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Optimizing the Number of Express Freight Trains on a High-speed Railway Corridor by the Departure Period

XIAOYAN JIA¹, RUICHUN HE, AND HUO CHAI¹

School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China

Corresponding author: Ruichun He (herc@mail.lzjtu.cn)

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ABSTRACT In China, the transportation capacity of high-speed railways is gradually sufficient to provide services for high-value express freight besides meeting passenger demands. However, similar as the passengers, express freight demands fluctuate and show clear peak and trough periods daily. Therefore, optimizing running numbers of express freight trains on high-speed railway by periods is quite necessary to guarantee the revenues of railway industry and to meet the various requirements of all consignees simultaneously. First, a space-period-pattern three-dimensional network with virtual arcs is built to describe the departure period selection and the stopping or skip-stopping operations. Second, by constructing the arcs' impedance function, the user equilibrium principle is introduced to optimize the express freight flow distribution for each pattern in each period. To elaborate the comprehensive goal of balancing the relationship between profits and the flow distribution, a bi-level programming model is established. The upper model addresses the railway industry's maximum profits, and the lower model addresses the minimum and similar impedance values of the final express freight flow distribution. Finally, through the use of a hybrid algorithm that combines the heuristic genetic algorithm with the Frank-Wolfe algorithm, an experimental case of 10 stations on the Beijing-Xi'an high-speed railway corridor is used to validate the study. The results show that the express freight volume is reasonably distributed to each kind of virtual arc, the impedance value of each pattern is minimized and almost equalized, and the profits of railway industry are maximized by optimizing the number and departure time of trains by each pattern in each period on the basis of meeting all the express freight demands.

INDEX TERMS High-speed railway corridor, express freight train, period-congestion, stopping or skip-stopping operating pattern, bi-level programming model.

I. INTRODUCTION

The express freight demands increase sharply and transporting by road, air and high-speed railways (HSRs) shares mainly this market. The competition among these three modes is serious but there are many challenges for HSR comparing with others. TABLE 1 lists some characteristics comparison considering the express freight flow. Clearly, delivering express freight by HSR does make sense.

As a transportation mode with high capacity, environmental friendliness and high safety, high-speed railways have quickly developed worldwide, especially in China. In addition to serving passengers, China Railway (CR) temporarily

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TABLE 1. Comparison between HSR, road and air in transporting express freight.

Item	HSR	Road	Air
Punctuality (%)	97 ^[1]	50	71
Premium distance (km)	[500, 1000]	≅ 500	≅ 1000
Moving speed (km/h)	250	100	600
Reception and delivering time (h) ^[2]	3	1	6
Carrying capacity	High	Low	Medium
Cost (CNY/km) ^[2]	1.5	0.3-0.4	2
Energy consumption (kJ/(t • km)) ^[3]	569	582	2989

transports express freight using modified passenger trains (seen in FIGURE 1) when there is a sharp demand increase

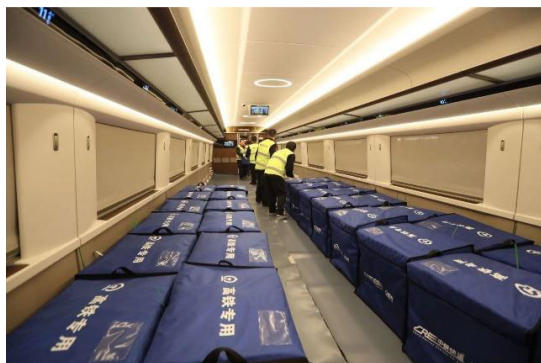


FIGURE 1. Inside of modified passenger trains disassembled seats and filled with parcels.

during typical online shopping carnivals. However, the disadvantages of modified passenger trains are obvious and include the inconvenient loading and unloading of parcels, the complex disassembly and reassembly of seats, and unprofessional fastening of parcels.

With the further construction of HSRs, the transportation capacity will be increased, and CR plans to run special high-speed express freight trains (HEFTs) sharing HSRs with passenger trains.

This specially designed HEFT (as shown in FIGURE 2) can load and unload parcels conveniently via the large-sized doors in the middle of the vehicle, and the parcels can be fastened easily as well. An HEFT can move parcels fast on HSRs according to dedicated operating patterns. Therefore, the carrying capacity of HEFTs is dramatically better than that of modified passenger trains.



FIGURE 2. Inside of HEFTs with large middle doors on both sides and fastening tracks on the floor.

The coming problem is to determine the optimal daily number of HEFTs needed to maximize the profits of CR while meeting the express freight demand without interrupting passenger trains' operations.

The remainder of our study is organized as follows. Section II is literature review. Section III presents the space-period-pattern 3D network for HEFTs and describes period-congestion using the UE principle. Moreover, the impedance

functions for all arcs are proposed. Section IV uses the skip-stopping pattern, the number of running HEFTs and the express freight flow distribution to form a bi-level programming model. Section V proposes a novel algorithm to solve the model. In section VI, a numerical example is given to evaluate the bi-level model and its algorithm, and Section VII summarizes the conclusions and suggests future research directions.

II. LITERATURE REVIEW

In this study, we optimize the number of running HEFTs via a bi-level programming model combined with the user equilibrium (UE) principle based on the experience of various train operating patterns on both HSRs and traditional railways.

A. TRAIN OPERATING PATTERN

An HEFT's operating pattern needs to be optimized since it is different from the pattern of both passenger trains on HSRs and normal-speed freight trains on other lines. For a long time, trains' stopping patterns and running times have been hot research issues around the world, especially the skip-stopping operating schedule. Cao *et al.* [4] studied the stop-skipping operating strategy according to passengers' perception and the objective of the stochastic binary integer programming was minimizing the trip and waiting time for passengers. Concerning the stopping pattern, Yue *et al.* [5] proposed a mixed 0-1 planning model with the goal of the maximum transportation revenue, considering both the stopping station choices and the stopping times. Integer coding and 0-1 coding were taken to solve the problem of train stop schedule by Lin and Ku [6]. Besides, minimizing both the running distance with empty seats and the total stopping times was taken as the goal of multi-objective linear programming model by Qi *et al.* [7]. Shang *et al.* [8] built a three-dimensional (3D) space-time-state network to show passenger travelling behavior, and the trains stopping scheme was optimized by this multi-commodity-flow network.

Others have advanced the study of this topic via multiple goals and models. Jamili and Aghaee [9] focused on the uncertain headway distribution and proposed a robust optimized stop-skipping plan model with the goal of increasing trains' speed and eliminating unnecessary cost. Wang *et al.* [10] studied a mixed-integer nonlinear programming concerning the passenger demands, and the objective was to reduce the passenger travel time and the energy consumption simultaneously. Chang *et al.* [11] considered the total operating cost and the passenger's total travel time loss, and built the multi-objective model to fix the train stop-schedule plan, service frequency and fleet size. Deng *et al.* [12] studied the game relation between stop schedule and passenger transfer choice, a bi-level programming model was built with the lower level of passenger flow assignment and the upper one of the generalized travel cost and stop quantity. Wang *et al.* [13] concerned both the profits of railway enterprises, passengers' waiting time and uncertain passenger flow, and determined the periodic train plan by the chance-constrained model.

Zhipeng [14] built a bi-level programming model considering both the maximizing operating profit and the reasonable passenger flow assignment by an uneven operation schedule.

These researches focused more on optimizing skip-stopping operating pattern considering cost, waiting time and stopping times, but seldom caring the difference among periods. However these experiences are partly helpful to determine the special HEFT patterns.

B. USER EQUILIBRIUM PRINCIPLE

Wardrop [15] proposed a theory of User Equilibrium in 1952, who defined the network equilibrium as: (i) each chosen route owes the minimum and equal driving time; (ii) others without any flow take more driving time. Its formula was then transferred to the integral function by Beckmann *et al.* [16]. Since it is easier to be solved, many researchers take it to address traffic flow management problems. Friesz *et al.* [17] determined the route choice and the departing time with the dynamic UE model. Cantarella [18] built a model to realize multi-mode multi-user equilibrium assignment with elastic demand. Huang and Lam [19] constructed a model considering a discrete-time and an equivalent “zero-extreme value”, and solved a simultaneous route and departure time choice equilibrium assignment. Yang and Huang [20] studied the network with a discrete set of value of time considering the different user classes, by multi-criteria and the cost-versus-time equilibrium. Ho *et al.* [21] minimized the total facility and the travel cost with a logit-type demand distribution function to present the destination choice by multi-class users and with facility externality to describe the congestion effect and the scale economies. Prakash *et al.* [22] formulated a consistent, reliability-based UE problem based on the stochastic link travel times, considering the flow-dependent and endogenous link travel time correlations.

Regarding UE and the flow assignment problem, some researchers concentrate mainly on the impedance function. Wong and Wong [23] constructed a spatially variable MBPR function to unveil the direct link between network topological metrics and the parameters of a specific macroscopic cost flow based on the macroscopic Bureau of Public Roads function.

