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On Stabilization of Linear Switched Singular Systems via P-D State Feedback

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ABSTRACT Impulsive phenomenon and state jumps at switching instants are inevitable control difficulties in the study of stability for switched singular systems. Proportional-derivative (P-D) state feedback may be an effective way to eliminate impulsive behaviors and state jumps. In this work, the problem of stabilization is studied for switched singular systems in the continuous-time case and discrete-time case. A synchronous design method of P-D state feedback controllers is proposed by introducing some free-weighting matrices. Based on P-D state feedback, some sufficient conditions, which can guarantee that the closed-loop systems are normal and stable (NS), are obtained by using multiple Lyapunov functions. Compared with step-by-step design, synchronous design brings more freedom to the design of P-D state feedback controllers and can better improve the dynamic performance of the systems. Finally, simulation examples are given to demonstrate the effectiveness of the proposed methods.

INDEX TERMS Switched singular systems, normal and stable, P-D state feedback, synchronous design.

I. INTRODUCTION

Switched singular systems (also named as switched descriptor systems), as higher-level abstractions of hybrid systems, contain a finite number of continuous-time (or discrete-time) subsystems and a switching signal specifying the switching among them [1]. Switching signal is a key part of a switched system, which can even determine the control performance of the system [2]. It is well known that a switched system consisting of stable subsystems may be unstable under improper switching signals [3]. Because of the existence of algebraic equations, switched singular systems can describe larger scope of actual dynamic systems than switched normal systems, so such systems have been widely concerned since they were proposed, and have been widely applied in aerospace, chemical systems, power electronic systems and other fields [4].

For switched singular systems, even if all subsystems are regular and impulse-free (continuous-time case) or regular and causal (discrete-time case), state jumps at switching

instants are unavoidable [5], [6], because the state at the end instant of the previous subsystem is generally not the compatible initial state of the next activated subsystem. This is one of the major differences between switched singular systems and switched normal systems [7], [8]. Instantaneous state jumps may cause the dynamic performance to deteriorate or even cause a system to collapse [6]. In order to eliminate the negative effect on system performance caused by state jumps, the hybrid impulsive controller consisting of a feedback controller and an impulsive controller was designed in [9]–[11] to study the stability of switched singular systems. It should be pointed out that the principal function of the impulsive controller is to reset the system states at switching instants, so as to reduce or eliminate the impulse. In fact, it is difficult to completely eliminate the state jumps by using state reset. In addition, it is also difficult to determine the switching time between any two subsystems in the complex situation, which will also bring difficulties to state reset. Until now, many research results have been reported under an assumption that the states do not jump at switching instants, see [6], [12]–[15]. How to eliminate state jumps completely is one of the

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challenging work in the study of switched singular systems [16].

Impulsivity and causality are unavoidable control problems in the study of continuous-time and discrete-time singular systems respectively. Because a switched singular system consists of a group of singular subsystems, impulsivity or causality is the control problem that must be faced in the study of switched singular systems. Due to the complex structure of switched singular systems, many research results were obtained under an assumption that the systems are impulse-free [7], [8], [14], [17], [18] or causal [4], [19], [20]. In the study of stability for switched singular systems, the closed-loop systems are usually guaranteed to be impulse-free or causal by proportional feedback controllers [6], [15], [21]–[25]. But in some cases, simple proportional controllers can not eliminate impulse and non-causality, so P-D state feedback is widely used in the study of singular systems, and many results have been reported [26]–[33]. It is well known that P-D state feedback can not only change the characteristics of singular systems, but also can be used to eliminate impulse behaviors of singular systems [16], [31]. Because of the complex structure of the system and the switching law involved, it is difficult to apply P-D state feedback directly to switched singular systems. So far, there are few results on P-D state feedback control of switched singular systems. In [16], P-D state feedback was applied for the first time, and the idea of step-by-step design was proposed. The output strictly passive H_∞ control problem was investigated for discrete-time switched singular systems. Although the step-by-step design reduces the difficulty of controller design, it also sacrifices the freedom of derivative controller design. To the best of the authors' knowledge, there are few results concerning the synchronous design of P-D state feedback controllers for switched singular systems in continuous-time case and discrete-time case, which motivates us for this study.

In this work, the stabilization problem of switched singular systems is studied respectively in the continuous-time case and discrete-time case by using P-D state feedback. Some sufficient conditions, which can ensure that the closed-loop systems are NS, are given by designing P-D state feedback controllers and state-dependent switching laws. Based on the above results, some sufficient conditions are also obtained by using derivative state feedback controllers alone to guarantee that the closed-loop systems are NS. The use of the free-weighting matrices reduces the strong coupling relationship between the proportional controller and the derivative controller, which greatly facilitates the design of the controllers. In addition, compared with the step-by-step design [16], synchronous design can promote the design freedom of the derivative controller, and can further improve the dynamic performance of the systems.

The rest of this work is organized as follows. Some of the relevant definitions, lemmas and preliminaries are briefly sketched in Sect. 2. The main results and some comments are given in Sect. 3. Simulation examples are given to

illustrate our main results in Sect. 4. Section 5 includes some concluding remarks.

II. PROBLEM FORMULATION

Notations: The superscript 'T' represents matrix transposition; \mathbf{R}^n denotes the n -dimensional Euclidean space; $\text{rank}(\cdot)$ represents the rank of a matrix; $\min\{\cdot\}$ stands for the minimum value in a collection; $\det\{\cdot\}$ represents the determinant of a matrix; $\deg\{\cdot\}$ represents the degree of a polynomial; $\arg \min_{i \in \{1, 2, \dots, m\}} \{f_i(t)\}$ is the subscript of the function that takes the minimum value in the function set $\{f_i(t)\}$ at time t ; 'I' represents the identity matrix with a appropriate dimension.

In this paper, we consider the following linear switched singular systems:

$$\begin{cases} E_{\sigma(t)} \delta x(t) = A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t) \\ x(0) = x_0 \end{cases} \quad (1)$$

where the symbol δ denotes the derivative operator in the continuous-time case ($\delta x(t) = dx(t)/dt$) and the shift-forward operator in the discrete-time case ($\delta x(t) = x(t+1)$). $\sigma(t)$ is a piecewise constant switching signal of t , which takes its values in a finite set $\mathbf{M} = \{1, 2, \dots, m\}$, and m is the total number of subsystems. $\sigma(t) = i$ denotes that the i th subsystem is activated at time t . $x(t) \in \mathbf{R}^n$ is the state vector. $u(t) \in \mathbf{R}^p$ is the control input. E_i , A_i and B_i are known real constant matrices with appropriate dimensions, and $\text{rank}(E_i) = r_i \leq n$. For convenience, the symbol t is replaced by k in discrete-time case. x_0 is the initial state of system (1).

