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Unit Sliding Mode Control for Disturbed Crowd Dynamics System Based on Integral Barrier Lyapunov Function

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ABSTRACT This paper presents a new tracking controller for a crowd dynamics system. The crowd dynamics are described by a continuum model in the macro-scale. An appropriate control variable is chosen to solve the problem of the multi-directionality of crowd movement. In order to stabilize the state density of the disturbed crowd dynamics system at a given reference density, a unit sliding mode controller based on the integral barrier Lyapunov function is designed, and the reaching law approach is used to avoid chattering. By ensuring the boundedness of the integral barrier Lyapunov function in the closed-loop, the global constraints of the system state variables are obtained. The design of the controller and the stability analysis of the closed-loop system are all done in the distributed parameter system. A numerical example illustrates the utility of the proposed controller.

INDEX TERMS Integral barrier Lyapunov function, unit sliding mode control, tracking control, disturbed crowd dynamics system, distributed parameter system.

I. INTRODUCTION

Pedestrian traffic is an important part of the urban traffic. It is of great significance to formulate measures to ensure people's safety, but the research on pedestrian traffic is far less than other modes of transportation [1]. Due to the complexity of pedestrian dynamics, pedestrian traffic management is also a challenging task.

Since the Crowd Safety Conference in 1993, the study of pedestrian traffic has gradually attracted people's attention. Various mathematical models represented by ordinary differential equations (ODEs) are used to describe the pedestrian dynamics in the micro-scale, such as social-force models [2], [3], cellular automata model [4], agent-based models [5], [6], where pedestrians are treated as physical particles. When the crowd density is low, these micro-models can accurately describe the crowd dynamics. However, when the crowd density is high, running the micro-scale models requires high computational costs and is even difficult to achieve. Therefore, the study of crowd dynamics in the macro-scale has become attractive, where the crowds are represented as a fluid because of the high similarity of their movements [7]. Mathematically, the crowd dynamics are represented by partial differential equation (PDE) and the average density is considered as the state variable. On the other hand, high-density crowds are more likely to cause dangerous accidents such as stampede, so the study of crowd dynamics in the macro-scale is of great significance.

In 2002, Hughes constructed a pedestrian dynamic model in the macro-scale [8], where pedestrians were considered as "thinking fluid", choosing the shortest path to their destination. Francesco *et al.* [9] solved the Hughes' model with a deterministic particle approach and got a global existence result. Hänseler *et al.* [10] developed a macroscopic model to describe the dynamic of congested, multi-directional and time-varying pedestrian flows, where the anisotropy was accurately considered in the macroscopic framework. A consistent continuum macroscopic fundamental diagram model was formulated and solved with a semi-Lagrangian scheme in [11], where Eulerian and Lagrangian representations were both used. The dynamic of two intersecting pedestrian flows was modelled in [12] by using nonlinear partial differential equation, and then was illustrated with macroscopic and microscopic simulations. Wadoo [13], [14] used a vehicle traffic model to describe

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the pedestrian dynamics, and chose the free flow speed as the control variable to solve the problem of the multi-directionality of the pedestrian movement. Qin *et al.* integrated the Lighthill-Whitham-Richards(LWR) model [15], [16] and the diffusion model to describe crowd dynamics, and designed boundary controller [17] and finite time controller [18], respectively. Qin *et al.* [19] used the unit sliding mode control method [20]–[22] to design tracking controllers for a disturbed crowd dynamics system, so that the crowd density can follow the given reference density to achieve different control purposes, such as maximum evacuation flow, maximum pedestrian movement speed, etc. However, due to the influence of external disturbances, the local density may be very high, which is very dangerous in actual evacuation and is prone to accidents such as stampede. Therefore, this article will design a controller to restrict the crowd density within a safe range while achieving tracking control goals.

The barrier Lyapunov function (BLF) was first defined in [23], which would grow to infinity as the variable approaches a certain value. Therefore, by ensuring the boundedness of the barrier Lyapunov function, the state variable can be constrained within a certain range. The BLF method was widely used in the finite-dimensional systems to achieve constraints on output variables. Reference [24] designed a adaptive fuzzy control scheme for a class of stochastic non-strict feedback nonlinear systems, and used BLF method to achieve the output constraint. A high-order BLF was applied to achieve output constraint for a category of highorder nonlinear systems in [25]. More valuable achievements could refer to [26]–[30]. Shuang [31] introduced the BLF method into the infinite-dimensional system, and designed a controller to ensure the tracking error bounded stable, that is, the tracking error is limited to a certain range, but it cannot asymptotically converge to 0.

In this paper, inspired by [19] and [31], a unit sliding mode controller will be designed for the disturbed crowd dynamics system based on the integral barrier Lyapunov function (IBLF). The design of the controller and the stability analysis of the closed-loop system are achieved directly within the distributed framework [32]–[37], which avoids the error caused by the spatial discretization. Compared with the existing works, the main contributions of this paper are summarized as follows.

