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TOPSIS Based Two-Sided Matching Under Interval-Valued Intuitionistic Fuzzy Environment in Virtual Reality Technology Transfer

QI YUE[1] AND LILI ZHANG[2]
[1]School of Information Management, Jiangxi University of Finance and Economics, Nanchang 330013, China
[2]School of Business, Dalian University of Technology–Panjin, Panjin 124221, China

Corresponding author: Lili Zhang (zhanglili18@126.com)

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ABSTRACT Bodies' behaviors remain important to the decision-making process of two-sided matching. The two-sided matching problem under interval-valued intuitionistic fuzzy environment is investigated from the perspective of bodies' behavior, i.e., the behavior of matching willingness. For solving the problem, a normalized interval-valued score function is firstly presented. Through using this function, interval-valued intuitionistic fuzzy number (IvIFN) preference matrices are converted to normalized interval-valued score matrices. The normalized interval-valued score matrices are then converted to score matrices. Based on score matrices, the matching willingness can be obtained by solving an optimal model. Based on score matrices and matching willingness matrix, the weighting score matrices are set up. Aiming at weighting score matrices, we use the TOPSIS method to calculate the closeness degrees. Based on closeness degree matrices and matching matrix, a two-sided matching (TsM) model is developed. Considering the same or different statuses of bodies, the TsM model is transformed to the one-goal TsM model. Through model solution, the optimal scheme of TsM can be obtained. Ultimately, the availability of TOPSIS based TsM method is illustrated by using a matching example of virtual reality technology.

INDEX TERMS Two-sided matching, interval-valued intuitionistic fuzzy number, bodies' behavior, model, virtual reality technology.

I. INTRODUCTION

Two-sided matching (TsM) is a well-known research direction of decision-making. It had been extensively employed in numerous fields, such as mechanical systems [1], complex product manufacturing tasks [2], green building technologies [3], content sharing in internet of vehicles [4], matching with the stars [5], stable job matching [6], marriage problems [7], and loan market [8]. As early as 1962, Gale and Shapley [9] have investigated two classical TsM model using preferences of ordinal numbers, i.e., stable marriage and college admissions. From reference [9], it is known that TsM focuses on obtaining the most suitable matching scheme(s) from two sets of bodies. Thereafter, various concepts, theories, methods, technologies, and applications are proposed [10]–[17]. Therefore, TsM research is very meaningful in theory and practice.

On the other hand, in decision-making process, owing to the increasing complexity of the society and economy, inadequacy of human knowledge and imprecise of judgement, the decision-makers or bodies may give their preferences using intuitionistic fuzzy sets (IFSs) rather than some exact values, such ordinal numbers, order relations, or linguistic variables [18]. Interval-valued intuitionistic fuzzy sets (IVIFSs) [19] can reflect human thinking more reasonable than IFSs, which is regarded as the generalization of IFSs. The degrees of membership, non-membership and hesitancy are expressed in interval numbers in [0, 1].

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Hence, TsM research using IVIFSs also has very important meanings.

The theory of IVIFSs proposed by Atanassov and Gargov [19] has gained a great attention in some research fields, such as distance measure of IVIFSs [18], ranking of IVIFSs [20], entropy of IVIFSs [21], aggregation operators of IVIFSs [22], correlation coefficient of IVIFSs [23], score functions of IVIFSs [24], envelopment analysis and preference fusion of IVIFSs [25]. And then, its application field is extended into decision-making. For example, the variable-based combinatorial optimization technique is presented to solve the IvIFN information MAGDM problems with combinatorial optimization characteristics [26]. With regarded to the multiple attribute decision making (MADM) problems under IVIF situations, the novel MADM approach is proposed [27]. A new interval-valued knowledge measure for the IvIFSs is presented firstly, and then it was applied into decision making problems in order to demonstrate it is simpler and more attractive in comparison with other existing measures [28].

However, the related theory of IFSs are rarely used in TsM research [29]. For example, a new similarity measure between TIFNs is proposed in reference [30], which has been used to develop an TsM approach. The developed new similarity measure between TrIFNs was also used to present an TsM approach [31]. The application of IVIFSs can be found in the following literature [29], [32]. A TsM model for the personnel-position matching problem is designed, which was based on several new intuitionistic fuzzy Choquet integral operators [32]. The TsM matching problem with IvIFNs and matching aspirations was studied [29]. But the TsM model developed in reference [32] may be only applied into the personnel-position matching problem. The interval-valued scores proposed in reference [29] may be less than zero, and the considered factors for determine matching aspirations in reference [29] may be incomplete. In addition, it is known that bodies' behaviors play an important role in TsM process. For this reason, this paper extended the application scope of IVIFSs to TsM field, and studied the TsM problem from bodies' behaviors using a matching example of virtual reality technology.

II. PRELIMINARIES

A. IvIFN

Definition 1 [19]: Let D be a domain of discourse, and P be a power set of interval [0, 1]. Then, an IvIFS could be represented in $\widehat{V} = \{< v,\ t_{\widehat{V}}(v),\ f_{\widehat{V}}(v) > \big|\ v \in D\}$, where $t_{\widehat{V}}(v) = [t^z_{\widehat{V}}(v), t^y_{\widehat{V}}(v)]$ and $f_{\widehat{V}}(v) = [f^z_{\widehat{V}}(v), f^y_{\widehat{V}}(v)]$ represent interval-valued membership degree and non-membership degree respectively, satisfying $t_{\widehat{V}}(v),\ f_{\widehat{V}}(v) \in P,\ t^y_{\widehat{V}}(v) + f^y_{\widehat{V}}(v) \in [0, 1]$.

Definition 2 [19]: If $h_{\widehat{V}}(v) = [h^z_{\widehat{V}}(v), h^y_{\widehat{V}}(v)] = 1 - t_{\widehat{V}}(v) - f_{\widehat{V}}(v)$, then $h_{\widehat{V}}(v)$ is known as hesitancy degree.

From Definition 2 and the operation rule of interval numbers, we know that for any $v \in D$, its hesitancy degree relative to $\widehat{V}$ can be computed below:

$$h_{\widehat{V}}(v) = [h^z_{\widehat{V}}(v), h^y_{\widehat{V}}(v)] = [1 - t^y_{\widehat{V}}(v) - f^y_{\widehat{V}}(v), 1 - t^z_{\widehat{V}}(v) - f^z_{\widehat{V}}(v)] \tag{1}$$

Evidently, $h_{\widehat{V}}(v) \in P,\ \forall v \in D$. In special, if $h_{\widehat{V}}(v) = [0, 0]$, then $\widehat{V}$ is degraded as a fuzzy set.

For later expedience, one element of IvIFS $\widehat{V}$ is taken out separately, which is noted as $\widehat{v} =< t_{\widehat{V}},\ f_{\widehat{V}} >$. Then, $\widehat{v} =< t_{\widehat{V}},\ f_{\widehat{V}} >$ is called an IvIFN.

Although some score functions of IvIFN are proposed, several minor errors may exist when comparing two IvIFNs. Furthermore, a normalized interval-valued score function of IvIFN, which belong to [0, 1], will be defined.

Definition 3: Let $\widehat{v} =< t_{\widehat{V}},\ f_{\widehat{V}} >$ be an IvIFN, an interval-valued score function of $\widehat{v}$ is expressed as

$$\tilde{s}_{\widehat{v}} = (t_{\widehat{V}} + \alpha_{\widehat{V}} h_{\widehat{V}}) - (f_{\widehat{V}} + \beta_{\widehat{V}} h_{\widehat{V}}) - \gamma_{\widehat{V}} h_{\widehat{V}} \tag{2}$$

where $\alpha_{\widehat{V}}$, $\beta_{\widehat{V}}$ and $\gamma_{\widehat{V}}$ $(0 \leq \alpha_{\widehat{V}},\ \beta_{\widehat{V}},\ \gamma_{\widehat{V}} \leq 1)$ represent support ratio, opposition ratio, and abstention ratio of $h_{\widehat{V}}$ respectively, satisfying

$$\alpha_{\widehat{V}} + \beta_{\widehat{V}} + \gamma_{\widehat{V}} = 1 \tag{3}$$

Remark 1: Ratios $\alpha_{\widehat{V}}$, $\beta_{\widehat{V}}$ and $\gamma_{\widehat{V}}$ could be determined on the basis of preferences of IvIFNs, which should reflect the result of TsM and will be displayed in Section 4.1.

From Eqs. (2) and (3), we have

$$\tilde{s}_{\widehat{v}} = (t_{\widehat{V}} - f_{\widehat{V}}) + (2\alpha_{\widehat{V}} - 1) h_{\widehat{V}} \tag{4}$$

It is easy to conclude that $\tilde{s}_{\widehat{v}} \subseteq [-1, 1]$. In order to eliminate the influence of negative interval-valued score functions when they are aggregated, the normalized interval-valued score function is introduced below:

Definition 4: Let $\widehat{v} =< t_{\widehat{V}},\ f_{\widehat{V}} >$ be an IvIFN, the normalized interval-valued score function can be expressed as

$$\bar{s}_{\widehat{v}} = \left(\frac{\tilde{s}_{\widehat{v}} + [1, 1]}{2}\right)^{\chi} \tag{5}$$

where parameter $\chi \geq 0$. Obviously, $\bar{s}_{\widehat{v}} \subseteq [0, 1]$.

