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An Extended Multi-Surface Sliding Control for Matched/Mismatched Uncertain Nonlinear Systems Through a Lumped Disturbance Estimator

HA LE NHU NGOC THANH¹ AND SUNG KYUNG HONG²

¹Faculty of Intelligent Mechatronics Engineering, Sejong University, Seoul 143-747(05006), South Korea

²Faculty of Mechanical and Aerospace Engineering, Sejong University, Seoul 143-747(05006), South Korea

Corresponding author: Sung Kyung Hong (skhong@sejong.ac.kr)

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ABSTRACT In this paper, we introduce a novel extended sliding mode control algorithm for a class of uncertain nonlinear system through a lumped disturbance estimator. The fundamental concept of this approach is to use multiple proportional-integral sliding surfaces to estimate the lumped disturbances generated by arbitrary matched/mismatched uncertainties. Following, an effectively extended sliding mode controller integrated with the estimated values is designed through the proposed series of sliding-surfaces to deal with the control problem of an uncertain nonlinear engineering model. The stability of both closed-loop control system and lumped disturbance estimator is achieved by Lyapunov theorem. The effectiveness of the proposed approach is demonstrated through simulation results of an illustrative example.

INDEX TERMS Multi-sliding surface, sliding mode control, lumped disturbance, disturbance estimator, matched/mismatched uncertain systems.

I. INTRODUCTION

Conventional sliding mode control (SMC) and extended SMCs are effective and well-known control techniques for nonlinear practical engineering systems. It is studied and developed in many decades due to its conceptual simplicity, and a great ability to eliminate disturbances and uncertainties [1]–[4]. Most of the conventional SMC methods describe the various control strategies to solve the nonlinear systems with matched disturbances/uncertainties [5], [6]. However, many practical engineering systems with unknown disturbances do not only consist of the matched condition but also the mismatched term, meaning that in these nonlinear systems the appearance of disturbances/uncertainties is on channels in which a control input is not available [7], [8]. If the traditional SMC techniques are used in the systems, the mismatched uncertainties may severely influence the tracking control performance or generate a divergent signal of the sliding control due to lumped disturbances induced

by inaccuracy mathematic model, external perturbations or uncertain parameters [9].

Because of the great significance of eliminating the matched/mismatched uncertainties and external disturbances in practical engineering nonlinear systems, this topic has attracted many researchers in recent years. Currently, many study activities keep going on solving the issue such as adaptive control [10], Riccati method [11], output feedback variable structure-based method [12], linear matrix inequality (LMI)-based method [13]–[15]. However, these approaches are not a realistic assumption in practical applications because the non-zero steady state value of mismatched uncertainties may occur in the real engineering systems. Hence, adaptive model compensation and adaptive controllers were proposed in [16]–[18] to overcome this disadvantage. However, the considered adaptive control techniques in these researches ignored the influences of high-frequency dynamics and nonlinearity. Other adaptive backstepping approaches also introduced in [19], [20] for controlling matched/mismatched uncertain systems. However, the most difficult requirement of these methods is to compute the

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derivatives of virtual inputs which are not easy to obtain, even these values are achieved through the analytical methods, they may generate a large control signal. Thus, the close-loop system may suffer from the so-called “explosion of term” [21]. The multi-surface sliding mode control (MSSC) was also introduced in [22]–[24] to control a nonlinear system under the presence of matched/mismatched disturbances/uncertainties. In MSSC, the series of sliding surfaces are established based on virtual desired trajectories and their derivatives. Thus, the problem of “explosion of term” also exists in the systems. However, this drawback is solved by a numerical differentiation method in [25], filter technique and dynamic surface control technique [21], [26].

Disturbance observer-based control is also another popular trend of research to solve the problem of mismatched uncertain systems. The concept of this method is to derive a controller integrated with the disturbance estimation value to eliminate the influences of mismatched uncertainties [27], [28]. A basic nonlinear disturbance observer based SMC and Integral SMC was proposed in [29], [30], this method exhibited a better performance than previous approaches. However, the disturbance observer technique may cause bias estimates once the mismatched disturbances/uncertainties are the unknown time-varying signals. A well-known research of the nonlinear disturbance observer-based control method was introduced in [31], [32]. However, the assumptions of an exogenous disturbance in this method are not general case because the effect of external disturbances on a nonlinear system may not be harmonic signals or constant signals.

Fuzzy structures and neural network (NN) techniques have been widely applied to control the matched/mismatched uncertain systems. The concept of this approach is to use a fusion algorithm between a fuzzy logic theorem and the NN technique to obtain the input-output data from the learning process [33]. However, it is not easy to obtain a partial derivative and prove the overall stability of a control system. Hence, this issue was solved by an adaptive SMC for fuzzy system in [34], [35], and by the integration of SMC and NN-fuzzy structure in [36].

The research’s motivation is to solve the aforementioned disadvantages of the previous methods. Thus, in this paper, we introduce a different approach based on the multiple proportional-integral sliding surfaces to control the nonlinear engineering systems with matched/mismatched uncertainties through the lumped disturbance estimators (LDE). The main contributions of this research are mentioned as the following statements:

- i) A novel multiple proportional-integral sliding surfaces are presented for a general single input and single output (SISO) nonlinear system of order n th, simultaneously containing matched and mismatched uncertainties.
- ii) An effective sliding controller is designed through the proposed multi-sliding surface and the LDE to alleviate the chattering effects and improve control performance.

- iii) The lumped disturbance estimator is introduced to estimate the undesirable effects of both arbitrarily matched and mismatched uncertainties in all channels of the system. The proposed LDE also overcomes the drawbacks of the previous researches such as in [29], the estimation method requires the bounded condition of uncertainties, while in [31], [32] the disturbance observer algorithm is only applied for estimating the harmonic signals and constant signals.
- iv) The proposed algorithm does not require any awareness of bounded information of the matched/mismatched uncertainties/disturbances in both controller design and disturbance estimation process.

