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Integration of Operational and Marketing Tools With Time Delays: Is Cooperation Possible?

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ABSTRACT We confine our interest to a supply chain consisting of a manufacturer offering the quality improving effort and a retailer performing the promotional activity. The quality effort positively contributes to brand goodwill while the promotion negatively affects brand goodwill. However, the influences of both strategies, positive or negative, on brand goodwill are not instantaneous but delayed. Hence, to explore the impact of time delay and figure out the feasibility of a cooperative program in this situation, we formulate a differential game model with dual time delays. By deriving and comparing the equilibrium strategies, profits under non-cooperative, two-subsidy and centralized patterns, we find that the cooperative program is implementable if the marginal profit of manufacturer and retailer satisfy particular relationships and can achieve Pareto-improving for both parties but cannot fully coordinate the supply chain. Consequently, revenue sharing and two-subsidy policy are introduced to coordinate the decentralized supply chain.

INDEX TERMS Quality improvement, promotion, delay, revenue sharing, cooperative program.

I. INTRODUCTION

Essentially, the formation of the supply chain originates from the division of labor, the fountain of the greater part of the skill, dexterity, and judgment according to Smith [1]. As a result, nowadays, members in the supply chain system specialize in their expertise to earn the maximized payoff, like manufacturers focusing on manufacturing and retailers concentrating on retailing. This fact naturally entails the integration of marketing and operational tools in the managerial practice the study of which has long been a primary domain in the management of dynamic supply chain. Numerous literatures contribute to this area. For example, literature [2] concerns sales, production, pricing and inventory, literature [3] focuses quality and advertising, literature [4] refers to quality, pricing and advertising and literature [5] applies to inventory control and pricing. In addition, the literature [4] argues that making decisions across functions enhances players' understanding of how to optimize their payoffs, choose the best strategies and gain a competitive advantage. Motivated by this prevalence, we intend to offer knowledge and insights necessary for managers to successfully manage the integration of multiple business functions, especially quality and promotion. We contribute to this area by putting forward a differential game model of marketing and operation management in which we characterize the manufacturer's quality improvement strategy and retailer's promotion strategy.

An important observation of quality unveiled by [6] is that certain goods or services, including medical services, new vacation resorts, unfranchised restaurant meals, repairs, and wine, are characterized by *asymmetric information*. Consumers do not know exactly the quality before purchasing. Consequently, the literature [6] distinguishes the qualities that consumers can ascertain by appraising before the purchase, denoted as search qualities, and qualities which can be assessed only after consumption, denoted as experience qualities. Hence, the studies of quality in a dynamic setting, which is triggered by [7] who calls for the need to integrate product quality in dynamic advertising models, concentrate on its influence upon the formation of brand goodwill, directly relevant to market demand. Subsequently, some contributions concerned have been made as responses [8]–[13]. Among those researches, literature [8] incorporates the insight into his dynamic model where monopolists maximize his monopoly profit from consumers being imperfectly acquainted with the product. Literature [9] bases his studies on the observation by assuming that the seller's reputation depends on both the quality-price ratio as perceived by former customers and the quantity already sold [10]. Literature [11] also considers an asymmetric situation in which the quality

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information of the product is hidden from consumers before purchasing. If brand goodwill has promised a higher quality than that indicated by experience, the brand goodwill will be reduced. If, on the other hand, the actual quality exceeds the expected level, the brand goodwill will be increased. Literature [14] determine the optimal quality decision when the demand is influenced by experience quality and advertising. In research concerning differential games, literature [15] and [3] initially incorporate the quality improvement in the management of dynamic supply chain. The literature [15] bridges the gap in the dynamic marketing literature by extending the Lanchester model to include quality strategy as a loyalty-building method of consumers to retain brand goodwill. The literature [3] presents a model in which the goodwill accumulation is not only dependent on advertising but also quality deemed as an essential factor of consideration in an organization. A series of subsequent papers extend this line of exploration [16]–[19].

In summary, the literature above hold that the formation of brand goodwill is driven by the quality perceived (reference quality) not just real quality because of the experience quality of a specific product or the unobservable property of quality. Therefore, this paper offers an alternative method to characterize the attribute of quality. Since the real quality of a product cannot be fully appreciated by consumers before consumption, the influence of quality on the evolution of brand goodwill is enacted in stages. The knowledge of quality improvement may enhance consumer's evaluation of the product but only after purchase, the consumer could form their attitude towards the product. Hence the quality influences the accumulation of brand goodwill step by step, not instantaneously, due to consumer's experience. Based on the assumption above, we use a delayed differential equation to depict the process.

Retailer's promotion also affects the evolution of brand goodwill. Extant literature has studied this issue [20]–[24]. Among those literature, [20], [21] and [22] assume that retailer's promotion has a positive effect on brand goodwill. However, some studies propose the opposite [25]–[30]. Consequently, subsequent researches include the negative impact of retailer's promotion in their differential game models, such as [23], [24] and [31]. All the papers mentioned above share one thing in common that the influence of promotion, whether it is positive or negative, on goodwill is instantaneous. However, given that the reverse effect of promotion is manifested by the lower probability of repurchasing according to [32], its consequence on goodwill might not assert itself instantly. Hence, another novel advance of this paper is to relax the assumption that the negative effect on goodwill is immediate and characterize the influence of promotion with a delayed differential equation.

In this paper, we analyze three scenarios. In the first scenario, both manufacturer and retailer determine their strategies independently and simultaneously, as the game is played à la Nash. In this case, both marketing and operational strategy are utilized so that the demand is operational and marketing driven. In another scenario, the manufacturer supports the retailer's promotion cost and retailer shoulder part of a manufacturer's quality improving cost. The literature [22] has proven that the cooperative program is feasible when the retailer's promotion damages brand goodwill, we introduce the program into this paper to see if it is also practical when manufacturer and retailers' strategies have delayed effects. This scenario is modeled as a Stackelberg game with both members being the dominators in parallel games. The last is the integrated scenario. We obtain and compare the equilibrium strategies, time trajectory of brand goodwill and profit under different scenarios, which indicate that cooperative program can stimulate the effort of both members and can achieve payoff Pareto improving, but cannot completely coordinate the supply chain. Hence, revenue sharing and two-subsidy contract are introduced to coordinate the decentralized supply chain.

The rest of the paper is organized as follows. In section II, we present the conceptual model and formulate the differential game. In section III, we obtain and compare the equilibrium strategies, time trajectory of brand goodwill and profit under different scenarios. In section VI, we introduce a mechanism to coordinate the supply chain. Section V concludes this paper.

II. MODEL FORMULATION

Consider a supply chain consisting of a manufacturer and a retailer. The manufacturer is responsible for production and the retailer purchases the products manufactured and sell them to the ultimate market. To establish and maintain the brand goodwill, closely relevant to the market demand, the manufacturer executes quality improvement strategy. To further boost sales over baseline market demand created by brand goodwill, the retailer performs the promotion strategy. In this sense, the manufacturer utilizes an operational tool to optimize his pay-off and retailer resorts to a marketing tool.

Former literature about the study of dynamic quality strategy commonly assume that the impact of quality improvement on brand goodwill is instantaneous or the quality of certain products is purely observable. Those assumptions are plausible because there are sorts of products the quality of which can be explicitly judged by aspect, established brand goodwill or previous experience before authentically buying. However, the literature [6] identifies the qualities of a brand that consumers can ascertain by inspection before purchase as search quality, and those evaluated only after purchase as so-called experience qualities. Put differently, it takes consumers some time to be fully aware of the quality of certain products. Or the quality of the product, in its nature, has two facets, superficial and interior. Hence, it is reasonable to presume that the assumption that the influence of quality improving upon brand goodwill is immediate neglects the carry-over effect of quality improvement which is deferred by experiencing. Consequently, our model relaxes those previous assumptions by simultaneously characterizing the instantaneous and delayed impacts of quality on the brand goodwill.

The instantaneous impact may also stem from the knowledge of quality improvement.

Another hypothesis of this paper is that the retailer's continuous promotion tends to exert potential damage on brand goodwill, despite its contribution to current market demand for the product. Although many studies on marketing assume the promotion activities to be beneficial to the accumulation of brand goodwill, many researchers propose the opposite. Literature [32] suggests that execution of promotion can give rise to lowered brand evolution manifested by the lower probability of repurchasing. One reason, raised by [22], behind the phenomenon is that consumers may associate promotion activities with inferior quality and come to believe that frequent promotions are used as a "cover-up" for insufficient quality, holding the view that high-quality products need little or no promotion. Given the fact, presented by [32], that promotion may deter consum ers who have already purchased from repurchasing, we have reason to hypothesize that the negative influence of promotion on goodwill is also deferred.