According to the above review, UE is a useful tool to solve the flow distribution problem and it is beneficial to deal with the express freight flow distribution in our study.

C. BI-LEVEL PROGRAMMING MODEL SOLVING TRAFFIC PROBLEM

The bi-level programming problem is a hierarchical optimization problem in which the upper level's constraints are defined partly by the lower level problem, and this problem was proposed first by Stackelberg in 1950. The bi-level programming model embedded with user equilibrium (UE) is a useful method to solve the network traffic problem worldwide. Maher *et al.* [24] incorporated the trip matrix estimation with the traffic signal optimization on a congested network and formulated them as a bi-level programming problem with

the stochastic UE assignment. To optimal signal control with UE route choice, Gao and Song [25] maximized the reserve capacity of a road network and the increase of traffic demand with a bi-level programming model simultaneously. Ceylan and Bell [26] built a bi-level programming model with upper level of an area traffic control by the delay and the number of stops and lower level of stochastic UE traffic assignment to cope with a congested area traffic control problem. Concerning the stochastic UE assignment with elastic demand, a bi-level programming model was built based on the logit stochastic UE assignment with elastic demand [27-29]. Carey [30] separated the flows on the time-space network by a bi-level UE framework to determine both the lengths of the links and the trip times on them. Migdalas [31] constructed a bi-level programming model and concerned the interests and rights of the network user and the public sector.

A bi-level programming model can handle the game relationship and balance the conflicts of two groups, which lays the foundation for solving the comprehensive problem of the number of running dedicated express freight trains and the flow assignment.

D. PROBLEM OF THE TRAIN PATTERN AND TIME-WINDOW

Previous researchers have built models with single or multiple objectives, including determining the transportation cost, the waiting time, and the flow assignment. However, consignees who choose HEFTs with higher moving fees are more sensitive to time, such as the departure time, moving time (speed) and arrival time.

A few researchers have studied the topics of both the train pattern and time sensitivity with a time-window. Tas *et al.* [32] studied a vehicle routing problem with soft time-dependent and stochastic travel time concerning both the service efficiency and the customers' reliability. To construct a sequence of journey-legs by public transport services according to a traveler's demand, Horn [33] designed a minimum cost journey-planning procedures when the traveler learned the fixed-schedule and the demand-responsive public transport modes. To determine the capacities of passenger food distribution centers and to optimize the number of meals delivered to stations, Wu *et al.* [34] proposed a three-echelon location routing model with time windows and time budget constraints, and generated a schedule for refrigerated cars traveling from the center to stations. Zografos and Androutsopoulos [35] studied the shortest path problem in a multimodal time-schedule network with time windows and time-dependent travel time.

Chang *et al.* [36] built a mixed-integer stochastic program to solve the vehicle routing and scheduling problems with the stochastic demands and within a given time window. Tong and Wong [37] formulated the transit network using the schedule-based approach, considering that both the passenger demand and scheduled time-table varied with time and the vehicles arrived punctually according to the schedule. Li *et al.* [38] studied the stochastic vehicle routing problems

with a time window and formulated it with a stochastic programming model. By use of a spatio-temporal stochastic process, Pavone *et al.* [39] proposed a stochastic and dynamic vehicle routing model with time window to fix the routing strategy of the minimal fraction of service requests missed. Lei *et al.* [40] formulated the capacitated vehicle routing problem with stochastic demands and the time window, and avoided a vertex failure and even a chain failures reaction in the route due to the time windows constraint.

According to a review of previous studies, a number of in-depth and systematic studies on operating patterns and the number of running trains have been put forward, but few studies have investigated congestion during certain periods in a day due to the fixed carrying capacity of each train and the constrained running capacity of the network.

E. CONTRIBUTION OF THIS STUDY

In our study, we consider the period-congestion because some express freight may possibly miss the best period choice and be placed on other trains departing later. However, the consignee, who pays more to move his or her freight via HEFTs, is significantly sensitive to time (the parcels' departure and arrival time). Congestion in the preferred periods causes higher freight protection costs (for example, wrapping cost), whereas taking a later train generates an additional loss related to the value of time. Therefore, we focus on the consignees' preferred HEFT departure period while optimizing the train operating patterns.

In our study, for an HSR corridor without a branch, we (i) construct an innovative space-period-pattern 3D network concentrating on the HEFT's departure period, its skip-stopping pattern and the number of each pattern in different periods simultaneously; (ii) design the corresponding impedance functions for each kind of operation by period (including the preferred departure period, traveling, stopping, and skip-stopping); and (iii) build a bi-level programming model with the upper level maximizing the railway enterprises' profits when running HEFTs and the lower level realizing the UE status for the express freight flow distribution.

III. PROBLEM DESCRIPTION

A. DEFINITION OF AN HSR CORRIDOR AND HEFTS

There is not a single definition of a high-speed railway in use worldwide; however, there are certain unique parameters, especially for this complex system, according to the UIC (International Union of Railways) [41]. The first index concerns the operating speed with new tracks at or above 250 km/h and existing tracks at or above 200 km/h. Other parameters include the modified rolling stock, the infrastructure, the station emplacement, and the specific operating rules.

According to the official website of CR Express Co. Ltd. [42], high-speed express freight transportation provides a wide range of customers with high efficiency and time-sensitive service. The express freight transported by HEFTs

needs to be sent fast, and speed is normally counted in hours; therefore, express freight must be delivered within the same day or the next day. Meanwhile, freight transported by normal trains is not as sensitive to time, and its delivery date is counted in days (per 24 hours). Additionally, there are still some other differences between HEFTs and normal-speed freight trains (TABLE 2).

TABLE 2. Comparison between HEFTs and normal-speed freight trains.

Item	HEFT	Normal-speed Train
Track	HSR	Normal speed railway
Speed	≥ 160 km/h	≤ 120 km/h
Freight	Valuable package/small size	Mass freight
Time sensitivity	High	Low
Due time	By hour	By day

B. DEFINITION OF PERIOD AND ITS CONGESTION

The demands of express freight transported by HEFTs are characterized by disequilibrium both in space and in time. In terms of space, the express freight flow from the departure station to other stations along the HSR corridor is uneven. Moreover, the peak and trough hours vary apparently but show clear regularity. We define these various hours as the 'period'. According to the existing operating regulations, trains are allowed to run in the range of 6 a.m. to 12 p.m. on HSRs. In this study, each hour covers one period; therefore, 6-7 a.m. is marked as 'period I', 7-8 a.m. as 'period II' and so on. Thus, there are 18 operating periods during a whole day. Since all HSRs need to be repaired at the very early morning (normally last from 0-6 a.m.) and all trains running along them must move to dedicated rolling stock depots at that time, we take one whole day to separate periods and to optimize HEFT patterns.

Because of the high time-sensitivity, the consignees substantially care about HEFTs' departure and arrival time. Their period preferences obviously care about the regularity. For example, period satisfaction remains constantly high when express freight is loaded and delivered in the preferred period (for example, between T_2 and T_3 in Figure 3) by HEFT₂, but it is much lower when express freight is delivered by HEFT₁ or may even be zero when express freight is delivered earlier than T_1 or later than T_4 .

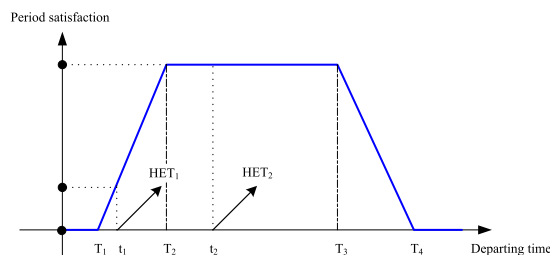


FIGURE 3. Period satisfaction varying with departure times.

This phenomenon may cause period-congestion because most consignees prefer only a few exact periods according

to the statistics. However, the number of departing HEFTs in each period is limited since they share the HSR corridor with passenger trains and the daily running capacity of HSRs is fixed by the periods. FIGURE 4 shows the running number of passenger trains departing from Beijing to Shijiazhuang and to Anyang (belonging to Beijing-Xi'an HSR corridor) in each period separately, which represents the clear peak periods and the peak congestion of this HSR corridor accordingly.

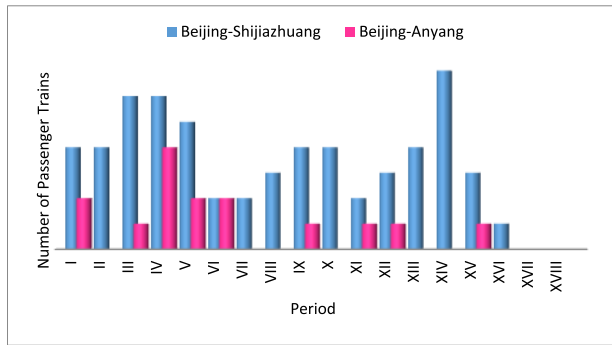


FIGURE 4. Numbers of HSR passenger trains departing from Beijing to Shijiazhuang and to Anyang in each period.