Remark 1: The derivative matrices E_i , $i \in \mathbf{M}$ can be divided into three cases. The first case is that all subsystems share the same derivative matrix (see, e.g. [4], [6], [10], [11], [13], [15], [18], [19], [22]–[25]). The second case is that derivative matrices of subsystems are not identical, but their ranks are the same ones (see, e.g. [7], [8], [17], [21]). The third case is that derivative matrices and their ranks of the subsystems are not identical. Obviously, the third case is more general and this work belongs to the case (see, e.g. [9], [14], [16]).

Definition 1 [16]: System (1) with $u(t) = 0$ is said to be NS if the derivative matrix $E_{\sigma(t)}$ is invertible and there exists a switching signal generated by $\sigma(t)$ such that the whole system is asymptotically stable.

Definition 2 [34]: For any $i \in \mathbf{M}$, the continuous-time singular system (E_i, A_i) is said to be

- (i) regular if $\det(sE_i - A_i)$ is not identically zero;
- (ii) impulse-free if $\deg(\det(sE_i - A_i)) = \text{rank}(E_i)$.

Definition 3 [22]: For any $i \in \mathbf{M}$, the discrete-time singular system (E_i, A_i) is said to be

- (i) regular if $\det(zE_i - A_i)$ is not identically zero;
- (ii) causal if $\deg(\det(zE_i - A_i)) = \text{rank}(E_i)$.

Remark 2: In continuous-time case, system (1) without control input is said to be regular and impulse-free if each subsystem is regular and impulse-free. It can be easy seen from Definitions 1 and 2 that if a switched system is NS,

it is regular, impulse-free and asymptotically stable. In discrete-time case, system (1) without control input is said to be regular and causal if each subsystem is regular and causal. It can be seen from Definitions 1 and 3 that if a switched system is NS, it is regular, causal and asymptotically stable.

Assumption 1: The states are measurable or predictable in this study.

Lemma 1.: (Schur complement lemma [35]) Given a symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$

where $S_{11} \in \mathbf{R}^{r \times r}$, the following three inequalities are equivalent.

- (1) $S < 0$,
- (2) $S_{11} < 0, \quad S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,
- (3) $S_{22} < 0, \quad S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

III. MAIN RESULTS

A. CONTINUOUS-TIME CASE

We design the P-D state feedback controller for system (1) in continuous-time case as follows.

$$u(t) = K_{a\sigma(t)}x(t) - K_{e\sigma(t)}\dot{x}(t) \quad (2)$$

First, we will study the stability condition of system (1) based on controller (2), and give the following results.

Theorem 1: Consider system (1) in continuous-time case, if there exist matrices $P_i > 0, F_i, G_i, K_{ai}, K_{ei}$ and scalars $\alpha_{ij} < 0, (i, j \in \mathbf{M}, i \neq j)$, such that

$$\Sigma_i = \begin{bmatrix} \Sigma_{i1} & \Sigma_{i2} \\ \Sigma_{i2}^T & \Sigma_{i3} \end{bmatrix} < 0 \quad (3)$$

where

$$\begin{aligned} A_{ci} &= A_i + B_i K_{ai}, E_{ci} = E_i + B_i K_{ei}, \\ \Sigma_{i1} &= A_{ci}^T F_i + F_i^T A_{ci} + \sum_{j=1}^m \alpha_{ij} (P_i - P_j), \\ \Sigma_{i2} &= P_i - F_i^T E_{ci} + A_{ci}^T G_i, \\ \Sigma_{i3} &= -E_{ci}^T G_i - G_i^T E_{ci}. \end{aligned}$$

The switching law is designed as

$$\sigma(t) = \min \left\{ \arg \min_{i \in \mathbf{M}} x^T(t) P_i x(t) \right\} \quad (4)$$

Then system (1) controlled by (2) is NS under the switching law in (4).

Proof: Note that inequality (3) implies that $E_{ci}, i \in \mathbf{M}$ is nonsingular. Substituting (2) into (1) gives

$$E_{c\sigma(t)}\dot{x}(t) = A_{c\sigma(t)}x(t) \quad (5)$$

For system (5), the following equation always holds for any weighting matrices F_i and G_i with appropriate dimensions.

$$2 \left[-x^T(t) F_i^T - \dot{x}^T(t) G_i^T \right] \times [E_{ci}\dot{x}(t) - A_{ci}x(t)] = 0 \quad (6)$$

We choose

$$V_{\sigma(t)}(x(t)) = x^T(t) P_{\sigma(t)} x(t) \quad (7)$$

as Lyapunov function of system (5).

When the i th subsystem is activated, from (5)-(7), we can get

$$\begin{aligned} \dot{V}_i(x(t)) &= \dot{x}^T(t) P_i x(t) + x^T(t) P_i \dot{x}(t) \\ &\quad - 2x^T(t) F_i^T E_{ci} \dot{x} + 2x^T(t) F_i^T A_{ci} x(t) \\ &\quad - 2\dot{x}^T(t) G_i^T E_{ci} \dot{x} + 2\dot{x}^T(t) G_i^T A_{ci} x(t) \\ &= \eta^T(t) \left(\Sigma_i - \begin{bmatrix} \sum_{j=1}^m \alpha_{ij} (P_i - P_j) & 0 \\ 0 & 0 \end{bmatrix} \right) \eta(t) \end{aligned} \quad (8)$$

where $\eta(t) = [x^T(t) \quad \dot{x}^T(t)]^T$.

From (4) and $\alpha_{ij} < 0$, we have

$$x^T(t) \sum_{j=1}^m \alpha_{ij} (P_i - P_j) x(t) > 0 \quad (9)$$

From (3), (8) and (9), we get $\dot{V}_i(x(t)) < 0$. According to the multiple Lyapunov functions theory, system (5) is asymptotically stable. From Definition 1, system (1) controlled by (2) is NS under the switching law in (4).

