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Teleportation of an Unknown Four-Qubit Cluster State Based on Cluster States With Minimum Resource

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ABSTRACT In this paper, we first present a scheme for teleporting an unknown four-qubit cluster state via a cluster state chain between two distant nodes, which do not share entanglement pairs directly. Adjacent nodes are linked by a partially entangled four-qubit cluster state with each other. In our scheme, we deduce the relationship between the coefficients of the entangled cluster states and the success probability of teleportation. Moreover, we derive the unitary matrixes for establishing direct channel between two distant nodes, which reduce the computational complexity and resource consumption significantly. By performing entanglement swapping simultaneously, our scheme is more flexible and efficient than most existing schemes.

INDEX TERMS Multi-hop teleportation scheme, four-qubit cluster state, quantum entanglement, minimum resource.

I. INTRODUCTION

Quantum teleportation is one of the most important branches in quantum communication and may have wide applications in quantum repeaters [1], [2], quantum dense coding [3], [4] and quantum networks [5]–[12]. In 1993, Bennett *et al.* [13] first proposed quantum teleportation with a Bell pair, which was later demonstrated in an experiment by Bouwmeester *et al.* [14]. Quantum teleportation has been developed rapidly both theoretically [15]–[26] and experimentally [14], [27]–[33]. To satisfy a variety of different quantum communication scenarios, a series of protocols for quantum teleportation have been proposed involving different quantum channels such as Bell states [15]–[18], GHZ states [19], [20], W states [21]–[23], etc. Several theoretical predictions were realized by various experiments with linear optical systems [27], cavity QED [28] and other kinds of physical systems [14], [29]–[33].

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Long-distance quantum communication between a sender and a receiver can be divided into multiple sections of short distance. In order to transmit quantum information between nodes that do not share direct entanglement, intermediate nodes are usually introduced where quantum channels are built through entanglement shared between adjacent nodes. In most existing quantum teleportation protocols, maximally entangled Bell pairs are used as the quantum channels between the nodes. However, in practical applications, due to the decoherence from the environment, the maximally entangled channel suffers distortion and readily evolves into non-maximally entangled states, leading to the loss of information. In order to achieve long distance and high-fidelity communication, several schemes have been proposed based on the quantum error rejection, the entanglement swapping, the entanglement purification and concentration [23], [34]–[55].

Quantum error rejection is a useful technique to faithfully transmit quantum states over large-scale quantum channels. In 2005, Kalamidas *et al.* [34] presented two linear-optical single-photon schemes to reject and correct arbitrary qubit errors without additional qubits. In 2007, Li *et al.* [35] proposed a setup for a single-photon qubit against collective noise without ancillary qubits, in which the success probability could be improved to 100%. In 2017, Jiang *et al.* [36] presented an original self-error-rejecting photonic qubit transmission scheme for both polarization and the spatial states of photon systems transmitted over collective noise channels. In 2019, Gao *et al.* [37] realized a faithful single-photon qubit transmission against the channel noise with error-rejecting coding. In recent studies, Guo *et al.* [38] reviewed the development of quantum error rejection and introduced several typical schemes for error-rejection transmission.

In long-distance quantum communication, entanglement purification is introduced to reduce the affect arisen from the noise. In 1996, Bennett *et al.* [39] firstly proposed the concept of the entanglement purification protocol based on the quantum CNOT logic operations. Subsequently, Deutsch *et al.* [40] reinvestigated and improved Bennett's protocols. In 2010, Sheng and Deng [41] presented a deterministic entanglement purification protocol with hyper-entanglement, which corrected the bit-flip error and the phase-flip error in quantum communication. In 2017, Zhou and Sheng [42] presented the first polarization entanglement purification protocol for concatenated GHZ state, resorting to the photon-atom interaction in low-quality cavity. In their study, Wang and Long [43] proposed an entanglement purification protocol for an entangled nitrogen-vacancy center pair based on the nondestructive parity-check detector.

Compared with entanglement purification, entanglement concentration is the method which distills less entangled pure states into maximally entangled states. In 1996, Bennett *et al.* [44] proposed the first entanglement concentration protocol, which was known as the Schmidt projection method. In 2001, Yamamoto *et al.* [45] and Zhao *et al.* [46] proposed two entanglement concentration protocols based on polarization beam splitters independently. In 2008, Sheng *et al.* [47] presented a nonlocal entanglement concentration scheme based on cross-Kerr nonlinearities to distinguish the parity of two polarization photons. Later in 2017, Du and Long [48] reported an entanglement concentration protocol for an unknown four-electron-spin cluster state by exploring the optical selection rules derived from the quantum-dot spins in one-sided optical microcavities. In their study, Wang *et al.* [49] proposed a hyper-entanglement concentration protocol for nonlocal two-photon six-qubit partially hyper-entangled Bell states with the parameter-splitting method.

On the other hand, multi-hop teleportation protocols provide a way to transmit qubits from source to destination via entanglement swapping and recovering operations. In 2015, Shi *et al.* [51] reported a quantum wireless multi-hop network in which the unknown information was teleported hop by hop via Werner states. To improve the transmission efficiency, Zou *et al.* [52] proposed a multi-hop teleportation protocol to implement the quantum teleportation of an unknown two-qubit state via the composite GHZ-Bell channel. Later in 2018, Zhou *et al.* [23] proposed an improved multi-hop teleportation scheme for an unknown state via W states.

Cluster state is one of the most important multi-particle entangled states discovered by Briegel and Raussendorf [56] in 2001. It is worth noting that cluster states have the properties of both GHZ and W states [57] and they have been proved that they are harder to be destroyed by local operations and less susceptible to decoherence than GHZ states [56], [58], which means that cluster states have the maximum connectivity and persistent entanglement. Due to these advantages, various quantum teleportation schemes have been put forward with cluster states [59]–[67]. For instance, in 2016 Li *et al.* [63] put forward a scheme for teleporting a four-qubit state via a six-qubit cluster state. In 2018, Zhao *et al.* [65] demonstrated that a eight-qubit cluster state could be teleported by a six-qubit cluster state. Subsequently, Sisodia and Pathak [66] reinvestigated and improved Zhao's protocol. In their protocol only two Bell states (not a six-qubit cluster state as in [65]) were utilized as the quantum channel. However, it is impossible to generate or maintain the maximally entangled state at one's disposal due to the inevitable influence of environmental noise.

To solve this problem, we present a scheme for teleporting an unknown four-qubit cluster state via partially entangled cluster states in a multi-hop teleportation network, where two distant nodes, the sender and the receiver, do not share the entanglement pairs directly. In our scheme, the required cluster states are distributed between adjacent nodes. All the intermediate nodes help these two distant nodes establish an entangled channel via entanglement swapping. In addition, we deduce the general unitary matrixes in the multi-hop scenario. The matrix relies only on the Bell state measurement results, so that both the computational complexity and the resource consumption are reduced significantly.

