reasoning, the regional differential price inevitably leads to a regional difference in product sales volume in different sub-regions. Thus, constructing a new sales model that adapts itself to variable regional market conditions and sale price instead of using fixed values is essential.

A variant of the aforementioned sales model is written as below. We let I be the number of regional granularity partitions for product use reliability and $P_i (1 \le i \le I)$ be the unit sale price for the ith sub-region. The product sales volume for the ith sub-region is given by

$$Q_i (P_i, W) = k_i (W + \tau)^\beta P_i^{-\gamma},$$
$$k_i, \tau > 0, 0 < \beta < 1, \gamma > 1; 1 \le i \le I. \quad (13)$$

where Q_i is the product sales volume for the ith sub-region, it is a function associated with warranty period W and sale price P_i. Thus, the total sales volume Q is equal to the sum of sales volume of products in each sub-region. It is given by

$$Q = \sum_{i=1}^{I} Q_i (P_i, W). \quad (14)$$

where $k_i (1 \le i \le I)$ is a scale factor for the ith sub-region to show the relative impact of competitors and other market factors, such as number of potential consumers and consumer purchasing power. τ, β and γ are similar to those defined above. These parameters can be easily obtained through a market survey and analysis by the marketing department. As shown in Eq. (13), warranty period positively affects sales volume, i.e., Q_i increases as W increases. By contrast, sale price negatively affects sales volume, i.e., Q_i decreases as P_i increases. Specifically, sales volume function Q_i based on variable regional market conditions and selling price is modified dynamically for each sub-region, as opposed to existing research that applied a single value to all regions.

F. OPTIMIZATION MODEL FOR THE SUB-REGION WARRANTY DIFFERENTIAL PRICING STRATEGY

On the basis of the discussion and analysis shown above, we develop a high-dimensional optimization model to express the expected integrated profit for the sub-region warranty differential pricing strategy based on product use reliability in different sub-regions.

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Specifically, the expected integrated profit for different sub-regions depends on the selling price for each sub-region, sales volume for each sub-region, unit R&D cost, unit production cost, and expected warranty cost for the unit product in each sub-region. The expression for expected integrated profit is

$$\max \pi_A = \sum_{i=1}^{I} \left[P_i - c_r - c_p - \omega(W, \lambda_i) \right] Q_i (P_i, W)$$

$$= \sum_{i=1}^{I} \left(P_i - c_r - c_p - c_m \lambda_i W \right) k_i (W + \tau)^\beta P_i^{-\gamma}. \quad (15)$$

where I is the number of regional granularity partitions of use reliability, c_r is the unit R&D cost, and c_p is the unit production cost. The other variables are similar to those defined above.

The main objective is to determine the optimal combination of use reliability λ_i $(i = 1, 2, \cdots, I)$, warranty period W^*, and selling price for different sub-regions P_i^* $(i = 1, 2, \cdots, I)$ that maximize the expected integrated profit π_A given by Eq. (15) for products sold with an FRW policy. The constraint conditions of P_i and W are as follows:

$$s.t. \begin{cases} W \in [0, L_{\max}], \\ P_i \in (C_{a\min}, P_{c\max}), \\ \sum_{i=1}^{I} c_m \lambda_i W k_i (W + \tau)^\beta P_i^{-\gamma} \in (0, C_{w\max}], \quad 1 \le i \le I. \end{cases}$$

$$\quad (16)$$

where $L_{\max}$ is the maximum service life of a product, $C_{a\min}$ is the minimum average cost of a product, $P_{c\max}$ is the maximum sale price that is acceptable to consumers, and $C_{w\max}$ is the maximum after-sales maintenance cost during the warranty period. Parameters $P_{c\max}$ and $C_{w\max}$ can be obtained through a market survey and profit analysis before new products are placed on the market.

IV. MODEL OPTIMIZATION ANALYSIS

The high-dimensional optimization model discussed above is considered for the two scenarios of product pricing and warranty, which are referred to as unified and partition warranty schemes.

A. UNIFIED WARRANTY (SCENARIO 1)

Under the unified warranty scheme, the selling price and warranty period for the same type of products in different sub-regions are considered unique and invariant, i.e., $P_X = P_i$ $(i = 1, 2, \cdots, I)$, $W = W_X$, $Q_i (P_X, W_X) = k_i (W_X + \tau)^\beta P_X^{-\gamma}$, and $\pi_A = \pi_X$. The main objective is to select the optimal values of P_X and W_X using Eqs. (15) and (16). The specific derivation is expressed below.

We obtain the Hessian matrix of the expected integrated profit function with respect to P_X and W_X from Eq. (15) as

follows:

$$D(\pi_X) = \begin{bmatrix} \dfrac{\partial^2 \pi_X}{\partial W_X^2} & \dfrac{\partial^2 \pi_X}{\partial W_X \partial P_X} \\ \dfrac{\partial^2 \pi_X}{\partial P_X \partial W_X} & \dfrac{\partial^2 \pi_X}{\partial P_X^2} \end{bmatrix}.$$

where

$$\frac{\partial^2 \pi_X}{\partial W_X^2} = -\beta (W_X + \tau)^{\beta-2} P_X^{-\gamma}$$

$$\times \sum_{i=1}^{I} k_i \left[(1 - \beta) (P_X - c_r - c_p - c_m \alpha_i \lambda_0 W_X) \right.$$

$$\left. + 2c_m \alpha_i \lambda_0 (W_X + \tau) \right] \quad (17)$$

$$\frac{\partial^2 \pi_X}{\partial W_X \partial P_X} = \frac{\partial^2 \pi_X}{\partial P_X \partial W_X}$$

$$= (W_X + \tau)^{\beta-1} P_X^{-\gamma-1}$$

$$\times \sum_{i=1}^{I} k_i \left[\beta P_X + \gamma c_m \alpha_i \lambda_0 (W_X + \tau) \right.$$

$$\left. - \beta \gamma (P_X - c_r - c_p - c_m \alpha_i \lambda_0 W_X) \right] \quad (18)$$

$$\frac{\partial^2 \pi_X}{\partial P_X^2} = -\gamma (W_X + \tau)^\beta P_X^{-\gamma-2}$$

$$\times \sum_{i=1}^{I} k_i \left[(P_X + c_r + c_p + c_m \alpha_i \lambda_0 W_X) \right.$$

$$\left. - \gamma (P_X - c_r - c_p - c_m \alpha_i \lambda_0 W_X) \right]. \quad (19)$$

We let D_1 be the first-order principal determinant and D_2 be the second-order principal determinant. Then,

$$D_1 = \frac{\partial^2 \pi_X}{\partial W_X^2}, \quad (20)$$

$$D_2 = \frac{\partial^2 \pi_X}{\partial W_X^2} \times \frac{\partial^2 \pi_X}{\partial P_X^2} - \frac{\partial^2 \pi_X}{\partial P_X \partial W_X} \times \frac{\partial^2 \pi_X}{\partial W_X \partial P_X}. \quad (21)$$

According to the computed principal and minor determinants, $D_1 < 0$. When $\gamma < \frac{P_X + c_r + c_p + c_m \alpha_i \lambda_o W_X}{P_X - c_r - c_p - c_m \alpha_i \lambda_o W_X}$, we have $D_2 > 0$. Thus, the expected integrated profit given by Eq. (15) has a strict concave function for P_X and W_X, which proves the existence of optimal solutions of the model under the unified warranty scheme.