1) A novel controller is designed with the help of IBLF, which can not only achieve the tracking control goals, but also can restrict the state density in a safe range, and effectively prevents the dangerous accidents caused by too high local density.

2) The designed controller in this paper can achieve the asymptotic stability of the tracking error, while the IBLF-based controller designed in article [31] can only achieve the bounded stability of the tracking error, that is, the tracking error is limited to a certain range. Therefore, this paper promotes the application of the IBLF method in the infinite-dimensional systems.

Notation: $H^2(0, L)$ is the Sobolev space with square integrable derivatives $\rho'' \in L^2(0, L)$ on the interval $(0, L)$, $L^2(0, L)$ is the Hilbert space with L_2 norm

$$\|\rho(x,t)\|_2 = \left[\int_0^L \rho^2(x,t)\,\mathrm{d}x\right]^{\frac{1}{2}}.$$

The following notation is used throughout this paper.

$$\rho_t(x,t) = \frac{\partial\rho(x,t)}{\partial t}, \quad \rho_x(x,t) = \frac{\partial\rho(x,t)}{\partial x},$$
$$\rho_{xx}(x,t) = \frac{\partial^2\rho(x,t)}{\partial x^2}.$$

II. PROBLEM FORMULATION

In this section, the crowd dynamics on a bounded interval $[0, L] \in R$ are modeled by a parabolic PDE. The average crowd density represented by $\rho(x, t)$ is the system state variable, where $x \in [0, L]$ and $t \in [0, +\infty)$ are the space and time variables. The PDE is given as

$$\rho_t(x,t) = D\rho_{xx}(x,t) - \frac{\partial}{\partial x}\left[\rho(x,t)\left(1-\frac{\rho(x,t)}{\rho_m}\right)u(x,t)\right] + \beta(x,t), \quad (x,t)\in\Omega, \tag{1}$$

where $\Omega = [0, L] \times (0, +\infty)$. D and ρ_m are constants, which represent the diffusion coefficient and the maximum crowd density, respectively. $\beta(x, t)$ represents the disturbance and $u(x, t) \in [-v_m, v_m]$ represents the controller, where v_m is the maximum velocity. The Dirichlet boundary conditions are given as

$$\rho(0,t) = \phi_0(t),$$
$$\rho(L,t) = \phi_L(t), \quad \forall t \in (0,\infty), \tag{2}$$

where $\phi_0(t)$ and $\phi_L(t)$ are the measured densities at boundaries $x = 0$ and $x = L$, respectively. The initial density $\rho(x, 0) = \rho_0(x)$ is assumed to meet the boundary conditions.

Remark 1: The system model is constructed based on the conservation law of mass, and the diffusion model is chosen to represent the relationship between the pedestrian speed and the crowd density, which shows the self-regulating nature of pedestrians, that is, pedestrians can adjust their moving speed in real-time according to the density ahead. The controller can not only control the pedestrians' moving speed but also its moving direction, which solves the problem of the multi-directionality of the pedestrian movement. The detailed modeling process can refer to [19].

In this paper, we are committed to designing a controller $u(x, t)$ to achieve the following goals,

(1) the crowd density $\rho(x, t)$ can track any given density $R(x, t)$;

(2) the crowd density $\rho(x, t)$ is globally constrained, i.e. $|\rho(x, t)| < C$, $\forall t \in (0, \infty)$, where C is a setting positive constant.

Clearly, the boundary value of the reference density $R(x, t)$ should be consistent with the boundary value of the state density $\rho(x,t)$, i.e.

$$R(0,t) = \phi_0(t),$$
$$R(L,t) = \phi_L(t), \quad \forall\, t \in (0,+\infty). \tag{3}$$

Furthermore, there exists a positive constant $B < C$ such that the reference density $|R(x,t)| \leq B$.

The disturbance $\beta(x,t)$ and its time derivative $\beta_t(x,t)$ are assumed to meet the following restrictions.

Assumption 1: There exists priori known constants η_1 and η_2, such that $\|\beta(x,t)\|_2 \leq \eta_1$, $\|\beta_t(x,t)\|_2 \leq \eta_2$, $\forall (x,t) \in \Omega$.

III. CONTROLLER DESIGN

In this section, an IBLF-based unit sliding mode controller is designed to achieve the control goals mentioned above.