Based on a survey being carried out in Shunfeng Express Company, the statistic results of accepting parcels in each period are listed in FIGURE 5 and, about two hours later, these express freight parcels are loaded and delivered. There are apparent peak periods again. Therefore, it is necessary to determine the number of HEFTs in each period based on the fluctuating and uneven express freight demands at the origin station.

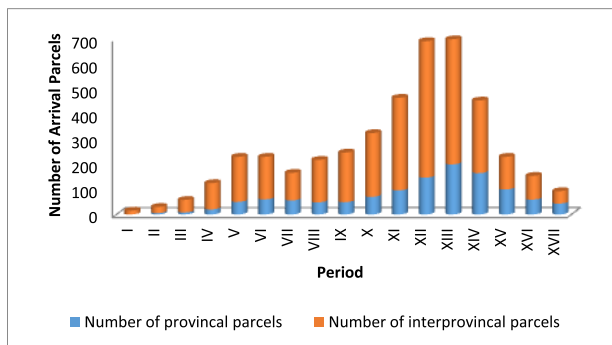


FIGURE 5. Statistic numbers of arrival parcels in each period in Shunfeng Express Company.

In addition, period-congestion may generate higher freight protection (or packaging) costs and delays. The consignees pay more due to choosing their preferred but congested periods, or they instead use later ones. Then, the level of consignees' satisfaction becomes somewhat lower.

C. SPACE-PERIOD-PATTERN 3D NETWORK FOR HEFTS

There are several traditional types of train operating patterns, including all-stopping, skip-stopping and nonstop, in CR. In the HSR corridor, both the skip-stopping and no-stop

patterns are commonly used to cut the overall operating cost, to decrease the total running time and to enhance efficiency. In daily operations, CR works out a set of skip-stopping patterns with fixed OD pairs and detailed stopping and skip-stopping stations so that the operations remain even and stable. Therefore, it is quite necessary to optimize the number of each exact pattern in each period to meet all express freight transportation demands with the minimal cost.

In this study, a few OD pairs connected by HEFTs are available along an HSR corridor and the skip-stopping patterns are given. A simple example is used to describe the problem clearly. FIGURE 6 shows the HSR corridor with six stations and four skip-stopping operating patterns. All six stations are covered by these patterns, and every two stations constitute an OD pair.

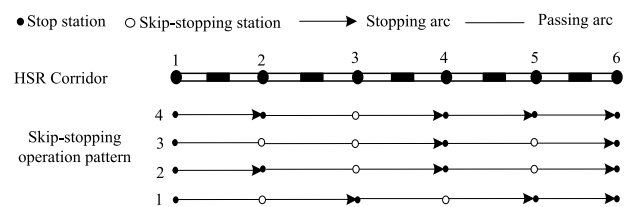


FIGURE 6. An HSR corridor and four skip-stopping operating patterns of HEFTs.

To illustrate the skip-stopping operating patterns clearly, a two dimensional diagram is drawn (FIGURE 7), and it shows the process of deriving a two dimensional skip-stopping operating pattern along the HSR corridor. For each station, two auxiliary lines are used to indicate the beginning and ending points of two possible operations at stations (stopping or passing). In addition, the skip-stopping virtual arc is introduced to replace the passing arc, and two virtual stopping arcs between the auxiliary lines replace one initial stopping arc in FIGURE 6.

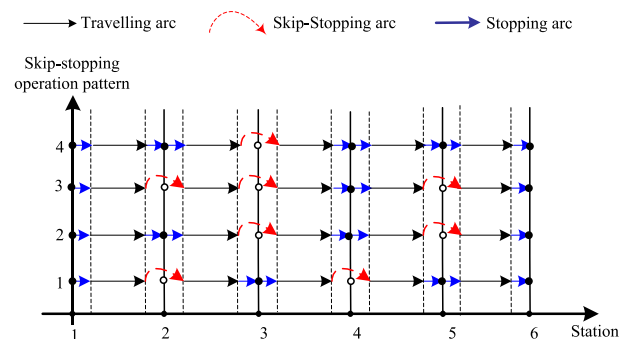


FIGURE 7. A Two-dimension diagram of skip-stopping operating patterns.

As mentioned above, the capacity of running HEFTs along the HSR corridor is finite, and the number of these skip-stopping operating patterns changes dynamically with the period. Therefore, the third dimension (the period) is used, and we design a space-period-pattern 3D network (seen in FIGURE 8).

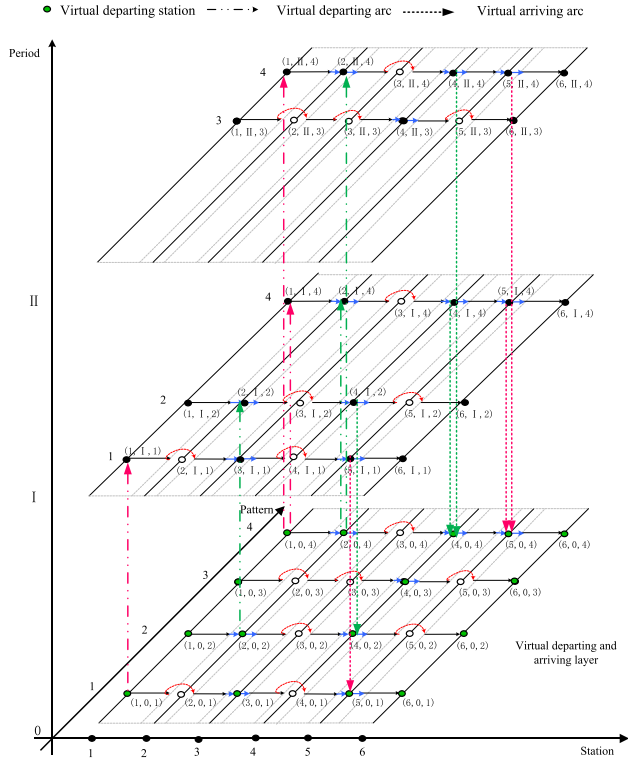


FIGURE 8. A space-period-pattern 3D network.

A virtual departure and arrival layer with six stations and all four skip-stop patterns is used to describe the choice of the preferred period behavior. This layer is named period 0. For simplicity, only two periods (I and II) are shown in the above figure. All the feasible transportation patterns are listed in TABLE III with two OD pairs, stations 1 to 5 and stations 2 to 4. Hence, each virtual departure arc can demonstrate both the departure period and the origin station, similar to the virtual arrival arc. Apparently, three skip-stopping operating patterns (1, 2 and 4) are available in the first period and two (3 and 4) are available in the second. The green virtual arcs clarify all three possible patterns connecting the station 1-5 OD pairs, while the pink arcs show the patterns connecting the station 2-4 OD pairs.

TABLE 3. Feasible patterns with arcs for two OD pairs.

OD station pairs	Period	Pattern	Patterns with arcs
1-5 (Pink lines)	I	1	(1,0,1);(1, I, 1);(3, I, 1); (5, I, 1); (5,0,1)
	I	4	(1,0,4);(1, I, 4); (2, I, 4);(4, I, 4); (5, I, 4);(5,0,4)
	II	4	(1,0,4);(1, II, 4); (2, II, 4);(4, II, 4); (5, II, 4);(5,0,4)
2-4 (Green lines)	I	2	(2,0,2);(2, I, 2);(4, I, 2); (4,0,2)
	I	4	(2,0,4);(2, I, 4);(4, I, 4); (4,0,4)
	II	4	(2,0,4);(2, II, 4);(4, II, 4); (4,0,4)

D. THE IMPEDANCE FUNCTION AND THE SKIP-STOPPING PATTERN CHOICE

In the above space-period-pattern 3D network, the impedance of various arcs is different. We construct the arc impedance functions according to the cost composition and influencing factors of each arc segment. TABLE 4 lists the subscripts and parameters used to formulate the relevant impedance functions.

TABLE 4. Subscripts and parameters used in the impedance function.