The rest of this paper is organized as follows. In Sect.II, we introduce the one-hop quantum teleportation of an unknown four-qubit cluster state via a non-maximally entangled cluster states. In Sect.III, we generalize the scheme described in Sect.II to a multi-hop scenario. The performance of our proposed scheme is discussed in Sect.IV. Conclusion is given in Sect.V.

II. ONE-HOP QUANTUM TELEPORTATION OF AN UNKNOWN FOUR-QUBIT CLUSTER STATE VIA PARTIALLY ENTANGLED CLUSTER STATE

Suppose that the sender Alice intends to transmit an unknown four-qubit cluster state to the receiver Bob. The unknown four-qubit cluster state can be expressed as follows:

$$|\chi\rangle_{1234} = (\alpha\,|0000\rangle + \beta\,|0011\rangle + \mu\,|1100\rangle - \nu\,|1111\rangle)_{1234}. \tag{1}$$

Here α, β, μ and ν are unknown parameters that satisfy the relationship: $|\alpha|^2+|\beta|^2+|\mu|^2+|\nu|^2=1$.

Assume the quantum channel shared by Alice and Bob is

$$|C\rangle_{A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} = (a\,|0000\rangle + b\,|0011\rangle + c\,|1100\rangle - d\,|1111\rangle)_{A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}. \tag{2}$$

Here the coefficients a, b, c, d are real and satisfy the normalization condition $a^2+b^2+c^2+d^2=1 (a \leqslant b \leqslant c \leqslant d)$. Alice possesses qubits 1, 2, 3, 4, $A_1^{(1)}$ and $A_2^{(1)}$. Bob possesses qubits $B_1^{(1)}$ and $B_2^{(1)}$, as schematically shown in Fig. 1.

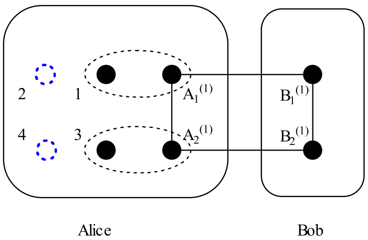

FIGURE 1. A diagram shows that Alice and Bob share a four-qubit entangled cluster state.

Now, the initial state that consists of qubits 1, 2, 3, 4, $A_1^{(1)}$, $B_1^{(1)}$, $A_2^{(1)}$ and $B_2^{(1)}$ can then be written as:

$$|\varphi\rangle_{1234A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} = |\chi\rangle_{1234} \otimes |C\rangle_{A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}. \tag{3}$$

In order to realize the teleportation of the unknown state described in *Eq.* (1), Alice and Bob perform the following operations, as shown in Fig. 2.

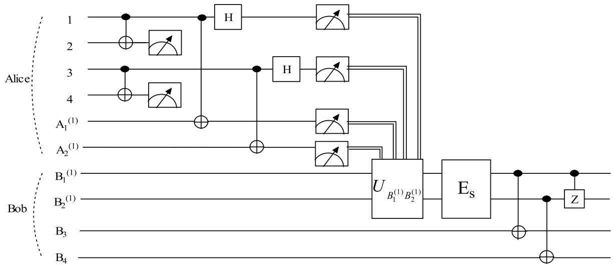

FIGURE 2. Quantum circuit for teleportation of an unknown four-qubit cluster state via partially entangled cluster states.

Step 1, Alice performs two CNOT operations on the selective qubit pairs $\{1, 2\}$and $\{3, 4\}$, which can be expressed as:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{4}$$

The state of the whole system become:

$$\begin{aligned} &|\varphi\rangle_{1234A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\ &= CNOT_{34}CNOT_{12} \\ &\quad \times \big((\alpha\,|0000\rangle + \beta\,|0011\rangle + \mu\,|1100\rangle \\ &\quad - \nu\,|1111\rangle)_{1234} \otimes |C\rangle_{A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}\big) \\ &= (\alpha\,|00\rangle + \beta\,|01\rangle + \mu\,|10\rangle - \nu\,|11\rangle)_{13}|00\rangle_{24} \\ &\quad \otimes |C\rangle_{A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}. \end{aligned} \tag{5}$$

It is obvious from *Eq.*(5) that Alice transfers the information of the initial unknown four-qubit cluster state described in *Eq.* (1) to qubits 1 and 3, which is expressed as $\alpha\,|00\rangle + \beta\,|01\rangle + \mu\,|10\rangle - \nu\,|11\rangle$. Now we just consider the state of qubits 1, 3, $A_1^{(1)}$, $A_2^{(2)}$, $B_1^{(1)}$ and $B_2^{(1)}$. The state can be expressed as:

$$|\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} = |\chi\rangle_{13} \otimes |C\rangle_{A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}. \tag{6}$$

Step 2, Alice performs two Bell state measurements on particle pairs $(1, A_1^{(1)})$ and $(3, A_2^{(1)})$ with the basis of $\{|\phi_{00}\rangle, |\phi_{01}\rangle, |\phi_{10}\rangle, |\phi_{11}\rangle\}$. Here $|\phi_{00}\rangle$, $|\phi_{01}\rangle$, $|\phi_{10}\rangle$ and $|\phi_{11}\rangle$ are determined as:

$$\begin{aligned} |\phi_{00}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), |\phi_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \\ |\phi_{10}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), |\phi_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle). \end{aligned} \tag{7}$$

When these two Bell state measurements are performed, there are sixteen possible collapsed states possessed by Bob, as follows:

$$\begin{aligned}
&{}_{3A_2^{(1)}}\langle\phi_{00}|\,{}_{1A_1^{(1)}}\langle\phi_{00}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha a\,|00\rangle + \beta b\,|01\rangle + \mu c\,|10\rangle + \nu d\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{10}|\,{}_{1A_1^{(1)}}\langle\phi_{00}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha a\,|00\rangle - \beta b\,|01\rangle + \mu c\,|10\rangle - \nu d\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{01}|\,{}_{1A_1^{(1)}}\langle\phi_{00}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha b\,|01\rangle + \beta a\,|00\rangle - \mu d\,|11\rangle - \nu c\,|10\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{11}|\,{}_{1A_1^{(1)}}\langle\phi_{00}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha b\,|01\rangle - \beta a\,|00\rangle - \mu d\,|11\rangle + \nu c\,|10\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{00}|\,{}_{1A_1^{(1)}}\langle\phi_{10}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha a\,|00\rangle + \beta b\,|01\rangle - \mu c\,|10\rangle - \nu d\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{10}|\,{}_{1A_1^{(1)}}\langle\phi_{10}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha a\,|00\rangle - \beta b\,|01\rangle - \mu c\,|10\rangle + \nu d\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{01}|\,{}_{1A_1^{(1)}}\langle\phi_{10}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha b\,|01\rangle + \beta a\,|00\rangle + \mu d\,|11\rangle + \nu c\,|10\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{11}|\,{}_{1A_1^{(1)}}\langle\phi_{10}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha b\,|01\rangle - \beta a\,|00\rangle + \mu d\,|11\rangle - \nu c\,|10\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{00}|\,{}_{1A_1^{(1)}}\langle\phi_{01}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}} \\
&= \frac{1}{2}(\alpha c\,|10\rangle - \beta d\,|11\rangle + \mu a\,|00\rangle - \nu b\,|01\rangle)_{B_1^{(1)}B_2^{(1)}}, \\
&{}_{3A_2^{(1)}}\langle\phi_{10}|\,{}_{1A_1^{(1)}}\langle\phi_{01}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}
\end{aligned}$$