Defining the tracking error as $e(x,t) = \rho(x,t) - R(x,t)$ and according to equation (1), the error dynamics are given by

$$\begin{aligned} e_t(x,t) &= De_{xx}(x,t) + DR_{xx}(x,t) - R_t(x,t) \\ &\quad - \frac{\partial}{\partial x}\left[\rho(x,t)\left(1 - \frac{\rho(x,t)}{\rho_m}\right)u(x,t)\right] \\ &\quad + \beta(x,t), \quad (x,t) \in \Omega, \end{aligned} \tag{4}$$

and subject to the boundary conditions

$$e(0,t) = 0,$$
$$e(L,t) = 0, \quad \forall t \in (0,+\infty). \tag{5}$$

The distributed sliding surface $s(x,t) \in H^2(0,L)$ is chosen as

$$s(x,t) = e_t(x,t) + \gamma e(x,t), \tag{6}$$

where γ is a positive constant. It is easy to get that $s(x,t)$ satisfies the following boundary conditions,

$$s(0,t) = 0,$$
$$s(L,t) = 0, \quad \forall t \in (0,+\infty). \tag{7}$$

Firstly, Eliminate the nonlinear term of the error dynamic system (4) using the idea of feedback linearization [38]. To this end, we design the following distributed controller

$$\begin{aligned} u(x,t) = \frac{\rho_m}{\rho(x,t)(\rho_m - \rho(x,t))}\int_0^x \Big[& g(\xi,t) + DR_{xx}(\xi,t) \\ & - R_t(\xi,t)\Big]\, \mathrm{d}\xi, \quad (x,t) \in \Omega, \end{aligned} \tag{8}$$

where $g(x,t)$ is a stabilizing function to be designed. Substituting (8) into (4) and invoking Leibniz integral rule, it is found that,

$$e_t(x,t) = De_{xx}(x,t) - g(x,t) + \beta(x,t), \quad (x,t) \in \Omega. \tag{9}$$

Denote $A = C - B > 0$ and consider the following IBLF

$$W(t) = \frac{1}{2}\int_0^L \ln\frac{A^2}{A^2 - e^2(x,t)}\mathrm{d}x + \frac{1}{2}\int_0^L s^2(x,t)\mathrm{d}x. \quad t \geq 0,$$

For $\frac{A^2}{A^2 - e^2(x,t)} > 1$ in the set of $|e(x,t)| < A$, the nature of the logarithmic function shows that $\frac{1}{2}\int_0^L \ln\frac{A^2}{A^2-e^2(x,t)}\, \mathrm{d}x > 0$. Invoking $\frac{1}{2}\int_0^L s^2(x,t)\, \mathrm{d}x \geq 0$, it is found that $W(t)$ is positive definite in the set of $|e(x,t)| < A$.

Remark 2: According to the definition of $W(t)$, when $|e(x,t)|$ approaches A, the IBLF $W(t) \to \infty$. If we can prove that there is a constant M such that $W(t) \leq M$, then $|e(x,t)|$ cannot approach A. Furthermore, by using a weak assumption $e(0,t) < A$, we can conclude that $|e(x,t)| < A$. Next, we prove the existence of the constant M.

The time derivative of $W(t)$ is given by

$$\dot{W}(t) = \int_0^L \frac{e(x,t)e_t(x,t)}{A^2 - e^2(x,t)}\mathrm{d}x + \int_0^L s(x,t)s_t(x,t)\mathrm{d}x, \quad t \geq 0. \tag{10}$$

Calculating the time derivative of $s(x,t)$ according to equation (6) and (9), we get

$$\begin{aligned} s_t(x,t) &= e_{tt}(x,t) + \gamma e_t(x,t) \\ &= De_{txx}(x,t) - g_t(x,t) + \beta_t(x,t) + D\gamma e_{xx}(x,t) \\ &\quad - \gamma g(x,t) + \gamma\beta(x,t) \\ &= Ds_{xx}(x,t) - P(x,t) + Q(x,t), \quad (x,t) \in \Omega, \end{aligned} \tag{11}$$

where

$$P(x,t) = g_t(x,t) + \gamma g(x,t), \quad (x,t) \in \Omega, \tag{12}$$

is a new control variable and

$$Q(x,t) = \beta_t(x,t) + \gamma\beta(x,t), \quad (x,t) \in \Omega,$$

is a new disturbance. Applying the Assumption 1 gives,

$$\|Q(x,t)\|_2 \leq \gamma\eta_1 + \eta_2 := \eta, \quad (x,t) \in \Omega. \tag{13}$$

In order to counteract disturbance and stabilize the error dynamic (4), the following unit sliding mode controller is designed,

$$P(x,t) = \eta\frac{s(x,t)}{\|s(x,t)\|_2} + ks(x,t) + \frac{e(x,t)}{A^2 - e^2(x,t)}, \tag{14}$$

where k is a positive constant.