Symbol	Definition
N	Set of stations in the high speed corridor
P	Set of periods for running HEFTs
G	Set of train skip-stopping patterns
N^V	Set of stations on the virtual departing and arriving layer in the space-period-pattern 3D network, $N^V \in N$
A	Set of arcs in space-period-pattern 3D network
A^D	Set of departing arcs in space-period-pattern 3D network, $A^D \in A$
A^T	Set of travelling arcs in space-period-pattern 3D network, $A^T \in A$
A^S	Set of stopping arcs in space-period-pattern 3D network, $A^S \in A$
A^A	Set of arriving arcs in space-period-pattern 3D network, $A^A \in A$
i, j	Indices of stations, $i, j \in N$
$(i, 0, g), (j, 0, g)$	Indices of stations on the virtual departing and arriving layer in the space-period-pattern 3D network, $(i, 0, g), (j, 0, g) \in N^V$
p	Indices of periods, $p \in P$
g	Indices of skip-stopping patterns, $g \in G$
(i, j)	Indices of OD pairs in an HSR corridor, $i, j \in N$
$(i, 0, g; j, 0, g)$	Indices of OD pairs by g pattern on the virtual departing and arriving layer in the space-period-pattern 3D network, $(i, 0, g), (j, 0, g) \in N^V$
α, β, γ	Indices of adjustment coefficient obtained by fitting relevant observation data
$(i, p, g; j, p', g)$	Indices of space-time-pattern arcs connecting (i, j) OD pairs by g pattern at period of p or p' , $(i, p, g; j, p', g) \in A$
ut	Unit transporting cost per kg and per km
L_{ij}	Distance between i and j station, $i, j \in N$
$R_{(i,0,g;i,p,g)}$	Departing operation and protection cost of the departing arc, $(i, 0, g; i, p, g) \in A^D$
$T_{(i,p,g;j,p,g)}$	Safety protection cost when moving along the travelling arc with zero-flow, $(i, p, g; j, p, g) \in A^T$
$S_{(i',p,g;i'',p,g)}$	Stopping protection cost on the stopping arc, $(i', p, g; i'', p, g) \in A^S$
$DW_{(i,0,g;i,p,g)}$	The impedance of departing arcs, $(i, 0, g; i, p, g) \in A^D$
$TW_{(i,p,g;j,p,g)}$	The impedance of travelling arcs, $(i, p, g; j, p, g) \in A^T$
$SW_{(i',p,g;i'',p,g)}$	The impedance of stopping arcs, $(i', p, g; i'', p, g) \in A^S$
$AW_{(i,p,g;i,0,g)}$	The impedance of arriving arcs, $(i, p, g; i, 0, g) \in A^A$
$H_{(i,p,g;j,p',g)}$	Indices of the maximum express freight flow volume on any arc, $(i, p, g; j, p', g) \in A$
$w_{(i,p,g;j,p',g)}$	The impedance of the arc in space-period-pattern 3D network, $(i, p, g; j, p', g) \in A$
$w_{(i,0,g;j,0,g)}$	The total impedance of OD pairs (i, j) in space-period-pattern 3D network, $(i, 0, g), (j, 0, g) \in N^V$
$x_{(i,p,g;j,p',g)}$	A variable being the express freight flow volume between (i, j) by g pattern in period p or p' , $(i, p, g; j, p', g) \in A$
$\lambda_{(i,p,g;j,p',g)}$	If the train by g pattern runs along the space-time-pattern arc, then $\lambda_{(i,p,g;j,p',g)} = 1$; otherwise, $\lambda_{(i,p,g;j,p',g)} = 0$, $(i, p, g; j, p', g) \in A$

The impedance function of each kind of virtual arc in the 3D network is built as follows.

1) DEPARTING ARC

In the space-period-pattern 3D network, the departure arc indicates that freight will be carried by a train using pattern g from station i in the selected period p . The costs of this virtual departing arc $(i, 0, g; i, p, g)$ include the departure operating cost $R_{(i,0,g;i,p,g)}$ and the departure congestion cost in the period for the pattern. The latter part increases as the freight flow along the departure arc grows.

$$DW_{(i,0,g;i,p,g)} = R_{(i,0,g;i,p,g)} \cdot (1 + \alpha \cdot (x_{(i,0,g;i,p,g)}/H_{(i,0,g;i,p,g)})) \quad (1)$$

Apparently, the departure congestion cost is a constant when the departure arc's flow $x_{(i,0,g;i,p,g)} = 0$, and its value is related to the departure operating capacity of station i .

2) TRAVELLING ARC

When the express freight flow is loaded on the travel arc $(i, p, g; j, p, g)$, the costs includes the transportation cost related to the spatial distance ($ut \cdot l_{ij}$) and the freight protection cost afforded by the consignees that prevents their freight from being damaged by others as the flow increases. Therefore, the impedance function of the travel arc is constructed as shown in formula (2):

$$TW_{(i,p,g;j,p,g)} = ut \cdot L_{i,j} + T_{(i,p,g;j,p,g)} \cdot (1 + \beta \cdot (x_{(i,p,g;j,p,g)}/H_{(i,p,g;j,p,g)})) \quad (2)$$

Similarly, $T_{(i,p,g;j,p,g)}$ represents the freight protection cost with zero flow on the arc and increases as the flow grows.

3) STOPPING ARC

The costs of the stopping arc $(i', p, g; i'', p, g)$ include two parts, the stopping operating cost at station i and the stopping protection cost, which change linearly according to the stopping flow $x_{(i',p,g;i'',p,g)}$. Then, the impedance function of the stopping arc can be described by formula (3):

$$SW_{(i',p,g;i'',p,g)} = S_{(i',p,g;i'',p,g)} \cdot (1 + \gamma \cdot (x_{(i',p,g;i'',p,g)}/H_{(i',p,g;i'',p,g)})) \quad (3)$$

where i' and i'' represent the beginning node and the ending node of the stopping arc, respectively. $S_{(i',p,g;i'',p,g)}$ is the stopping protection cost when there is no flow on the stopping arc, and it increases as the stopping flow grows.

4) ARRIVING ARC

In the space-period-pattern 3D network, the arrival arc $(i, p, g; i, 0, g)$ is only determined by the departure arc since all the express trains run according to the settled skip-stopping train pattern. That is, once a train departs from station i according to pattern g in period p , its arrival state will not be changeable. Therefore, the arrival arc does not produce an actual cost and its impedance function is as follows:

$$AW_{(i,p,g;i,0,g)} = 0 \quad (4)$$

To clarify, the comprehensive impedance function of all the arcs in the space-period-pattern 3D network can be summarized as follows:

$$w_{(i,p,g;j,p',g)} = \begin{cases} DW_{(i,0,g;i,p,g)} & (i, 0, g; i, p, g) \in A^D \\ TW_{(i,p,g;j,p,g)} & (i, p, g; j, p, g) \in A^T \\ SW_{(i',p,g;i'',p,g)} & (i', p, g; i'', p, g) \in A^S \\ AW_{(i,p,g;i,0,g)} & (i, p, g; i, 0, g) \in A^A \end{cases} \quad (5)$$

Therefore, the total cost of the whole pattern with all the arcs in the space-period-pattern 3D network is described by Eq.(6):

$$w_{(i,0,g;j,0,g)} = \sum_{(i,p,g;j,p',g) \in A} w_{(i,p,g;j,p',g)} \cdot \lambda_{(i,p,g;j,p',g)} \quad (6)$$

If the express train runs using skip-stopping pattern g in period p , that is, the train runs along the space-period-pattern arc $(i, p, g; j, p', g)$ in the 3D network, then $\lambda_{(i,p,g;j,p',g)} = 1$; otherwise, $\lambda_{(i,p,g;j,p',g)} = 0$.

In the section discussing previous literatures ([15]~[21]), we learned that UE is an effective tool for handling network flow assignment problems. In our study, the choice among all feasible skip-stopping patterns and the final flow assignment abide by the UE principle. When transporting express freight, pattern g in period p with the minimal overall impedance value is always chosen by the consignee, which may cause "congestion" in the space-period-pattern network in the certain period. Then, as the impedance value increases, that consignee would naturally give up their initial choice and make a new lower impedance choice. This is a typical cycling process whose eventual situation is the UE result, that is, the impedance value in each period for each pattern is similar to the others and remains at the minimum level. This UE result means the least congestion in the space-period-pattern 3D network with the optimal impedance value for each pattern in each period. Thus, an HEFT runs with the minimal energy consumption while meeting all the express freight demands. We set this part in the lower level of the bi-level programming model in the next section.

IV. MODEL BUILDING

A. PROBLEM FORMULATION AND HYPOTHESE

Both the HSR organizers and the consignees are involved in express freight transportation decisions. The organizers pursue their maximum benefits. Therefore, they desire the pattern with the minimum travel and stopping costs depending on the period-congestion. Meanwhile, the consignees hope that their freight is safely sent to the destination at the exact time with the minimum cost. However, the safety and packaging costs of express freight fluctuate as the freight volume changes on the train for each pattern and each period. To meet these two demands simultaneously, a bi-level programming model is built, with the upper model focusing on the HSR organizers' benefits and the lower on the freight protection cost concerns of the consignees.

The following hypotheses are given in this study to clarify the problem:

- All the express freights are served by only one HEFT with one single pattern;
- Once the HEFT departs from the origin station, it will run exactly according to one skip-stopping operating pattern along the HSR corridor;
- All the HEFTs run at the same speed along the HSR corridor and use the same train with the same carrying capacity.

B. DECISION VARIABLES AND PARAMETERS

TABLE 5 lists the subscripts, parameters used in the bi-level programming model.

TABLE 5. Subscripts and parameters used in the bi-level model.