Remark 3: If the free-weighting matrices F_i and G_i are not introduced, the strong coupling relationship between the gains of proportional controllers and the gains of derivative controllers will appear as follows (see [16]):

$$(E_i + B_i K_{ei})^{-1} (A_i + B_i K_{ai}) \quad (10)$$

It can be seen from (10) that it is difficult to design the gains of P-D state feedback controllers synchronously. In order to reduce the difficulty, the step-by-step design was adopted in [16] to design P-D state feedback controllers. However, this design method brings some conservatism because it is difficult to determine whether the gains of derivative controllers designed in advance meet the requirements of control performance. It can be seen from inequality (3) that using free-weighting matrices makes the gains of the proportional controller and the derivative controller uncoupled, which makes it possible to design the gains of P-D state feedback controllers synchronously.

Remark 4: Depending on Theorem 1, under the action of the controllers designed in this work, the closed-loop system is a switched normal system. In theory, for any initial condition, the states of switched normal systems do not jump at switching instants. Therefore, the initial state can be selected arbitrarily according to the actual demand and the compatibility problem does not need to be considered.

Next, the gains of P-D state feedback controllers are designed by Lemma 1, and the following theorem is given.

Theorem 2: Consider system (1) in continuous-time case, if there exist matrices $V_{1i} > 0, V_{2i}, V_{3i}$ with $\det(V_{3i}) \neq 0$,

S_{1i} , S_{2i} and scalars $\alpha_{ij} < 0$, ($i, j \in \mathbf{M}, i \neq j$), such that

$$\begin{bmatrix} V_{2i} + V_{2i}^T + \sum_{j=1}^m \alpha_{ij} V_{1i} & \Xi_{i2}^T & \alpha_{i1} V_{1i} & \cdots \\ \Xi_{i2} & \Xi_{i3} & 0 & \cdots \\ \alpha_{i1} V_{1i} & 0 & \alpha_{i1} V_{11} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \alpha_{i(i-1)} V_{1i} & 0 & 0 & \cdots \\ \alpha_{i(i+1)} V_{1i} & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \alpha_{im} V_{1i} & 0 & 0 & \cdots \\ \alpha_{i(i-1)} V_{1i} & \alpha_{i(i+1)} V_{1i} & \cdots & \alpha_{im} V_{1i} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{i(i-1)} V_{1(i-1)} & 0 & \cdots & 0 \\ 0 & \alpha_{i(i+1)} V_{1(i+1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{im} V_{1m} \end{bmatrix} < 0 \quad (11)$$

where

$$\begin{aligned} \Xi_{i2} &= V_{3i}^T + A_i V_{1i} - E_i V_{2i} + B_i S_{1i}, \\ \Xi_{i3} &= -E_i V_{3i} - V_{3i}^T E_i^T + B_i S_{2i} + S_{2i}^T B_i^T. \end{aligned}$$

The gains of controller (2) are given as follows.

$$K_{ai} = (S_{1i} - S_{2i} V_{3i}^{-1} V_{2i}) V_{1i}^{-1}, \quad K_{ei} = -S_{2i} V_{3i}^{-1} \quad (12)$$

The switching law is designed as

$$\sigma(t) = \min \left\{ \arg \min_{i \in \mathbf{M}} x^T(t) V_{1i}^{-1} x(t) \right\} \quad (13)$$

Then system (1) controlled by (2) is NS under the switching law in (13).

Proof: Pre- and post-multiplying the left-hand-side matrix of (3) by

$$\begin{bmatrix} P_i & 0 \\ F_i & G_i \end{bmatrix}^{-T}$$

and its transpose, respectively, and letting

$$\begin{bmatrix} P_i & 0 \\ F_i & G_i \end{bmatrix}^{-1} = \begin{bmatrix} V_{1i} & 0 \\ V_{2i} & V_{3i} \end{bmatrix} \quad (14)$$

we get

$$\begin{bmatrix} \Psi_{i1} & \Psi_{i2} \\ \Psi_{i2}^T & -E_{ci} V_{3i} - V_{3i}^T E_{ci}^T \end{bmatrix} < 0 \quad (15)$$

where

$$\begin{aligned} \Psi_{i1} &= V_{2i} + V_{2i}^T + V_{1i}^T \sum_{j=1}^m \alpha_{ij} (V_{1i}^{-1} - V_{1j}^{-1}) V_{1i} \\ \Psi_{i2} &= V_{3i} + V_{1i}^T A_{ci}^T - V_{2i}^T E_{ci}^T \end{aligned}$$

From (12) and (15), we can easily get (11) by using Lemma 1. From the above proving process, it can be seen that inequality (11) is equal to inequality (3). Switching law (13) is equal to (4). According to Theorem 1, system (1) controlled by (2) is NS under the switching law in (13).

Remark 5: In Theorem 2, it can be seen that the gains of P-D state feedback controllers are designed synchronously. Compared with the step-by-step design in [16], synchronous design brings more freedom of controller design and reduces the conservatism to a certain extent. In addition, Lemma 1 is used to deal with the strong nonlinear term caused by switching law in the process of proof instead of inequality scaling skills.

Remark 6: In some cases, the control goals can also be achieved by using derivative state feedback alone. Compared with the P-D state feedback controllers, the costs of designing and maintaining for derivative state feedback controllers are smaller.

Next, we design the following derivative state feedback controller for system (1), and give a corollary based on Theorem 2.

$$u(t) = -K_{e\sigma(t)} \dot{x}(t) \quad (16)$$

Corollary 1: Consider system (1) in continuous-time case, if there exist matrices $V_{1i} > 0$, X_i , S_i , and scalars $\alpha_{ij} < 0$, ($i, j \in \mathbf{M}, i \neq j$), such that

$$\begin{bmatrix} X_i + X_i^T + \sum_{j=1}^m \alpha_{ij} V_{1i} & \Gamma_{i2}^T & \alpha_{i1} V_{1i} & \cdots \\ \Gamma_{i2} & \Gamma_{i3} & 0 & \cdots \\ \alpha_{i1} V_{1i} & 0 & \alpha_{i1} V_{11} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \alpha_{i(i-1)} V_{1i} & 0 & 0 & \cdots \\ \alpha_{i(i+1)} V_{1i} & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \alpha_{im} V_{1i} & 0 & 0 & \cdots \\ \alpha_{i(i-1)} V_{1i} & \alpha_{i(i+1)} V_{1i} & \cdots & \alpha_{im} V_{1i} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{i(i-1)} V_{1(i-1)} & 0 & \cdots & 0 \\ 0 & \alpha_{i(i+1)} V_{1(i+1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{im} V_{1m} \end{bmatrix} < 0 \quad (17)$$

where

$$\begin{aligned} \Gamma_{i2} &= X_i^T + A_i V_{1i} - E_i X_i + B_i S_i, \\ \Gamma_{i3} &= -E_i X_i - X_i^T E_i^T + B_i S_i + S_i^T B_i^T. \end{aligned}$$

The gains of controller (16) are given as follows.

$$K_{ei} = -S_i X_i^{-1} \quad (18)$$

Then system (1) controlled by (16) is NS under the switching law in (13).