B. TsM

The concept of TsM is provided in many literatures. This paper uses the following related notations. Let $X = \{X_1, \ldots, X_k, \ldots, X_m\}$ and $Y = \{Y_1, \ldots, Y_l, \ldots, Y_n\}$ be two independent body set, where X_k (Y_l) signifies the kth body on side X (the lth body on side Y). Give this hypothesis $2 \leq m \leq n$, and set $M = \{1, \ldots, k, \ldots, m\}, N = \{1, \ldots, l, \ldots, n\}$.

Definition 5 [33]: Let $\Omega : X \cup Y \rightarrow X \cup Y$ be a one-one mapping. If mapping Ω satisfies the following properties:

i) $\Omega(X_k) \in Y$, ii) $\Omega(Y_l) \in X \cup \{Y_l\}$, iii) $\Omega(X_k) = Y_l$ iff $\Omega(Y_l) = X_k, \forall X_k \in X, \forall Y_l \in Y$, then Ω is called a TsM. Thereinto, $\Omega(X_k) = Y_l$ means that $\Omega(X_k, Y_l)$ is a matching pair (MP), which indicates body X_k is matched with body Y_l, $\Omega(Y_l) = Y_l$ means that $\Omega(Y_l, Y_l)$ is a single matching pair (SMP), which indicates body Y_l is single.

Definition 6 [33]: For a TsM $\Omega : X \cup Y \rightarrow X \cup Y$, it also could be represented in $\Omega = \Omega_{MP} \cup \Omega_{SMP}$, where Ω_{MP} signifies MP set, Ω_{SMP} signifies SMP set.

III. TOPSIS BASED TsM DECISION FOR IvIFNs CONSIDERING MATCHING WILLINGNESS

A. TsM PROBLEM FOR IvIFNs CONSIDERING MATCHING WILLINGNESS

The following problem of TsM is involved. Let $\widehat{V}^X = [\widehat{v}_{kl}^X]_{m\times n}$ be IvIFN matrix of side X, where IvIFN $\widehat{v}_{kl}^X =< [t_{\widehat{v}_{kl}^X}^z, t_{\widehat{v}_{kl}^X}^y], [f_{\widehat{v}_{kl}^X}^z, f_{\widehat{v}_{kl}^X}^y] >$. Thereinto, $[t_{\widehat{v}_{kl}^X}^z, t_{\widehat{v}_{kl}^X}^y]$ indicates the interval-valued satisfied degree of body X_k towards body Y_l, and $[f_{\widehat{v}_{kl}^X}^z, f_{\widehat{v}_{kl}^X}^y]$ indicates the interval-valued dissatisfied degree of body X_k towards body Y_l. Let $\widehat{V}^Y = [\widehat{v}_{kl}^Y]_{m\times n}$ be IvIFN matrix of side Y, where IvIFN $\widehat{v}_{kl}^Y =< [t_{\widehat{v}_{kl}^Y}^z, t_{\widehat{v}_{kl}^Y}^y], [f_{\widehat{v}_{kl}^Y}^z, f_{\widehat{v}_{kl}^Y}^y] >$. Thereinto, $[t_{\widehat{v}_{kl}^Y}^z, t_{\widehat{v}_{kl}^Y}^y]$ indicates the interval-valued satisfied degree of body Y_l towards body X_k, and $[f_{\widehat{v}_{kl}^Y}^z, f_{\widehat{v}_{kl}^Y}^y]$ indicates the interval-valued dissatisfied degree of body Y_l towards body X_k. Let $W = [w_{kl}]_{m\times n}$ be matching willingness matrix between X and Y. Thereinto, w_{kl} indicates the matching willingness between body X_k and body Y_l. Let Ω^* be the "optimum" TsM.

Remark 2: In this paper, bodies' behaviors are represented by the matching willingness. Matching willingness w_{kl} could be computed on the basis of satisfied degrees of bodies X_k and Y_l, which should satisfy the features of non-negativity and standardization. Its computation approach will be displayed in Section 4.2.

Motived by the above statement, the problem displayed here is how to acquire "optimum" TsM Ω^* on the basis of IvIFN matrices $\widehat{V}^X = [\widehat{v}_{kl}^X]_{m\times n}$ and $\widehat{V}^Y = [\widehat{v}_{kl}^Y]_{m\times n}$, matching willingness matrix $W = [w_{kl}]_{m\times n}$. The research procedure for the above-mentioned problem is presented in Figure 1.

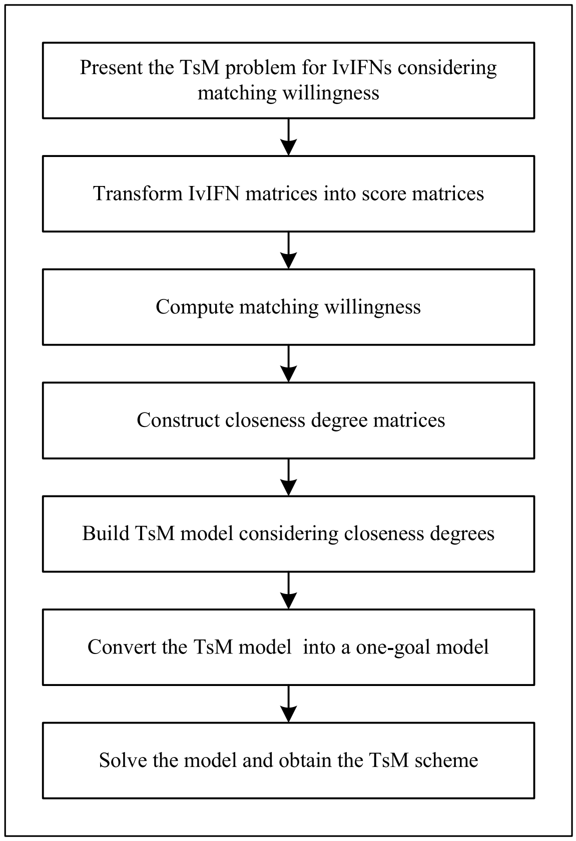

FIGURE 1. Research procedure for the above-mentioned problem.

B. ESTABLISHMENT OF SCORE MATRICES

First, by using Eqs. (4) and (5), IvIFN matrices $\widehat{V}^X = [\widehat{v}_{kl}^X]_{m\times n}$ and $\widehat{V}^Y = [\widehat{v}_{kl}^Y]_{m\times n}$ are converted to normalized interval-valued score matrices $\bar{S}_{\widehat{V}^X} = [\bar{s}_{\widehat{v}_{kl}^X}]_{m\times n} = [[s_{\widehat{v}_{kl}^X}^z, s_{\widehat{v}_{kl}^X}^y]]_{m\times n}$ and $\bar{S}_{\widehat{V}^Y} = [\bar{s}_{\widehat{v}_{kl}^Y}]_{m\times n} = [[s_{\widehat{v}_{kl}^Y}^z, s_{\widehat{v}_{kl}^Y}^y]]_{m\times n}$, where normalized interval-valued scores $\bar{s}_{\widehat{v}_{kl}^X} = [s_{\widehat{v}_{kl}^X}^z, s_{\widehat{v}_{kl}^X}^y]$ and $\bar{s}_{\widehat{v}_{kl}^Y} = [s_{\widehat{v}_{kl}^Y}^z, s_{\widehat{v}_{kl}^Y}^y]$ can be computed by (6) and (7), as shown at the bottom of the next page.

