

Received April 17, 2020, accepted April 22, 2020, date of publication April 27, 2020, date of current version May 14, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2990644

A Platoon Control Strategy for Autonomous Vehicles Based on Sliding-Mode Control Theory

BO PENG^{ID}, DEXIN YU^{ID}, HUXING ZHOU^{ID}, XUE XIAO^{ID}, AND YUNFENG FANG^{ID}

School of Transportation, Jilin University, Changchun 130022, China

Jilin Engineering Research Center for Intelligent Transportation, Jilin University, Changchun 130022, China

Jilin Province Key Laboratory of Road Traffic, Jilin University, Changchun 130022, China

Corresponding author: Huxing Zhou (zhouhx@jlu.edu.cn)

This work was supported in part by the Jilin Special Fund for Industrial Innovation under Grant 2019C024, in part by the Jilin Science and Technology Development Project under Grant 20190101023JH, and in part by the Jilin Education Department 13th Five-Year Science and Technology Project under Grant JJKH20180153KJ.

ABSTRACT Platooning is one of the innovations in the automotive industry, which aims to improve the safety and efficiency of automobiles, while alleviating traffic congestion, reducing pollution, and reducing passenger pressure. According to the car-following (CF) theory, a platoon control strategy for autonomous vehicles based on sliding-mode control (SMC) theory is proposed. This strategy can be applied to achieve the rapid platoon forming of multiple autonomous vehicles and maintain the stable state of the vehicle platoon. The Multiple Velocity Difference (MVD) model is selected to describe the positional state of vehicle platoon changing over time. The control target is to converge the error between the actual headway (the distance between front tips of two neighboring cars) and the expected headway to zero while ensuring the stable velocity and acceleration of the platoon. In addition, a hypothetical first car strategy is proposed to improve the control efficiency. Numerical simulation experiments for urban roads and highways are designed, the space-time states of vehicle platoon under different MVD model parameters (non-control strategy) and sliding-mode control strategies are compared. The results show: proposed improved vehicle platoon sliding-mode control strategy can provide a shorter time of forming a platoon and better stability in the simulated environment, and its control effect is better than that of non-control strategy and conventional sliding-mode control strategy. Besides the proposed strategy allows vehicle platoon to quickly reach a stable and controllable state, and it provides an idea for collaborative control of autonomous vehicles.

INDEX TERMS Platoon control, autonomous vehicles, sliding-mode control, Simulink, acceleration.

I. INTRODUCTION

With the development and application of autonomous driving technology, vehicles are accelerating the transition from manually controlled mechanical products to intelligent products controlled by intelligent systems. Vehicle platoon has become a research hotspot in the field of autonomous driving. Under the premise of intelligent vehicle control and information exchange, multi-vehicle platoon driving can provide a smaller safety distance (higher road utilization), lower risk for drivers, and less wind resistance (lower fuel consumption) [1]. From a microscopic perspective, the vehicle platoon is more like

a car-following (CF) behavior that can be automatically adjusted.

The research on the CF model has been very detailed. Bando *et al.* put forward the Optimal Velocity (OV) Model, which served as the foundation for a series of CF models proposed by scholars [2]. Nagatani *et al.* discovered the model of traffic flow based on the headway (the distance between front tips of two neighboring cars) [3]. Helbing *et al.* compared the fluctuating vehicle statistics of urban car-following behavior and micro stimulation model, based on which the Generalized Force (GF) Model was proposed [4]. Jiang *et al.* put forward the Full Velocity Difference (FVD) Model based on the CF model, and applied it to describe the phase transition of traffic flow to estimate the congestion evolution [5]. On the basis of FVD Model, Wang *et al.* attempted to establish an expanded

The associate editor coordinating the review of this manuscript and approving it for publication was Shihong Ding^{ID}.

model with a large number of previous cars' information, and proposed the Multiple Velocity Difference (MVD) Model [6]. From the perspective of practical engineering, there are two important behaviors for vehicle platoon i.e., the formation and the maintenance of the platoon. This process cannot be achieved by following only the CF model. The convergence speed of the CF model cannot meet the efficiency requirements of vehicle formation.

Sliding-mode control is a classic convergence control method. So far, the sliding-mode control theory has been widely applied on fault diagnosis and fault tolerant control [7]–[9]. He *et al.* explored a vehicle stability control strategy based on sliding-mode variable structure control theory [10]. Gao *et al.* presented a distributed sliding mode control strategy for platoon control of multiple autonomous vehicles [11]. Lu *et al.* presented a novel neural network adaptive sliding-mode control method to design the dynamic control system for an omnidirectional vehicle [12]. Sun *et al.* studied the leader-follower platoon control of under actuated surface vehicles with model uncertainties and environmental disturbances [13]. There has been some research on platoon maintenance. Kwon *et al.* proposed a platoon-control method for an interconnected vehicular system using a coupled sliding-mode control to guarantee string stability [14]. Yan *et al.* developed a neural adaptive sliding-mode control algorithm to investigate the output feedback control of a vehicle platoon [15]. Li *et al.* described the features of traffic flow by adopting the FVD Model, and designed a sliding-mode controller, verifying that the strategy of the sliding-mode control is better than the reflection control [16].

From the existing research, it seems that there is less attention to the formation process of vehicle platoon. The formation of the vehicle platoon is the first process of platoon driving. Therefore, this study is dedicated to finding a fast and safe platoon control strategy. This strategy should meet these conditions: first, the formation and maintenance of the platoon can be achieved according to the preset speed and headway; second, the safety of the driver and the compliance with road traffic regulations are ensured during the formation of the platoon; third, this strategy must have practical engineering significance.

For the above conditions, first, we combined the classic MVD model and an improved sliding-mode control method to realize the process that the vehicles quickly formed a stable platoon from irregular positions and speeds, and observed the performance of the platoon in continuous disturbance; second, we fully considered the drivers' experience and traffic regulations during the formation of the platoon, limiting the speed and acceleration of the vehicles during the formation of the platoon; third, we improved the conventional sliding-mode controller, which greatly weakened the chatter and made it practical. In order to verify the control effect of the proposed strategy, a numerical experiment was designed. Scenes with different velocity requirements were designed to verify the platoon state under the no-control strategy, the conventional sliding-mode control strategy, and the improved

sliding-mode control strategy. The results show that the effect of improved sliding-mode control strategy is better than other strategies. Which can achieve the formation and stable driving of vehicle platoon, while maintaining reasonable speed and acceleration.

The main advantage compared to previous ones is that the formation of the platoon is fast and stable, and it can ensure stability under continuous interference. And while maintaining a small amount of calculation, the chatter is greatly weakened. Besides, a hypothetical first car strategy is proposed which greatly improves the stability of the platoon and control efficiency. With the gradual popularization of autonomous driving, the proposed platoon control strategy can provide necessary scientific guidance in improving transportation efficiency and safety.

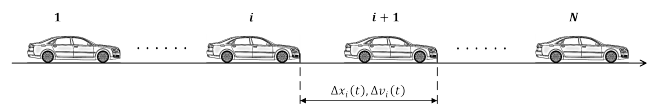


FIGURE 1. Schematic diagram of a single lane vehicle platoon scene.

II. PRELIMINARIES

Shown in Fig.1 is a car-following scene where the vehicle platoon in a single lane has a total number of N cars. Based on the FVD Model, Wang *et al.* proposed the MVD Model [6], which enhances the stability of the vehicle platoon according to the information from multiple vehicles. This study selected the MVD Model to describe the spatial state of car platoon in a single lane and configure the controlled objective: to converge the difference between the actual headway and the expected headway into zero within a short time span. Apart from considering the velocity difference between the previous and the current vehicle, the velocity differences between multiple (n) previous vehicles were also utilized to describe the traffic flow characteristics of the vehicle platoon. The model expression is as shown in (1).

$$\ddot{x}_i(t) = a [V(\Delta x_i(t)) - v_i(t)] + \sum_{j=1}^n \lambda_j \Delta v_{i+j-1}(t) \quad (1)$$

where t represents the moment; a is the sensitivity coefficient; $x_i(t)$, $v_i(t)$, and $\ddot{x}_i(t)$ respectively represent the position, velocity, and acceleration of the i^{th} car at t moment; j indicates the current number of vehicles considering the velocity difference, and $j = 1, 2, \dots, n, n \ll N$.

Hence, $\Delta x_i(t) = x_{i+1}(t) - x_i(t)$ represents the headway between the $i + 1^{\text{th}}$ and i^{th} car at t moment, and $\Delta v_{i+j-1}(t) = v_{i+j}(t) - v_{i+j-1}(t)$ represents the velocity difference between the $i + j^{\text{th}}$ and $i + j - 1^{\text{th}}$ car at t moment, and λ_j are the response coefficients of the difference of velocity. If the actual conditions are also considered, and $\lambda_j > \lambda_{j+1}$. It can be apparently seen when n is taken as 1, and the model will become the FVD model.

