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The Perturbations Estimation in Two Gas Plants

JOSÉ DE JESÚS RUBIO[1], (Member, IEEE), DANY IVAN MARTINEZ[1], VICTOR GARCIA[1], GUADALUPE JULIANA GUTIERREZ[1], TOMAS MIGUEL VARGAS[1], GENARO OCHOA[1], RICARDO BALCAZAR[1], JAIME PACHECO[1], JESUS ALBERTO MEDA-CAMPAÑA[2], (Member, IEEE), AND DANTE MUJICA-VARGAS[3]

[1]Sección de Estudios de Posgrado e Investigación, Esime Azcapotzalco, Instituto Politécnico Nacional, Mexico City 02250, Mexico
[2]Sección de Estudios de Posgrado e Investigación, Esime Zacatenco, Instituto Politécnico Nacional, Mexico 07738, Mexico
[3]Department of Computer Science, Tecnológico Nacional de México/CENIDET, Cuernavaca 62490, Mexico

Corresponding author: José de Jesús Rubio (rubio.josedejesus@gmail.com)

ABSTRACT The perturbations are the unwanted and unknown inlets in nonlinear plants which can affect the outlets. In this article, an estimator is studied for the variables and perturbations estimation in nonlinear plants. The saturation map is used in our estimator instead of the signum map to decrease the chattering, and we ensure the estimator convergence by the Lyapunov analysis. The conditions required by our estimator gains are found to reach the variables error convergence, and these gains are used for the perturbations estimation. An algorithm is proposed to choose the gains for achieving a satisfactory performance in our estimator. The studied estimator is applied for the variables and perturbations estimation in the gas turbine and gasification plants.

INDEX TERMS Variables, perturbations, estimation, gas turbine, gasification, convergence.

I. INTRODUCTION

The perturbations are the unwanted and unknown inlets in nonlinear plants which can affect the outlets. This issue has occurred in many nonlinear plants. Since perturbations can affect the sensors, actuators, or plants yielding additional costs, and since most nonlinear plants regulators require the knowledge of perturbations; an approach for the perturbations estimation is welcome.

There are some studies about regulators for perturbed plants. In [1] and [2], the active strategy for the perturbations attenuation is mentioned. In [3] and [4], the singular perturbations approach for the perturbations attenuation is considered. The variables and perturbations estimation in plants is focused in [5]–[7], and [8]. In [9]–[12], and [13], the robust analysis for the perturbed plants stabilization is focused. The perturbations estimation with fuzzy regulators is mentioned in [14] and [15]. In [16]–[18], and [19], the authors use the neuro-fuzzy approximations for the perturbed plants regulation. The adaptive laws for the perturbed plants regulation are focused in [20], [21], and [22]. In [23] and [24], the structure theory for the perturbations attenuation is mentioned. From the above studies, in [1], [2], [5], [6], [7]–[11], [12], [13], [17], and [19], the authors use approaches for the variables or perturbations estimation in nonlinear plants; then it would be welcome to be focused on this issue. The novelty of this article is that each nonlinear plant has a different structure; consequently, a special estimator with the structure of the gas plants must be discussed.

There are various estimators who use the plant outlets for the variables estimation [1], [2], [5], [6], but there are not many estimators who use the plant outlets for the perturbations estimation. In [7], [8], [11], [13], [17], [19], previous studies of estimators are focused for the perturbations estimation, but with the two below differences: in the previous studies estimators with adaptive or feedback outlets are used, while in this article an estimator with sliding modes outlets is used, in the plants of previous studies the noise is not used, while in the plants of this article the noise is used.

In this article, the sliding mode approach is utilized in our estimator for the variables and perturbations estimation in nonlinear plants. Since the sliding mode approach uses the signum map, it can yield the unwanted chattering [3], [12], [14], [15].

The first contribution of this article is that an estimator is designed, it is described by the following characteristics: 1) the saturation map is used in our estimator instead of the signum map to decrease the chattering; and later, we ensure

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the estimator convergence by the Lyapunov analysis, 2) find the conditions required by our estimator gains that allow it to reach the variables error convergence; it yields an acceptable performance in the variables and perturbations estimation.

The second contribution of this article is that an algorithm is proposed to choose the gains for achieving a satisfactory performance in our estimator, it is described as follows: 1) we choose a value for the gain 1, 2) we obtain the estimator matrix, 3) we obtain the eigenvalues of the estimator matrix, if the real parts of all the eigenvalues of the estimator matrix are negative, then the gain 1 is correctly chosen and we can go to the step 4, otherwise, we must return to the step 1, 4) we choose the matrix 1 of the Lyapunov equation, 5) we substitute matrix 1 and estimator matrix into the Lyapunov equation, and we find the matrix 2 of the Lyapunov equation, if matrix 1 and matrix 2 of the Lyapunov equation are positive definite, then matrix 1 is correctly chosen and we can go to the step 6, otherwise, we must return to step 1, 6) we choose the gain 2, gain 3, and gain 4 to solve the estimator, if the estimator reaches an acceptable exactness for the variables and perturbations estimation in the nonlinear plant, then the algorithm finishes, otherwise, we must return to step 6.

Our estimator is applied for the variables and perturbations estimation in the gas turbine and gasification plants. The gas turbine plant is used for the electrical energy generation from the gas [25], while the gasification plant is used for the gas generation from biomass [26].

The rest of the article is described below. Section II presents the estimator design containing the variables error convergence, and the perturbations estimation in nonlinear plants, later, an algorithm is proposed to choose the gains for achieving a satisfactory performance in our estimator. Sections III and IV estimators are studied for the variables and perturbations estimation in the gas turbine and gasification plants. Section V express the conclusion and future work.

II. THE ESTIMATOR FOR THE VARIABLES AND PERTURBATIONS ESTIMATION

In this section, a variables estimator, and a perturbations estimator, which are termed as estimator will be studied for the variables and perturbations estimation in nonlinear plants.

In this article, a special nonlinear plant will be used in which the outlets have a linear combination with the variables, the variables have a nonlinear combination with the variables, and the perturbations are entered additively. The nonlinear plant is [25], [26]:

$$\begin{aligned} \dot{h} &= Ah + f(h,v) + Bu + B\varkappa \\ y &= Ch \end{aligned} \tag{1}$$

$h \in \Re^n$ as plant variables, $v \in \Re^m$ as the plant inlets, $y \in \Re$ as the plant outlets, $u \in \Re$ as the perturbations, $f(h,v) \in \Re^n$ as a nonlinear map, $\varkappa \in \Re$ as the noise, $A \in \Re^{n\times n}$, $B \in \Re^{n\times 1}$, $C \in \Re^{p\times n}$ as matrices.

A. THE VARIABLES ESTIMATOR

The goal of the variables estimator is that using the inlets and outlets, the variables of the variables estimator should estimate the nonlinear plant variables.

The estimator error $\widetilde{y}$ is:

$$\widetilde{y} = y - \widehat{y} = C\widetilde{h} \tag{2}$$

$\widehat{y}$ as the variables estimator outlet, $\widetilde{h} = h - \widehat{h}$ as the variables error, $\widehat{h}$ as the estimator variables. The variables estimator is:

$$\begin{aligned} \dot{\widehat{h}} &= A\widehat{h} + f(\widehat{h},v) + K\widetilde{y} + Esat(M\widetilde{y}) \\ \widehat{y} &= C\widehat{h} \\ sat(M\widetilde{y}) &= \begin{cases} [c]cc1 & M\widetilde{y} > 1 \\ M\widetilde{y} & |M\widetilde{y}| \le 1 \\ -1 & M\widetilde{y} < -1, \end{cases} \end{aligned} \tag{3}$$

$\widehat{h}$ as the estimator variables, $\widehat{y}$ as the estimator outlets, $sat(\cdot)$ as the saturation map, M as a matrix where $MC \in \Re^{n\times n}$ is a positive semi-definite constant, $K \in \Re^{n\times 1}$ and $E \in \Re^{n\times 1}$.