Remark 3: From Remark 1, we know that $\alpha_{\widehat{v}_{kl}^X}$, $\beta_{\widehat{v}_{kl}^X}$ and $\gamma_{\widehat{v}_{kl}^X}$ should reflect TsM result $\Omega(X_k) = Y_l$. Furthermore, the allocation ratio of $\bar{h}_{\widehat{v}_{kl}^X} = [h_{\widehat{v}_{kl}^X}^z, h_{\widehat{v}_{kl}^X}^y]$ may be provided as follows:

$$\alpha_{\widehat{v}_{kl}^X} = \frac{(1-\theta_k^X)t_{\widehat{v}_{kl}^X}^z + \theta_k^X t_{\widehat{v}_{kl}^X}^y}{\sum_{j=1}^{n}((1-\theta_k^X)t_{\widehat{v}_{kj}^X}^z + \theta_k^X t_{\widehat{v}_{kj}^X}^y)} \tag{8}$$

$$\beta_{\widehat{v}_{kl}^X} = \frac{(1-\theta_k^X)f_{\widehat{v}_{kl}^X}^z + \theta_k^X f_{\widehat{v}_{kl}^X}^y}{\sum_{j=1}^{n}((1-\theta_k^X)f_{\widehat{v}_{kj}^X}^z + \theta_k^X f_{\widehat{v}_{kj}^X}^y)} \tag{9}$$

$$\gamma_{\widehat{v}_{kl}^X} = 1 - \alpha_{\widehat{v}_{kl}^X} - \beta_{\widehat{v}_{kl}^X} \tag{10}$$

Thereinto, $\alpha_{\widehat{v}_{kl}^X}$ can be interpreted as the support ratio of $\Omega(X_k) = Y_l$, $\beta_{\widehat{v}_{kl}^X}$ can be interpreted as the opposition ratio of $\Omega(X_k) = Y_l$ and $\gamma_{\widehat{v}_{kl}^X}$ can be interpreted as the neutral ratio of $\Omega(X_k) = Y_l$; θ_k^X represents the optimism coefficient of body X_k. Similarly, the allocation ratio of $\bar{h}_{\widehat{v}_{kl}^Y} = [h_{\widehat{v}_{kl}^Y}^z, h_{\widehat{v}_{kl}^Y}^y]$ may be also provided as follows:

$$\alpha_{\widehat{v}_{kl}^Y} = \frac{(1-\theta_l^Y)t_{\widehat{v}_{kl}^Y}^z + \theta_l^Y t_{\widehat{v}_{kl}^Y}^y}{\sum_{i=1}^{m}((1-\theta_l^Y)t_{\widehat{v}_{il}^Y}^z + \theta_l^Y t_{\widehat{v}_{il}^Y}^y)} \tag{11}$$

$$\beta_{\overset{\frown}{v}^{Y}_{kl}} = \frac{(1-\theta_l^Y) f^z_{\overset{\frown}{v}^{Y}_{kl}} + \theta_l^Y f^y_{\overset{\frown}{v}^{Y}_{kl}}}{\sum_{i=1}^{m} ((1-\theta_l^Y) f^z_{\overset{\frown}{v}^{Y}_{il}} + \theta_l^Y f^y_{\overset{\frown}{v}^{Y}_{il}})} \tag{12}$$

$$\gamma_{\overset{\frown}{v}^{Y}_{kl}} = 1 - \alpha_{\overset{\frown}{v}^{Y}_{kl}} - \beta_{\overset{\frown}{v}^{Y}_{kl}} \tag{13}$$

Thereinto, $\alpha_{\overset{\frown}{v}^{Y}_{kl}}$ can be interpreted as the support ratio of $\Omega(X_k) = Y_l$, $\beta_{\overset{\frown}{v}^{Y}_{kl}}$ can be interpreted as the opposition ratio of $\Omega(X_k) = Y_l$, $\gamma_{\overset{\frown}{v}^{Y}_{kl}}$ can be interpreted as the neutral ratio of $\Omega(X_k) = Y_l$; θ_l^Y represents the optimism coefficient of body Y_l.

Furthermore, normalized interval-valued score matrices $\bar{S}_{\overset{\frown}{V}^{X}} = [\bar{s}_{\overset{\frown}{v}^{X}_{kl}}]_{m\times n}$ and $\bar{S}_{\overset{\frown}{V}^{Y}} = [\bar{s}_{\overset{\frown}{v}^{Y}_{kl}}]_{m\times n}$ is converted to score matrices $S_{\overset{\frown}{V}^{X}} = [s_{\overset{\frown}{v}^{X}_{kl}}]_{m\times n}$ and $S_{\overset{\frown}{V}^{Y}} = [s_{\overset{\frown}{v}^{Y}_{kl}}]_{m\times n}$, where scores $s_{\overset{\frown}{v}^{X}_{kl}}$ and $s_{\overset{\frown}{v}^{Y}_{kl}}$ is expressed by:

$$s_{\overset{\frown}{v}^{X}_{kl}} = (1-\theta^X) s^z_{\overset{\frown}{v}^{X}_{kl}} + \theta^X s^y_{\overset{\frown}{v}^{X}_{kl}} \tag{14}$$

$$s_{\overset{\frown}{v}^{Y}_{kl}} = (1-\theta^Y) s^z_{\overset{\frown}{v}^{Y}_{kl}} + \theta^Y s^y_{\overset{\frown}{v}^{Y}_{kl}} \tag{15}$$

Thereinto, θ^X represents the compositive optimism coefficient of bodies on side X, and θ^Y represents the compositive optimism coefficient of bodies on side Y.

C. COMPUTATION OF MATCHING WILLINGNESS

In above analysis, matching willingness X_5 is unknown. For obtaining w_{kl}, an analysis is provided as follows.

On one hand, if absolute difference $\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|$ gets larger and larger, then the difference of satisfied degree between body X_k and body Y_l also gets larger and larger. Hence, matching willingness w_{kl} should get smaller and smaller, and vice versa. On the other hand, if $s_{\overset{\frown}{v}^{X}_{kl}}$ (or $s_{\overset{\frown}{v}^{Y}_{kl}}$) gets larger and larger, then the satisfied degree of body X_k towards body Y_l (or the satisfied degree of body Y_1 towards body X_k also gets larger and larger. Hence, matching willingness w_{kl} should get larger and larger, and vice versa. Therefore, matching willingness w_{kl} is inverse proportion to $\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|$, and is proportional to $s_{\overset{\frown}{v}^{X}_{kl}}$ and $s_{\overset{\frown}{v}^{Y}_{kl}}$. Furthermore, selection of w_{kl} should enable the following total weighting score for all bodies of two sides (noted as $R_{X\leftrightarrow Y}$) greatest:

$$R_{X\leftrightarrow Y} = \sum_{k=1}^{m}\sum_{l=1}^{n} \frac{s_{\overset{\frown}{v}^{X}_{kl}} s_{\overset{\frown}{v}^{Y}_{kl}} w_{kl}}{\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|} \tag{16}$$

It is noted that $\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right| = 0$ occurs in some cases. In this case, Eq. (16) has no meaning. For dealing with the case, we change $\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|$ into $\eta^{\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|}$, where η is given in advance, $\eta > 1$. Hence, Eq. (16) can be represented by:

$$R_{X\leftrightarrow Y} = \sum_{k=1}^{m}\sum_{l=1}^{n} \frac{s_{\overset{\frown}{v}^{X}_{kl}} s_{\overset{\frown}{v}^{Y}_{kl}} w_{kl}}{\eta^{\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|}} \tag{17}$$

Therefore, an optimum model (T-1) for obtaining w_{kl} could be set up, i.e.,

$$\text{(T-1)}\begin{cases} \max R_{X\leftrightarrow Y} = \sum_{k=1}^{m}\sum_{l=1}^{n} \frac{s_{\overset{\frown}{v}^{X}_{kl}} s_{\overset{\frown}{v}^{Y}_{kl}} w_{kl}}{\eta^{\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|}} \\ \text{s.t.} \sum_{i=1}^{m}\sum_{l=1}^{n} w_{kl}^2 = 1, k \in M; w_{kl} \in [0,1], k \in M, l \in N \end{cases}$$

Theorem 1: The optimum solution of model (T-1) is expressed by the following:

$$w_{kl} = \frac{s_{\overset{\frown}{v}^{X}_{kl}} s_{\overset{\frown}{v}^{Y}_{kl}}}{\eta^{\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|} \sqrt{\sum_{k=1}^{m}\sum_{l=1}^{n} \frac{\left(s_{\overset{\frown}{v}^{X}_{kl}} s_{\overset{\frown}{v}^{Y}_{kl}}\right)^2}{\eta^{2\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|}}}} \tag{18}$$

Proof: Let Lagrange function $L = \sum_{k=1}^{m}\sum_{l=1}^{n} \frac{s_{\overset{\frown}{v}^{X}_{kl}} s_{\overset{\frown}{v}^{Y}_{kl}} w_{kl}}{\eta^{\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|}} + \lambda\left(\sum_{i=1}^{m}\sum_{l=1}^{n} w_{kl}^2 - 1\right)$. Furthermore, let $\frac{\partial L}{\partial w_{kl}} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$, then we have

$$\frac{s_{\overset{\frown}{v}^{X}_{kl}} s_{\overset{\frown}{v}^{Y}_{kl}}}{\eta^{\left| s_{\overset{\frown}{v}^{X}_{kl}} - s_{\overset{\frown}{v}^{Y}_{kl}} \right|}} + 2\lambda w_{kl} = 0 \tag{19}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{n} w_{kl}^2 - 1 = 0 \tag{20}$$

$$[\bar{s}^z_{\overset{\frown}{v}^{X}_{kl}}, \bar{s}^y_{\overset{\frown}{v}^{X}_{kl}}] = \left(\frac{([t^z_{\overset{\frown}{v}^{X}_{kl}}, t^y_{\overset{\frown}{v}^{X}_{kl}}] - [f^z_{\overset{\frown}{v}^{X}_{kl}}, f^y_{\overset{\frown}{v}^{X}_{kl}}]) + (2\alpha_{\overset{\frown}{v}^{X}_{kl}} - 1)[h^z_{\overset{\frown}{v}^{X}_{kl}}, h^y_{\overset{\frown}{v}^{X}_{kl}}] + [1,1]}{2}\right)^{\chi} \tag{6}$$

$$[\bar{s}^z_{\overset{\frown}{v}^{Y}_{kl}}, \bar{s}^y_{\overset{\frown}{v}^{Y}_{kl}}] = \left(\frac{[t^z_{\overset{\frown}{v}^{Y}_{kl}}, t^y_{\overset{\frown}{v}^{Y}_{kl}}] - [f^z_{\overset{\frown}{v}^{Y}_{kl}}, f^y_{\overset{\frown}{v}^{Y}_{kl}}]) + (2\alpha_{\overset{\frown}{v}^{Y}_{kl}} - 1)[h^z_{\overset{\frown}{v}^{Y}_{kl}}, h^y_{\overset{\frown}{v}^{Y}_{kl}}] + [1,1]}{2}\right)^{\chi} \tag{7}$$