$V(\cdot)$ is the optimal velocity function [5], a monotonically increasing function with an upper boundary, and is as shown

in (2).

$$V(\Delta x_i(t)) = \frac{v_m}{2} [\tanh(\Delta x_i(t) - \Delta x_c) + \tanh(\Delta x_c)] \quad (2)$$

$V(\Delta x_i(t))$ represents the expected velocity; $\tanh(\cdot)$ is a hyperbolic tangent function; v_m is the highest velocity; Δx_c is the safe headway between two neighboring vehicles in the platoon.

According to the expression of MVD Model, (1) can be rewritten into the differential form as shown in (3).

$$\begin{cases} \frac{dv_i(t)}{dt} = a[V(\Delta x_i(t)) - v_i(t)] + \sum_{j=1}^n \lambda_j \Delta v_{i+j-1}(t) \\ \frac{d\Delta x_i(t)}{dt} = v_{i+1}(t) - v_i(t) \end{cases} \quad (3)$$

If the first car travels at the expected velocity v_0 , the position of the first car at t moment can be described as (4).

$$x_N(t) = v_0 t + x_N(0) \quad (4)$$

where $x_N(t)$ represents the position of the first car at t moment; $x_N(0)$ is the initial position of the said first car.

Hence, in terms of the vehicle states of the whole platoon, the inverse function of the optimal velocity function can be used to obtain the expected headway between the cars. In other words, (5) can be derived from (2).

$$\Delta x_d = V^{-1}(v_0) \quad (5)$$

where Δx_d represents the expected headway between neighboring vehicles if the first car travels at the expected velocity v_0 ; $V^{-1}(\cdot)$ equals the inverse function of the optimal velocity function.

Therefore, the stability state of the vehicles in the platoon can be expressed as (6) [17]–[19].

$$[v_i^*(t), \Delta x_i^*(t)]^T = [v_0, V^{-1}(v_0)]^T \quad (6)$$

Equation (7) can be derived from (3) by linearizing the latter in its stable region.

$$\begin{cases} \frac{d\delta v_i(t)}{dt} = a[\Lambda \delta \Delta x_i(t) - \delta v_i(t)] + \sum_{j=1}^n \lambda_j \delta \Delta v_{i+j-1}(t) \\ \frac{d\delta \Delta x_i(t)}{dt} = \delta v_{i+1}(t) - \delta v_i(t) \end{cases} \quad (7)$$

where $\delta v_i(t) = v_i(t) - v_0$, $\delta \Delta x_i(t) = \Delta x_i(t) - V^{-1}(v_0)$, Λ represents the slopes of the model when $\Delta x_i(t) = V^{-1}(v_0)$, and Λ is as shown in Equation (8).

$$\begin{aligned} \Lambda &= \left. \frac{dV(\Delta x_i(t))}{d\Delta x_i(t)} \right|_{\Delta x_i(t)=V^{-1}(v_0)} \\ &= \frac{1}{[\cosh(V^{-1}(v_0) - x_c)]^2} \end{aligned} \quad (8)$$

By introducing the control term of the sliding-mode controller into (7), the spatial expression for the required state

of the vehicle platoon system can be obtained and is shown as (9).

$$\begin{cases} \frac{d\delta v_i(t)}{dt} = a[\Lambda \delta \Delta x_i(t) - \delta v_i(t)] + u_i(t) \\ \quad + \sum_{j=1}^n \lambda_j \delta \Delta v_{i+j-1}(t) \\ \frac{d\delta \Delta x_i(t)}{dt} = \delta v_{i+1}(t) - \delta v_i(t) \end{cases} \quad (9)$$

where $u_i(t)$ is the control signal term. $u_i(t)$ as a control item is to ensure the error and convergence speed of the vehicle spacing. (9) can be used to solve the vehicle state at different moments. The car's following state has been described so far. The next step is to design a control module.

III. METHDODOLOGY

A. THE DESCRIPTION OF THE SYSTEM

According to the fundamental principles of the variable sliding-mode structural control [20]–[22], the design would be applicable for the sliding-mode controller of the system mentioned above. Besides, a velocity feedback control module is added. The control strategy is as shown in Fig.2.

Firstly, the control objective is determined. For the vehicle platoon in which the first car is traveling at the expected velocity, the control objective shall be the headway. (5) can be used to obtain the error between the actual headway and the expected headway which is represented as e , as is shown in (10).

$$e = \delta \Delta x_i(t) = \Delta x_i(t) - V^{-1}(v_0) \quad (10)$$

Therefore, the control objective within a bounded time parameter τ can be expressed as $\lim_{t \rightarrow \tau} e = 0, \tau > 0$. This expression can also be elaborated as the requirement for the actual headway and the expected headway to be equal within a given limited time.

B. THE DESIGN OF SLIDING-MODE CONTROL MODULE

1) CONVENTIONAL SMC METHOD

The uncertainty of the system needs to be considered in the modeling so that the designed controller can meet the design requirements under the interference of uncertainty. With theoretical and engineering practical significance, the characteristics of sliding-mode control (SMC) are suitable for traffic flow platoon stability control [23], [24]. Based on the fundamental theory of sliding-mode control, the sliding-mode surface can be defined as (11).

$$s = ce + \dot{e} \quad (11)$$

where e and \dot{e} are respectively the error and its rate of change; c must meet the Hurwitz criterion i.e., $c > 0$.

It can be observed from (11) that when $s(t) = 0, ce(t) + \dot{e}(t) = 0$ and the model will converge to $e(t) = e(0)e^{-ct}$. That is to say, when $t \rightarrow \infty$, the error exponent will converge to 0 at a rate that is determined by the value of c . Thus, the convergence of the error function s can be inferred as the convergence of both the position tracking error e and that

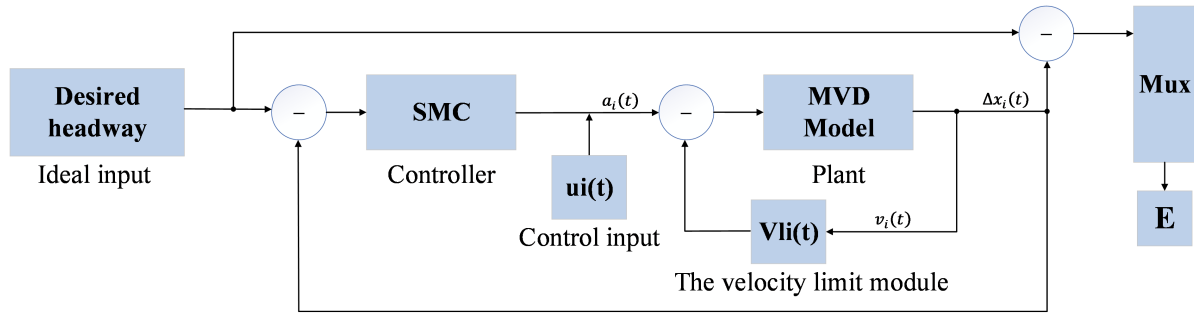


FIGURE 2. Platoon control strategy flow chart.

of the velocity tracking error \dot{e} , during which s will be the sliding-mode function.

The definition of the Lyapunov function is shown in (12).

$$L = 0.5s^2 \quad (12)$$

Thus, the differential form of (12) can be obtained, as shown in (13).

$$\dot{L} = s\dot{s} \quad (13)$$

\dot{s} can be obtained as shown in (14).

$$\dot{s} = c\dot{e} + \ddot{e} = c\delta v_{i+1}(t) - c\delta v_i(t) + \frac{d\delta v_{i+1}(t)}{dt} - \frac{d\delta v_i(t)}{dt} \quad (14)$$

Combined with the state space expression (9) of the system, \dot{s} can be further rewritten as (15).

$$\dot{s} = c\delta v_{i+1}(t) - c\delta v_i(t) + \frac{d\delta v_{i+1}(t)}{dt} - a\Lambda\delta\Delta x_i(t) + a\delta v_i(t) - \sum_{j=1}^n \lambda_j\delta\Delta v_{i+j-1}(t) - u_i(t) \quad (15)$$

Bring (15) into (13) and get the expression of \dot{L} as (16).

$$\dot{L} = s \left[c\delta v_{i+1}(t) - c\delta v_i(t) + \frac{d\delta v_{i+1}(t)}{dt} - a\Lambda\delta\Delta x_i(t) + a\delta v_i(t) - \sum_{j=1}^n \lambda_j\delta\Delta v_{i+j-1}(t) - u_i(t) \right] \quad (16)$$

To ensure that $\dot{L} \leq 0$, the conventional sliding-mode control rate can be designed as shown in (17).

$$\begin{aligned} u_i(t) &= c\delta v_{i+1}(t) - c\delta v_i(t) + \frac{d\delta v_{i+1}(t)}{dt} - a\Lambda\delta\Delta x_i(t) \\ &\quad + a\delta v_i(t) - \sum_{j=1}^n \lambda_j\delta\Delta v_{i+j-1}(t) + ks + \eta \operatorname{sgn}(s) \\ &= c\delta v_{i+1}(t) - c\delta v_i(t) + \frac{d\delta v_{i+1}(t)}{dt} - a\Lambda\delta\Delta x_i(t) \\ &\quad + a\delta v_i(t) - \lambda_1\delta v_{i+1}(t) + \lambda_1\delta v_i(t) - \lambda_2\delta v_{i+2}(t) \\ &\quad + \lambda_2\delta v_{i+1}(t) - \dots - \lambda_n\delta v_{i+n}(t) + \lambda_n\delta v_{i+n-1}(t) \\ &\quad + ks + \eta \operatorname{sgn}(s) \end{aligned} \quad (17)$$

where $k > 0$, $\eta > 0$, $\operatorname{sgn}(\cdot)$ is a sign function.

