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PIO Output Fault Diagnosis by ARX-Laguerre Model Applied to 2^{nd} Order Electrical System

CHAKIB BEN NJIMA[1] AND TAREK GARNA[1,2]
[1]National Engineering School of Monastir, University of Monastir, Monastir 5000, Tunisia
[2]Higher Institute of Applied Science and Technology of Sousse, University of Sousse, Sousse 4003, Tunisia

Corresponding author: Chakib Ben Njima (chakib.bennjima@gmail.com)

ABSTRACT The novelty of this work consists in the synthesis of a new structure of proportional-integral observer (PIO) reformulated from the new linear ARX-Laguerre representation with filters on system input and output. This is in order to estimate the unknown outputs presented as faults and to detect the time instant corresponding to the system malfunction. The stability and the convergence properties of the proposed PIO are ensured by using Linear Matrix Inequality. Furthermore an optimal identification of both Laguerre poles is achieved by a genetic algorithm approach where a parametric significant reduction is ensured to guarantee a reduced observer. The performances of the identification approach and the resulting PIO are tested on an experimental 2^{nd} order electrical system.

INDEX TERMS ARX-Laguerre model, genetic algorithm, proportional-integral observer.

I. INTRODUCTION

Observer-based fault diagnosis is still current key research focus and shows great potentials in the area of model-based fault diagnosis. One of the current research axes is the design of observer associated to the process with unknown inputs entitled Unknown Input Observer (UIO). In this case, the sensors and actuator faults can be considered as unknown inputs as well which are not available from measurements. Therefore, the UIO received great attention in the recent years by using geometric theory to decouple the effects from the unknown inputs [5], [19] or by exploiting the sliding mode approach for the unknown input observer [7], [9] in order to the state estimation as well as the fault. In this context, Proportional-Integral observer (PIO) is considered as an UIO with a simple structure. This last is developed and amply applied by many researchers in linear case by using a high-gain observer associated to the both terms proportional and integral applied to the output estimation error. In fact, the PIO allows to have an unbiased estimate of the state as well as the detection and estimation of unknown inputs despite the presence of uncertainties or unknown inputs [6], [8], [20]. The PIO requires knowledge of the suitable model for representing the dynamic behavior of the system. As long as the model is characterized by a reduced parametric complexity we obtain a reduced observer with a simple structure. This parametric reduction has practical and technical benefits for the reduced-order observer design for large-scale linear systems with high dimension and complexity. In the practical case the systems are generally of a nonlinear and complex type which are mainly represented by a complex and nonlinear differential equations linearized under certain linearization assumptions. The aim of this linearization is to work in the vicinity of an operating point. However, these hypotheses can degrade the performances of the obtained complex linear model as well as that of the developed PI observer for the fault diagnosis in the vicinity of the operating point. Therefore, it is important to obtain a linear model ensuring faithfully the representation of the system, even if we work around an operating point, and also that it is characterized by a significant reduction in parametric complexity. This result also allows the design of observer-based reduced order controllers. Consequently, in order to circumvent the differential equations or the knowledge of the system's state space representation and moreover in order to satisfy the compromise between parametric complexity reduction and the representation of complex linear systems we propose in this work to exploit the discrete new linear representation entitled ARX-Laguerre model developed by expanding the SISO ARX model on two Laguerre bases. We note that Laguerre functions have been used by filtering the process input and output. Based on the Laguerre functions properties [14], [17]

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the resulting ARX-Laguerre model is not sensitive to the sampling rate choice. Furthermore, for model development and compared to the traditional ARX model, no explicit knowledge is required for the system time constant and time delay. Also, the ARX-Laguerre model allows an easy representation and a good approximation of complex linear system. In fact, the proof of the parametric reduction of ARX-Laguerre model is presented in Bouzrara *et al.* [3], [4] where the expansion parsimony is highly linked to the poles choice defining both Laguerre bases. Thus we exploit the genetic algorithm [10]–[12] in this paper to present a Laguerre pole optimization algorithm is proposed. The optimum values are determined automatically in the genetic evolutionary process. Such a process is one of the stochastic optimization algorithms by expressing the complex structure problems with its hierarchy and can determine the feasible solving space automatically without giving any superstructure. Furthermore, the identification of the ARX-Laguerre model only requires a set of input/ouput measurements while eliminating a theoretical study of the system to reach the state space representation. For consequence, those performances are powerful tools for an attractive modeling framework to the fault diagnosis methods development which are utilized for fault detection/identification. The most important issue in model-based fault detection and identification (FDI) concerns the accuracy and the simplicity of the model describing the behavior of the monitored system. In this case, a recent work is presented in [16] allocated to the detection and identification of input faults by developing a proportional-integral observer based on ARX-Laguerre model. The major advantage lay on the one hand in the synthesis of a reduced observer and on the other hand in the effectiveness of the proposed PIO diagnosis with respect to the unknown input as fault. However, like to the input fault, in the practical case the output of the system can also suffer from the presence of an unknown fault. Consequently, we exploit the principle of filtering the output residing in the ARX-Laguerre model in order to extend the work of Najeh *et al.* [16]. Indeed, filtering of the output makes it possible to develop an augmented system with output fault based on the ARX-Laguerre model. Taking into account the obtained configuration, we propose a PI observer for the detection/ identification of faults at the output. Furthermore, in a similar way to [16] we present the optimization of the observer's gains using linear matrix inequality. This latter allows to ensure stability and convergence properties of the proposed PIO. Therefore, this work focuses on the detection and estimation of the sensor faults, modeled as unknown outputs of the system, by considering the design and the development of model-based FDI scheme from the ARX-Laguerre model. The key issue of the proposed diagnosis method is to introduce a new structure design of a PIO by using the recursive representation of the ARX-Laguerre model and the augmented system with output fault. In this case, the proposed ARX-Laguerre PI observation approach is exploited to modify the traditional PI observer by applying ARX-Laguerre model. Then, the PIO exploits the input/output measurements to reconstruct only the Laguerre filter outputs without estimate the states of the system. The purpose of this note is to extend the principle of the PIO used in the linear system framework [1], [18] to the ARX-Laguerre model [2] not to the knowledge of the state but only to set up a new FDI scheme. This latter allows to generate fault indicators (residuals) to the faults detection by specifying the time instant of appearance in which the unknown outputs are estimated simultaneously. In this case, sufficient conditions of the proposed PIO for ensuring stability and the estimation error convergence are given in LMI form. Besides we note that when the parametric dimension of ARX-Laguerre model is less than the order of the system we get a reduced observer which may have better properties than a full-order observer to develop a control algorithm in an observer-based design [15], [18]. The main contributions of this paper are mainly fourfold. 1) We present a genetic algorithm as in [16] to optimize Laguerre poles to ensure the parameter number reduction. 2) We develop a new structure design of a PIO based on the new linear representation ARX-Laguerre model and if the parametric reduction is less than the proposed observer is a reduced observer. 3) We present a LMI formulation in order to guarantee the stability and the estimation error convergence of the proposed PIO as well as to compute the observer's gains. 4) We present a residual generation structure in order to propose an algorithm detecting and estimating output faults for the new FDI scheme.

The paper is organized as follows. The section 2, will be dedicated to the ARX-Laguerre model presentation with its simple and recursive representation. The section 3 will detail the method for Laguerre pole optimization by exploiting the genetic algorithm. Section 4 illustrates the designed of PIO output fault diagnosis for output fault identification and estimation characterizing the proposed FDI scheme. In this case, the development of a PIO and a LMI formulation is achieved in order to present the PIO output fault diagnosis algorithm. Finally, section 5 evaluates from an experimental application on a 2^{nd} order electrical system, the performances and the efficiency of the Laguerre pole optimization and the proposed PIO diagnosis.