Symbol	Definition
q_{ij}	Freight transport demand from i to j station, $i, j \in N$
$V_{p,g}$	Unit transporting charge by g pattern running at p period, $g \in G, p \in P$
$CO_{p,g}$	HEFT's travelling cost of g pattern running at p period, $g \in G, p \in P$
$CS_{p,g}$	HEFT's stopping cost of g pattern running at p period, $g \in G, p \in P$
ψ_{ij}	Proportion of one train's carrying capacity allocated by CR between i and j station, $i, j \in N$
N_p	Permissive numbers of travelling trains at p period along arcs, $p \in P$
M_i	Maximum train numbers of stopping at i station during a whole day, $i \in N$
θ_g	Carrying capacity of the train by g pattern, $g \in G$
$\xi_{i,g}$	=1, if the train by g pattern stops at i station; =0, otherwise.
$\pi_{(i,0,g;j,0,g)}^{p,(i,p,g;j,p',g)}$	=1 if the arc $(i,p,g;j,p',g)$ is part of the link connecting the OD pair $(i,0,g;j,0,g)$; =0, otherwise.

TABLE 6 lists the variables used in the bi-level programming model.

TABLE 6. Variables used in the bi-level model.

Symbol	Definition
$y_{p,g}$	Variable of the upper level model, representing the number of the trains by g pattern running at p period, $g \in G, p \in P$
$f_{(i,0,g;j,0,g)}^p$	Intermediate variable meaning the express freight flow volume of (i,j) OD pairs by the train of g pattern at p period, $p \in P, (i,0,g),(j,0,g) \in N^v$
$x_{(i,p,g;j,p',g)}$	Variable of the lower level model, representing the express freight flow volume of the arc by the train of g pattern at p or p' period, $(i,p,g;j,p',g) \in A$

C. BI-LEVEL PROGRAMMING MODEL

1) UPPER LEVEL MODEL

The goal of the upper model is to ensure the maximum profits of HST organizers. The income from transporting express freight is mainly obtained by charging the consignees, and the costs include a series of basic and overall payments for transportation, operation, power, staff and vehicles. In this study, only two main kinds of costs (travel and stopping) are considered because they are highly relevant to the skip-stopping operating patterns and change dynamically in each period.

a: THE INCOME OF EXPRESS FREIGHT TRANSPORTATION

The total income of transporting express freight using an HEFT depends on the unit transportation fee, the moving distance and the express freight volume on the train of pattern g .

$$\sum_{i,j \in N} \sum_{p \in P} \sum_{g \in G} V_{p,g} \cdot L_{i,j} \cdot f_{(i,0,g;j,0,g)}^p \cdot y_{p,g} \quad (7)$$

b: THE TRAVELLING COST

When moving express freight, the organizer must run the HEFT and pay for the power, rolling stock, HSR usage, etc. This complex cost is named the travel cost and changes with the skip-stopping train pattern and the chosen period. Eq. (8) describes this.

$$\sum_{p \in P} \sum_{g \in G} CO_{p,g} \cdot y_{p,g} \quad (8)$$

c: THE STOPPING COST

There is a stopping cost when any HEFT stops at intermediate stations on an HSR, but it varies according to the period and the pattern as well. For example, the stopping cost is higher when there are more passengers or express freight congested at the station during some special periods compared to others, which is presented in Eq.(9).

$$\sum_{p \in P} \sum_{g \in G} CS_{p,g} \cdot y_{p,g} \quad (9)$$

Therefore, the objective function of the upper model is stated as Eq. (10).

$$Z = \max \sum_{i,j \in N} \sum_{p \in P} \sum_{g \in G} V_{p,g} \cdot L_{i,j} \cdot f_{(i,0,g;j,0,g)}^p \cdot y_{p,g} - \sum_{p \in P} \sum_{g \in G} CO_{p,g} \cdot y_{p,g} - \sum_{p \in P} \sum_{g \in G} CS_{p,g} \cdot y_{p,g} \quad (10)$$

$$\text{Subject to: } \sum_{g \in G} y_{p,g} \leq N_p \quad \forall p \quad (11)$$

$$\sum_{i,j \in N} \xi_{i,g} \cdot \xi_{j,g} \cdot q_{i,j} \leq \sum_{p \in P} \sum_{g \in G} y_{p,g} \cdot \psi_{ij} \cdot \theta_g \quad \forall i, j, p, g \quad (12)$$

$$\sum_{p \in P} \sum_{g \in G} y_{p,g} \cdot \xi_{i,g} \leq M_i \quad \forall i, p, g \quad (13)$$

$$y_{p,g} \geq 0, \quad \text{and is an integer} \quad (14)$$

Constraint (11) promises that the total number of departing trains during each period p is not more than the station's maximum capacity. Constraint (12) represents that the express freight transportation demands for an HSR between stations i and j should not exceed the maximum carrying capacity of all running trains. Constraint (13) means that the number of trains stopping at station i must be less than its daily permissive stopping capacity. Constraint (14) ensures that the number of trains using pattern g in period p is a positive integer.

2) LOWER LEVEL MODEL

When the impedance of any space-period-pattern arc remains high, the relevant safety and package costs increase. Then, the consignees prefer choosing other arcs with lower costs, and the impedance of the previous arc decreases as a result. The fluctuation of this dynamic selection can finally result in the UE of the express freight flow distribution. Let O_g be the minimal impedance of pattern g . Then, the UE principle can be written as follows:

$$\begin{cases} w_{(i,p,g;j,p',g)}(x_{(i,p,g;j,p',g)}) = O_g & x_{(i,p,g;j,p',g)} > 0 \\ w_{(i,p,g;j,p',g)}(x_{(i,p,g;j,p',g)}) \geq O_g & x_{(i,p,g;j,p',g)} = 0 \end{cases} \forall g \in G \quad (15)$$

The objective formula of the lower model is stated as Eq.(16).

$$\min E = \sum_{i,j \in N} \sum_{p,p' \in P} \sum_{g \in G} \int_0^{x_{(i,p,g;j,p',g)}} w_{(i,p,g;j,p',g)} \cdot (x_{(i,p,g;j,p',g)}) (u) du \quad (16)$$

$$\text{Subject to: } \sum_{p \in P} f_{(i,0,g;j,0,g)}^p = q_{ij} \quad \forall i, j, g \quad (17)$$

$$f_{(i,0,g;j,0,g)}^p \geq 0 \quad \forall p, (i, 0, g; j, 0, g) \quad (18)$$

$$x_{(i,p,g;j,p',g)} = \sum_{(i,0,g) \in N^V} \sum_{(j,0,g) \in N^V} \sum_{p \in P} f_{(i,0,g;j,0,g)}^p$$

$$\cdot \pi_{(i,0,g;j,0,g)}^{p,(i,p,g;j,p',g)} \quad \forall p, (i, 0, g), (j, 0, g) \quad (19)$$

Constraint (17) is the flow conservation principle that ensures that all the express freight demands are reasonably distributed over all periods and patterns. Constraint (18) keeps the express freight volume positive. Eq.(19) shows each arc's freight flow composition for all periods and all patterns.

V. ALGORITHM

The bi-level programming model is known as one of the most difficult problems to solve. Proposed initially by Holland in 1975, the genetic algorithm (GA) is generally acknowledged as an efficient method because of its global superiority and dramatic convergence and because it simulates the survival of the fittest in nature. We solve the upper model using the GA. The Frank-Wolfe algorithm is introduced to solve the lower model because it is an efficient way to solve a UE problem.

Integer coding adopts chromosome coding, including p gene-segments representing all the periods in the space-period-pattern 3D network. Each gene-segment owns g gene positions representing all feasible patterns. There are $p \times g$ total gene positions in a chromosome, representing the possible HEFTs with g patterns and p periods ($\{y_{p,g} | p \in P, g \in G\}$). The value of each gene position ranges from $[0, M_i]$ to constrain the number of HEFTs to be not more than the capacity of station i .

The heuristic algorithm of the GA combined with Frank-Wolfe is implemented as follows.

A. INITIALIZING

The following relevant parameters are assigned: the maximum number of generations Ge , the crossover probability Pc , and the mutation rate Mr . The random initial population is generated within the scale of the population size Ps . Let the iteration number be $\varepsilon=0$.

B. ASSIGNING EXPRESS FREIGHT FLOW BY UE

The accumulative express freight volume $x_{(i,p,g;j,p',g)}$ for pattern g in period p is calculated using the Frank-Wolfe algorithm. The process is as follows.

(a) Initialization. Let the impedance value of each arc in the space-period-pattern network be $w_{(i,p,g;j,p',g)}(\cdot) = w_{(i,p,g;j,p',g)}(0)$, and calculate the zero-flow impedance $w_{(i,0,g;j,0,g)}^p(0)$ of each OD pair using Eq.(5). Using the All-or-Nothing assignment method, all the express freight demands $\{q_{ij}\}$ of all the OD pairs are loaded on pattern g with the minimal impedance value. Then, we obtain the flow of each arc $\{x_{(i,p,g;j,p',g)}^{(1)}\}$ and let the iteration number be $\hat{u}=1$.

(b) Update the impedance value of each arc $w_{(i,p,g;j,p',g)}(\cdot) = \{w_{(i,p,g;j,p',g)}(x_{(i,p,g;j,p',g)})\}$ with \hat{u} iterations.