The necessary conditions for P_X^* and W_X^* to be optimal are

$$\frac{\partial \pi_X}{\partial W_X} = \sum_{i=1}^{I} \left[-c_m \alpha_i \lambda_0 k_i (W_X + \tau)^\beta P_X^{-\gamma} \right.$$

$$\left. + \beta (P_X - c_r - c_p - c_m \alpha_i \lambda_0 W_X) k_i (W_X + \tau)^{\beta-1} P_X^{-\gamma} \right]$$

$$= 0, \quad (22)$$

$$\frac{\partial \pi_X}{\partial P_X} = \sum_{i=1}^{I} \left[k_i (W_X + \tau)^\beta P_X^{-\gamma} \right.$$

$$\left. - \gamma (P_X - c_r - c_p - c_m \alpha_i \lambda_0 W_X) k_i (W_X + \tau)^\beta P_X^{-\gamma-1} \right]$$

$$= 0. \quad (23)$$

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From Eqs. (22) and (23), we can obtain P_X^* and W_X^* without the constraints. They are calculated as

$$W_X^* = \frac{I\beta\left(c_r + c_p\right) - (\gamma - 1)\,c_m\lambda_0\tau\sum\limits_{i=1}^{I}\alpha_i}{(\gamma - \beta - 1)\,c_m\lambda_0\sum\limits_{i=1}^{I}\alpha_i}, \qquad (24)$$

$$P_X^* = \frac{\gamma}{\gamma - \beta - 1}\left(c_r + c_p - \frac{c_m}{I}\lambda_0\tau\sum\limits_{i=1}^{I}\alpha_i\right). \qquad (25)$$

If P_X^* and W_X^* satisfy the constraints given by Eq. (16), then P_X^* and W_X^* are the optimal selling price and warranty period, respectively. Otherwise, the optimal selling price P^* is given as

$$P_X^* = \begin{cases} C_{a\min} & P_X < C_{a\min} \\ P_{c\max} & P_X > P_{c\max}. \end{cases} \qquad (26)$$

The obtained P_X^* in this case is substituted into Eq. (16). Then, the optimal warranty period W_X^* can be determined.

B. PARTITION WARRANTY (SCENARIO 2)

Under the partition warranty scheme, the selling price and warranty period for the same type of products in different sub-regions are considered variables, i.e., $P_1 \neq P_2 \neq \cdots P_i \neq \cdots \neq P_I$ $(i = 1, 2, \cdots, I)$, $W = W_Y$, $Q_i(P_i, W_Y) = k_i(W_Y + \tau)^\beta P_i^{-\gamma}$, and $\pi_A = \pi_Y$. The main objective is to determine the optimal values of P_i $(i = 1, 2, \cdots, I)$ and W_Y simultaneously by using Eqs. (15) and (16). The specific derivation is described below.

We obtain the Hessian matrix of the expected profit function with respect to P_i $(i = 1, 2, \cdots, I)$ and W_Y from Eq. (15) as follows:

$$D(\pi_Y) = \begin{bmatrix} \dfrac{\partial^2\pi_Y}{\partial P_1^2} & \dfrac{\partial^2\pi_Y}{\partial P_1\partial P_2} & \cdots\cdots & \dfrac{\partial^2\pi_Y}{\partial P_1\partial P_I} & \dfrac{\partial^2\pi_Y}{\partial P_1\partial W_Y} \\[2mm] \dfrac{\partial^2\pi_Y}{\partial P_2\partial P_1} & \dfrac{\partial^2\pi_Y}{\partial P_2^2} & \cdots\cdots & \dfrac{\partial^2\pi_Y}{\partial P_2\partial P_I} & \dfrac{\partial^2\pi_Y}{\partial P_2\partial W_Y} \\[2mm] \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \dfrac{\partial^2\pi_Y}{\partial P_I\partial P_1} & \dfrac{\partial^2\pi_Y}{\partial P_I\partial P_2} & \cdots\cdots & \dfrac{\partial^2\pi_Y}{\partial P_I^2} & \dfrac{\partial^2\pi_Y}{\partial P_I\partial W_Y} \\[2mm] \dfrac{\partial^2\pi_Y}{\partial W_Y\partial P_1} & \dfrac{\partial^2\pi_Y}{\partial W_Y\partial P_2} & \cdots\cdots & \dfrac{\partial^2\pi_Y}{\partial W_Y\partial P_I} & \dfrac{\partial^2\pi_Y}{\partial W_Y^2} \end{bmatrix}.$$

where

$$\frac{\partial^2\pi_Y}{\partial P_i\partial P_j} = \frac{\partial^2\pi_Y}{\partial P_j\partial P_i} = 0 \quad (i \neq j), \qquad (27)$$

$$\frac{\partial\pi_Y}{\partial P_i^2} = -\gamma k_i(W_Y + \tau)^\beta P_i^{-\gamma-2}[(P_i + c_r + c_p + c_m\alpha_i\lambda_0 W_Y) - \gamma(P_i - c_r - c_p - c_m\alpha_i\lambda_0 W_Y)], \qquad (28)$$

$$\frac{\partial^2\pi_Y}{\partial W_Y^2} = -\beta\,(W_Y + \tau)^{\beta-2}$$
$$\times \sum\limits_{i=1}^{I}k_iP_i^{-\gamma}\left[(1-\beta)\left(P_i - c_r - c_p - c_m\alpha_i\lambda_0 W_Y\right)\right.$$
$$\left. + 2c_m\alpha_i\lambda_0\,(W_Y + \tau)\right], \qquad (29)$$

$$\frac{\partial^2\pi_Y}{\partial P_i\partial W_Y} = \frac{\partial^2\pi_Y}{\partial W_Y\partial P_i}$$
$$= k_iP_i^{-\gamma-1}(W_Y + \tau)^{\beta-1}[\beta P_i + \gamma c_m\alpha_i\lambda_0(W_Y + \tau)$$
$$- \gamma\beta(P_i - c_r - c_p - c_m\alpha_i\lambda_0 W_Y)]. \qquad (30)$$