Take Eq. (19) into Eq. (20), then we find:

$$\lambda = -\frac{1}{2}\sqrt{\sum_{k=1}^{m}\sum_{l=1}^{n}\frac{\left(s_{\overset{\frown}{v}_{kl}^{X}}s_{\overset{\frown}{v}_{kl}^{Y}}\right)^2}{\eta^{2\left|s_{\overset{\frown}{v}_{kl}^{X}}-s_{\overset{\frown}{v}_{kl}^{Y}}\right|}}} \tag{21}$$

Take Eq. (21) into Eq. (19), then we find:

$$w_{kl} = \frac{s_{\overset{\frown}{v}_{kl}^{X}}s_{\overset{\frown}{v}_{kl}^{Y}}}{\eta^{\left|s_{\overset{\frown}{v}_{kl}^{X}}-s_{\overset{\frown}{v}_{kl}^{Y}}\right|}\sqrt{\sum_{k=1}^{m}\sum_{l=1}^{n}\frac{\left(s_{\overset{\frown}{v}_{kl}^{X}}s_{\overset{\frown}{v}_{kl}^{Y}}\right)^2}{\eta^{2\left|s_{\overset{\frown}{v}_{kl}^{X}}-s_{\overset{\frown}{v}_{kl}^{Y}}\right|}}}} \tag{22}$$

Through solving model (T-1), Eq. (22) is obtained. If the constraint of normalization is considered, then w_{kl}^* can be further computed by:

$$w_{kl}^* = \frac{w_{kl}}{\sum_{j=1}^{n} w_{kj}} \tag{23}$$

Using Eq. (23), matching willingness matrix $W = [w_{kl}^*]_{m\times n}$ is built.

D. CONSTRUCTION OF CLOSENESS DEGREE MATRICES

In this subsection, the TOPSIS technology is adopted to construct the closeness degree matrices. On the basis of score matrices $S_{\overset{\frown}{V}^X} = [s_{\overset{\frown}{v}_{kl}^{X}}]_{m\times n}$ and $S_{\overset{\frown}{V}^Y} = [s_{\overset{\frown}{v}_{kl}^{Y}}]_{m\times n}$, and matching willingness matrix $W = [w_{kl}^*]_{m\times n}$, the weighting score matrices $\bar{S}_{\overset{\frown}{V}^X} = [\bar{s}_{\overset{\frown}{v}_{kl}^{X}}]_{m\times n}$ and $\bar{S}_{\overset{\frown}{V}^Y} = [\bar{s}_{\overset{\frown}{v}_{kl}^{Y}}]_{m\times n}$ are constructed, where $\bar{s}_{\overset{\frown}{v}_{kl}^{X}}$ and $\bar{s}_{\overset{\frown}{v}_{kl}^{Y}}$ are expressed by

$$\bar{s}_{\overset{\frown}{v}_{kl}^{X}} = w_{kl}^* s_{\overset{\frown}{v}_{kl}^{X}} \tag{24}$$

$$\bar{s}_{\overset{\frown}{v}_{kl}^{Y}} = w_{kl}^* s_{\overset{\frown}{v}_{kl}^{Y}} \tag{25}$$

Aiming at weighting score matrix $\bar{S}_{\overset{\frown}{V}^X} = [\bar{s}_{\overset{\frown}{v}_{kl}^{X}}]_{m\times n}$, its specific form can be demonstrated by

$$\bar{S}_{\overset{\frown}{V}^X} = \begin{array}{c} \\ X_1 \\ X_2 \\ \cdots \\ X_m \end{array}\begin{array}{c} \begin{array}{cccc} Y_1 & Y_2 & \cdots & Y_n \end{array} \\ \begin{bmatrix} \bar{s}_{\overset{\frown}{v}_{11}^{X}} & \bar{s}_{\overset{\frown}{v}_{12}^{X}} & \cdots & \bar{s}_{\overset{\frown}{v}_{1n}^{X}} \\ \bar{s}_{\overset{\frown}{v}_{21}^{X}} & \bar{s}_{\overset{\frown}{v}_{22}^{X}} & \cdots & \bar{s}_{\overset{\frown}{v}_{2n}^{X}} \\ \cdots & \cdots & \cdots & \cdots \\ \bar{s}_{\overset{\frown}{v}_{m1}^{X}} & \bar{s}_{\overset{\frown}{v}_{m2}^{X}} & \cdots & \bar{s}_{\overset{\frown}{v}_{mn}^{X}} \end{bmatrix}_{m\times n} \end{array}$$

Then, the positive ideal vector from X to Y can be obtained, where $s^{*X} = \left(s_1^{*X}, s_2^{*X}, \ldots, s_m^{*X}\right)^T$. Here, the element s_k^{*X} can be computed as

$$s_k^{*X} = \max_{l\in N}\left\{\bar{s}_{\overset{\frown}{v}_{kl}^{X}}\right\} \tag{26}$$

The negative ideal vector from X to Y can also be obtained, where $s^{\circ X} = \left(s_1^{\circ X}, s_2^{\circ X}, \ldots, s_m^{\circ X}\right)^T$. Here, the element $s_k^{\circ X}$ can also be computed as

$$s_k^{\circ X} = \min_{l\in N}\left\{\bar{s}_{\overset{\frown}{v}_{kl}^{X}}\right\} \tag{27}$$

Moreover, the ideal distance matrix from X to Y is obtained, where $D^{*X} = [d_{kl}^{*X}]_{m\times n}$. Here, d_{kl}^{*X} is computed as

$$d_{kl}^{*X} = s_k^{*X} - \bar{s}_{\overset{\frown}{v}_{kl}^{X}} \tag{28}$$

The negative distance matrix from X to Y is also obtained, where $D^{\circ X} = [d_{kl}^{\circ X}]_{m\times n}$. Here, $d_{kl}^{\circ X}$ is computed as

$$d_{kl}^{\circ X} = \bar{s}_{\overset{\frown}{v}_{kl}^{X}} - s_k^{\circ X} \tag{29}$$

Then, the closeness degree matrix from X to Y is constructed, where $C^X = [c_{kl}^X]_{m\times n}$. Here, closeness degree c_{kl}^X is computed as

$$c_{kl}^X = \frac{d_{kl}^{\circ X}}{d_{kl}^{\circ X} + d_{kl}^{*X}} \tag{30}$$

From Eqs. (26)-(30), it is known that the larger c_{kl}^X is, the larger satisfied degree of X_k to Y_l will be.

In a similar way, aiming at weighting score matrix $\bar{S}_{\overset{\frown}{V}^Y} = [\bar{s}_{\overset{\frown}{v}_{kl}^{Y}}]_{m\times n}$, its specific form can be demonstrated by

$$\bar{S}_{\overset{\frown}{V}^Y} = \begin{array}{c} \\ X_1 \\ X_2 \\ \cdots \\ X_m \end{array}\begin{array}{c} \begin{array}{cccc} Y_1 & Y_2 & \cdots & Y_n \end{array} \\ \begin{bmatrix} \bar{s}_{\overset{\frown}{v}_{11}^{Y}} & \bar{s}_{\overset{\frown}{v}_{12}^{Y}} & \cdots & \bar{s}_{\overset{\frown}{v}_{1n}^{Y}} \\ \bar{s}_{\overset{\frown}{v}_{21}^{Y}} & \bar{s}_{\overset{\frown}{v}_{22}^{Y}} & \cdots & \bar{s}_{\overset{\frown}{v}_{2n}^{Y}} \\ \cdots & \cdots & \cdots & \cdots \\ \bar{s}_{\overset{\frown}{v}_{m1}^{Y}} & \bar{s}_{\overset{\frown}{v}_{m2}^{Y}} & \cdots & \bar{s}_{\overset{\frown}{v}_{mn}^{Y}} \end{bmatrix}_{m\times n} \end{array}$$

Then, the positive ideal vector from Y to X is obtained, where $s^{*Y} = \left(s_1^{*Y}, s_2^{*Y}, \ldots, s_n^{*Y}\right)$. Here, the element s_l^{*Y} can be computed as

$$s_l^{*Y} = \max_{k\in M}\left\{\bar{s}_{\overset{\frown}{v}_{kl}^{Y}}\right\} \tag{31}$$