II. ARX-LAGUERRE MODEL

The new linear ARX-Laguerre model results from the ARX model decomposition on orthonormal and independent Laguerre bases defined as follows [4]:

$$y(k) = \sum_{n=0}^{Na-1} g_{n,a} x_{n,y}(k) + \sum_{n=0}^{Nb-1} g_{n,b} x_{n,u}(k) \tag{1}$$

characterized by the parameter number:

$$M = Na + Nb \tag{2}$$

where $g_{n,a}$, $g_{n,b}$ are the Fourier coefficients with Na and Nb the truncating orders and $x_{n,y}(k)$ and $x_{n,u}(k)$ are respectively the filtered output and the filtered input by Laguerre functions:

$$x_{n,y}(k) = \sum_{j=1}^{\infty} \ell_n^a(j, \xi_a) y(k-j) = \ell_n^a(k, \xi_a) * y(k) \quad (3)$$

$$x_{n,u}(k) = \sum_{j=1}^{\infty} \ell_n^b(j, \xi_b) u(k-j) = \ell_n^b(k, \xi_b) * u(k) \quad (4)$$

such as $y(k)$ and $u(k)$ are respectively the system's input and output and $*$ is the convolution product where $\ell_n^a(j, \xi_a)$ and $\ell_n^b(j, \xi_b)$ are the Laguerre orthonormal functions which are given by their Z-transform:

$$L_n^i(z) = \frac{\sqrt{1-\xi_i^2}}{z-\xi_i}\left(\frac{1-\xi_i z}{z-\xi_i}\right)^n, \quad i = a, b \text{ and } n = 0, 1, 2, \ldots \quad (5)$$

with Laguerre poles $\xi_i (|\xi_i| < 1)$, $i = a, b$. Furthermore, the ARX-Laguerre model can be represented by the following recursive representation:

$$\begin{cases} X(k) = A\, X(k-1) + b_y\, y(k-1) + b_u\, u(k-1) \\ y(k) = c^T\, X(k) \end{cases} \quad (6)$$

where:

✓ $X(k)$ is a vector with the filtered output/ input.

$$X(k) = \left[X_a^T(k), X_b^T(k)\right]^T \in \Re^M \quad (7)$$

with:

$$\begin{cases} X_a(k) = \left[x_{0,y}(k), \ \ldots, \ x_{Na-1,y}(k)\right]^T \in \Re^{Na} \\ X_b(k) = \left[x_{0,u}(k), \ \ldots, \ x_{Nb-1,u}(k)\right]^T \in \Re^{Nb} \end{cases} \quad (8)$$

✓ A is a square matrix:

$$A = \begin{pmatrix} A_y & 0_{Na,Nb} \\ 0_{Nb,Na} & A_u \end{pmatrix} \in \Re^{M\times M} \quad (9)$$

with $0_{i,j}$ a null matrix of dimension $(i \times j)$ and A_y and A_u are two square matrices (10) and (11), as shown at the bottom of the next page.

✓ $b_y \in \Re^M$ and $b_u \in \Re^M$ are two column vectors:

$$b_y = \begin{bmatrix} b_a \\ 0_{Nb,1} \end{bmatrix} \in \Re^M, \quad b_u = \begin{bmatrix} 0_{Na,1} \\ b_b \end{bmatrix} \in \Re^M \quad (12)$$

with b_a and b_b two column vectors with dimension Na and Nb respectively:

$$b_i = \sqrt{1-\xi_i^2}\begin{bmatrix} 1 \\ -\xi_i \\ (-\xi_i)^2 \\ \vdots \\ (-\xi_i)^{Ni-1} \end{bmatrix}, \quad i = a, b \quad (13)$$

✓ c is the Fourier coefficients parameters vector:

$$c = \left[c_a^T, c_b^T\right]^T \in \Re^M \quad (14)$$

such as:

$$\begin{cases} c_a = \left[g_{0,a}, \ \ldots, \ g_{Na-1,a}\right]^T \in \Re^{Na} \\ c_b = \left[g_{0,b}, \ \ldots, \ g_{Nb-1,b}\right]^T \in \Re^{Nb} \end{cases} \quad (15)$$

III. GENETIC ALGORITHM FOR LAGUERRE POLE IDENTIFICATION

An optimal identification of Laguerre poles ξ_a and ξ_b guarantees an important reduction of the parameter number M [4]. A nonlinear optimization problem could be applied to the Laguerre poles identification such as the GA. This latter is an evolutionary optimization method [10]–[12] which is applied to a large class of optimization problems. Infact, initial population of N_{ind} individuals is created by involving probabilistic steps. In order to converge to the optimal solution of the optimization problem, this initial population is treated using some genetic operations. Then, by minimizing an objective function known as *fitness*, the Laguerre poles are identified. In our case we consider the Normalized Mean Square Error (NMSE) computing the quadratic error between the ARX–Laguerre model output $y(k)$ and the measured output $y_m(k)$ as follows:

$$NMSE(\underline{\xi}) = \sum_{k=1}^{H}\left[y_m(k) - c^T X(k)\right]^2 \Big/ \sum_{k=1}^{H}[y_m(k)]^2 \quad (16)$$

where H is a measurement window and

$$\underline{\xi} = [\xi_a, \xi_b] \quad (17)$$

Therefore, we present as follows the GA for Laguerre poles optimization:

ALGORITHM 1: LAGUERRE POLES OPTIMIZATION BY GENETIC ALGORITHM

1. We consider H couples $(u(k), y_m(k))$ of input/output.
2. We fix the truncating orders Na and Nb.
3. Specify a threshold ε as a prespecified stopping criterion with the genetic parameters P_c and P_m as the crossover rate and the mutation probability respectively.
4. We generate randomly N_{ind} initial values of Laguerre poles $\underline{\xi}_i = \left[\xi_a^i, \xi_b^i\right], i = 1, \ldots, N_{ind}$.
5. Evaluation phase:
   (a) For each $\underline{\xi}_i = \left[\xi_a^i, \xi_b^i\right], i = 1, \ldots, N_{ind}$: compute the Fourier coefficients by RLS method and also the fitness $NMSE(\underline{\xi}_i)$.
   (b) Obtain the minimal fitness
   $$NMSE_{\min} = \min_{i=1,\ldots,N_{\text{ind}}}\left(NMSE(\underline{\xi}_i)\right)$$
6. Selection phase: Select solutions after evaluation phase according to the fitness.
7. Crossover and Mutation phase: Apply crossover and mutation to the selected solutions.
8. Evaluation phase: Determine the fitness $NMSE$ as in 5 according to the new generated solutions.
9. If $NMSE_{\min} \leq \varepsilon$, stop the algorithm. Else, return to Step 6.

IV. PIO OUTPUT FAULT DIAGNOSIS

A. THE PROPOSED PI OBSERVER

Taking into account (8) and (15), the recursive representation (6) can be written in this new decomposed vector representation with respect to $y(k)$ and $u(k)$:

$$\begin{cases} X_a(k) = A_y\, X_a(k-1) + b_a\, y(k-1) \\ X_b(k) = A_u\, X_b(k-1) + b_b\, u(k-1) \\ y_a(k) = c_a^T\, X_a(k) \\ y_b(k) = c_b^T\, X_b(k) \\ y(k) = y_a(k) + y_b(k) \end{cases} \tag{18}$$

Thus, the discrete-time ARX-Laguerre filter network can be represented by Figure 1.