The dynamics of the brand goodwill, henceforth denoted by $G(t)$, is described by the following differential equation

$$\dot{G}(t)=\theta_1 q(t)+\theta_2 q(t-d_1)-\theta_3 p(t-d_2)-\delta G(t), \quad G(0)=G_0 \tag{2.1}$$

where $G_0 > 0$ is the initial level of goodwill. $q(t) > 0$ is the manufacturer's quality improving effort at the time t and positive parameter θ_1 measures the instantaneous effect of the quality improvement. $\theta_2 q(t-d_1)$ characterizes the carry-over effect of quality improvement. The first two terms in (2.1) conclude the inherent attribute of quality effort and its overall impacts on goodwill. $\theta_3 p(t-d_2)$ stands for the lagged negative effect of the retailer's promotion effort. $d_1 > 0$ and $d_2 > 0$ are the delayed time of quality and promotion respectively. $\delta > 0$ is decay rate of the brand goodwill. We can find from (2.1) that the change rate of brand goodwill at time t is negatively affected by retailer's activity at $t-d_2$. Hence, in order to offset the adverse effect undesirable to the manufacturer, he should increase his quality effort.

Enlightened by [33]–[36], the demand rate $D(t)$ is

$$D(t)=\alpha G(t)+\beta p(t) \tag{2.2}$$

where α and β are both positive constants, representing the contribution of per unit of brand goodwill and promotion to demand. We can learn from (2.2) that the market demand is directly boosted by the retailer's promotion and indirectly influenced by the manufacturer's quality effort through its commitment to brand goodwill. The demand specification in (2.2) also implies the retailer's trade-off: promoting strongly and continually the brand will result in the rise of current sales, but, due to (2.1), damages the brand goodwill as well.

Since the costs of manufacturer's quality improving and retailer's promotion strategies are characteristic of convex increasing, we take quadratic form for simplicity, i.e.,

$$C(q)=\frac{1}{2}k_M q^2, \quad C(p)=\frac{1}{2}k_R p^2$$

where k_M and k_R are positive constants.

Let $\rho_M > 0$ and $\rho_R > 0$ denote the marginal profits of the manufacturer and the retailer. We assume that the marginal profits of both members are constant for the reason that in the perfect competitive market the companies are only price taker instead of price settler, making the marginal profit constant. Previous literature, such as [31], [37] and [38], also take this assumption. Hence, during infinite horizon, the objective functional of the manufacturer is

$$\max_{q(\cdot)} J_M=\int_0^{+\infty} e^{-rt}\left\{\rho_M D(t)-\frac{1}{2}k_M q^2(t)\right\}\mathrm{d}t \tag{2.3}$$

and that of the retailer is

$$\max_{p(\cdot)} J_R=\int_0^{+\infty} e^{-rt}\left\{\rho_R D(t)-\frac{1}{2}k_R p^2(t)\right\}\mathrm{d}t \tag{2.4}$$

where $r > 0$ denotes discount rate.

When the manufacturer and retailer are integrated as a whole firm, the objective functional of the whole firm is

$$\max_{p(\cdot),q(\cdot)} J_C=\int_0^{+\infty} e^{-rt}\left\{(\rho_R+\rho_M)D(t)-\frac{1}{2}k_R p^2(t)-\frac{1}{2}k_M q^2(t)\right\}\mathrm{d}t \tag{2.5}$$

According to the modelling above, the differential game of the manufacturer and the retailer is formulated. In this paper, we assume that both supply chain members conduct open-loop equilibrium strategies, indicating that both of the manufacturer and the retailer pre-commit to their decisions throughout the game. Given the deterministic situations in our paper, open-loop assumption may be reasonable [39]. From now on, we omit time argument for clarity where no confusion can arise.

III. EQUILIBRIUM AND ANALYSIS

A. NON-COOPERATIVE PATTERN

This scenario is characterized by Nash game, with both supply chain members independently and simultaneously determining their strategies. Since neither the manufacturer nor the retailer will bear the cost of their counterparts, the sequence of decision events will not exert any influence on their ultimate decisions, which indicates that the outcome under this scenario is as same as those under Stackelberg game. We use superscript "N" to stand for this pattern.

In order to derive the equilibrium outcomes, we should first introduce the following lemma.

Lemma: The necessary conditions for the following optimal control problem with delayed time

$$\max_{u(t)} \int_0^{\infty} F(t,x(t),u(t))\,\mathrm{d}t \tag{1}$$

$$\text{subject to } \dot{x}(t)=g(t,x(t),u(t),u(t-\tau))$$

$$u(t)=u_0, \quad \text{for } t\in[-\tau,0]$$

$$x(\infty)\text{ free} \tag{2}$$

are

$$\partial H/\partial u_t + \partial H/\partial u_{t-\tau}|_{t+\tau} = 0, \quad 0 \le t < \infty$$
$$\dot{\lambda} = -\partial H/\partial x_t, \quad 0 \le t < \infty$$

where $H(t, x_t, u_t, u_{t-\tau}, \lambda_t) = F(t, x_t, u_t) + \lambda_t g(t, x_t, u_t, u_{t-\tau})$.

In this problem, F and g are continuously differentiable functions of three arguments. The control variable $u(t)$ must be a continuous function of time. The state variable $x(t)$ moves over time according to the differential equation (2) which governs its dynamics.

With the lemma above, we can have proposition 3.1-3.3. Detailed proof the lemma sees appendices.

Proposition 3.1: The equilibrium quality improvement strategy of the manufacturer is

$$q_M^N = \rho_M Q \tag{3.1}$$

and the equilibrium promotion strategy of the retailer is

$$p_R^N = \rho_R P \tag{3.2}$$

The profit of the manufacturer is ($d_1 \le d_2$ *or* $d_1 > d_2$)

$$J_M^N = \frac{\alpha \rho_M G_0}{r+\delta} + \frac{k_R \rho_M \rho_R P^2}{r} + \frac{k_M \rho_M^2 Q^2}{2r} \tag{3.3}$$

The profit of the retailer is ($d_1 \le d_2$ *or* $d_1 > d_2$)

$$J_R^N = \frac{\alpha \rho_R G_0}{r+\delta} + \frac{k_R \rho_R^2 P^2}{2r} + \frac{k_M \rho_R \rho_M Q^2}{r} \tag{3.4}$$

where

$$Q = \frac{\alpha\left(\theta_1 + \theta_2 e^{-rd_1}\right)}{k_M(r+\delta)}, \quad P = \frac{\beta(r+\delta) - \theta_3 \alpha e^{-rd_2}}{k_R(r+\delta)} \tag{3.5}$$

Through the analysis of Proposition 3.1, we can have the following observations:

(i) We can discover from Proposition 3.2 that both optimal strategies are constants and are functions of current marginal cost and current marginal profit which is in line with [24]. However, the equilibrium results are also affected by the delayed time d_1 and d_2. For example, the optimal quality decision consists of two parts which are $\alpha\rho_M\theta_1/k_M(r+\delta)$ and $\alpha\rho_M\theta_2 e^{-rd_1}/k_M(r+\delta)$, respectively. The first term derives from its direct and instantaneous effect on the brand goodwill which is closely associated with demand. The second term, with a discount factor e^{-rd_1}, is due to its postponed influence to the goodwill through dynamic equation (2.1).

(ii) The retailer's optimal strategy also consists of two parts featuring a positive and a negative term. The first term stems from the contribution of the local effort to the immediate elevation of demand, since, according to the assumption, the retailer's effort can be helpful to the expansion of market demand through the term $\beta p(t)$ in the demand function. The negativity of the second term is due to its potential harm to the goodwill in the future through the term $-\theta_3 p(t-d_2)$ in the dynamic equation and the discount factor e^{-rd_2} implies that this harm is also not instantaneous. If the retailer's activity will not exert any undesirable influence on goodwill, which means that the parameter $\theta_3 = 0$, the best strategy will be $\rho_R\beta/k_R$. Hence, reasonable retailer should lower his spending on the promotion to achieve the optimum profit desired when fully aware of the dual impact of his practice. Moreover, to ensure the non-negativity of the retailer's strategy, we should impose the following condition

$$\beta(\delta + r) - \theta_3 \alpha e^{-rd_2} > 0$$

Since the manufacturer's quality improving effort does not directly boost the demand, the first term in (3.2) is excluded in (3.1).

