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Fuzzy-Modeled Prescribed Performance Integral Controller Design for Nonlinear Descriptor System With Uncertainties

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ABSTRACT A suitable solution for a nonlinear descriptor system with uncertainties is considered in this paper by designing a fuzzy-modeled prescribed performance integral controller. The system with a parasitic parameter ε is known as the descriptor system, and this parasitic parameter is used for specifying a fast mode of such a system. Based on a linear matrix inequality (LMI) approach, the interaction of fast and slow dynamic modes which causes ill-conditioned LMI result has normally occurred in the nonlinear descriptor system with uncertainties. Therefore, with the design of the fuzzy-modeled prescribed performance integral controller, such a system with parametric uncertainties is represented by a Takagi-Sugeno fuzzy model, and the H_∞ fuzzy state-feedback controller is designed to achieve an adequate condition for overcoming the effects of the parasitic parameter, the uncertainties, and the exogenous input disturbance. Moreover, the integral controller is added to increase the performances of stability. In summary, the design process of the proposed controller and the numerical example serve to illustrate various performance results of the proposed controller.

INDEX TERMS H_∞ fuzzy controller, integral controller, linear matrix inequality (LMI), Takagi-Sugeno fuzzy model, uncertain nonlinear descriptor system.

I. INTRODUCTION

For general engineering, the descriptor system has been an active area of research for five decades. The descriptor system is widely known as a reliable numerical algorithm for complex systems that have a parasitic parameter. The parasitic parameter is normally a small value capable of causing a number of problems that are unable to be controlled or unsolvable in any system. For the control engineer, the most pressing problem is that the system has high dimensionality, so these problems can be alleviated by using the mathematical framework of the descriptor system, which is called a reduction technique. The concept of such a technique is defining the parasitic parameter as the ε symbol to separate the fast mode system from the slow mode system into the descriptor system that is arranged as the state-space form. The so-called

technique has been studied by many researchers in area of control systems [1]–[11].

The descriptor system can be separated into a linear and a nonlinear system that provide an advantage for solving different characteristics of general systems. For the linear descriptor system, a dynamic linear system sometimes cannot avoid the variant parameters such as the internal disturbance, external disturbance, variant temperature, and other uncertainties; consequently, several successful studies have analyzed an approach to solving the problem of the existence of variant parameters by an H_∞ control method over the past two decades [6]–[9]. On the other hand, the nonlinear system has become more complex characteristics compared to the linear one. Therefore, the powerful tools known as the switching controllers [12]–[14] have been employed to deal with the nonlinear system with disturbances and uncertainties; however, the nonlinear descriptor system is more complex than general nonlinear system because of the involved parameter

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of the state-space model that produces an interaction of fast and slow dynamic modes. However, while researchers can use the H_∞ control method to solve the effect of the nonlinearity, this method can be only be designed for the slow dynamic mode of the nonlinear descriptor system [15]–[18]. Thus, the study of the H_∞ control design for the nonlinearity of the descriptor system requires more the development.

The method that is the most powerful tool for the nonlinear descriptor system is the Takagi-Sugeno fuzzy model. The concept of the Takagi-Sugeno fuzzy model relies on using the linear system model, substituting it into the nonlinear model by sublinear models, and combining it with IF-THEN rules and fuzzy membership functions to approximate the nonlinearity [19]–[41]. Over the past two decades, the Takagi-Sugeno fuzzy model has been used in combination with H_∞ control design [17]–[24] and has been employed to describe the nonlinear descriptor system for easy analysis by many studies [35]–[39]. Moreover, the nonlinear descriptor system with uncertain characteristics achieved by the robust H_∞ control design addresses both state-feedback and output-feedback [36]. However, although the uncertain nonlinear descriptor system has been applied in many applications, different applications always have different problems to address.

For improvement of in the control area, an integral control design has been developed for improving and achieving performance in controlling various control field [42]–[51]. Based on the LMI approach, the asymptotic stability has been ensured by using the integral sliding-mode control, such that this method can remove a restrictive fuzzy assumption of the integral sliding-mode control [48], [49]. Considering the next application, the doubly fed induction generator (DFIG) wind energy system has been guaranteed to have asymptotic stability under the nonlinearity, uncertain parameters, and the disturbance by employing the robust H_∞ fuzzy integral controller [50]. Moreover, an H_∞ fuzzy integral controller has been considered for the case of the nonlinear descriptor system [51]; however, an issue regarding the existence of uncertain parameters remains to be considered in the literature.

Briefly, the uncertain nonlinear descriptor system with disturbance remains to be gently considered in points of various methods which are mentioned above. So far, the combination of three powerful methods; the robust H_∞ control, the Takagi-Sugeno fuzzy model, and the integral control, for such a system has not yet been examined in the literature, so the contributions of this study are concisely explained as follows:

- 1) The uncertain nonlinear descriptor system with disturbance is described and approximated by Takagi-Sugeno fuzzy model.
- 2) According to computational point of views, the design of proposed controller for uncertain nonlinear descriptor system has been examined by using the Lyapunov function and has been described in terms of LMIs.
- 3) The proposed controller can overcome the uncertainty and the disturbance effect, and also obtains the better

transient response of the system when compared with [35] and [37], which is validated through an example.

Thus, this paper emphasizes designing the fuzzy-modeled prescribed performance integral controller based on the LMI approach for the case of the uncertain nonlinear descriptor system. First, the nonlinear descriptor system with parametric uncertainties is described in the mathematical framework of the Takagi-Sugeno fuzzy model and thoroughly illustrated in the problem statement and modeling section. Second, the main findings of this paper are shown in the main results section. The mathematical framework of the Takagi-Sugeno fuzzy model describing the nonlinear descriptor system with parametric uncertainties in the first section will be considered for designing a suitable controller that can overcome the effect of the existence of parasitic parameters, nonlinearity, disturbances, and uncertainties. Based on the LMI approach, the fuzzy-modeled prescribed performance integral controller must achieve a set of adequate conditions, such that the L_2 -gain of mapping from the regulated output energy to the exogenous input disturbance energy is less than or equal to definable value γ . In Lemma 2, as established by the concept of designing a conventional controller, the ill-conditioned LMI occurs because of the interaction of fast and slow dynamic modes, so Theorem 1 has been established to show how to solve the ill-conditioned LMI by separating the ε -independent LMI from the ε -dependent LMI. Third, the numerical example has been simulated with the proposed controller and the results are demonstrate with many examples in the example section. In summary, the results of the examples will be further explained in brief in the conclusions section.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. THE UNCERTAIN NONLINEAR DESCRIPTOR SYSTEM

In this study, the significant system is considered in the following uncertain nonlinear descriptor system.

$$\dot{x}_1(t) = f(x_1(t), x_2(t), u(t), w_1(t)) \quad (1)$$

$$\varepsilon \dot{x}_2(t) = f(x_1(t), x_2(t), u(t), w_2(t)) \quad (2)$$

$$y(t) = f(x_1(t), x_2(t)) \quad (3)$$

$$z(t) = f(x_1(t), x_2(t)) \quad (4)$$

where $x_1(t) \in \mathfrak{R}^a$ and $x_2(t) \in \mathfrak{R}^b$ are the state vectors, $u(t) \in \mathfrak{R}^c$ is the input, $w_1(t) \in \mathfrak{R}^d$ and $w_2(t) \in \mathfrak{R}^e$ are the disturbances, $y(t) \in \mathfrak{R}^l$ is the measured output, $z(t) \in \mathfrak{R}^m$ is the controlled output, and $\varepsilon (\varepsilon > 0)$ is the parasitic parameter.

Based on Takagi-Sugeno fuzzy modeling, the uncertain nonlinear descriptor system (1)-(4) can be rewritten as follows:

Plant Rule r :

IF $s_1(t)$ is M_{r1} and , . . . and $s_p(t)$ is M_{rp} THEN

$$\begin{aligned} \dot{x}_1(t) = & (A_{11,r} + \Delta A_{11,r}) x_1(t) + (A_{12,r} + \Delta A_{12,r}) x_2(t) \\ & + (B_{1,r} + \Delta B_{1,r}) u(t) \\ & + (B_{w_1,r} + \Delta B_{w_1,r}) w_1(t) \end{aligned} \quad (5)$$

$$\begin{aligned} \varepsilon \dot{x}_2(t) &= (A_{21_r} + \Delta A_{21_r})x_1(t) + (A_{22_r} + \Delta A_{22_r})x_2(t) \\ &\quad + (B_{2_r} + \Delta B_{2_r})u(t) \\ &\quad + (B_{w_{2_r}} + \Delta B_{w_{2_r}})w_2(t) \end{aligned} \quad (6)$$

$$\begin{aligned} y(t) &= (C_{y_{1_r}} + \Delta C_{y_{1_r}})x_1(t) \\ &\quad + (C_{y_{2_r}} + \Delta C_{y_{2_r}})x_2(t) \end{aligned} \quad (7)$$

$$\begin{aligned} z(t) &= (C_{z_{1_r}} + \Delta C_{z_{1_r}})x_1(t) \\ &\quad + (C_{z_{2_r}} + \Delta C_{z_{2_r}})x_2(t) \end{aligned} \quad (8)$$

where $r = 1, 2, 3, \dots, j$, j represents the IF-THEN rules, $M_{io}(o = 1, 2, 3, \dots, p)$ represents the fuzzy set, $s_1(t), \dots, s_p(t)$ are the premise variables, the matrices $A_{11_r}, A_{12_r}, A_{21_r}, A_{22_r}, B_{1_r}, B_{2_r}, B_{w_{1_r}}, B_{w_{2_r}}, C_{y_{1_r}}, C_{y_{2_r}}, C_{z_{1_r}}, C_{z_{2_r}}$ are the appropriate matrices, and the matrices $\Delta A_{11_r}, \Delta A_{12_r}, \Delta A_{21_r}, \Delta A_{22_r}, \Delta B_{1_r}, \Delta B_{2_r}, \Delta B_{w_{1_r}}, \Delta B_{w_{2_r}}, \Delta C_{y_{1_r}}, \Delta C_{y_{2_r}}, \Delta C_{z_{1_r}}, \Delta C_{z_{2_r}}$ are the system uncertainties which satisfy an assumption.