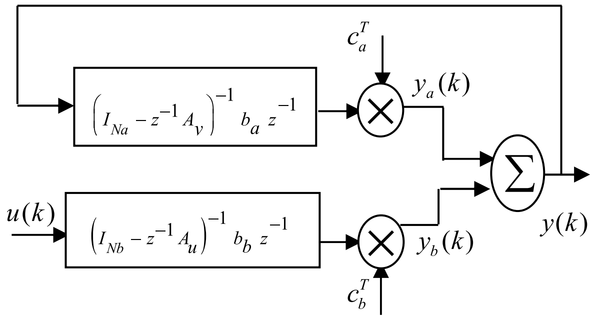

FIGURE 1. Decomposed discrete-time ARX-Laguerre filter network with respect to $y(k)$ and $u(k)$.

In the following, we neglect the noise applied to the system and the measured output system y_m presents an additive fault V (For example, a sensor fault) i.e. $y + V$. This is illustrated by this configuration:

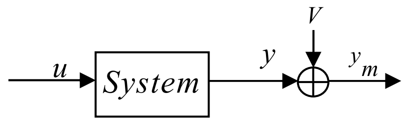

FIGURE 2. Representation with output fault.

Considering the ARX-Laguerre model (18) and the fault $V(k)$ the configuration of the Figure 1 is modified as follows:

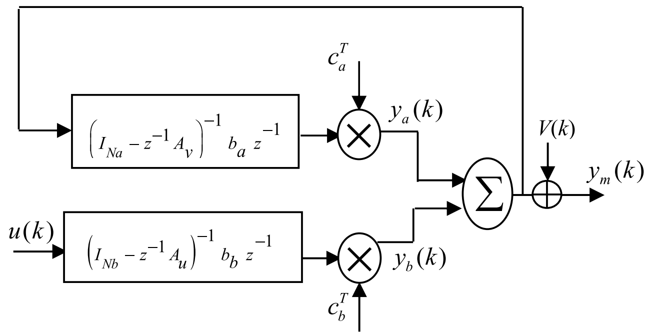

FIGURE 3. Representation with fault with respect to the system output.

According to the configurations of Figures 1 and 3 we see the filtering of the input $u(k)$ and the output $y(k)$. So we propose to extend this filtering principle to the output $y(k)$ with fault $V(k)$ i.e. the filtering of $y_m(k) = y(k) + V(k)$ instead of $y(k)$. In this case, we propose an augmented system with output fault based on the ARX-Laguerre model. Taking into account the configuration of Figure 3, the proposed augmented system can be illustrated by this configuration:

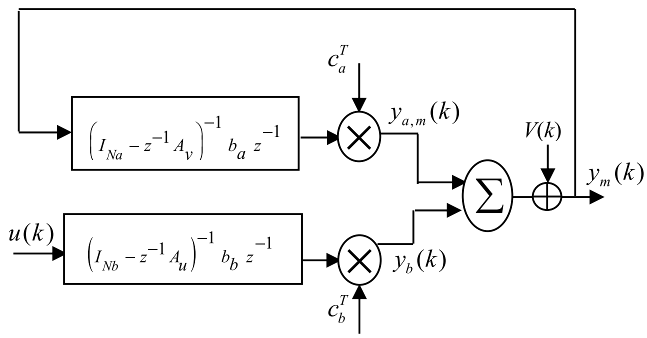

FIGURE 4. Proposed augmented system with output fault based on the ARX-Laguerre model.

In this case, taking account of Figure 4 and the representation (18) of ARX-Laguerre model we deduce the following augmented system with output fault $V(k)$:

$$\begin{cases} X_{a,m}(k) = A_y\, X_{a,m}(k-1) + b_a\, y_m(k-1) \\ X_b(k) = A_u\, X_b(k-1) + b_b\, u(k-1) \\ y_{a,m}(k) = c_a^T\, X_{a,m}(k) \\ y_b(k) = c_b^T\, X_b(k) \\ y_m(k) = y_{a,m}\,(k) + y_b(k) + V(k) \end{cases} \tag{19}$$

$$A_y = \begin{bmatrix} \xi_a & 0 & 0 & \cdots & 0 \\ 1-\xi_a^2 & \xi_a & 0 & \cdots & 0 \\ -\xi_a(1-\xi_a^2) & 1-\xi_a^2 & \xi_a & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-\xi_a)^{Na-2}(1-\xi_a^2) & (-\xi_a)^{Na-3}(1-\xi_a^2) & (-\xi_a)^{Na-4}(1-\xi_a^2) & \cdots & \xi_a \end{bmatrix} \in \Re^{Na\times Na} \tag{10}$$

$$A_u = = \begin{bmatrix} \xi_b & 0 & 0 & \cdots & 0 \\ 1-\xi_b^2 & \xi_b & 0 & \cdots & 0 \\ -\xi_b(1-\xi_b^2) & 1-\xi_b^2 & \xi_b & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-\xi_b)^{Nb-2}(1-\xi_b^2) & (-\xi_b)^{Nb-3}(1-\xi_b^2) & (-\xi_b)^{Nb-4}(1-\xi_b^2) & \cdots & \xi_b \end{bmatrix} \in \Re^{Nb\times Nb} \tag{11}$$

or:

$$\begin{cases} X_{a,m}(k) = \left(A_y + b_a c_a^T\right) X_{a,m}(k-1) + b_a c_b^T X_b(k-1) \\ \qquad + b_a V(k-1) \\ X_b(k) = A_u X_b(k-1) + b_b u(k-1) \\ y_{a,m}(k) = c_a^T X_{a,m}(k) \\ y_b(k) = c_b^T X_b(k) \\ y_m(k) = y_{a,m}(k) + y_b(k) + V(k) \end{cases} \tag{20}$$

such as:

$$X_{a,m}(k) = \left[x_{0,y_m}(k), \ \ldots, \ x_{Na-1,y_m}(k) \right]^T \in \Re^{Na} \tag{21}$$

To reconstitute or estimate the augmented representation (20) with output fault, we propose the development of a proportional-integral observer based on the ARX-Laguerre model defined by (18). In this case, the representation (18) is written in this form:

where $x_{n,y_m}(k)$ is defined in (3) by replacing only $y(k)$ with $y_m(k)$.

$$\begin{cases} X_a(k) = \left(A_y + b_a c_a^T\right) X_a(k-1) + b_a c_b^T X_b(k-1) \\ X_b(k) = A_u X_b(k-1) + b_b u(k-1) \\ y_a(k) = c_a^T X_a(k) \\ y_b(k) = c_b^T X_b(k) \\ y(k) = y_a(k) + y_b(k) \end{cases} \tag{22}$$

Therefore, the proposed PIO is developed based on the vectors $X_a(k)$ and $X_b(k)$ of the relation (22) whose structure is shown in Figure 5:

$$\begin{cases} \hat{X}_a(k) = \left(A_y + b_a c_a^T\right) \hat{X}_a(k-1) + b_a c_b^T \hat{X}_b(k-1) \\ \qquad + L_a \left(y_m(k-1) - \hat{y}(k-1)\right) + b_a \hat{V}(k-1) \\ \hat{V}(k) = \hat{V}(k-1) + K_V \left(y_m(k-1) - \hat{y}(k-1)\right) \\ \hat{X}_b(k) = A_u \hat{X}_b(k-1) + b_b u(k-1) \\ \qquad + L_b \left(y_m(k-1) - \hat{y}(k-1)\right) \\ \hat{y}_b(k) = c_b^T \hat{X}_b(k) \\ \hat{y}_a(k) = c_a^T \hat{X}_a(k) \\ \hat{y}(k) = \hat{y}_a(k) + \hat{y}_b(k) + \hat{V}(k) \end{cases} \tag{23}$$

where:

✓ $\hat{X}_a(k)$ and $\hat{X}_b(k)$ are the estimates of the vectors $X_{a,m}(k)$ and $X_b(k)$ such as:

$$\begin{cases} \hat{X}_a(k) = \left[\hat{x}_{0,\hat{y}}, \ \ldots, \ \hat{x}_{Na-1,\hat{y}} \right]^T \in \Re^{Na} \\ \hat{X}_b(k) = \left[\hat{x}_{0,u}, \ \ldots, \ \hat{x}_{Nb-1,u} \right]^T \in \Re^{Nb} \end{cases} \tag{24}$$

with $\hat{x}_{n,\hat{y}}(k)$ and $\hat{x}_{n,u}(k)$ the estimated filtered output and output of $x_{n,y_m}(k)$ and $x_{n,u}(k)$ respectively.