B. THE CONVERGENCE ANALYSIS OF THE VARIABLES ESTIMATOR

In this sub-section, the convergence of the variables estimator applied to nonlinear plants is analyzed based on the Lyapunov approach [3], [4].

The closed loop model of the variables estimator is the subtraction of (3) to (1) and using the estimator error (2) as:

$$\begin{aligned} \dot{\widetilde{h}} \;=\; & A\widetilde{h} + f(h,v) + Bu + B\varkappa - f(\widehat{h},v) \\ & -K\widetilde{y} - Esat(M\widetilde{y}) \\ \Longrightarrow\; & \dot{\widetilde{h}} = A\widetilde{h} + \widetilde{f} + Bu + B\varkappa - KC\widetilde{h} - Esat(MC\widetilde{h}) \\ \Longrightarrow\; & \dot{\widetilde{h}} = A_s\widetilde{h} + \widetilde{f} + Bu + B\varkappa - Esat(MC\widetilde{h}) \end{aligned} \tag{4}$$

$\widetilde{f} = f(h,v) - f(\widehat{h},v)$, $A_s = A - KC$. The nonlinear map $\widetilde{f}$ is bounded as $\left|\widetilde{f}\right| \le \overline{f}$, $|\cdot|$ as the absolute value.

The below theorem analyzes the variables estimator convergence.

Theorem 1: The variables error of the variables estimator (2)-(3) applied to estimate the nonlinear plant variables h (1) is convergent, $\gamma = \lambda_{\min}(Q_s P_s^{-1})$, $\left|\widetilde{f} + Bu + B\varkappa\right| \le \overline{u}$, $\overline{u} \le E$, $\|\cdot\|$ as the Euclidean norm in $\Re^n$, $|\cdot|$ as the absolute value, $P_s \in \Re^{n\times n}$ and $Q_s \in \Re^{n\times n}$ are positive definite matrices which meet:

$$A_s^T P_s + P_s A_s = -Q_s \tag{5}$$

A_s as is expressed in (4).

Proof: The Lyapunov candidate map is:

$$L = \widetilde{h}^T P_s \widetilde{h} \tag{6}$$

The derivative of (4) is:

$$\begin{aligned} \dot{L} \;=\; & \dot{\widetilde{h}}^T P_s\widetilde{h} + \widetilde{h}^T P_s \dot{\widetilde{h}} \\ \Longrightarrow\; & \dot{L} = \widetilde{h}^T \left(A_s^T P_s + P_s A_s\right)\widetilde{h} \\ & +2\widetilde{h}^T P_s\left[\widetilde{f} + Bu + B\varkappa - Esat(MC\widetilde{h})\right] \end{aligned} \tag{7}$$

Using the second term of (7), $sat(M C\widetilde{h}) = sat(\widetilde{h})$, and $\left|\widetilde{f} + Bu + B\varkappa\right| \leq \overline{u}$, is:

$$2\widetilde{h}^T P_s \left[\widetilde{f} + Bu - Esat(M C\widetilde{h})\right] = 2\widetilde{h}^T P_s \left[\widetilde{f} + Bu + B\varkappa\right] - 2\widetilde{h}^T P_s Esat(\widetilde{h}) \quad (8)$$

Using (8) in (7) is:

$$\dot{L} = \widetilde{h}^T \left(A_s^T P_s + P_s A_s\right)\widetilde{h} + 2\widetilde{h}^T P_s \left[\widetilde{f} + Bu + B\varkappa\right] - 2\widetilde{h}^T P_s Esat(\widetilde{h}) \quad (9)$$

Using $A_s^T P_s + P_s A_s + \mu P_s = -Q_s$ of (5), is: The equation of (9) can be expressed as:

$$\dot{L} = -\widetilde{h}^T Q_s \widetilde{h} + 2\widetilde{h}^T P_s \left[\widetilde{f} + Bu + B\varkappa\right] - 2\widetilde{h}^T P_s Esat(\widetilde{h}) \quad (10)$$

Since (5), $\left|\widetilde{f} + Bu + B\varkappa\right| \leq \overline{u}$, $\overline{u} \leq E$, and since (3), (7), $sat(\widetilde{h}) = \begin{cases} 1 & \widetilde{h} > 1 \\ \widetilde{h} & \left|\widetilde{h}\right| \leq 1 \\ -1 & \widetilde{h} < -1 \end{cases}$, we notice that there are three cases of the saturation map. 1) If $\widetilde{h} > 1$, then $sat(\widetilde{h}) = 1$ and $\widetilde{h} = \left|\widetilde{h}\right|$, we replace in (10) as:

$$\begin{aligned} \dot{L} &= -\widetilde{h}^T Q_s \widetilde{h} + 2\widetilde{h}^T P_s \left[\widetilde{f} + Bu + B\varkappa\right] - 2\widetilde{h}^T P_s Esat(\widetilde{h}) \\ &\Rightarrow \dot{L} \leq -\widetilde{h}^T Q_s \widetilde{h} + 2\widetilde{h}^T P_s \overline{u} - 2\left|\widetilde{h}\right|^T P_s E \\ &\Rightarrow \dot{L} \leq -\widetilde{h}^T Q_s \widetilde{h} \end{aligned} \quad (11)$$

2) If $\left|\widetilde{h}\right| \leq 1$, then $sat(\widetilde{h}) = \widetilde{h}$ and $\widetilde{h}^T\widetilde{h} = \left|\widetilde{h}\right|^T \left|\widetilde{h}\right|$, we replace in (10) as:

$$\begin{aligned} \dot{L} &= -\widetilde{h}^T Q_s \widetilde{h} + 2\widetilde{h}^T P_s \left[\widetilde{f} + Bu + B\varkappa\right] - 2\widetilde{h}^T P_s Esat(\widetilde{h}) \\ &\Rightarrow \dot{L} = -\widetilde{h}^T Q_s \widetilde{h} + 2\widetilde{h}^T P_s \overline{u} - 2\widetilde{h}^T \widetilde{h} P_s E \\ &\Rightarrow \dot{L} = -\widetilde{h}^T Q_s \widetilde{h} + 2\widetilde{h}^T P_s \overline{u} - 2\left|\widetilde{h}\right|^T \left|\widetilde{h}\right| P_s E \\ &\Rightarrow \dot{L} = -\widetilde{h}^T Q_s \widetilde{h} - 2\left|\widetilde{h}\right|^T \left[\left|\widetilde{h}\right| P_s E - P_s \overline{u}\right] \\ &\Rightarrow \dot{L} = -\widetilde{h}^T Q_s \widetilde{h} \end{aligned} \quad (12)$$

since in this case $\left|\widetilde{h}\right| \leq 1$, $\left|\widetilde{h}\right| P_s E - P_s \overline{u} \geq 0 \Rightarrow P_s \overline{u} \leq \left|\widetilde{h}\right| P_s E \leq P_s E$. 3) If $\widetilde{h} < -1$, then $sat(\widetilde{h}) = -1$ and $\widetilde{h} = -\left|\widetilde{h}\right|$, we replace in (10) as:

$$\begin{aligned} \dot{L} &= -\widetilde{h}^T Q_s \widetilde{h} + 2\widetilde{h}^T P_s \left[\widetilde{f} + Bu + B\varkappa\right] - 2\widetilde{h}^T P_s Esat(\widetilde{h}) \\ &\Rightarrow \dot{L} = -\widetilde{h}^T Q_s \widetilde{h} - 2\left(-\left|\widetilde{h}\right|^T\right) P_s \left[-\left(\widetilde{f} + Bu + B\varkappa\right)\right] \\ &\quad -2\left(-\left|\widetilde{h}\right|^T\right) P_s E\,(-1) \\ &\Rightarrow \dot{L} \leq -\widetilde{h}^T Q_s \widetilde{h} + 2\left|\widetilde{h}\right|^T P_s \overline{u} - 2\left|\widetilde{h}\right|^T P_s E \\ &\Rightarrow \dot{L} \leq -\widetilde{h}^T Q_s \widetilde{h} \end{aligned} \quad (13)$$

since (11), (12), (13), the three cases we have the same inequality expressed as:

$$\dot{L} \leq -\widetilde{h}^T Q_s \widetilde{h} \quad (14)$$

Using $\gamma = \lambda_{\min}(Q_s P_s^{-1})$ (14) becomes to:

$$\dot{L} \leq -\gamma L \quad (15)$$

Since (15), it concludes that the variables error of the variables estimator applied to estimate the nonlinear plants variables is convergent. □

C. THE PERTURBATIONS ESTIMATOR

The goal of the perturbations estimator is that using the outlets, the perturbations of the perturbations estimator should estimate the nonlinear plant perturbations.