Then, the final outputs of Takagi-Sugeno fuzzy modeling given a set of $(x_1(t), x_2(t), u(t))$ in (5)-(8) are expressed as follows:

$$\begin{aligned} \dot{x}_1(t) &= \sum_{r=1}^j \rho_r(s(t)) ((A_{11_r} + \Delta A_{11_r})x_1(t) \\ &\quad + (A_{12_r} + \Delta A_{12_r})x_2(t) + (B_{1_r} + \Delta B_{1_r})u(t) \\ &\quad + (B_{w_{1_r}} + \Delta B_{w_{1_r}})w_1(t)) \end{aligned} \quad (9)$$

$$\begin{aligned} \varepsilon \dot{x}_2(t) &= \sum_{r=1}^j \rho_r(s(t)) ((A_{21_r} + \Delta A_{21_r})x_1(t) \\ &\quad + (A_{22_r} + \Delta A_{22_r})x_2(t) + (B_{2_r} + \Delta B_{2_r})u(t) \\ &\quad + (B_{w_{2_r}} + \Delta B_{w_{2_r}})w_2(t)) \end{aligned} \quad (10)$$

$$\begin{aligned} y(t) &= \sum_{r=1}^j \rho_r(s(t)) ((C_{y_{1_r}} + \Delta C_{y_{1_r}})x_1(t) \\ &\quad + (C_{y_{2_r}} + \Delta C_{y_{2_r}})x_2(t)) \end{aligned} \quad (11)$$

$$\begin{aligned} z(t) &= \sum_{r=1}^j \rho_r(s(t)) ((C_{z_{1_r}} + \Delta C_{z_{1_r}})x_1(t) \\ &\quad + (C_{z_{2_r}} + \Delta C_{z_{2_r}})x_2(t)) \end{aligned} \quad (12)$$

where

$$\begin{aligned} \rho_r(s(t)) &= \frac{\zeta_r(s(t))}{\sum_{r=1}^j \zeta_r(s(t))}, \\ \zeta_r(s(t)) &= \prod_{o=1}^n M_{io}(s_o(t)). \end{aligned}$$

for all t . $M_{io}(s_o(t))$ is the grade of membership of $s_o(t)$ in M_{io} . Let

$$\begin{aligned} \sum_{r=1}^j \zeta_r(s(t)) &> 0, \quad r = 1, 2, \dots, j; \\ \zeta_i(s(t)) &\geq 0. \end{aligned}$$

and

$$\begin{aligned} \sum_{r=1}^j \rho_r(s(t)) &= 1, \quad r = 1, 2, \dots, j; \\ \rho_r(s(t)) &\geq 0. \end{aligned}$$

for all t .

B. PROBLEM STATEMENT

Here, a novel controller for the system is considered in this subsection. The fuzzy-modeled prescribed performance integral controller for the nonlinear descriptor system with uncertainties (9)-(12) is inferred as follows:

Controller Rule f :

IF $s_1(t)$ is M_{f1} and \dots and $s_p(t)$ is M_{fp} THEN

$$u(t) = (K_{1_f}x_1(t) + K_{2_f}x_2(t) + K_{I_f}q(t)) \quad (13)$$

where $f = 1, 2, 3, \dots, j$, j represents the IF-THEN rules, $q(t)$ is the integral-state vector, K_{1_f} and K_{2_f} are the gains of state-feedback controllers, and K_{I_f} is the gain of the state feedback of integral controller.

Based on the fuzzy-modeled design approach, the complete form of the fuzzy-modeled prescribed performance integral controller is inferred as follows:

$$u(t) = \sum_{f=1}^j \rho_f(s(t)) (K_{1_f}x_1(t) + K_{2_f}x_2(t) + K_{I_f}q(t)) \quad (14)$$

Next, the uncertain nonlinear descriptor systems described by the Takagi-Sugeno fuzzy model (9)-(12) with the fuzzy-modeled prescribed performance integral controller (14), as shown in Fig. 1, can be rewritten as follows:

$$\begin{aligned} E_\varepsilon \dot{\check{x}}(t) &= \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \left((\check{A}_{rf} + \Delta \check{A}_{rf})\check{x}(t) \right. \\ &\quad \left. + (\check{B}_{w_r} + \Delta \check{B}_{w_r})\varpi(t) \right) \end{aligned} \quad (15)$$

$$\check{y}(t) = \sum_{r=1}^j \rho_r(s(t)) \left((\check{C}_{y_r} + \Delta \check{C}_{y_r})\check{x}(t) \right) \quad (16)$$

$$\check{z}(t) = \sum_{r=1}^j \rho_r(s(t)) \left((\check{C}_{z_r} + \Delta \check{C}_{z_r})\check{x}(t) \right) \quad (17)$$

with

$$\begin{aligned} \check{A}_{rf} &= \begin{bmatrix} A_{11_r} + B_{1_r}K_{1_f} & A_{12_r} + B_{1_r}K_{2_f} & B_{1_r}K_{I_f} \\ A_{21_r} + B_{2_r}K_{1_f} & A_{22_r} + B_{2_r}K_{2_f} & B_{2_r}K_{I_f} \\ C_{y_{1_r}} & C_{y_{2_r}} & 0 \end{bmatrix}, \\ \check{B}_{w_r} &= \begin{bmatrix} B_{w_{1_r}} & 0 & 0 \\ 0 & B_{w_{2_r}} & 0 \\ 0 & 0 & I \end{bmatrix}, \\ \check{C}_{y_r} &= [C_{y_{1_r}} \quad C_{y_{2_r}} \quad 0], \\ \check{C}_{z_r} &= [C_{z_{1_r}} \quad C_{z_{2_r}} \quad 0], \end{aligned}$$

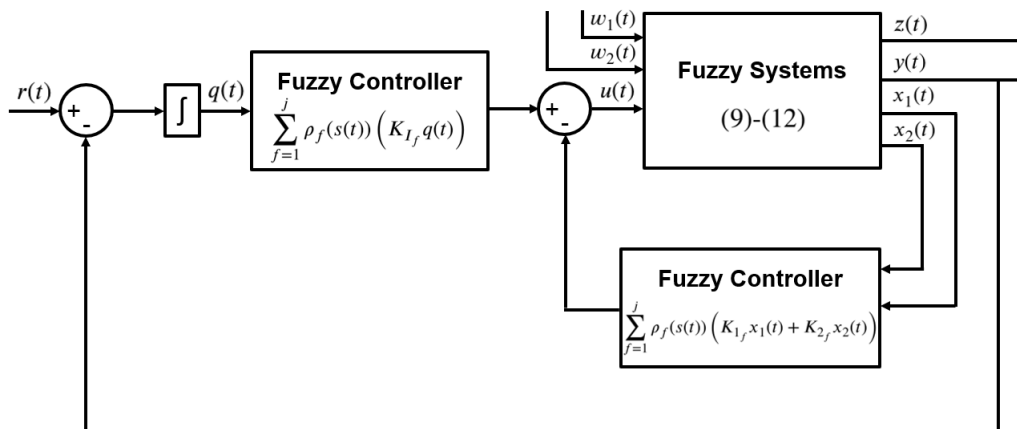


FIGURE 1. The block diagram of the fuzzy-modeled prescribed performance integral controller for the uncertain nonlinear descriptor systems.

$$\check{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix}, \quad \varpi(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ 0 \end{bmatrix},$$

$$E_\varepsilon = \begin{bmatrix} I & 0 & 0 \\ 0 & \varepsilon I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

where the matrices $\Delta\check{A}_{rf}$, $\Delta\check{B}_{wr}$, $\Delta\check{C}_{yr}$, and $\Delta\check{C}_{zr}$ represent the uncertainties of such a system that satisfies the following assumption.

Assumption 1:

$$\Delta\check{A}_{rf} = G(\check{x}(t), t)F_{1r}, \Delta\check{B}_{wr} = G(\check{x}(t), t)F_{2r},$$

$$\Delta\check{C}_{yr} = G(\check{x}(t), t)F_{3r}, \Delta\check{C}_{zr} = G(\check{x}(t), t)F_{4r}.$$

where $F_k, k = 1, 2, 3, 4$ are matrix functions featuring the structure of such uncertainties. In addition, the inequality of such matrix functions holds as follows:

$$\|G(\check{x}(t), t)\| \leq \beta.$$

for any positive constant β . Subsequently, let us consider the following definition.