✓ $\hat{V}(k)$ is the estimation of the unknown output fault $V(k)$.
✓ $\hat{y}_a(k)$ is the estimation of $y_{a,m}(k)$ by the PIO.
✓ $\hat{y}_b(k)$ is the estimation of $y_b(k)$ by the PIO.
✓ L_a, L_b and K_V are the PIO's gains.

B. CALCULATION OF GAINS BY LMI OPTIMIZATION

From relation (20) we consider:

✓ $X_m(k)$ a column vector:

$$X_m(k) = \left[X_{a,m}^T(k), X_b^T(k) \right]^T \in \Re^M \tag{25}$$

✓ $\underline{A}_m$ a square matrix:

$$\underline{A}_m = \begin{pmatrix} A_y + b_a c_a^T & b_a c_b^T \\ 0_{Nb,Na} & A_u \end{pmatrix} \in \Re^{M \times M} \tag{26}$$

Moreover, by exploiting (12), (13) and (14) we propose to write the measured output $y_m(k)$ from the augmented system (19), under this compact form:

$$\begin{cases} X_m(k) = \underline{A}_m X_m(k-1) + b_u u(k-1) + b_y V(k-1) \\ y_m(k) = c^T X_m(k) + V(k) \end{cases} \tag{27}$$

In this same way and from (23); the estimated output $\hat{y}(k)$ of $y_m(k)$ can be written:

$$\begin{cases} \hat{X}(k) = \underline{A}_m \hat{X}(k-1) + b_u u(k-1) \\ \qquad + F \left(y_m(k-1) - \hat{y}(k-1)\right) + b_y \hat{V}(k-1) \\ \hat{V}(k) = \hat{V}(k-1) + K_V \left(y_m(k-1) - \hat{y}(k-1)\right) \\ \hat{y}(k) = c^T \hat{X}(k) + \hat{V}(k) \end{cases} \tag{28}$$

where:

✓ $\hat{X}(k)$ and F are two column vectors:

$$\hat{X}(k) = \left[\hat{X}_a^T(k), \hat{X}_b^T(k) \right]^T \in \Re^M \tag{29}$$

$$F = \begin{pmatrix} L_a \\ L_b \end{pmatrix} \in \Re^M \tag{30}$$

We consider the state and output fault reconstruction errors $e(k)$ and $\varepsilon(k)$ respectively:

$$e(k) = X_m(k) - \hat{X}(k) \tag{31}$$

$$\varepsilon(k) = V(k) - \hat{V}(k) \tag{32}$$

By exploiting (27) and (28), the expression of $e(k)$ is deducted as follows:

$$e(k) = \left(\underline{A}_m - F c^T\right) e(k-1) + \left(b_y - F\right) \varepsilon(k-1) \tag{33}$$

If we consider that the unknown input $V(k)$ is constant or very slow dynamics over time i.e.:

$$V(k) - V(k-1) = 0 \tag{34}$$

We can deduce from the second equation of the system (28)that,

$$\varepsilon(k) = (1 - K_V)\, \varepsilon(k-1) - K_V c^T e(k-1) \tag{35}$$

Using relations (33) and (35) we can obtain:

$$\begin{pmatrix} e(k) \\ \varepsilon(k) \end{pmatrix} = \begin{pmatrix} \underline{A}_m - F c^T & b_y - F \\ -K_V c^T & 1 - K_V \end{pmatrix} \begin{pmatrix} e(k-1) \\ \varepsilon(k-1) \end{pmatrix} \tag{36}$$

According to (33) and (35), we propose to consider the global reconstruction error $e_\varepsilon(k)$:

$$e_\varepsilon(k) = \begin{pmatrix} e(k) \\ \varepsilon(k) \end{pmatrix} \tag{37}$$

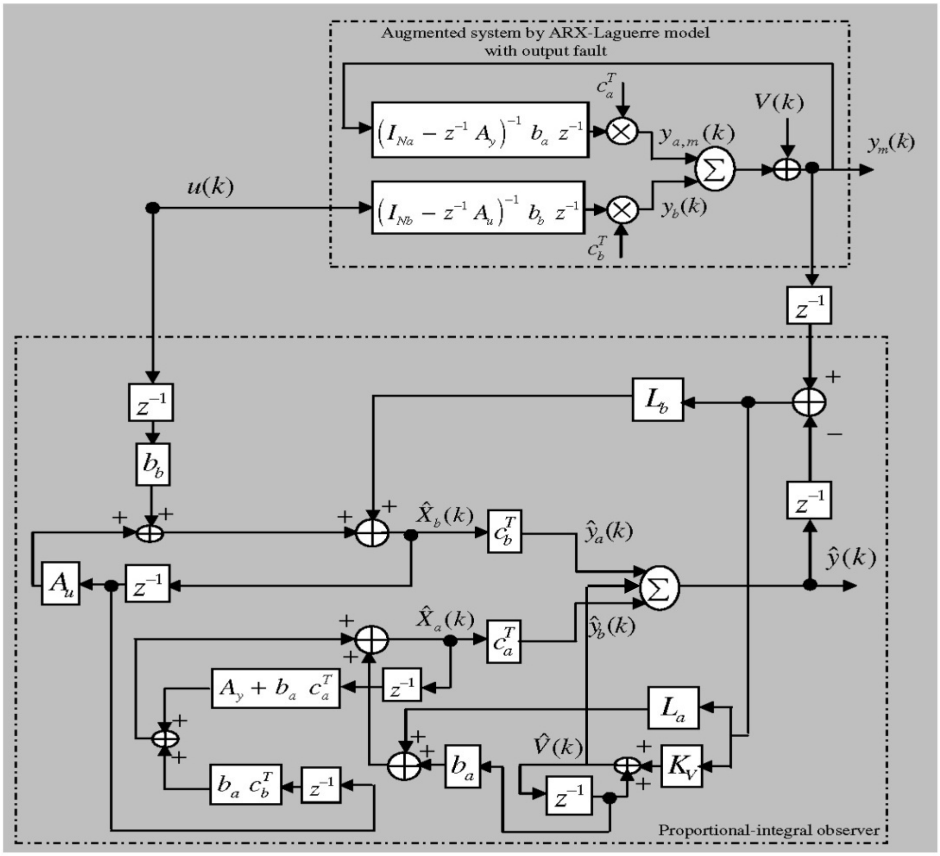

FIGURE 5. Structure of PIO based on the ARX-Laguerre model.