The rest of the article is organized as follows. The problem statement is given in Section 2. The controller design and LDE are given in Section 3. The overall stability is analyzed in Section 4. The numerical simulation results of an illustrative example are presented in Section 5. Finally, the conclusion of the study is provided in Section 6.

II. PROBLEM STATEMENT

In this section, it is proposed to consider an n th order single-input and single-output nonlinear engineering system simultaneously influenced by both non-zero matched and mismatched disturbances/uncertainties as the following equation:

$$\begin{aligned}
 \dot{x}_1 &= x_2 + d_1(\mathbf{x}, t) \\
 \dot{x}_2 &= x_3 + d_2(\mathbf{x}, t) \\
 \dot{x}_3 &= x_4 + d_3(\mathbf{x}, t) \\
 &\vdots \\
 \dot{x}_{n-1} &= x_n + d_{n-1}(\mathbf{x}, t) \\
 \dot{x}_n &= f(\mathbf{x}, t) + b(\mathbf{x}, t)u + d_n(\mathbf{x}, t) \\
 y &= x_1
 \end{aligned} \tag{1}$$

where \mathbf{x} and u are represented instead of $\mathbf{x}(t)$ and $u(t)$ for convenience, $\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ is a variable state vector, $u \in \mathbb{R}$ denotes the control input signal, and $y \in \mathbb{R}$ represents the output of the system. The unknown continuous functions $d_i(\mathbf{x}, t) \in \mathbb{R}$, $i = 1, 2, \dots, n$ are the matched and mismatched disturbances/uncertainties. $\forall t \geq 0$, the math expressions of both $f(\mathbf{x}, t)$ and $b(\mathbf{x}, t) \neq 0$ are smooth functions in term of \mathbf{x} . The objective is to design a controller u such that the output feedback x_1 converges to the desired trajectory x_{1d} with a small tracking error in the presence of unknown non-zero mismatched and matched uncertainties.

III. MAIN RESULTS

A. CONTROLLER DESIGN

A general procedure of designing the multiple sliding surfaces and control law u are derived in this section. A novel proportional–integral sliding surface is proposed by modifying the integral sliding surface described in [29] as

follows:

$$s_i = \beta_i \tilde{x}_i + \alpha_i \int_0^t \tilde{x}_i dt - \tilde{x}_i(0) e^{-\gamma_i t} \quad (2)$$

$$\tilde{x}_i = x_i - x_{id}, \quad (i = 1, 2, \dots, n) \quad (3)$$

where \tilde{x}_i denote tracking errors. x_{id} represent the desired trajectories. β_i , α_i and γ_i are positive constants. It can be seen that the term $\tilde{x}_i(0)e^{-\gamma_i t} \rightarrow 0, t \rightarrow \infty$. Therefore, if a controller u is designed in such a way that the sliding surface variables, s_i in Eq.(2), converge to a small neighborhood of origin zero for all time $t > 0$, then the output feedback x_i will exponentially converge to a small neighborhood of the desired trajectories x_{id} .

From Eq.(2), the derivative of sliding surface is computed as follows:

$$\dot{s}_i = \beta_i \dot{\tilde{x}}_i + \alpha_i \tilde{x}_i + \gamma_i \tilde{x}_i(0) e^{-\gamma_i t} \quad (4)$$

The proposed controller for the given SISO system based on the LDE is presented by several steps:

Step $i=1$: The derivative of the 1st sliding surface, \dot{s}_1 , is obtained from Eq.(4):

$$\dot{s}_1 = \beta_1 \dot{\tilde{x}}_1 + \alpha_1 \tilde{x}_1 + \gamma_1 \tilde{x}_1(0) e^{-\gamma_1 t} \quad (5)$$

where the tracking error, \tilde{x}_1 , and its first derivative, $\dot{\tilde{x}}_1$, are achieved from Eqs.(1),(3), $\tilde{x}_1 = x_1 - x_{1d}$, $\dot{\tilde{x}}_1 = \dot{x}_1 - \dot{x}_{1d}$. Thus, \dot{s}_1 can be re-written as follows:

$$\dot{s}_1 = \beta_1 (\dot{x}_2 + \dot{x}_{2d} - \dot{x}_{1d} + \xi_1) + \alpha_1 \tilde{x}_1 \quad (6)$$

where the lumped disturbance ξ_1 is given by:

$$\xi_1 = d_1 + \frac{\gamma_1}{\beta_1} \tilde{x}_1(0) e^{-\gamma_1 t} \quad (7)$$

The virtual desired trajectory x_{2d} can be chosen as:

$$x_{2d} = \dot{x}_{1d} - \hat{\xi}_1 - c_1 s_1 - \frac{\alpha_1}{\beta_1} \tilde{x}_1 \quad (8)$$

where $\hat{\xi}_1$ is an approximate value of the lumped disturbance ξ_1 , and $c_1 \in \mathbb{R}^+$ is a given constant. Eq.(6) can be simplified by using the Eq.(8)

$$\dot{s}_1 = \beta_1 (\dot{x}_2 + \tilde{\xi}_1 - c_1 s_1) \quad (9)$$

where the estimate error $\tilde{\xi}_1$ is defined by $\tilde{\xi}_1 = \xi_1 - \hat{\xi}_1$.