The negative ideal vector from Y to X can also be obtained, where $s^{\circ Y} = \left(s_1^{\circ Y}, s_2^{\circ Y}, \ldots, s_n^{\circ Y}\right)$. Here, the element $s_l^{\circ Y}$ can also be computed as

$$s_l^{\circ Y} = \min_{k\in M}\left\{\bar{s}_{\overset{\frown}{v}_{kl}^{Y}}\right\} \tag{32}$$

Moreover, the ideal distance matrix from Y to X is obtained, where $D^{*Y} = [d_{kl}^{*Y}]_{m\times n}$. Here, d_{kl}^{*Y} is computed as

$$d_{kl}^{*Y} = s_l^{*Y} - \bar{s}_{\overset{\frown}{v}_{kl}^{Y}} \tag{33}$$

The negative distance matrix from Y to X is also obtained, where $D^{\circ Y} = [d_{kl}^{\circ Y}]_{m\times n}$. Here, $d_{kl}^{\circ Y}$ is computed as

$$d_{kl}^{\circ Y} = \bar{s}_{\overset{\frown}{v}_{kl}^{Y}} - s_l^{\circ Y} \tag{34}$$

Then, the closeness degree matrix from Y to X is constructed, where $C^Y = [c_{kl}^Y]_{m\times n}$. Here, closeness degree c_{kl}^Y is computed as

$$c_{kl}^Y = \frac{d_{kl}^{\circ Y}}{d_{kl}^{\circ Y} + d_{kl}^{*Y}} \tag{35}$$

From Eqs. (31)-(35), it is known that the larger c_{kl}^{Y} is, the larger satisfied degree of Y_l to X_k will be.

E. BUILDING OF TsM MODEL CONSIDERING CLOSENESS DEGREES

A 0-1 variable z_{ij} is given in the first place, where $z_{kl} = \begin{cases} 1, \Omega(X_k) = Y_l \\ 0, \Omega(X_k) = Y_l \end{cases}$. After that, a matching matrix $Z = [z_{kl}]_{m\times n}$ is established. On the basis of closeness degree matrices $C^X = [c_{kl}^X]_{m\times n}$ and $C^Y = [c_{kl}^Y]_{m\times n}$, and matching matrix $Z = [z_{kl}]_{m\times n}$, a TsM model will be developed. To maximize the closeness degrees, the following TsM model (T-2) under the TsM constraints is built, i.e.,

$$
\text{(T-2)} \begin{cases} \max T_{X_k} = \sum_{l=1}^{n} c_{kl}^X z_{kl}, k \in M \\ \max T_{Y_l} = \sum_{k=1}^{m} c_{kl}^Y z_{kl}, l \in N \\ \text{s.t.} \sum_{l=1}^{n} z_{kl} = 1, k \in M; \\ \sum_{k=1}^{m} z_{kl} \le 1, l \in N; \\ z_{kl} = 0 \text{ or } 1, k \in M, l \in N \end{cases}
$$

F. SOLUTION OF TsM MODEL CONSIDERING CLOSENESS DEGREES

If we consider that all the bodies are in the same position, then model (T-2) is converted to the one-goal TsM model (T-3):

$$
\text{(T-3)} \begin{cases} \max T = \sum_{k=1}^{m} \sum_{l=1}^{n} (c_{kl}^X + c_{kl}^Y) z_{kl} \\ \text{s.t.} \sum_{l=1}^{n} z_{kl} = 1, k \in M; \\ \sum_{k=1}^{m} z_{kl} \le 1, l \in N; \\ z_{kl} = 0 \text{ or } 1, k \in M, l \in N \end{cases}
$$

If we consider that all the bodies are not exactly equal, then the technique of linear weighting can be employed. Let w_k^X and w_l^Y be the weights for bodies X_k and Y_l respectively, such that $\sum_k w_k^X + \sum_l w_l^Y = 1$, then model (T-2) is turned into the one-goal TsM Model (T-4):

$$
\text{(T-4)} \begin{cases} \max T = \sum_{k=1}^{m} \sum_{l=1}^{n} (w_k^X c_{kl}^X + w_l^Y c_{kl}^Y) z_{kl} \\ \text{s.t.} \sum_{l=1}^{n} z_{kl} = 1, k \in M; \\ \sum_{k=1}^{m} z_{kl} \le 1, l \in N; \\ z_{kl} = 0 \text{ or } 1, k \in M, l \in N \end{cases}
$$

Via solving Model (T-3) or (T-4), optimum TsM matrix $Z^* = [z_{kl}^*]_{m\times n}$ is acquired. On the basis of matrix $Z^* = [z_{kl}^*]_{m\times n}$, optimum TsM scheme Ω^* can be acquired.

G. PROCEDURE FOR TOPSIS BASED TsM

Above all, a procedure for TOPSIS based TsM for IvIFNs is put forward below:

Step 1: Convert IvIFN matrix $\widehat{V}^X = [\widehat{v}_{kl}^X]_{m\times n}$ into normalized interval-valued score matrix $\bar{S}_{\widehat{V}^X} = [\bar{s}_{\widehat{v}_{kl}^X}]_{m\times n}$ via using Eqs. (6) and (8); Convert IvIFN matrix $\widehat{V}^Y = [\widehat{v}_{kl}^Y]_{m\times n}$ into normalized interval-valued score matrix $\bar{S}_{\widehat{V}^Y} = [\bar{s}_{\widehat{v}_{kl}^Y}]_{m\times n}$ via Eqs. (7) and (11).

Step 2: Convert normalized interval-valued score matrices $\bar{S}_{\widehat{V}^X} = [\bar{s}_{\widehat{v}_{kl}^X}]_{m\times n}$ and $\bar{S}_{\widehat{V}^Y} = [\bar{s}_{\widehat{v}_{kl}^Y}]_{m\times n}$ into score matrices $S_{\widehat{V}^X} = [s_{\widehat{v}_{kl}^X}]_{m\times n}$ and $S_{\widehat{V}^Y} = [s_{\widehat{v}_{kl}^Y}]_{m\times n}$ via Eqs. (14) and (15) respectively.

Step 3: Acquire matching willingness matrix $W = [w_{kl}^*]_{m\times n}$ via Eqs. (22) and (23).

Step 4: Construct the weighting score matrices $\bar{S}_{\widehat{V}^X} = [\bar{s}_{\widehat{v}_{kl}^X}]_{m\times n}$ and $\bar{S}_{\widehat{V}^Y} = [\bar{s}_{\widehat{v}_{kl}^Y}]_{m\times n}$ on the basis of score matrices $S_{\widehat{V}^X} = [s_{\widehat{v}_{kl}^X}]_{m\times n}$ and $S_{\widehat{V}^Y} = [s_{\widehat{v}_{kl}^Y}]_{m\times n}$, matching willingness matrix $W = [w_{kl}^*]_{m\times n}$ via Eqs. (24) and (25), respectively.

Step 5: Compute the positive and negative ideal vectors $s^{*X} = (s_1^{*X}, s_2^{*X}, \ldots, s_m^{*X})^T$ and $s^{\circ X} = (s_1^{\circ X}, s_2^{\circ X}, \ldots, s_m^{\circ X})^T$ on the basis of weighting score matrix $\bar{S}_{\widehat{V}^X} = [\bar{s}_{\widehat{v}_{kl}^X}]_{m\times n}$ via Eqs. (26) and (27) respectively.

Step 6: Compute the closeness degree matrix $C^X = [c_{kl}^X]_{m\times n}$ via Eqs. (28)-(30).

Step 7: Compute the positive and negative ideal vectors $s^{*Y} = (s_1^{*Y}, s_2^{*Y}, \ldots, s_n^{*Y})$ and $s^{\circ Y} = (s_1^{\circ Y}, s_2^{\circ Y}, \ldots, s_n^{\circ Y})$ on the basis of weighted score matrix $\bar{S}_{\widehat{V}^Y} = [\bar{s}_{\widehat{v}_{kl}^Y}]_{m\times n}$ via Eqs. (31) and (32) respectively.

Step 8: Compute the closeness degree matrix $C^Y = [c_{kl}^Y]_{m\times n}$ via Eqs. (33)-(35).

Step 9: Construct a TsM Model (T-2) on the basis of closeness degree matrices $C^X = [c_{kl}^X]_{m\times n}$ and $C^Y = [c_{kl}^Y]_{m\times n}$, and matching matrix $Z = [z_{kl}]_{m\times n}$.

Step 10: Transform TsM Model (T-2) into TsM Model (T-3).

Step 11: Acquire optimum TsM scheme Ω^* via solving model (T-3).