At this time, the expression of the sliding-mode control rate is as shown in the (18).

$$\dot{s} = -ks - \eta \operatorname{sgn}(s) \quad (18)$$

Further, \dot{L} under the conventional sliding-mode control strategy can be obtained, as shown in (19).

$$\dot{L}_c = s[-ks - \eta \operatorname{sgn}(s)] = -ks^2 - \eta |s| \quad (19)$$

2) IMPROVED SMC METHOD

Equation (17) is a conventional sliding-mode controller design method. Using conventional SMC strategies, the system will generate chattering when the interference is large. In this study, it is specifically expressed as the discontinuity of acceleration. For the drivers, this chattering will be extremely serious, which will affect the driving experience and safety. To eliminate the adverse effects of chatter during driving, the following improved SMC strategy is proposed. The switching term $\eta \operatorname{sgn}(s)$ in the original control function is the main cause of chattering. After several attempts, the term was eventually replaced by the hyperbolic tangent function. Rewritten as the continuous switching term $\eta \tanh\left(\frac{s}{\varepsilon}\right)$, the control function is shown in (20).

$$\begin{aligned} u_i(t) &= c\delta v_{i+1}(t) - c\delta v_i(t) + \frac{d\delta v_{i+1}(t)}{dt} - a\Lambda\delta\Delta x_i(t) \\ &\quad + a\delta v_i(t) - \sum_{j=1}^n \lambda_j\delta\Delta v_{i+j-1}(t) + ks + \eta \tanh\left(\frac{s}{\varepsilon}\right) \end{aligned} \quad (20)$$

At this time, $\dot{s} = -ks - \eta \tanh\left(\frac{s}{\varepsilon}\right)$, Further, \dot{L} under the improved sliding-mode control strategy can be obtained, as shown in (21).

$$\dot{L}_i = s \left[-ks - \eta \tanh\left(\frac{s}{\varepsilon}\right) \right] \quad (21)$$

3) THE PROOF OF CONVERGENCE

Lyapunov analysis has been utilized to test the finite-time stability of the closed-loop sliding-mode dynamics [25]. The convergence of conventional SMC strategy and improved SMC strategy is proved as follows.

For traditional SMC strategy. From (17) it can be seen that $\dot{s} = -ks$ is the limit convergence term [26], whose root is $s = s(0)e^{-kt}$. The rate of limit convergence is as shown by (22).

$$\dot{L}_c = -ks^2 - \eta |s| \leq -\frac{k}{2}L - \eta |s| \leq -\frac{k}{2}L \quad (22)$$

For improved SMC strategy. Separating the term $s\eta \tanh\left(\frac{s}{\varepsilon}\right)$ from (21), (23) can be obtained.

$$s\eta \tanh\left(\frac{s}{\varepsilon}\right) = s\eta \frac{e^{\frac{s}{\varepsilon}} - e^{-\frac{s}{\varepsilon}}}{e^{\frac{s}{\varepsilon}} + e^{-\frac{s}{\varepsilon}}} = \frac{\eta}{e^{2\frac{s}{\varepsilon}} + 1} s(e^{2\frac{s}{\varepsilon}} - 1) \quad (23)$$

Because $\begin{cases} e^{2\frac{s}{\varepsilon}} - 1 \geq 0, s \geq 0 \\ e^{2\frac{s}{\varepsilon}} - 1 < 0, s < 0 \end{cases}$, which is $s(e^{2\frac{s}{\varepsilon}} - 1) \geq 0$.

Using this condition, we can prove that the term is not less than 0, as shown in (24).

$$s\eta \tanh\left(\frac{s}{\varepsilon}\right) = \frac{\eta}{e^{2\frac{s}{\varepsilon}} + 1} s(e^{2\frac{s}{\varepsilon}} - 1) \geq 0 \quad (24)$$

So we can also get the upper limit of \dot{L}_i as shown in (25).

$$\dot{L}_i = -ks^2 - s\eta \tanh\left(\frac{s}{\varepsilon}\right) \leq -\frac{k}{2}L - s\eta \tanh\left(\frac{s}{\varepsilon}\right) \leq -\frac{k}{2}L \quad (25)$$

Considering Lemma 1 for $Y : [0, \infty) \in R$, the root(s) for the inequality $\dot{Y} \leq -\alpha Y + f, \forall t \geq t_0 \geq 0$ is as represented in (26) [27].

$$Y(t) \leq e^{-\alpha(t-t_0)}Y(t_0) + \int_{t_0}^t e^{-\alpha(t-\tau)}f(\tau)d\tau \quad (26)$$

where α is an arbitrary constant. The proof process of Lemma 1 is as follows [28].

Assume $\omega(t) \dot{Y} + \alpha Y - f$, then $\omega(t) \leq 0$, and $\dot{Y} = -\alpha Y + f + \omega$. Solving this formula gives (27).

$$Y(t) = e^{-\alpha(t-t_0)}Y(t_0) + \int_{t_0}^t e^{-\alpha(t-\tau)}f(\tau)d\tau + \int_{t_0}^t e^{-\alpha(t-\tau)}\omega(\tau)d\tau \quad (27)$$

Due to $\omega(t) < 0, \forall t \geq t_0 \geq 0$, (27) can be converted to (28).

$$Y(t) = e^{-\alpha(t-t_0)}Y(t_0) + \int_{t_0}^t e^{-\alpha(t-\tau)}f(\tau)d\tau \quad (28)$$

Therefore, referring to Lemma 1, for the inequality $\dot{L} \leq -\frac{k}{2}L$, there are $\alpha = \frac{k}{2}, f = 0$. The roots are as shown in (29).

$$L(t) \leq e^{-\frac{k}{2}(t-t_0)}L(t_0) \quad (29)$$

It can be seen that if the exponent of $L(t)$ converges to zero i.e., s is exponentially convergent, the exponential convergence and rate of convergence e and \dot{e} will depend on k . This implies that the exponential term ks can guarantee that, when s approaches a very large value, the system state can converge to the sliding-mode state at a relatively high rate.

C. THE DESIGN OF VELOCITY LIMIT MODULE

In these control strategies, the goal of the SMC module is to ensure fast convergence. However, in the actual environment, vehicle performance and traffic management regulations must be considered. The output constraints widely exist in the physical systems [29]. In this study, the speed and

acceleration limits can be converted into a controller output limit. The acceleration and velocity of the vehicle need to be controlled in a range of changes. To this end, a velocity limit module is designed.

As for the acceleration of the vehicle. Since these control strategies are aimed at autonomous vehicles, the acceleration control of the vehicle is more accurate. So the acceleration output value of the vehicle is limited. To ensure safety and comfort, the acceleration range is set to $[-3.00, 3.00]m/s^2$.

As for the velocity of the vehicle, if the output value of the vehicle velocity is directly restricted, it will cause the two sides of the differential equation to be unequal, and the convergence time will tend to be infinite. Therefore, the vehicle acceleration is feedback-controlled by setting a velocity threshold, and the control method is shown in the (30).

$$\begin{cases} a_i(t) = -\gamma_i \text{sgn}[v_i(t)] & (v_i(t) \in V_{limit}) \\ a_i(t) = \frac{dv_i(t)}{dt} & (v_i(t) \in V_{limit}) \end{cases} \quad (30)$$

where $a_i(t)$ represents the acceleration output value of vehicle i at t moment. V_{limit} represents the velocity limit range. γ_i is an adjustment coefficient set according to the velocity limit range.

IV. THE HYPOTHETICAL FIRST CAR STRATEGY

For convenience, the car at the front of the platoon is marked as “car1,” and so on to “car20”. In the process of forming the platoon, the application of control strategy can be simply understood as a recursive problem. Therefore, the parameter setting of “car1” is critical and can be regarded as an example in the platoon adjustment process. In order to achieve the desired headway between the rear vehicles, their velocity will converge towards the velocity of “car1”. However, as the initial setting in this study, “car1” which is the first information receiver cannot be subjected to an ideal state of disturbance and acceleration, but it should be larger than other vehicles. To solve this problem, a hypothetical first car strategy is proposed.

As shown in Fig.3, in front of “car1,” the hypothetical first car “car0” is generated. And it is driven by the ideal headway and ideal velocity. At this point, “car0” is the first car for the entire platoon. The advantages of this method are not only to enhance the stability of the platoon, but also to adjust the “ideal state” of the hypothetical vehicle to adjust the final running state of the whole platoon. In combination with the fast convergence characteristics of the proposed control strategy, the fast and stable control of the platoon can be realized.