From (36) we deduce:

$$e_\varepsilon(k) = (A_\varepsilon - K_\varepsilon c_\varepsilon) e_\varepsilon(k-1) \tag{38}$$

with:

✓ A_ε a square matrix:

$$A_\varepsilon = \begin{pmatrix} A_m & b_y \\ 0_{1,M} & 1 \end{pmatrix} \in \Re^{(M+1)\times(M+1)} \tag{39}$$

✓ K_ε is a column vector:

$$K_\varepsilon = \begin{bmatrix} F \\ K_V \end{bmatrix} \in \Re^{M+1} \tag{40}$$

✓ c_ε is a line vector:

$$c_\varepsilon = \begin{bmatrix} c^T \, 1 \end{bmatrix} \in \Re^{1\times M} \tag{41}$$

So, a theorem characterizing sufficient conditions and ensuring the exponential convergence of the global reconstruction error $e_\varepsilon(k)$ is proposed as in [16] by taking into account that the unknown input $V(k)$ is constant:

Theorem 1: If there exists a positive symmetric matrix P, the global reconstruction error $e_\varepsilon(k)$ characterizing the proportional-integral observer converges to zero exponentially and defines a matrix G_ε satisfying the following LMIs:

$$\begin{bmatrix} (1-2\alpha)P & A_\varepsilon^T P - c_\varepsilon^T G_\varepsilon^T \\ PA_\varepsilon - G_\varepsilon c_\varepsilon & P \end{bmatrix} > 0 \text{ with } \alpha \in]0, 0.5[\tag{42}$$

such as:

$$K_\varepsilon = P^{-1} G_\varepsilon \tag{43}$$

Proof in appendix ∎

C. DETECTION AND OUTPUT FAULT ESTIMATION

In terms of performance for the fault detection at the output, the following residues are calculated:

➢ the residue $e_y(k)$ as the error between the measured output $y_m(k)$ and the estimated output $\hat{y}(k)$:

$$e_y(k) = \hat{y}(k) - y_m(k) \tag{44}$$

➢ the residue $e_y^a(k)$ as the error between $\hat{y}_a(k)$ and $y_{a,m}(k)$:

$$e_y^a(k) = \hat{y}_a(k) - y_{a,m}(k) \tag{45}$$

Therefore, the general residual structure is proposed in Figure 6.

Considering the relations (20) and (23), we deduce that:

✓ for $\hat{V}(k) \neq V(k)$:

$$\hat{y}(k) \neq y_m(k) \Leftrightarrow e_y(k) \neq 0 \tag{46}$$

$$\begin{aligned} \hat{X}_a(k) - X_{a,m}(k) &= A_y \left(\hat{X}_a(k-1) - X_{a,m}(k-1)\right) \\ &\quad + (L_a + b_a)\left(y_m(k-1) - \hat{y}(k-1)\right) \\ &\neq 0 \Leftrightarrow e_y^a(k) \neq 0 \end{aligned} \tag{47}$$

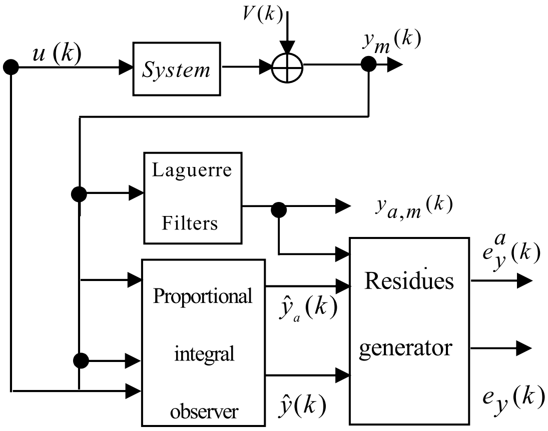

FIGURE 6. Residual generation structure.

✓ for or $\hat{V}(k) \simeq 0$ (without fault):

$$\hat{y}(k) \simeq y_m(k) \Rightarrow e_y(k) \simeq 0 \tag{48}$$

$$\hat{X}_a(k) - X_{a,m}(k) \simeq A_y\left(\hat{X}_a(k-1) - X_{a,m}(k-1)\right) \simeq 0 \Leftrightarrow e^a_y(k) \simeq 0 \tag{49}$$

Therefore, we can get the following proposition:

Proposition 1: The output fault detection and estimation conditions using PI observer, based on the ARX-Laguerre model are given by:

✓ For $\hat{V}(k) \neq V(k)$:

$$\hat{y}(k) \neq y_m(k) \text{ and } e^a_y(k) \neq 0 \tag{50}$$

✓ For $\hat{V}(k) \simeq V(k)$ or $\hat{V}(k) \simeq 0$ (without fault):

$$\hat{y}(k) \simeq y_m(k) \text{ and } e^a_y(k) \simeq 0 \tag{51}$$

Therefore, by considering proposition 1 and the gains L_a, L_b and K_V calculated using the LMI formulations (42) and (43), an algorithm for the PIO output fault diagnosis is proposed.

ALGORITHM 2: PIO OUTPUT FAULT DIAGNOSIS ALGORITHM

A. Offline calculation phase:

(1) Fix the truncating orders (N_a, N_b) and we assume that the Laguerre poles (ξ_a, ξ_b), and the Fourier coefficients c are identified.

(2) Calculate A, b_y and b_u from (9) and (12).

(3) Fix α and calculate the gains L_a, L_b and K_V by (42) and (43).

B. Online calculation phase:

(4) For each increment of time instant $k \geq 1$:

(a) Measure $u(k)$ and $y_m(k)$.

(b) Calculate $\hat{y}(k)$ and $\hat{V}(k)$ from (23).

(c) If $\hat{V}(k) \simeq 0$ then $k = k+1 \Rightarrow$ step 4.a.

Else if $\hat{V}(k) \neq 0$

(c.1) Fault detection.

(c.2) $k = k+1 \Rightarrow$ step 4.a.

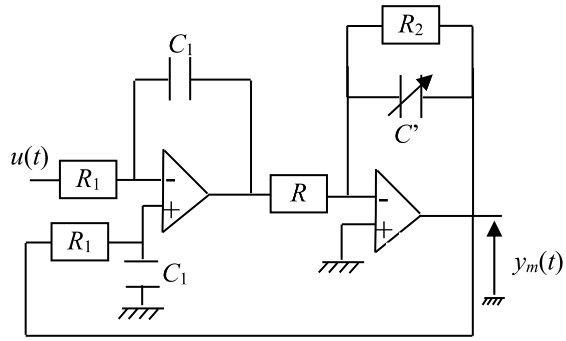

FIGURE 7. 2^{nd} order electrical system.

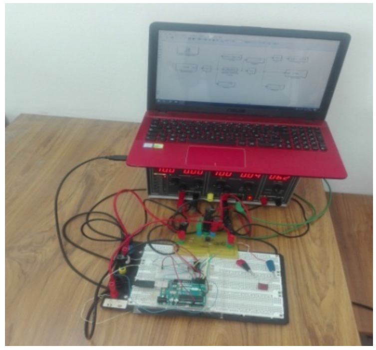

FIGURE 8. Photograph of the experimental 2^{nd} order electrical system.

V. EXPERIMENTAL APPLICATION ON A 2^{nd} ORDER ELECTRICAL SYSTEM

This section presents experimental results on the 2^{nd} order electrical system to illustrate the efficiency of the Laguerre pole identification based on genetic algorithm and the performance of the proposed PIO. We consider the following electrical system with $C_1 = 10nF$, $C' = 25n$F and $R_1 = R = R_2 = 68k\Omega$:

The experimental platform in Figure 8 is characterized by:

- The Arduino card for the acquisition of $y_m(t)$ and the control signal $u(t)$.
- The MATLAB/Simulink to transfer by USB experiment the control signal $u(t)$ to the system from DAC0800 as a 8-bit digital to analog converter. Also Simulink allows the acquisition of $y_m(t)$ from the ADC of Arduino card.