Step $i=2$: The derivative of the 2nd sliding surface, \dot{s}_2 , is computed by:

$$\dot{s}_2 = \beta_2 \dot{\tilde{x}}_2 + \alpha_2 \tilde{x}_2 + \gamma_2 \tilde{x}_2(0) e^{-\gamma_2 t} \quad (10)$$

where the virtual tracking error, \tilde{x}_2 , and its first derivative, $\dot{\tilde{x}}_2$, can be computed by $\tilde{x}_2 = x_2 - x_{2d}$, and

$$\dot{\tilde{x}}_2 = \tilde{x}_3 + \dot{x}_{3d} + d_2 - \left(\ddot{x}_{1d} - \hat{\xi}_1 - c_1 \dot{s}_1 - \frac{\alpha_1}{\beta_1} \dot{\tilde{x}}_1 \right) \quad (11)$$

Thus, \dot{s}_2 can be re-written from Eq.(10) and Eq.(11)

$$\dot{s}_2 = \beta_2 (\tilde{x}_3 + \dot{x}_{3d} - \ddot{x}_{1d} + \xi_2) + \alpha_2 \tilde{x}_2 \quad (12)$$

where the lumped disturbance ξ_2 is described by:

$$\xi_2 = d_2 + \frac{\gamma_2}{\beta_2} \tilde{x}_2(0) e^{-\gamma_2 t} + \hat{\xi}_1 + c_1 \dot{s}_1 + \frac{\alpha_1}{\beta_1} \dot{\tilde{x}}_1 \quad (13)$$

The virtual desired trajectory x_{3d} can be selected as:

$$x_{3d} = \ddot{x}_{1d} - \hat{\xi}_2 - c_2 s_2 - \frac{\alpha_2}{\beta_2} \tilde{x}_2 \quad (14)$$

where $\hat{\xi}_2$ is an approximate value of the lumped disturbance ξ_2 , and $c_2 \in \mathbb{R}^+$ is a given constant.

The Eq.(12) can be simplified by using the Eq.(14)

$$\dot{s}_2 = \beta_2 (\tilde{x}_3 + \tilde{\xi}_2 - c_2 s_2) \quad (15)$$

where the estimate error $\tilde{\xi}_2$ is defined by $\tilde{\xi}_2 = \xi_2 - \hat{\xi}_2$.

Step $i=3$: The derivative of the 3rd sliding surface, \dot{s}_3 , is described by:

$$\dot{s}_3 = \beta_3 \dot{\tilde{x}}_3 + \alpha_3 \tilde{x}_3 + \gamma_3 \tilde{x}_3(0) e^{-\gamma_3 t} \quad (16)$$

where the virtual tracking error, \tilde{x}_3 , and its first derivative, $\dot{\tilde{x}}_3$, can be computed by $\tilde{x}_3 = x_3 - x_{3d}$, and

$$\dot{\tilde{x}}_3 = \tilde{x}_4 + \dot{x}_{4d} + d_3 - \left(\ddot{x}_{1d} - \hat{\xi}_2 - c_2 \dot{s}_2 - \frac{\alpha_2}{\beta_2} \dot{\tilde{x}}_2 \right) \quad (17)$$

Thus, \dot{s}_3 , can be re-written from Eq.(16) and Eq.(17)

$$\dot{s}_3 = \beta_3 (\tilde{x}_4 + \dot{x}_{4d} - \ddot{x}_{1d} + \xi_3) + \alpha_3 \tilde{x}_3 \quad (18)$$

where the lumped disturbance ξ_3 is given by:

$$\xi_3 = d_3 + \frac{\gamma_3}{\beta_3} \tilde{x}_3(0) e^{-\gamma_3 t} + \hat{\xi}_2 + c_2 \dot{s}_2 + \frac{\alpha_2}{\beta_2} \dot{\tilde{x}}_2 \quad (19)$$

The virtual desired trajectory x_{4d} can be chosen as:

$$x_{4d} = \ddot{x}_{1d} - \hat{\xi}_3 - c_3 s_3 - \frac{\alpha_3}{\beta_3} \tilde{x}_3 \quad (20)$$

where $\hat{\xi}_3$ is an approximate value of the lumped disturbance ξ_3 , and $c_3 \in \mathbb{R}^+$ is a given constant. The Eq.(18) can be simplified by using the Eq.(20),

$$\dot{s}_3 = \beta_3 (\tilde{x}_4 + \tilde{\xi}_3 - c_3 s_3) \quad (21)$$

where the estimate error $\tilde{\xi}_3$ is defined by $\tilde{\xi}_3 = \xi_3 - \hat{\xi}_3$.

Step $i = 1, 2, \dots, n - 1$: The analysis process is entirely similar to the previous steps. We can obtain the following results:

The derivative of the i th sliding surface, \dot{s}_i , is described by:

$$\begin{aligned} \dot{s}_i &= \beta_i (\tilde{x}_{i+1} + x_{(i+1)d} - x_{1d}^{(i)} + \xi_i) + \alpha_i \tilde{x}_i \\ &= \beta_i (\tilde{x}_{i+1} + \tilde{\xi}_i - c_i s_i) \end{aligned} \quad (22)$$

where $x_{1d}^{(i)}$ represents the i th derivative of x_{1d} , $c_i > 0$ are given constants, $\xi_i, \hat{\xi}_i, \tilde{\xi}_i \in \mathbb{R}$ are the lumped disturbances in the i th channel, its estimate values, and the approximate errors, respectively. $x_{(i+1)d}$ are desired trajectories of $(i + 1)$ th channel. The functions of $\xi_i, \tilde{\xi}_i$, and $x_{(i+1)d}$ are described by

the following equations:

$$\tilde{\xi}_i = \xi_i - \hat{\xi}_i \quad (23)$$

$$\dot{\xi}_i = d_i + \frac{\gamma_i}{\beta_i} \tilde{x}_i(0)e^{-\gamma_i t} + \dot{\hat{\xi}}_{i-1} + c_{i-1} \dot{s}_{i-1} + \frac{\alpha_{i-1}}{\beta_{i-1}} \dot{\tilde{x}}_{i-1} \quad (24)$$

$$x_{(i+1)d} = x_{1d}^{(i)} - \hat{\xi}_i - c_i s_i - \frac{\alpha_i}{\beta_i} \tilde{x}_i \quad (25)$$