Remark 3: In the infinite-dimensional space, the sliding mode control term $sign(s(t))$ used in the finite-dimensional space cannot be simply translated to the form $sign(s(x,t))$ [39], so the component $\eta\frac{s(x,t)}{\|s(x,t)\|_2}$ is designed according to the unit control [40], which is a continuous function beyond the sliding surface $s(x,t) = 0$ and does not subject to the spatial dimension. Furthermore, the unit sliding mode controller $\eta\frac{s(x,t)}{\|s(x,t)\|_2} + ks(x,t)$ is constructed by using the reaching law approach [41] to suppress chattering.

Substituting (11) and (14) into (10), yields

$$\begin{aligned} \dot{W}(t) &= \int_0^L \frac{e(x,t)e_t(x,t)}{A^2 - e^2(x,t)}\, \mathrm{d}x + D\int_0^L s(x,t)s_{xx}(x,t)\, \mathrm{d}x \\ &\quad - \eta\int_0^L \frac{s^2(x,t)}{\|s(x,t)\|_2}\, \mathrm{d}x - k\int_0^L s^2(x,t)\, \mathrm{d}x \\ &\quad - \int_0^L \frac{e(x,t)e_t(x,t)}{A^2 - e^2(x,t)}\, \mathrm{d}x - \gamma\int_0^L \frac{e^2(x,t)}{A^2 - e^2(x,t)}\, \mathrm{d}x \\ &\quad + \int_0^L s(x,t)Q(x,t)\, \mathrm{d}x \end{aligned}$$

$$
\begin{aligned}
&= D\int_0^L s(x,t)s_{xx}(x,t)\,\mathrm{d}x - \eta\|s(x,t)\|_2 \\
&\quad - k\int_0^L s^2(x,t)\,\mathrm{d}x - \gamma\int_0^L \frac{e^2(x,t)}{A^2-e^2(x,t)}\,\mathrm{d}x \\
&\quad + \int_0^L s(x,t)Q(x,t)\,\mathrm{d}x
\end{aligned} \tag{15}
$$

Integrating $D\int_0^L s(x,t)s_{xx}(x,t)\,\mathrm{d}x$ by parts, yields

$$
\begin{aligned}
&D\int_0^L s(x,t)s_{xx}(x,t)\,\mathrm{d}x \\
&\quad = s(x,t)s_x(x,t)\Big|_0^L - D\int_0^L s_x^2(x,t)\,\mathrm{d}x, \quad t\geq 0.
\end{aligned} \tag{16}
$$

Considering the boundary condition (7), one has

$$
D\int_0^L s(x,t)s_{xx}(x,t)\mathrm{d}x = -D\int_0^L s_x^2(x,t)\mathrm{d}x, \quad t\geq 0. \tag{17}
$$

By the Poincaré Inequality [42]

$$
\int_0^L s^2(x,t)\,\mathrm{d}x \leq 2Ls^2(L,t) + 4L^2\int_0^L s_x^2(x,t)\,\mathrm{d}x, \quad t\geq 0,
$$

one can derive

$$
D\int_0^L s(x,t)s_{xx}(x,t)\mathrm{d}x \leq -\frac{D}{4L^2}\int_0^L s^2(x,t)\mathrm{d}x, \quad t\geq 0. \tag{18}
$$

For the item $\int_0^L s(x,t)Q(x,t)\,\mathrm{d}x$, employing Cauchy-Schwarz Inequality [43], it can be written as

$$
\begin{aligned}
&\int_0^L s(x,t)Q(x,t)\,\mathrm{d}x \\
&\quad \leq \left(\int_0^L s^2(x,t)\,\mathrm{d}x\right)^{\frac{1}{2}}\left(\int_0^L Q^2(x,t)\,\mathrm{d}x\right)^{\frac{1}{2}} \\
&\quad = \|s(x,t)\|_2\|Q(x,t)\|_2, \quad t\geq 0.
\end{aligned} \tag{19}
$$

Combination of (18), (19) and (15) gives

$$
\begin{aligned}
\dot{W}(t) \leq -\left(\frac{D}{4L^2}+k\right)\int_0^L s^2(x,t)\,\mathrm{d}x - \eta\|s(x,t)\|_2 \\
+ \|s(x,t)\|_2\|Q(x,t)\|_2, \quad t\geq 0.
\end{aligned}
$$

It follows from (13) that

$$
\dot{W}(t) \leq -\left(\frac{D}{4L^2}+k\right)\int_0^L e^2(x,t)\,\mathrm{d}x \leq 0, \quad t\geq 0.
$$

Therefore, it can reach the sliding surface $s(t)$ in a finite time. Moreover, as soon as the sliding surface is reached, the tracking error $e(x,t)$ will converge exponentially to 0 according to the the definition (6).