Proof: In Theorem 2, setting $V_{2i} = V_{3i} = X_i$ and $S_{1i} = S_{2i} = S_i$, we can get (17) directly and $K_{ai} = 0$.

B. DISCRETE-TIME CASE

We design the P-D state feedback controller for system (1) in discrete-time case as follows.

$$u(k) = K_{a\sigma(k)}x(k) - K_{e\sigma(k)}x(k+1) \tag{19}$$

Remark 7: The P-D state feedback in discrete-time case was first proposed in [32] and has been used to solve various related control problems (see, e.g. [16], [28], [29], [33]). Assume that instant $k + 1$ is the current time and k is the previous time, then $x(k + 1)$ can be estimated by the previous state information (see, [16] and [32]). In the case of high accuracy of the state observer, it is reasonable to use the estimated value of $x(k + 1)$ when design P-D state feedback controllers. The simple state feedback we are familiar with also uses the approximate value of the state instead of the actual value, because the state data measured by the measuring instrument also have errors.

First, we will study the stability condition of system (1) based on controller (19), and give the following results.

Theorem 3: Consider system (1) in discrete-time case, if there exist matrices $P_i > 0$, F_i , G_i , K_{ai} , K_{ei} and scalars $\alpha_{ij} < 0$, ($i, j \in \mathbf{M}, i \neq j$), such that

$$\Omega_i = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} \\ \Omega_{i2}^T & \Omega_{i3} \end{bmatrix} < 0 \tag{20}$$

where

$$\begin{aligned} A_{ci} &= A_i + B_i K_{ai}, E_{ci} = E_i + B_i K_{ei}, \\ \Omega_{i1} &= -P_i + A_{ci}^T F_i + F_i^T A_{ci} + \sum_{j=1}^m \alpha_{ij}(P_i - P_j), \\ \Omega_{i2} &= -F_i^T E_{ci} + A_{ci}^T G_i, \\ \Omega_{i3} &= P_i - E_{ci}^T G_i - G_i^T E_{ci}. \end{aligned}$$

The switching law is designed as

$$\sigma(k) = \min \left\{ \arg \min_{i \in \mathbf{M}} x^T(k) P_i x(k) \right\} \tag{21}$$

Then system (1) controlled by (19) is NS under the switching law in (21).

Proof: Note that inequality (20) implies that E_{ci} , $i \in \mathbf{M}$ is nonsingular. Substituting (19) into (1) gives

$$E_{c\sigma(k)}x(k+1) = A_{c\sigma(k)}x(k) \tag{22}$$

For system (22), the following equation always holds for any weighting matrices F_i and G_i with appropriate dimensions.

$$2 \left[-x^T(k) F_i^T - x^T(k+1) G_i^T \right] \times [E_{ci}x(k+1) - A_{ci}x(k)] = 0 \tag{23}$$

We choose

$$V(x(k)) = x^T(k) P_\sigma x(k) \tag{24}$$

as the Lyapunov function of system (22).

Let $\sigma(k+1) = j$, $\sigma(k) = i$. From (21)-(24), we have

$$\begin{aligned} \Delta V_i(k) &= V_j(x(k+1)) - V_i(x(k)) \\ &\leq V_i(x(k+1)) - V_i(x(k)) \\ &= x^T(k+1) P_i x(k+1) - x^T(k) P_i x(k) \\ &\quad + 2 \left[-x^T(k) F_i^T - x^T(k+1) G_i^T \right] \\ &\quad \times [E_{ci}x(k+1) - A_{ci}x(k)] \\ &= \eta^T(k) \left(\Sigma_i - \begin{bmatrix} \sum_{j=1}^m \alpha_{ij}(P_i - P_j) & 0 \\ 0 & 0 \end{bmatrix} \right) \eta(k) \end{aligned} \tag{25}$$

where $\eta(k) = [x^T(k) \ x^T(k+1)]^T$.

From (21), we have

$$x^T(k) \sum_{j=1}^m \alpha_{ij}(P_i - P_j)x(k) > 0 \tag{26}$$

From (20), (25) and (26), we get $\Delta V_i(k) < 0$. According to the multiple Lyapunov functions theory, system (22) is asymptotically stable. From Definition 1, system (1) controlled by (19) is NS under the switching law in (21).

Next, the gains of P-D state feedback controllers are designed by Lemma 1, and the following theorem is given.

Theorem 4: Consider system (1) in discrete-time case, if there exist matrices $V_{1i} > 0$, V_{2i} , V_{3i} with $\det(V_{3i}) \neq 0$, S_{1i} , S_{2i} and scalars $\alpha_{ij} < 0$, ($i, j \in \mathbf{M}, i \neq j$), such that

$$\begin{bmatrix} -V_{1i} + \sum_{j=1}^m \alpha_{ij} V_{1i} & \Theta_{i2}^T & V_{2i}^T & \alpha_{i1} V_{1i} & \cdots \\ \Theta_{i2} & \Theta_{i3} & V_{3i}^T & 0 & \cdots \\ V_{2i} & V_{3i} & -V_{1i} & 0 & \cdots \\ \alpha_{i1} V_{1i} & 0 & 0 & \alpha_{i1} V_{11} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ \alpha_{i(i-1)} V_{1i} & 0 & 0 & 0 & \cdots \\ \alpha_{i(i+1)} V_{1i} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ \alpha_{im} V_{1i} & 0 & 0 & 0 & \cdots \\ \alpha_{i(i-1)} V_{1i} & \alpha_{i(i+1)} V_{1i} & \cdots & \alpha_{im} V_{1i} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{i(i-1)} V_{1(i-1)} & 0 & \cdots & 0 \\ 0 & \alpha_{i(i+1)} V_{1(i+1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{im} V_{1m} \end{bmatrix} < 0 \tag{27}$$

where

$$\begin{aligned} \Theta_{i2} &= A_i V_{1i} - E_i V_{2i} + B_i S_{1i}, \\ \Theta_{i3} &= -E_i V_{3i} - V_{3i}^T E_i^T + B_i S_{2i} + S_{2i}^T B_i^T. \end{aligned}$$

The gains of controller (19) are given as follows.

$$K_{ai} = (S_{1i} - S_{2i}V_{3i}^{-1}V_{2i})V_{1i}^{-1}, K_{ei} = -S_{2i}V_{3i}^{-1} \quad (28)$$

The switching law is designed as

$$\sigma(k) = \min \left\{ \arg \min_{i \in \mathbf{M}} x^T(k)V_{1i}^{-1}x(k) \right\} \quad (29)$$

Then system (1) controlled by (19) is NS under the switching law in (29).