Step $i = n$: The derivative of the n th sliding surface, \dot{s}_n , is computed from Eq.(4)

$$\dot{s}_n = \beta_n \left(\dot{x}_n - \dot{x}_{nd} + \frac{\gamma_n}{\beta_n} \tilde{x}_n(0) e^{-\gamma_n t} \right) + \alpha_n \tilde{x}_n \quad (26)$$

where \dot{x}_n can be obtained by Eq.(1) and Eq.(24)

$$\begin{aligned} \dot{x}_n = f(\mathbf{x}, t) + b(\mathbf{x}, t)u + \xi_n - \frac{\gamma_n}{\beta_n} \tilde{x}_n(0)e^{-\gamma_n t} - \dot{\hat{\xi}}_{n-1} \\ - c_{n-1} \dot{s}_{n-1} - \frac{\alpha_{n-1}}{\beta_{n-1}} \dot{\tilde{x}}_{n-1} \end{aligned} \quad (27)$$

and the term of \dot{x}_{nd} is computed by Eq.(25) with $i = n - 1$,

$$\dot{x}_{nd} = x_{1d}^{(n)} - \dot{\hat{\xi}}_{n-1} - c_{n-1} \dot{s}_{n-1} - \frac{\alpha_{n-1}}{\beta_{n-1}} \dot{\tilde{x}}_{n-1} \quad (28)$$

Substituting \dot{x}_n and \dot{x}_{nd} from Eqs.(27),(28) into Eq.(26), the term of \dot{s}_n can be obtained as follows:

$$\dot{s}_n = \beta_n \left(f(\mathbf{x}, t) + b(\mathbf{x}, t)u - x_{1d}^{(n)} + \xi_n \right) + \alpha_n \tilde{x}_n \quad (29)$$

To stabilize the nonlinear systems with matched/mismatched disturbances/uncertainties, the actual control law u is designed as follows:

$$\begin{aligned} u = -\frac{1}{b(\mathbf{x}, t)} \left[f(\mathbf{x}, t) + \hat{\xi}_n - x_{1d}^{(n)} + c_n s_n \right. \\ \left. + \frac{1}{k_s} |s_n| \operatorname{sgn}(s_n) + \frac{\alpha_n}{\beta_n} \tilde{x}_n \right] \end{aligned} \quad (30)$$

where $k_s > 0$ is a constant. $\hat{\xi}_n$ is an estimate of the lumped disturbance ξ_n . The function $\operatorname{sgn}(s_n)$ is defined by [37]:

$$\operatorname{sgn}(s_n) = \begin{cases} +1, & \text{if } s_n > 0 \\ 0, & \text{if } s_n = 0 \\ -1, & \text{if } s_n < 0 \end{cases} \quad (31)$$

B. LUMPED DISTURBANCE ESTIMATOR (LDE)

In this section, the lumped disturbance estimator is designed to estimate the lumped disturbances/uncertainties, ξ_i , following steps:

Step 1: The LDE is proposed for channel i th ($i = 1, 2, \dots, n - 1$) as follows:

$$\dot{\hat{\xi}}_i = p_{i1} + \delta_{i1} \int_0^t \hat{\xi}_i dt + l_{i1} s_i \quad (32)$$

$$\dot{p}_{i1} = -l_{i1} \left(\beta_i (\tilde{x}_{i+1} + x_{(i+1)d}) - x_{1d}^{(i)} + \hat{\xi}_i \right) + \alpha_i \tilde{x}_i \quad (33)$$

$$\dot{\hat{\xi}}_i = p_{i2} + \delta_{i2} \int_0^t \hat{\xi}_i dt + l_{i2} s_i \quad (34)$$

$$\dot{p}_{i2} = -l_{i2} \left(\beta_i (\tilde{x}_{i+1} + x_{(i+1)d}) - x_{1d}^{(i)} + \hat{\xi}_i \right) + \alpha_i \tilde{x}_i \quad (35)$$

where $\hat{\xi}_i$ and $\dot{\hat{\xi}}_i$ are estimates of ξ_i and $\dot{\xi}_i$ respectively; p_{i1} and p_{i2} are auxiliary variables, l_{i1} , l_{i2} , δ_{i1} , δ_{i2} are positive constants. The errors in the estimations of ξ_i and $\dot{\xi}_i$ is computed by:

$$\tilde{\xi}_i = \xi_i - \hat{\xi}_i \quad (36)$$

$$\ddot{\xi}_i = \dot{\xi}_i - \dot{\hat{\xi}}_i \quad (37)$$

From Eqs.(32),(33) and Eq.(22), we can see that:

$$\dot{\hat{\xi}}_i = l_{i1} \beta_i \tilde{\xi}_i + \delta_{i1} \dot{\hat{\xi}}_i \quad (38)$$

The function of $\dot{\hat{\xi}}_i$ is achieved from Eq.(36), (37) and Eq.(38).

$$\dot{\hat{\xi}}_i = -l_{i1} \beta_i \tilde{\xi}_i + \delta_{i1} \dot{\xi}_i + (1 - \delta_{i1}) \dot{\hat{\xi}}_i \quad (39)$$

Similar process, from Eqs.(34), (35), and Eq.(22)

$$\dot{\hat{\xi}}_i = l_{i2} \beta_i \tilde{\xi}_i + \delta_{i2} \dot{\hat{\xi}}_i \quad (40)$$

The term of $\dot{\hat{\xi}}_i$ can be computed from Eqs.(36), (37) and Eq.(40).

$$\dot{\hat{\xi}}_i = -(l_{i2} \beta_i - \delta_{i2}) \tilde{\xi}_i - \delta_{i2} \xi_i + \ddot{\xi}_i \quad (41)$$