$$= \frac{1}{2}(\alpha c\,|10\rangle + \beta d\,|11\rangle + \mu a\,|00\rangle + \nu b\,|01\rangle)_{B_1^{(1)}B_2^{(1)}},$$
$${}_{3A_2^{(1)}}\langle\phi_{01}|\,{}_{1A_1^{(1)}}\langle\phi_{01}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}$$
$$= \frac{1}{2}(-\alpha d\,|11\rangle + \beta c\,|10\rangle + \mu b\,|01\rangle - \nu a\,|00\rangle)_{B_1^{(1)}B_2^{(1)}},$$
$${}_{3A_2^{(1)}}\langle\phi_{11}|\,{}_{1A_1^{(1)}}\langle\phi_{01}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}$$
$$= \frac{1}{2}(-\alpha d\,|11\rangle - \beta c\,|10\rangle + \mu b\,|01\rangle + \nu a\,|00\rangle)_{B_1^{(1)}B_2^{(1)}},$$
$${}_{3A_2^{(1)}}\langle\phi_{00}|\,{}_{1A_1^{(1)}}\langle\phi_{11}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}$$
$$= \frac{1}{2}(\alpha c\,|10\rangle - \beta d\,|11\rangle - \mu a\,|00\rangle + \nu b\,|01\rangle)_{B_1^{(1)}B_2^{(1)}},$$
$${}_{3A_2^{(1)}}\langle\phi_{10}|\,{}_{1A_1^{(1)}}\langle\phi_{11}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}$$
$$= \frac{1}{2}(\alpha c\,|10\rangle + \beta d\,|11\rangle - \mu a\,|00\rangle - \nu b\,|01\rangle)_{B_1^{(1)}B_2^{(1)}},$$
$${}_{3A_2^{(1)}}\langle\phi_{01}|\,{}_{1A_1^{(1)}}\langle\phi_{11}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}$$
$$= \frac{1}{2}(-\alpha d\,|11\rangle + \beta c\,|10\rangle - \mu b\,|01\rangle + \nu a\,|00\rangle)_{B_1^{(1)}B_2^{(1)}},$$
$${}_{3A_2^{(1)}}\langle\phi_{11}|\,{}_{1A_1^{(1)}}\langle\phi_{11}\mid\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}$$
$$= \frac{1}{2}(-\alpha d\,|11\rangle - \beta c\,|10\rangle - \mu b\,|01\rangle - \nu a\,|00\rangle)_{B_1^{(1)}B_2^{(1)}}. \quad (8)$$

Next, Alice tells her measurement outcomes to Bob via classical communication. According to Alice's measurement results, Bob performs corresponding unitary operations on his qubits $B_1^{(1)}$ and $B_2^{(1)}$. The unitary operations $U_{B_1^{(1)}B_2^{(1)}}$ can be represented by Pauli matrices:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \; Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

,and

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The detailed relationship between the measurement outcomes $|\phi_{m_1m_2}\rangle_{1A_1^{(1)}}$, $|\phi_{n_1n_2}\rangle_{3A_2^{(1)}}$ $(m_1, m_2, n_1, n_2 \in [0, 1])$ and the corresponding unitary operations is given in Table 1.

According to the unitary operations mentioned above, Bob's qubits $|\varphi_m\rangle$ $(m = 0, 1, 2, 3)$ collapse into one of the following states:

$$|\varphi_0\rangle = \frac{1}{\sqrt{p_0}}(\alpha a\,|00\rangle + \beta b\,|01\rangle + \mu c\,|10\rangle + \nu d\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}, \quad (9)$$

with the probability $p_0 = |\alpha a|^2 + |\beta b|^2 + |\mu c|^2 + |\nu d|^2$ or

$$|\varphi_1\rangle = \frac{1}{\sqrt{p_1}}(\alpha b\,|00\rangle + \beta a\,|01\rangle + \mu d\,|10\rangle + \nu c\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}, \quad (10)$$

with the probability $p_1 = |\alpha b|^2 + |\beta a|^2 + |\mu d|^2 + |\nu c|^2$ or

$$|\varphi_2\rangle = \frac{1}{\sqrt{p_2}}(\alpha c\,|00\rangle + \beta d\,|01\rangle + \mu a\,|10\rangle + \nu b\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}, \quad (11)$$

TABLE 1. The relationship between the measurements outcomes of Alice and the unitary operation performed by Bob.

Measurement results	Unitary operations
$\|\phi_{00}\rangle_{1A_1^{(1)}}\|\phi_{00}\rangle_{3A_2^{(1)}}$	$I_{B_1^{(1)}} \otimes I_{B_2^{(1)}}$
$\|\phi_{00}\rangle_{1A_1^{(1)}}\|\phi_{01}\rangle_{3A_2^{(1)}}$	$Z_{B_1^{(1)}} \otimes X_{B_2^{(1)}}$
$\|\phi_{00}\rangle_{1A_1^{(1)}}\|\phi_{10}\rangle_{3A_2^{(1)}}$	$I_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}$
$\|\phi_{00}\rangle_{1A_1^{(1)}}\|\phi_{11}\rangle_{3A_2^{(1)}}$	$Z_{B_1^{(1)}} \otimes X_{B_2^{(1)}}Z_{B_2^{(1)}}$
$\|\phi_{01}\rangle_{1A_1^{(1)}}\|\phi_{00}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}$
$\|\phi_{01}\rangle_{1A_1^{(1)}}\|\phi_{01}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}}Z_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}X_{B_2^{(1)}}$
$\|\phi_{01}\rangle_{1A_1^{(1)}}\|\phi_{10}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}} \otimes I_{B_2^{(1)}}$
$\|\phi_{01}\rangle_{1A_1^{(1)}}\|\phi_{11}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}}Z_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}X_{B_2^{(1)}}Z_{B_2^{(1)}}$
$\|\phi_{10}\rangle_{1A_1^{(1)}}\|\phi_{00}\rangle_{3A_2^{(1)}}$	$Z_{B_1^{(1)}} \otimes I_{B_2^{(1)}}$
$\|\phi_{10}\rangle_{1A_1^{(1)}}\|\phi_{01}\rangle_{3A_2^{(1)}}$	$I_{B_1^{(1)}} \otimes X_{B_2^{(1)}}$
$\|\phi_{10}\rangle_{1A_1^{(1)}}\|\phi_{10}\rangle_{3A_2^{(1)}}$	$Z_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}$
$\|\phi_{10}\rangle_{1A_1^{(1)}}\|\phi_{11}\rangle_{3A_2^{(1)}}$	$X_{B_2^{(1)}}Z_{B_2^{(1)}}$
$\|\phi_{11}\rangle_{1A_1^{(1)}}\|\phi_{00}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}}Z_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}$
$\|\phi_{11}\rangle_{1A_1^{(1)}}\|\phi_{01}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}X_{B_2^{(1)}}$
$\|\phi_{11}\rangle_{1A_1^{(1)}}\|\phi_{10}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}}Z_{B_1^{(1)}} \otimes I_{B_2^{(1)}}$
$\|\phi_{11}\rangle_{1A_1^{(1)}}\|\phi_{11}\rangle_{3A_2^{(1)}}$	$X_{B_1^{(1)}} \otimes Z_{B_2^{(1)}}X_{B_2^{(1)}}Z_{B_2^{(1)}}$