Proof: The proving process of Theorem 4 is similar to that of Theorem 2, which is omitted here.

Next, we design the following derivative state feedback controller for system (1), and give a corollary based on Theorem 4.

$$u(k) = -K_{e\sigma(k)}x(k+1) \quad (30)$$

Corollary 2: Consider system (1) in discrete-time case, if there exist matrices $V_{1i} > 0$, X_i , S_i , and scalars $\alpha_{ij} < 0$, ($i, j \in \mathbf{M}$, $i \neq j$), such that

$$\begin{bmatrix} -V_{1i} + \sum_{j=1}^m \alpha_{ij}V_{1j} & \Phi_{i2}^T & X_i^T & \alpha_{i1}V_{1i} & \cdots \\ \Phi_{i2} & \Phi_{i3} & X_i^T & 0 & \cdots \\ X_i & X_i & -V_{1i} & 0 & \cdots \\ \alpha_{i1}V_{1i} & 0 & 0 & \alpha_{i1}V_{11} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ \alpha_{i(i-1)}V_{1i} & 0 & 0 & 0 & \cdots \\ \alpha_{i(i+1)}V_{1i} & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ \alpha_{im}V_{1i} & 0 & 0 & 0 & \cdots \\ \alpha_{i(i-1)}V_{1i} & \alpha_{i(i+1)}V_{1i} & \cdots & \alpha_{im}V_{1i} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{i(i-1)}V_{1(i-1)} & 0 & \cdots & 0 \\ 0 & \alpha_{i(i+1)}V_{1(i+1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{im}V_{1m} \end{bmatrix} < 0 \quad (31)$$

where

$$\begin{aligned} \Phi_{i2} &= A_iV_{1i} - E_iX_i + B_iS_i, \\ \Phi_{i3} &= -E_iX_i - X_i^TE_i^T + B_iS_i + S_i^TB_i^T. \end{aligned}$$

The gains of controller (19) are given as follows.

$$K_{ei} = -S_iX_i^{-1} \quad (32)$$

Then system (1) controlled by (30) is NS under the switching law in (29).

Proof: The proving process of Corollary 2 is similar to that of Corollary 1, which is omitted here.

Remark 8: It is worth noting that all the results obtained in this paper are valid for switched normal systems and switched

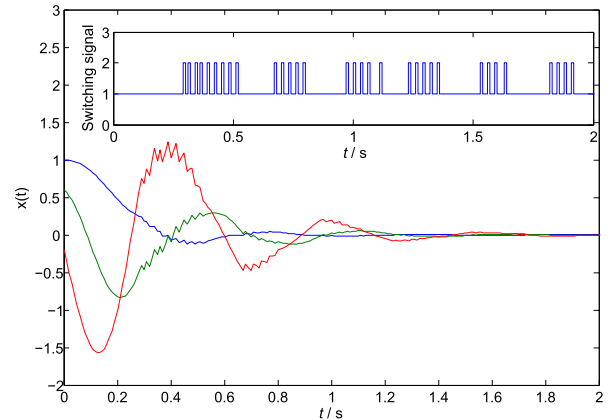


FIGURE 1. State trajectory of the closed-loop system based on Theorem 2.

singular systems consisting of normal subsystems and singular subsystems. In addition, for P-D state feedback and derivative state feedback, how to choose the control strategy depends on the specific situation.

Remark 9: The P-D state feedback controllers designed in this paper do not need to guarantee that all subsystems are asymptotically stable, because the stability of the closed-loop systems depends on both the designed controllers and the switching laws.

IV. NUMERICAL EXAMPLES

Example 4.1: Consider a continuous-time linear switched singular system in (1) with two subsystems.

$$\begin{aligned} E_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} -1 & 1 & 4 \\ 2 & -5 & 3 \\ 1 & -3 & 6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ -1 \\ 15 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 2 & 1 & 5 \\ 0 & -3 & 4 \\ -1 & 2 & -5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \\ -0.5 \end{bmatrix}. \end{aligned}$$

Choose $\alpha_{12} = \alpha_{21} = -0.5$, we can get K_{ai} and K_{ei} in (2) by solving (11) in Theorem 2 as follows.

$$\begin{aligned} K_{a1} &= [1.8166 \ 3.9341 \ -4.8756], \\ K_{a2} &= [6.7909 \ 7.1528 \ 7.4696], \\ K_{e1} &= [-1.4638 \ 0.5937 \ -0.3350], \\ K_{e2} &= [-0.7490 \ -1.4840 \ -0.2699]. \end{aligned}$$

Select the same parameters as above, we can get K_{ei} in (16) by solving (17) in Corollary 1 as follows.

$$\begin{aligned} K_{e1} &= [-0.0008 \ -0.0004 \ -1.0000], \\ K_{e2} &= [-0.0002 \ 0.0002 \ 2.0000]. \end{aligned}$$

We choose the initial condition $x(0) = [1 \ 0.6 \ -0.2]^T$. The continuous-time switched singular system in (1) controlled by (2) and (16) respectively is NS. The state trajectories and the switching laws are depicted in Figs. 1 and 2 respectively.

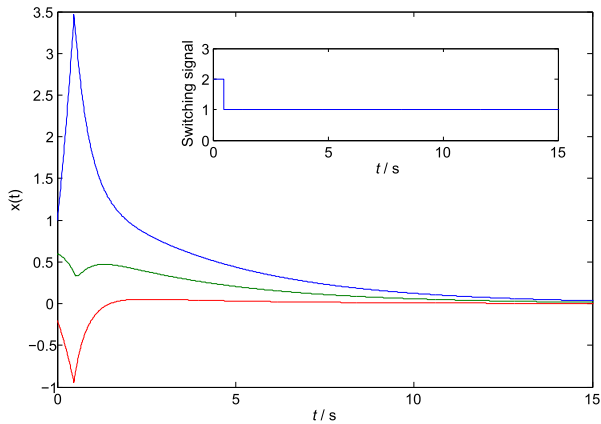


FIGURE 2. State trajectory of the closed-loop system based on Corollary 1.