We let $D_1, D_2, \cdots, D_I, D_{I+1}$ be the first-order principal determinant to the $I + 1$ order principal determinant. Then,

$$D_1 = \frac{\partial^2\pi_Y}{\partial P_1^2}$$
$$= -\gamma k_1P_1^{-\gamma-2}(W_Y + \tau)^\beta[(P_1 + c_r + c_p + c_m\alpha_1\lambda_0 W_Y) - \gamma(P_1 - c_r - c_p - c_m\alpha_1\lambda_0 W_Y)], \qquad (31)$$

$$D_2 = \frac{\partial^2\pi_Y}{\partial P_1^2} \times \frac{\partial^2\pi_Y}{\partial P_2^2} - \frac{\partial^2\pi_Y}{\partial P_1\partial P_2} \times \frac{\partial^2\pi_Y}{\partial P_2\partial P_1}$$
$$= \prod\limits_{i=1}^{2}\gamma k_iP_i^{-\gamma-2}(W_Y + \tau)^\beta[(P_i + c_r + c_p + c_m\alpha_1\lambda_0 W_Y) - \gamma(P_i - c_r - c_p - c_m\alpha_1\lambda_0 W_Y)], \qquad (32)$$

By recursively implementing these equations, we obtain

$$D_I = (-1)^I\prod\limits_{i=1}^{I}\gamma k_iP_i^{-\gamma-2}\,(W_Y + \tau)^\beta$$
$$\times \left[(P_i + c_r + c_p + c_m\alpha_i\lambda_0 W_Y)\right.$$
$$\left. - \gamma\,(P_i - c_r - c_p - c_m\alpha_i\lambda_0 W_Y)\right], \qquad (33)$$

$$D_{I+1} = D_I \times \frac{\partial^2\pi_Y}{\partial W_Y^2} = -D_I\beta\,(W_Y + \tau)^{\beta-2}$$
$$\times \sum\limits_{i=1}^{I}k_iP_i^{-\gamma}\left[(1-\beta)\left(P_i - c_r - c_p - c_m\alpha_i\lambda_0 W_Y\right)\right.$$
$$\left. + 2c_m\alpha_i\lambda_0\,(W_Y + \tau)\right] \qquad (34)$$

According to the derivation, $\frac{\partial^2\pi_Y}{\partial W_Y^2} < 0$. When $\gamma < \frac{P_i+c_r+c_p+c_m\alpha_i\lambda_o W_Y}{P_i-c_r-c_p-c_m\alpha_i\lambda_o W_Y}$ and if I is considered an odd number, then $D_I < 0$; thus, $D_{I+1} = D_I \times \frac{\partial^2\pi_Y}{\partial W_Y^2} > 0$. On the contrary, if I is an even number, then $D_I > 0$; thus, $D_{I+1} = D_I \times \frac{\partial^2\pi_Y}{\partial W_Y^2} < 0$. The Hessian matrix in this case is therefore negative, i.e., the expected integrated profit given by Eq. (15) has a strict concave function for W_Y and P_i $(i = 1, 2, \cdots, I)$, which proves the existence of optimal solutions of the model under the partition warranty scheme.

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The necessary conditions for W_Y^* and P_i^* $(i = 1, 2, \cdots, I)$ to be optimal are

$$\frac{\partial \pi_Y}{\partial W_Y} = \sum_{i=1}^{I} \left[-c_m \alpha_i \lambda_0 k_i (W_Y + \tau)^\beta P_i^{-\gamma} \right.$$

$$+ \beta \left(P_i - c_r - c_p - c_m \alpha_i \lambda_0 W_Y \right) k_i (W_Y + \tau)^{\beta-1} P_i^{-\gamma} \Big]$$

$$= 0, \qquad (35)$$

$$\frac{\partial \pi_Y}{\partial P_i} = k_i (W_Y + \tau)^\beta P_i^{-\gamma} - \gamma (P_i - c_r$$

$$- c_p - c_m \alpha_i \lambda_0 W_Y) k_i (W_Y + \tau)^\beta P_i^{-\gamma-1} = 0. \qquad (36)$$

By analyzing the set of equations above, we determine W_Y^* and P_i^* $(i = 1, 2, \cdots, I)$ that maximize the expected profit given by Eq. (15) without the constraints. If W_Y^* and P_i^* $(i = 1, 2, \cdots, I)$ satisfy the constraints given by Eq. (16), then W_Y^* and P_i^* $(i = 1, 2, \cdots, I)$ are the optimal warranty period and selling price for different sub-regions, respectively. Otherwise, the optimal selling price P_i^* $(i = 1, 2, \cdots, I)$ is taken as follows:

$$P_i^* = \begin{cases} C_{a\,\min} & P_i < C_{a\,\min} \\ P_{c\max} & P_i > P_{c\max}. \end{cases} \qquad (37)$$

In this case, the obtained P_i^* $(i = 1, 2, \cdots, I)$ is substituted into Eq. (16) to determine the optimal warranty period W_Y^*.

V. PRACTICAL CASE STUDY AND DISCUSSION

In this section, a practical case of the air-conditioning industry in China is presented to illustrate the validity and applicability of the proposed models. The models are applied to evaluate the use reliability of air-conditioners and optimize warranty decisions. On this basis, we obtain the optimal combination of use reliability, warranty period, and selling price of air-conditioners. We provide a detailed discussion of the influence rules of model parameters on warranty period, selling price, expected warranty cost and profit for different sub-regions via a sensitivity analysis.

In accordance with the regional granularity partition results of use reliability of air-conditioners (Fig. 1), we set $I = 10$. Then, we calculate k_i through the following steps. First, the sales volume of air-conditioners in different regions Q_i is computed using the quantity of owned air-conditioners published by the National Bureau of Statistics of China [56]. Second, in accordance with warranty period W and average sales price P commonly used by the current air-conditioning industry, k_i is computed as $k_i = Q_i (P_i, W) (W + \tau)^{-\beta} P_i^\gamma$. The values used in the actual example are shown in Table 1.