with the probability $p_2 = |\alpha c|^2 + |\beta d|^2 + |\mu a|^2 + |\nu b|^2$ or

$$|\varphi_3\rangle = \frac{1}{\sqrt{p_3}}(\alpha d\,|00\rangle + \beta c\,|01\rangle + \mu b\,|10\rangle + \nu a\,|11\rangle)_{B_1^{(1)}B_2^{(1)}}. \quad (12)$$

with the probability $p_3 = |\alpha d|^2 + |\beta c|^2 + |\mu b|^2 + |\nu a|^2$.

Step 3, Bob performs the generalized measurement given by Kraus operators [55] for $|\varphi_m\rangle$ $(m = 0, 1, 2, 3)$

$$E_{Sm} = \sum_{i,j=0}^{1} b_{ij}\,|ij\rangle\,\langle ij|\,, \quad (13a)$$

$$E_{Fm} = \sum_{i,j=0}^{1} \sqrt{1 - b_{ij}^{\,2}}\,|ij\rangle\,\langle ij|\,. \quad (13b)$$

TABLE 2. The relationship between all the possible state of qubits $B_1^{(1)}$, $B_2^{(1)}$ and the coefficients $b_{ij}(i,j=0,1)$ for the Kraus operators.

$\|\varphi_m\rangle_{B_1^{(1)}B_2^{(1)}}$	$b_{00}, b_{01}, b_{10}, b_{11}$
$\|\varphi_0\rangle_{B_1^{(1)}B_2^{(1)}}$	$b_{00}=1, b_{01}=\frac{a}{b}, b_{10}=\frac{a}{c}, b_{11}=\frac{a}{d}.$
$\|\varphi_1\rangle_{B_1^{(1)}B_2^{(1)}}$	$b_{00}=\frac{a}{b}, b_{01}=1, b_{10}=\frac{a}{d}, b_{11}=\frac{a}{c}.$
$\|\varphi_2\rangle_{B_1^{(1)}B_2^{(1)}}$	$b_{00}=\frac{a}{c}, b_{01}=\frac{a}{d}, b_{10}=1, b_{11}=\frac{a}{b}.$
$\|\varphi_3\rangle_{B_1^{(1)}B_2^{(1)}}$	$b_{00}=\frac{a}{d}, b_{01}=\frac{a}{c}, b_{10}=\frac{a}{b}, b_{11}=1.$

The relationship between all the possible states of qubits $B_1^{(1)}$, $B_2^{(1)}$ and $b_{ij}(i,j=0,1)$ for the Kraus operators is given in Table 2.

For example, if Bob's qubits collapse into the state $|\varphi_0\rangle$, when E_{S0} is obtained, the state of qubits $B_1^{(1)}$ and $B_2^{(1)}$ will collapse into:

$$|\varphi\rangle_{B_1^{(1)}B_2^{(1)}} = a(\alpha|00\rangle+\beta|01\rangle+\mu|10\rangle+\nu|11\rangle)_{B_1^{(1)}B_2^{(1)}}. \quad (14)$$

The success probability can be calculated as $p = p_0\langle\varphi_0|E_{S0}^{\dagger}E_{S0}|\varphi_0\rangle = |a|^2$. To obtain the initial four-qubit cluster state described in *Eq.* (1), Bob introduces another two ancillary qubits B_3 and B_4 with the initial state $|00\rangle_{B_3B_4}$ and then executes two CNOT operations on the selective qubit pairs $\{B_1^{(1)}, B_3\}$ and $\{B_2^{(1)}, B_4\}$.

$$\begin{aligned}&\left|\varphi^{(1)}\right\rangle_{B_1^{(1)}B_3B_2^{(1)}B_4}\\&= CNOT_{B_2^{(1)}B_4}CNOT_{B_1^{(1)}B_3}\\&\quad\times(\alpha|00\rangle+\beta|01\rangle+\mu|10\rangle+\nu|11\rangle)_{B_1^{(1)}B_2^{(1)}}|00\rangle_{B_3B_4}\\&=(\alpha|0000\rangle+\beta|0011\rangle+\mu|1100\rangle+\nu|1111\rangle)_{B_1^{(1)}B_3B_2^{(1)}B_4}.\end{aligned} \quad (15)$$

Finally, Bob applies a CZ gate on qubits $B_1^{(1)}$ and $B_2^{(1)}$, as follows:

$$CZ=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}. \quad (16)$$

We obtain the following state:

$$\begin{aligned}&\left|\varphi^{(2)}\right\rangle_{B_1^{(1)}B_3B_2^{(1)}B_4}\\&=CZ_{B_1^{(1)}B_2^{(1)}}(\alpha|0000\rangle+\beta|0011\rangle+\mu|1100\rangle\\&\quad+\nu|1111\rangle)_{B_1^{(1)}B_3B_2^{(1)}B_4}\\&=(\alpha|0000\rangle+\beta|0011\rangle+\mu|1100\rangle-\nu|1111\rangle)_{B_1^{(1)}B_3B_2^{(1)}B_4}.\end{aligned} \quad (17)$$

In this way, the unknown four-qubit cluster state shown in *Eq.* (1) is teleported to the remote receiver Bob successfully.

Similarly, combining all the situations shown in *Eq.* (9) - *Eq.* (12) and Table 2, Bob can obtain the teleported state with a certain probability. The total success probability of the teleportation can be calculated as:

$$P_{total}=\sum_{m=0}^{3}p_m\langle\varphi_m|E_{Sm}^{\dagger}E_{Sm}|\varphi_m\rangle=4|a|^2. \quad (18)$$

Note that if we use maximally entangled quantum channel, *i.e.*, $|a|=|b|=|c|=|d|=\frac{1}{2}$, the total success probability reaches maximum 100%.