IV. A MATCHING EXAMPLE OF VIRTUAL REALITY TECHNOLOGY

An instance of supply-demand matching for virtual reality technology is provided in the section. An intermediary company in nanchang mainly provides the intermediary services for virtual reality technology to small and medium-sized manufacturing enterprises. Initially, five demanders on side X (i.e., investors $X_1, X_2, \ldots, X_5$) release demand and self-preference information through the intermediary company in order to obtain the new technology for virtual reality. After three weeks, the intermediary company have received the supply and self-preference information of six suppliers on side Y (i.e., enterprises $Y_1, Y_2, \ldots, Y_6$). Moreover, investors $X_1, X_2, \ldots, X_5$ evaluate enterprises $Y_1, Y_2, \ldots, Y_6$ by mainly considering the technical level, service level and credibility, and provide an IvIFN matrix of side X ($\widehat{V}^X = [\widehat{v}_{kl}^X]_{5\times 6}$),

TABLE 1. IvIFN matrix $\widehat{V}^X = [\widehat{v}^X_{kl}]_{5\times 6}$.

investor	enterprise					
	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	<[0.5,0.6], [0.3,0.4]>	<[0.4,0.5], [0.3,0.3]>	<[0.4,0.45], [0.2,0.4]>	<[0.4,0.5], [0.25,0.3]>	<[0.4,0.55], [0.3,0.35]>	<[0.4,0.5], [0.3,0.4]>
X_2	<[0.4,0.5], [0.3,0.4]>	<[0.4,0.6], [0.3,0.4]>	<[0.3,0.5], [0.3,0.3]>	<[0.4,0.6], [0.3,0.4]>	<[0.5,0.5], [0.35,0.4]>	<[0.5,0.55], [0.3,0.4]>
X_3	<[0.5,0.6], [0.3,0.35]>	<[0.4,0.55], [0.3,0.4]>	<[0.4,0.5], [0.35,0.4]>	<[0.3,0.5], [0.25,0.3]>	<[0.5,0.6], [0.3,0.4]>	<[0.5,0.5], [0.35,0.4]>
X_4	<[0.3,0.5], [0.2,0.3]>	<[0.4,0.6], [0.35,0.4]>	<[0.4,0.5], [0.2,0.4]>	<[0.4,0.55], [0.3,0.35]>	<[0.4,0.5], [0.2,0.4]>	<[0.4,0.55], [0.3,0.35]>
X_5	<[0.4,0.5], [0.2,0.4]>	<[0.3,0.5], [0.2,0.3]>	<[0.4,0.6], [0.3,0.4]>	<[0.4,0.6], [0.35,0.4]>	<[0.4,0.5], [0.35,0.4]>	<[0.4,0.45], [0.2,0.3]>

TABLE 2. IvIFN matrix $\widehat{V}^Y = [\widehat{v}^Y_{kl}]_{5\times 6}$.

investor	enterprise					
	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	<[0.4,0.5], [0.4,0.4]>	<[0.4,0.6], [0.3,0.35]>	<[0.45,0.5], [0.35,0.4]>	<[0.45,0.5], [0.35,0.45]>	<[0.4,0.55], [0.3,0.4]>	<[0.3,0.5], [0.2,0.3]>
X_2	<[0.5,0.55], [0.3,0.4]>	<[0.3,0.5], [0.2,0.3]>	<[0.45,0.55], [0.3,0.4]>	<[0.4,0.55], [0.3,0.4]>	<[0.35,0.5], [0.25,0.45]>	<[0.4,0.5], [0.35,0.4]>
X_3	<[0.35,0.5], [0.3,0.45]>	<[0.4,0.5], [0.3,0.45]>	<[0.4,0.6], [0.3,0.35]>	<[0.4,0.5], [0.25,0.3]>	<[0.3,0.45], [0.35,0.45]>	<[0.35,0.5], [0.35,0.45]>
X_4	<[0.4,0.55], [0.35,0.4]>	<[0.4,0.55], [0.35,0.4]>	<[0.5,0.5], [0.35,0.45]>	<[0.35,0.5], [0.2,0.4]>	<[0.5,0.55], [0.3,0.3]>	<[0.4,0.45], [0.2,0.4]>
X_5	<[0.45,0.5], [0.35,0.4]>	<[0.45,0.5], [0.35,0.45]>	<[0.4,0.55], [0.3,0.4]>	<[0.5,0.55], [0.3,0.4]>	<[0.4,0.55], [0.3,0.35]>	<[0.4,0.6], [0.3,0.35]>

TABLE 3. Normalized interval-valued score matrix $\bar{S}_{\widehat{V}^X} = [[s^z_{\widehat{v}^X_{kl}}, s^y_{\widehat{v}^X_{kl}}]]_{5\times 6}$ when $\theta^X_k = 0.6$ and $\chi = 1$.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	[0.4889,0.6500]	[0.4479,0.5319]	[0.3597,0.5724]	[0.4750,0.5583]	[0.4260,0.5920]	[0.3979,0.5660]
X_2	[0.3968,0.5656]	[0.4029,0.6500]	[0.3569,0.5285]	[0.4029,0.6500]	[0.5004,0.5419]	[0.4859,0.6090]
X_3	[0.5125,0.6344]	[0.3992,0.6082]	[0.4135,0.5404]	[0.3382,0.5531]	[0.4875,0.6500]	[0.5001,0.5417]
X_4	[0.3239,0.5796]	[0.4208,0.6250]	[0.3648,0.6162]	[0.4268,0.5923]	[0.3648,0.6162]	[0.4268,0.5923]
X_5	[0.3655,0.6164]	[0.3247,0.5799]	[0.4055,0.6500]	[0.4213,0.6250]	[0.4159,0.5414]	[0.4112,0.5383]

which is demonstrated by Table 1. Enterprises $Y_1, Y_2, \ldots, Y_6$ can evaluate investors $X_1, X_2, \ldots, X_5$ by mainly considering ease of implementation, credibility and the potential for cooperation, and provide an IvIFN matrix of side Y ($\widehat{V}^Y = [\widehat{v}^Y_{kl}]_{5\times 6}$), which is demonstrated by Table 2. Ultimately, the intermediary company needs to decide the optimum TsM Ω^* combined with the above-presented preference information of investors and enterprises.

TABLE 4. Normalized interval-valued score matrix $\bar{S}_{\frown{V}^Y} = [[s^z_{\frown{v}^Y_{kl}}, s^y_{\frown{v}^Y_{kl}}]]_{5\times6}$ when $\theta^X_k = 0.6$ and $\chi = 1$.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	[0.4383,0.5192]	[0.4408,0.6360]	[0.4634,0.5442]	[0.4400,0.5600]	[0.4128,0.6105]	[0.3425,0.5870]
X_2	[0.4942,0.6110]	[0.3386,0.5854]	[0.4510,0.6102]	[0.4113,0.6102]	[0.3252,0.6094]	[0.4257,0.5453]
X_3	[0.3392,0.5842]	[0.3832,0.5847]	[0.4374,0.6354]	[0.4421,0.5633]	[0.3083,0.5167]	[0.3581,0.5597]
X_4	[0.4261,0.5852]	[0.4267,0.5853]	[0.4800,0.5600]	[0.3325,0.6183]	[0.5453,0.5840]	[0.3758,0.5784]
X_5	[0.4650,0.5450]	[0.4405,0.5601]	[0.4088,0.6098]	[0.4942,0.6110]	[0.4378,0.5959]	[0.4437,0.6365]

TABLE 5. Score matrix $\bar{S}_{\frown{V}^X} = [[s^z_{\frown{v}^X_{kl}}, s^y_{\frown{v}^X_{kl}}]]_{5\times6}$ when $\theta^X = 0.6$.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	0.5856	0.4983	0.4873	0.5250	0.5256	0.4988
X_2	0.4981	0.5512	0.4599	0.5512	0.5253	0.5598
X_3	0.5856	0.5246	0.4896	0.4671	0.5850	0.5251
X_4	0.4773	0.5433	0.5156	0.5261	0.5156	0.5261
X_5	0.5160	0.4778	0.5522	0.5435	0.4912	0.4874

TABLE 6. Score matrix $S_{\frown{V}^Y} = [s_{\frown{v}^Y_{kl}}]_{5\times6}$ when $\theta^Y = 0.6$.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	0.4868	0.5579	0.5119	0.5120	0.5314	0.4892
X_2	0.5643	0.4867	0.5465	0.5306	0.4957	0.4975
X_3	0.4862	0.5041	0.5562	0.5148	0.4333	0.4791
X_4	0.5216	0.5219	0.5280	0.5040	0.5685	0.4974
X_5	0.5130	0.5123	0.5294	0.5643	0.5327	0.5594

A brief solution process for the TsM problem with IvIFNs is demonstrated below.

Step 1: Convert the IvIFN matrix $\frown{V}^X = [\frown{v}^X_{kl}]_{5\times6}$ into normalized interval-valued score matrix $\bar{S}_{\frown{V}^X} = [[s^z_{\frown{v}^X_{kl}}, s^y_{\frown{v}^X_{kl}}]]_{5\times6}$ via using Eqs. (6) and (8) when $\theta^X_k = 0.6$ and $\chi = 1$, as demonstrated in Table 3; Convert the IvIFN matrix $\frown{V}^Y = [\frown{v}^Y_{kl}]_{5\times6}$ into normalized interval-valued score matrix $\bar{S}_{\frown{V}^Y} = [[s^z_{\frown{v}^Y_{kl}}, s^y_{\frown{v}^Y_{kl}}]]_{5\times6}$ via using Eqs. (7) and (11) when $\theta^Y_l = 0.6$, as demonstrated in Table 4.