(iii) Although the delayed time of quality decision may differ from the delayed time of local promotion, profits of the manufacturer and the retailer under different scenarios are identical. We can also find, from (3.3), that the profit of the manufacturer consists of three parts, the first term of which is $\alpha\rho_M G_0/(r+\delta)$ indicating that this part of profit is obtained from the initial level of goodwill. The second part of profit can be divided into $\beta\rho_M\rho_R P/r$ and $-\alpha\theta_3\rho_M\rho_R e^{-rd_2}P/[r(r+\delta)]$ according to (3.5). The former term is the profit derived from the direct contribution of retailer's effort to market demand and the latter is the loss in profit due to its potential negative effect of promotion upon goodwill. The third part actually encompasses the cost of the manufacturer $-k_M\rho_M^2 Q^2/2r$ and the profit get from the accumulation of goodwill caused by his quality effort $k_M\rho_M^2 Q^2/r$. All the three parts constitute the whole profit gained by the manufacturer and the formation of retailer's profit is very similar.

By substituting (3.1) and (3.2) into dynamic equation (2.1), we can acquire the time trajectory of brand goodwill as when $d_1 \le d_2$

$$G^N(t) = \begin{cases} \left(G_0 - \dfrac{\rho_M\theta_1 Q}{\delta}\right)e^{-\delta t} + \dfrac{\rho_M\theta_1 Q}{\delta}, \quad t < d_1 \\ \left[G_0 - \dfrac{\rho_M Q\left(\theta_1 + \theta_2 e^{\delta d_1}\right)}{\delta}\right]e^{-\delta t} + \dfrac{\rho_M Q(\theta_1+\theta_2)}{\delta}, \\ d_1 \le t \le d_2 \\ \left[G_0 - \dfrac{\rho_M Q(\theta_1+\theta_2)e^{\delta d_1} - \rho_R\theta_3 P e^{\delta d_2}}{\delta}\right]e^{-\delta t} \\ + G_\infty^N(t), \quad t \ge d_2 \end{cases}$$

when $d_1 > d_2$

$$G^N(t) = \begin{cases} \left(G_0 - \dfrac{\rho_M\theta_1 Q}{\delta}\right)e^{-\delta t} + \dfrac{\rho_M\theta_1 Q}{\delta}, \quad t < d_2 \\ \left(G_0 - \dfrac{\rho_M\theta_1 Q - \rho_R\theta_3 P e^{\delta d_2}}{\delta}\right)e^{-\delta t} + \dfrac{\rho_M\theta_1 Q - \rho_R\theta_3 P}{\delta}, \\ d_2 \le t \le d_1 \\ \left[G_0 - \dfrac{\rho_M Q\left(\theta_1 + \theta_2 e^{\delta d_1}\right) - \rho_R\theta_3 P e^{\delta d_2}}{\delta}\right]e^{-\delta t} + G_\infty^N(t), \\ t \ge d_1 \end{cases}$$

where $G_\infty^N(t) = \frac{\theta_1\rho_M Q + \theta_2\rho_M Q - \theta_3\rho_R P}{\delta}$.

Obviously, the steady-state of the brand goodwill turns out to be $(\theta_1\rho_M Q+\theta_2\rho_M Q-\theta_3\rho_R P)/\delta$ irrespective of the relation between d_1 and d_2. To ensure the non-negativity of steady-state, we should impose the following condition

$$\theta_1 Q+\theta_2 Q>\theta_3 P$$

Actually, the inequality above states that the global contribution of quality effort, whether it is deferred or not, should outweigh the impact of promotion on the goodwill.

From Proposition 3.1, it is natural to have Corollary 3.1

Corollary 3.1: The impacts of key parameters on the optimal quality and promotion strategies can be given by

1) $\frac{\partial q_M^N}{\partial \alpha}>0$, $\frac{\partial q_M^N}{\partial \theta_1}>0$, $\frac{\partial q_M^N}{\partial \theta_2}>0$, $\frac{\partial q_M^N}{\partial d_1}<0$, $\frac{\partial q_M^N}{\partial r}<0$, $\frac{\partial q_M^N}{\partial \delta}<0$

2) $\frac{\partial p_R^N}{\partial \alpha}<0$, $\frac{\partial p_R^N}{\partial \beta}>0$, $\frac{\partial p_R^N}{\partial \theta_3}<0$, $\frac{\partial p_R^N}{\partial d_2}>0$, $\frac{\partial p_R^N}{\partial r}>0$, $\frac{\partial p_R^N}{\partial \delta}>0$

Through the analysis of Corollary 3.1, we will have the following results:

(i) In our model, the parameter α measures the efficacy of goodwill on the market demand and the increase of the parameter implies the enhancement of effectiveness which leads to higher investment in quality but lower investment in promotion due to its negative effect on goodwill. Although retailer's promotion effort can hurt goodwill, it can directly boost the demand leading to the fact that retailer's investment increases with the increase of β.

(ii) The manufacturer should increase his quality effort when either of θ_1 and θ_2 increases. The two parameters assess the effect of quality on the brand goodwill which directly contributes to the demand and therefore the increase of them actually indicate the increase of effectiveness of quality effort on demand. While the retailer should lower the promotion effort to alleviate its negative effect on goodwill when θ_3 increases.

(iii) The manufacturer should reduce the quality effort when δ increases because the parameter δ is the decay rate the increase of which hinders the accumulation of goodwill and actually undermines the efficiency of quality effort in its establishment of goodwill closely associated with future profitability. As a contrast, the retailer's promotion is detrimental to the goodwill and the increase of δ, actually, counterbalances the negative effect incurred by promotion on the goodwill. Therefore, the retailer increases its promotional expenditure with the increase of δ. Meanwhile the discount rate r, which assesses the time preference of players, also affects the decision-making of both members. If $r=0$, it means that both members will not discount the future profit and the future profit is equivalent to the current. As for the manufacturer whose quality effort is aimed at boosting goodwill to acquire profit, the rise of discount rate will make the manufacturer more impatient and put more emphasis on short-term profit instead of the long-term. Hence, the manufacturer will decrease the quality effort with the increase of r. The retailer will increase the promotion effort as the discount rate increases.

(iv) The optimal strategies are also influenced by delayed time d_1 and d_2. According to the analysis of Proposition 3.1, the optimal quality strategy of the manufacturer consists of two parts. The first term of stems from its immediate contribution to goodwill and the second term with e^{-rd_1} is due to its delayed contribution. Hence the manufacturer should decrease his spending on quality effort when d_1 becomes large. However, retailer should increase the promotion effort when d_2 increases because the promotion has negative effect on the goodwill and the longer d_2 is, the less impacts the promotion effort will produce.

B. TWO-SUBSIDY POLICY

The literature [35] raise the two-subsidy mechanism to achieve Pareto improvement in the non-cooperative supply chain. In this section, we are about to introduce this policy into the supply chain with dual time delays. Since both members share part of their counterpart's cost, the game structure is Stackelberg game the sequence of which is as follows: the manufacturer and the retailer firstly announce their subsidy policy and the retailer and the manufacturer determine their promotion and quality decisions separately based on the other's participation rate. Actually, these are two parallel Stackelberg games with the manufacturer and the retailer being the dominator in their separate games. We use superscript "S" to represent this pattern.

Proposition 3.2 exhibits the equilibrium strategies in this decision pattern.

Proposition 3.2: The equilibrium quality improving strategy of the manufacturer is

$$q_M^S=\frac{\alpha\rho_M Q}{1-\phi_q} \tag{3.6}$$

the retailer's local promotion strategy is

$$p_R^S=\frac{\rho_R P}{1-\phi_p} \tag{3.7}$$

The manufacturer and the retailer's participation rates are

$$\phi_p=\frac{2\rho_M-\rho_R}{2\rho_M+\rho_R},\quad \phi_q=\frac{2\rho_R-\rho_M}{2\rho_R+\rho_M} \tag{3.8}$$

The profit of the manufacturer is ($d_1\le d_2$ *or* $d_1>d_2$)

$$J_M^S=\frac{\alpha\rho_M G_0}{r+\delta}+\frac{k_M\rho_M\left(2\rho_R+\rho_M\right)Q^2}{4r}+\frac{k_R\left(2\rho_M+\rho_R\right)^2P^2}{8r}$$

The profit of the retailer is ($d_1\le d_2$ *or* $d_1>d_2$)

$$J_R^S=\frac{\alpha\rho_R G_0}{r+\delta}+\frac{k_R\rho_R\left(2\rho_M+\rho_R\right)P^2}{4r}+\frac{k_M\left(2\rho_R+\rho_M\right)^2Q^2}{8r}$$

Through the analysis of Proposition 3.2, it is natural to have the following observations:

(i) The values of delayed time d_1 and d_2 do not affect both members' participation rates and the subsidy policies are identical to those with no time delays.