Considering the particular type of air-conditioners in this case study, we set the following values: inherent failure rate $\lambda_0 = 0.00803$, expected cost of each minimal repair $c_m = ¥400$, unit R&D cost $c_r = ¥500$, unit production cost $c_p = ¥1100$, maximum service life $L_{\max} = 12$ years, minimum average cost $C_{a\,\min} = ¥2000$, and maximum price $P_{c\max} = ¥4000$. In the sales model given by Eq. (13), we set $\beta = 0.02$, $\gamma = 2$, and $\tau = 0.5$. In view of the actual ownership

TABLE 1. Values of k_i for different sub-regions.

i	sub-regions	k_i
1	Inner Mongolia, Gansu, Ningxia, Sinkiang, Qinghai, Tibet	1.7169×10^{12}
2	Liaoning, Jilin, Heilongjiang	4.4838×10^{12}
3	Beijing, Tianjin, Hebei, Shanxi	3.3659×10^{13}
4	Yunnan	1.3140×10^{11}
5	Sichuan, Chongqing, Guizhou	1.9380×10^{13}
6	Shandong, Henan, Shaanxi	4.1549×10^{13}
7	Jiangxi, Hunan	1.6485×10^{13}
8	Jiangsu, Anhui, Hubei	6.4338×10^{13}
9	Guangdong, Fujian, Guangxi, Hainan	6.6748×10^{13}
10	Shanghai, Zhejiang	5.1508×10^{13}

quantity of air-conditioners in Mainland China, we set the maximum after-sales maintenance cost during the warranty period to $C_{w\,\max} = ¥100$ million.

A. ESTIMATION OF USE RELIABILITY

In accordance with the regional granularity partition results and existing after-sales failure data of the same type of air-conditioners, we obtain the use failure rate of the same type of air-conditioners in certain sub-regions. The integrated distance between these sub-regions is determined with Eq. (9). On this basis, we obtain $\delta = 0.0018$ and $\xi = -0.0024$ by using the least squares method. The quantitative relationship model between the use failure rate of air-conditioners and the integrated distance for different sub-regions is given as

$$|\Delta \lambda_{ij}| = |\lambda_i - \lambda_j| = 0.0018 D_{ij} - 0.0024.$$

TABLE 2. Actual use failure rate and its influence coefficients for different sub-regions.

i	sub-regions	λ_i	α_i
1	Inner Mongolia, Gansu, Ningxia, Sinkiang, Qinghai, Tibet	0.00740	0.92154
2	Liaoning, Jilin, Heilongjiang	0.00781	0.97202
3	Beijing, Tianjin, Hebei, Shanxi	0.00806	1.00374
4	Yunnan	0.00827	1.02989
5	Sichuan, Chongqing, Guizhou	0.00840	1.04608
6	Shandong, Henan, Shaanxi	0.00856	1.06600
7	Jiangxi, Hunan	0.00892	1.11083
8	Jiangsu, Anhui, Hubei	0.00922	1.14819
9	Guangdong, Fujian, Guangxi, Hainan	0.00975	1.21420
10	Shanghai, Zhejiang	0.01023	1.27397

Furthermore, we obtain actual use failure rate λ_i and its influence coefficients for different sub-regions (Table 2). For a thorough comparison of the inherent failure rate and actual use failure rate, we record the results of the failure rate of products in different sub-regions. We present the results of six typical regions as an example (Fig. 3).

As shown in Table 2, an evident regional difference is found in the actual use failure rate and influence coefficients.
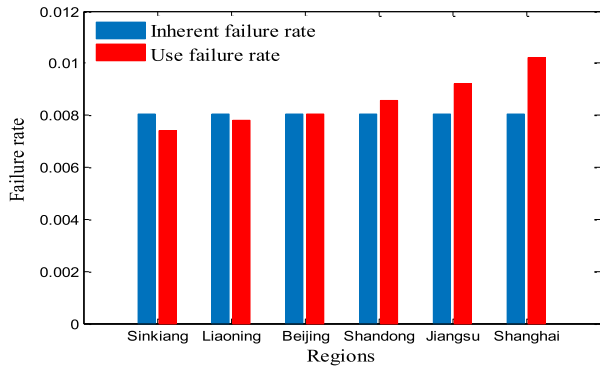
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FIGURE 3. Use failure rate in six regions.

This finding is mainly due to the difference in natural environmental conditions, economic conditions, and user lifestyle of different sub-regions, all of which result in increased diversity of use reliability. In addition, the results demonstrate that large influence coefficients equate to a high actual use failure rate, i.e., low use reliability level.

As depicted in Fig. 3, the actual use failure rate for different regions fluctuates around the inherent failure rate. The actual use failure rate for Sinkiang and Liaoning is lower than the inherent failure rate. The actual use failure rate is close to the inherent failure rate for Beijing. By contrast, the actual use failure rate for the three other regions is significantly higher than the inherent failure rate (the failure rate is increased by about 3% to 27%). This analysis implies that a fluctuation in the actual use failure rate causes a difference in the expected warranty cost per unit. Hence, considering the regional difference in use reliability is necessary in making scientific and rational warranty decisions.

B. RESULTS OBTAINED BY UNIFIED WARRANTY (SCENARIO 1)

The obtained parameter values are substituted into Eqs. (24) and (25) to further determine the optimal combination of warranty period and selling price for the unified warranty scheme. Then, the combination is compared with traditional warranty optimization methods that are based on inherent reliability and inherent reliability growth. In Table 3, — indicates that the objective functions do not consider the decision variable.

TABLE 3. Comparison with other inherent reliability-based models (the best results are highlighted in bold).

	inherent reliability	inherent reliability growth	use reliability
W_X^*	6.02	7.97	**8.37**
P_X^*	—	—	**3261.50**

Table 3 shows that compared with the other inherent reliability-based methods, the unified warranty scheme based on use reliability obtains the desired value and extended

warranty period, thus revealing its superiority. The primary reason for the desired results is that our proposed scheme considers the diversity in the use reliability of products in different sub-regions. The inherent reliability growth-based method achieves the second largest value, and the inherent reliability-based method has the smallest value. The main reason for the difference in optimization results is that the two inherent reliability-based methods assume that the use reliability of products in different sub-regions is equal to their inherent reliability. However, the inherent reliability-based method does not consider the growth of inherent reliability of products in the manufacture and use processes. Thus, it is the worst approach in terms of the length of the warranty period.

In addition, inherent reliability-based methods only establish warranty benefit prediction functions with the warranty period as the decision variable; hence, the corresponding price variable cannot be obtained. However, our proposed model comprehensively considers the influence of selling price, warranty period, and regional difference on actual use reliability to achieve the joint optimization of the three parameters. Therefore, the warranty period must be designed based on use reliability to match the actual situation of products in different regions.