Definition 1: H_∞ performance can be guaranteed to overcome disturbances and exhibit asymptotically robust stability when the closed-loop systems (15)-(17) have been achieved, such that the condition is met that the L_2 -gain must be less than or equal to positive real number γ :

$$\int_0^{T_f} \check{z}^T(t)\check{z}(t) dt \leq \gamma^2 \int_0^{T_f} \varpi^T(t)\varpi(t) dt \quad (18)$$

for all $T_f \geq 0$ and $\varpi(t) \in L_2[0, T_f]$.

Lemma 1 [53]: Considering in a normed vector space, the triangle inequality is written as follow:

$$\|O + U\| \leq \|O\| + \|U\| \quad (19)$$

with

$$\tilde{S} \leq O + U.$$

where O, U, \tilde{S} are the lengths of the sides of the triangle, with no side being greater than \tilde{S} .

C. PRELIMINARIES

Using Assumption 1, the closed-loop of the nonlinear descriptor system with uncertainties described by the Takagi-Sugeno fuzzy model with the fuzzy-modeled prescribed performance integral controller (15)-(17) can be expressed as follows:

$$E_\varepsilon \dot{\chi}(t) = \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t))\rho_f(s(t)) (\check{A}_{rf} \chi(t)) + \check{B}_{wr} \varpi(t) \quad (20)$$

$$\dot{y}(t) = \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t))\rho_f(s(t)) (\check{C}_{yrf} \chi(t)) \quad (21)$$

$$\dot{z}(t) = \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t))\rho_f(s(t)) (\check{C}_{zrf} \chi(t)) \quad (22)$$

where \check{B}_{wr} and \check{C}_{zrf} are the appropriate matrices defined for Lemma 2 and Theorem 1, $\chi(t) = [x_1(t) \ x_2(t) \ q(t)]^T$, and $\varpi(t)$ is the disturbance relating to Assumption 1 which is defined as follows:

$$\varpi(t) = \begin{bmatrix} \frac{1}{\delta} G(\check{x}(t), t)F_{1r}\check{x}(t) \\ G(\check{x}(t), t)F_{2r}\varpi(t) \\ \varpi(t) \end{bmatrix} \quad (23)$$

Based on the LMI approach, the process of designing the fuzzy-modeled prescribed performance integral controller is provided in this subsection. The Lyapunov function is used to derive the adequate condition, which addresses the condition of the fuzzy-modeled prescribed performance integral controller, by achieving Definition 1. In the following lemma, the symmetric terms that exist in the symmetric block matrices are replaced by a symbol (*).

Lemma 2: Given the prescribed scalars $\gamma > 0$ and $\delta > 0$, the closed-loop systems of the uncertain fuzzy descriptor system (15)-(17) exhibit asymptotically robust stability and guarantee the H_∞ criterion (18) if there exist a matrix $P_\varepsilon = P_\varepsilon^T$ and matrices $Y_{1f}(\varepsilon), Y_{2f}(\varepsilon), Y_{I_f}(\varepsilon), f = 1, 2, \dots, j$

that satisfy the ε -dependent linear matrix inequalities as follows:

$$P_\varepsilon > 0 \tag{24}$$

$$\Theta_{rr}(\varepsilon) < 0, \quad r = 1, 2, \dots, j \tag{25}$$

$$\Theta_{rf}(\varepsilon) + \Theta_{fr}(\varepsilon) < 0, \quad r < f \leq j \tag{26}$$

where

$$\Theta_{rf}(\varepsilon) = \begin{bmatrix} (\Omega_{rf}(\varepsilon) + \Omega_{rf}^T(\varepsilon)) & (*)^T & (*)^T \\ E_\varepsilon^{-1} \check{B}_{w_r}^T & -\gamma^2 I & (*)^T \\ \check{C}_{z_{rf}} P_\varepsilon & 0 & -I \end{bmatrix}$$

with

$$\Omega_{rf}(\varepsilon) = \begin{bmatrix} \varphi_{rf_1}(\varepsilon) & \varphi_{rf_2}(\varepsilon) & B_{1_r} Y_{1_f}(\varepsilon) \\ \varphi_{rf_3}(\varepsilon) & \varphi_{rf_4}(\varepsilon) & B_{2_r} Y_{1_f}(\varepsilon) \\ C_{y_{1_r}} P_\varepsilon & C_{y_{2_r}} P_\varepsilon & 0 \end{bmatrix}$$

$$\varphi_{rf_1}(\varepsilon) = A_{11_r} E_\varepsilon^{-1} P_\varepsilon + B_{1_r} Y_{1_f}(\varepsilon)$$

$$\varphi_{rf_2}(\varepsilon) = A_{12_r} E_\varepsilon^{-1} P_\varepsilon + B_{1_r} Y_{2_f}(\varepsilon)$$

$$\varphi_{rf_3}(\varepsilon) = A_{21_r} E_\varepsilon^{-1} P_\varepsilon + B_{2_r} Y_{1_f}(\varepsilon)$$

$$\varphi_{rf_4}(\varepsilon) = A_{22_r} E_\varepsilon^{-1} P_\varepsilon + B_{2_r} Y_{2_f}(\varepsilon)$$

$$\check{B}_{w_r} = [\delta I \quad I \quad \check{B}_{w_r}]$$

$$\check{C}_{z_{rf}} = \left[\frac{\gamma\beta}{\delta} F_{1_r}^T \quad 0 \quad \sqrt{2}\beta\lambda F_{4_r}^T \quad \sqrt{2}\lambda\check{C}_{z_r}^T \right]^T$$

$$\lambda = \left(1 + \beta^2 \sum_{r=1}^j \left[\|F_{2_r}^T F_{2_r}\| \right] \right)^{\frac{1}{2}}$$

Remark 1: The proof of Lemma 2 is available in appendix.

III. FUZZY-MODELED PRESCRIBED PERFORMANCE INTEGRAL CONTROLLER DESIGN

From the case of ε -dependent LMI in Lemma 2, the existence of the parasitic parameter ε occurs because the LMI given in Lemma 2 becomes ill-conditioned. In the uncertain non-linear descriptor systems, this ill-conditioned LMI can occur naturally, so this situation can be alleviated by the following theorem.

Theorem 1: Given prescribed scalars $\gamma > 0$ and $\delta > 0$, the closed-loop systems of the uncertain fuzzy descriptor system (15)-(17) have asymptotically robust stability and guarantee the H_∞ criterion (18) if there exist matrix P and matrices $Y_{1_f}, Y_{2_f}, Y_{I_f}, f = 1, 2, \dots, j$ that satisfy the ε -independent linear matrix inequalities as follows:

$$VPV + WPW + XPX > 0 \tag{27}$$

$$\Theta_{rr} < 0, \quad r = 1, 2, \dots, j \tag{28}$$

$$\Theta_{rf} + \Theta_{fr} < 0, \quad r < f \leq j \tag{29}$$

where

$$\Theta_{rf} = \begin{bmatrix} \left(\begin{matrix} \Omega_{11_{rf}} & (*)^T & (*)^T \\ \Omega_{21_{rf}} & \Omega_{22_{rf}} & (*)^T \\ \Omega_{31_{rf}} & \Omega_{32_{rf}} & \Omega_{33_{rf}} \end{matrix} \right) & (*)^T & (*)^T \\ \check{B}_{w_r}^T & -\gamma^2 I & (*)^T \\ \check{C}_{z_{rf}} P & 0 & -I \end{bmatrix},$$

$$VPV = VP^T V,$$

$$WPW = WP^T W,$$

$$XPX = XP^T X.$$

with

$$P = \begin{bmatrix} P_1 & 0 & P_3 \\ P_2 & P_1 & P_2 \\ P_3 & 0 & P_1 \end{bmatrix}, \quad V = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix},$$

$$\Omega_{11_{rf}} = A_{11_r} P_1 + P_1 A_{11_r}^T + A_{12_r} P_2 + P_2 A_{12_r}^T + B_{1_r} Y_{1_f} + Y_{1_f}^T B_{1_r}^T$$

$$\Omega_{21_{rf}} = A_{21_r} P_1 + P_1 A_{21_r}^T + A_{22_r} P_2 + B_{2_r} Y_{1_f} + Y_{2_f}^T B_{1_r}^T$$

$$\Omega_{22_{rf}} = A_{22_r} P_1 + P_1 A_{22_r}^T + B_{2_r} Y_{2_f} + Y_{2_f}^T B_{2_r}^T$$

$$\Omega_{31_{rf}} = C_{y_{1_r}} P_1 + C_{y_{2_r}} P_2 + P_2 A_{12_r}^T + P_3 A_{11_r}^T + Y_{3_f}^T B_{1_r}^T$$

$$\Omega_{32_{rf}} = C_{y_{2_r}} P_1 + P_2 A_{22_r}^T + P_3 A_{21_r}^T + Y_{3_f}^T B_{2_r}^T$$

$$\Omega_{33_{rf}} = C_{y_{2_r}} P_2 + P_2 C_{y_{2_r}}^T + C_{y_{1_r}} P_3 + P_3 C_{y_{1_r}}^T$$

$$\check{B}_{w_r} = [\delta I \quad I \quad \check{B}_{w_r}]$$

$$\check{C}_{z_{rf}} = \left[\frac{\gamma\beta}{\delta} F_{1_r}^T \quad 0 \quad \sqrt{2}\beta\lambda F_{4_r}^T \quad \sqrt{2}\lambda\check{C}_{z_r}^T \right]^T$$

$$\lambda = \left(1 + \beta^2 \sum_{r=1}^j \left[\|F_{2_r}^T F_{2_r}\| \right] \right)^{\frac{1}{2}}$$