(ii) The participation rates ϕ_q and ϕ_p are dependent on the marginal profits of both members, implying that the determination of subsidy policy is based on the full knowledge of profitability in the supply chain system. Differentiating ϕ_q and ϕ_p with regard to ρ_M and ρ_R, we can get the following results

$$\frac{\partial \phi_p}{\partial \rho_M} = \frac{4\rho_R}{(2\rho_M+\rho_R)^2} > 0, \quad \frac{\partial \phi_p}{\partial \rho_R} = \frac{-4\rho_M}{(2\rho_M+\rho_R)^2} < 0 \tag{3.9}$$

$$\frac{\partial \phi_q}{\partial \rho_M} = \frac{-4\rho_R}{(2\rho_R+\rho_M)^2} < 0, \quad \frac{\partial \phi_q}{\partial \rho_R} = \frac{4\rho_M}{(2\rho_R+\rho_M)^2} > 0 \tag{3.10}$$

The results above imply that the increase of one's own marginal profit can enhance the willingness to bear the cost while the increase of the other's marginal profit can attenuate this willingness.

(iii) If the participation rates are reduced to zero, the results will be as same as those in proposition 3.1, indicating that the Nash equilibrium strategies can be deemed as a special case of a two-subsidy policy. To guarantee the feasibility of the mechanism, we should impose the following conditions

$$2\rho_M - \rho_R > 0, \quad 2\rho_R - \rho_M > 0 \tag{3.11}$$

The participation rates $0 < \phi_i < 1, i \in \{p, q\}$ if the conditions above hold. Differentiating both supply chain member's optimal strategies with regard to ϕ_q and ϕ_p respectively, we can obtain the following results

$$\frac{\partial q_M^S}{\partial \phi_q} = \frac{\alpha \rho_M Q}{(1-\phi_q)^2} > 0 \tag{3.12}$$

$$\frac{\partial p_R^S}{\partial \phi_p} = \frac{\rho_R P}{(1-\phi_p)^2} > 0 \tag{3.13}$$

The results above indicate that the subsidy policy can stimulate investment of both members and the more the participation rates are, the more investment will be channeled to quality and promotion.

By substituting (3.6) and (3.7) into the dynamic equation (2.1), we obtain the time trajectory of the brand goodwill as when $d_1 \le d_2$

$$G^S(t) = \begin{cases} \left[G_0 - \dfrac{\rho_M \theta_1 Q}{\delta(1-\phi_q)}\right] e^{-\delta t} + \dfrac{\rho_M \theta_1 Q}{\delta(1-\phi_q)}, & t < d_1 \\ \left[G_0 - \dfrac{\rho_M Q\left(\theta_1 + \theta_2 e^{\delta d_1}\right)}{\delta(1-\phi_q)}\right] e^{-\delta t} + \dfrac{\rho_M Q(\theta_1+\theta_2)}{\delta(1-\phi_q)}, & \\ \quad d_1 \le t \le d_2 & \\ \left[G_0 - \dfrac{\rho_M Q\left(\theta_1+\theta_2 e^{\delta d_1}\right)}{\delta(1-\phi_q)} + \dfrac{\rho_R \theta_3 P e^{\delta d_2}}{\delta(1-\phi_p)}\right] e^{-\delta t} & \\ \quad + G_\infty^S(t), \quad t \ge d_2 & \end{cases}$$

when $d_1 > d_2$

$$G^S(t) = \begin{cases} \left[G_0 - \dfrac{\rho_M \theta_1 Q}{\delta(1-\phi_q)}\right] e^{-\delta t} + \dfrac{\rho_M \theta_1 Q}{\delta(1-\phi_q)}, & t < d_2 \\ \left(G_0 - \dfrac{\rho_M \theta_1 Q}{\delta(1-\phi_q)} + \dfrac{\rho_R \theta_3 P e^{\delta d_2}}{\delta(1-\phi_p)}\right) e^{-\delta t} + \dfrac{\rho_M \theta_1 Q}{\delta(1-\phi_q)} & \\ \quad - \dfrac{\rho_R \theta_3 P}{\delta(1-\phi_p)}, \quad d_2 \le t \le d_1 & \\ \left[G_0 - \dfrac{\rho_M Q\left(\theta_1+\theta_2 e^{\delta d_1}\right)}{\delta(1-\phi_q)} + \dfrac{\rho_R \theta_3 P e^{\delta d_2}}{\delta(1-\phi_p)}\right] e^{-\delta t} & \\ \quad + G_\infty^S(t), \quad t \ge d_1 & \end{cases}$$

where $G_\infty^S(t) = \frac{\rho_M Q(\theta_1+\theta_2)}{\delta(1-\phi_q)} - \frac{\rho_R \theta_3 P}{\delta(1-\phi_p)}$.

To ensure the non-negativity of steady-state, we should impose the following condition

$$\frac{\theta_1 Q + \theta_2 Q}{1-\phi_q} > \frac{\theta_3 P}{1-\phi_p}$$

From Proposition 3.2, it is natural to have Corollary 3.2.

Corollary 3.2: The equilibrium subsidized quality strategy increases in retailer's marginal profit and the equilibrium subsidized promotion strategy increases in manufacturer's marginal profit.

From Corollary 3.2, we find that the retailer's marginal profit also affects manufacturer's quality decision and the manufacturer's marginal profit also exerts influence on retailer's promotion decision. Substituting the participation rates ϕ_q and ϕ_p into (3.6) and (3.7), we can obtain

$$q_M^S = \frac{\alpha(2\rho_R + \rho_M) Q}{2} \tag{3.14}$$

$$p_R^S = \frac{(2\rho_M + \rho_R) P}{2} \tag{3.15}$$

Differentiating (3.14) and (3.15) with regard to ρ_R and ρ_M, we can get

$$\frac{\partial q_M^S}{\partial \rho_R} = \alpha Q, \quad \frac{\partial p_R^S}{\partial \rho_M} = 2P \tag{3.16}$$

The results presented above is different from those in Corollary 3.1 in which the retailer's marginal profit has no influence on quality improving strategy and neither does the manufacturer's marginal profit. However, Corollary 3.2 shows different results which is due to the two-subsidy mechanism conducted by both the manufacturer and the retailer. According to the analysis in Proposition 3.2, the increase of manufacturer's marginal profit can enhance his willingness to shoulder more promotion cost which exactly gives rise to the increase of retailer's promotion investment. Likewise, when the retailer's marginal profit increases, he will also heighten his participation rate.

Corollary 3.3: The impacts of key parameters on the optimal profits can be given by

1) $\frac{\partial J_M^{N/S}}{\partial G_0} > 0$, $\frac{\partial J_M^{N/S}}{\partial \rho_M} > 0$, $\frac{\partial J_M^{N/S}}{\partial \rho_R} > 0$, $\frac{\partial J_M^{N/S}}{\partial d_1} < 0$

2) $\frac{\partial J_R^{N/S}}{\partial G_0} > 0$, $\frac{\partial J_R^{N/S}}{\partial \rho_R} > 0$, $\frac{\partial J_R^{N/S}}{\partial \rho_M} < 0$, $\frac{\partial J_R^{N/S}}{\partial d_2} > 0$

We know, from the analysis of Proposition 3.1, that the manufacturer and the retailers' profits derive from three sources: the initial level of goodwill, the quality investment of the manufacturer and the promotion effort of the retailer. Hence, the profits will increase with higher initial level of goodwill. Meanwhile, the increase of the marginal profits can also boost profits of both parties due to the fact that the increase of marginal profits can not only stimulate directly the inputs of both members but also, in the two-subsidy scenario, can enhance the participation rates which propel investments further. Moreover, the delayed time d_1 and d_2 can also influence the profits. Since we have had the observation that the quality effort will decrease with the increase of d_1 and the promotion effort will increase with the increase of d_2, the profits varies in accordance with the variation of delayed time.

C. INTEGRATED PATTERN

In the integrated pattern, both members share the same and solitary goal of maximizing the profit of the supply chain system without caring about the distribution issue. Here, we will derive and analyze the optimal strategies of both members when they are perfectly coordinated. We use superscript "I" to represent this pattern.

Proposition 3.3 exhibits the equilibrium strategies in this decision pattern.