It can be seen from Figs. 1 and 2 that both control strategies can stabilize the system in (1). From the comparison of these two control strategies, we can find that P-D state feedback control makes the settling time shorter, the overshoot smaller and the response speed faster through frequent switching between subsystems. Therefore, the control effect of P-D state feedback is better than that of derivative state feedback alone in this example.

Example 4.2: Consider a discrete-time linear switched singular system in (1) with two subsystems.

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0.6 & 0.2 & 0.3 \\ -0.5 & -0.3 & 0.3 \\ -0.4 & 0.2 & 0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.3 \\ -1 \\ 0.7 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.4 & -0.5 & 0.3 \\ 0.5 & -0.1 & 0.3 \\ 0.1 & 0.2 & -0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.8 \\ 0.3 \\ -0.5 \end{bmatrix}.$$

Choose $\alpha_{12} = -0.3$ and $\alpha_{21} = -0.5$, we can get K_{ai} and K_{ei} in (19) by solving (27) in Theorem 4 as follows.

$$K_{a1} = [-9.7923 \quad -0.9304 \quad 5.2958],$$

$$K_{a2} = [24.4530 \quad 3.3048 \quad 5.6533],$$

$$K_{e1} = [0.2573 \quad -0.4364 \quad -15.2819],$$

$$K_{e2} = [212.1340 \quad 11.9886 \quad -3.7401].$$

Select the same parameters as above, we can get K_{ei} in (30) by solving (31) in Corollary 2 as follows.

$$K_{e1} = [-0.2281 \quad -0.0943 \quad 3.6723],$$

$$K_{e2} = [-123.5762 \quad -37.4837 \quad -37.1265].$$

We choose the initial condition $x(0) = [1 \ 0 \ -1]^T$. The discrete-time switched singular system in (1) controlled by (19) and (30) respectively is NS. The state trajectories and the switching laws are depicted in Figs. 3 and 4 respectively.

It can be seen from Figs. 3 and 4 that both control strategies can stabilize the system in (1). From the comparison of these two control strategies, we can find that P-D state feedback

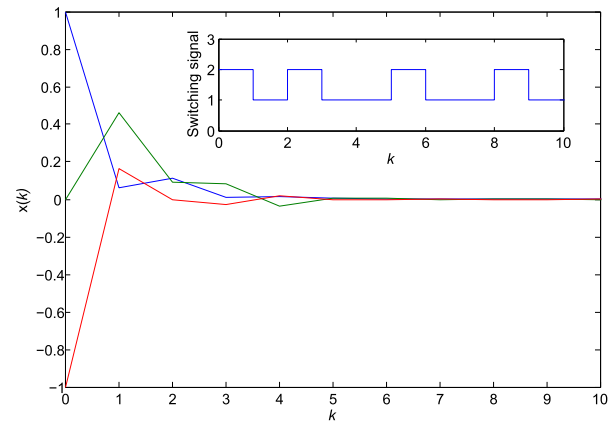


FIGURE 3. State trajectory of the closed-loop system based on Theorem 4.

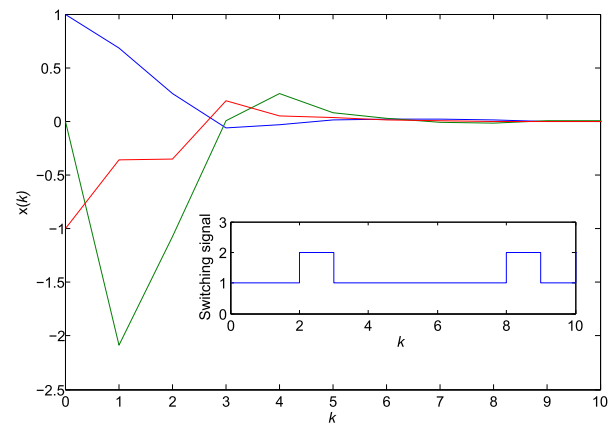


FIGURE 4. State trajectory of the closed-loop system based on Corollary 2.

control makes the settling time shorter, the response speed faster, the overshoot smaller and the stationarity of dynamic process better through frequent switching between subsystems. Therefore, the control effect of P-D state feedback is better than that of derivative state feedback in this example.

Select the same parameters as above and other parameters as in [16], we can construct the gains of derivative controllers as $K_{e1} = [0 \ 0 \ 0.7]$, $K_{e2} = [0.8 \ 0 \ 0]$. According to Theorem 1 of [16], we can get

$$K_{a1} = [1.7991 \quad -0.3777 \quad -1.4226],$$

$$K_{a2} = [0.3707 \quad 0.6706 \quad -0.6993].$$

We choose the same initial condition as above, and the state trajectory and switching law are depicted in Fig 5.

It can be seen from Figs. 3 and 5 that the P-D state feedback controller designed in this work makes the settling time shorter, the response speed faster and the stationarity of dynamic process better compared with the P-D state feedback controller designed in [16]. This shows that designing the gains of derivative controllers in advance may bring some conservatism.

Example 4.3: In this example, we introduce a PWM driven boost converter model (see [36]) to illustrate the effectiveness of the proposed methods. The boost converter is shown in Fig. 6.

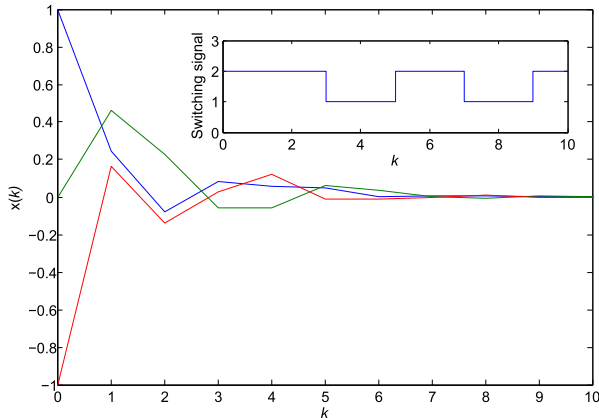


FIGURE 5. State trajectory of the closed-loop system based on Theorem 1 of [16].