Since the Theorem 1, it is:

$$\lim_{T\to\infty} \left|\widetilde{h}\right| \cong 0 \quad (16)$$

Using, $\lim_{T\to\infty} \dot{\widetilde{h}} - \lim_{T\to\infty} B\varkappa \leq \lim_{T\to\infty} \overline{h}$, $\lim_{T\to\infty} \widetilde{f} \cong 0$, $\overline{h}$ as the upper bound of $\dot{\widetilde{h}}$, in the first equality of (4) is:

$$\begin{aligned} &\lim_{T\to\infty} A\widetilde{h} + \lim_{T\to\infty} \widetilde{f} + \lim_{T\to\infty} Bu - \lim_{T\to\infty} K\widetilde{y} \\ &\quad - \lim_{T\to\infty} Esat(M\widetilde{y}) = \lim_{T\to\infty} \dot{\widetilde{h}} - \lim_{T\to\infty} B\varkappa \leq \lim_{T\to\infty} \overline{h} \\ &\Longrightarrow \lim_{T\to\infty} Bu - \lim_{T\to\infty} K\widetilde{y} - \lim_{T\to\infty} Esat(M\widetilde{y}) \leq \lim_{T\to\infty} \overline{h} \\ &\Longrightarrow \lim_{T\to\infty} u \leq \lim_{T\to\infty} B^{-1}\left[K\widetilde{y} + Esat(M\widetilde{y}) + \overline{h}\right] \end{aligned} \quad (17)$$

B^{-1} as the pseudo-inverse of B. Since all the terms of (17) are bounded independently of T, it becomes to:

$$\widehat{u} \cong B^{-1}\left[K\widetilde{y} + Esat(M\widetilde{y}) + \overline{h}\right] \quad (18)$$

$\widehat{u}$ as the perturbations of the perturbations estimator, u as the nonlinear plant perturbations, $\overline{h}$ as the upper bound of $\dot{\widetilde{h}} - B\varkappa$.

Remark 1: Note that the usage of the map $sat(\cdot)$ yields that the derivative $\dot{\widetilde{h}}$ does not tend to zero even the map $\widetilde{h}$ tends to zero, but it could be said that $\dot{\widetilde{h}}$ is bounded, this fact is used to express $\widehat{u}$ as the estimated perturbations of u.

Remark 2: Note that in (2), (18), $\widehat{u}$ only allows to estimate perturbations u, and since in this article the map $sat(\cdot)$ is used, it can reduce the unwanted chattering. The possibility of changing the non-continuous map $sat(\cdot)$ for a softer one also could reduce the chattering.

The Figure 1 shows the variables estimator and perturbations estimator for the variables and perturbations estimation. The variables estimator of (2), (3) and perturbations estimator of (2), (18) are termed as the estimator of (2), (3), (18).

Remark 3: Note it is not required that the nonlinear plant (1) must be convergent to achieve a satisfactory performance in our estimator.

Remark 4: For the satisfactory operation of the estimator (2), (3), (18), the theory and application conditions mentioned below must be met: a) propose a gain K that meets the theory condition (5) such that the variables $\widehat{h}$ of the variables estimator (2), (3) must reach as soon as possible the nonlinear plant variables h of (1), b) propose the gain

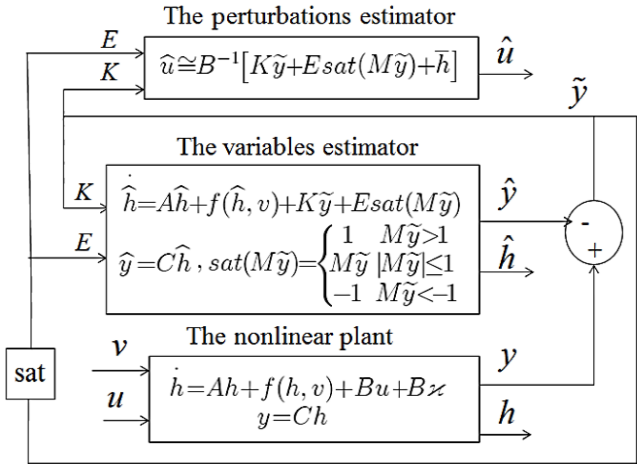

FIGURE 1. **The estimator for the perturbations estimation.**

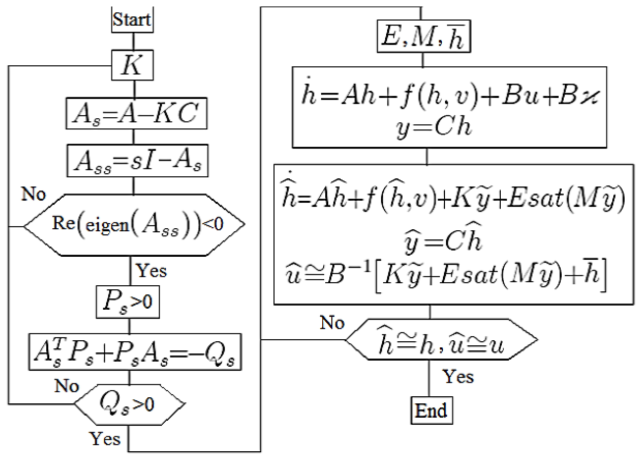

FIGURE 2. **The proposed algorithm of our estimator.**

E of the perturbations estimator (2), (18) that meets the application condition such that the estimated perturbations $\widehat{u}$ of (2), (18) should reach as soon as possible the nonlinear plant perturbations u of (1). In case that the proposed gains K, E do not work, you have to start over.

The Figure 2 shows the proposed algorithm to choose the gains K, E, M, $\overline{h}$ for achieving a satisfactory performance in our estimator, the request $Re\,(eigen\,(A_{ss})) < 0$ represents if the real part in the eigenvalues of A_{ss} are negative, the request $Q_s > 0$ represents if Q_s is positive definite, and the request $\widehat{h} \cong h, \widehat{u} \cong u$ represents of $\widehat{h}, \widehat{u}$ achieve a satisfactory performance in the estimation of h, u. The proposed algorithm of Figure 2 detailed as follows: 1) we choose a value for K, 2) we obtain $A_s = A - KC$, 3) we obtain the eigenvalues of $A_{ss} = sI - A_s$, if the real parts af all the eigenvalues of A_{ss} are negative, then the gain K is correctly chosen and we can go to the step 4, otherwise, we must return to the step 1, 4) we choose P_s, 5) we substitute P_s and A_s into the Lyapunov equation $A_s^T P_s + P_sA_s = -Q_s$, and we find Q_s, if P_s and Q_s are positive definite, then P_s is correctly chosen, the Theorem 1 is met, and we can go to the step 6, otherwise, we must return to step 1, 6) we choose the gains E, M, and $\overline{h}$ to solve the proposed estimator as $\dot{\widehat{h}} = A\widehat{h} + f(\widehat{h}, v) + K\widetilde{y} + Esat(M\widetilde{y})$, $\widehat{y} = C\widehat{h}$, $\widehat{u} \cong B^{-1}\left[K\widetilde{y} + Esat(M\widetilde{y}) + \overline{h}\right]$, if the proposed estimator reaches an acceptable exactness for the variables and perturbations estimation in the nonlinear plant $\dot{h} = Ah + f(h, v) + Bu + B\varkappa$, $y = Ch$, then the algorithm finishes, otherwise, we must return to step 6.