III. MULTI-HOP QUANTUM TELEPORTATION OF AN UNKNOWN FOUR-QUBIT CLUSTER STATE VIA PARTIALLY ENTANGLED CLUSTER STATE

The above scheme can be generalized to a multi-hop scenario via non-maximally entangled cluster states, in which there is no direct channel between the sender Alice and the receiver Bob. In detail, we suppose there are totally T ($T \geqslant 1$) intermediate nodes between Alice and Bob. As shown in Fig. 3, all the participants are linked by one channel with its neighboring nodes, which can be expressed as:

$$\begin{aligned}&|C\rangle_{A_1^{(i)}B_1^{(i)}A_2^{(i)}B_2^{(i)}}\\&=(a_{00}^{(i)}|0000\rangle+a_{01}^{(i)}|0011\rangle+a_{10}^{(i)}|1100\rangle\\&\quad-a_{11}^{(i)}|1111\rangle)_{A_1^{(i)}B_1^{(i)}A_2^{(i)}B_2^{(i)}}.\end{aligned} \quad (19)$$

$$\begin{aligned}&|\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}\ldots\ldots A_1^{(T+1)}B_1^{(T+1)}A_2^{(T+1)}B_2^{(T+1)}}\\&=|\chi\rangle_{13}\otimes_{i=1}^{T+1}\left|C'\right\rangle_{A_1^{(i)}B_1^{(i)}A_2^{(i)}B_2^{(i)}}\\&=|\chi\rangle_{13}\otimes(a_{00}^{(1)}|0000\rangle+a_{01}^{(1)}|0011\rangle+a_{10}^{(1)}|1100\rangle+a_{11}^{(1)}|1111\rangle)_{A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}}\\&\quad\otimes(a_{00}^{(2)}|0000\rangle+a_{01}^{(2)}|0011\rangle+a_{10}^{(2)}|1100\rangle+a_{11}^{(2)}|1111\rangle)_{A_1^{(2)}B_1^{(2)}A_2^{(2)}B_2^{(2)}}\\&\quad\otimes\cdots\otimes(a_{00}^{(T+1)}|0000\rangle+a_{01}^{(T+1)}|0011\rangle+a_{10}^{(T+1)}|1100\rangle+a_{11}^{(T+1)}|1111\rangle)_{A_1^{(T+1)}B_1^{(T+1)}A_2^{(T+1)}B_2^{(T+1)}}.\end{aligned} \quad (20)$$

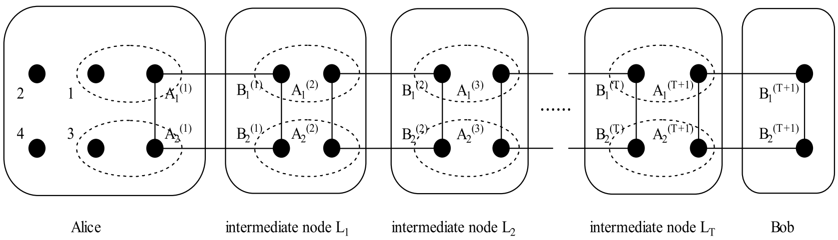

FIGURE 3. $T+1$ **cluster states are shared between Alice, Bob and the intermediate nodes.** T **is the number of the intermediate node.**

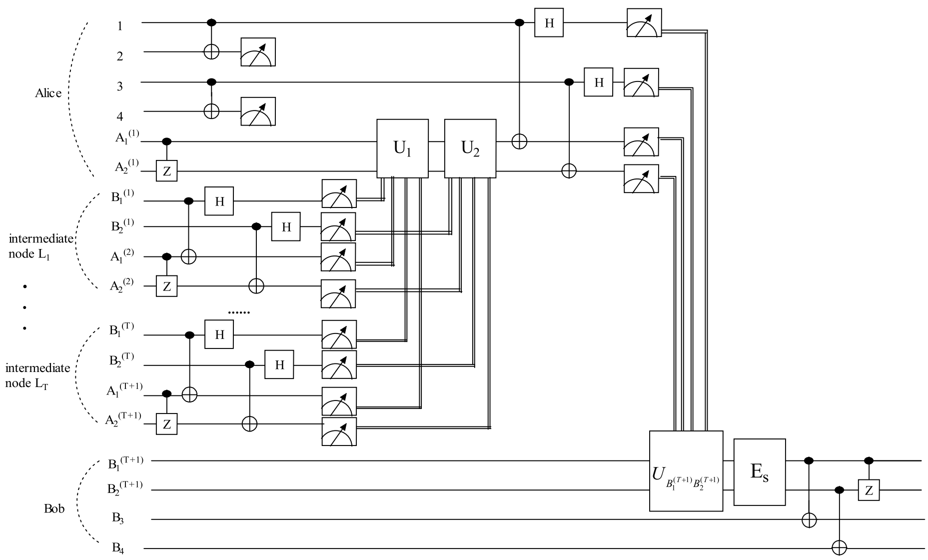

FIGURE 4. Quantum circuit for multi-hop teleportation of an unknown four-qubit cluster state via non-maximally entangled cluster states.

Here the coefficients $a_{00}^{(i)}$, $a_{01}^{(i)}$, $a_{10}^{(i)}$, $a_{11}^{(i)}$ $(i = 1, 2, \cdots, T+1)$ satisfy the normalization condition $a_{00}^{(i)} \leqslant a_{01}^{(i)} \leqslant a_{10}^{(i)} \leqslant a_{11}^{(i)}$ and $\left(a_{00}^{(i)}\right)^2+\left(a_{01}^{(i)}\right)^2+\left(a_{10}^{(i)}\right)^2+\left(a_{11}^{(i)}\right)^2=1$.

Step 1, Alice and all the intermediate nodes perform CZ gates on their own qubit pair $\{A_1^{(i)}, A_2^{(i)}\}(i = 1, 2, 3, \ldots, T+1)$ to preprocess the channel, where qubit $A_1^{(i)}$ works as the controlling qubit and qubit $A_2^{(i)}$ works as the target qubit. Now the state of the entire system can be decomposed as (20), as shown at the bottom of the previous page.

Step 2, T intermediate nodes perform two Bell state measurements on particle pairs $\{B_1^{(i)}, A_1^{(i+1)}\}$and $\{B_2^{(i)}, A_2^{(i+1)}\}(i = 1, 2, 3, \ldots, T)$ with the basis of $\{|\phi_{00}\rangle, |\phi_{01}\rangle, |\phi_{10}\rangle, |\phi_{11}\rangle\}$, respectively, as shown in Fig. 4.

Now the entire state can be written as (21), shown at the bottom of the next page.

Here, the unitary matrixes $U1$ and $U2$ can be expressed by:

$$U_1 = [L_{00}^1, L_{01}^1, L_{10}^1, L_{11}^1] \cdot [I_{A_1^{(1)}}, X_{A_1^{(1)}}, Z_{A_1^{(1)}}, Z_{A_1^{(1)}}X_{A_1^{(1)}}]',$$
$$U_2 = [L_{00}^2, L_{01}^2, L_{10}^2, L_{11}^2] \cdot [I_{A_2^{(1)}}, X_{A_2^{(1)}}, Z_{A_2^{(1)}}, Z_{A_2^{(1)}}X_{A_2^{(1)}}]'. \quad (22)$$

with

$$L_{00}^1=\overline{\oplus_{i=1}^T m_1^{(i)}} \cdot \overline{\oplus_{i=1}^T n_1^{(i)}}, \quad L_{01}^1=\overline{\oplus_{i=1}^T m_1^{(i)}} \cdot \oplus_{i=1}^T n_1^{(i)},$$
$$L_{10}^1=\oplus_{i=1}^T m_1^{(i)} \cdot \overline{\oplus_{i=1}^T n_1^{(i)}}, \quad L_{11}^1=\oplus_{i=1}^T m_1^{(i)} \cdot \oplus_{i=1}^T n_1^{(i)}. \quad (23)$$

$$L_{00}^2=\overline{\oplus_{i=1}^T m_2^{(i)}} \cdot \overline{\oplus_{i=1}^T n_2^{(i)}}, \quad L_{01}^2=\overline{\oplus_{i=2}^T m_2^{(i)}} \cdot \oplus_{i=2}^T n_2^{(i)},$$
$$L_{10}^2=\oplus_{i=1}^T m_2^{(i)} \cdot \overline{\oplus_{i=1}^T n_2^{(i)}}, \quad L_{11}^2=\oplus_{i=1}^T m_2^{(i)} \cdot \oplus_{i=1}^T n_2^{(i)}. \quad (24)$$