Proposition 3.3: The optimal quality improving and local promotion strategies in integrated scenario are

$$q^I = (\rho_R + \rho_M) Q \tag{3.17}$$

$$p^I = (\rho_R + \rho_M) P \tag{3.18}$$

The total profit of supply chain system is ($d_1 \le d_2$ *or* $d_1 > d_2$)

$$J^I = \frac{\alpha(\rho_R+\rho_M) G_0}{r+\delta} + \frac{k_M (\rho_R+\rho_M)^2 Q^2}{2r} + \frac{k_R (\rho_R+\rho_M)^2 P^2}{2r}$$

By substituting (3.17) and (3.18) into (2.1), we can obtain the time trajectory of goodwill as

when $d_1 \le d_2$

$$G^I(t) = \begin{cases} \left(G_0 - \dfrac{\rho_{MR}\theta_1 Q}{\delta}\right) e^{-\delta t} + \dfrac{\rho_{MR}\theta_1 Q}{\delta}, & 0 \le t < d_1 \\ \left[G_0 - \dfrac{\rho_{MR}\left(\theta_1 + \theta_2 e^{\delta d_1}\right) Q}{\delta}\right] e^{-\delta t} & \\ \quad + \dfrac{\rho_{MR}(\theta_1+\theta_2) Q}{\delta}, & d_1 \le t < d_2 \\ \left\{G_0 - \dfrac{\rho_{MR}\left[\left(\theta_1 + \theta_2 e^{\delta d_1}\right) Q - \theta_3 e^{\delta d_2} P\right]}{\delta}\right\} e^{-\delta t} & \\ \quad + G^I_\infty(t), & t \ge d_2 \end{cases}$$

when $d_1 > d_2$

$$G^I(t) = \begin{cases} \left(G_0 - \dfrac{\rho_{MR}\theta_1 Q}{\delta}\right) e^{-\delta t} + \dfrac{\rho_{MR}\theta_1 Q}{\delta}, & 0 \le t < d_2 \\ \left[G_0 - \dfrac{\rho_{MR}\left(\theta_1 Q - \theta_3 e^{\delta d_2} P\right)}{\delta}\right] e^{-\delta t} & \\ \quad + \frac{\rho_{MR}(\theta_1 Q - \theta_3 P)}{\delta}, \ d_2 \le t < d_1 & \\ \left\{G_0 - \frac{\rho_{MR}\left[\left(\theta_1 + \theta_2 e^{\delta d_1}\right) Q - \theta_3 e^{\delta d_2} P\right]}{\delta}\right\} e^{-\delta t} + G^I_\infty(t), & \\ \quad t \ge d_1 & \end{cases}$$

where $G^I_\infty(t) = \frac{\rho_{MR}(\theta_1 Q + \alpha\theta_2 Q - \theta_3 P)}{\delta}$, $\rho_{MR} = \rho_R + \rho_M$.

To ensure the non-negativity of steady-state value, we should impose the following condition

$$\alpha\theta_1 Q + \alpha\theta_2 Q > \theta_3 P$$

D. COMPARISONS AND ANALYSES

This section is aimed to compare the optimal strategies, steady-state goodwill, and profits of supply chain members under three scenarios including integrated, non-cooperative and the two-subsidy scenarios. The results of comparisons are presented in the following propositions

Proposition 3.4: The steady-state of brand goodwill under different scenarios satisfy

$$G^I_\infty(t) > G^N_\infty(t), \quad G^I_\infty(t) > G^S_\infty(t) \tag{4.1}$$

$$G^S_\infty(t) - G^N_\infty(t) \begin{cases} \ge 0, & \rho_M/\rho_R \le \pi \\ < 0, & \rho_M/\rho_R > \pi \end{cases} \tag{4.2}$$

where $\pi = \frac{2\theta_1 Q + 2\theta_2 Q + \theta_3 P}{\theta_1 Q + \theta_2 Q + 2\theta_3 P}$.

It can be observed from Proposition 3.4 that the steady-state of brand goodwill under integrated scenario is the biggest among those three scenarios. This conclusion is definite regardless of the value of parameters. However, the relationship between $G^N_\infty(t)$ and $G^S_\infty(t)$ is circumstantial. The evolution of brand goodwill, manifested by the dynamic equation, is pushed by two opposing forces deriving respectively from manufacturer's quality improving effort which boosts goodwill and retailer's promotional effort which impairs the goodwill. And the two-subsidy policy can incentivize the input of both members and therefore simultaneously amplify the two opposing forces. However, which force is dominant is dependent upon the relationship between the ratio of marginal profits of members and the threshold. If $\rho_M/\rho_R > \pi$, the negative force triumph over the positive and therefore the goodwill affected by the two-subsidy policy will be lower than that under non-cooperative pattern. If $\rho_M/\rho_R \le \pi$, the goodwill will be higher.

Proposition 3.5: The relationships of profits under different scenarios satisfy

$$J^I - \left(J^S_M + J^S_R\right) = \frac{k_M \rho_M^2 Q^2}{8r} + \frac{k_R \rho_R^2 P^2}{8r} > 0 \tag{4.3}$$

$$J_M^S - J_M^N = \frac{k_M \rho_M (2\rho_R - \rho_M) Q^2}{4r} + \frac{k_R (2\rho_M - \rho_R)^2 P^2}{8r} > 0 \quad (4.4)$$

$$J_R^S - J_R^N = \frac{k_R \rho_R (2\rho_M - \rho_R) P^2}{4r} + \frac{k_M (2\rho_R - \rho_M)^2 Q^2}{8r} > 0 \quad (4.5)$$

Despite the fact that the goodwill influenced by the two-subsidy policy may be lower than that under the non-cooperative scenario, profits of supply chain members obtained from cost-sharing mechanism is greater than those in the non-cooperative scenario. Because, even though the increase of promotion effort by retailer may further damage the goodwill, it also directly boosts the market demand as compensation and the results turn out to be desirable to both of them. Hence, the two-subsidy scheme is effective in enhancing profits and both members will be acceptable to the execution of the scheme. However, the total profit of supply chain system under two-subsidy scenario is still lower than that in the integrated scenario, urging us to come up with a mechanism to perfectly coordinate the supply chain which unfolds in the next section.

IV. COORDINATION MECHANISM

We can draw a conclusion from the analyses above that decentralization prevents efforts and jeopardizes pay-offs for both supply chain members, necessitating the need to devise a mechanism to coordinate the supply chain when both manufacturer and retailer make decision separately. To this end, in the section, we are going to introduce the revenue sharing and two-subsidy mechanism, ever used by [31], to coordinate the supply chain. The superscript "D" is used to represent this scenario.

Under the framework of "revenue sharing and two-subsidy mechanism", the retailer will be responsible for part of the manufacturer's quality cost with participation rate ϕ_q and the manufacturer will shoulder part of retailer's promotion cost with participation rate ϕ_p. At the same time, the manufacturer shares part of his profit to the retailer with sharing rate φ. Consequently, both the manufacturer and the retailer should set their own individual participation rates after negotiating a proper revenue sharing rate. Hence, the objective of the manufacturer in this scenario is

$$\max_{q(\cdot)} J_M = \int_0^{+\infty} e^{-rt} \left\{ (1-\varphi)\, \rho_M D(t) - \frac{1}{2}\left(1-\phi_q\right) k_M q^2(t) - \frac{1}{2}\phi_p k_R p^2(t) \right\} \mathrm{d}t$$

and that of retailer is

$$\max_{p(\cdot)} J_R = \int_0^{+\infty} e^{-rt} \left\{ (\rho_R + \varphi\rho_M)\, D(t) - \frac{1}{2}\left(1-\phi_p\right) k_R p^2(t) - \frac{1}{2}\phi_q k_M q^2(t) \right\} \mathrm{d}t$$

Proposition 4 exhibits the optimal quality improving and promotion strategies and their profits when participation and revenue sharing rates are certain.

Proposition 4.1: Under this mechanism, for any negotiated revenue sharing rate $0 < \varphi < 1$, the incentivized equilibrium quality improving and promotion strategies are

$$q^D = \frac{(1-\varphi)\, \alpha \rho_M Q}{1-\phi_q}, \quad p^D = \frac{(\rho_R + \varphi\rho_M) P}{1-\phi_p} \quad (4.6)$$

the cost-sharing rates of the manufacturer and the retailer are

$$\phi_p^D = \frac{(1-\varphi)\, \rho_M}{\rho_R + \rho_M}, \quad \phi_q^D = \frac{\rho_R + \varphi\rho_M}{\rho_R + \rho_M} \quad (4.7)$$

The profit of the manufacturer is ($d_1 \le d_2$ or $d_1 > d_2$)

$$J_M^D = \rho_M (1-\varphi) \left[\frac{\alpha G_0}{r+\delta} + \frac{k_M (\rho_R + \rho_M) Q^2}{2r} + \frac{k_R (\rho_R + \rho_M) P^2}{2r} \right]$$

The profit of the retailer is ($d_1 \le d_2$ or $d_1 > d_2$)

$$J_R^D = (\rho_R + \varphi\rho_M) \left[\frac{\alpha G_0}{r+\delta} + \frac{k_M (\rho_R + \rho_M) Q^2}{2r} + \frac{k_R (\rho_R + \rho_M) P^2}{2r} \right]$$

It can be discovered from the results above that the determination of participation rates of both members are quite dependent on the revenue sharing rate φ, which can ensure that the quality improving and promotion strategies within this mechanism can be elevated to the integrated level. Hence the total profit of decentralized supply chain is identical to that in the integrated supply chain. However, only if the manufacturer and the retailer can acquire more profits from this mechanism, will them accept this scheme, entailing the necessity to restrict the domain of φ in order to guarantee $J_M^D \ge J_M^S$ and $J_R^D \ge J_R^S$. Hence, we can have the following proposition.