In the below sections, the root mean squared error (MSE) is used for comparisons, it is:

$$J = \left(\frac{1}{T}\int_0^T h^2 u\tau\right)^{\frac{1}{2}} \tag{19}$$

$J = J_h$, $h^2 = \widetilde{h}^2$ for variables, $J = J_y$, $h^2 = \widetilde{y}^2$ for outlets, $J = J_u$, $h^2 = \widetilde{u}^2 = (u - \widehat{u})^2$ for perturbations.

III. THE GAS TURBINE PLANT

The Figure 3 shows the gas turbine plant.

FIGURE 3. **The gas turbine plant.**

The gas turbine plant consists of a compressor, a combustion chamber, a turbine, and a power turbine. T_3, T_4, T_5, T_6 as the temperatures in the different stages, M_3, M_4, M_5, M_6 as the amounts of mass. m_f as the flow rate of the fuel. The inlets are $v_1 = m_f$, $v_2 = T_2$, $v_3 = P_2$, the variables are $h_1 = m_3$, $h_2 = m_4$, $h_3 = m_5$, $h_4 = m_6$, $h_5 = T_3$, $h_6 = T_4$, $h_7 = T_5$, $h_8 = T_6$, $h_9 = P_3$, $h_{10} = P_4$, $h_{11} = P_5$, $h_{12} = P_6$, and the outlet is $y = P_6$. Table 1 shows the gas turbine plant constants.

The gas turbine plant is represented in the form (1) with $A = \text{diag}[a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}]$, $a_1 = -1$, $a_2 = -1$, $a_3 = -1$, $a_4 = -1$, $a_5 = -\frac{R}{C_v}$, $a_6 = -\frac{R}{C_{vg}}$, $a_7 = -\frac{R}{C_v}$, $a_8 = -\frac{R}{C_v}$, $a_9 = -\frac{R}{V}$, $a_{10} = -DP$, $a_{11} = -\frac{R}{V}$,

TABLE 1. The gas turbine plant constants.

Constant	Constant
$MFDP = 1 \times 10^{-2}$kg/s	$m_a = 76$kg/s
$m_{3cor} = 7.731$kg/s	$T_{2ref} = 288.15$K
$C_p = 1005$(J/kg)K	$\eta_c = 1.46966$
$C_v = 718$(J/kg)K	$V = 0.8\text{m}^3$
$R = 287$(J/kg)K	$\eta_b = 0.99$
$LHV = 44124$(J/kg)K	$\eta_t = 1 \times 10^{-4}$
$P_{2ref} = 103.125$Pa	$DP = 0.99$Pa
$m_g = 32.94229$kg/s	$\eta_{pt} = 1 \times 10^{-4}$
$P_{4ref} = 680.125$Pa	$T_{4ref} = 1400$K
$C_{vg} = 863$(J/kg)K	$\gamma = 1.4$
$P_{5ref} = 416.0$Pa	$T_{5ref} = 1000$K
$m_{5cor} = 2.7184$kg/s	$\sigma = 1 \times 10^{-6}$
$m_{6cor} = 0.3912$kg/s	$\gamma_g = 1.33$

$a_{12} = -\frac{R}{V}$, $f(h,v) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}]^T$, $C = [0,0,0,0,0,0,0,0,0,0,0,1]$, $h = [h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}, h_{12}]^T$, $v = [v_1, v_2, v_3]^T$, $B = [1,1,1,1,1,1,1,1,1,1,1,1]^T$, u as the perturbation, the terms of $f(h,v)$ are [25]:

$$f_1 = MFDPm_{3cor}\left(\frac{v_3/P_{2ref}}{\left(v_2/T_{2ref}\right)^{0.5}}\right)$$
$$f_2 = (h_1 + v_1)$$
$$f_3 = MFDPm_{5cor}\left(\frac{h_{10}/P_{4ref}}{\left(h_6/T_{4ref}\right)^{0.5}}\right)$$
$$f_4 = MFDPm_{6cor}\left(\frac{h_{11}/P_{5ref}}{\left(h_7/T_{5ref}\right)^{0.5}}\right)$$
$$f_5 = \frac{1}{h_1 C_v}\left\{MFDPm_{3cor}\left(\frac{v_3/P_{2ref}}{\left(v_2/T_{2ref}\right)^{0.5}}\right) * \left(C_p v_2\left\{1+\frac{1}{\eta_c}\left[\left(\frac{h_9}{v_3}\right)^{\frac{\gamma-1}{\gamma}}-1\right]\right\} - C_v h_5\right)\right\}$$
$$f_6 = \frac{(C_p h_6 - C_{vg} h_6)m_a + (LHV\eta_b - C_{vg}h_6)v_1 - m_g R h_6 + R h_2 h_6}{h_2 C_{vg}}$$
$$f_7 = \frac{1}{h_3 C_v}\left\{MFDPm_{5cor}\left(\frac{h_{10}/P_{4ref}}{\left(h_6/T_{4ref}\right)^{0.5}}\right) * \left(C_p h_6\left\{1+\eta_t\left[1-\left(\frac{h_{11}}{h_{10}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]\right\} - C_v h_7\right)\right\}$$
$$f_8 = \frac{1}{h_4 C_v}\left\{MFDPm_{6cor}\left(\frac{h_{11}/P_{5ref}}{\left(h_7/T_{5ref}\right)^{0.5}}\right) * \left(C_p h_7\left\{1+\eta_{pt}\left[1-\left(\frac{h_{12}}{h_{11}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]\right\} - C_v h_8\right)\right\}$$
$$f_9 = \frac{R}{V}\left(h_9 + h_1\dot{h}_5 + h_5\dot{h}_1\right)$$
$$f_{10} = DP\left(h_{10} + \dot{h}_9\right)$$
$$f_{11} = \frac{R}{V}\left(h_{11} + h_3\dot{h}_7 + h_7\dot{h}_3\right)$$
$$f_{12} = \frac{R}{V}\left(h_{12} + h_4\dot{h}_8 + h_8\dot{h}_4\right)$$

A. RESULTS

In this sub-section, the estimator of this research called Estimator 1 is compared against the estimator of [7], [8], called Estimator 2. The initial conditions of the plant are $h_0 = [75.9, 75.9, 75.9, 75.9, 691.7694, 1.6164 \times 10^3, 1.2708 \times 10^3, 932.4205, 2.0561\times10^6, 2.0355\times10^6, 5.7420\times10^5, 1.2679\times10^5]^T$. The goal of the estimators is that the variables $\widehat{h}$ of the variables estimator have to reach the plant variables h and that the perturbations $\widehat{u}$ of the perturbations estimator have to reach the plant perturbations u. The plant inlets are $v_1 = 1.6 + 0.1\sin(2T)$ kg/s, $v_2 = 288 + 1\sin(2T)$ K, and $v_3 = 1.013 \times 10^5 + 1\sin(2T)$ Pa from 0 s to 20 s. The perturbation is $u = 7.6923 \times 10^{-3}\sin(2T)$ from 0 s to 20 s. The noise is $\varkappa = 0.1$ rand from 0 s to 20 s. rand are random values between 0 and 1.

Estimator 2 is expressed as [7], [8] with initial conditions as $\widehat{h}_0 = [76, 76, 76, 76, 691, 1.616\times10^3, 1.27\times10^3, 932, 2.05\times10^6, 2.03\times10^6, 5.74\times10^5, 1.24\times10^5]^T$ and gains as $L = [4\times10^{-8}, 4\times10^{-8}, 2\times10^{-8}, 2\times10^{-8}, 2\times10^{-8}, 4\times10^{-8}, 4\times10^{-8}, 4\times10^{-8}, 6\times10^{-6}, 6\times10^{-6}, 6\times10^{-6}, 6\times10^{-6}]^T$.