We note that the experimental system is defined by the following continuous-time transfer function:

$$G_c(s) = \frac{kw_0^2}{s^2 + 2mw_0 s + w_0^2} \tag{52}$$

such as $w_0 = \sqrt{\frac{1}{RR_1C_1C'}} = 930.08$ rad/s is the natural frequency, $m = \frac{1}{2R_2}\sqrt{\frac{RR_1C_1}{C'}} = 0.3162$ is the damping ratio and $k = 1$ is the static gain.

A. ARX-LAGUERRE MODEL IDENTIFICATION

Using the process at a sampling time of 0.01s, 500 input/output observations were collected. The voltage input $u(k)$ is in the range of 1V and 3.37 V which is a pseudo-random sequence. The evolution of $u(k)$ is presented in Figure 9.

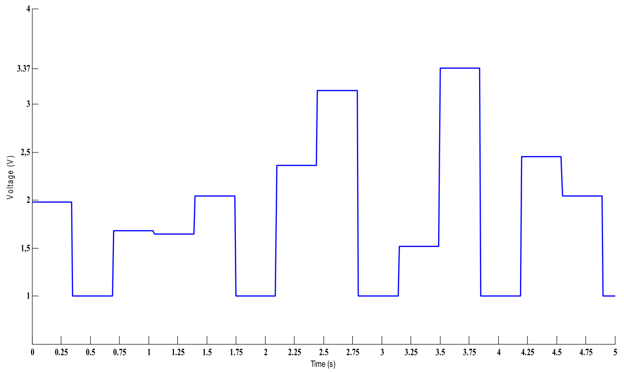

FIGURE 9. Input signal.

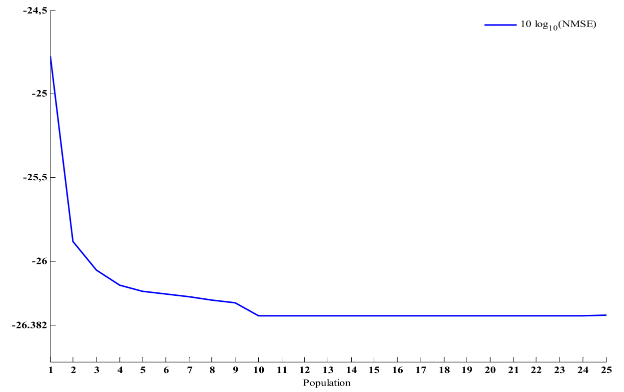

FIGURE 10. Evolution of the NMSE.

We adopt the model structure for which the truncating orders are $Na = Nb = 1$ i.e. $M = 2$ which ensures a PRR (Parameter Reduction Ratio) equal to $(1 - 2/4) \times 100 = 50\%$. We note that the 4 parameters characterize the discrete transfer function using Z-transform between output and input obtained by taking into account the DAC0800 and the continuous-time transfer function $G_c(s)$. Indeed, by considering theoretically that the DAC0800 can be handled as being a Zero Order Hold (ZOH), we obtain the following discrete transfer function:

$$G_D(z) = Z\left[H_{ZOH}(s).G_c(s)\right] = \frac{11.1110^{-4}z + 6.05410^{-5}}{z^2 + 87.1110^{-3}z + 2.7910^{-3}} \quad (53)$$

The genetic parameters are fixed as $N_{ind} = 150$, $\varepsilon = 10^{-3}$, $P_c = 0.85$ and $P_m = 0.08$. By applying algorithm 1, the evolution of the NMSE and the identification of the Laguerre are presented in Figures 10 and 11 respectively such as $\xi_a = 0.631$ and $\xi_b = -0.348$. The system and the ARX-Laguerre outputs are represented in Figure 12 with a NMSE = 2.3 10^{-3}.

B. OUTPUT FAULT DETECTION AND ESTIMATION

For the generation of the output faults, we disconnect the polarization of the two operational amplifiers corresponding to ±10V. Consequently, in certain time intervals a zero output to the electrical system level is obtained. In this case, it notes that $y(k) + V(k) = 0V$. Therefore, we obtain from the

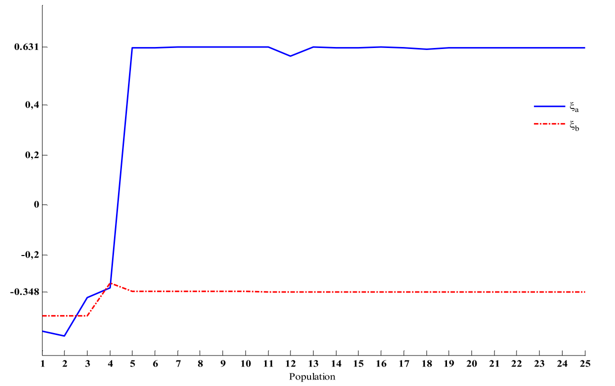

FIGURE 11. Identification of Laguerre poles ξ_a and ξ_b for $Na = 1$ and $Nb = 1$.

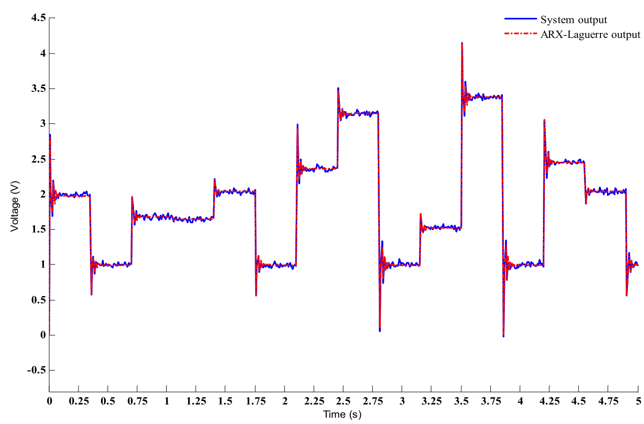

FIGURE 12. System and ARX-Laguerre model outputs for $Na = 1$ and $Nb = 1$.

experiment manipulation the following condition defining the presence of faults at output:

$$y(k) + V(k) = 0V \quad \text{for } k \in [90, 400] \text{ or } k \in [820, 1150] \quad (54)$$

We fix $\alpha = 0.18$ for the output fault detection and estimation by algorithm 2. Then the observer gains L_a, L_b and K_V are obtained by solving the LMI formulations (42) and (43):

$$L_a = 0.6269, L_b = 0.05959, K_V = 0.38 \quad (55)$$

The Figures 13 and 14 show the signal of the unknown output $\hat{V}(k)$ as a fault as well the system and the estimate system outputs. Figure 15 presents the residual signals evaluation $e_y(k)$ and $e_y^a(k)$. Then from Figure 13 the fault signal is defined as follows:

$$V(k) = \begin{cases} \left.\begin{array}{ll} -2.467V & \text{if} k \in [90, 232] \\ -1.69V & \text{if } k \in [233, 400] \end{array}\right\} : 1^{st} \text{ fault} \\ \left.\begin{array}{ll} -1.69V & \text{if } k \in [820, 921] \\ -2.815V & \text{if } k \in [922, 1150] \end{array}\right\} : 2^{nd} \text{ fault} \\ 0 \quad \text{else} \end{cases} \quad (56)$$