Note that for the sufficiently small $\hat{\varepsilon} > 0$, the inequality (18) holds for $\varepsilon \in (0, \hat{\varepsilon}]$, so the appropriate fuzzy controller for the uncertain fuzzy descriptor system is as follows:

$$u(t) = \sum_{f=1}^j \rho_f(s(t))(K_{1_f} x_1(t) + K_{2_f} x_2(t) + K_{I_f} q(t)) \tag{30}$$

where

$$K_f = Y_f P^{-1}$$

with

$$K_f = [K_{1_f} \quad K_{2_f} \quad K_{I_f}]$$

Proof of Theorem 1. Define a matrix P that is the positive-definite matrix (27) and holds for ε -independent linear matrix inequalities (27)-(29) as follows:

$$P = \begin{bmatrix} P_1 & 0 & P_3 \\ P_2 & P_1 & P_2 \\ P_3 & 0 & P_1 \end{bmatrix} \tag{31}$$

with $P_1 = P_1^T > 0$. Let

$$P_\varepsilon = E_\varepsilon (P + \varepsilon \tilde{P}) \tag{32}$$

with

$$\tilde{P} = \begin{bmatrix} 0 & P_2 & 0 \\ 0 & 0 & 0 \\ 0 & P_2 & 0 \end{bmatrix} \tag{33}$$

Substituting (31) and (33) into (32) gains

$$P_\varepsilon = \begin{bmatrix} P_1 & \varepsilon P_2 & P_3 \\ \varepsilon P_2 & \varepsilon P_1 & \varepsilon P_2 \\ P_3 & \varepsilon P_2 & P_1 \end{bmatrix} \quad (34)$$

With respect to equality (34), the matrix $P_\varepsilon = P_\varepsilon^T$ and there is a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, $P_\varepsilon > 0$. Using the matrix inversion lemma, the obtained equalities that support the existence of matrix P_ε are

$$P_\varepsilon^{-1} = (P^{-1} + \varepsilon Z_\varepsilon) E_\varepsilon^{-1} \quad (35)$$

where

$$Z_\varepsilon = -P^{-1} \tilde{P} (I + \varepsilon P^{-1} \tilde{P})^{-1} P^{-1},$$

$$E_\varepsilon = \begin{bmatrix} I & 0 & 0 \\ 0 & \varepsilon I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Using Assumption 1, the closed-loop fuzzy system (15)-(17) can be expressed by (20)-(22). Let us consider the following Lyapunov function

$$V(\chi(t)) = \chi^T(t) E_\varepsilon Q_\varepsilon \chi(t) \quad (36)$$

where $Q_\varepsilon = (P^{-1} + \varepsilon Z_\varepsilon)$. Using the matrix inversion lemma, it can be shown simply as $E_\varepsilon Q_\varepsilon = Q_\varepsilon^T E_\varepsilon$ and there is a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, $P_\varepsilon > 0$, $E_\varepsilon Q_\varepsilon > 0$. Differentiating $V(\chi(t))$ along the nonlinear descriptor system with uncertainties and the controller (30) gains

$$\begin{aligned} \dot{V}(\chi(t)) &= \dot{\chi}^T(t) E_\varepsilon Q_\varepsilon \chi(t) + \chi^T(t) E_\varepsilon Q_\varepsilon \dot{\chi}(t) \\ \dot{V}(\chi(t)) &= \dot{\chi}^T(t) E_\varepsilon Q_\varepsilon \chi(t) + \chi^T(t) Q_\varepsilon^T E_\varepsilon \dot{\chi}(t) \\ \dot{V}(\chi(t)) &= [\dot{x}_1^T(t) \ \varepsilon \dot{x}_2^T(t) \ \dot{q}^T(t)] Q_\varepsilon \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} \\ &\quad + [x_1^T(t) \ x_2^T(t) \ q^T(t)] Q_\varepsilon^T \begin{bmatrix} \dot{x}_1(t) \\ \varepsilon \dot{x}_2(t) \\ \dot{q}(t) \end{bmatrix} \\ \dot{V}(\chi(t)) &= \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) (\chi^T(t) \check{A}_{rf}^T Q_\varepsilon \chi(t)) \\ &\quad + \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) (\chi^T(t) Q_\varepsilon^T \check{A}_{rf} \chi(t)) \\ &\quad + \dot{\omega}^T(t) \check{B}_{w_r}^T Q_\varepsilon \chi(t) \\ &\quad + \chi^T(t) Q_\varepsilon^T \check{B}_{w_r} \dot{\omega}(t) \end{aligned} \quad (37)$$

Adding and subtracting $-\dot{z}^T(t) \dot{z}(t) + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \rho_r(s(t)) \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) [\dot{\omega}^T(t) \dot{\omega}(t)]$ to and from (37), one obtains

$$\begin{aligned} \dot{V}(\chi(t)) &= \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ &\quad \times [\chi^T(t) \quad \dot{\omega}^T(t)] \end{aligned}$$

$$\begin{aligned} &\times \begin{bmatrix} \check{A}_{rf}^T Q_\varepsilon + Q_\varepsilon^T \check{A}_{rf} + \check{C}_{z_{rf}}^T \check{C}_{z_{hg}} & Q_\varepsilon^T \check{B}_{w_r} \\ \check{B}_{w_r}^T Q_\varepsilon & -\gamma^2 I \end{bmatrix} \\ &\times \begin{bmatrix} \chi(t) \\ \dot{\omega}(t) \end{bmatrix} - \dot{z}^T(t) \dot{z}(t) \\ &+ \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \rho_f(s(t)) \rho_h(s(t)) \\ &\quad \times \rho_g(s(t)) (\dot{\omega}^T(t) \dot{\omega}(t)) \end{aligned} \quad (38)$$

Using the fact $Q_\varepsilon = (P^{-1} + \varepsilon Z_\varepsilon)$, $Q = P^{-1}$, and $Z_\varepsilon = -P^{-1} \tilde{P} (I + \varepsilon P^{-1} \tilde{P})^{-1} P^{-1}$ equation (38) becomes

$$\begin{aligned} \dot{V}(\chi(t)) &= \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ &\quad \times [\chi^T(t) \quad \dot{\omega}^T(t)] \\ &\quad \times \begin{bmatrix} \check{A}_{rf}^T Q + Q^T \check{A}_{rf} + \check{C}_{z_{rf}}^T \check{C}_{z_{hg}} & Q^T \check{B}_{w_r} \\ \check{B}_{w_r}^T Q & -\gamma^2 I \end{bmatrix} \\ &\quad \times \begin{bmatrix} \chi(t) \\ \dot{\omega}(t) \end{bmatrix} + \varepsilon \Delta \Theta_{rf} - \dot{z}^T(t) \dot{z}(t) \\ &\quad + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \rho_f(s(t)) \rho_h(s(t)) \\ &\quad \times \rho_g(s(t)) (\dot{\omega}^T(t) \dot{\omega}(t)) \end{aligned} \quad (39)$$

where

$$\varepsilon \Delta \Theta_{rf} = \begin{bmatrix} \check{A}_{rf}^T Z_\varepsilon + M_\varepsilon^T \check{A}_{rf} & Z_\varepsilon \check{B}_{w_r} \\ \check{B}_{w_r}^T Z_\varepsilon & 0 \end{bmatrix} \quad (40)$$

For ε -multiplying, the term variable (40) will disappear when the parasitic parameter (ε) tends to zero. Next, recall Theorem 1 and by substituting $K_f = Y_f P^{-1}$ into (28) and (29), the linear matrix inequalities are rewritten as follows:

$$\Theta_{rr} < 0, \quad r = 1, 2, \dots, j \quad (41)$$

$$\Theta_{rf} + \Theta_{fr} < 0, \quad r < f \leq j \quad (42)$$

where

$$\Theta_{rf} = \begin{bmatrix} \begin{pmatrix} \Omega_{11_{rf}} & (*)^T & (*)^T \\ \Omega_{21_{rf}} & \Omega_{22_{rf}} & (*)^T \\ \Omega_{31_{rf}} & \Omega_{32_{rf}} & \Omega_{33_{rf}} \end{pmatrix} & (*)^T & (*)^T \\ \check{B}_{w_r}^T & -\gamma^2 I & (*)^T \\ \check{C}_{z_{rf}} P & 0 & -I \end{bmatrix} \quad (43)$$

with

$$\begin{aligned} \Omega_{11_{rf}} &= A_{11_r} P_1 + P_1 A_{11_r}^T + A_{12_r} P_2 + P_2 A_{12_r}^T \\ &\quad + B_{1_r} (K_{1_f} P_1 + K_{2_f} P_2 + K_{I_f} P_3) \\ &\quad + (P_1 K_{1_f} + P_2 K_{2_f} + P_3 K_{I_f}) B_{1_r}^T \\ \Omega_{21_{rf}} &= A_{21_r} P_1 + P_1 A_{12_r}^T + A_{22_r} P_2 \\ &\quad + B_{2_r} (K_{1_f} P_1 + K_{2_f} P_2 + K_{I_f} P_3) + P_1 K_{2_f} B_{1_r}^T \end{aligned}$$