Combining equation (8), (12) and (14), yields the distributed controller

$$
\begin{aligned}
u(x,t) = &\frac{\rho_m}{\rho(x,t)(\rho_m-\rho(x,t))}\int_0^x \Bigg[e^{-\gamma t}\int_0^t \Bigg(\eta\frac{s(\xi,\zeta)}{\|s(\xi,\zeta)\|_2} \\
&+ ks(\xi,\zeta) + \frac{e(\xi,\zeta)}{A^2-e^2(\xi,\zeta)}\Bigg)e^{\gamma\zeta}\,\mathrm{d}\zeta + DR_{xx}(\xi,t) \\
&- R_t(\xi,t)\Bigg]\,\mathrm{d}\xi, \quad (x,t)\in\Omega.
\end{aligned} \tag{20}
$$

From the above statement, the following theorem is obtained.

Theorem 1: Consider the disturbed crowd dynamics system (1) - (2) with the disturbance $\beta(x,t)$ satisfies Assumption 1, if the reference density $R(x,t) < B$ subjects to restrictions (3), the distributed controller (20) guarantees that

(1) The tracking error $e(x,t)$ is asymptotically stable.

(2) If the initial error $e(x,0)$ satisfies $|e(x,0)| < A$, $e(x,t)$ is globally constrained by A, i.e., $|e(x,t)| < A$, $(x,t)\in\Omega$.

(3) If the initial density $\rho(x,0)$ satisfies $|\rho(x,0)| < C$, $\rho(x,t)$ is globally constrained by C, i.e., $|\rho(x,t)| < C$, $(x,t)\in\Omega$.

Proof: (1) From the above analysis, the asymptotic stability of the tracking error $e(x,t)$ is obvious.

(2) From $\dot{W}(t)\leq 0$, $t\geq 0$, it follows that $W(t)\leq W(0)$. For $\int_0^L \ln\frac{A^2}{A^2-e^2(x,t)}\,\mathrm{d}x$ and $\int_0^L s^2(x,t)\,\mathrm{d}x$ are non-negative function, one can derive that

$$
\ln\frac{A^2}{A^2-e^2(x,t)} \leq W(0), \quad \forall x\in[0,L].
$$

From the fact that

$$
\ln\frac{A^2}{A^2-e^2(x,t)} \to \infty, \quad \text{as } |e(x,t)|\to A,
$$

we have $|e(x,t)| \neq A$, $(x,t)\in\Omega$. Because of the initial error $e(x,0)$ satisfies $|e(x,0)| < A$ and $e(x,t)$ is a continuous function, one can derive

$$
|e(x,t)| < A, \quad (x,t)\in\Omega.
$$

(3) From $|R(x,t)|\leq B$,

$$
\begin{aligned}
|\rho(x,t)| &= |e(x,t)+R(x,t)| \leq |e(x,t)| + |R(x,t)| \\
&< A+B = C, \quad (x,t)\in\Omega,
\end{aligned}
$$

i.e., $\rho(x,t)$ is globally constrained by C.

Remark 4: In the article [19], Qin *et al.* used the unit sliding mode control method to solve the tracking control problem, but did not consider the constraints on the state density. External disturbances may cause the local density to be too high, leading to dangerous accidents such as stampede. In this paper, the BLF method is used to design the controller, which can track the target and constrain the state density in a safe range.

Remark 5: A diffusion system which is frequently used in industrial processes is studied in [31], which is shown as

$$
\rho_t(x,t) = D\rho_{xx}(x,t) + u(x,t) + \beta(x,t), \tag{21}
$$

subjects to the initial and Dirichlet boundary conditions

$$
\begin{aligned}
\rho(x,t_0) &= \rho_0(x), \quad \forall x\in(0,L), \\
\rho(0,t) &= \phi_0(t), \\
\rho(L,t) &= \phi_L(t), \quad \forall t\in(0,\infty).
\end{aligned} \tag{22}
$$

The bounded stable of the tracking error is got, that is, the tracking error is limited to a certain range, but it cannot asymptotically converge to 0. In this paper, a more effective controller can be derived from Theorem 1 to make the tracking error of the diffusion system (21)-(22) asymptotically stable, as shown in the following corollary.

Corollary 1: Consider the diffusion system (21) - (22) with the disturbance $\beta(x,t)$ satisfies Assumption 1, if the reference density $R(x,t) < B$ subjects to restrictions (3), the distributed controller

$$u(x,t) = e^{-\gamma t}\int_0^t \left(\eta\frac{s(x,\zeta)}{\|s(x,\zeta)\|_2} + ks(x,\zeta) + \frac{e(x,\zeta)}{A^2 - e^2(x,\zeta)}\right)e^{\gamma\zeta}\,\mathrm{d}\zeta + DR_{xx}(x,t) - R_t(x,t), \quad (x,t) \in \Omega. \tag{23}$$

guarantees that

(1) The tracking error $e(x,t)$ is asymptotically stable.

(2) If the initial error $e(x,0)$ satisfies $|e(x,0)| < A$, $e(x,t)$ is globally constrained by A, i.e., $|e(x,t)| < A$, $(x,t) \in \Omega$.