(c) Find the iteration direction. Based on the value of $\{w_{(i,p,g;j,p',g)}(x_{(i,p,g;j,p',g)})\}$, assign the express freight demand $\{q_{ij}\}$ of all OD pairs on the current pattern with the minimal impedance value. Then, we obtain a set of additional flow volumes $\{y_{p,g}^{\hat{u}}\}$.

(d) Calculate the step of iteration σ using Eq.(20).

$$\sum_{(i,p,g;j,p',g) \in N^V} \left(y_{(i,p,g;j,p',g)}^{(\hat{u})} - x_{(i,p,g;j,p',g)}^{(\hat{u})} \right) \cdot w_{(i,p,g;j,p',g)} \left[x_{(i,p,g;j,p',g)}^{(\hat{u})} + \sigma \cdot \left(y_{(i,p,g;j,p',g)}^{(\hat{u})} - x_{(i,p,g;j,p',g)}^{(\hat{u})} \right) \right] = 0 \quad (20)$$

(e) Calculate the accumulation flow volume of each arc using Eq.(21).

$$x_{(i,p,g;j,p',g)}^{(\hat{u}+1)} = x_{(i,p,g;j,p',g)}^{(\hat{u})} + \sigma \cdot \left(y_{(i,p,g;j,p',g)}^{(\hat{u})} - x_{(i,p,g;j,p',g)}^{(\hat{u})} \right) \quad (21)$$

(f) Stop the iterations if the convergence principle is met (Eq.(22)).

$$\frac{\sqrt{\sum_{(i,p,g;j,p',g) \in N^V} (x_{(i,p,g;j,p',g)}^{(\hat{u}+1)} - x_{(i,p,g;j,p',g)}^{(\hat{u})})^2}}{\sum_{(i,p,g;j,p',g) \in N^V} x_{(i,p,g;j,p',g)}^{(\hat{u})}} < \Omega \quad (22)$$

where Ω is the given limitation of the deviation.

Then, calculate the fitness of each individual chromosome of the upper level using Eq.(23), and go to Step 3. Otherwise, let $\hat{u} = \hat{u} + 1$ and return to Step (b).

$$fit = \frac{Z_{\max} - Z}{Z_{\max} - Z_{\min}} \quad (23)$$

C. UPDATING POPULATION

The new population is obtained based on the initial population operation (roulette selection, two gene position crossing, or uniform mutation). Let the iteration number $\varepsilon = \varepsilon + 1$, and then check whether the individuals in the current generation population meet constraints (12) and (13). Go to Step E if so; otherwise, go to Step D.

D. REPAIRING CHROMOSOME

Randomly select one of the nonzero positions of the chromosome to repair the chromosomes and to retain the feasibility of the population. Let $y_{p,g} = y_{p,g} - 1$, and test if this new repaired chromosome obeys those constraints. Return to Step C if so; otherwise, repeat Step D.

E. TERMINATING TEST

If the iteration number $\varepsilon > Ge$, then go to Step F; otherwise, return to Step B.

F. TERMINATING TEST

Output the final calculation results of the upper model $y_{p,g}$ and of the lower model $x_{(i,p,g;j,p',g)}$ with the maximum fitness. FIGURE 9 shows the whole genetic algorithm process.

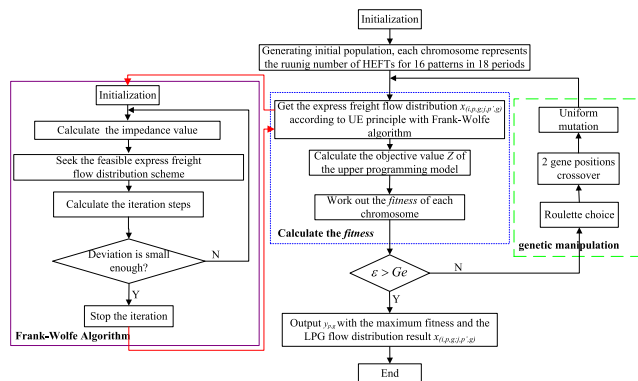


FIGURE 9. Genetic algorithm process.

VI. EXPERIMENTAL CASE

A. PARAMETER VALUES

China has built an HSR network that is approximately 50 000 km. The HSR corridor connecting Beijing and Xi’an was opened in 2010. Comparing with Beijing-Shanghai or Shanghai-Hangzhou, this corridor has much more remaining capacity for running HEFTs. Additionally, transportation demands of express freight along this HSR corridor keep high. So the HSR corridor of Beijing-Xi to Xi’an-Bei (seen in FIGURE 10) is taken as an experimental case to test the bi-level model and the heuristic GA algorithm combined with the Frank-Wolfe one.

1) VALUES OF PARAMETERS IN THE IMPEDANCE FUNCTIONS

There are ten HSR stations in this corridor, and the express freight volume is forecasted based on statistical data

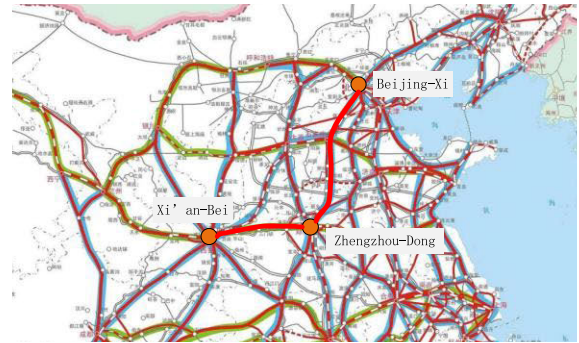


FIGURE 10. The HSR network and the Beijing-Xi to Xi’an-Bei HSR corridor.

from four express companies and three railway companies (Beijing, Zhengzhou and Xi’an Railway Administrations) covering the area. FIGURE 11 shows the details of the Beijing-Xi’an HSR corridor, and we number each station from 1 to 10.



FIGURE 11. The Beijing-Xi to Xi’an-Bei HSR corridor with distances between close stations.

This numerical example takes only the single direction from Beijing-Xi to Xi’an-Bei as the objective. Therefore, the OD pair demands of the express freight are also one-way travel demands.

There are some constant values, including the total skip-stopping pattern g , the number of periods p , the unit transportation cost ut and three adjustment coefficients (α , β and γ). They are listed in TABLE 7.

TABLE 7. Values of constant parameters.

N	g	p	$ut(\text{CNY}/\text{t}\cdot\text{km})$	α	β	γ
10	16	18	4.4	0.8	0.8	0.8

In this Beijing-Xi’an HSR corridor, some stations are defined as stop-stations for any train, such as stations 1, 2, 5, 6, 9, and 10; others are stop-stations for only a few trains or are skip-stopping stations for the remaining trains, such as stations 3, 4, 7 and 8. Therefore, all sixteen patterns differ from each other based on the situations of these skip-stopping stations. The initial patterns are listed in FIGURE 14 with the final results later.

According to experience and statistics, CR allocates a corresponding percentage (ψ_{ij}) of each HEFT’s carrying capacity to every OD pair to attract the surrounding express freight demand. Correspondingly, each arc has a maximum express freight flow volume that varies with each OD pair. These values are listed in TABLE 8 and TABLE 9.

We confirm the values of the departure protection cost ($R_{(i,0,g;i,p,g)}$) on the departure arcs and the stopping

TABLE 8. Percentage of each HEFT's carrying capacity ψ_{ij} for OD pairs.

Station	ψ_{ij} (%)									
	2	3	4	5	6	7	8	9	10	
1	1	9.7	1.4	1.1	1.2	8.9	1.3	1.2	11.2	
2	-	1.7	0.2	0.1	0.2	1.7	0.4	0.5	2.3	
3	-	-	0.5	2.1	2.4	8	2.1	2	9.2	
4	-	-	-	0.2	0.1	1.3	0.1	0.4	1.5	
5	-	-	-	-	0.2	1.5	0.4	0.6	1.2	
6	-	-	-	-	-	1.3	0.2	0.2	1.4	
7	-	-	-	-	-	-	2.1	1.3	12.5	
8	-	-	-	-	-	-	-	0.3	1.6	
9	-	-	-	-	-	-	-	-	1.2	

TABLE 9. The maximum express freight flow volume $H_{(i,p,g;j,p',g)}$ on the arcs.

Station	$H_{(i,p,g;j,p',g)}$ (kg)									
	2	3	4	5	6	7	8	9	10	
1	56	85	62	83	75	111	45	50	65	
2	-	66	78	57	67	82	49	57	81	
3	-	-	35	35	50	90	56	80	120	
4	-	-	-	36	36	61	37	37	130	
5	-	-	-	-	35	60	34	48	92	
6	-	-	-	-	-	54	30	23	154	
7	-	-	-	-	-	-	42	35	193	
8	-	-	-	-	-	-	-	45	96	
9	-	-	-	-	-	-	-	-	87	

protection cost ($S_{(i',p,g;i'',p',g)}$) on the stopping ones. These two parameters are mostly affected by the period-congestion and the stations' levels. Four stations (1, 3, 7 and 10) in this HSR corridor are first-class HSR stations, station 2 is a second-class station, and the others (4, 5, 6, 8 and 9) are normal. Based on these factors, their values are listed in TABLE 10.