Step 2: The LDE is proposed for the channel n th ($i = n$) as follows:

$$\dot{\hat{\xi}}_n = p_{n1} + \delta_{n1} \int_0^t \hat{\xi}_n dt + l_{n1} s_n \quad (42)$$

$$\dot{p}_{n1} = -l_{n1} \left(\beta_n \left(f(\mathbf{x}, t) + b(\mathbf{x}, t)u - x_{1d}^{(n)} + \hat{\xi}_n \right) + \alpha_n \tilde{x}_n \right) \quad (43)$$

$$\dot{\hat{\xi}}_n = p_{n2} + \delta_{n2} \int_0^t \hat{\xi}_n dt + l_{n2} s_n \quad (44)$$

$$\dot{p}_{n2} = -l_{n2} \left(\beta_n \left(f(\mathbf{x}, t) + b(\mathbf{x}, t)u - x_{1d}^{(n)} + \hat{\xi}_n \right) + \alpha_n \tilde{x}_n \right) \quad (45)$$

where $\hat{\xi}_n$ and $\dot{\hat{\xi}}_n$ are estimates of ξ_n and $\dot{\xi}_n$ respectively; p_{n1} , p_{n2} are auxiliary states; l_{n1} , l_{n2} , δ_{n1} , δ_{n2} are positive constants; $\tilde{\xi}_n$ and $\ddot{\xi}_n$ are the errors in the estimations of ξ_n and $\dot{\xi}_n$, and their values are computed from Eq.(36) and Eq.(37) with $i = n$. From Eqs.(42),(43), and Eq.(29)

$$\dot{\hat{\xi}}_n = l_{n1} \beta_n \tilde{\xi}_n + \delta_{n1} \dot{\hat{\xi}}_n \quad (46)$$

The term of $\dot{\hat{\xi}}_n$ can be achieved from Eqs.(36), (37) and Eq.(46).

$$\dot{\hat{\xi}}_n = -l_{n1} \beta_n \tilde{\xi}_n + \delta_{n1} \dot{\xi}_n + (1 - \delta_{n1}) \dot{\hat{\xi}}_n \quad (47)$$

Similar process, from Eqs.(44), (45), and Eq.(29)

$$\dot{\hat{\xi}}_n = l_{n2} \beta_n \tilde{\xi}_n + \delta_{n2} \dot{\hat{\xi}}_n \quad (48)$$

The function of $\dot{\hat{\xi}}_n$ can be computed from Eqs.(36), (37) and Eq.(48).

$$\dot{\hat{\xi}}_n = -(l_{n2} \beta_n - \delta_{n2}) \tilde{\xi}_n - \delta_{n2} \xi_n + \ddot{\xi}_n \quad (49)$$

Let $\zeta \in \mathbb{R}^{n \times 1}$ and $\tilde{e} \in \mathbb{R}^{2n \times 1}$ denote the lumped disturbance vector and its estimation error vector defined as

$$\zeta = [\xi_1 \quad \xi_2 \quad \dots \quad \xi_n]^T \tag{50}$$

$$\tilde{e} = [\tilde{\xi}_1 \quad \tilde{\xi}_1 \quad \tilde{\xi}_2 \quad \tilde{\xi}_2 \quad \dots \quad \tilde{\xi}_n \quad \tilde{\xi}_n]^T \tag{51}$$

Assumption 1: The lumped disturbances, ξ_i , are j th times differentiable functions and satisfy the following expression:

$$\left\| \frac{d^j \zeta}{dt^j} \right\| \leq \eta \quad \text{for } j = 0, 1, 2 \tag{52}$$

where η is an unknown positive constant.

The mathematic model of the lumped disturbance estimator can be described from Eqs.(39), (41), (47), (49), (50) and Eq.(51) as follows:

$$\dot{\tilde{e}} = A\tilde{e} + B\zeta + C\dot{\zeta} + D\ddot{\zeta} \tag{53}$$

where A is a matrix $2n \times 2n$; B, C , and D are matrices $2n \times n$, see (54)–(56), as shown at the bottom of this page.

IV. STABILITY ANALYSIS

In this section, the stability of LDE and the overall control system is analyzed. From Eqs.(53), (54), (55) and Eq.(56), obviously, it is always possible to choose the constants of $\beta_i, l_{i1}, l_{i2}, \delta_{i1}$, and $\delta_{i2}(i = 1, 2, \dots, n)$ in such a way that eigenvalues of matrix A are arbitrarily placed in the left half complex-plane (LHP). Therefore, it is always possible to determine a positive symmetric matrix P such that

$$A^T P + PA = -Q \tag{57}$$

where Q is a positive definite matrix. Let $\lambda_{\min}\{Q\}$ and $\lambda_{\max}\{Q\}$ represent the smallest and largest eigenvalue of the matrix Q respectively.

$$\lambda_{\min}\{Q\} \|\tilde{e}\|^2 \leq \tilde{e}^T Q \tilde{e} \leq \lambda_{\max}\{Q\} \|\tilde{e}\|^2 \tag{58}$$

A Lyapunov function candidate is considered as follows:

$$V(\tilde{e}) = \tilde{e}^T P \tilde{e} \tag{59}$$

Derivative of $V(\tilde{e})$ is computed and evaluated as follows:

$$\begin{aligned} \dot{V}(\tilde{e}) &= \dot{\tilde{e}}^T P \tilde{e} + \tilde{e}^T P \dot{\tilde{e}} \\ &= \tilde{e}^T (A^T P + PA) \tilde{e} + 2\tilde{e}^T P B \zeta + 2\tilde{e}^T P C \dot{\zeta} + 2\tilde{e}^T P D \ddot{\zeta} \\ &\leq -\tilde{e}^T Q \tilde{e} + 2 \|PB\| \|\tilde{e}\| \|\zeta\| \\ &\quad + 2 \|PC\| \|\tilde{e}\| \|\dot{\zeta}\| + 2 \|PD\| \|\tilde{e}\| \|\ddot{\zeta}\| \\ &\leq -\lambda_{\min}\{Q\} \|\tilde{e}\|^2 + 2 (\|PB\| + \|PC\| + \|PD\|) \|\tilde{e}\| \eta \\ &\leq -\|\tilde{e}\| (\lambda_{\min}\{Q\} \|\tilde{e}\| - 2\eta (\|PB\| + \|PC\| + \|PD\|)) \end{aligned} \tag{60}$$