(3) If the initial density $\rho(x,0)$ satisfies $|\rho(x,0)| < C$, $\rho(x,t)$ is globally constrained by C, i.e., $|\rho(x,t)| < C$, $(x,t) \in \Omega$.

IV. NUMERICAL SIMULATION

To illustrate the effectiveness of the controller (20), a numerical example is given based on the finite difference method. Consider the system (1) - (2) with parameters $L = 6$, $D = 0.1, \eta = 14, k = 6, \rho_m = 5, \gamma = 1$, and initial density

$$\rho(x,0) = 2.5 + cos(\frac{\pi x}{2} + \frac{\pi}{2}).$$

The disturbance $\beta(x,t)$ and reference density $R(x,t)$ are set as

$$\beta(x,t) = \sin(2\pi x) + \sin(2\pi t),$$
$$R(x,t) = 2.5.$$

The control objectives are:

1. The state density $\rho(x,t)$ asymptotically converges to the reference density $R(x,t)$;

2. The tracking error $e(x,t)|$ is constrained globally within 1.1, i.e. $|e(x,t)| < 1.1$. From $R(x,t) = 2.5$, the state density $\rho(x,t)$ is limited within 3.6, i.e. $|\rho(x,t)| < 3.6$.

The uncontrolled density evolution of system (1) is illustrated in Fig. 1. As can be seen from the figure, it is a slow diffusion process, but the density profile can not stabilized to the reference density because of the influence of the disturbance $\beta(x,t)$.

Fig. 2 illustrates the density response with the designed controller (20). The density changes smoothly, reaches the reference density in about 6 seconds, and is globally constrained within 3.6, i.e. $|\rho(x,t)| < 3.6$. The tracking error is graphed in Fig. 3, and we can see that $e(x,t)$ is limited within 1.1. The control input is illustrated in Fig. 4.

For a clearer demonstration, we select some spatial and temporal points to show the density evolution process. Fig. 5 – Fig. 6 show the density evolution of crowd dynamics systems at spatial locations $x = 1$ and $x = 3$, respectively.

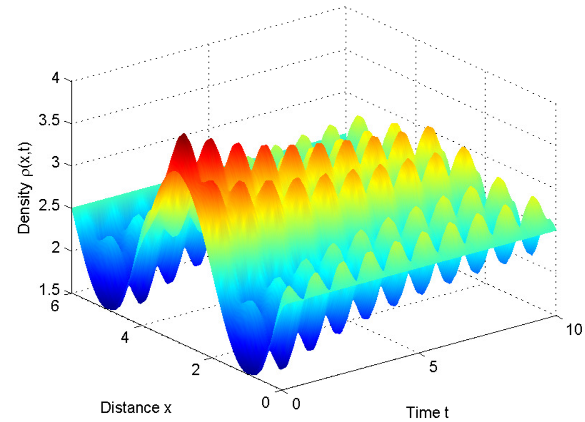

FIGURE 1. **Density evolution without control input.**

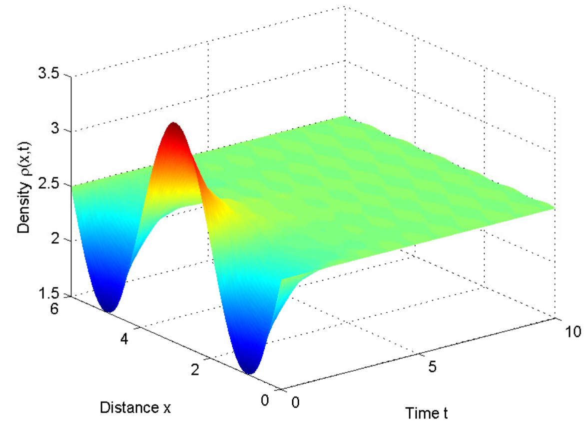

FIGURE 2. **Density response with the designed control.**

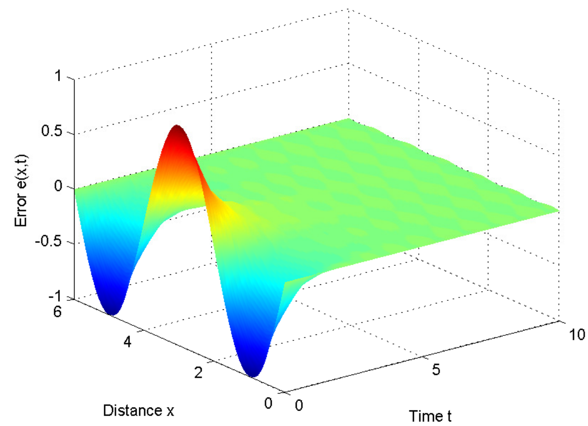

FIGURE 3. **Error $e(x,t)$.**

By inspection of Fig. 5 and Fig. 6, we see that the density profile can not reach the reference density without control, and may break through the constraints due to the influence of disturbance term, such as the density at $t = 0.5$ in Fig. 6. On the contrary, under the action of the controller (20), the density profile asymptotically stabilizes to the $R(x,t) = 2.5$ without violating the constraint.