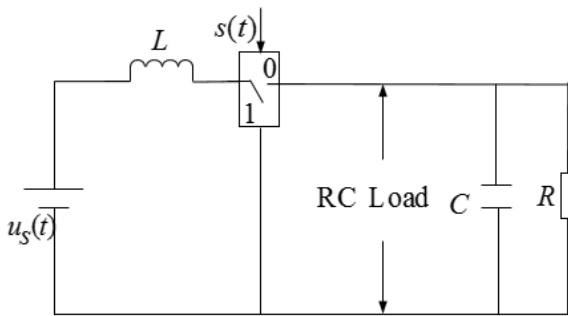


FIGURE 6. Boost converter.

The mathematical model of the boost converter can be built as follows by using Kirchhoff laws.

$$\begin{cases} \dot{u}_C(t) = -\frac{1}{RC}u_C(t) + (1 - s(t))\frac{1}{C}i_L(t), \\ \dot{i}_L(t) = -(1 - s(t))\frac{1}{L}u_C(t) + s(t)\frac{1}{L}u_s(t), \end{cases}$$

which can be further expressed by

$$E_i \dot{x}(t) = A_i x(t), \quad i \in \{1, 2\}$$

where $E_1 = E_2 = I$. The purpose of this example is to verify the effectiveness of the proposed methods, so we directly use the parameters after discretization in [36].

$$A_1 = \begin{bmatrix} 0.94 & 0.10 & 0.06 \\ -0.30 & 0.95 & -0.30 \\ -0.25 & -0.06 & 0.63 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.93 & 0.08 & 0.07 \\ -0.14 & 0.66 & -0.20 \\ -0.16 & -0.04 & 0.66 \end{bmatrix}.$$

Other system matrices can be supposed as

$$B_1 = \begin{bmatrix} 0.3 \\ 0.5 \\ 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.8 \\ -0.3 \\ 0.2 \end{bmatrix}.$$

Choose $\alpha_{12} = \alpha_{21} = -0.5$, we can get K_{ai} and K_{ei} in (19) by solving (27) in Theorem 4 as follows.

$$K_{a1} = \begin{bmatrix} -4.1189 & 1.9839 & -4.7146 \\ 0.0681 & -0.1805 & -2.1966 \\ 44.0182 & 53.8449 & 10.7300 \end{bmatrix},$$

$$K_{a2} = \begin{bmatrix} 25.2387 & -8.3784 & 5.5778 \end{bmatrix},$$

$$K_{e1} = \begin{bmatrix} 25.2387 & -8.3784 & 5.5778 \end{bmatrix},$$

$$K_{e2} = \begin{bmatrix} 25.2387 & -8.3784 & 5.5778 \end{bmatrix}.$$

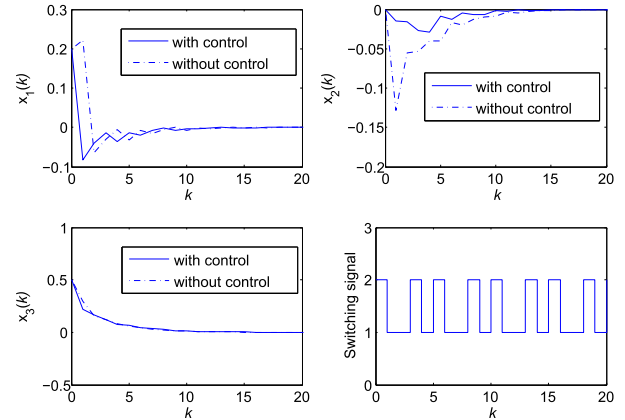


FIGURE 7. State trajectory of the boost converter based on Theorem 4.

The comparison of state trajectories for the input-free system and the closed-loop system is shown in Fig. 7. It can be seen from Fig. 7 that the transient response of the closed-loop system is much better than that of input-free system. It shows that the proposed method in this paper is also effective for the linear switched normal systems.

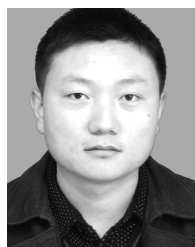
V. CONCLUSION

In this paper, the problem of stabilization is studied for switched singular systems by designing P-D state feedback controllers. Some sufficient conditions for the stabilization of the systems are given, and the gains of P-D state feedback controllers are designed synchronously. The advantages of synchronous design compared with step-by-step design are analyzed theoretically. The effectiveness of the proposed methods is verified by some simulations. The results of this work can be easily extended to switched singular systems with uncertainties and time-delays.

REFERENCES

- [1] X. Zhao, P. Shi, Y. Yin, and S. K. Nguang, "New results on stability of slowly switched systems: A multiple discontinuous Lyapunov function approach," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3502–3509, Jul. 2017.
- [2] Z. Gao, Z. Wang, and D. Wu, "Input and output strictly passive H_∞ control of discrete-time switched systems," *Int. J. Syst. Sci.*, vol. 50, no. 15, pp. 2776–2784, 2019.
- [3] J. P. Zhou, "Weighted H_∞ consensus design for stochastic multi-agent systems subject to external disturbances and ADT switching topologies," *Nonlinear Dyn.*, vol. 96, no. 2, pp. 853–868, 2019.
- [4] S. Li and H. Lin, "On l_1 stability of switched positive singular systems with time-varying delay," *J. Robust Nonlinear Control*, vol. 27, no. 16, pp. 2798–2812, 2017.
- [5] D. Liberzon and S. Trenn, "On stability of linear switched differential algebraic equations," presented at the Proc. 48th IEEE Conf. Decis. Control Held Jointly 28th Chin. Control Conf., Shanghai, China, Dec. 15-18, 2009.
- [6] J. Lin, S. Fei, Z. Gao, and Y. Shi, "Reliable dissipative control of discrete-time switched singular systems with mixed time delays and stochastic actuator failures," *IET Control Theory Appl.*, vol. 7, no. 11, pp. 1447–1462, Jul. 2013.
- [7] L. Zhou, D. W. C. Ho, and G. Zhai, "Stability analysis of switched linear singular systems," *Automatica*, vol. 49, no. 5, pp. 1481–1487, May 2013.
- [8] J. Li, R. Ma, G. M. Dimirovski, and J. Fu, "Dwell-time-based stabilization of switched linear singular systems with all unstable-mode subsystems," *J. Franklin Inst.*, vol. 354, no. 7, pp. 2712–2724, May 2017.
- [9] S. Shi, "Hybrid impulsive control for switched singular systems," *IET Control Theory Appl.*, vol. 5, no. 1, pp. 103–111, 2011.
- [10] G. Feng and J. Cao, "Stability analysis of impulsive switched singular systems," *IET Control Theory Appl.*, vol. 9, no. 6, pp. 863–870, Apr. 2015.