C. RESULTS OBTAINED BY PARTITION WARRANTY (SCENARIO 2)

In this case, the selling price and warranty period vary from sub-region to sub-region, and we consider the case $I = 10$. We let $P_1, P_2, \cdots, P_{10}, W_Y$ be the prices and warranty period for the 10 different sub-regions. Thus, we have an optimization problem involving 11 variables ($P_1, P_2, \cdots, P_{10}$, and W_Y) that need to be selected optimally. Parameters k_i and λ_i ($i = 1, 2, \cdots, I$) are similar to those used in Scenario 1. All parameter values are substituted into Eqs. (15) and (16).

TABLE 4. Optimal solutions for the partition warranty scheme.

i	$\omega\left(W_Y^*, \lambda_i\right)$	$Q_i\left(P_i^*, W_Y^*\right)$	$\pi_Y\left(P_i^*, Q_i\right)$	P_i^*	W_Y^*	π_Y
1	24.7604	1.6985 10^5	2.7597 10^8	3249.5		
2	26.1676	4.4281 10^5	7.2009 10^8	3252.3		
3	27.0053	3.3207 10^6	5.4028 10^9	3254.0		
4	27.7089	1.2952 10^4	2.1083 10^7	3255.4		
5	28.1445	1.9093 10^6	3.1086 10^9	3256.3	8.37	4.804299 10^{10}
6	28.6805	4.0906 10^6	6.6623 10^9	3257.4		
7	29.8867	1.6206 10^6	2.6415 10^9	3259.8		
8	30.8919	6.3172 10^6	1.0303 10^{10}	3261.8		
9	32.6677	6.5396 10^6	1.0677 10^{10}	3265.3		
10	34.2759	5.0365 10^6	8.2311 10^{10}	3268.6		

Given the complex nature of the high-dimensional optimization model, the MATLAB program is used to evaluate the optimal prices and warranty period. The optimal solutions for the partition warranty scheme are shown in Table 4.

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As indicated in Table 4, the warranty period based on use reliability is significantly extended, given that $W = 6$ years is commonly used by the current air-conditioning industry. When the warranty period is extended appropriately, customer satisfaction and brand influence are improved correspondingly, thereby affecting the sales volume and total profit. In addition, the results reveal that the regional difference in the actual use failure rate causes a regional difference in prices and expected warranty cost per unit. As the use failure rate increases, the expected warranty cost per unit increases gradually, and the prices show an increasing trend with a corresponding extent. Specifically, an evident price difference exists between different sub-regions, indicating a good advantage in differential pricing. Therefore, the proposed partition warranty scheme makes full use of the high diversity of use reliability. It can overcome the shortcomings of existing warranty decisions based on inherent reliability.

TABLE 5. Comparison of the two scenarios (the best results are highlighted in bold).

i	$Q_i(P^*, W_X^*)$	$Q_i(P_i^*, W_Y^*)$	P_X^*	$\boldsymbol{P_i^*}$	π_X	π_Y
1	$1.6860\ 10^5$	$\mathbf{1.6985\ 10^5}$		**3249.5**		
2	$4.4031\ 10^5$	$\mathbf{4.4281\ 10^5}$		**3252.3**		
3	$3.3054\ 10^6$	$\mathbf{3.3207\ 10^6}$		**3254.0**		
4	$1.2904\ 10^4$	$\mathbf{1.2952\ 10^4}$		**3255.4**		
5	$1.9031\ 10^6$	$\mathbf{1.9093\ 10^6}$	3261.5	**3256.3**	$4.804287\ 10^{10}$	$\mathbf{4.804299\ 10^{10}}$
6	$4.0801\ 10^6$	$\mathbf{4.0906\ 10^6}$		**3257.4**		
7	$1.6188\ 10^6$	$\mathbf{1.6206\ 10^6}$		**3259.8**		
8	$6.3180\ 10^6$	$\mathbf{6.3172\ 10^6}$		**3261.8**		
9	$6.5547\ 10^6$	$\mathbf{6.5396\ 10^6}$		**3265.3**		
10	$5.0581\ 10^6$	$\mathbf{5.0365\ 10^6}$		**3268.6**		

For a quantitative comparison of the two scenarios, Table 5 summarizes the results of the unified and partition warranty schemes. Compared with the unified warranty scheme, the partition warranty scheme obtains better results on expected total profit. Under the partition warranty scheme, the company can increase its sales volume by appropriately reducing the price of products with a high use reliability level, as shown in sub-regions $i = 1, 2, 3, 4, 5, 6, 7$. On the contrary, the company can reduce its warranty cost pressure by correspondingly increasing the price of products with a low use reliability level, as shown in sub-regions $i = 8, 9, 10$. This analysis implies that the values of $\boldsymbol{P_i^*}$ based on the partition warranty scheme are modified dynamically for each sub-region, as opposed to the unified warranty scheme in which a single fixed value of P_X^* is selected for different sub-regions.

A comparison of Tables 3, 4, and 5 shows that the partition warranty scheme obtains better results than the unified warranty scheme. Therefore, the partition warranty scheme

is more suitable for achieving an effective trade-off among use reliability, warranty period, and selling price.

D. SENSITIVITY ANALYSIS OF MODEL PARAMETERS

To comprehensively analyze the effect of warranty elasticity and price elasticity parameters on the optimal values of warranty period, selling price, and expected warranty cost and profit for different sub-regions, we perform a sensitivity analysis by changing one parameter at a time (parameters β and γ) while keeping the remaining parameters similar to those used in Scenarios 1 and 2.
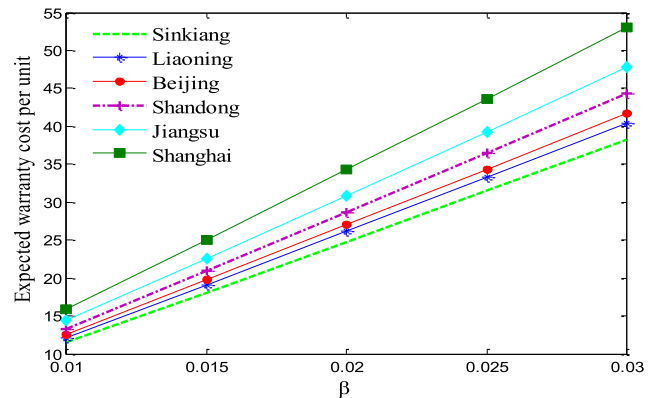
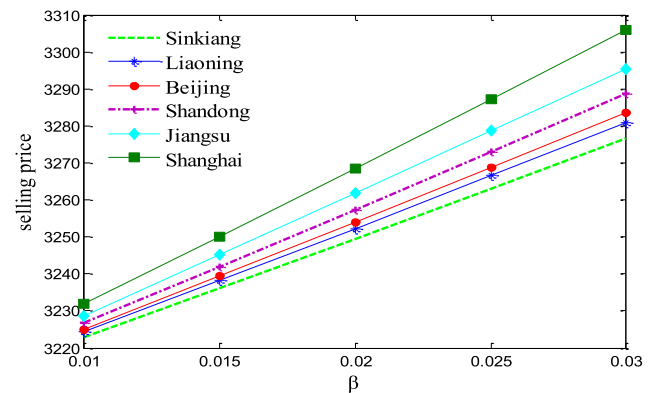**FIGURE 4.** Expected warranty cost per unit for Scenario 2 with β change.