Here, $m_1^{(i)}$, $n_1^{(i)}$, $m_2^{(i)}$, $n_2^{(i)}$ $(i = 1, 2, 3, \cdots, T)$ are used to denote the Bell state measurement results 0 or 1. The symbols " $\oplus$ ", " $\cdot$ " and "-" represent logic XOR, AND and negation, respectively.

According to the unitary operations mentioned above, a desired entangled channel between the source node Alice and the destination node Bob is established successfully. The state of qubits $A_1^{(1)}$, $A_2^{(1)}$, $B_1^{(T+1)}$, $B_2^{(T+1)}$ collapses into the following state:

$$\begin{aligned}
&|\varphi\rangle_{A_1^{(1)}B_1^{(T+1)}A_2^{(1)}B_2^{(T+1)}} \\
&=\frac{1}{\sqrt{p_{n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}}}}(a_{00}^{T+1} a^T_{n_1^{(T)},n_2^{(T)}} a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}} \\
&\quad\cdots\ a^1_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}}|0000\rangle \\
&\quad+a_{01}^{T+1} a^T_{n_1^{(T)},n_2^{(T)}\oplus 1} a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus 1} \\
&\quad\cdots\ a^1_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus 1}|0011\rangle \\
&\quad+a_{10}^{T+1} a^T_{n_1^{(T)}\oplus 1,n_2^{(T)}} a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}} \\
&\quad\cdots\ a^1_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}}|1100\rangle
\end{aligned}$$

$$+a_{11}^{T+1}a^{T}_{n_1^{(T)}\oplus 1,n_2^{(T)}\oplus 1}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus 1}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus 1}\left|1111\right\rangle). \tag{25}$$

For simplicity, we redefine the coefficients as follows:

$$\begin{aligned}
\kappa_{00} &= a_{00}^{T+1}a^{T}_{n_1^{(T)},n_2^{(T)}}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}},\\
\kappa_{01} &= a_{01}^{T+1}a^{T}_{n_1^{(T)},n_2^{(T)}\oplus 1}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus 1}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus 1},\\
\kappa_{10} &= a_{10}^{T+1}a^{T}_{n_1^{(T)}\oplus 1,n_2^{(T)}}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}},\\
\kappa_{11} &= a_{11}^{T+1}a^{T}_{n_1^{(T)}\oplus 1,n_2^{(T)}\oplus 1}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus 1}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus 1}.
\end{aligned} \tag{26}$$

The probability of state given by *Eq.* (25) can be calculated as:

$$p_{n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}}=|\kappa_{00}|^2+|\kappa_{01}|^2+|\kappa_{10}|^2+|\kappa_{11}|^2. \tag{27}$$

Step 3, Alice performs a CZ gate operation on qubit pair $\{A_1^{(1)}, A_2^{(1)}\}$. Then the state will become (28), shown at the bottom of the next page.

For example, suppose the Bell state measurement outcomes of all the intermediate nodes are $|\phi_{00}\rangle_{B_1^{(i)}A_1^{(i+1)}}$ and $|\phi_{00}\rangle_{B_2^{(i)}A_2^{(i+1)}}(i = 1, 2, 3, \cdots, T)$. The state of qubits $A_1^{(1)}$, $A_2^{(1)}$, $B_1^{(T+1)}$, $B_2^{(T+1)}$ collapses into:

$$\begin{aligned}
&|\varphi\rangle_{13A_1^{(1)}B_1^{(T+1)}A_2^{(1)}B_2^{(T+1)}}\\
&= |\chi\rangle_{13}\otimes(a_{00}^{T+1}a_{00}^{T}a_{00}^{T-1}\cdots a_{00}^{1}\left|0000\right\rangle\\
&\quad +a_{01}^{T+1}a_{01}^{T}a_{01}^{T-1}\cdots a_{01}^{1}\left|0011\right\rangle\\
&\quad +a_{10}^{T+1}a_{10}^{T}a_{10}^{T-1}\cdots a_{10}^{1}\left|1100\right\rangle\\
&\quad -a_{11}^{T+1}a_{11}^{T}a_{11}^{T-1}\cdots a_{11}^{1}\left|1111\right\rangle)_{A_1^{(1)}B_1^{(T+1)}A_2^{(1)}B_2^{(T+1)}}.
\end{aligned} \tag{29}$$

According to *Eq.* (26), we redefine the coefficients as follows:

$$\begin{aligned}
\kappa_{00} &= a_{00}^{T+1}a_{00}^{T}a_{00}^{T-1}\cdots a_{00}^{1}\\
\kappa_{01} &= a_{01}^{T+1}a_{01}^{T}a_{01}^{T-1}\cdots a_{01}^{1},\\
\kappa_{10} &= a_{10}^{T+1}a_{10}^{T}a_{10}^{T-1}\cdots a_{10}^{1}\\
\kappa_{11} &= a_{11}^{T+1}a_{11}^{T}a_{11}^{T-1}\cdots a_{11}^{1}.
\end{aligned} \tag{30}$$

Step 4, Alice performs two Bell state measurements. The state $|\varphi_m\rangle$ $(m = 0, 1, 2, 3)$ obtained by Bob collapses into one of the following states:

$$|\varphi_0\rangle=\frac{1}{\sqrt{p_0}}\left(\alpha\kappa_{00}\left|00\right\rangle+\beta\kappa_{01}\left|01\right\rangle+\mu\kappa_{10}\left|10\right\rangle+\nu\kappa_{11}\left|11\right\rangle\right), \tag{31}$$