Step 2: Convert normalized interval-valued score matrices $\bar{S}_{\frown{V}^X} = [[s^z_{\frown{v}^X_{kl}}, s^y_{\frown{v}^X_{kl}}]]_{5\times6}$ and $\bar{S}_{\frown{V}^Y} = [[s^z_{\frown{v}^Y_{kl}}, s^y_{\frown{v}^Y_{kl}}]]_{5\times6}$ into score matrices $S_{\frown{V}^X} = [s_{\frown{v}^X_{kl}}]_{5\times6}$ and $S_{\frown{V}^Y} = [s_{\frown{v}^Y_{kl}}]_{5\times6}$ via using Eqs. (14) and (15) respectively when $\theta^X = 0.6$ and $\theta^Y = 0.6$, as demonstrated in Table 5 and Table 6.

Step 3: Acquire matching willingness matrix $W = [w^*_{kl}]_{5\times6}$ via Eqs. (22) and (23), as demonstrated in Table 7.

TABLE 7. Matching willingness matrix $W = [w^*_{kl}]_{5\times6}$.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	0.1701	0.1704	0.1567	0.1702	0.1777	0.1549
X_2	0.1708	0.1632	0.1506	0.1834	0.1623	0.1697
X_3	0.1782	0.1749	0.1744	0.156	0.153	0.1634
X_4	0.1518	0.1756	0.1697	0.1641	0.1776	0.1612
X_5	0.1644	0.1487	0.1791	0.1881	0.1582	0.1614

Step 4. Construct the weighting score matrices $\bar{S}_{\frown{V}^X} = [\bar{s}_{\frown{v}^X_{kl}}]_{5\times6}$ and $\bar{S}_{\frown{V}^Y} = [\bar{s}_{\frown{v}^Y_{kl}}]_{5\times6}$ on the basis of score matrices $S_{\frown{V}^X} = [s_{\frown{v}^X_{kl}}]_{5\times6}$ and $S_{\frown{V}^Y} = [s_{\frown{v}^Y_{kl}}]_{5\times6}$, matching willingness matrix $W = [w^*_{kl}]_{5\times6}$ via Eqs. (24) and (25), respectively.

Step 5-6. Compute the positive and negative ideal vectors $s^{*X} = \left(s^{*X}_1, s^{*X}_2, \ldots, s^{*X}_5\right)^T$ and $s^{\circ X} = \left(s^{\circ X}_1, s^{\circ X}_2, \ldots, s^{\circ X}_5\right)^T$ on the basis of weighting score matrix $\bar{S}_{\frown{V}^X} = [\bar{s}_{\frown{v}^X_{kl}}]_{5\times6}$ via Eqs. (26) and (27) respectively. Compute the closeness degree matrix $C^X = [c^X_{kl}]_{5\times6}$ via Eqs. (28)-(30).

Step 7-8. Compute the positive and negative ideal vectors $s^{*Y} = \left(s^{*Y}_1, s^{*Y}_2, \ldots, s^{*Y}_6\right)$ and $s^{\circ Y} = \left(s^{\circ Y}_1, s^{\circ Y}_2, \ldots, s^{\circ Y}_6\right)$ on the basis of weighting score matrix $\bar{S}_{\frown{V}^Y} = [\bar{s}_{\frown{v}^Y_{kl}}]_{5\times6}$ via Eqs. (31) and (32) respectively. Compute the closeness degree matrix $C^Y = [c^Y_{kl}]_{5\times6}$ via Eqs. (33)-(35).

Step 9-10: Construct a TsM Model (T-2) on the basis of closeness degree matrices $C^X = [c^X_{kl}]_{5\times6}$ and $C^Y = [c^Y_{kl}]_{5\times6}$, and matching matrix $Z = [z_{kl}]_{5\times6}$. Transform TsM Model (T-2) into TsM Model (T-3):

$$\text{(T-3)}\quad \begin{cases} \max T = \sum_{k=1}^{5}\sum_{l=1}^{6}(c^X_{kl} + c^Y_{kl})z_{kl} \\ \text{s.t.} \sum_{l=1}^{6} z_{kl} = 1, k \in M; \\ \sum_{k=1}^{5} z_{kl} \le 1, l \in N; \\ z_{kl} = 0 \text{ or } 1, k \in M, l \in N \end{cases}$$

where matrix $\Psi = [c^X_{kl} + c^Y_{kl}]_{5\times6}$ is depicted by Table 8.

Step 11: Solve Model (T-3), optimum TsM matrix $Z = [z^*_{kl}]_{5\times6}$ is acquired, as shown in Table 9.

TABLE 8. Coefficient matrix $\Psi = [c_{kl}^X + c_{kl}^Y]_{5\times 6}$.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	1.2093	1.3664	0	0.8239	1.5426	0.0388
X_2	1.4969	0.8203	0.125	1.6589	0.9124	1.4013
X_3	1.4302	1.2349	1.3968	0	0.527	0.5819
X_4	0	1.8148	1.2145	0.6956	1.8341	0.8406
X_5	0.7388	0	1.7633	2	0.7335	1.2468

TABLE 9. Optimum TsM matrix $Z = [z_{kl}^*]_{5\times 6}$.

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
X_1	0	0	0	0	1	0
X_2	1	0	0	0	0	0
X_3	0	0	1	0	0	0
X_4	0	1	0	0	1	0
X_5	0	0	0	1	0	0

TABLE 10. 11 cases for weights w_X andw_Y.

Case	weight	Case	weight
Case I	$w_X = 1, w_Y = 0$	Case II	$w_X = 0.9, w_Y = 0.1$
Case III	$w_X = 0.8, w_Y = 0.2$	Case IV	$w_X = 0.7, w_Y = 0.3$
Case V	$w_X = 0.6, w_Y = 0.4$	Case VI	$w_X = w_Y = 0.5$
Case VII	$w_X = 0.4, w_Y = 0.6$	Case VIII	$w_X = 0.3, w_Y = 0.7$
Case IX	$w_X = 0.2, w_Y = 0.8$	Case X	$w_X = 0.1, w_Y = 0.9$
Case XI	$w_X = 0, w_Y = 1$		

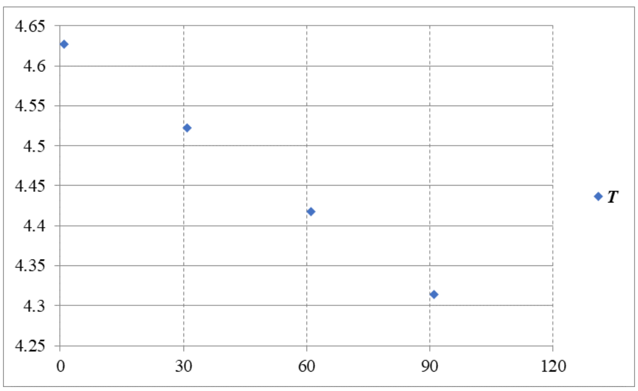

FIGURE 2. Relationship of T from Case I to IV.

On the basis of matrix $Z = [z_{kl}^*]_{5\times 6}$, optimum TsM scheme Ω^* can be acquired, i.e., $\Omega^* = \Omega_{MP}^* \cup \Omega_{SMP}^*$, where $\Omega_{MP}^* = \{(X_1, Y_5), (X_2, Y_1), (X_3, Y_3), (X_4, Y_5), (X_5, Y_4)\}$, $\Omega_{SMP}^* = \{(Y_6, Y_6)\}$. In other words, investor X_1 matches with enterprise Y_5, investor X_2 matches with enterprise Y_1, investor X_3 matches with enterprise Y_3, investor X_4 matches with enterprise Y_5, investor X_5 matches with enterprise Y_4; enterprise Y_6 is not matched.

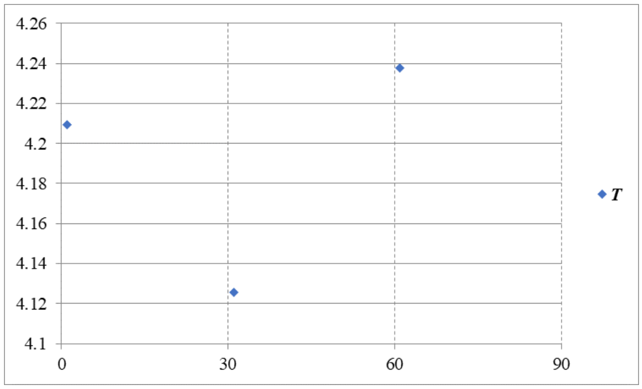

FIGURE 3. Relationship of T from Case V to VII.