FIGURE 5. Selling price for Scenario 2 with β change.

Six typical regions are regarded as examples to investigate the expected warranty cost per unit and the selling price of Scenario 2 under five different parameters. The results for different regions with β change are depicted in Figs. 4 and 5. For a careful comparison of Scenarios 1 and 2, the corresponding results on the optimal warranty period, selling price, sales volume, and profit obtained by the two scenarios with β change are also provided in Tables 6 to 8.

As shown in Fig. 4, a significant upward trend is observed in the expected warranty cost per unit. The key reason for this result is that the warranty period increases as β increases (Table 6). In particular, when the value of β increases from 0.010 to 0.030, the expected warranty cost per unit increases by at least 2.3 times. As depicted in Fig. 5, the selling price increases sharply due to the increase in the warranty period

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TABLE 6. Optimal solutions obtained by the two scenarios with β change.

		Scenario 1			
	$\beta = 0.010$	$\beta = 0.015$	$\beta = 0.020$	$\beta = 0.025$	$\beta = 0.030$
P_X^*	3228.6	3245.0	3261.5	3278.3	3295.2
W_X^*	3.89	6.12	8.37	10.65	12.95
π_X	$4.715224\ 10^{10}$	$4.755446\ 10^{10}$	$4.804287\ 10^{10}$	$4.859890\ 10^{10}$	$4.921213\ 10^{10}$

		Scenario 2			
	$\beta = 0.010$	$\beta = 0.015$	$\beta = 0.020$	$\beta = 0.025$	$\beta = 0.030$
P_i^*					
$i = 1$	3223.0	3236.2	3249.5	3263.0	3276.6
$i = 2$	3224.3	3238.3	3252.3	3266.6	3280.9
$i = 3$	3225.1	3239.5	3254.0	3268.7	3283.5
$i = 4$	3225.8	3240.5	3255.4	3270.5	3285.7
$i = 5$	3226.2	3241.1	3256.3	3271.6	3287.0
$i = 6$	3226.7	3241.9	3257.4	3273.0	3288.7
$i = 7$	3227.8	3243.7	3259.8	3276	3292.4
$i = 8$	3228.7	3245.2	3261.8	3278.6	3295.5
$i = 9$	3230.4	3247.8	3265.3	3283.1	3301.0
$i = 10$	3231.9	3250.1	3268.6	3287.2	3306.0
W_Y^*	3.89	6.12	8.37	10.65	12.95
π_Y	$4.715228\ 10^{10}$	$4.755452\ 10^{10}$	$4.804299\ 10^{10}$	$4.859908\ 10^{10}$	4.92124110^{10}

and expected warranty cost per unit. The results also reveal large differences in the expected warranty cost per unit and selling price in different regions due to the regional difference in use reliability.

Table 6 shows that the warranty period and price obtained by the two scenarios increase as β increases. The price difference under Scenario 2 increases with β as well. For instance, when $\beta = 0.010$, the range of P_i^* is always within the interval of [3323, 3231.9]. However, when $\beta = 0.030$, the range of P_i^* is always within the interval of [3276.6, 3306.0]. Accordingly, the appropriate ranges of P_i^* based on Scenario 2 are adjusted adaptively in accordance with its use reliability level.

Table 7 shows that extension of the warranty period helps increase consumers' purchasing power. Thus, the sales volume for different sub-regions also increases as β increases. Specifically, when the value of β increases from 0.010 to 0.030, the sales volume for each sub-region increases by 2% to 3%. Regardless of the value of β, Scenario 2 always has large values for sales volume in sub-regions $i = 1, 2, 3, 4, 5, 6, 7$ and small values in sub-regions $i = 8, 9, 10$ due to the price variations.

Table 8 indicates that the profit for each sub-region increases as β increases due to the increase in sales volume. The profit of Scenario 2 is greater than that of Scenario 1, indicating that Scenario 2 outperforms Scenario 1. In addition, the profit in sub-regions $i = 3, 6, 8, 9, 10$ accounts for more than 85% of the expected total profit. We can conclude from Tables 6, 7, and 8 that the company should exert its best effort to satisfy consumers in these sub-regions, moderately extend the warranty period, and increase the sales volume to increase total profit π_Y.

A sensitivity analysis is also performed for parameter γ by adjusting the value of γ from 1.8 to 2.2. Several typical regions are used as examples to analyze the expected warranty cost per unit and the selling price of Scenario 2 under five different parameters, i.e., $\gamma = 1.8, 1.9, 2.0, 2.1, 2.2$. The results are illustrated in Figs. 6 and 7. For a careful

TABLE 7. Sales volume for different sub-regions with β change.

Q_i	$\beta = 0.010$	$\beta = 0.015$	$\beta = 0.020$	$\beta = 0.025$	$\beta = 0.030$
	Scenario 1/Scenario 2	Scenario 1/Scenario 2	Scenario 1/Scenario 2	Scenario 1/Scenario 2	Scenario 1/Scenario 2
$i = 1$	167163/167744	167738/168652	168600/169852	169681/171278	170941/172889
$i = 2$	436558/437720	438058/439885	440311/442813	443134/446323	446425/450315
$i = 3$	3277192/3284333	3288454/3299676	3305364/3320727	3326559/3346136	3351261/3375134
$i = 4$	12794/12816	12838/12873	12904/12952	12986/13048	13083/13158
$i = 5$	1886902/1889772	1893386/1897895	1903122/1909293	1915326/1923186	1929548/1939130
$i = 6$	4045330/4050234	4059232/4066932	4080106/4090641	4106269/4119685	4136760/4153111
$i = 7$	1605064/1605893	1610580/1611881	1618862/1620640	1629243/1631506	1641341/1644096
$i = 8$	6264163/6263773	6285689/6285072	6318012/6317160	6358526/6357431	6405742/6404395
$i = 9$	6498828/6491785	6521161/6510100	6554695/6539562	6596727/6577454	6645711/6622222
$i = 10$	5015011/5004943	5032245/5016443	5058122/5036517	5090557/5063060	5128357/5094866