$$\begin{aligned} \Omega_{22_{rf}} &= A_{22_r} P_1 + P_1 A_{22_r}^T + B_{2_r} K_{2_f} P_1 + P_1 K_{2_f} B_{2_r}^T \\ \Omega_{31_{rf}} &= C_{y1_r} P_1 + C_{y2_r} P_2 + P_2 A_{12_r}^T + P_3 A_{11_r}^T \\ &\quad + (P_3 K_{1_f} + P_2 K_{2_f} + P_1 K_{1_f}) B_{1_r}^T \\ \Omega_{32_{rf}} &= C_{y2_r} P_1 + P_2 A_{22_r}^T + P_3 A_{21_r}^T \\ &\quad + (P_3 K_{1_f} + P_2 K_{2_f} + P_1 K_{1_f}) B_{2_r}^T \\ \Omega_{33_{rf}} &= C_{y2_r} P_2 + P_2 C_{y2_r}^T + C_{y1_r} P_3 + P_3 C_{y1_r}^T \\ \check{B}_{w_r} &= [\delta I \quad I \quad \check{B}_{w_r}] \\ \check{C}_{z_{rf}} &= \left[\frac{\gamma \beta}{\delta} F_{1_r}^T \quad 0 \quad \sqrt{2} \beta \lambda F_{4_r}^T \quad \sqrt{2} \lambda \check{C}_{z_r}^T \right]^T \\ \lambda &= \left(1 + \beta^2 \sum_{r=1}^j [\|F_{2_r}^T F_{2_r}\|] \right)^{\frac{1}{2}} \end{aligned}$$

Pre- and post-multiplying (41)-(42) by $\begin{bmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and using the fact $Q = P^{-1}$ gains

$$\begin{bmatrix} \check{A}_{rr}^T Q + Q^T \check{A}_{rr} & (*)^T & (*)^T \\ \check{B}_{w_r}^T Q & -\gamma^2 I & (*)^T \\ \check{C}_{z_{rr}} & 0 & -I \end{bmatrix} < 0 \quad (44)$$

and

$$\begin{bmatrix} \check{A}_{rf}^T Q + Q^T \check{A}_{rf} & (*)^T & (*)^T \\ \check{B}_{w_r}^T Q & -\gamma^2 I & (*)^T \\ \check{C}_{z_{rf}} & 0 & -I \end{bmatrix} + \begin{bmatrix} \check{A}_{fr}^T Q + Q^T \check{A}_{fr} & (*)^T & (*)^T \\ \check{B}_{w_r}^T Q & -\gamma^2 I & (*)^T \\ \check{C}_{z_{fr}} & 0 & -I \end{bmatrix} < 0 \quad (45)$$

Applying the Schur complement to (44)-(45) and rewriting the equation as follows:

$$\begin{bmatrix} \check{A}_{rr}^T Q + Q^T \check{A}_{rr} + \check{C}_{z_{rr}}^T \check{C}_{z_{rr}} & (*)^T \\ \check{B}_{w_r}^T Q & -\gamma^2 I \end{bmatrix} < 0 \quad (46)$$

and

$$\begin{bmatrix} \check{A}_{rf}^T Q + Q^T \check{A}_{rf} + \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} & (*)^T \\ \check{B}_{w_r}^T Q & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \check{A}_{fr}^T Q + Q^T \check{A}_{fr} + \check{C}_{z_{fr}}^T \check{C}_{z_{fr}} & (*)^T \\ \check{B}_{w_r}^T Q & -\gamma^2 I \end{bmatrix} < 0 \quad (47)$$

Applying (46)-(47) with the following fact

$$\begin{aligned} &\sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ &\quad \times (H_{rf}^T S_{hg}) \\ &\leq \frac{1}{2} \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) (H_{rf}^T H_{rf} + S_{rf}^T S_{rf}) \end{aligned} \quad (48)$$

The result is

$$\begin{bmatrix} \check{A}_{rf}^T Q + Q^T \check{A}_{rf} + \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} & (*)^T \\ \check{B}_{w_r}^T Q & -\gamma^2 I \end{bmatrix} < 0 \quad (49)$$

Accordingly, (49) is less than zero; then, using the fact that $\rho_r(s(t)) \geq 0$ and $\sum_{r=1}^j \rho_r(s(t)) = 1$, (39) becomes

$$\begin{aligned} \dot{V}(\chi(t)) &\leq -\dot{z}^T(t) \dot{z}(t) + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \\ &\quad \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) (\dot{\varpi}^T(t) \dot{\varpi}(t)) \end{aligned} \quad (50)$$

Integrating both sides of (50) yields

$$\begin{aligned} \int_0^{T_f} \dot{V}(\chi(t)) dt &\leq \int_0^{T_f} (-\dot{z}^T(t) \dot{z}(t)) \\ &\quad + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \\ &\quad \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ &\quad \times (\dot{\varpi}^T(t) \dot{\varpi}(t)) dt \end{aligned} \quad (51)$$

$$\begin{aligned} V(\chi(T_f)) - V(\chi(0)) &\leq \int_0^{T_f} (-\dot{z}^T(t) \dot{z}(t)) \\ &\quad + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \\ &\quad \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ &\quad \times (\dot{\varpi}^T(t) \dot{\varpi}(t)) dt \end{aligned} \quad (52)$$

Using the fact that $\chi(0) = 0$ and $V(\chi(T_f)) \geq 0$ for all $T_f \neq 0$, then (50) becomes

$$\begin{aligned} \int_0^{T_f} \dot{z}^T(t) \dot{z}(t) dt &\leq \gamma^2 \int_0^{T_f} \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \\ &\quad \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ &\quad \times (\dot{\varpi}^T(t) \dot{\varpi}(t)) dt \end{aligned} \quad (53)$$

Substituting $\dot{z}(t)$ and $\dot{\varpi}(t)$ given in (22) and (23), respectively, into (53) and using the fact that $\|G(\check{x}(t), t)\| \leq \beta$,

$$\varpi(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ 0 \end{bmatrix}, \chi = \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix}, \text{ and inequality (48) yields}$$

$$\begin{aligned} &\int_0^{T_f} \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ &\quad \times (2\lambda^2 \beta^2 \check{x}^T(t) F_{4_r}^T F_{4_r} \check{x}(t) + 2\lambda^2 \check{x}^T(t) \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} \check{x}(t)) dt \\ &\leq \gamma^2 \lambda^2 \int_0^{T_f} \varpi^T(t) \varpi(t) dt \end{aligned} \quad (54)$$

Adding and subtracting

$$\begin{aligned} \lambda^2 \check{z}^T(t) \check{z}(t) &= \lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ &\times \left(\check{x}^T(t) \left(\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r} \right) \right. \\ &\times \left. \left(\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r} \right) \check{x}(t) \right) \end{aligned}$$

to and from (54), one has

$$\begin{aligned} \gamma^2 \lambda^2 \int_0^{T_f} \varpi^T(t) \varpi(t) dt &\geq \int_0^{T_f} \lambda^2 \check{z}^T(t) \check{z}(t) \\ &+ \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ &\times (2\lambda^2 \beta^2 \check{x}^T(t) F_{4_r}^T F_{4_r} \check{x}(t) \\ &+ 2\lambda^2 \check{x}^T(t) \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} \check{x}(t)) \\ &- \lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ &\times (\check{x}^T(t) \\ &\times \left(\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r} \right)^T \\ &\times \left(\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r} \right) \\ &\times \check{x}(t)) \end{aligned} \quad (55)$$

Employing the triangle inequality (19) and the fact that $\|G(\check{x}(t), t)\| \leq \beta$, one obtains

$$\begin{aligned} 2\lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) &\left(\beta^2 \check{x}^T(t) F_{4_r}^T F_{4_r} \check{x}(t) \right. \\ &\left. + \check{x}^T(t) \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} \check{x}(t) \right) \\ &\leq \lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ &\times \left(\check{x}^T(t) \left(\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r} \right) \right. \\ &\times \left. \left(\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r} \right) \check{x}(t) \right) \end{aligned} \quad (56)$$

Using (56) on (55), one obtains

$$\int_0^{T_f} \check{z}^T(t) \check{z}(t) dt \leq \gamma^2 \int_0^{T_f} \varpi^T(t) \varpi(t) dt \quad (57)$$

Finally, inequality (18) holds for $\varepsilon \in (0, \hat{\varepsilon}]$. This completes the proof.

IV. NUMERICAL EXAMPLE

This section illustrates how to apply the proposed controller with the application of the uncertain nonlinear descriptor system. A circuit of the separately excited dc motor with parametric uncertainties [52] is shown in Fig. 2, and the equations of such a system are characterized as follows:

$$J \frac{d\omega(t)}{dt} = -D\omega(t) + K_m L_f i^2(t) - \tau_L(t) \quad (58)$$

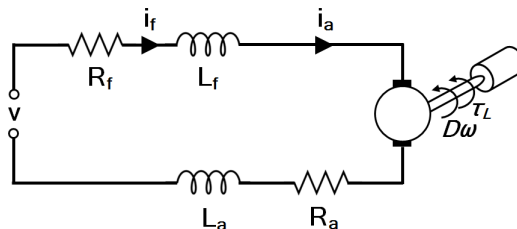


FIGURE 2. The separately excited dc motor circuit.

TABLE 1. Parameters of the Separately Excited DC Motor.