$$\begin{aligned}
&|\varphi\rangle_{13A_1^{(1)}B_1^{(1)}A_2^{(1)}B_2^{(1)}\ldots\ldots A_1^{(T+1)}B_1^{(T+1)}A_2^{(T+1)}B_2^{(T+1)}}\\
&= |\chi\rangle_{13}\otimes\frac{1}{2^T}\sum_{\substack{m_1^{(1)}n_1^{(1)}\\ m_2^{(1)}n_2^{(1)}=0}}^{1}\ \sum_{\substack{m_1^{(2)}n_1^{(2)}\\ m_2^{(2)}n_2^{(2)}=0}}^{1}\cdots\sum_{\substack{m_1^{(T)}n_1^{(T)}\\ m_2^{(T)}n_2^{(T)}=0}}^{1}\left|\phi_{m_1^{(1)}n_1^{(1)}}\right\rangle_{B_1^{(1)}A_1^{(2)}}\left|\phi_{m_2^{(1)}n_2^{(1)}}\right\rangle_{B_2^{(1)}A_2^{(2)}}\\
&\quad\cdots\left|\phi_{m_1^{(T)}n_1^{(T)}}\right\rangle_{B_1^{(T)}A_1^{(T+1)}}\left|\phi_{m_2^{(T)}n_2^{(T)}}\right\rangle_{B_2^{(T)}A_2^{(T+1)}}U_1U_2\\
&\quad(a_{00}^{T+1}a^{T}_{n_1^{(T)},n_2^{(T)}}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}}\left|0000\right\rangle\\
&\quad+a_{01}^{T+1}a^{T}_{n_1^{(T)},n_2^{(T)}\oplus 1}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus 1}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus 1}\left|0011\right\rangle\\
&\quad+a_{10}^{T+1}a^{T}_{n_1^{(T)}\oplus 1,n_2^{(T)}}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}}\left|1100\right\rangle\\
&\quad+a_{11}^{T+1}a^{T}_{n_1^{(T)}\oplus 1,n_2^{(T)}\oplus 1}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus 1}\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus 1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus 1}\left|1111\right\rangle)_{A_1^{(1)}B_1^{(T+1)}A_2^{(1)}B_2^{(T+1)}}.
\end{aligned} \tag{21}$$

with the probability $p_0=\frac{|\alpha\kappa_{00}|^2+|\beta\kappa_{01}|^2+|\mu\kappa_{10}|^2+|\nu\kappa_{11}|^2}{P_{n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}}}$ or

$$|\varphi_1\rangle=\frac{1}{\sqrt{p_1}}\left(\alpha\kappa_{01}\left|00\right\rangle+\beta\kappa_{00}\left|01\right\rangle+\mu\kappa_{11}\left|10\right\rangle+\nu\kappa_{10}\left|11\right\rangle\right), \tag{32}$$

with the probability $p_1=\frac{|\alpha\kappa_{01}|^2+|\beta\kappa_{00}|^2+|\mu\kappa_{11}|^2+|\nu\kappa_{10}|^2}{P_{n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}}}$ or

$$|\varphi_2\rangle=\frac{1}{\sqrt{p_2}}\left(\alpha\kappa_{10}\left|00\right\rangle+\beta\kappa_{11}\left|01\right\rangle+\mu\kappa_{00}\left|10\right\rangle+\nu\kappa_{01}\left|11\right\rangle\right), \tag{33}$$

with the probability $p_2=\frac{|\alpha\kappa_{10}|^2+|\beta\kappa_{11}|^2+|\mu\kappa_{00}|^2+|\nu\kappa_{01}|^2}{P_{n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}}}$ or

$$|\varphi_3\rangle=\frac{1}{\sqrt{p_3}}\left(\alpha\kappa_{11}\left|00\right\rangle+\beta\kappa_{10}\left|01\right\rangle+\mu\kappa_{01}\left|10\right\rangle+\nu\kappa_{00}\left|11\right\rangle\right). \tag{34}$$

with the probability $p_3=\frac{|\alpha\kappa_{11}|^2+|\beta\kappa_{10}|^2+|\mu\kappa_{01}|^2+|\nu\kappa_{00}|^2}{P_{n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}}}$.

In the multi-hop scenario, the operators performed by Bob can be expressed as:

$$E_{S\left(n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}\right)}=\sum_{i,j=0}^{1}b_{ij}\left|ij\right\rangle\left\langle ij\right|, \tag{35a}$$

$$E_{F\left(n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}\right)}=\sum_{i,j=0}^{1}\sqrt{1-b_{ij}{}^2}\left|ij\right\rangle\left\langle ij\right|. \tag{35b}$$

TABLE 3. The relationship between all the possible state of qubits $B_1^{(T+1)}$, $B_2^{(T+1)}$ and the coefficients $b_{ij}(i,j=0,1)$ for the Kraus operators.

$\lvert\varphi_m\rangle_{B_1^{(T+1)}B_2^{(T+1)}}$	$b_{00},b_{01},b_{10},b_{11}$
$\lvert\varphi_0\rangle_{B_1^{(T+1)}B_2^{(T+1)}}$	$b_{00}=\frac{\kappa_{\min}}{\kappa_{00}},b_{01}=\frac{\kappa_{\min}}{\kappa_{01}},b_{10}=\frac{\kappa_{\min}}{\kappa_{10}},b_{11}=\frac{\kappa_{\min}}{\kappa_{11}}.$
$\lvert\varphi_1\rangle_{B_1^{(T+1)}B_2^{(T+1)}}$	$b_{00}=\frac{\kappa_{\min}}{\kappa_{01}},b_{01}=\frac{\kappa_{\min}}{\kappa_{00}},b_{10}=\frac{\kappa_{\min}}{\kappa_{11}},b_{11}=\frac{\kappa_{\min}}{\kappa_{10}}.$
$\lvert\varphi_2\rangle_{B_1^{(T+1)}B_2^{(T+1)}}$	$b_{00}=\frac{\kappa_{\min}}{\kappa_{10}},b_{01}=\frac{\kappa_{\min}}{\kappa_{11}},b_{10}=\frac{\kappa_{\min}}{\kappa_{01}},b_{11}=\frac{\kappa_{\min}}{\kappa_{00}}.$
$\lvert\varphi_3\rangle_{B_1^{(T+1)}B_2^{(T+1)}}$	$b_{00}=\frac{\kappa_{\min}}{\kappa_{11}},b_{01}=\frac{\kappa_{\min}}{\kappa_{10}},b_{10}=\frac{\kappa_{\min}}{\kappa_{01}},b_{11}=\frac{\kappa_{\min}}{\kappa_{00}}.$

Here $\kappa_{\min}=\min\kappa_{ij},(i,j=0,1)$.

The relationship between all the possible states of qubit $B_1^{(T+1)}$, $B_2^{(T+1)}$ and $b_{ij}(i,j=0,1)$ for the Kraus operators is given in Table 3.

Here $\kappa_{\min}=\min\kappa_{ij},(i,j=0,1)$.