TABLE 8. Profit for different sub-regions with β change.

| π_i	$\beta = 0.010$ Scenario 1/Scenario 2	$\beta = 0.015$ Scenario 1/Scenario 2	$\beta = 0.020$ Scenario 1/Scenario 2	$\beta = 0.025$ Scenario 1/Scenario 2	$\beta = 0.030$ Scenario 1/Scenario 2
$i = 1$	$2.703204/2.703213 \ 10^8$	$2.728929/2.728950 \ 10^8$	$2.759650/2.759690 \ 10^8$	$2.794314/2.794379 \ 10^8$	$2.832333/2.832430 \ 10^8$
$i = 2$	$7.056744/7.056758 \ 10^8$	$7.122275/7.122309 \ 10^8$	$7.200817/7.200880 \ 10^8$	$7.289611/7.289713 \ 10^8$	$7.387117/7.387268 \ 10^8$
$i = 3$	$5.296146/5.296153 \ 10^9$	$5.344602/5.344619 \ 10^9$	$5.402807/5.402840 \ 10^9$	$5.468689/5.468741 \ 10^9$	$5.541090/5.541167 \ 10^9$
$i = 4$	$2.067098/2.067100 \ 10^7$	$2.085772/2.085777 \ 10^7$	$2.108247/2.108256 \ 10^7$	$2.133712/2.133726 \ 10^7$	$2.161715/2.161735 \ 10^7$
$i = 5$	$3.048352/3.0484354 \ 10^9$	$3.075673/3.075678 \ 10^9$	$3.108595/3.108605 \ 10^9$	$3.145921/3.145936 \ 10^9$	$3.186983/3.187006 \ 10^9$
$i = 6$	$6.534356/6.534359 \ 10^9$	$6.592349/6.592356 \ 10^9$	$6.662334/6.662347 \ 10^9$	$6.741745/6.741766 \ 10^9$	$6.829148/6.829180 \ 10^9$
$i = 7$	$2.591700/2.591734 \ 10^9$	$2.614224/2.614225 \ 10^9$	$2.641459/2.641460 \ 10^9$	$2.672421/2.672423 \ 10^9$	$2.706538/2.706541 \ 10^9$
$i = 8$	$1.011196/1.011196 \ 10^{10}$	$1.019804/1.019804 \ 10^{10}$	$1.030261/1.030261 \ 10^{10}$	$1.042166/1.042166 \ 10^{10}$	$1.055299/1.055299 \ 10^{10}$
$i = 9$	$1.048541/1.048542 \ 10^{10}$	$1.057161/1.057162 \ 10^{10}$	$1.067692/1.067693 \ 10^{10}$	$1.079718/1.079720 \ 10^{10}$	$1.093007/1.093010 \ 10^{10}$
$i = 10$	$8.087626/8.087633 \ 10^9$	$8.151980/8.151998 \ 10^9$	$8.231025/8.231058 \ 10^9$	$8.321552/8.321606 \ 10^9$	$8.421762/8.421842 \ 10^9$
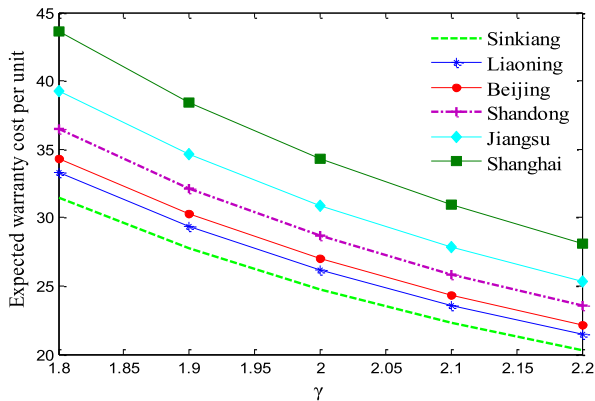
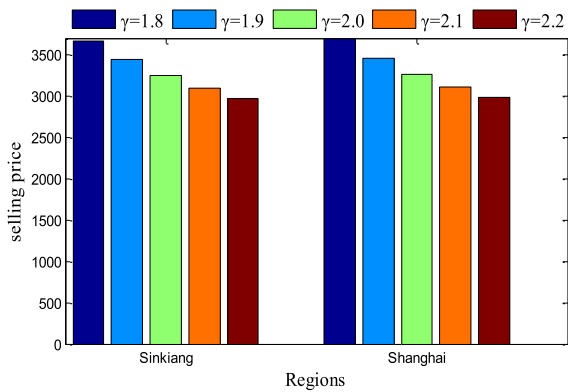**FIGURE 6.** Expected warranty cost per unit for Scenario 2 with γ change.

FIGURE 7. Selling price of Scenario 2 with γ change.

comparison of Scenarios 1 and 2, the corresponding optimal warranty period, selling price, and total profit obtained by the two scenarios with γ change are presented in Table 9. The sales volume and profit for each sub-region are shown in Figs. 8 and 9, respectively.

TABLE 9. Optimal solutions obtained by the two scenarios with γ change.

	Scenario 1				
	$\gamma = 1.8$	$\gamma = 1.9$	$\gamma = 2.0$	$\gamma = 2.1$	$\gamma = 2.2$
P_X^*	3688.1	3450.6	3261.5	3107.5	2979.6
W_X^*	10.65	9.38	8.37	7.55	6.87
π_X	$2.451065 \ 10^{11}$	$1.081838 \ 10^{11}$	$4.804287 \ 10^{10}$	$2.144679 \ 10^{10}$	$9.617321 \ 10^9$
	Scenario 2				
	$\gamma = 1.8$	$\gamma = 1.9$	$\gamma = 2.0$	$\gamma = 2.1$	$\gamma = 2.2$
P_i^*					
$i = 1$	3670.9	3436.3	3249.5	3097.2	2970.6
$i = 2$	3674.9	3439.7	3252.3	3099.6	2972.7
$i = 3$	3677.3	3441.7	3254.0	3101.0	2973.9
$i = 4$	3679.3	3443.3	3255.4	3102.3	2975.0
$i = 5$	3680.5	3444.3	3256.3	3103.0	2975.7
$i = 6$	3682.1	3445.6	3257.4	3103.9	2976.5
$i = 7$	3685.5	3448.5	3259.8	3106.0	2978.3
$i = 8$	3688.4	3450.8	3261.8	3107.7	2979.8
$i = 9$	3693.5	3455.0	3265.3	3110.8	2982.5
$i = 10$	3698.1	3458.9	3268.6	3113.6	2984.9
W_Y^*	10.65	9.38	8.37	7.55	6.87
π_Y	$2.451072 \ 10^{11}$	$1.081841 \ 10^{11}$	$4.804299 \ 10^{10}$	$2.144685 \ 10^{10}$	$9.617341 \ 10^9$