Parameters	Name	Value	Units
J	Moment of inertia	0.705	kg·m ²
K_m	Torque/back EMF constant	1.348	N·m/A
L_f	Field inductance	0.917	H
D	Viscous-friction coefficient	4.230	N·m/rad/s
R	Resistance	7.2	Ω

$$L \frac{di(t)}{dt} = - (K_m L_f i) \omega(t) - (R \pm \Delta R) i(t) + v(t) \quad (59)$$

where the parameters in Fig. 2 including $L_f, L_a, R_f, R_a, i_f, i_a$ are the field inductance, the armature inductance, the field resistance, the armature resistance, the field current, and the armature current respectively, so some parameters in (58)-(59) related to [52] can be defined as $L \triangleq L_f + L_a$, $R \triangleq R_f + R_a$, and $i = i_f = i_a$. The state equation variables are defined as follows: the state vectors $x_1(t)$ and $x_2(t)$ replace the angular speed $\omega(t)$ and the electric current $i(t)$, respectively; the controlled input $u(t)$ replaces the voltage $v(t)$; the disturbance $w(t)$ replaces the load torque $\tau_L(t)$; and the parasitic parameter ε replaces the inductance L . Therefore, the state equation of the separately excited dc motor circuit (58)-(59) can be rewritten as follows:

$$\dot{x}_1(t) = -\frac{D}{J} x_1(t) + \frac{K_m L_f}{J} x_2^2(t) - \frac{w(t)}{J} \quad (60)$$

$$\begin{aligned} \varepsilon \dot{x}_2(t) &= - (K_m L_f x_2(t)) x_1(t) \\ &- (R \pm \Delta R) x_2(t) + u(t) \end{aligned} \quad (61)$$

$$y(t) = x_2(t) \quad (62)$$

$$z_1(t) = x_1(t) \quad (63)$$

$$z_2(t) = x_2(t) \quad (64)$$

Given the parameters in the separately excited dc motor circuit with their values defined in Table 1, the equation (60)-(64) can be rewritten as follows:

$$\dot{x}_1(t) = -6x_1(t) + 1.753x_2^2(t) - 1.42w(t) \quad (65)$$

$$\begin{aligned} \varepsilon \dot{x}_2(t) &= - (1.236x_2(t)) x_1(t) \\ &- (7.2 \pm \Delta R) x_2(t) + u(t) \end{aligned} \quad (66)$$

$$y(t) = x_2(t) \quad (67)$$

$$z_1(t) = x_1(t) \quad (68)$$

$$z_2(t) = x_2(t) \quad (69)$$

The uncertain nonlinear descriptor systems that are formed in the Takagi-Sugeno fuzzy model can be used to describe

equations (65)-(69). The Takagi-Sugeno fuzzy model of the separately excited dc motor circuit can be described as follows:

Plant rule 1:

IF $s_1(t)$ is $M_1(s_1(t))$ and $s_2(t)$ is $N_1(s_2(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{\tilde{x}}(t) &= (A_1 + \Delta A_1) \tilde{x}(t) + B_1 u(t) + B_w w(t) \\ \tilde{z}(t) &= C_z \tilde{x}(t) \\ \tilde{y}(t) &= C_y \tilde{x}(t) \end{aligned}$$

Plant rule 2:

IF $s_1(t)$ is $M_1(s_1(t))$ and $s_2(t)$ is $N_2(s_2(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{\tilde{x}}(t) &= (A_2 + \Delta A_2) \tilde{x}(t) + B_2 u(t) + B_w w(t) \\ \tilde{z}(t) &= C_z \tilde{x}(t) \\ \tilde{y}(t) &= C_y \tilde{x}(t) \end{aligned}$$

Plant rule 3:

IF $s_1(t)$ is $M_2(s_1(t))$ and $s_2(t)$ is $N_1(s_2(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{\tilde{x}}(t) &= (A_3 + \Delta A_3) \tilde{x}(t) + B_3 u(t) + B_w w(t) \\ \tilde{z}(t) &= C_z \tilde{x}(t) \\ \tilde{y}(t) &= C_y \tilde{x}(t) \end{aligned}$$

Plant rule 4:

IF $s_1(t)$ is $M_2(s_1(t))$ and $s_2(t)$ is $N_2(s_2(t))$ THEN

$$\begin{aligned} E_\varepsilon \dot{\tilde{x}}(t) &= (A_4 + \Delta A_4) \tilde{x}(t) + B_4 u(t) + B_w w(t) \\ \tilde{z}(t) &= C_z \tilde{x}(t) \\ \tilde{y}(t) &= C_y \tilde{x}(t) \end{aligned}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} -6 & 1.753 \\ -1.236 & 7.2 \end{bmatrix}, & A_2 &= \begin{bmatrix} -6 & 1.753 \\ 1.236 & 7.2 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -6 & -1.753 \\ -1.236 & 7.2 \end{bmatrix}, & A_4 &= \begin{bmatrix} -6 & -1.753 \\ 1.236 & 7.2 \end{bmatrix}, \\ B_1 = B_2 = B_3 = B_4 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & B_w &= \begin{bmatrix} -1.42 \\ 0 \end{bmatrix}, \\ C_y &= [0 \quad 1], & C_z &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \tilde{x}(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, & \tilde{x}_0(t) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, & E_\varepsilon &= \begin{bmatrix} I & 0 \\ 0 & \varepsilon \end{bmatrix}, \\ \Delta A_1 &= G(\tilde{x}(t), t)F_{11}, & \Delta A_2 &= G(\tilde{x}(t), t)F_{12}, \\ \Delta A_3 &= G(\tilde{x}(t), t)F_{13}, & \Delta A_4 &= G(\tilde{x}(t), t)F_{14} \end{aligned}$$

Assuming that $\|G(\tilde{x}(t), t)\| \leq \beta$, $\beta = 1$ and the values of R are uncertain and bounded within 30% of their nominal values, then

$$F_{11} = F_{12} = F_{13} = F_{14} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3 \end{bmatrix}$$

Next, such a nonlinear system can be defined by the Takagi-Sugeno fuzzy model, so the membership functions can be defined as in Fig. 3 and 4. Let the X-axis be the state $s_1(t)$ and $s_2(t)$, the solid line and dashed line are the first fuzzy set $M_1(s_1(t))$ and the second fuzzy set $M_2(s_1(t))$, respectively,

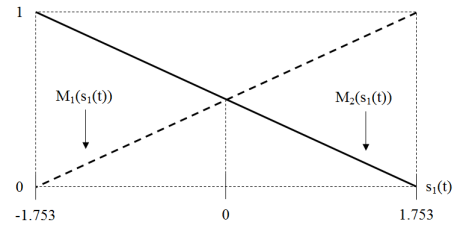


FIGURE 3. Membership function $M_1(s_1(t))$ and $M_2(s_1(t))$.

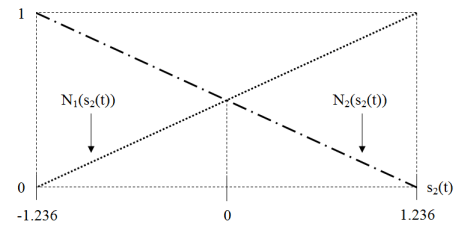


FIGURE 4. Membership function $N_1(s_2(t))$ and $N_2(s_2(t))$.

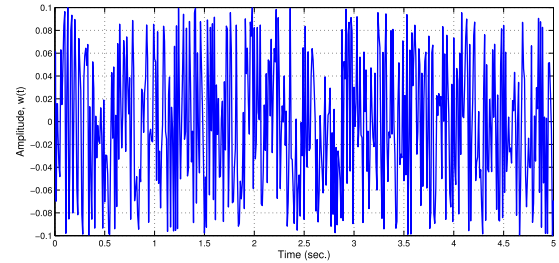


FIGURE 5. The disturbance input signal $w(t)$.

and the dotted line and dot-dashed line are the third fuzzy set $N_1(s_2(t))$ and the last fuzzy set $N_2(s_2(t))$, respectively.

$$\begin{aligned} M_1(s_1(t)) &= \frac{s_1(t) + 1.753}{3.506}, & M_2(s_1(t)) &= \frac{1.753 - s_1(t)}{3.506} \\ N_1(s_2(t)) &= \frac{s_2(t) + 1.236}{2.472}, & N_2(s_2(t)) &= \frac{1.236 - s_2(t)}{2.472} \end{aligned}$$

The example, which applies Theorem 1, has been specified employing the MATLAB LMI solver, and the results of the LMI optimization with $\varepsilon = 0.01$ and $\gamma = 1$ are shown as follows:

$$\begin{aligned} P &= \begin{bmatrix} 2.893 & 0 & 0.258 \\ -2.299 & 2.893 & -2.299 \\ 0.258 & 0 & 2.893 \end{bmatrix}, \\ Y_1 &= [20.085 \quad -21.092 \quad 16.849], \\ Y_2 &= [12.934 \quad -21.092 \quad 16.212], \\ Y_3 &= [20.186 \quad -21.092 \quad 16.849], \\ Y_4 &= [13.035 \quad -21.092 \quad 16.213], \\ K_1 &= [1.155 \quad -7.291 \quad -0.073], \\ K_2 &= [-1.317 \quad -7.291 \quad -0.073], \\ K_3 &= [1.190 \quad -7.291 \quad -0.076], \\ K_4 &= [-1.282 \quad -7.291 \quad -0.076] \end{aligned}$$

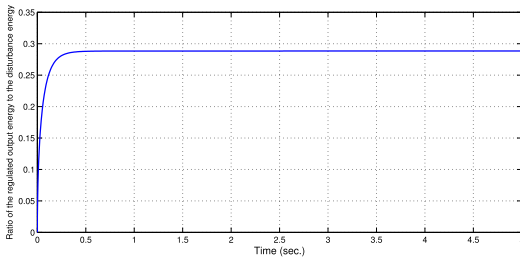


FIGURE 6. The ratio of the regulated output energy to the disturbance energy ($\epsilon = 0.01$).