From Figure 15 we have $e_y(k) \neq 0$ and $e_y^a(k) \neq 0$ for faults detection. At time instant 90, 233, 820 and 922,

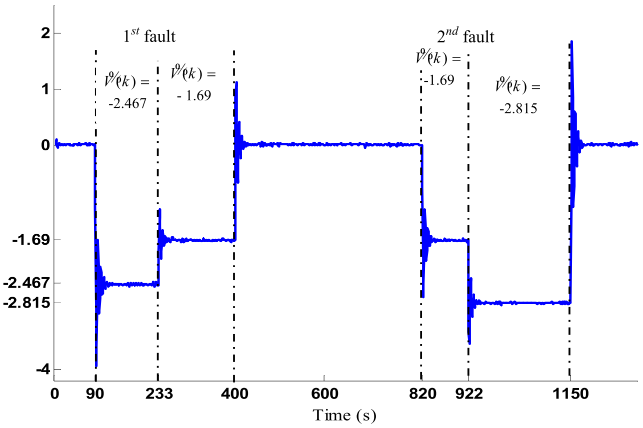

FIGURE 13. Signal $\hat{V}(k)$ which estimates the output fault $V(k)$.

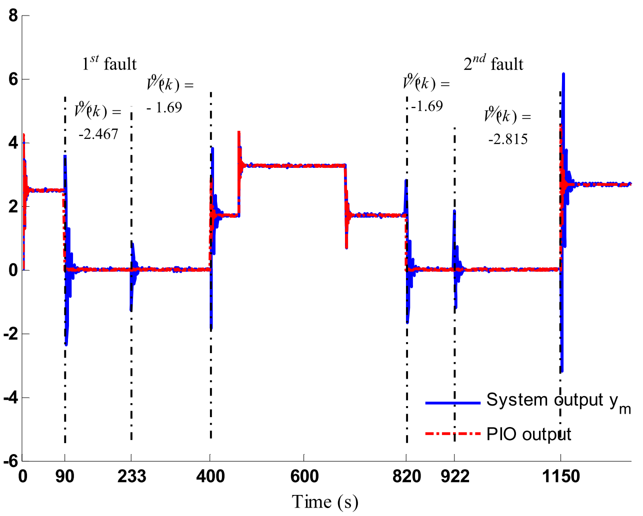

FIGURE 14. System output $y_m(k)$ and the estimate system output $\hat{y}(k)$ by the PIO.

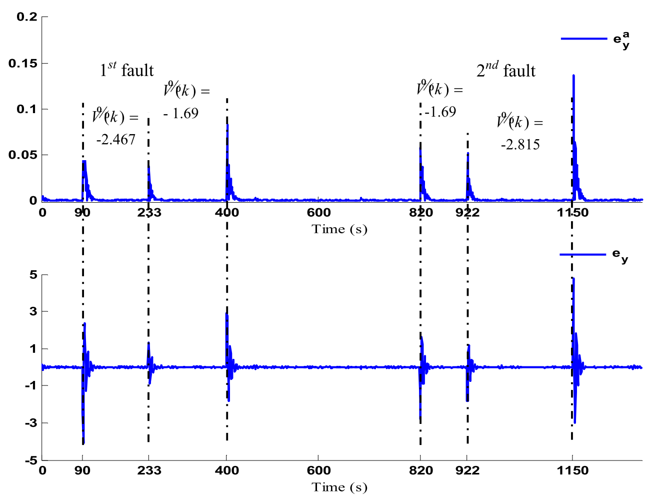

FIGURE 15. Residual signals $e_y(k)$ and $e_y^a(k)$.

we see overshoots which are the limiting instants for the faults detection. The overshoots are results from the fact that $V(k+1) \neq V(k)$. From the Figures 13 and 15, the faults appearance is characterized by:

- ✓ a falling edge at time instant $k = 90, 233, 820, 922$.
- ✓ a rising edge at time instant $k = 400, 1150$.

In this case, it is not complied with the condition $V(k+1) = V(k)$ for observer gains computation. For that purpose, the algorithm 2 does not ensure the estimation of faults for the corresponding times. Therefore, the appearance of transitional regimes with overshoots in Figures 14 and 15 with $y_m(k) \neq \hat{y}(k)$. Furthermore, from Figure 15 we note that we have $e_y(k) \simeq 0$ and $e_y^a(k) \simeq 0$ despite the presence of faults. This is due to the fact that the proposed PIO offers faults estimation at the output i.e. $\hat{V}(k) \simeq V(k)$. This estimate is illustrated in Figure 14 and which is guaranteed since according to (56) we have $V(k+1) = V(k)$. Then, we get $y(k)+V(k) \simeq y(k)+\hat{V}(k)$ and it results $\hat{y}(k) \simeq y_m(k)$. For this why we notice in Figure 15 the concordance between $y_m(k)$ and $\hat{y}(k)$. Considering Figures 13, 14 and 15, for $V(k) \neq \hat{V}(k)$ we can deduce that:

$$\hat{y}(k) \neq y_m(k) \text{ and } e_y^a(k) \neq 0 \tag{57}$$

This is consistent with the result of the relation (50). Furthermore, comparing the order system equal to two with the parameter number of the ARX-Laguerre ($M = 2$) we can say that in this case the proposed PIO has the same order of the system i.e. is not a reduced observer.

VI. CONCLUSION

In this article, we developed a PI observer based on the new linear representation ARX-Laguerre. Therefore, a new FDI scheme was proposed for the output faults detection and identification. We noted that if the ARX-Laguerre model parameters number is lower than the system order so we can consider that the resulting PIO is a reduced observer. Furthermore, for pole optimization of ARX-Laguerre model we exploited the genetic algorithm. The pole optimization procedure as well as the resulting PIO were tested and validated on an experimental 2nd order electrical system. In fact, the performances of the ARX-Laguerre model in terms of parameter complexity reduction and quality approximation were appreciated. Also, the results confirm the efficiency of the proposed PIO for the output faults detection and identification. We will propose later a possible extension of this work to an adaptive identification of the Laguerre poles and the Fourier coefficients on a sliding window to improve the proposed PI observer by automatically updating the gain values L_a, L_b and K_V. Another possible research direction would be the exploitation of other types observer-based fault diagnosis to achieve a comparative study like in [7].

APPENDIX

PIO'S GAINS BY LMI OPTIMIZATION

To ensure the exponential convergence of the global reconstruction error $e_\varepsilon(k)$ to zero, a quadratic Lyapunov function $V_\varepsilon(k)$ is restrained [14]:

$$V_\varepsilon(k) = e_\varepsilon^T(k) P e_\varepsilon(k) \tag{58}$$

such as P is a Lyapunov positive define and symmetric matrix. We note that the following condition guarantees the exponential stability of $V_\varepsilon(k)$:

$$\Delta V_\varepsilon(k) + 2\alpha V_\varepsilon(k) < 0 \tag{59}$$

where α is the decay rate and $\Delta V_\varepsilon(k) = V_\varepsilon(k) - V_\varepsilon(k-1)$ is the increment of $V_\varepsilon(k)$. From (38) and (59) we obtain:

$$e_\varepsilon^T(k-1)\left\{(1+2\alpha)(A_\varepsilon - K_\varepsilon c_\varepsilon)^T P(A_\varepsilon - K_\varepsilon c_\varepsilon) - P\right\} e_\varepsilon(k-1) < 0 \tag{60}$$

According to Lyapunov theory we can write:

$$(1+2\alpha)(A_\varepsilon - K_\varepsilon c_\varepsilon)^T P(A_\varepsilon - K_\varepsilon c_\varepsilon) - P < 0 \tag{61}$$