Estimator 1 is expressed as (2), (3), (18) with $\widehat{h} = [\widehat{h}_1, \widehat{h}_2, \widehat{h}_3, \widehat{h}_4, \widehat{h}_5, \widehat{h}_6, \widehat{h}_7, \widehat{h}_8, \widehat{h}_9, \widehat{h}_{10}, \widehat{h}_{11}, \widehat{h}_{12}]^T$, $f(\widehat{h}, v) = [\widehat{f}_1, \widehat{f}_2, \widehat{f}_3, \widehat{f}_4, \widehat{f}_5, \widehat{f}_6, \widehat{f}_7, \widehat{f}_8, \widehat{f}_9, \widehat{f}_{10}, \widehat{f}_{11}, \widehat{f}_{12}]^T$, the terms of $f(\widehat{h}, v)$ are:

$$\widehat{f}_1 = MFDPm_{3cor}\left(\frac{v_3/P_{2ref}}{\left(v_2/T_{2ref}\right)^{0.5}}\right)$$
$$\widehat{f}_2 = (\widehat{h}_1 + v_1)$$
$$\widehat{f}_3 = MFDPm_{5cor}\left(\frac{\widehat{h}_{10}/P_{4ref}}{\left(\widehat{h}_6/T_{4ref}\right)^{0.5}}\right)$$
$$\widehat{f}_4 = MFDPm_{6cor}\left(\frac{\widehat{h}_{11}/P_{5ref}}{\left(\widehat{h}_7/T_{5ref}\right)^{0.5}}\right)$$
$$\widehat{f}_5 = \frac{1}{\widehat{h}_1 C_v}\left\{MFDPm_{3cor}\left(\frac{v_3/P_{2ref}}{\left(v_2/T_{2ref}\right)^{0.5}}\right) * \left(C_p v_2\left\{1+\frac{1}{\eta_c}\left[\left(\frac{\widehat{h}_9}{v_3}\right)^{\frac{\gamma-1}{\gamma}}-1\right]\right\} - C_v\widehat{h}_5\right)\right\}$$
$$\widehat{f}_6 = \frac{(C_p\widehat{h}_6 - C_{vg}\widehat{h}_6)m_a + (LHV\eta_b - C_{vg}\widehat{h}_6)v_1 - m_g R\widehat{h}_6 + R\widehat{h}_2\widehat{h}_6}{\widehat{h}_2 C_{vg}}$$
$$\widehat{f}_7 = \frac{1}{\widehat{h}_3 C_v}\left\{MFDPm_{5cor}\left(\frac{\widehat{h}_{10}/P_{4ref}}{\left(\widehat{h}_6/T_{4ref}\right)^{0.5}}\right) * \left(C_p\widehat{h}_6\left\{1+\eta_t\left[1-\left(\frac{\widehat{h}_{11}}{\widehat{h}_{10}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]\right\} - C_v\widehat{h}_7\right)\right\}$$

$$\widehat{f}_8 = \frac{1}{\widehat{h}_4 C_v}\left\{MFDPm_{6cor}\left(\frac{\widehat{h}_{11}/P_{5ref}}{\left(\widehat{h}_7/T_{5ref}\right)^{0.5}}\right)\right.$$
$$\left. * \left(C_p\widehat{h}_7\left\{1+\eta_{pt}\left[1-\left(\frac{\widehat{h}_{12}}{\widehat{h}_{11}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]\right\}-C_v\widehat{h}_8\right)\right\}$$
$$\widehat{f}_9 = \frac{R}{V}\left(\widehat{h}_9+\widehat{h}_1\dot{\widehat{h}}_5+\widehat{h}_5\dot{\widehat{h}}_1\right)$$
$$\widehat{f}_{10} = DP\left(\widehat{h}_{10}+\dot{\widehat{h}}_9\right)$$
$$\widehat{f}_{11} = \frac{R}{V}\left(\widehat{h}_{11}+\widehat{h}_3\dot{\widehat{h}}_7+\widehat{h}_7\dot{\widehat{h}}_3\right)$$
$$\widehat{f}_{12} = \frac{R}{V}\left(\widehat{h}_{12}+\widehat{h}_4\dot{\widehat{h}}_8+\widehat{h}_8\dot{\widehat{h}}_4\right)$$

the initial conditions are $\widehat{h}_0 = [76, 76, 76, 76, 691, 1.616 \times 10^3, 1.27 \times 10^3, 932, 2.05 \times 10^6, 2.03 \times 10^6, 5.74 \times 10^5, 1.24 \times 10^5]^T$.

The gains $K \in \Re^{12}$, $E \in \Re^{12}$, $M \in \Re^{12}$, and $\overline{h} \in \Re$ for the Estimator 1 are chosen using the proposed algorithm of Figure 2 detailed as follows: 1) we choose a value for $K = [4\times10^{-9}, 4\times10^{-9}, 2\times10^{-9}, 2\times10^{-9}, 2\times10^{-9}, 4\times10^{-9}, 4\times10^{-9}, 4\times10^{-9}, 6\times10^{-7}, 6\times10^{-7}, 6\times10^{-7}, 6\times10^{-7}]^T$, 2) we obtain $A_s = A - KC = [a_{sij}] \in \Re^{12\times12}$, $a_{s11} = -1$, $a_{s22} = -1$, $a_{s33} = -1$, $a_{s44} = -1$, $a_{s55} = -\frac{287}{718}$, $a_{s66} = -\frac{287}{863}$, $a_{s77} = -\frac{287}{718}$, $a_{s88} = -\frac{287}{718}$, $a_{s99} = -358.75$, $a_{s1010} = -0.99$, $a_{s1111} = -358.75$, $a_{s1212} = -358.75$, $a_{s112} = -\frac{1}{500000000}$, $a_{s212} = -\frac{1}{500000000}$, $a_{s312} = -\frac{1}{500000000}$, $a_{s412} = -\frac{1}{500000000}$, $a_{s512} = -\frac{1}{500000000}$, $a_{s612} = -\frac{1}{500000000}$, $a_{s712} = -\frac{1}{500000000}$, $a_{s812} = -\frac{1}{500000000}$, $a_{s912} = -\frac{3}{500000000}$, $a_{s1012} = -\frac{3}{500000000}$, $a_{s1112} = -\frac{3}{500000000}$, the other terms of A_s have a value of 0, 3) we obtain the eigenvalues of $A_{ss} = sI - A_s$ as $-0.42028+0.11615i$, $-0.42028-0.11615i$, -347.89, -1.3013, -18.240, $-364.18-9.7741i$, $-364.18+9.7741i$, $-0.31231-4.1895\times10^{-2}i$, $-0.31231+4.1895\times10^{-2}i$, $-0.75333+0.25701i$, $-0.75333-0.25701i$, -1.124, if the real parts of all the eigenvalues of A_{ss} are negative, then the gain $K \in \Re^{12}$ is correctly chosen and we can go to the step 4, otherwise, we must return to the step 1, 4) we choose $P_s = [p_{ij}] \in \Re^{12\times12}$, $p_{11} = 0.5$, $p_{22} = 0.5$, $p_{33} = 0.5$, $p_{44} = 0.5$, $p_{55} = 1.2509$, $p_{66} = 1.5035$, $p_{77} = 1.2509$, $p_{88} = 1.2509$, $p_{99} = 1.3937\times10^{-3}$, $p_{1010} = 0.50505$, $p_{1111} = 1.3937\times10^{-3}$, $p_{1212} = 1.3937\times10^{-3}$, the other terms of P_s have a value of 0, 5) we substitute $P_s = [p_{ij}] \in \Re^{12\times12}$ and $A_s = A - KC$ into the Lyapunov equation $A_s^T P_s + P_s A_s = -Q_s$, and we find $Q_s = [q_{ij}] \in \Re^{12\times12}$, $q_{11} = 1.0$, $q_{22} = 1.0$, $q_{33} = 1.0$, $q_{44} = 1.0$, $q_{55} = 1.0$, $q_{66} = 1.0$, $q_{77} = 1.0$, $q_{88} = 1.0$, $q_{99} = 0.99998$, $q_{1010} = 1.0$, $q_{1111} = 0.99998$, $q_{1212} = 0.99998$, the other terms of Q_s have a value of 0, if $P_s = [p_{ij}] \in \Re^{12\times12}$ and $Q_s = [q_{ij}] \in \Re^{12\times12}$ are positive definite, then $P_s = [p_{ij}] \in \Re^{12\times12}$ is correctly chosen, the Theorem 1 is met, and we can go to the step 6, otherwise, we must return to step 1, 6) we choose the gains $E = [1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}, 1\times10^{-10}]^T$, $M = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]^T$, and $\overline{h} = 2.14$ to solve the Estimator 1 as $\dot{\widehat{h}} = A\widehat{h} + f(\widehat{h}, v) + K\widetilde{y} + Esat(M\widetilde{y})$, $\widehat{y} = C\widehat{h}$, $\widehat{u} \cong B^{-1}\left[K\widetilde{y} + Esat(M\widetilde{y}) + \overline{h}\right]$, if the Estimator 1 reaches an acceptable exactness for the variables and perturbations estimation in the nonlinear plant $\dot{h} = Ah + f(h, v) + Bu + B\varkappa$, $y = Ch$, then the algorithm finishes, otherwise, we must return to step 6.