Therefore, after a sufficiently long time, the norm of lumped disturbance estimation error vector will be bounded by:

$$\|\tilde{e}\| \leq \vartheta \tag{61}$$

where

$$\vartheta = \frac{2\eta (\|PB\| + \|PC\| + \|PD\|)}{\lambda_{\min}\{Q\}} \tag{62}$$

From the Eq.(61) and Eq.(62), it can be seen that the norm of estimation error vector, $\|\tilde{e}\|$, will be ultimately bounded if the constant parameters $\beta_i, l_{i1}, l_{i2}, \delta_{i1}$, and $\delta_{i2} (i = 1, 2, \dots, n)$

$$A = \begin{bmatrix} -l_{11}\beta_1 & \delta_{11} & 0 & 0 & \dots & 0 & 0 \\ -(l_{12}\beta_1 - \delta_{12}) & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & -l_{21}\beta_2 & \delta_{21} & \dots & 0 & 0 \\ 0 & 0 & -(l_{22}\beta_2 - \delta_{22}) & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -l_{n1}\beta_n & \delta_{n1} \\ 0 & 0 & 0 & 0 & \dots & -(l_{n2}\beta_n - \delta_{n2}) & 0 \end{bmatrix} \tag{54}$$

$$B = \begin{bmatrix} 0 & 0 & \dots & 0 \\ -\delta_{12} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & -\delta_{22} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & -\delta_{n2} \end{bmatrix}, \quad C = \begin{bmatrix} (1 - \delta_{11}) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & (1 - \delta_{21}) & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & (1 - \delta_{n1}) \\ 0 & 0 & \dots & 0 \end{bmatrix} \tag{55}$$

$$D = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 \end{bmatrix} \tag{56}$$

are appropriately selected in such a way that the eigenvalues of matrix A are placed in the LHP. Thus, the errors, $\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_n$, will also be all bounded. In other words, the lumped disturbance estimation errors, $\tilde{\xi}_i$, will converge to small balls containing the origin zero.

$$\left| \tilde{\xi}_i \right| \leq \|\tilde{e}\| \leq \vartheta, \quad i = 1, 2, \dots, n \quad (63)$$

Following, the bound of s_n can be found as follows

The derivative of n th sliding surface, \dot{s}_n in Eq.(29), can be simplified by using the controller u in Eq.(30)

$$\dot{s}_n = \beta_n \left(-c_n s_n - \frac{1}{k_s} |s_n| \operatorname{sgn}(s_n) + \tilde{\xi}_n \right) \quad (64)$$

Consider a Lyapunov function

$$V_n(s_n) = \frac{1}{2} s_n^2 \quad (65)$$

The derivative of $V_n(s_n)$ can be obtained and analyzed from Eq.(64) and Eq.(65)

$$\begin{aligned} \dot{V}_n(s_n) &= s_n \dot{s}_n \\ &= \beta_n \left(-c_n s_n^2 - \frac{1}{k_s} |s_n| s_n \operatorname{sgn}(s_n) + s_n \tilde{\xi}_n \right) \\ &\leq \beta_n \left(-c_n s_n^2 - \frac{1}{k_s} |s_n|^2 + |s_n| \vartheta \right) \\ &\leq -\beta_n |s_n| \left[\left(c_n + \frac{1}{k_s} \right) |s_n| - \vartheta \right] \end{aligned} \quad (66)$$

From the result of Eq.(66), it can be seen that after a sufficiently long time, the value of s_n will be bounded by

$$|s_n| \leq \frac{\vartheta k_s}{k_s c_n + 1} \quad (67)$$

Therefore, the tracking error \tilde{x}_n will also be bounded and converged to a small neighborhood of zero

$$|\tilde{x}_n| \leq \mu_n \quad (68)$$

where $\mu_n > 0$ is an ultimately bounded of \tilde{x}_n .

Next, the bounds of s_i , ($i = 1, 2, \dots, n - 1$) are also found as follows:

Consider a Lyapunov function $V_i(s_i)$

$$V_i(s_i) = \frac{1}{2} s_i^2 \quad (69)$$

Using Eq.(22), the derivative of $V_i(s_i)$ is evaluated as follows

$$\begin{aligned} \dot{V}_i(s_i) &= \beta_i \left(s_i \tilde{x}_{i+1} + s_i \tilde{\xi}_i - c_i s_i^2 \right) \\ &\leq \beta_i \left(|s_i| |\tilde{x}_{i+1}| + |s_i| \vartheta - c_i s_i^2 \right) \end{aligned} \quad (70)$$

• If $i = n - 1$, from Eq.(68): $|\tilde{x}_{i+1}| = |\tilde{x}_n| \leq \mu_n$, the Eq.(70) becomes

$$\begin{aligned} \dot{V}_{n-1}(s_{n-1}) &\leq \beta_{n-1} \left(|s_{n-1}| \mu_n + |s_{n-1}| \vartheta - c_{n-1} s_{n-1}^2 \right) \\ &\leq -\beta_{n-1} |s_{n-1}| \left(-\mu_n - \vartheta + c_{n-1} |s_{n-1}| \right) \end{aligned} \quad (71)$$