Table 9 indicates that the price and warranty period obtained by the two scenarios decrease as γ increases. When $\gamma = 1.8$, the range of P_i^* under Scenario 2 is
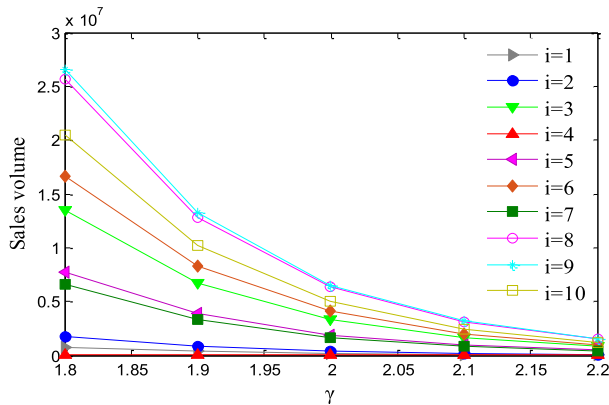
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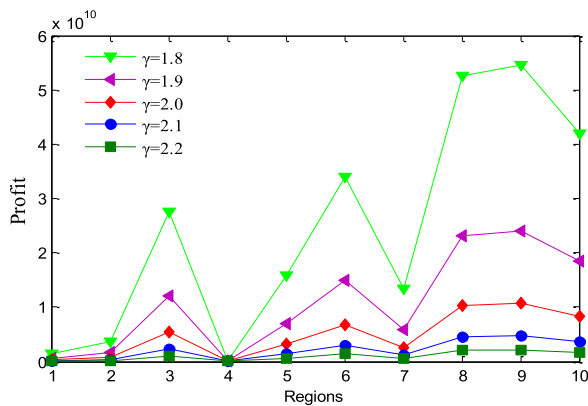**FIGURE 8.** Sales volume for different sub-regions with γ change.

FIGURE 9. Profit for different sub-regions with γ change.

always within the interval of [3670.9, 3698.1]. However, when $\gamma = 2.2$, the range of P_i^* is always within the interval of [2970.6, 2984.9]. This analysis implies that the price difference under Scenario 2 decreases with γ as well.

Fig. 6 shows a significant reduction in the expected warranty cost per unit, and the magnitude of the reduction is gradually decreasing. The key reason for this result is that the warranty period decreases as β increases (Table 9). Specifically, when the value of γ increases from 1.8 to 2.2, the expected warranty cost per unit for each sub-region decreases by about 35.5%. Fig. 7 shows that the selling price decreases sharply due to the decrease in warranty period and expected warranty cost per unit. In addition, the expected warranty cost per unit and selling price exhibit regional differences, which is consistent with the results in Figs. 4 and 5.

As depicted in Fig. 8, the decrease in warranty period causes a reduction in sales volume. Accordingly, the sales volume for each sub-region decreases as γ increases, but the magnitude of the reduction is gradually decreasing. When the value of γ increases from 1.8 to 2.2, the sales volume for each sub-region decreases by 94%. As shown in Fig. 9, the profit for each sub-region decreases as γ increases due to the decrease in sales volume. The profit is mainly concentrated in sub-regions $i = 3, 6, 8, 9, 10$, which is consistent with the results in Table 8. Therefore, the warranty period and price should be determined scientifically and reasonably in

accordance with the actual conditions in these sub-regions in order to increase customer satisfaction and the manufacturer's profit simultaneously.

VI. CONCLUSIONS

We developed a novel sub-region warranty differential pricing optimization strategy based on the regional granularity of use reliability and revealed the benefits of achieving balance among use reliability, warranty, and selling price.

We began with an overview of the development of reliability-based warranty decisions and a discussion of the limitations of current reliability evaluation methods and reliability-based warranty modeling. Then, we introduced a general formula of expected warranty cost and modified the sales model for each sub-region. Moreover, we established a new use reliability prediction model on the basis of the regional granularity partition results and after-sales failure data to accurately evaluate use reliability in different sub-regions. The proposed approach can ensure that the economic benefit of the regional difference in use reliability is explored to the greatest extent. Next, we built a high-dimensional decision model that considers the regional difference in use reliability, warranty, and price to optimize the regional warranty differential pricing strategy. We discussed two scenarios for pricing and warranty period: a unified warranty scheme (with a constant selling price and warranty period in all regions) and a partition warranty scheme (with the selling price and warranty period varying from region to region). We also separately derived the necessary conditions that characterize the optimal combination of use reliability, warranty period, and selling price to analyze the interactions among the variables of the proposed model. Afterward, we conducted a practical case study and a detailed sensitivity analysis of model parameters.

The validity and applicability of our model were demonstrated through comparisons between the proposed model and other alternatives. The results showed that our proposed scheme contributes to a longer warranty period and might be more suitable for an effective trade-off among use reliability, warranty, and selling price. In addition, manufacturers can make the most profitable decisions on use reliability, warranty period, and selling price by using the developed model under the partition warranty scheme. In particular, a moderate reduction in the unit sale price of sub-regions with high use reliability and an increase in the unit sale price of sub-regions with low use reliability are effective methods to increase profit and can provide a good reference for manufacturers when formulating warranty strategies.

The relevant results of this study can increase these methods' usefulness in practical applications compared with other inherent reliability-based methods. Notably, we provide several recommendations for using the model in practice. First, the number of regional granularity partitions of use reliability must be known in advance. Second, the scale factor for each sub-region must be pre-computed to show the relative impact of competitors and other market factors.

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Third, determining the actual use reliability in different sub-regions is a key step and an important prerequisite for a successful model application.

However, the proposed model still has several limitations because of the complexity and difficulty of the addressed problem. The model can be further extended in different directions. For instance, channel investment, advertising investment, and brand influence are not considered in this model. To obtain a more realistic model, future studies could consider these factors and modify the presented model reasonably. Moreover, the current model could be extended to warranty optimization issues with multiple objectives, such as cost minimization, market share maximization, sales revenue maximization, and profit maximization. Establishing a warranty optimization model with multiple objectives and constraints would likewise be interesting. In addition, studying this model with other types of common warranty policies (PRW or RFRW) and different types of repair actions is another topic for future work. The application of the proposed model or its extension to other consumer electronics can also be explored.

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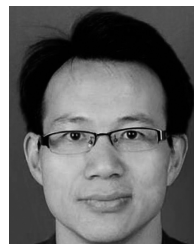
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