V. NUMERICAL EXPERIMENT

The simulation experiment is designed to verify the effectiveness of the proposed strategies, in which a vehicle platoon consisting of 20 vehicles is traveling in different scenes. MATLAB Simulink is employed in the numerical simulation to establish an iterative subsystem for the 20 vehicles to obtain the real-time state of each one. A 4-stage Runge-Kutta

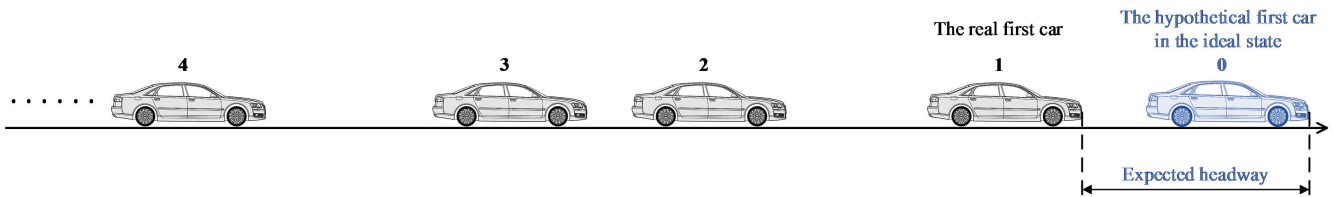


FIGURE 3. Schematic diagram of the hypothetical first car strategy.

algorithm is adopted to solve the differential equations, and the time step size is set as 0.01 s.

A. SCENES DESIGN AND CHOICE OF PARAMETERS

Two scenes of urban road and highway are simulated by numerical simulation. Random noise with a maximum value of $\pm 10^{-2} m/s^2$ is added to the acceleration of each vehicle to express the difference between different vehicles. A sine wave is added to the acceleration of the first car as an environmental disturbance (this is done to maintain the continuity of the disturbance, and irregular waves can also be added).

1) SCENE A:URBAN ROAD SCENE

Velocity setting takes China’s velocity requirements as an example, and can be adjusted according to the requirements of different regions. v_m and Δx_c are set as $v_m = 20.00 m/s$, $\Delta x_c = 20.00 m$, with its initial states set as $v_i(0) = [8.80, 10.00] m/s$, $\Delta x_i(0) = [14.00, 24.00] m$, The velocity of “car0” is set to 9.40 m/s, through which the expected headway becomes $V^{-1}(v_0) = 19.9399 m$. The noise disturbance of “car1” is set as a sine wave.

2) SCENE B:HIGHWAY SCENE

According to highway velocity requirements, v_m and Δx_c are set as $v_m = 33.00 m/s$, $\Delta x_c = 50.00 m$, with its initial states set as $v_{i0} = [21.00, 25.00] m/s$, $\Delta x_i(0) = [40.00, 60.00] m$, The velocity of “car0” is set to 23.00 m/s, through which the expected headway becomes $V^{-1}(v_0) = 40.4156 m$. The peak of the sine waves as a disturbance of “car1” is magnified 2.5 times.

3) CHIOCE OF PARAMETERS

The parametric configuration of the MVD Model is as shown in TABLE 1.

TABLE 1. The parametric configuration of the MVD model.

N	a	λ_1	λ_2	λ_3
20	0.10	0.50	0.45	0.40

The setting of the controller parameters affects the velocity and stability of the forming of the platoon. In combination with the actual engineering environment, it is set to

TABLE 2. The value of η_i .

SCENE	Car1	Car2 to Car20
SCENE A	1.001	0.011
SCENE B	2.501	0.011

$k_i = 0.20$, $\varepsilon = 0.05$. The value of η_i is selected according to the disturbance of each vehicle. According to the disturbance settings in this study, the value of η_i is selected as TABLE 2. The values of γ_i in scene A and scene B are selected as 0.30 m/s and 1.00 m/s, respectively.

B. PLATOON EVOLUTION PROCESS UNDER DIFFERENT CONTROL STRATEGIES IN SCENE A

1) COMPARISON OF HEADWAY VARIATION UNDER EACH CONTROL STRATEGY

Figure 4 shows the space-time distribution of the headway in the urban road scene. In this case, no control strategies were introduced, and the parameter selection for the MVD Model is taken as $n = 1/2/3$, corresponds to Fig.4 (a), Fig.4 (b), and Fig.4 (c). Thus, by applying the settings of previous parameters, the differential equation system of every individual vehicle is built. By separately extracting the states of the 20 vehicles (including vehicle position, velocity, and acceleration), all the headway distributions ranging $t \in [0, 150] s$ can be obtained.

The hypothetical vehicle (“car0”) moves in a straight line at a uniform velocity, so when the headways between all vehicles reach the desired value, it can be concluded that the platoon is already in the desired ideal state. As shown in Fig.4, when all the headways maintain a small fluctuation near 19.94 m, it means that the vehicle platoon has been formed. Without adding a control strategy and the model parameter $n = 1/2/3$, the time required for the headways of vehicle platoon to reach a stable state is greater than 50 s. In the first 50 s, the fluctuations occur when the headways are between 13.00 m – 28.00 m. Observing the curve of the headways, it can be found that when $n = 1$ and $n = 2$, the time required for the platoon to reach a stable state is basically the same. The platoon forming time when $n = 3$ is longer than when $n = 1$ or 2. And after the forming of the platoon, there are small sinusoidal disturbances in all three cases i.e., the disturbance of “car1” has not been completely

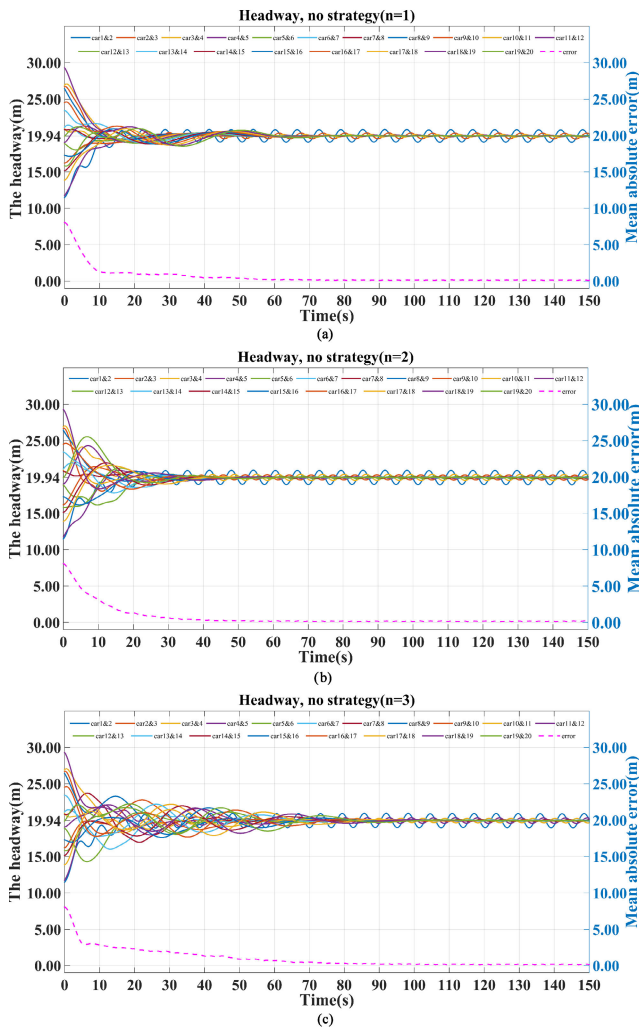


FIGURE 4. Non-controlled strategy, the space-time distribution of the headway when $n = 1/2/3$ in the urban road scene.

eliminated. By observing their mean absolute error curve, it can be concluded that when no control strategy is adopted, the efficiency of the model parameter $n = 1$ or 2 is better than $n = 3$, and the mean absolute error decreases faster when $n = 1$. Therefore, the model parameter $n = 1$ is selected to compare with the control effect after adding the control strategies.

Figure 5 is a comparison of headway variation before and after the control strategy is adopted in the urban road scene. Fig.5 (a) corresponds to the no-control strategy ($n = 1$), and Fig.5 (b) corresponds to the conventional SMC strategy, and Fig.5 (c) corresponds to improved SMC strategy.

The time required to form and stabilize the platoon is about 20 s under conventional SMC strategy and improved SMC strategy, the mean absolute headway error curve changes smoothly, and the sinusoidal disturbance of the headway is eliminated. The control effects of the two control strategies on the headway are the same, and both are significantly better than the state without a control strategy.