Proposition 4.2: If $\rho_M/\rho_R > \sqrt{k_R (P/Q)^2 / 2k_M}$ and the revenue sharing rate φ satisfies

$$\varphi_{\min} \le \varphi \le \varphi_{\max} \quad (4.8)$$

where

$$\varphi_{\min} = \max\left\{0, \frac{L_1}{L_3}\right\}, \quad \varphi_{\max} = \frac{L_2}{L_3}$$

$$L_1 = (r+\delta)\left(k_M \rho_M^2 Q^2 - 2k_R \rho_R^2 P^2\right)$$

$$L_2 = (r+\delta)\left(2k_M \rho_M^2 Q^2 - k_R \rho_R^2 P^2\right)$$

$$L_3 = 4\rho_M \left[2r\alpha G_0 + (r+\delta)(\rho_R + \rho_M)\left(k_R P^2 + k_M Q^2\right)\right]$$

then, the decentralized system can be fully coordinated.

The upper limit $\varphi_{\max}$ derives from the condition $J_M^D \ge J_M^S$ and the lower limit $\varphi_{\min}$ derives from the condition $J_R^D \ge J_R^S$. Apparently, the inequality $L_3 > L_2 > L_1$ holds and whether the mechanism is feasible or not depend on the value of

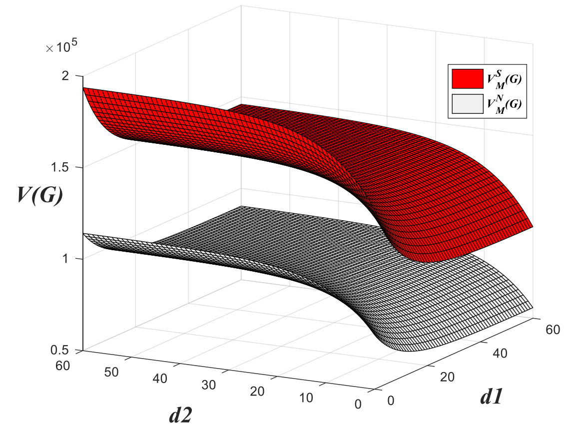

FIGURE 1. The impacts of d_1 and d_2 upon the manufacturer's profit.

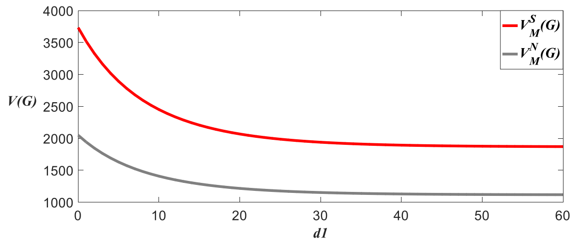

FIGURE 2. The impact of d_1 on the manufacturer's profits under the non-cooperative and the two-subsidy scenarios.

upper limit $\varphi_{\max}$. If $\rho_M/\rho_R < \sqrt{k_R (P/Q)^2/2k_M}$, which will cause the upper limit $\varphi_{\max} < 0$, the revenue sharing rate $0 \leq \varphi \leq 1$ cannot assure the manufacturer of the profit higher than J_M^S and the manufacturer will naturally refuse to conduct the scheme. If $\rho_M/\rho_R > \sqrt{k_R (P/Q)^2/2k_M}$, any value of φ within its domain is able to make both parties get profits no smaller than those they can get under decentralized pattern. In this condition, both the manufacturer and the retailer are readily to accept the contract and the supply chain can be fully coordinated. Hence, we can draw a conclusion that there is a threshold $\sqrt{k_R (P/Q)^2/2k_M}$ and the feasibility of this mechanism depend on the relationship between the ratio of members' marginal profit and the threshold.

V. NUMERICAL ANALYSIS

In this section, we use numerical example to analyze the impacts of delayed time and the two-subsidy policy on profits of supply chain member. Consider the following parameters: $G_0 = 0; r = 0.1; \delta = 0.2; \alpha = 1; \beta = 2; \theta_1 = 0.8; \theta_2 = 0.5; \theta_3 = 1; \rho_M = 4; \rho_R = 6; k_M = 1; k_R = 1; d_1 = 20; d_2 = 10.$

Fig.1 shows that the manufacturer's profit under the two-subsidy scenario is higher than that under the non-cooperative scenario which is in line with the results in proposition 3.5 and this conclusion still holds with the presence of delayed time. Meanwhile, the manufacturer's profit decrease with the increase of d_1 as is shown in Fig.2 and increase with the increase of d_2 as is illustrated in Fig.3. This is because

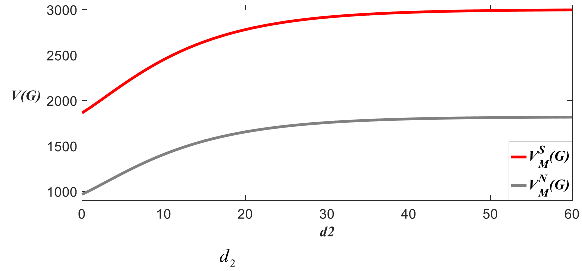

FIGURE 3. The impact of d_2 on the manufacturer's profits under the non-cooperative and the two-subsidy scenarios.

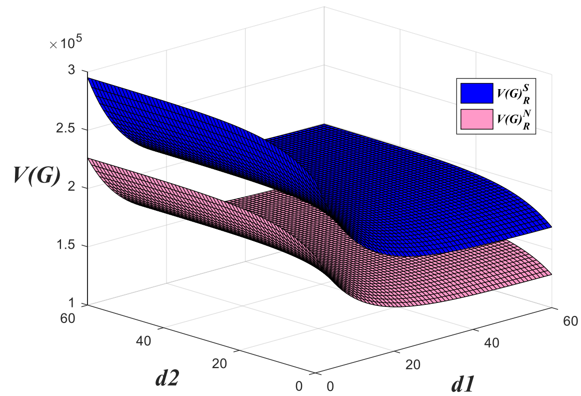

FIGURE 4. The impacts of d_1 and d_2 upon retailer's profit.

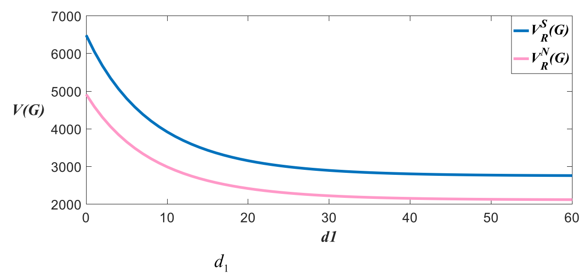

FIGURE 5. The impact of d_1 on the retailer's profits under the non-cooperative and the two-subsidy scenarios.

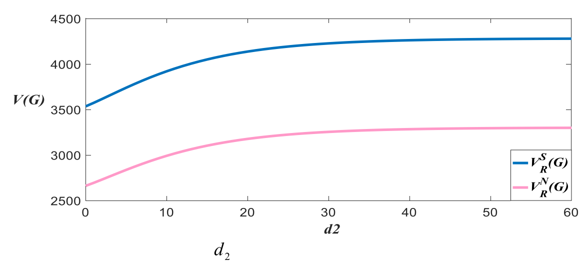

FIGURE 6. The impact of d_2 on the retailer's profits under the non-cooperative and the two-subsidy scenarios.

the increase of d_1 force the manufacturer to lower his quality effort but the increase of d_2 stimulate the retailer to enhance his promotion. It also worth noting that when d_1 and d_2 diverge to infinity, the profit will finally reach the steady-state for the reason that if d_1 and d_2 are infinite, the delayed influences of quality and promotion effort on goodwill are infinitesimal.