TABLE 10. Values of departing protection cost and stopping protection cost (CNY/ t).

Congested level	Period	$R_{(i,0,g;i,p,g)}$		
		1 st -class station	2 nd -class station	Normal
Peak	2, 3,11, 12	1.5	1.3	0.8
Secondary peak	4, 6, 7, 13	1.2	1	0.7
Medium	5, 8, 9, 10, 14, 15, 16	1	0.8	0.6
Trough	1, 17,18	0.8	0.6	0.5

Congested level	Period	$S_{(i',p,g;i'',p',g)}$		
		1 st -class station	2 nd -class station	Normal
Peak	2, 3,11, 12	1.2	1	0.9
Secondary peak	4, 6, 7, 13	1	0.9	0.7
Medium	5, 8, 9, 10, 14, 15, 16	0.8	0.7	0.6
Trough	1, 17,18	0.6	0.5	0.5

Due to congestion, the cost of freight safety protection increases when moving on the travel arc with zero-flow ($T_{(i,p,g;j,p',g)}$). This parameter is affected less by the HEFT's pattern and its values are listed in TABLE 11.

TABLE 11. Values of $T_{(i,p,g;j,p',g)}$.

Period	1	2	3	4	5	6	7	8	9
$T_{(i,p,g;j,p',g)}$ (CNY/t)	8	8.9	8.9	8.6	8.2	8.6	8.6	8.2	8.2

Period	10	11	12	13	14	15	16	17	18
$T_{(i,p,g;j,p',g)}$ (CNY/t)	8.2	8.9	8.9	8.6	8.2	8.2	8.2	8	8

2) VALUES OF PARAMETERS IN THE BI-LEVEL PROGRAMMING MODEL

Suppose that the predicted total volume of express freight each day in 2025 is 600 tons in the single-direction HSR corridor. Each OD pair demand of the express freight is listed in TABLE 12.

TABLE 12. Values of express freight demand $q_{i,j}$ of each OD pair in 2025 (ton).

Station	2	3	4	5	6	7	8	9	10
1	6	42	18	25.8	30	54	14.4	18	61.2
2	-	4.8	1.2	1.8	1.2	18	0.6	1.2	9
3	-	-	4.2	7.2	4.8	39	6.6	7.2	19.2
4	-	-	-	2.4	1.2	2.4	0.6	1.8	10.8
5	-	-	-	-	3	1.2	1.8	3	42
6	-	-	-	-	-	5.4	3	3.6	48
7	-	-	-	-	-	-	9	8.4	36
8	-	-	-	-	-	-	-	3	12
9	-	-	-	-	-	-	-	-	6

The unit transportation charge $V_{p,g}$ paid by consignees changes with the period-congestion level. During the peak hours, they must pay more to move their freight. However, the charge is affected less by the HEFT's pattern. Meanwhile, the permitted numbers of running HEFTs N_p along the arcs change with the period-congestion, too. These parameters' values are shown in TABLE 13.

TABLE 13. Values of $V_{p,g}$ and N_p .

Period	1	2	3	4	5	6	7	8	9
$V_{p,g} \times 10^{-3}$ (CRY/km)	2.5	4.3	4.5	3.5	2.7	4.0	4.1	3.6	3.0
Number of N_p	10	4	4	6	9	5	5	7	9

Period	10	11	12	13	14	15	16	17	18
$V_{p,g} \times 10^{-3}$ (CRY/km)	3.0	4.5	4.5	3.7	3.2	3.0	2.8	2.5	2.2
Number of N_p	8	4	4	6	7	8	9	10	12

Each HEFT's capacity Q_g is calculated by referring to the annual statistics in 2018 for CR. That is, each wagon can carry 17 700 kg of express freight and each HEFT is normally constituted by 8 wagons. Then, $Q_g=141\ 600$ kg per HEFT.

When running HEFTs, CR must pay the usage costs of the rolling stock and the HSR, which is only determined by the skip-stopping pattern and each HEFT's number of stops because CR globally controls all the trains and the

HSR network by itself. Therefore, the value of the HEFT's travel cost ($CO_{p,g}$) changes only with the pattern. Similarly, the stopping operating cost of each HEFT ($CS_{p,g}$) has nothing to do with the period. Their values are listed in TABLE 14.

TABLE 14. Values of parameters $CO_{p,g}$ and $CS_{p,g}$ (THS CNY/HEFT).

g	1	2	3	4	5	6	7	8
$CO_{p,g}$	80	83	83	83	83	86	86	86
$CS_{p,g}$	1	2	2	2	2	3	3	3
g	9	10	11	12	13	14	15	16
$CO_{p,g}$	86	86	86	89	89	89	89	92
$CS_{p,g}$	3	3	3	4	4	4	4	5

In daily operations, each station in the HSR corridor permits the maximum stopping number (M_i) of HEFTs, and these values are listed in TABLE 15.

TABLE 15. Maximum HEFTs' daily stopping numbers at each station.

station	1	2	3	4	5	6	7	8	9	10
M_i	154	37	120	46	39	50	136	36	30	143

3) VALUES OF PARAMETERS IN GA

Let the population size be $Ps=200$, the crossover probability be $Pc=0.8$, the mutation probability be $Mr=0.07$, and the maximum number of iterations be $\epsilon=100$.

B. ANALYSIS OF THE CALCULATION RESULTS

The model and algorithm proposed in this study are solved using the MATLAB 2014a software. The solution time is 48.05 min. All the data experiments are completed on the same computer configured with an Intel(R) Core(TM) 2 Duo P7450 CPU. The objective function value of the upper level programming tends to converge after 35 generations, and the optimal value appears at 68 generations. With this optimal value, the corresponding lower model is solved with the stable accuracy Ω of $1e-4$ after 79 iterations (seen in FIGURE 12).

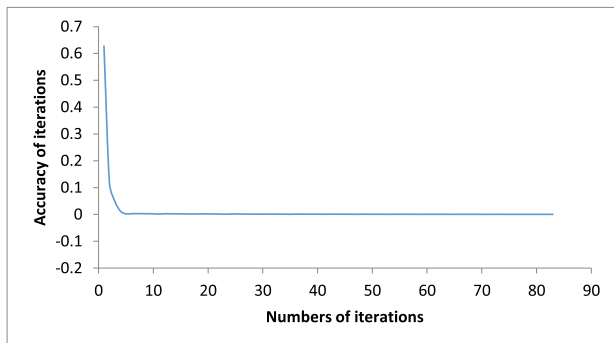


FIGURE 12. Accuracy changing with iterations.

With this small accuracy, the minimal and similar impedance value of each period is guaranteed. For example, the approximate impedance value of station 3-8 OD pair is 1.166 (seen in FIGURE 13). The fact, that impedance values

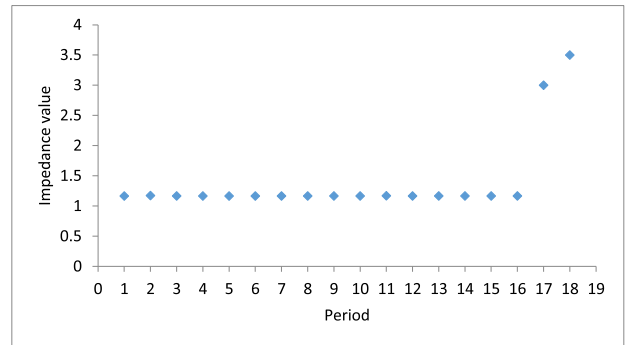


FIGURE 13. Impedance value of Station 3-8 OD pair.

of Period 17 and 18 for any OD pair, are much higher than others, represents these two periods without carrying any express freight flow. It is because that these last two periods can't provide enough time for running HEFTs before the HSR corridor being closed.

1) THE RESULTS ANALYSIS OF THE UPPER LEVEL MODEL

To meet the transportation demands of 600 tons of express freight, 115 HEFTs need to run along the Beijing-Xi'an HSR corridor daily. The detailed running plan with 16 patterns and 18 periods is shown in FIGURE 14.

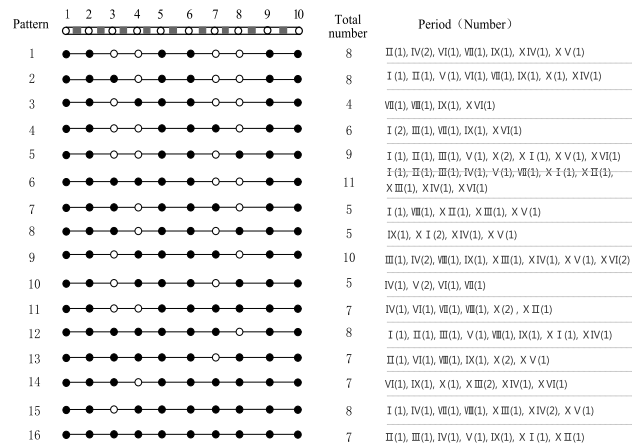


FIGURE 14. The number of HEFTs of each pattern at each period.