- [11] I. Zamani, M. Shafiee, and A. Ibeas, "On singular hybrid switched and impulsive systems," *Int. J. Robust Nonlinear Control*, vol. 28, no. 2, pp. 437–465, Jan. 2018.
- [12] I. Zamani and M. Shafiee, "On the stability issues of switched singular time-delay systems with slow switching based on average dwell-time: Switched singular time-delay systems with slow switching," *Int. J. Robust Nonlinear Control*, vol. 24, no. 4, pp. 595–624, Mar. 2014.
- [13] I. Zamani, M. Shafiee, and A. Ibeas, "Switched nonlinear singular systems with time-delay: Stability analysis," *Int. J. Robust Nonlinear Control*, vol. 25, no. 10, pp. 1497–1513, Jul. 2015.
- [14] Z. Gao, K. Yang, Y. Shen, and Z. Ji, "Input-to-state stability analysis for a class of nonlinear switched descriptor systems," *Int. J. Syst. Sci.*, vol. 46, no. 16, pp. 2973–2981, Dec. 2015.
- [15] K. Yang, Y. X. Shen, and Z. C. Ji, "Uniform finite-time stability for a class of continuous-time switched descriptor systems with time-varying delay," *Control Decis.*, vol. 29, no. 7, pp. 1316–1320, 2014.
- [16] Z. Gao, Z. Wang, Z. Ji, and Y. Liu, "Output strictly passive H_∞ control of discrete-time linear switched singular systems via proportional plus derivative state feedback," *Circuits, Syst., Signal Process.*, to be published, doi: 10.1007/s00034-020-01353-6.
- [17] Y. Wang, Y. Zou, Y. Liu, X. Shi, and Z. Zuo, "Average dwell time approach to finite-time stabilization of switched singular linear systems," *J. Franklin Inst.*, vol. 352, no. 7, pp. 2920–2933, Jul. 2015.
- [18] I. Zamani, M. Shafiee, and A. Ibeas, "Exponential stability of hybrid switched nonlinear singular systems with time-varying delay," *J. Franklin Inst.*, vol. 350, no. 1, pp. 171–193, Feb. 2013.
- [19] J. Lian, C. Li, and D. Liu, "Input-to-state stability for discrete-time nonlinear switched singular systems," *IET Control Theory Appl.*, vol. 11, no. 16, pp. 2893–2899, Nov. 2017.
- [20] S. Li and Z. Xiang, "Stability, l_1 -gain and l_∞ -gain analysis for discrete-time positive switched singular delayed systems," *Appl. Math. Comput.*, vol. 275, pp. 95–106, Feb. 2016.
- [21] M. Darouach and M. Chadli, "Admissibility and control of switched discrete-time singular systems," *Syst. Sci. Control Eng.*, vol. 1, no. 1, pp. 43–51, Dec. 2013.
- [22] J. Lin, S. Fei, and Z. Gao, "Stabilization of discrete-time switched singular time-delay systems under asynchronous switching," *J. Franklin Inst.*, vol. 349, no. 5, pp. 1808–1827, 2012.
- [23] S. Ma, C. Zhang, and Z. Wu, "Delay-dependent stability and control for uncertain discrete switched singular systems with time-delay," *Appl. Math. Comput.*, vol. 206, no. 1, pp. 413–424, Dec. 2008.
- [24] Z. M. Fu and S. M. Fei, "H-infinity control for a class of switched linear singular systems," *Control Theory Appl.*, vol. 25, no. 4, pp. 693–698, 2008.
- [25] Z. M. Fu and S. M. Fei, "Robust H_∞ dynamic output feedback stabilization for a class of uncertain switched singular systems," *Acta Autom. Sinica*, vol. 34, no. 4, pp. 482–487, 2008.
- [26] C. Lin, Q.-G. Wang, and T. Heng Lee, "Robust normalization and stabilization of uncertain descriptor systems with norm-bounded perturbations," *IEEE Trans. Autom. Control*, vol. 50, no. 4, pp. 515–520, Apr. 2005.
- [27] J. Li, Q. Zhang, J. Ren, and Y. Zhang, "Robust decentralised stabilisation of uncertain large-scale interconnected nonlinear descriptor systems via proportional plus derivative feedback," *Int. J. Syst. Sci.*, vol. 48, no. 14, pp. 2997–3006, Oct. 2017.
- [28] R. Bru, C. Coll, and N. Thome, "Balancing singular discrete-time systems," *Appl. Math. Lett.*, vol. 13, no. 8, pp. 7–13, Nov. 2000.
- [29] A. Bunse-Gerstner, V. Mehrmann, and N. K. Nichols, "Regularization of descriptor systems by derivative and proportional state feedback," *SIAM J. Matrix Anal. Appl.*, vol. 13, no. 1, pp. 46–67, Jan. 1992.
- [30] J. Ren and Q. Zhang, "Non-fragile PD state control for a class of uncertain descriptor systems," *Appl. Math. Comput.*, vol. 218, no. 17, pp. 8806–8815, May 2012.
- [31] A.-G. Wu, S.-M. Zhao, and G.-R. Duan, "Impulsive-mode controllability in descriptor linear systems," *IET Control Theory Appl.*, vol. 1, no. 3, pp. 558–563, May 2007.
- [32] D. W. Pearson, M. J. Chapman, and D. N. Shields, "Partial singular-value assignment in the design of robust observers for discrete-time descriptor systems," *IMA J. Math. Control Inf.*, vol. 5, no. 3, pp. 203–213, 1988.
- [33] Y.-C. Kuo, W.-W. Lin, and S.-F. Xu, "Regularization of linear discrete-time periodic descriptor systems by derivative and proportional state feedback," *SIAM J. Matrix Anal. Appl.*, vol. 25, no. 4, pp. 1046–1073, Jan. 2004.
- [34] W. Qi, G. Zong, and H. R. Karimi, "Sliding mode control for nonlinear stochastic singular semi-Markov jump systems," *IEEE Trans. Autom. Control*, vol. 65, no. 1, pp. 361–368, Jan. 2020.
- [35] L. Yu, *Robust Control: Linear Matrix Inequality Method*. Beijing, China: Tsinghua Univ. Press, 2002, pp. 8–9.
- [36] J. Wen, S. K. Nguang, P. Shi, and X. Zhao, "Stability and H_∞ control of discrete-time switched systems via one-step ahead Lyapunov function approach," *IET Control Theory Appl.*, vol. 12, no. 8, pp. 1141–1147, 2018.



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