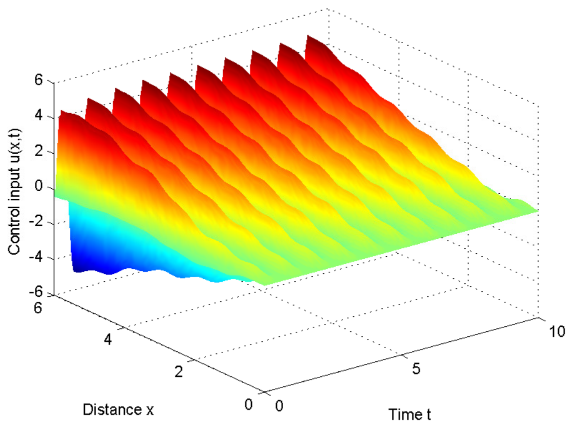

FIGURE 4. Control input $u(x, t)$.

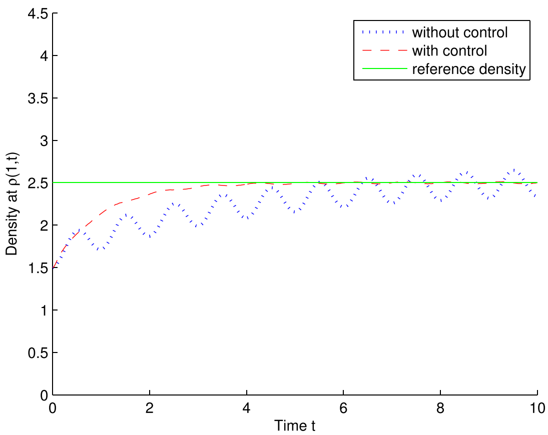

FIGURE 5. Density variation at $x = 1$.

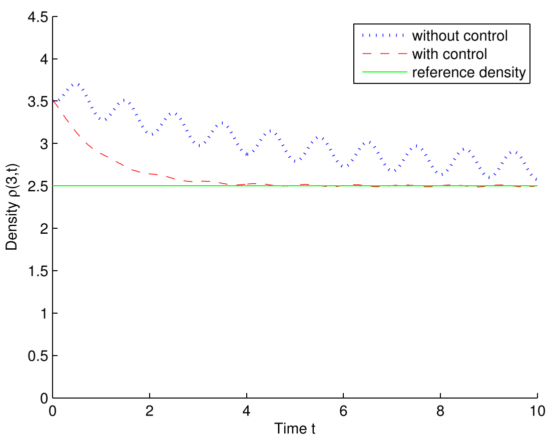

FIGURE 6. Density variation at $x = 3$.

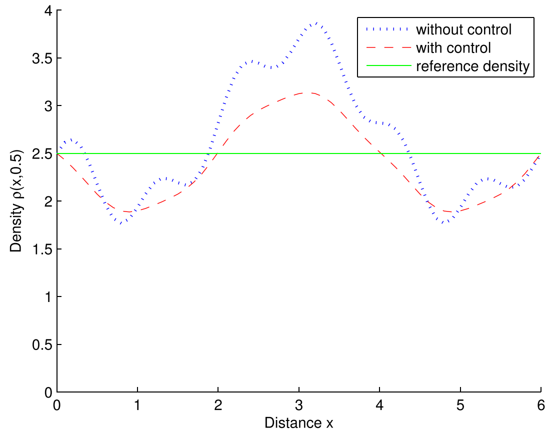

FIGURE 7. Density distribution when $t = 0.5s$.

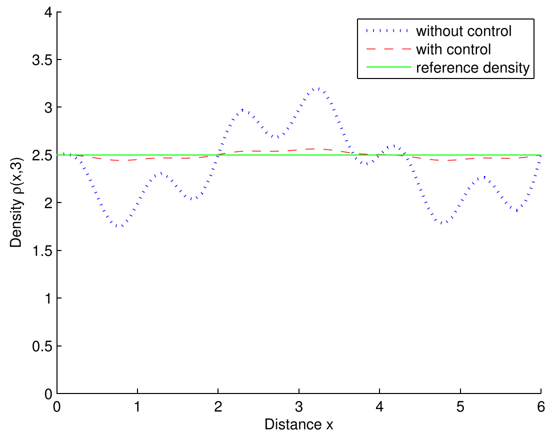

FIGURE 8. Density distribution when $t = 3s$.