The change law of retailer's profit is similar to the that of the manufacturer as is shown in Fig.4-6. Hence, we can observe from the figures above that the two-subsidy policy is able to achieve the payoff-Pareto-improvement situation. But the total profit of the supply chain system under the two-subsidy scenario is still lower than that in integrated scenario which entails the need to devise a coordination mechanism.

VI. CONCLUSIONS AND MANAGERIAL INSIGHTS

A. CONCLUSIONS

We concentrate on a supply chain system consisting of a manufacturer and a retailer. The manufacturer invests on quality improvement effort which will positively affect the goodwill and the retailer undertakes the promotion effort which has negative effect on the goodwill. Meanwhile, the influence of both member's investment on the goodwill is not instantaneous but delayed due to the inherent attributes of quality and promotion effort. Based on the assumption above, we construct and analyze three different scenarios which are non-cooperative scenario, two-subsidy scenario and integrated scenario respectively. Furthermore, we also modify the two-subsidy mechanism by introducing revenue sharing to coordinate the supply chain system. Through the analyses and comparisons, we can find that 1) the delayed time of quality and promotion effort is an important factor in decision-making for both supply chain members. When d_1 increases, the manufacturer should lower his quality investment but when d_2 increase, the retailer should enhance his promotion effort. 2) the optimal strategies and profits under the integrated scenario are highest among different scenarios and the presence of delayed phenomenon do not change this conclusion. The application of the two-subsidy policy can elevate the strategies and profits of both supply chain members under non-cooperative scenario and achieve the pareto improvement but cannot completely coordinate the supply chain. 3) when certain conditions are satisfied, the two-subsidy and revenue sharing contract can fully coordinate the supply chain.

B. MANAGERIAL INSIGHTS

Based on the analyses and conclusions above, we can have the following managerial insights. Firstly, due to the presence of delayed response of the goodwill to the quality and the promotion effort, both the manufacturer and the retailer should make adjustment in managerial practice. For example, the manufacturer should not blindly increase his quality effort when the goodwill is not immediately boosted. The right move for the manufacturer is to appropriately lower his investment and be patient. The retailer should increase his effort when facing this phenomenon due to the fact that the retailer's promotion has negative effect on the goodwill and the delayed response of the goodwill to the promotion is actually an alleviation of the impact. Secondly, the two-subsidy policy alone cannot coordinate the supply chain, because cost-sharing can only stimulate investment but cannot properly solve the distribution problem. Hence, in the practice, we should not only pay attention to the stimulation problem but also to appropriately distribute the additional profits brought out by cost-sharing.

APPENDIX

Proof of Lemma:

Proof: Suppose the optimal control and the corresponding trajectory to (1) and (2) is $u^*(t)$ and $x^*(t)$. Let $h(t)$ be an arbitrary but fixed function and any feasible path of the problem is $u(t) = u^*(t) + \alpha h(t)$, where a is a scalar. The solution of $u(t)$ is denoted by $y(t, a)$ which is smooth function of a and t. Since $a = 0$ must yield $x^*(t)$, then we can get $y(t, 0) = x^*(t)$. By using an arbitrary continuously differentiable function $\lambda(t)$ and equation (2), we can obtain

$$\int_0^\infty F(t, x(t), u(t))\,\mathrm{d}t = \int_0^\infty F[t, x(t), u(t)] + \lambda(t)\, g[t, x(t), u(t), u(t-\tau)] - \lambda(t)\dot{x}(t)\,\mathrm{d}t \tag{A.1}$$

Integrate the last term on the right of (A.1) by parts

$$\int_0^\infty \lambda(t)\dot{x}(t)\mathrm{d}t = \lambda(\infty)x(\infty) - \lambda(0)x(0) - \int_0^\infty \dot{\lambda}(t)x(t)\mathrm{d}t \tag{A.2}$$

Substituting from (A.2) into (A.1) gives

$$\begin{aligned}\int_0^\infty F(t, x, u)\mathrm{d}t &= \int_0^\infty F[t, x(t), u(t)] + \lambda(t)g[t, x(t), u(t), u(t-\tau)] \\ &\quad + \dot{\lambda}(t)x(t)\mathrm{d}t - \lambda(\infty)x(\infty) + \lambda(0)x(0)\end{aligned} \tag{A.3}$$

Equation (A.3) holds for any feasible control path $u(t)$ and any differentiable $\lambda(t)$. If we calculate (A.3) with control variable $u(t)$ and the corresponding state variable $y(t, a)$, the value of (A.3) turns out to be a function of the parameter a. Since $u^*(t)$ and $x^*(t)$ are fixed, we can have

$$\begin{aligned}J(a) = \lambda(0)y(0, a) - \lambda(\infty)y(\infty, a) + \int_0^\infty F[t, y(t, a), u^*(t) \\ + \alpha h(t)] + \lambda(t)g[t, y(t, a), u^*(t) + \alpha h(t), u^*(t-\tau) \\ + \alpha h(t-\tau)] + \dot{\lambda}(t)y(t, a)\mathrm{d}t\end{aligned} \tag{A.4}$$

Hence the first-order condition of (A.4) emerges as

$$\begin{aligned}\frac{\mathrm{d}J(a)}{\mathrm{d}a} &= \int_0^\infty \left\{ \begin{array}{l} F_{x(t)}[t, y(t, a), u^*(t) + \alpha h(t)]y_a(t, a)+ \\ F_{u(t)}[t, y(t, a), u^*(t) + \alpha h(t)]h(t) \end{array} \right\} \mathrm{d}t \\ &\quad + \int_0^\infty \lambda(t) \left\{ \begin{array}{l} g_{x(t)}[t, y(t, a), u^*(t) + \alpha h(t), u^*(t-\tau) \\ + \alpha h(t-\tau)]y_a(t, a) + g_{u(t)}[t, y(t, a), u^*(t) \\ + \alpha h(t), u^*(t-\tau) + \alpha h(t-\tau)]h(t) \\ + g_{u(t-\tau)}[t, y(t, a), u^*(t) + \alpha h(t), u^*(t-\tau) \\ + \alpha h(t-\tau)]h(t-\tau) \end{array} \right\} \mathrm{d}t\end{aligned}$$

$$+\int_0^{\infty}\dot{\lambda}(t)y_a(t,a)\mathrm{d}t-\lambda(\infty)y_a(\infty,a)+\lambda(0)y_a(0,a) \tag{A.5}$$

By setting $\xi = t-\tau$, so that $\xi+\tau = t$, one has

$$\begin{aligned}\int_0^{\infty}&\lambda(t)\{g_{u(t-\tau)}[t,y(t,a),u^*(t)\\&+\alpha h(t),u^*(t-\tau)+\alpha h(t-\tau)]h(t-\tau)\}\mathrm{d}t\\=\int_{-\tau}^{\infty}&\lambda(\xi+\tau)g_{u(t-\tau)}[\xi+\tau,y(\xi+\tau,a),u^*(\xi+\tau)\\&+\alpha h(\xi+\tau),u^*(\xi)+\alpha h(\xi)]\times h(\xi)\mathrm{d}\xi\end{aligned} \tag{A.6}$$

Since $u(t)$ and $x(t)$ are fixed before 0, change of a exerts no influence on $x(t)$ which means that $h(t) = 0$ if $t\in[-\tau,0]$. Hence, we can increase the lower limit of the integral in (A.6) by τ. Substituting (A.5) by (A.6) with the change in lower limit gives

$$\begin{aligned}\frac{\mathrm{d}J(a)}{\mathrm{d}a}&=\int_0^{\infty}\left\{\begin{array}{l}F_{x(t)}[t,y(t,a),u^*(t)+\alpha h(t)]y_a(t,a)+\\F_{u(t)}[t,y(t,a),u^*(t)+\alpha h(t)]h(t)\end{array}\right\}\mathrm{d}t\\&+\int_0^{\infty}\lambda(t)\left\{\begin{array}{l}g_{x(t)}[t,y(t,a),u^*(t)+\alpha h(t),\\u^*(t-\tau)+\alpha h(t-\tau)]y_a(t,a)\\+g_{u(t)}[t,y(t,a),u^*(t)+\alpha h(t),\\u^*(t-\tau)+\alpha h(t-\tau)]h(t)\end{array}\right\}\mathrm{d}t\\&+\int_0^{\infty}\lambda(t+\tau)g_{u(t-\tau)}[t+\tau,y(t+\tau,a),\\&u^*(t+\tau)+\alpha h(t+\tau),u^*(t)+\alpha h(t)]h(t)\mathrm{d}t\\&+\int_0^{\infty}\dot{\lambda}(t)y_a(t,a)\mathrm{d}t-\lambda(\infty)y_a(\infty,a)+\lambda(0)y_a(0,a)\end{aligned} \tag{A.7}$$