Knowing that $(P^{-1})^T = P^{-1}$ the inequality (61) is reformulated as follows:

$$(1+2\alpha)\left\{A_\varepsilon^T P A_\varepsilon - A_\varepsilon^T G_\varepsilon c_\varepsilon - c_\varepsilon^T G_\varepsilon^T A_\varepsilon + c_\varepsilon^T G_\varepsilon^T P^{-1} G_\varepsilon c_\varepsilon\right\} - P < 0 \tag{62}$$

such as $G_\varepsilon = PK_\varepsilon$. For $\alpha \in]0, 0.5[$ we can deduce that:

$$\begin{cases} 1 - 2\alpha > 0 \\ 4\alpha^2 \approx 0 \end{cases} \tag{63}$$

and inequality (62) is simplified and rewritten as:

$$(1-2\alpha)P - \left\{A_\varepsilon^T P A_\varepsilon - A_\varepsilon^T G_\varepsilon c_\varepsilon - c_\varepsilon^T G_\varepsilon^T A_\varepsilon + c_\varepsilon^T G_\varepsilon^T P^{-1} G_\varepsilon c_\varepsilon\right\} > 0 \tag{64}$$

By taking into account that:

$$A_\varepsilon^T P A_\varepsilon - A_\varepsilon^T G_\varepsilon c_\varepsilon - c_\varepsilon^T G_\varepsilon^T A_\varepsilon + c_\varepsilon^T G_\varepsilon^T P^{-1} G_\varepsilon c_\varepsilon = (A_\varepsilon^T P - c_\varepsilon^T G_\varepsilon^T) P^{-1}(PA_\varepsilon - G_\varepsilon c_\varepsilon) \tag{65}$$

we conclude that:

$$(1-2\alpha)P - (A_\varepsilon^T P - c_\varepsilon^T G_\varepsilon^T) P^{-1}(PA_\varepsilon - G_\varepsilon c_\varepsilon) > 0 \quad \text{for } \alpha \in]0, 0.5[\tag{66}$$

Finally, from (66) the exponential convergence conditions of the global estimation error $e_\varepsilon(k)$ can be obtained by exploiting the Schur complement and classical methods for LMI resolution and which are summarized in Theorem 1.

REFERENCES

[1] S. Beale and B. Shafai, "Robust control system design with a proportional integral observer," *Int. J. Control*, vol. 50, no. 1, pp. 97–111, Jul. 1989.

[2] H. Bedoui, T. Garna, K. Ben Othman, and H. Messaoud, "Design of a proportional observer based on the ARX-Laguerre model," *Stud. Informat. Control*, vol. 24, no. 4, pp. 471–476, Dec. 2015.

[3] K. Bouzrara, T. Garna, J. Ragot, and H. Messaoud, "Online identification of the ARX model expansion on Laguerre orthonormal bases with filters on model input and output," *Int. J. Control*, vol. 86, no. 3, pp. 369–385, Mar. 2013.

[4] K. Bouzrara, T. Garna, J. Ragot, and H. Messaoud, "Decomposition of an ARX model on Laguerre orthonormal bases," *ISA Trans.*, vol. 51, no. 6, pp. 848–860, Nov. 2012.

[5] J. Chen, R. J. Patton, and H.-Y. Zhang, "Design of unknown input observers and robust fault detection filters," *Int. J. Control*, vol. 63, no. 1, pp. 85–105, 2007.

[6] C. Chen, Y. Wang, and S. Li, "Generalized proportional integral observer–based composite control method for robotic thermal tactile sensor with disturbances," *Int. J. Adv. Robotic Syst.*, vol. 14, no. 3, May 2017, Art. no. 172988141771003.

[7] C. Edwards and C. P. Tan, "A comparison of sliding mode and unknown input observers for fault reconstruction," *Eur. J. Control*, vol. 12, no. 3, pp. 245–260, Jan. 2006.

[8] G.-R. Duan and A.-G. Wu, "Robust fault detection in linear systems based on PI observers," *Int. J. Syst. Sci.*, vol. 37, no. 12, pp. 809–816, Oct. 2006.

[9] T. Floquet, C. Edwards, and S. Spurgeon, "On sliding mode observers for systems with unknown inputs," *Int. J. Adapt. Control Signal Process.*, vol. 21, nos. 8–9, pp. 638–656, 2007.

[10] K. Gallagher and M. Sambridge, "Genetic algorithms: A powerful tool for large-scale nonlinear optimization problems," *Comput. Geosci.*, vol. 20, nos. 7–8, pp. 1229–1236, Aug. 1994.

[11] J. R. Koza, *Genetic Programming: On the Programming of Computer by Means of Natural Selection*. Cambridge, MA, USA: MIT Press, 1992.

[12] J. R. Koza, *Genetic Programming II: Automatic Discovery of Reusable Programs*. Cambridge, MA, USA: MIT Press, 1994.

[13] K. Ogata, *Discrete-Time Control Systems*. Upper Saddle River, NJ, USA: Prentice-Hall, 2010.

[14] R. Orjuela, B. Marx, D. Maquin, and J. Ragot, "State estimation of nonlinear discrete-time systems based on the decoupled multiple model approach," in *Proc. 5th Workshop Adv. Control Diagnosis*, Grenoble, France, 2007, pp. 1–8.

[15] V. Sundarapandian, "Distributed control schemes for large-scale interconnected discrete-time linear systems," *Math. Comput. Model.*, vol. 41, nos. 2–3, pp. 313–319, Jan. 2005.

[16] T. Najeh, C. Ben Njima, T. Garna, and J. Ragot, "Input fault detection and estimation using PI observer based on the ARX-Laguerre model," *Int. J. Adv. Manuf. Technol.*, vol. 90, nos. 5–8, pp. 1317–1336, May 2017.

[17] B. Wahlberg, "System identification using Laguerre models," *IEEE Trans. Autom. Control*, vol. 36, no. 5, pp. 551–562, May 1991.

[18] A. Weinmann, *Uncertain Models and Robust Control*. New York, NY, USA: Springer-Verlag, 1991.

[19] F. Xu, J. Tan, X. Wang, V. Puig, H. Liu, B. Liang, and B. Yuan, "Robust state estimation and fault detection combining unknown input observer and set-membership approach," in *Proc. 3rd Conf. Control Fault-Tolerant Syst. (SysTol)*, Barcelona, Spain, Sep. 2016, pp. 67–72.

[20] Y. X. Yao and A. V. Radun, "Proportional integral observer design for linear systems with time delay," *IET Control Theory Appl.*, vol. 1, no. 4, pp. 887–892, Jul. 2007.

CHAKIB BEN NJIMA was born in Sousse, Tunisia, in 1979. He received the Master Diploma degree from the National School of Engineers of Monastir, Tunisia, in 2007, and the Ph.D. degree from the National Engineering School of Monastir (ENIM), in 2013. He is currently an Associate Professor with the Higher Institute of Transport and Logistics of Sousse, University of Sousse, Tunisia. He is also the Founder of the IEEE International Conference of Control, Automation, and Diagnosis (ICCAD) and a member of the Laboratory of Automatic Control and Signal Processing (LARATSI).

TAREK GARNA was born in Sousse, Tunisia, in 1978. He received the Ph.D. degree in automatic control from the National Engineering School of Monastir, in 2009, and the Habilitation degree in electrical engineering from the High School of Science and Technology of Hammam Sousse, in June 2015. He is currently an Assistant Professor with the Higher Institute of Applied Sciences and Technology of Sousse and a member of the Laboratory LARATSI.