When $E_{S\left(n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}\right)}$ is obtained, Bob performs the same operations as that shown in Sec.II to obtain the initial

$$\begin{aligned}
&|\varphi\rangle_{13A_1^{(1)}B_1^{(T+1)}A_2^{(1)}B_2^{(T+1)}}\\
&=|\chi\rangle_{13}\otimes(a_{00}^{T+1}a^{T}_{n_1^{(T)},n_2^{(T)}}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}}\\
&\quad\cdots\ a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}}\left|0000\right\rangle\\
&\quad+a_{01}^{T+1}a^{T}_{n_1^{(T)},n_2^{(T)}\oplus1}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus1}\\
&\quad\cdots a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)},n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus1}\left|0011\right\rangle\\
&\quad+a_{10}^{T+1}a^{T}_{n_1^{(T)}\oplus1,n_2^{(T)}}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus1,n_2^{(T)}\oplus n_2^{(T-1)}}\\
&\quad\cdots\ a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}}\left|1100\right\rangle\\
&\quad-a_{11}^{T+1}a^{T}_{n_1^{(T)}\oplus1,n_2^{(T)}\oplus1}a^{T-1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus1}\\
&\quad\cdots\ a^{1}_{n_1^{(T)}\oplus n_1^{(T-1)}\oplus\cdots\oplus n_1^{(1)}\oplus1,n_2^{(T)}\oplus n_2^{(T-1)}\oplus\cdots\oplus n_2^{(1)}\oplus1}\left|1111\right\rangle)_{A_1^{(1)}B_1^{(T+1)}A_2^{(1)}B_2^{(T+1)}}.
\end{aligned} \tag{28}$$

$$\begin{aligned}
P_{total}&=\sum_{\substack{n_1^{(1)},n_1^{(2)},\cdots n_1^{(T)}\\ n_2^{(1)},n_2^{(2)},\cdots n_2^{(T)}=0}}^{1}\ \sum_{m=0}^{3}p_m\left\langle\varphi_m\right|E^{\dagger}_{S\left(n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}\right)}E_{S\left(n_1^{(1)},\ldots,n_1^{(T)},n_2^{(1)},\ldots,n_2^{(T)}\right)}\left|\varphi_m\right\rangle\\
&=\sum_{\substack{n_1^{(1)},n_1^{(2)},\cdots n_1^{(T)}\\ n_2^{(1)},n_2^{(2)},\cdots n_2^{(T)}=0}}^{1}4|\kappa_{\min}|^2
\end{aligned} \tag{36}$$

TABLE 4. Comparisons between four quantum teleportation protocols.

Schemes	Quantum resource consumption	Classical resource consumed per qubit	Necessary operation	Teleported quantum state
Wang[50]	$T+1$ Bell state	$2(T+1)$	Two-qubit measurement	One-qubit state
Zou[52]	$T+1$ five-qubit composite GHZ-Bell state	$\frac{5}{2}(T+1)$	Five-qubit measurement	Two-qubit state
Qi[53]	$T+1$ four-vertex graph state	$2(T+1)$	Four-qubit measurement	Two-qubit state
Choudhury[54]	$4(T+1)$ Bell state	$2(T+1)$	Eight-qubit measurement	Four-qubit state
ours	$T+1$ four-qubit cluster state	$T+1$	Bell state measurement	Four-qubit state

four-qubit cluster state. The total probability of successfully recovering the original state is (36), shown at the bottom of the previous page.

Obviously, if we use the maximally entangled quantum channel, *i.e.*, $\left|a_{00}^{(i)}\right| = \left|a_{01}^{(i)}\right| = \left|a_{10}^{(i)}\right| = \left|a_{11}^{(i)}\right| = \frac{1}{2}$ $(i = 1, 2, \cdots, T+1)$, the total success probability is:

$$P = 4 * 2^{2T} * \frac{1}{\left(2^{T+1}\right)^2} = 1. \tag{37}$$

IV. EFFICIENCY ANALYSIS

In quantum teleportation scheme, classical communication cost and quantum communication delay are usually used to evaluate the efficiency of the protocol.

First, we discuss the usage of classical information in our scheme. Here, the classical communication cost is defined as the number of data transmission required. In our scheme, each intermediate node needs to perform two Bell state measurements and then send measurement outcomes via classical communication. Moreover, after establishing the quantum entangled channel between source node and destination node successfully, Alice needs to publish two Bell state measurement outcomes to Bob. Therefore, the total classical information cost can be expressed as:

$$C = 4 * (T+1)\,. \tag{38}$$

Second, we discuss the quantum communication delay in our scheme. Quantum communication delay usually occurs in quantum measurements, unitary operations and measurement outcomes transmission. In our scheme, all intermediate nodes perform Bell state measurement independently and transmit measurement results simultaneously, which introduces the delay of Bell measurement d_{meas} and measurement outcomes transmission delay d_{trans}. After that, Alice performs a series of unitary operations to adjust the entangled quantum channel, and executes Bell state measurements and transmits measurement outcomes to Bob, which introduces the delay of unitary operation d_{oper}, Bell measurement d_{meas}, and measurement outcomes transmission delay d_{trans}. Finally, Bob performs a series of the unitary operations to recover the target four-qubit cluster state, which introduces unitary operation delay d_{oper}. Therefore, the total quantum communication delay can be expressed as:

$$d_{tatol} = 2(d_{meas} + d_{trans} + d_{oper}). \tag{39}$$

If we take use of the hop-by-hop transmission [51], [55], the measurement and outcome transmission are performed one by one. The total communication delay in the hop-by-hop quantum teleportation can be written as:

$$d_{tatol} = (T+1)(d_{meas} + d_{trans} + d_{oper}). \tag{40}$$

It is obvious from *Eq.* (39) and *Eq.* (40) that the delay of our multi-hop protocol is much less than the hop-by-hop case, especially when the amount of intermediate nodes is huge.

In Table 4, we discuss the efficiency of our scheme with T intermediate nodes and compare with other quantum teleportation schemes in the following aspects; the quantum resource consumption, the classical resource consumption, the complexity of necessary operation and the quantum state to be teleported.

It is clear from Table 4 that our scheme has several merits. First, our aim is to transfer a four-qubit state while only two-qubit state is prepared in schemes given in [52], [53]. Second, if we utilize Choudhury's scheme to transfer an unknown four-qubit state, it needs at least $4(T+1)$ Bell states as quantum channels. Only $T+1$ four-qubit cluster states are required in our scheme, indicating the quantum resources used in our scheme are more effective. Third, in terms of classical resource consumption, our scheme only needs $T+1$ bit classical resources to teleport each qubit. It is noteworthy that our scheme did not consider the influence of noise during the transmission. In real systems, the quantum noise is unavoidable which reduces the fidelity of quantum states. Therefore, we hope the scheme can be improved later by considering the noise effect on multi-hop teleportation network.

V. CONCLUSION

In summary, we propose a novel scheme for multi-hop teleportation of an arbitrary four-qubit cluster state between two distant nodes. These two nodes have no entanglement pairs shared directly. First, we make detailed calculations on one-hop teleportation of an arbitrary four-qubit cluster state and then generalized the scheme to the multi-hop case. Moreover, we deduce the relationship between the coefficients of the entangled cluster states and the probability of the successful teleportation. The success probability and the fidelity of our scheme can reach 100% when the maximally entangled channel is applied. Finally, we compare our scheme with other schemes on quantum and classical resource consumption, the complexity of necessary operation and the quantum state to be teleported. We believe our scheme is efficient. We hope our findings will stimulate more investigations on the development of quantum teleportation.

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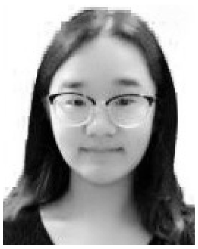

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