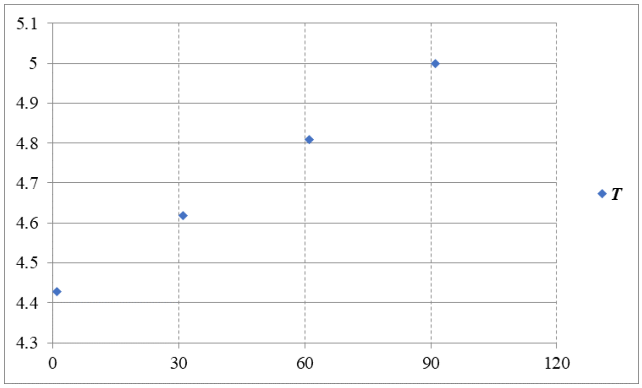

FIGURE 4. Relationship of T from Case VIII to XI.

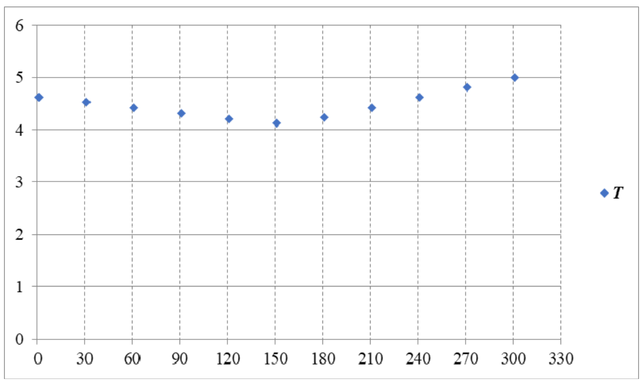

FIGURE 5. Relationship of T from Case I to XI.

Next, model 3 is discussed further. Let $c_{kl} = w_X c_{kl}^X + w_Y c_{kl}^Y$, then 11 cases of c_{kl} can be listed in Table 10. From Table 10, it is known that the above Table 9 corresponds to Case VI. Other cases will be discussed from the view of relationships among weights w_X and w_Y, objective function T, synthetical closeness degrees c_{kl} and matching value z_{kj}.

Figure 2 reveals relationship of the values of object function T from Case I to IV. Figure 3 reveals relationship of the values of object function T from Case V to VII. Figure 4 reveals relationship of the values of object function T from Case VIII to XI.

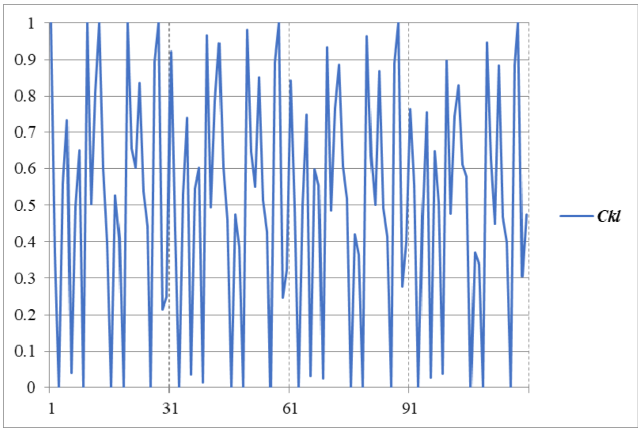

FIGURE 6. Relationship of synthetical closeness degrees c_{kl} from Case I to IV.

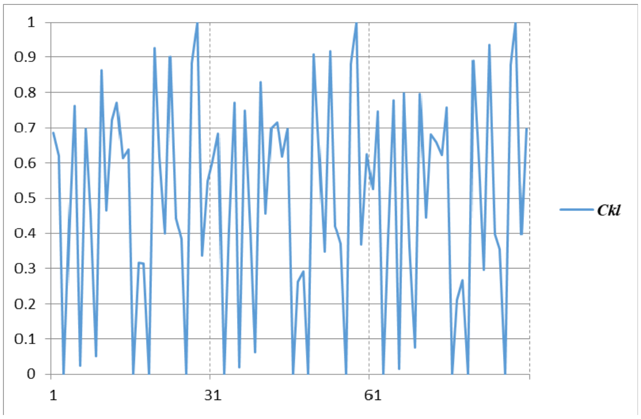

FIGURE 7. Relationship of synthetical closeness degrees c_{kl} from Case V to VII.

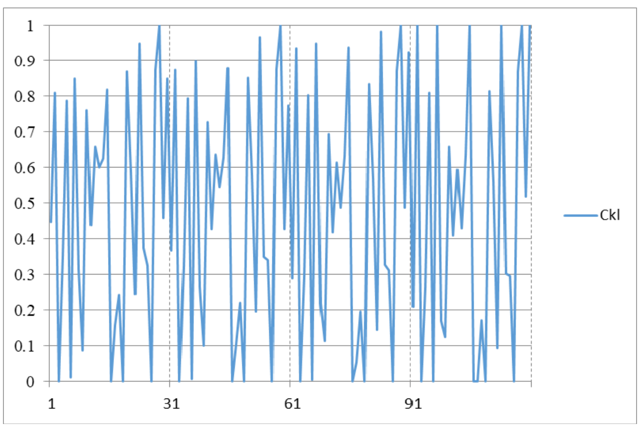

FIGURE 8. Relationship of synthetical closeness degrees c_{kl} from Case VIII to XI.

According to Figures 2-4, the relationship of the values of object function T from Case I to XI can be determined, as displayed in Figure 5. From Figure 5, the values of object function T decreases first and then increases to five.

Figure 6 reveals relationship of synthetical closeness degrees c_{kl} from Case I to IV. Figure 7 reveals relationship of synthetical closeness degrees c_{kl} from Case V to VII. Figure 8 reveals relationship of synthetical closeness degrees c_{kl} from Case VIII to XI.

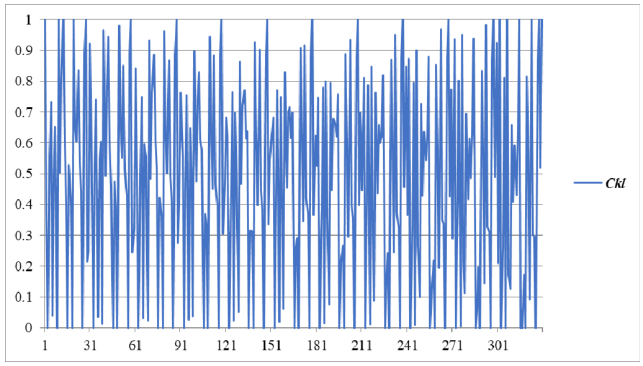

FIGURE 9. Relationship of synthetical closeness degrees c_{kl} from Case I to XI.

According to Figures 6-8, the relationship of synthetical closeness degrees c_{kl} from Case I to XI can be determined, as depicted in Figure 9. From Figure 9, the synthetical closeness degrees c_{kl} are different in many cases.

V. CONCLUSION

An interval-valued intuitionistic fuzzy two-sided matching decision-making approach is put forward, where bodies' behaviors are involved. Bodies' behaviors here are characterized by the matching willingness. The IvIFN matrix is converted into normalized interval-valued score matrices, and then into score matrices. Based on score matrices, the matching willingness can be obtained. Based on score matrices and matching willingness matrix, the weighting score matrices are constructed. Furthermore, according to the TOPSIS technology, the closeness degree matrices are computed. Then a TsM model based on closeness degree matrices and matching matrix can be developed. Optimum TsM scheme is acquired through model solution. An instance of supply-demand matching for virtual reality technology is used to verify the validity of the presented approach.

Compared with previous studies, this paper makes contribution to the following two areas: (1) the application of IvIFSs was extended into TsM field, which are often neglected in previous studies; (2) the normalized interval-valued score function was given, which can make full use of hesitancy and eliminate the influence of negative interval-valued scores; (3) the view is based on bodies' behaviors, and thus the obtained TsM scheme can reflect the matching willingness of bodies; (4) this paper puts forward a decision theory and method for TsM with IvIFNs. Limitations of the present study can be summarized below: it only investigated the TsM problem with IvIFSs preferences; but the related theories of stable matching with IvIFSs and other intuitionistic fuzzy information are not studied.

Therefore, the two aspects can be further studied. For one thing, when bodies' preferences are other intuitionistic fuzzy forms, this type of TsM problem is worthy of attention. For another thing, the related theory of stable matching under interval-valued intuitionistic fuzzy environment should also be considered.

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QI YUE was born in 1983. He received the Ph.D. degree from the School of Business Administration, Northeastern University, in 2011. He is currently an Associate Professor with the School of Information Management, Jiangxi University of Finance and Economics. He has contributed more than 70 journal articles to professional journals. His current interests include management decision analysis and operations research.

LILI ZHANG received the Ph.D. degree in management science and engineering from Northeastern University, in 2010. She is currently with the School of Business, Dalian University of Technology–Panjin. She has contributed more than 20 articles to professional journals and academic conferences, and received 20 projects funded, such as NSFC. Her current research interest includes decision theory and applications.