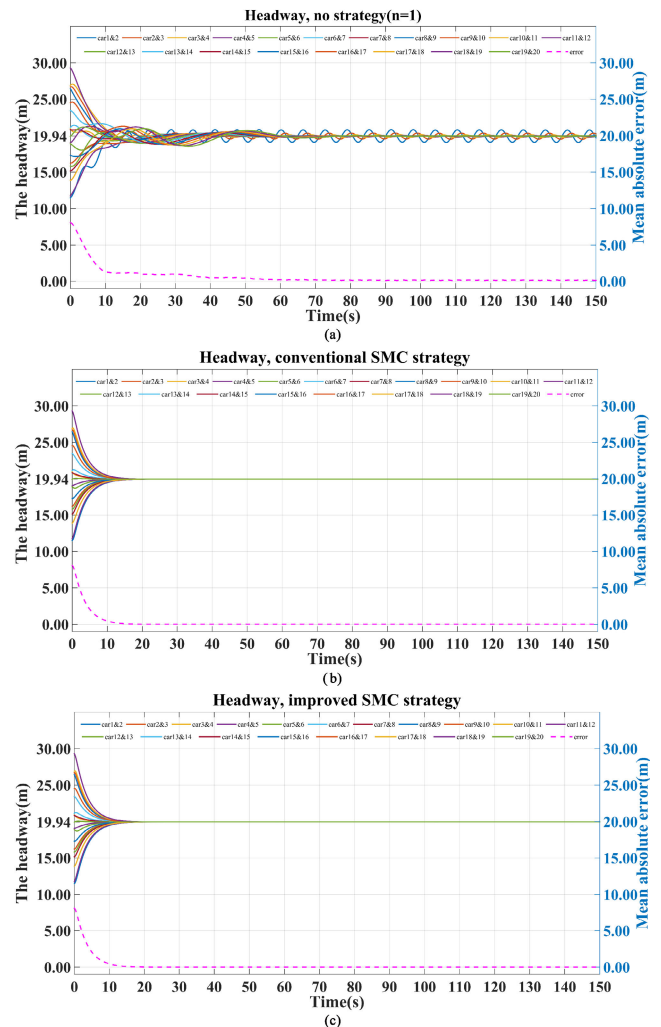


FIGURE 5. Comparison of headway variation before and after the control strategy is adopted in the urban road scene.

2) COMPARISON OF CONTROLLER OUTPUT VALUE UNDER EACH CONTROL STRATEGY

Figure 6 shows the comparison of controller output using conventional SMC strategy and improved SMC strategy in urban road scenes, corresponding to Fig.6 (a) and Fig.6 (b).

Take the first 100 s controller output of “car1” and “car20” for comparison. The controller output of conventional SMC strategy is discontinuous, and there is obvious chattering. This will stimulate the high frequency in the system as a modeling dynamic in actual engineering, which will cause the control system to collapse, which will appear as a discontinuity in acceleration in this study. The controller output of improved SMC strategy is continuous and relatively stable, and the chattering is eliminated.

3) COMPARISON OF VELOCITY VARIATION UNDER EACH CONTROL STRATEGY

In Fig.7, Fig.8, and Fig.9, the velocity variation of vehicles in the platoon under no control strategy, conventional SMC

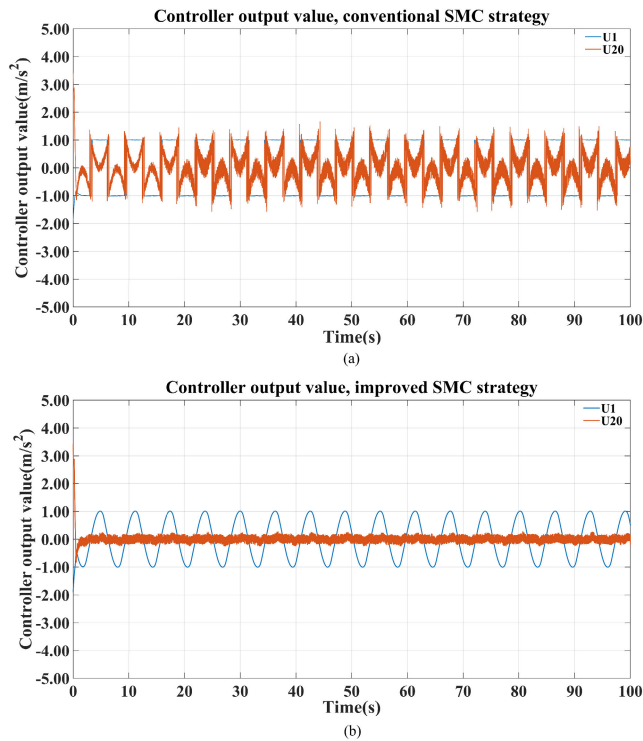


FIGURE 6. Comparison of controller output value under each control strategy in the urban road scene.

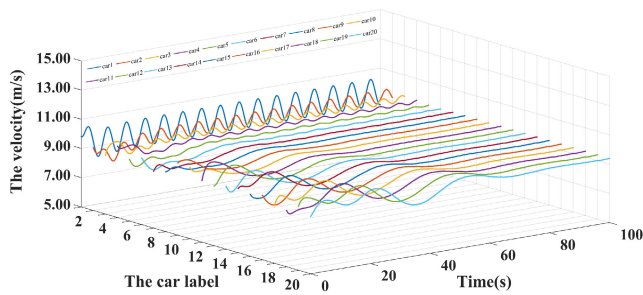


FIGURE 7. Velocity variation of vehicles in the platoon under no control strategy in the urban road scene.

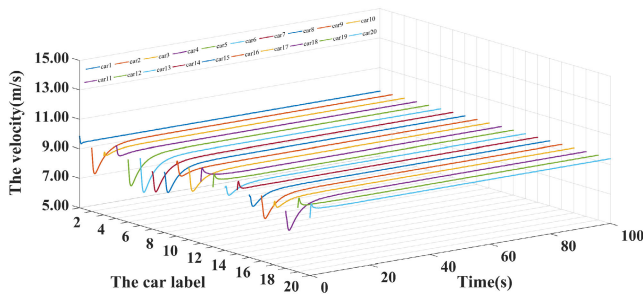


FIGURE 8. Velocity variation of vehicles in the platoon under conventional SMC strategy in the urban road scene.

strategy, and improved SMC strategy in the urban road scene are shown, respectively.

According to the ideal velocity setting of “car0,” the velocity of all vehicles in the platoon should finally stabilize near

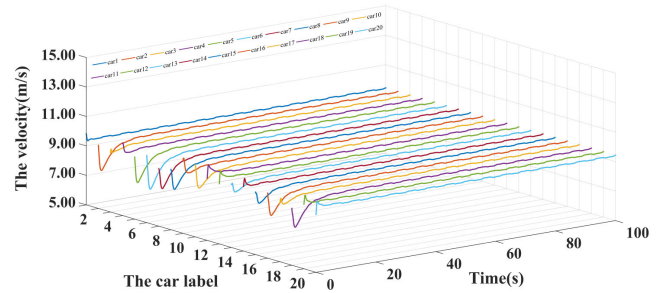


FIGURE 9. Velocity variation of vehicles in the platoon under improved SMC strategy in the urban road scene.

9.40 m/s. Without the control strategy, the stabilization time of the entire platoon velocity is about 70 s. The vehicles at the front of the platoon are disturbed by a sine wave. By observing the wave trend of the velocity curve, it can be found that the disturbance propagates from front to back. Under conventional SMC and improved SMC strategies, the stabilization time of the entire platoon is about 20 s, and velocity of all vehicles have not reached the velocity limit value. The velocity curve is stable and the sine wave disturbance is eliminated. It is worth noting that waves with amplitude less than ± 0.10 m/s can be observed on the velocity change curve under improved SMC strategy. This is because the acceleration change under improved SMC strategy is more continuous than that under conventional SMC strategy, which will be seen in the next section. And this continuous slight velocity fluctuation is actually difficult to detect in actual driving.

4) COMPARISON OF ACCELERATION VARIATION UNDER EACH CONTROL STRATEGY

Figure 10, Fig.11, and Fig.12 show the acceleration variation under no control strategy, conventional SMC strategy, and improved SMC strategy in the urban road scene, respectively.

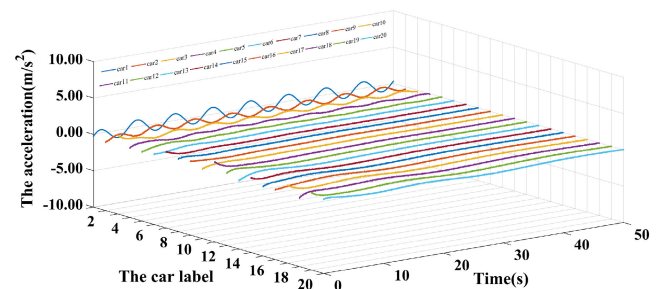


FIGURE 10. Acceleration variation of vehicles in the platoon under no control strategy in the urban road scene.

The acceleration of all vehicles tends to be stable at 30 s without a control strategy being adopted. They all fluctuate around 0 m/s². Similar to the velocity curve, the vehicle at the front of the platoon is disturbed by a sine wave. For the conventional SMC strategy, a chattering phenomenon appears after the stabilizing of the acceleration as predicted

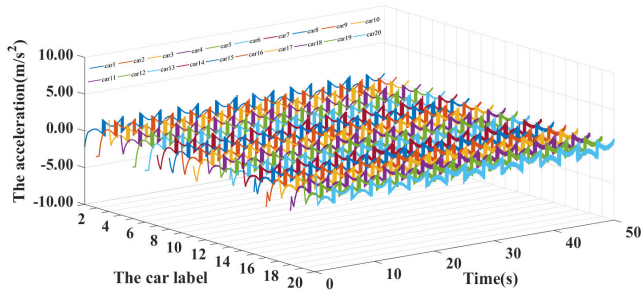


FIGURE 11. Acceleration variation of vehicles in the platoon under conventional SMC strategy in the urban road scene.