Obviously, after a sufficiently long time, s_{n-1} is bounded by

$$|s_{n-1}| \leq \frac{\mu_n + \vartheta}{c_{n-1}} \quad (72)$$

Thus, the tracking errors \tilde{x}_{n-1} is also bounded by

$$|\tilde{x}_{n-1}| \leq \mu_{n-1}, \quad \text{where } \mu_{n-1} > 0 \quad (73)$$

• If $i = n - 2$, from Eq.(73): $|\tilde{x}_{i+1}| = |\tilde{x}_{n-1}| \leq \mu_{n-1}$, the Eq.(70) becomes

$$\dot{V}_{n-2}(s_{n-2}) \leq -\beta_{n-2} |s_{n-2}| \left(-\mu_{n-1} - \vartheta + c_{n-2} |s_{n-2}| \right) \quad (74)$$

After a sufficiently long time, s_{n-2} is bounded by

$$|s_{n-2}| \leq \frac{\mu_{n-1} + \vartheta}{c_{n-2}} \quad (75)$$

Thus, the tracking errors \tilde{x}_{n-2} is also bounded by

$$|\tilde{x}_{n-2}| \leq \mu_{n-2}, \quad \text{where } \mu_{n-2} > 0 \quad (76)$$

• The stability of other steps is similarly demonstrated. As a general rule, when $i = 1, 2, \dots, n - 1$, we can see that,

$$\dot{V}_i(s_i) \leq -\beta_i |s_i| \left(-\mu_{i+1} - \vartheta + c_i |s_i| \right) \quad (77)$$

Thus, after a sufficiently long time, s_i is bounded by

$$|s_i| \leq \frac{\mu_{i+1} + \vartheta}{c_i}, \quad \text{where } \mu_{i+1} > 0, \quad i = 1, 2, \dots, n - 1 \quad (78)$$

In summary, from results of Eq.(67), and Eq.(78), it can be seen that the multiple proportional–integral sliding surfaces s_i ($i = 1, 2, \dots, n$) are always bounded and converged to small balls containing the origin. Thus, the tracking errors, \tilde{x}_i , are also bounded and exponentially converged to small neighborhoods of zero.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, the proposed controller is verified through an illustrative example considered in [29] as shown in Eq.(79). The performance of simulation result is compared with dynamic surface control (DSC) in [21], and integral-type sliding mode control (ISMC) in [30].

$$\begin{aligned} \dot{x}_1 &= x_2 + d_1(\mathbf{x}, t), \\ \dot{x}_2 &= -2x_1 - x_2 + e^{x_1} + u + d_2(\mathbf{x}, t), \\ y &= x_1 \end{aligned} \quad (79)$$

where $d_1(\mathbf{x}, t)$ and $d_2(\mathbf{x}, t)$ are disturbances/uncertainties presented by $d_1 = \theta x_1^4 + \sin 3\pi t$, where θ is an unknown parameter selected of $\theta = 1.25$, and $d_2 = -x_1 + x_2 \sin 2\pi t$. The objective of this example is to design a controller in such a way that the output feedback, x_1 , converges to a reference trajectory $x_{1d} = 1 + \sin 2\pi t$. The coefficients of multi-sliding surfaces s_1, s_2 are chosen as $\alpha_1 = 30, \beta_1 = 1.0, \gamma_1 = 0.001$, and $\alpha_2 = 80, \beta_2 = 1.0, \gamma_2 = 0.001$, respectively. The controller gains of the presented method are selected as $c_1 = c_2 = 50$, and the switching gain is given as $k_s = 0.002$. The gains of lumped disturbance estimators for channels of the system (79) are appropriately chosen in such a way that the eigenvalues of matrix A are in the LHP:

$$A = \begin{bmatrix} -l_{11}\beta_1 & \delta_{11} & 0 & 0 \\ -(l_{12}\beta_1 - \delta_{12}) & 0 & 0 & 0 \\ 0 & 0 & -l_{21}\beta_2 & \delta_{21} \\ 0 & 0 & -(l_{22}\beta_2 - \delta_{22}) & 0 \end{bmatrix}$$

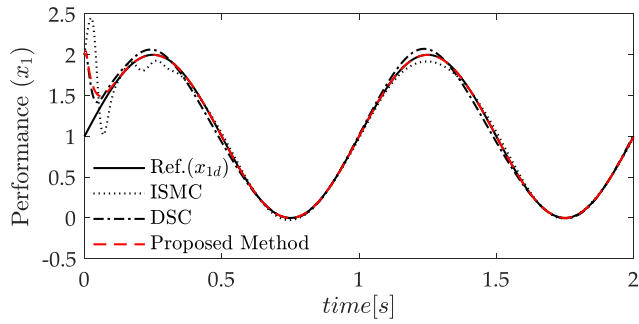


FIGURE 1. Tracking control performance.

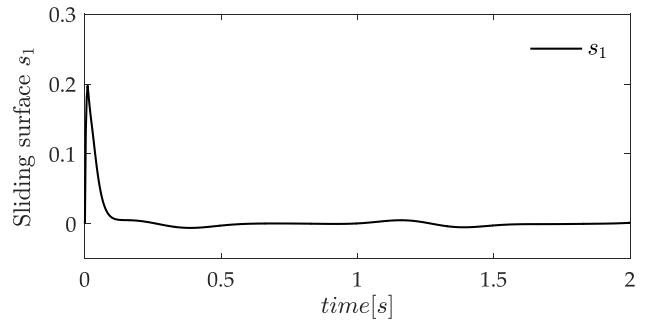


FIGURE 4. Sliding surface s_1 .