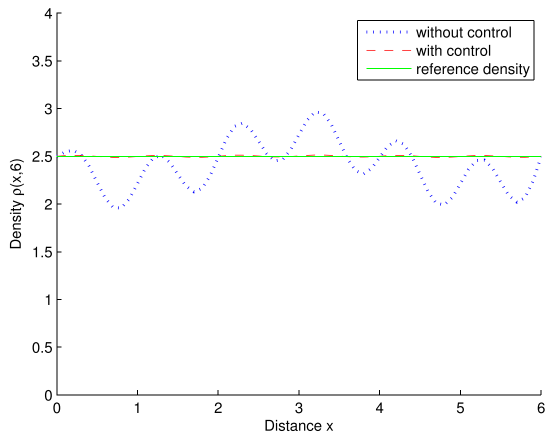

FIGURE 9. Density distribution when $t = 6s$.

Fig. 7 – Fig. 9 demonstrate the density distribution when $t = 0.5s$, $t = 3s$ and $t = 6s$, respectively. From Fig. 7, we note that the uncontrolled state density $\rho(x, t)$ reaches about 3.9 at x = 3.4, while the state density under the control input (20) is strictly constrained within the target range. Comparing Fig. 7 – Fig. 9, we see that the density under the control input approaches the reference density at a faster speed and reaches the reference density at 6s.

To illustrate the superiority of the proposed control, the sliding mode controller (SMC) designed in article [19] is used to control system (1) - (2) under the same initial conditions and reference density. The disturbance $\beta(x, t)$ is set as $\beta(x, t) = 1.5[\sin(2\pi x) + \sin(2\pi t)]$. Under the action of the two controllers, the density distributions at time $t = 0.5s$ and $t = 8s$ are shown in Fig. 10 and Fig. 11, respectively. When $t = 0.5s$, the maximum density with

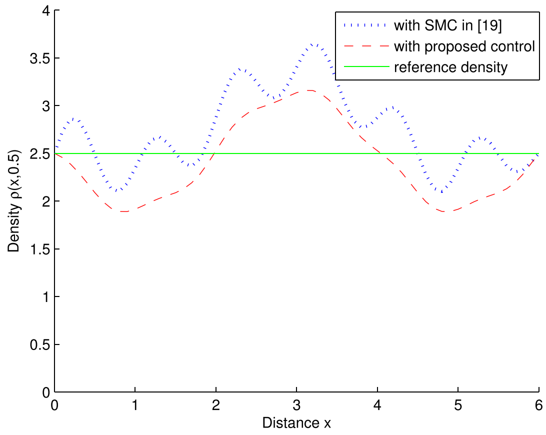

FIGURE 10. Density comparison when $t = 0.5s$.

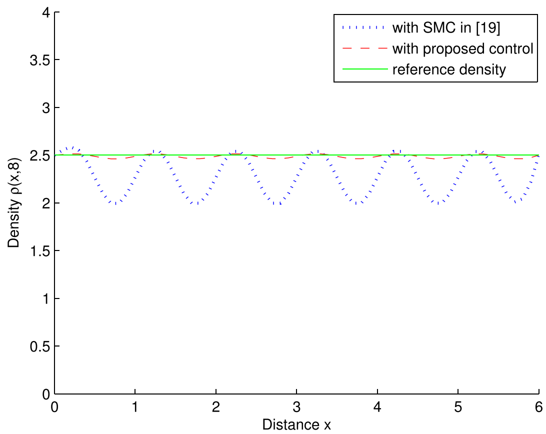

FIGURE 11. Density comparison when $t = 8s$.

the controller in [19] is 3.6896, which violates the density constraint $|\rho(x, t)| < 3.6$, and cannot stabilize to the reference density when $t = 8s$, as shown in Fig. 11. However, the density with the proposed controller in this paper is globally constrained within 3.6. Despite the increase of the disturbance, it can still stabilize to the reference density in about 8 seconds.

In summary, the control input (20) not only stabilizes the state density to the reference density, but also constrains it within the target range, i.e. $|\rho(x, t)| < 3.6$, which effectively avoids the problem of excessive local density caused by external disturbances and completes the established control target.

V. CONCLUSION

Tracking control for a disturbed crowd dynamics system described by partial differential equations has achieved. By designing a novel controller, the state density is asymptotically stabilized to the reference density and constrained in a safe range. The design idea of the unit sliding mode controller based on IBLF can provide a reference for solving the output constraint problem of other distributed parameter systems. In addition, the high-order sliding mode controllers have strong chatter suppression capabilities, so the design of high-order sliding mode controllers based on IBLF is an interesting topic.

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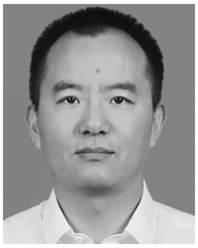

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