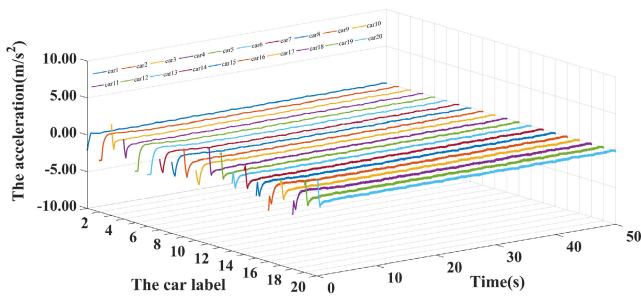


FIGURE 12. Acceleration variation of vehicles in the platoon under improved SMC strategy in the urban road scene.

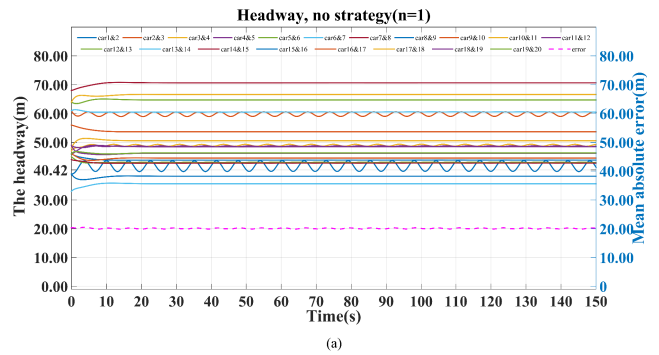
by the results output of the controller. The acceleration is discontinuous and fluctuating. Under the improved SMC strategy, the acceleration tends to be stable within 10 s. The acceleration of all vehicles in the platoon converges to zero and remains stable. Under the action of the velocity limiting, the absolute values of the accelerations of the two control strategies are kept within 3 m/s^2 .

C. PLATOON EVOLUTION PROCESS UNDER DIFFERENT CONTROL STRATEGIES IN SCENE B

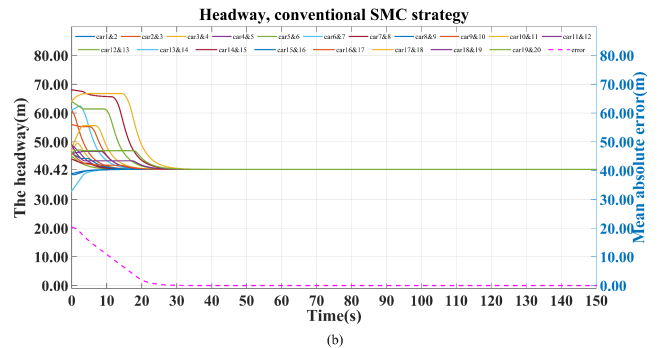
1) COMPARISON OF HEADWAY VARIATION UNDER EACH CONTROL STRATEGY

In this case, when no controllers are introduced, and the parameter selection for the MVD model is taken as $n = 1$ i.e., the velocity differences of previous vehicle and the current vehicle are considered. For $t \in [0, 500] \text{ s}$, the headway distribution of all the vehicles under each strategy are as shown in Fig.13.

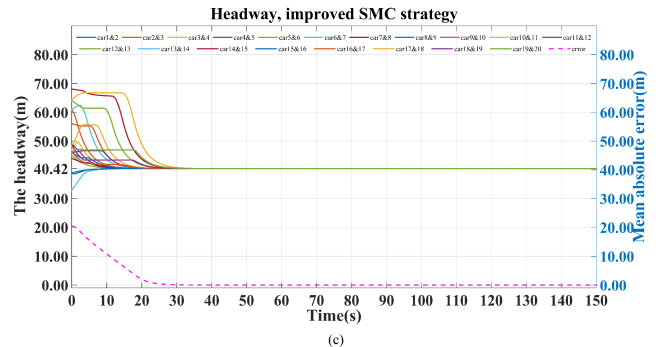
From Fig.13, without the control strategy, the vehicles can not be successfully formed into a platoon in 150 s, and the disturbances suffered by some vehicles are not eliminated. The mean absolute error of the headways remain at about 20.00 m . Under conventional SMC and improved SMC strategies, the time required to form and stabilize the platoon is about 35 s, the curve of mean absolute error changes smoothly, and the sinusoidal disturbances of the headways are eliminated. The control effects of the two control strategies on the headways are basically the same, and both are significantly better than the state without a control strategy.



(a)



(b)



(c)

FIGURE 13. Comparison of headway variation before and after the control strategy is adopted in the highway scene.

2) COMPARISON OF CONTROLLER OUTPUT VALUE UNDER EACH CONTROL STRATEGY

Figure 14 shows the comparison of controller output using conventional SMC strategy and improved SMC strategy in highway scenes, corresponding to Fig.14 (a) and Fig.14 (b).

Take the first 100 s controller output of “car1” and “car20” for comparison. In the highway scene, the control output of conventional SMC strategy is also discontinuous, and there is obvious chatter. Improved SMC strategy also eliminates chatter at high velocity.

3) COMPARISON OF VELOCITY VARIATION UNDER EACH CONTROL STRATEGY

In Fig.15, Fig.16, and Fig.17, the velocity variation of vehicles in the platoon under no control strategy, conventional SMC strategy, and improved SMC strategy in highway scene is shown, respectively.

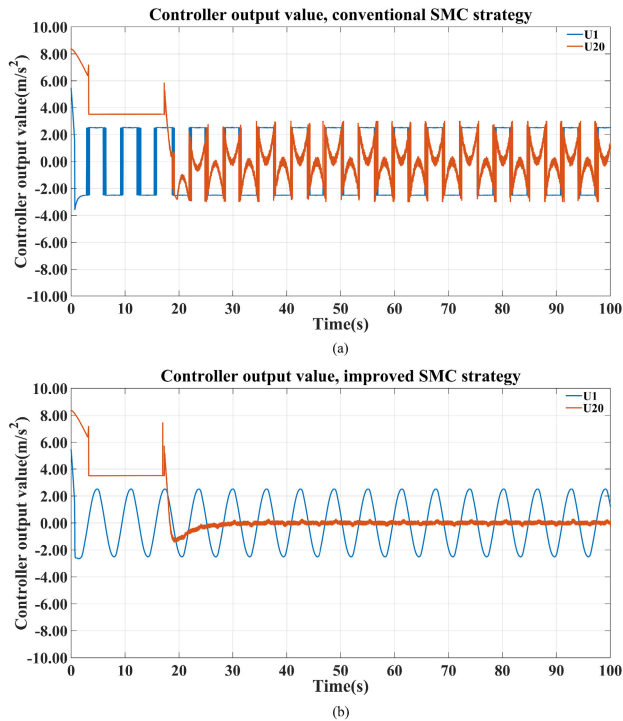


FIGURE 14. Comparison of controller output value under each control strategy in the highway scene.

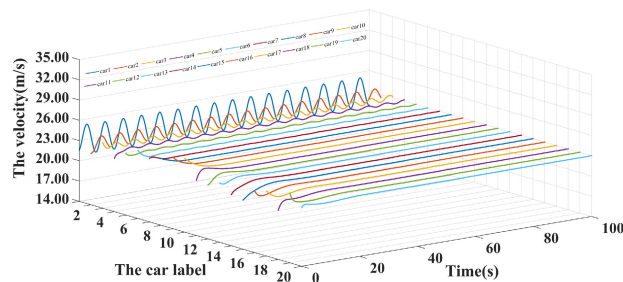


FIGURE 15. Velocity variation of vehicles in the platoon under no control strategy in the highway scene.

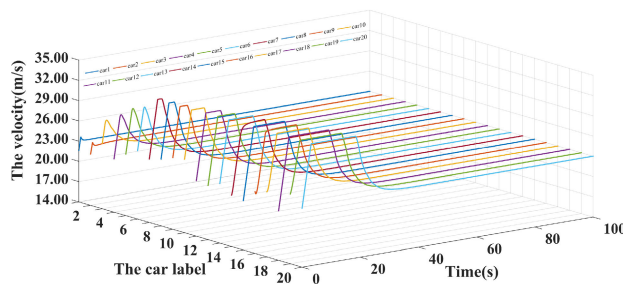


FIGURE 16. Velocity variation of vehicles in the platoon under conventional SMC strategy in the highway scene.