The number of HEFTs departing from Station 1 varies with the situation of each period and is shown in FIGURE 15.

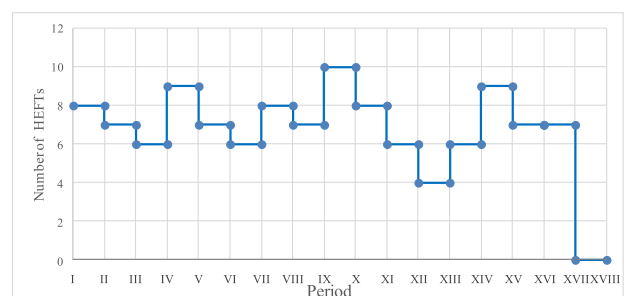


FIGURE 15. Number of HEFTs varying with the period.

From the above figures, we can see the following: (i) pattern 1 stops only at stop-stations, and 8 HEFTs are run daily; (ii) the pattern 16 is all-stop HEFT, and 7 are run; (iii) the maximum running pattern is pattern 6 which has 11 trains moving; and (iv) the minimum running pattern is pattern 3. These 115 HEFTs depart from Station 1 in periods I to XVI and service all the transportation demands arriving at the other stations in the corridor. The last two periods (Periods XVII and XVIII) do not have any departing HEFTs. This is because the remaining operating time is not enough to finish any whole pattern.

Stopping at the station is done to unload and load the express freight. The number of daily stops at each station on the Beijing-Xi'an HSR corridor is listed in FIGURE 16.

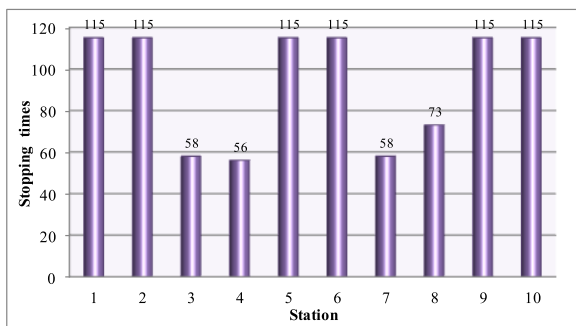


FIGURE 16. HEFTs stopping times of each station in the Beijing-Xi'an HSR corridor.

Here, the number of stops at those stop-stations is equal to the total number of HEFTs. In addition, the numbers for only four skip-stopping stations vary with the periods. These numbers fluctuate with the transportation demands for express freight at the skip-stopping stations (Stations 3, 4, 7 and 8).

In addition, the number of stops for these four skip-stopping stations differs by the period as well. FIGURE 17 shows the difference in the number of stops in each period for them.

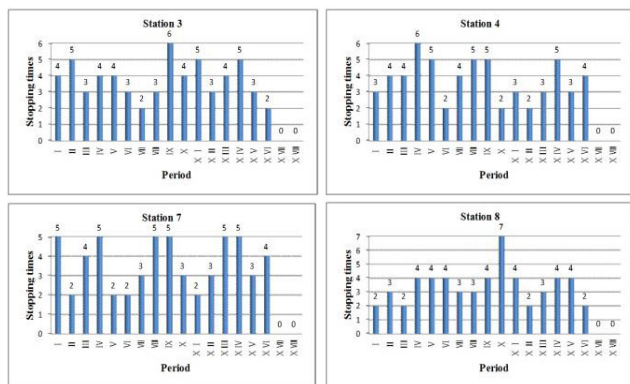


FIGURE 17. Stopping times of four skip-stopping stations at each period.

Clearly, the difference in the number of stops among these stations is caused by the peak period-congestion. In addition, there is no stopping operation in the last two periods.

2) THE RESULTS ANALYSIS OF THE LOWER LEVEL MODEL

The objective of the lower level model is to distribute the total express freight flow to all the patterns in all periods according to the UE principle. In this case, the transportation demands of all 45 OD pairs are assigned to 115 HEFTs (with 16 patterns in 18 periods). The concrete flow distribution result of these OD pairs is shown in FIGURE 18.

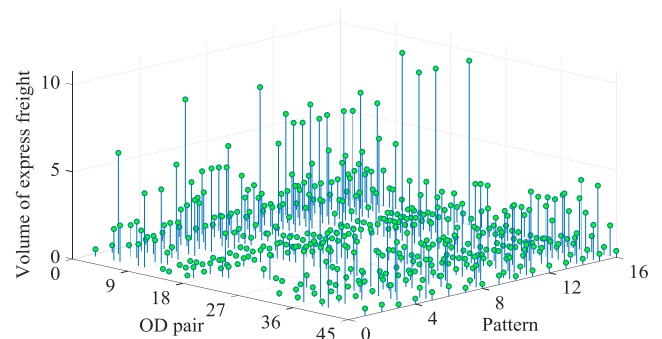


FIGURE 18. Flow distribution results of OD pairs.

In fact, the transportation demands are met by the HEFTs with different patterns in 18 periods, and the flow is distributed with the minimum and similar impedance value. In this optimal result, the flow distribution varies with both the pattern and the period (FIGURE 19).

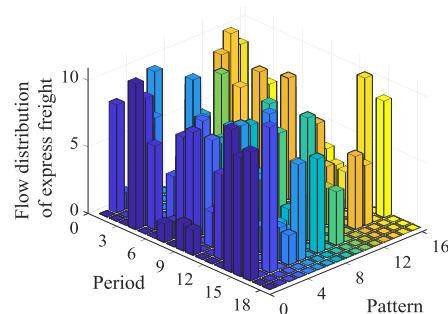


FIGURE 19. Express freight flow distribution results with both the pattern and the period.

Additionally, to meet the demands of each OD pair and retain the optimal impedance value, the flow distributions of all the OD pairs differ from each other. We take the stop-station pair (1-10) and the skip-stopping station pair (3-5) as examples to show the difference (FIGURE 20). The flow pair 1-10 is carried out in each period, but that of pair 3-5 is only carried out in certain periods due to the period-congestion and high impedance value.

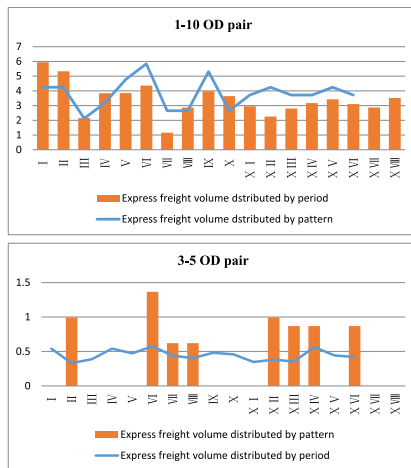


FIGURE 20. Comparison of the express freight volume between two OD pairs.

To meet the express freight transportation demands and to maintain the minimum period impedance level, the suitable number of HEFTs is determined according to each pattern in each period. Obviously, due to sharing the HSR with passenger trains, the impedance value remains high and only a few HEFTs can run. The flow distribution of the express freight is similarly affected for each pattern and in each period, especially for those four skip-stopping stations.

VII. CONCLUSION

A bi-level programming model is built in this study to optimize the number of HEFTs on each skip-stopping pattern considering the period-congestion in the HSR corridor. In the upper level model, we maximize the profits of CR on the basis of meeting all the express freight demands with the minimal number of HEFTs. Meanwhile, we propose a novel method to distribute the express freight flow on different HEFT patterns and in various periods using the UE principle in the lower level model. Hence, the impedance values of each pattern and in each period are similar and minimal. The advantages of the bi-level model can be summarized as follows.

- The bi-level model defines the period-congestion and its characteristics when transporting express freight on an HSR corridor by analyzing the daily transportation regulations.
- The bi-level model designs a space-period-pattern 3D network describing HEFTs' running and stopping, in which the virtual departure arc is used to represent the period choice with a changeable impedance value. The stopping or skip-stopping operations are shown by the corresponding virtual arcs.
- The bi-level model constructs the impedance functions for each kind of virtual arc. These functions can reflect the dynamic relationships between the consignees' preferred period choices and their payments for transporting the express freight.
- The impedance value of the express freight is determined by the departure period and these impedance

functions can quite well show the game relationship between the consignees' choices.

- The number of HEFTs and the flow distribution during the peak periods remain as low as possible due to both the period-congestion and high impedance value.

The bi-level model can realize the minimum number of HEFTs, cut down the general costs, and guarantee the optimal flow distribution of the express freight for each skip-stopping pattern in each period. Therefore, the CR profits and the financial burden of consignees are considered simultaneously. This novel model and algorithm are feasible and adaptive when determining the optimal number of running HEFTs and selecting the optimal period and the optimal pattern for the whole railway industry.

It must be clarified, however, that this study has not fully addressed the issues of express freight transportation by HEFTs. We should improve the adjustment coefficients of the impedance functions using a better fitting method, and we should deeply consider the interference caused by passenger trains on an HSR corridor. These will be emphasized in our future works.

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