The result of the proposed controller is

$$u(t) = \sum_{f=1}^4 \rho_f(s(t))(K_{1f}x_1(t) + K_{2f}x_2(t) + K_{I_f}q(t)) \quad (70)$$

where

$$\begin{aligned} \rho_1(s(t)) &= M_1(s_1(t)), & \rho_2(s(t)) &= M_2(s_1(t)), \\ \rho_3(s(t)) &= N_1(s_2(t)), & \rho_4(s(t)) &= N_2(s_2(t)), \end{aligned}$$

Remark 2: Regarding the simulation part, Fig. 5 was used as the system disturbance signal $w(t)$, and the ratio of the regulated output energy to the exogenous input disturbance energy of the example, which was simulated by the proposed controller with $\epsilon = 0.01$, is shown in Fig. 6. First, the result of Fig. 6 shows that the value of the ratio tends to a constant

TABLE 2. The different values of ϵ that can be controlled by the proposed controller.

ϵ	γ	Positive-definition condition of	
		P_ϵ (Lemma 2)	P (Theorem 1)
0.0001	0.505	Passed	Passed
0.001	0.506	Passed	Passed
0.01	0.506	Passed	Passed
0.10	0.509	Passed	Passed
0.44	0.553	Failed	Passed
0.50	0.597	Failed	Passed
1.00	0.616	Failed	Passed
1.12	0.652	Failed	Failed

value γ of 0.256 after 0.5 sec, so the square root of the constant value γ is 0.506, which is less than the prescribed value.

Moreover, the proposed controller can achieve the H_∞ condition based on the LMI approach with different values of the small parameter ϵ by reaching the positive-definite condition of matrix P , which is shown in Table 2. The value of ϵ at 0.44 makes the ill-conditioned LMI that cannot obtain the positive-definite condition of matrix P_ϵ based on Lemma 2, whereas the proposed controller related to ϵ -independent LMI according to Theorem 1 can reach the positive-definite condition of matrix P . In addition, the advantages of the proposed controller related to integral controller can reduce both the steady-state error and the overshoot for such a system. Fig. 7 and Fig. 8 are shown that the response performances of the proposed controller quickly tend to reach the equilibrium

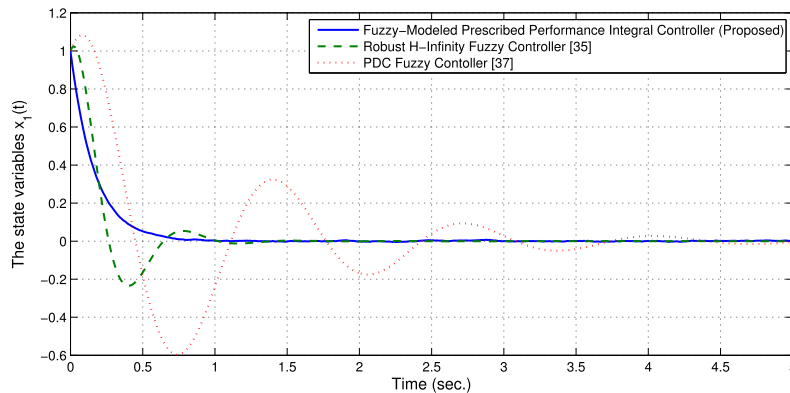


FIGURE 7. The response of the motor's controlled angular speed $x_1(t)$ ($\epsilon = 0.01$).

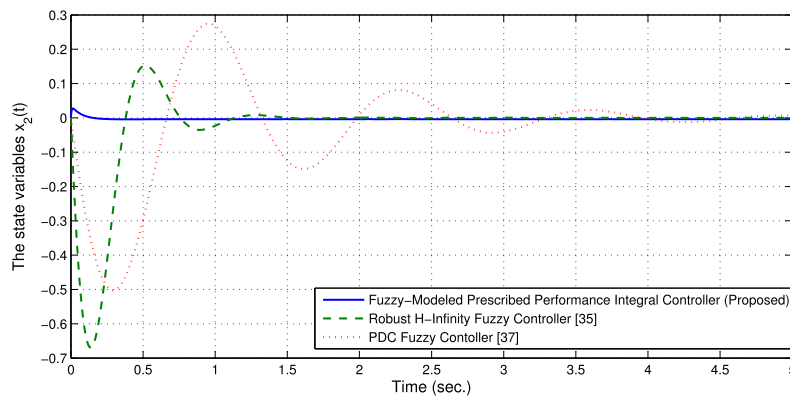


FIGURE 8. The response of the motor's controlled current $x_2(t)$ ($\epsilon = 0.01$).

point without the steady-state error and the overshoot when compared with the other methods [35] and [37].

Remark 3: According to Theorem 1, if the higher order nonlinear system which exists widely in many engineering descriptor systems, is considered, it may take time to obtain the result due to the complexity of the calculation and it is sometimes difficult to obtain the feasible solution. The high dimension of the ε -independent LMI may be a cause of this situation. In addition, if the nonlinear descriptor system with stochastic disturbance and time-delay is examined, this control problem will not be solved in our proposed technique and is also the limitation of the proposed control method. This problem is still an open problem.

V. CONCLUSIONS

The aim of this paper is to present the fuzzy-modeled prescribed performance integral controller for the uncertain nonlinear descriptor system described by the Takagi-Sugeno fuzzy model. Based on the LMI approach, the adequate condition of this proposed controller is given to guarantee the L_2 -gain and the stability of H_∞ performance under the existence of the parasitic parameter, parametric uncertainties, and the disturbance in such a system. The Lyapunov function is employed to prove the achieved condition of such a controller, and the ill-conditioned LMI can be alleviated by separating the ε -independent LMI from the ε -dependent LMI, such that when ε tends to zero, the ε -dependent LMI also tends to zero. In addition, numerous simulation results of an example show that the proposed controller can overcome many important factors; namely, the proposed controller can reduce the steady-state error, the overshoot, the robust effect, and the equilibrium point's time. Thus, the fuzzy-modeled prescribed performance integral controller is an efficient and suitable controller for the uncertain nonlinear descriptor system. Due to the complexity of the calculation, it may be difficult to obtain the feasible solution. However, to reduce the computational complexity and the number of variables, these problems are interesting and important, and can be considered under this scheme of the existing research results in our future research work. In addition, the unknown external disturbances and the time-varying delay may increase the complexity of the descriptor control problem which can be also investigated in our future research work.

**APPENDIX
PROOF OF LEMMA 2**

Consider the following Lyapunov function

$$V(\chi(t)) = \chi^T(t)Q_\varepsilon\chi(t) \tag{71}$$

where $Q_\varepsilon = P_\varepsilon^{-1}$. Differentiating $V(\chi(t))$ along the nonlinear descriptor system with uncertainties and the controller (14) yields

$$\begin{aligned} \dot{V}(\chi(t)) &= \dot{\chi}^T(t)Q_\varepsilon\chi(t) + \chi^T(t)Q_\varepsilon\dot{\chi}(t) \\ \dot{V}(\chi(t)) &= \begin{bmatrix} \dot{x}_1^T(t) & \dot{x}_2^T(t) & \dot{q}^T(t) \end{bmatrix} Q_\varepsilon \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &+ \begin{bmatrix} x_1^T(t) & x_2^T(t) & q^T(t) \end{bmatrix} Q_\varepsilon^T \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{q}(t) \end{bmatrix} \\ \dot{V}(\chi(t)) &= \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t))\rho_f(s(t)) \left(\chi^T(t)E_\varepsilon^{-1}\check{A}_{rf}^T Q_\varepsilon \chi(t) \right) \\ &+ \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t))\rho_f(s(t)) (\chi^T(t)Q_\varepsilon^T \check{A}_{rf} E_\varepsilon^{-1} \chi(t) \\ &+ \check{\omega}^T(t)E_\varepsilon^{-1}\check{B}_{wr}^T Q_\varepsilon \chi(t) \\ &+ \chi^T(t)Q_\varepsilon^T \check{B}_{wr} E_\varepsilon^{-1} \check{\omega}(t) \end{aligned} \tag{72}$$