The ideal velocity is set to 23.00 m/s. When no control strategy is adopted, the velocity of all vehicles fluctuates steadily from about 20 s. And the vehicles at the front of

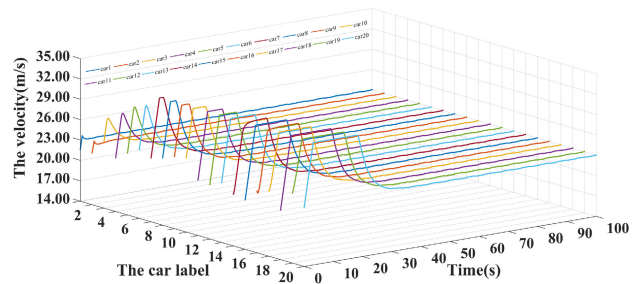


FIGURE 17. Velocity variation of vehicles in the platoon under improved SMC strategy in the highway scene.

the platoon are disturbed by a sine wave. By combining the vehicle distance curve, it can be seen that the vehicle platoon is not formed at this time. Under the conventional SMC strategy and the improved SMC strategy, the time for the velocity of the entire platoon to stabilize is about 40 s. During the velocity adjustment of the platoon, the maximum velocity is reached, the overall velocity curve is stable, and the sine wave disturbance is eliminated.

4) COMPARISON OF ACCELERATION VARIATION UNDER EACH CONTROL STRATEGY

Figure 18, Fig.19, and Fig.20 show the acceleration variation under no control strategy, conventional SMC strategy, and improved SMC strategy in the highway scene, respectively.

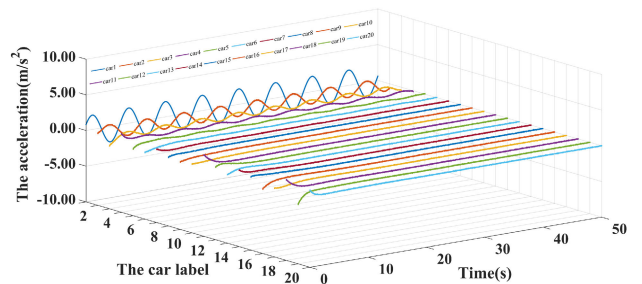


FIGURE 18. Acceleration variation of vehicles in the platoon under no control strategy in the highway scene.

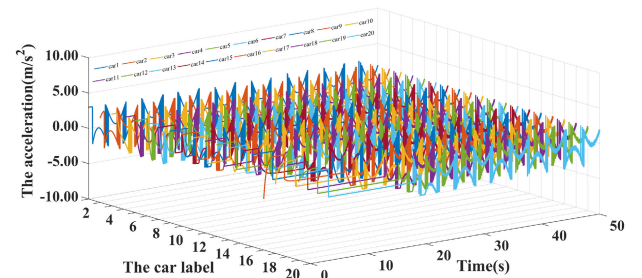


FIGURE 19. Acceleration variation of vehicles in the platoon under conventional SMC strategy in the highway scene.

With no control strategy being adopted, the acceleration of all vehicles stabilizes at 30 s and fluctuates around 0 m/s².

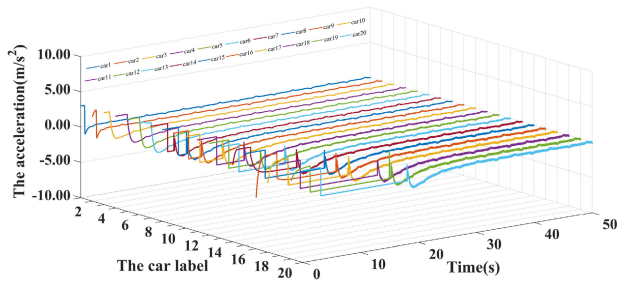


FIGURE 20. Acceleration variation of vehicles in the platoon under improved SMC strategy in the highway scene.

The vehicles at the front of the platoon are disturbed by a sine wave. Under the conventional SMC strategy, it is the same as that shown in the low-velocity state, after the acceleration of the entire platoon stabilizes, a chattering phenomenon similar to the output of the controller appears. The acceleration under conventional SMC strategy is discontinuous and fluctuant. Under the improved SMC strategy, the acceleration of all vehicles in the platoon converges to 0 m/s^2 and remains stable within 30 s. During the forming process of the platoon, the acceleration of the vehicles changes continuously and the acceleration process is stable. Under the action of the velocity limit module, the absolute values of the acceleration of the two control strategies are kept within 3.00 m/s^2 .

VI. EFFECT ANALYSIS OF CONTROL STRATEGIES

In all cases, the hypothetical vehicle (“car0”) moves in a straight line at a uniform velocity, so when the headways between all vehicles reach the desired value, it can be concluded that the platoon is already in the desired ideal state. What can be judged in the previous sections is that both the conventional SMC strategy and improved SMC strategy can ensure that the platoon reaches the ideal state in a short time. To visually observe the platoon control efficiency of improved SMC strategy, the ideal platoon trajectory coordinates and the actual platoon trajectory coordinates under improved SMC strategy in scene B are plotted.

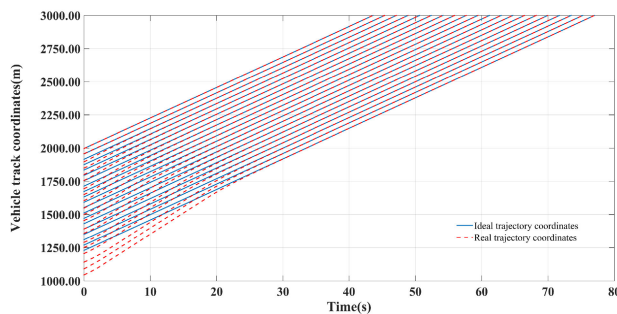


FIGURE 21. The ideal platoon trajectory coordinates and the actual platoon trajectory coordinates under improved SMC strategy.

As shown in Fig.21, the solid blue line is the ideal vehicle trajectory (all vehicles have formed a platoon at the initial

moment, and they are driving at the ideal velocity and ideal headway), the red dotted line is the actual vehicle trajectory under improved SMC strategy. It can be seen that the forming process of the proposed strategy is smooth and rapid. When the initial vehicle headways are not equal, these trajectories gradually become parallel and equidistant straight lines within 30s, which coincides with the ideal trajectory.

To illustrate the superiority of improved SMC strategy more intuitively, vehicles marked as “car1,” “car10,” and “car20” in scene B were selected, their trajectory errors and coefficient of variation of their acceleration under different strategies were compared.

TABLE 3. The average error between the actual trajectory and the ideal trajectory of the vehicle(m).

Strategy	Car1	Car10	Car20
No control strategy	-1.8250	77.5094	185.1636
Conventional SMC	5.5347×10^{-4}	1.0357	4.3954
Improved SMC	4.3757×10^{-4}	1.0390	4.4010

As shown in Table 3, when the control strategy is not adopted, the error value is far greater than that when the strategy is adopted. The trajectory error values of the two control strategies are the same, and both are very small. It can be seen that the trajectory error from the front to the back of the platoon shows an increasing trend. This is because the initial headway is greater than the ideal value, which can also be observed from Fig.21.

TABLE 4. Standard deviation of vehicle acceleration(m/s^2).

Strategy	Car1	Car10	Car20
No control strategy	1.5153	0.0648	0.0612
Conventional SMC	1.2433	1.2526	1.2532
Improved SMC	0.1202	0.2772	0.3467

In the scene designed in this study, we limited the upper and lower limits of acceleration. However, under the influence of the chattering, the acceleration may still jump from the threshold boundary to another threshold boundary, and this situation may occur continuously. For example, the acceleration under the conventional SMC strategy is in this transition state. To confirm whether the improved SMC strategy guarantees the continuity of acceleration, we extracted the acceleration data during the 500 s of these vehicles, and calculated the standard deviation of these accelerations under each strategy, as shown in Table 4. Standard deviation can reflect the degree of dispersion of a data set. These standard deviations can reflect two meanings. On the one hand, a large standard deviation value may indicate that the sine wave disturbance has not been eliminated. On the other hand, a larger standard

deviation value means that the data is more discrete and the acceleration changes too frequently and discontinuously.

As shown in Table 4, when no control strategy is adopted, because the sine wave disturbance is not eliminated, the acceleration standard deviation of “car1” is large. Combined with the acceleration image in scene B, the sine wave disturbance of “car10” and “car20” is weakened due to the queue transfer. And because they did not line up successfully, the acceleration adjustment was very small. So their acceleration standard deviation was very small.

When the conventional SMC strategy is adopted, the acceleration standard deviation values of “car1,” “car10,” and “car20” are similar. And there is no decreasing trend, indicating that the sine wave disturbance has been eliminated. Because the acceleration of all vehicles is in a high-frequency change state, the standard deviation of acceleration is large. Combining images can also confirm these conclusions.

When the improved SMC strategy is adopted, the acceleration standard deviation values of “car1,” “car10,” and “car20” are similar and small, and there is no downward trend. This shows that the sine wave disturbance has been eliminated and the continuity of the acceleration is good. It is proved that the proposed improved SMC strategy eliminates the high frequency switching of acceleration in the conventional SMC strategy under the premise of achieving efficient control.