Since $u^*(t)$ is the optimal control by the hypothesis, the function $J(a)$ must achieve its maximum at $a=0$ which means $J'(0)=0$. Hence, considering boundary conditions, we can obtain

$$\begin{aligned}J'&(0)\\&=\int_0^{\infty}\left\{\begin{array}{l}\dot{\lambda}(t)+F_{x(t)}\left[t,x^*(t),u^*(t)\right]\\+\lambda(t)g_{x(t)}\left[t,x^*(t),u^*(t),u^*(t-\tau)\right]\end{array}\right\}y_a(t,a)\,\mathrm{d}t\\&+\int_0^{\infty}\left\{\begin{array}{l}F_{u(t)}[t,x^*(t),u^*(t)]\\+\lambda(t)g_{u(t)}[t,x^*(t),u^*(t),u^*(t-\tau)]\\+\lambda(t+\tau)g_{u(t-\tau)}[t+\tau,x^*(t+\tau),\\u^*(t+\tau),u^*(t)]\end{array}\right\}h(t)\,\mathrm{d}t\end{aligned} \tag{A.8}$$

Since we hypothesize that the function $h(t)$ is arbitrary, to ensure $J'(0)=0$, we can have the following necessary conditions

$$\begin{aligned}&\dot{\lambda}(t)=-F_{x_t}(t,x_t,u_t)-\lambda_t g_{x_t}(t,x_t,u_t,u_{t-\tau}),\quad 0\le t<\infty\\&F_{u_t}(t,x_t,u_t)+\lambda_t g_{u_t}(t,x_t,u_t,u_{t-\tau})\\&\quad+\left[\lambda_t g_{u_{t-\tau}}(t,x_t,u_t,u_{t-\tau})\right]\Big|_{t+\tau}=0,\quad 0\le t<\infty\end{aligned}$$

If the Hamiltonian is written as

$$H(t,x_t,u_t,u_{t-\tau})=F(t,x_t,u_t)+\lambda_t g(t,x_t,u_t,u_{t-\tau})$$

then it can be easily verified that

$$\begin{aligned}\dot{\lambda}&=-\partial H/\partial x_t,\quad 0\le t<\infty\\\partial H/\partial u_t+\partial H/\partial u_{t-\tau}|_{t+\tau}&=0,\quad 0\le t<\infty\end{aligned}$$

Moreover, the conditions above become sufficient for optimality if the functions F and g are both jointly concave in $x(t)$ and $u(t)$. Hence, the necessary conditions for the problem in this paper are sufficient. The derivation of the necessary conditions is based on [40] and [41] and the sufficiency condition can be found in [42].

Proof of Proposition 3.1:

Proof: The Hamiltonian function of manufacturer is

$$\begin{aligned}H_M^N&=e^{-rt}\left\{\rho_M\left[\alpha G(t)+\beta p(t)\right]-\frac{1}{2}k_M q^2(t)\right\}\\&+\lambda_M(t)\left[\theta_1 q(t)+\theta_2 q(t-d_1)-\theta_3 p(t-d_2)-\delta G(t)\right]\end{aligned} \tag{A.9}$$

According to the lemma

$$\frac{\partial H_M(t)}{\partial q(t)}+\frac{\partial H_M(t+d_1)}{\partial q(t)}=0 \tag{A.10}$$

$$\dot{\lambda}_M(t)+\frac{\partial H_M}{\partial G(t)}=0 \tag{A.11}$$

Solve the differential equation in (A.11), we can get

$$\lambda_M(t)=\frac{\alpha\rho_M}{\delta+r}e^{-rt}+c_0e^{\delta t} \tag{A.12}$$

According to transversality condition

$$\lim_{t\to\infty}\lambda_M(t)=0\to c_0=0 \tag{A.13}$$

Hence the we can get the costate variable

$$\lambda_M(t)=\frac{\alpha\rho_M}{\delta+r}e^{-rt} \tag{A.14}$$

Insert (A.14) into (A.10) and we will have

$$q_M^N=\frac{\alpha\rho_M\left(\theta_1+\theta_2e^{-rd_1}\right)}{k_M(r+\delta)} \tag{A.15}$$

Similarly, we will get

$$p_R^N=\frac{\rho_R\left[\beta(r+\delta)-\theta_3\alpha e^{-rd_2}\right]}{k_R(r+\delta)} \tag{A.16}$$

Insert (A.15) and (A.16) into the dynamic equation of goodwill and we will have the time trajectory of goodwill when $d_1\le d_2$

$$G^N(t)=\begin{cases}\left(G_0-\dfrac{\rho_M\theta_1Q}{\delta}\right)e^{-\delta t}+\dfrac{\rho_M\theta_1Q}{\delta},\ t<d_1\\\left[G_0-\dfrac{\rho_MQ\left(\theta_1+\theta_2e^{\delta d_1}\right)}{\delta}\right]e^{-\delta t}+\dfrac{\rho_MQ(\theta_1+\theta_2)}{\delta},\\\quad d_1\le t\le d_2\\\left[G_0-\dfrac{\rho_MQ(\theta_1+\theta_2)e^{\delta d_1}-\rho_R\theta_3Pe^{\delta d_2}}{\delta}\right]e^{-\delta t}\\\quad+G_\infty^N(t),\quad t\ge d_2\end{cases}$$

when $d_1 > d_2$

$$G^N(t) = \begin{cases} \left(G_0 - \frac{\rho_M \theta_1 Q}{\delta}\right) e^{-\delta t} + \frac{\rho_M \theta_1 Q}{\delta}, \quad t < d_2 \\ \left(G_0 - \frac{\rho_M \theta_1 Q - \rho_R \theta_3 P e^{\delta d_2}}{\delta}\right) e^{-\delta t} \\ \quad + \frac{\rho_M \theta_1 Q - \rho_R \theta_3 P}{\delta}, \quad d_2 \le t \le d_1 \\ \left[G_0 - \frac{\rho_M Q\left(\theta_1 + \theta_2 e^{\delta d_1}\right) - \rho_R \theta_3 P e^{\delta d_2}}{\delta}\right] e^{-\delta t} \\ \quad + G_\infty^N(t), \quad t \ge d_1 \end{cases}$$

where $G_\infty^N(t) = \frac{\theta_1 \rho_M Q + \theta_2 \rho_M Q - \theta_3 \rho_R P}{\delta}$

Then the profit of the manufacturer is ($d_1 \le d_2$ or $d_1 > d_2$)

$$J_M^N = \frac{\alpha \rho_M G_0}{r+\delta} + \frac{k_R \rho_M \rho_R P^2}{r} + \frac{k_M \rho_M^2 Q^2}{2r}$$

the profit of the retailer is ($d_1 \le d_2$ or $d_1 > d_2$)

$$J_R^N = \frac{\alpha \rho_R G_0}{r+\delta} + \frac{k_R \rho_R^2 P^2}{2r} + \frac{k_M \rho_R \rho_M Q^2}{r}$$

Proofs of Proposition 3.2, 3.3:

Proof: The proofs of proposition 3.2 and 3.3 are similar to that of proposition 3.1.

Proof of Proposition 3.4:

Proof: Computing the difference of $G_\infty^S(t) - G_\infty^N(t)$, one gets

$$G_\infty^S(t) - G_\infty^N(t) = \frac{(2\rho_R - \rho_M)(\theta_1 Q + \theta_2 Q) - (2\rho_M - \rho_R)\theta_3 P}{2\delta} \tag{A.17}$$

Hence, the value of $G_\infty^S(t) - G_\infty^N(t)$ depends on the relationship between ρ_M/ρ_R and π. If $\rho_M/\rho_R \le \pi$, then $G_\infty^S(t) \ge G_\infty^N(t)$ and if $\rho_M/\rho_R > \pi$ then $G_\infty^S(t) < G_\infty^N(t)$, where $\pi = \frac{2\theta_1 Q + 2\theta_2 Q + \theta_3 P}{\theta_1 Q + \theta_2 Q + 2\theta_3 P}$.

Proof of Proposition 4.1:

Proof: To guarantee that both of the manufacturer and the retailer are willing to accept the coordination mechanism, we should ensure that the profits that both members can get from the mechanism are not lower than those they can get under the two-subsidy scenario. Hence we can get the upper limit $\varphi_{\max}$ from the condition $J_M^D \ge J_M^S$ and the lower limit $\varphi_{\min}$ from the condition $J_R^D \ge J_R^S$. Meanwhile we also have to ensure the upper limit $\varphi_{\max} > 0$ from which we can get the condition $\rho_M/\rho_R > \sqrt{k_R (P/Q)^2 / 2k_M}$.

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