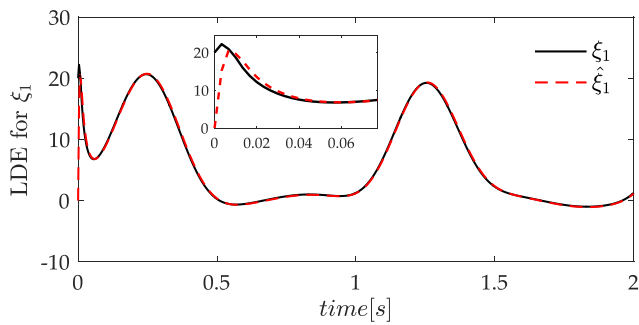


FIGURE 2. Lumped disturbance in channel 1 and its estimation.

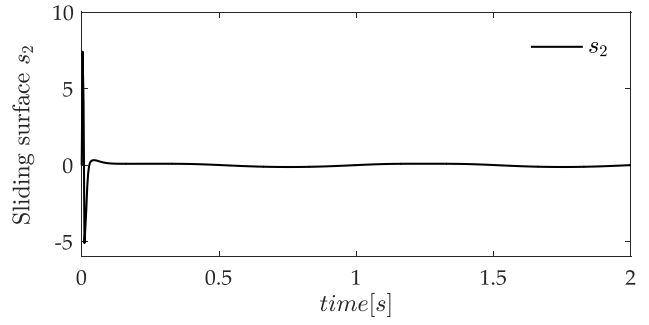


FIGURE 5. Sliding surface s_2 .

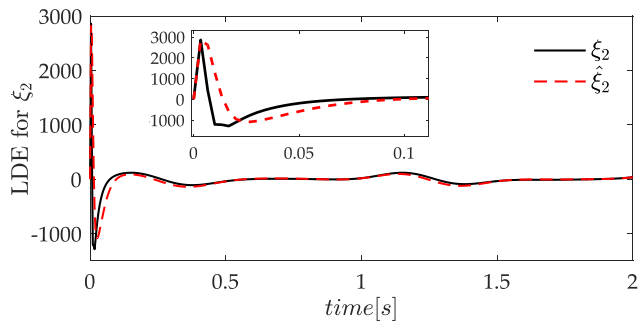


FIGURE 3. Lumped disturbance in channel 2 and its estimation.

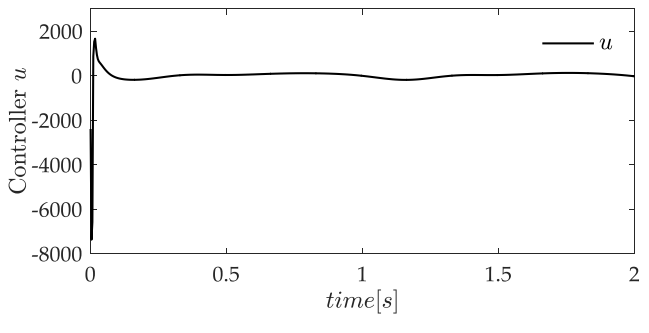


FIGURE 6. Controller signal u .

We can select the parameters given by $l_{11} = 400, l_{12} = 150, l_{21} = 150, l_{22} = 150, \delta_{11} = \delta_{12} = \delta_{21} = \delta_{22} = 0.7$; and the $Eigen(A) = \{-399.74, -0.26, -149.3, -0.7\}$. The initial states are given by $x_1(0) = 2$, and $x_2(0) = 0$. The results of simulation are shown in Figures 1-6.

As shown in Figure 1, the output feedback x_1 well converges to the desired trajectory x_{1d} although we do not have any knowledge of bounded information of the mismatched/matched uncertainties d_1 and d_2 , while the ISMC and DSC do not show a good tracking performance. Figure 2 and Figure 3 exhibit approximations of the lumped disturbance in channel 1 and 2. It can be seen that the estimated values $\hat{\xi}_1$, and $\hat{\xi}_2$ rapidly converge to their true signal ξ_1 and ξ_2 , respectively. The trajectories of sliding surfaces s_1 and s_2 converge to a small neighborhood of zero as shown in Figures 4 and 5. The controller u exhibited a strong chattering elimination as shown in Figure 6.

VI. CONCLUSIONS

In this research, we introduced a novel extended sliding mode control algorithm based on the multiple proportional-integral sliding surfaces for controlling the nonlinear systems of order n th with matched/mismatched uncertainties. The fundamental concept of this method is to use multiple sliding surfaces to estimate the lumped disturbances influenced on all channels of the system without knowing the bounded information of uncertainties. In addition, an effective sliding controller integrated with the estimated value is designed to stabilize the system. The overall stability of the control system is demonstrated by the Lyapunov theorem. The results of simulation exhibited that the control system is possible of well handling in both tracking performance and chattering alleviation.

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HA LE NHU NGOC THANH received the B.S. and M.S. degrees in mechanical engineering from the HCMC University of Technology and Education and the HCMC University of Technology, Vietnam, in 2011 and 2015, respectively, and the Ph.D. degree from the Mechanical and Aerospace Engineering, Sejong University, Seoul, South Korea, in 2019. He is currently an Assistant Professor with the Faculty of Intelligent Mechatronics Engineering, Sejong University. His research interests include nonlinear control, mechatronics, and robotics. He received an outstanding research award in Sejong University, in 2019. He currently serves as a Guest Editor of *Sensor* journal (MDPI) and *Applied Sciences* journal (MDPI).



SUNG KYUNG HONG received the B.S. and M.S. degrees in mechanical engineering from Yonsei University, Seoul, South Korea, in 1987 and 1989, respectively, and the Ph.D. degree from Texas A&M University, College Station, in 1998. From 1989 to 2000, he was with the Unmanned Aerial Vehicle System Division, and the Flight Dynamics and Control Laboratory of the Agency for Defense Development (ADD) in South Korea. He is currently a Full Professor with the Department of Aerospace Engineering, Sejong University, South Korea. His research interests include fuzzy logic controls, inertial sensor applications, and flight control systems.

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