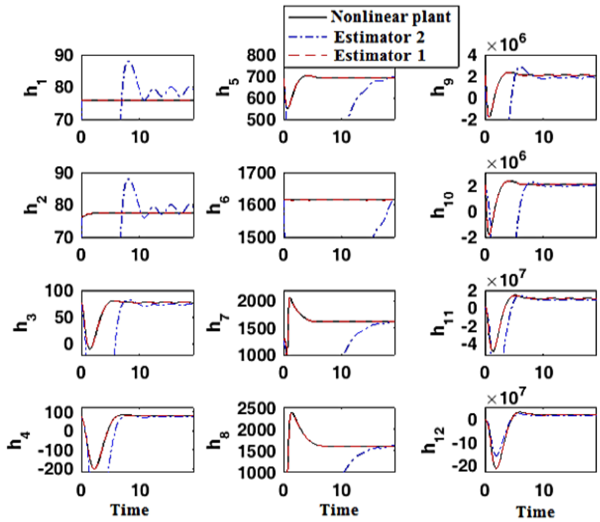

FIGURE 4. The variables estimation in the gas turbine plant.

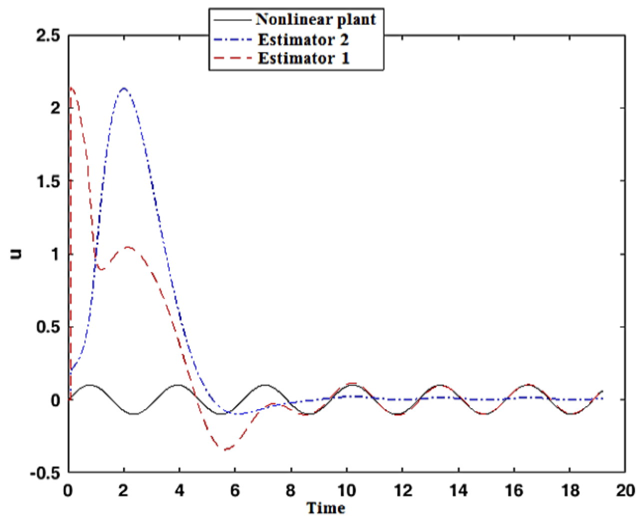

FIGURE 5. The perturbations estimation in the gas turbine plant.

The Figures 4 and 5 show the variables and perturbations estimation of the Estimator 1 and Estimator 2 applied to the gas turbine plant. The Table 2 shows the MSE of (19).

In the Figures 4 and 5, since the Estimator 1 reaches better variables and perturbations estimation of the plant than the Estimator 2, it is seen that the Estimator 1 reaches better performance. In addition, the Figures 4 and 5 show that in the variables and perturbations estimation of Estimator 1,

TABLE 2. The gas turbine plant results.

	Estimator 2	Estimator 1
J_h	4.4084	0.0255
J_y	3.2915	0.0248
J_u	0.1134	0.0387

the unwanted chattering is not presented. In the Table 2, since the MSE is smaller for the Estimator 1 than for the Estimator 2, it is seen that the Estimator 1 reaches better exactness for the variables and perturbations estimation.

IV. THE GASIFICATION PLANT

The Figure 6 shows the gasification plant.

FIGURE 6. The gasification plant.

The gasification plant model consists of drying, pyrolysis, oxidation or combustion, and reduction. C_{CO}, C_{CO_2}, C_{H_2}, C_{H_4}, C_{Tar}, C_{H_2O}, C_{O_2}, C_{N_2} as concentrations of gases CO, CO_2, H_2, H_4, Tar, H_2O, O_2, N_2 in mol/cm^3, ρ_{coal}, ρ_{char} as the coal solids densities, char in g/cm^3, T as the gases temperature in K, Ts as the solids temperature in K, u as the injected gases flow in mols/cm^2/s, δ as the steam flow in mols/cm^2/s, R_1, R_2, R_3 as the chemical reactions, m_{O_2}, m_{H_2O} as the internal molar fractions of O_2 and H_2O. The variables are $h_1 = \rho_{coal}$, $h_2 = \rho_{char}$, $h_3 = Ts$, $h_4 = C_{CO}$, $h_5 = C_{CO_2}$, $h_6 = C_{H_2}$, $h_7 = C_{H_4}$, $h_8 = C_{Tar}$, $h_9 = C_{H_2O}$, $h_{10} = C_{O_2}$, $h_{11} = C_{N_2}$, the inlet is $v = u$, the perturbation is δ, and the outlet is $y = C_{H_2O}$. Table 3 shows the gasification plant constants.

TABLE 3. The gasification plant constants.

Constant	Constant
$p_{R_1char} = 0.766$	$C_s = 7.3920$cal/g/K
$p_{R_1CO} = 0.008$	$h_t = 0.001$cal/s/K/cm^3
$p_{R_1CO_2} = 0.058$	$\beta = 7 \times 10^{-6}$mol/cm^3/s
$p_{R_1H_2} = 0.083$	$\Delta q_2 = -93929$cal/mol
$p_{R_1CH_4} = 0.044$	$\Delta q_3 = 31309.7$cal/mol
$p_{R_1Tar} = 0.0138$	$\Pi = 0.77$
$p_{R_1H_2O} = 0.055$	$\Xi = 0.154$
$p_{R_2H_2O} = 0.075$	$\Psi = 7.6$
$p_{R_3H_2O} = 0.925$	$M_1 = 54.9916$g/mol
$p_{R_2O_2} = 1.02$	$M_2 = 52.9666$g/mol
$P = 4.83$	$T = 430$K
$L = 100$cm	