Adding and subtracting $-\dot{z}^T(t)\dot{z}(t) + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \rho_r(s(t))\rho_f(s(t))\rho_h(s(t))\rho_g(s(t)) [\check{\omega}^T(t)\check{\omega}(t)]$ to and from (72), one obtains

$$\begin{aligned} \dot{V}(\chi(t)) &= \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t))\rho_f(s(t))\rho_h(s(t))\rho_g(s(t)) \\ &\times \begin{bmatrix} \chi^T(t) & \check{\omega}^T(t) \\ \left(\begin{array}{c} E_\varepsilon^{-1}\check{A}_{rf}^T Q_\varepsilon \\ + Q_\varepsilon^T \check{A}_{rf} E_\varepsilon^{-1} \\ + \check{C}_{zrf}^T \check{C}_{zhg} \\ E_\varepsilon^{-1}\check{B}_{wr}^T Q_\varepsilon \end{array} \right) & Q_\varepsilon^T \check{B}_{wr} E_\varepsilon^{-1} \\ & \times \begin{bmatrix} \chi(t) \\ \check{\omega}(t) \end{bmatrix} - \dot{z}^T(t)\dot{z}(t) \\ & + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t))\rho_f(s(t))\rho_h(s(t)) \\ & \times \rho_g(s(t)) \left(\check{\omega}^T(t)\check{\omega}(t) \right) \end{bmatrix} \end{aligned} \tag{73}$$

Pre- and post-multiplying (25)-(26) by $\begin{bmatrix} Q_\varepsilon & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ on gains

$$\begin{bmatrix} E_\varepsilon^{-1}\check{A}_{rr}^T Q_\varepsilon + Q_\varepsilon^T \check{A}_{rr} E_\varepsilon^{-1} & (*)^T & (*)^T \\ E_\varepsilon^{-1}\check{B}_{wr}^T Q_\varepsilon & -\gamma^2 I & (*)^T \\ \check{C}_{zrr} & 0 & -I \end{bmatrix} < 0 \tag{74}$$

and

$$\begin{aligned} &\begin{bmatrix} E_\varepsilon^{-1}\check{A}_{rf}^T Q_\varepsilon + Q_\varepsilon^T \check{A}_{rf} E_\varepsilon^{-1} & (*)^T & (*)^T \\ E_\varepsilon^{-1}\check{B}_{wr}^T Q_\varepsilon & -\gamma^2 I & (*)^T \\ \check{C}_{zrf} & 0 & -I \end{bmatrix} \\ &+ \begin{bmatrix} E_\varepsilon^{-1}\check{A}_{fr}^T Q_\varepsilon + Q_\varepsilon^T \check{A}_{fr} E_\varepsilon^{-1} & (*)^T & (*)^T \\ E_\varepsilon^{-1}\check{B}_{fr}^T Q_\varepsilon & -\gamma^2 I & (*)^T \\ \check{C}_{zfr} & 0 & -I \end{bmatrix} < 0 \end{aligned} \tag{75}$$

Using the Schur complement applied to (74)-(75) and rewriting the equation as follows:

$$\begin{bmatrix} \left(\begin{array}{c} E_\varepsilon^{-1}\check{A}_{rr}^T Q_\varepsilon \\ + Q_\varepsilon^T \check{A}_{rr} E_\varepsilon^{-1} \\ + \check{C}_{zrr}^T \check{C}_{zrr} \\ E_\varepsilon^{-1}\check{B}_{wr}^T Q_\varepsilon \end{array} \right) & (*)^T \\ & -\gamma^2 I \end{bmatrix} < 0 \tag{76}$$

and

$$\begin{bmatrix} \left(\begin{array}{c} E_\varepsilon^{-1} \check{A}_{rf}^T Q_\varepsilon \\ + Q_\varepsilon^T \check{A}_{rf} E_\varepsilon^{-1} \\ + \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} \end{array} \right) & (*)^T \\ E_\varepsilon^{-1} \check{B}_{w_r}^T Q_\varepsilon & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \left(\begin{array}{c} E_\varepsilon^{-1} \check{A}_{fr}^T Q_\varepsilon \\ + Q_\varepsilon^T \check{A}_{fr} E_\varepsilon^{-1} \\ + \check{C}_{z_{fr}}^T \check{C}_{z_{fr}} \end{array} \right) & (*)^T \\ E_\varepsilon^{-1} \check{B}_{w_f}^T Q_\varepsilon & -\gamma^2 I \end{bmatrix} < 0 \quad (77)$$

Applying (76)-(77) with the following fact

$$\begin{aligned} & \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ & \times (H_{rf}^T S_{hg}) \\ & \leq \frac{1}{2} \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) (H_{rf}^T H_{rf} + S_{rf}^T S_{rf}) \end{aligned} \quad (78)$$

The result is

$$\begin{bmatrix} E_\varepsilon^{-1} \check{A}_{rf}^T Q_\varepsilon + Q_\varepsilon^T \check{A}_{rf} E_\varepsilon^{-1} + \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} & (*)^T \\ E_\varepsilon^{-1} \check{B}_{w_r}^T Q_\varepsilon & -\gamma^2 I \end{bmatrix} < 0 \quad (79)$$

Accordingly, (79) is less than zero; then, using the fact that $\rho_r(s(t)) \geq 0$ and $\sum_{r=1}^j \rho_r(s(t)) = 1$, (73) becomes

$$\begin{aligned} \dot{V}(\chi(t)) & \leq -\check{z}^T(t) \check{z}(t) + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \\ & \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) (\check{\omega}^T(t) \check{\omega}(t)) \end{aligned} \quad (80)$$

Integrating both sides of (80) yields

$$\begin{aligned} \int_0^{T_f} \dot{V}(\chi(t)) dt & \leq \int_0^{T_f} (-\check{z}^T(t) \check{z}(t) \\ & + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \\ & \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ & \times (\check{\omega}^T(t) \check{\omega}(t))) dt \end{aligned} \quad (81)$$

$$\begin{aligned} V(\chi(T_f)) - V(\chi(0)) & \leq \int_0^{T_f} (-\check{z}^T(t) \check{z}(t) \\ & + \gamma^2 \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t)) \\ & \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ & \times (\check{\omega}^T(t) \check{\omega}(t))) dt \end{aligned} \quad (82)$$

Using the fact that $\chi(0) = 0$ and $V(\chi(T_f)) \geq 0$ for all $T_f \neq 0$, then (82) becomes

$$\int_0^{T_f} \check{z}^T(t) \check{z}(t) dt \leq \gamma^2 \int_0^{T_f} \sum_{r=1}^j \sum_{f=1}^j \sum_{h=1}^j \sum_{g=1}^j \rho_r(s(t))$$

$$\begin{aligned} & \times \rho_f(s(t)) \rho_h(s(t)) \rho_g(s(t)) \\ & \times (\check{\omega}^T(t) \check{\omega}(t)) dt \end{aligned} \quad (83)$$

Substituting $\check{z}(t)$ and $\check{\omega}(t)$ given in (22) and (23), respectively, into (83) and using the fact that $\|G(\check{x}(t), t)\| \leq \beta$,

$$\check{\omega}(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \\ 0 \end{bmatrix}, \chi = \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix}, \text{ and inequality (78) yields}$$

$$\begin{aligned} & \int_0^{T_f} \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ & \times \left(2\lambda^2 \beta^2 \check{x}^T(t) F_{4_r}^T F_{4_r} \check{x}(t) + 2\lambda^2 \check{x}^T(t) \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} \check{x}(t) \right) dt \\ & \leq \gamma^2 \lambda^2 \int_0^{T_f} \check{\omega}^T(t) \check{\omega}(t) dt \end{aligned} \quad (84)$$

Adding and subtracting

$$\begin{aligned} \lambda^2 \check{z}^T(t) \check{z}(t) & = \lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ & \times (\check{x}^T(t) (\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r}) \\ & \times (\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r}) \check{x}(t)) \end{aligned}$$

to and from (84), one has

$$\begin{aligned} \gamma^2 \lambda^2 \int_0^{T_f} \check{\omega}^T(t) \check{\omega}(t) dt & \geq \int_0^{T_f} \lambda^2 \check{z}^T(t) \check{z}(t) \\ & + \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ & \times (2\lambda^2 \beta^2 \check{x}^T(t) F_{4_r}^T F_{4_r} \check{x}(t) \\ & + 2\lambda^2 \check{x}^T(t) \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} \check{x}(t)) \\ & - \lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \\ & \times (\check{x}^T(t) \\ & \times (\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r}))^T \\ & \times (\check{C}_{z_r} + G(\check{x}(t), t) F_{4_r}) \\ & \times \check{x}(t)) \end{aligned} \quad (85)$$

Employing the triangle inequality (19) and the fact that $\|G(\check{x}(t), t)\| \leq \beta$, one obtains

$$\begin{aligned} & 2\lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \left(\beta^2 \check{x}^T(t) F_{4_r}^T F_{4_r} \check{x}(t) \right. \\ & \left. + \check{x}^T(t) \check{C}_{z_{rf}}^T \check{C}_{z_{rf}} \check{x}(t) \right) \end{aligned}$$

$$\leq \lambda^2 \sum_{r=1}^j \sum_{f=1}^j \rho_r(s(t)) \rho_f(s(t)) \left(\dot{\tilde{x}}^T(t) \left(\check{C}_{z_r} + G(\tilde{x}(t), t) F_{4_r} \right)^T \right. \\ \left. \times \left(\check{C}_{z_r} + G(\tilde{x}(t), t) F_{4_r} \right) \tilde{x}(t) \right) \quad (86)$$

Using (86) on (85), one obtains

$$\int_0^{T_f} \tilde{z}^T(t) \tilde{z}(t) dt \leq \gamma^2 \int_0^{T_f} \varpi^T(t) \varpi(t) dt \quad (87)$$

Finally, the inequality (18) holds for $\varepsilon \in (0, \hat{\varepsilon}]$. This completes the proof.

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