VII. CONCLUSIONS

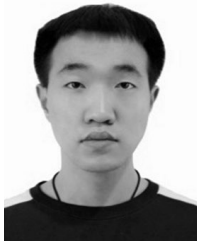
A vehicle platoon control strategy in the simulated environment is proposed. This study designed numerical experiments to verify the superiority of the proposed strategy. The results show that the effect of the improved sliding-mode control strategy is better than other strategies. Applying this strategy can achieve the formation and stable driving of vehicle platoon, while maintaining reasonable velocity and acceleration. This strategy can improve the efficiency of transportation, and ensure the safety and comfort of the drivers.

Due to the lack of a real autonomous vehicle experimental site, the verification method is only limited to numerical simulation. In future research, we hope to make more improvements through the verification of real scene or driving simulators to verify the practicality. Besides, the driving environment of the vehicle platoon is complex and changeable in the actual traffic scene, so the application of the method proposed in this study in the multi-lane scene or intersections should be discussed further, and the application of other sliding-mode control methods in vehicle platoon should also be considered.

REFERENCES

- [1] V. Turri, B. Besselink, and K. H. Johansson, “Cooperative look-ahead control for fuel-efficient and safe heavy-duty vehicle platooning,” *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 1, pp. 12–28, Jan. 2017.
- [2] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, “Dynamical model of traffic congestion and numerical simulation,” *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 51, no. 2, pp. 1035–1042, Feb. 1995.
- [3] T. Nagatani, “Density waves in traffic flow,” *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 61, no. 4, pp. 3564–3570, Apr. 2000.
- [4] D. Helbing and B. Tilch, “Generalized force model of traffic dynamics,” *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 58, no. 1, pp. 133–138, Jul. 1998.
- [5] R. Jiang, Q. Wu, and Z. Zhu, “Full velocity difference model for a car-following theory,” *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 64, no. 1, Jun. 2001, Art. no. 017101.
- [6] T. Wang, Z. Gao, and X. Zhao, “Multiple velocity difference model and its stability analysis,” *Acta Phys. Sinica*, vol. 55, no. 5, pp. 634–638, 2006.
- [7] Y. Wu, B. Jiang, and N. Lu, “A descriptor system approach for estimation of incipient faults with application to high-speed railway traction devices,” *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 49, no. 10, pp. 2108–2118, Oct. 2019.
- [8] Y. Wu, B. Jiang, and Y. Wang, “Incipient winding fault detection and diagnosis for squirrel-cage induction motors equipped on CRH trains,” *ISA Trans.*, vol. 99, pp. 488–495, Apr. 2020, doi: [10.1016/j.isatra.2019.09.020](https://doi.org/10.1016/j.isatra.2019.09.020).
- [9] S. Ding, W.-H. Chen, K. Mei, and D. J. Murray-Smith, “Disturbance observer design for nonlinear systems represented by input-output models,” *IEEE Trans. Ind. Electron.*, vol. 67, no. 2, pp. 1222–1232, Feb. 2020.
- [10] L. He, C. Yang, and J. Wang, “Simulation research on vehicle stability based on sliding mode variable structure control,” in *Proc. 3rd Int. Conf. Mechatronics Eng. Inf. Technol. (ICMEIT)*, Dalian, China, Mar. 2019, pp. 1–8.
- [11] F. Gao, B. Liu, J. Qi, and C. Wang, “Distributed sliding mode control for formation of multiple nonlinear AVs coupled by uncertain topology,” *Social Netw. Appl. Sci.*, vol. 1, no. 4, pp. 374–378, Apr. 2019.
- [12] X. Lu, X. Zhang, G. Zhang, J. Fan, and S. Jia, “Neural network adaptive sliding mode control for omnidirectional vehicle with uncertainties,” *ISA Trans.*, vol. 86, pp. 201–214, Mar. 2019.
- [13] Z. Sun, G. Zhang, Y. Lu, and W. Zhang, “Leader-follower formation control of underactuated surface vehicles based on sliding mode control and parameter estimation,” *ISA Trans.*, vol. 72, pp. 15–24, Jan. 2018.
- [14] J.-W. Kwon and D. Chwa, “Adaptive bidirectional platoon control using a coupled sliding mode control method,” *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 5, pp. 2040–2048, Oct. 2014.
- [15] M. Yan, J. Song, L. Zuo, and P. Yang, “Neural adaptive sliding-mode control of a vehicle platoon using output feedback,” *Energies*, vol. 10, no. 11, p. 1906, 2017.
- [16] Y. Li, Y. Kang, B. Yang, S. Peeta, L. Zhang, T. Zheng, and Y. Li, “A sliding mode controller for vehicular traffic flow,” *Phys. A, Stat. Mech. Appl.*, vol. 462, no. 15, pp. 38–47, Nov. 2016.
- [17] K. Konishi, H. Kokame, and K. Hirata, “Delayed-feedback control of an optimal velocity traffic model,” in *Proc. Control Oscillations Chaos*, Saint Petersburg, Russia, Oct. 2000, pp. 1–4.
- [18] X. Zhao and Z. Gao, “Controlling traffic jams by a feedback signal,” *Eur. Phys. J. B*, vol. 43, no. 4, pp. 565–572, Feb. 2005.
- [19] Y. Li, D. Sun, and W. Liu, “Feedback control of traffic jam based on the full velocity difference car-following model,” *J. Inf. Comput. Sci.*, vol. 9, no. 3, pp. 719–730, 2012.
- [20] V. Utkin, “Variable structure systems with sliding modes,” *IEEE Trans. Autom. Control*, vol. 22, no. 2, pp. 212–222, Apr. 1977.
- [21] K. D. Young, V. I. Utkin, and U. Ozguner, “A control engineer’s guide to sliding mode control,” *IEEE Trans. Control Syst. Technol.*, vol. 7, no. 3, pp. 328–342, May 1999.
- [22] K. Erbatun and A. Kawamura, “Chattering elimination via fuzzy boundary layer tuning,” in *Proc. Conf. IEEE Ind. Electron. Soc.*, Seville, Spain, Nov. 2002, pp. 2131–2136.
- [23] L. Jinkun and E. Lianjie, “Sliding mode controller design for position and speed control of flight simulator servo system with large friction,” *J. Syst. Eng. Electron.*, vol. 14, no. 3, pp. 59–62, 2003.
- [24] J.-K. Liu and F.-C. Sun, “Research and development on theory and algorithms of sliding mode control,” *Control Theory Appl.*, vol. 24, no. 3, pp. 407–418, 2016.
- [25] K. Mei and S. Ding, “Second-order sliding mode controller design subject to an upper-triangular structure,” *IEEE Trans. Syst., Man, Cybern. Syst.*, early access, Nov. 5, 2018, doi: [10.1109/TSMC.2018.2875267](https://doi.org/10.1109/TSMC.2018.2875267).
- [26] L. Jinkun, *Sliding Mode Control Design and MATLAB Simulation, the Basic Theory and Design Method*, 3rd ed. Beijing, China: Tsinghua Press, 2015, p. 39.
- [27] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ, USA: Prentice-Hall, 1996, pp. 1574–1578.

- [28] A. Petros and L. Jinkun, *Robust Adaptive Control*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1996, p. 24.
- [29] S. Ding, J. H. Park, and C.-C. Chen, "Second-order sliding mode controller design with output constraint," *Automatica*, vol. 112, Feb. 2020, Art. no. 108704.



BO PENG received the bachelor's degree in measurement and control technology and instruments from Jilin University, Changchun, China, in 2017, where he is currently pursuing the Ph.D. degree with the College of Transportation. His research interest includes intelligent transportation systems.



DEXIN YU received the Ph.D. degree in traffic information engineering and control from Jilin University, Jilin, China, in 2006. He is currently a Professor with the Transportation College, Jilin University. He has more than ten articles and seven patents in recent years. His research interests include traffic information processing and traffic system analysis. As the person in charge of the project, he has presided over and completed more than 20 national, provincial and ministerial and engineering industrialization projects in China, including six national high-tech research plans (863 plan) and one national natural fund project. As the main finisher, he won two second prizes for national scientific and technological progress, and three provincial and ministerial science and technology awards.



HUXING ZHOU received the Ph.D. degree in traffic information engineering and control from Jilin University, Jilin, China, in 2013. He is currently a Lecturer with the School of Transportation, Jilin University. He has more than eight articles and three patents in recent years. His research interests include intelligent transportation systems and the Internet of Vehicles. He has presided over one basic scientific research project of the Jilin Education Department Science and Technology Project, and participated in more than ten projects, including the National 863 Program, the National Natural Science Foundation, and the Science and Technology Development Project of Jilin Province.



XUE XIAO received the bachelor's degree in traffic transportation from Shenyang Jianzhu University, Shenyang, China, in 2017. She is currently pursuing the master's degree with the College of Transportation, Jilin University. Her research interest includes intelligent parking systems.



YUNFENG FANG received the bachelor's degree in traffic engineering from Jilin University, Changchun, China, in 2017, where he is currently pursuing the master's degree with the College of Transportation. His research interest includes intelligent transportation systems.

...