The gasification plant has the form of (1) with $h = [h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}]^T$, $v = u$, $A = -\beta diag(1) \in \Re^{11\times 11}$, $diag(1)$ as a diagonal matrix with a number 1 in the main diagonal, $C = [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$, $B = [0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{L}, 0, 0]^T$, $f(h, v) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}]^T$, the terms of $f(h, v)$ are [26]:

$$
\begin{aligned}
f_1 &= -M_1 R_1 \\
f_2 &= M_2 \left[p_{R_1char} R_1 - R_2 - R_3 \right] \\
f_3 &= \frac{1}{C_s} \left[h_t \left(T - h_3 \right) - \Delta q_2 R_2 - \Delta q_3 R_3 \right] \\
f_4 &= p_{R_1CO} R_1 + R_3 \\
f_5 &= p_{R_1CO_2} R_1 + R_2 \\
f_6 &= = p_{R_1H_2} R_1 + R_3 \\
f_7 &= p_{R_1CH_4} R_1 \\
f_8 &= p_{R_1Tar} R_1 \\
f_9 &= p_{R_1H_2O} R_1 + p_{R_2H_2O} R_2 - p_{R_3H_2O} R_3 + \frac{\Pi}{L} v \\
f_{10} &= -p_{R_2O_2} R_2 + \frac{\Xi}{L} v \\
f_{11} &= \frac{\Psi}{L} v \\
R_1 &= 5 \frac{h_1}{M_1} \exp\left(\frac{-6039}{h_3} \right) \\
R_{m2} &= \frac{1}{10} h_t m_{O_2} \\
R_{c2} &= \frac{1}{M_2} \left[9.55 \times 10^8 h_2 m_{O_2} P \exp\left(\frac{-22142}{h_3} \right) h_3^{-0.5} \right] \\
R_2 &= \frac{1}{\frac{1}{R_{c2}} + \frac{1}{R_{m2}}} \\
R_{m3} &= \frac{1}{10} h_t m_{H_2O} \\
R_{c3} &= \frac{h_2 m_{H_2O}^2 P^2 \exp\left(5.052 - \frac{12908}{h_3} \right)}{M_2 \left[m_{H_2O} P + \exp\left(-22.216 + \frac{24880}{h_3} \right) \right]^2}
\end{aligned}
$$

$$R_3 = \frac{1}{\frac{1}{R_{c3}} + \frac{1}{R_{m3}}}$$
$$m_{O_2} = \frac{h_5}{C_T + h_9}$$
$$m_{H_2O} = \frac{h_9}{C_T + h_9}$$
$$C_T = h_4 + h_5 + h_6 + h_7 + h_8 + h_{10} + h_{11}$$

A. RESULTS

In this sub-section, the estimator of this research called Estimator 1 is compared against the estimator of [7], [8], called Estimator 2. The plant initial conditions are $h_0 = [0.48, 0, 480, 0, 0, 0, 0, 0, 0, 4.2 \times 10^{-4}, 1.6 \times 10^{-6}]^T$. The goal of the estimators is that the variables $\widehat{h}$ of the variables estimator have to reach the plant variables h and that the perturbations $\widehat{u}$ of the perturbations estimator have to reach the plant perturbations u. The plant inlet is $v = 0.2 \times 10^{-3}\sin(2T)$ moles/cm^2/s from 0 s to 20 s. The perturbation is $u = 1.5 \times 10^{-5}\sin(2T)$ moles/cm^2/s from 0 s to 20 s. The noise is $\varkappa = 1 \times 10^{-5}$ rand from 0 s to 20 s. rand are random values between 0 and 1.

Estimator 2 is expressed as [7], [8] with initial conditions as $\widehat{h}_0 = [0.48, 6 \times 10^0, 480, 1 \times 10^{-9}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-9}, 1 \times 10^{-7}, 4.2 \times 10^4, 1.6 \times 10^{-6}]^T$ and gains as $L = [1\times10^{-1}, 1\times10^{1}, 1\times10^{-1}, 2\times10^{-3}, 1\times10^{-2}, 2\times10^{-2}, 1\times10^{-2}, 4\times10^{-3}, 1\times10^{-2}, 1\times10^{-3}, 1\times10^{-1}]^T$.

Estimator 1 is expressed as (2), (3), (18) with $\widehat{h} = [\widehat{h}_1, \widehat{h}_2, \widehat{h}_3, \widehat{h}_4, \widehat{h}_5, \widehat{h}_6, \widehat{h}_7, \widehat{h}_8, \widehat{h}_9, \widehat{h}_{10}, \widehat{h}_{11}]^T$, $f(\widehat{h}, v) = [\widehat{f}_1, \widehat{f}_2, \widehat{f}_3, \widehat{f}_4, \widehat{f}_5, \widehat{f}_6, \widehat{f}_7, \widehat{f}_8, \widehat{f}_9, \widehat{f}_{10}, \widehat{f}_{11}]^T$, the terms of $f(\widehat{h}, v)$ are:

$$\widehat{f}_1 = -M_1\widehat{R}_1$$
$$\widehat{f}_2 = M_2\left[p_{R_1char}\widehat{R}_1 - \widehat{R}_2 - \widehat{R}_3\right]$$
$$\widehat{f}_3 = \frac{1}{C_s}\left[h_t\left(T - \widehat{h}_3\right) - \Delta q_2\widehat{R}_2 - \Delta q_3\widehat{R}_3\right]$$
$$\widehat{f}_4 = p_{R_1CO}\widehat{R}_1 + \widehat{R}_3$$
$$\widehat{f}_5 = p_{R_1CO_2}\widehat{R}_1 + \widehat{R}_2$$
$$\widehat{f}_6 = p_{R_1H_2}\widehat{R}_1 + \widehat{R}_3$$
$$\widehat{f}_7 = p_{R_1CH_4}\widehat{R}_1$$
$$\widehat{f}_8 = p_{R_1Tar}\widehat{R}_1$$
$$\widehat{f}_9 = p_{R_1H_2O}\widehat{R}_1 + p_{R_2H_2O}\widehat{R}_2 - p_{R_3H_2O}\widehat{R}_3 + \frac{\Pi}{L}v$$
$$\widehat{f}_{10} = -p_{R_2O_2}\widehat{R}_2 + \frac{\Xi}{L}v$$
$$\widehat{f}_{11} = \frac{\Psi}{L}v$$
$$\widehat{R}_1 = 5\frac{\widehat{h}_1}{M_1}\exp\left(\frac{-6039}{\widehat{h}_3}\right)$$
$$\widehat{R}_{m2} = \frac{1}{10}h_t\widehat{m}_{O_2}$$
$$\widehat{R}_{c2} = \frac{1}{M_2}\left[9.55 \times 10^8\widehat{h}_2 m_{O_2}P\exp\left(\frac{-22142}{\widehat{h}_3}\right)\widehat{h}_3^{-0.5}\right]$$
$$\widehat{R}_2 = \frac{1}{\frac{1}{\widehat{R}_{c2}} + \frac{1}{\widehat{R}_{m2}}}$$
$$\widehat{R}_{m3} = \frac{1}{10}h_t\widehat{m}_{H_2O}$$
$$\widehat{R}_{c3} = \frac{\widehat{h}_2\widehat{m}_{H_2O}^2P^2\exp\left(5.052 - \frac{12908}{\widehat{h}_3}\right)}{M_2\left[\widehat{m}_{H_2O}P + \exp\left(-22.216 + \frac{24880}{\widehat{h}_3}\right)\right]^2}$$
$$\widehat{R}_3 = \frac{1}{\frac{1}{\widehat{R}_{c3}} + \frac{1}{\widehat{R}_{m3}}}$$
$$\widehat{m}_{O_2} = \frac{\widehat{h}_5}{\widehat{C}_T + \widehat{h}_9}$$
$$\widehat{m}_{H_2O} = \frac{\widehat{h}_9}{\widehat{C}_T + \widehat{h}_9}$$
$$\widehat{C}_T = \widehat{h}_4 + \widehat{h}_5 + \widehat{h}_6 + \widehat{h}_7 + \widehat{h}_8 + \widehat{h}_{10} + \widehat{h}_{11}$$

the initial conditions as $\widehat{h}_0 = [0.48, 1 \times 10^{-5}, 480, 1 \times 10^{-9}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-8}, 1 \times 10^{-9}, 1 \times 10^{-7}, 4.2 \times 10^4, 1.6 \times 10^{-6}]^T$.

The gains $K \in \Re^{11}$, $E \in \Re^{11}$, $M \in \Re^{11}$, and $\overline{h} \in \Re$ for the Estimator 1 are chosen using the proposed algorithm of Figure 2 detailed as follows: 1) we choose a value for $K = [1\times10^{-1}, 1\times10^{0}, 1\times10^{-1}, 1\times10^{-4}, 1\times10^{-4}, 1\times10^{-4}, 1 \times 10^{-4}, 1 \times 10^{-4}, 6 \times 10^{-1}, 1 \times 10^{-2}, 1 \times 10^{-1}]^T$, 2) we obtain $A_s = A - KC = [a_{sij}] \in \Re^{11\times11}$, $a_{s11} = -\frac{7}{1000000}$, $a_{s22} = -\frac{7}{1000000}$, $a_{s33} = -\frac{7}{1000000}$, $a_{s44} = -\frac{7}{1000000}$, $a_{s55} = -\frac{7}{1000000}$, $a_{s66} = -\frac{7}{1000000}$, $a_{s77} = -\frac{7}{1000000}$, $a_{s88} = -\frac{7}{1000000}$, $a_{s99} = -\frac{600007}{1000000}$, $a_{s1010} = -\frac{7}{1000000}$, $a_{s1111} = -\frac{7}{1000000}$, $a_{s19} = -\frac{1}{10}$, $a_{s29} = -1$, $a_{s39} = -\frac{1}{10}$, $a_{s49} = -\frac{1}{10000}$, $a_{s59} = -\frac{1}{10000}$, $a_{s69} = -\frac{1}{10000}$, $a_{s79} = -\frac{1}{10000}$, $a_{s89} = -\frac{1}{10000}$, $a_{s109} = -\frac{1}{100}$, $a_{s119} = -\frac{1}{10}$, the other terms of A_s have a value of 0, 3) we obtain the eigenvalues of $A_{ss} = sI - A_s$ as -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -7.0×10^{-6}, -0.60001, if the real parts of all the eigenvalues of A_{ss} are negative, then the gain $K \in \Re^{12}$ is correctly chosen and we can go to the step 4, otherwise, we must return to the step 1, 4) we choose $P_s = [p_{ij}] \in \Re^{11\times11}$, $p_{11} = 1$, $p_{22} = 71429$, $p_{33} = 1$, $p_{44} = 1$, $p_{55} = 1$, $p_{66} = 1$, $p_{77} = 1$, $p_{88} = 1$, $p_{99} = 1\times10^8$, $p_{1010} = 1$, $p_{1111} = 1$, $p_{19} = p_{91} = -0.125$, $p_{29} = p_{92} = -375$, $p_{39} = p_{93} = -0.125$, $p_{49} = p_{94} = -8.7499 \times 10^{-2}$, $p_{59} = p_{95} = -0.62499$, $p_{69} = p_{96} = -8.7499\times10^{-2}$, $p_{79} = p_{97} = -0.5$, $p_{89} = p_{98} = -0.125$, $p_{109} = p_{910} = -0.125$, $p_{119} = p_{911} = -0.125$, the other terms of P_s have a value of 0, 5) we substitute $P_s = [p_{ij}] \in \Re^{11\times11}$ and $A_s = A - KC$ into the Lyapunov equation $A_s^T P_s + P_s A_s = -Q_s$, and we find $Q_s = [q_{ij}] \in \Re^{11\times11}$, $q_{11} = 1.4 \times 10^{-5}$, $q_{22} = 1.4 \times 10^{-5}$, $q_{33} = 1.4 \times 10^{-5}$, $q_{44} = 1.4 \times 10^{-5}$, $q_{55} = 1.4 \times 10^{-5}$, $q_{66} = 1.4 \times 10^{-5}$, $q_{77} = 1.4 \times 10^{-5}$, $q_{88} = 1.4 \times 10^{-5}$, $q_{99} = 1.5978\times10^8$, $q_{1010} = 1.4\times10^{-5}$, $q_{1111} = 1.4\times10^{-5}$, $q_{19} = q_{91} = -8.7499\times10^{-7}$, $q_{29} = q_{92} = -2.6250\times10^{-3}$, $q_{39} = q_{93} = -8.7499\times10^{-7}$, $q_{49} = q_{94} = -6.1249\times10^{-7}$, $q_{59} = q_{95} = -4.3750 \times 10^{-6}$, $q_{69} = q_{96} = 0.63$, $q_{79} = q_{97} = -3.5000\times10^{-6}$, $q_{89} = q_{98} = -8.7499\times10^{-7}$,

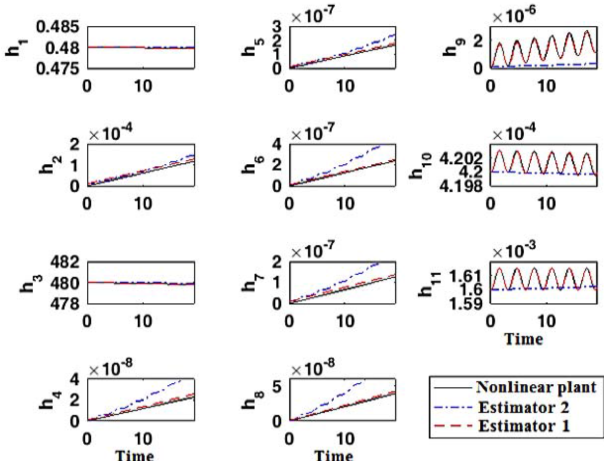

FIGURE 7. The variables estimation in the gasification plant.

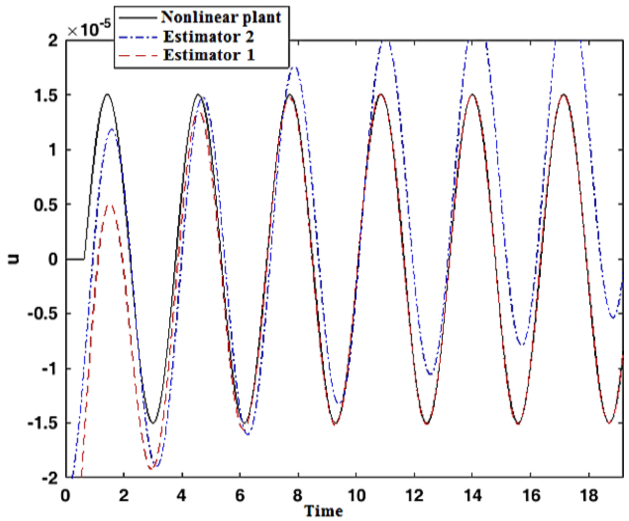

FIGURE 8. The perturbations estimation in the gasification plant.

TABLE 4. The gasification plant results.

	Estimator 2	Estimator 1
J_h	0.0529	7.5237×10^{-6}
J_y	9.5523×10^{-7}	6.4086×10^{-8}
J_u	1.1859×10^{-5}	1.0118×10^{-5}

$q_{109} = q_{910} = -8.7499 \times 10^{-7}$, $q_{119} = q_{911} = -8.7499 \times 10^{-7}$, the other terms of Q_s have a value of 0, if $P_s = [p_{ij}] \in \Re^{11\times 11}$ and $Q_s = [q_{ij}] \in \Re^{11\times 11}$ are positive definite, then $P_s = [p_{ij}] \in \Re^{11\times 11}$ is correctly chosen, the Theorem 1 is met, and we can go to the step 6, otherwise, we must return to step 1, 6) we choose the gains $E = [1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}, 1 \times 10^{-10}]^T$, $M = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]^T$, and $\overline{h} = 1.73 \times 10^{-5}$ to solve the Estimator 1 as $\dot{\widehat{h}} = A\widehat{h} + f(\widehat{h}, v) + K\widetilde{y} + Esat(M\widetilde{y})$, $\widehat{y} = C\widehat{h}$, $\widehat{u} \cong B^{-1}\left[K\widetilde{y} + Esat(M\widetilde{y}) + \overline{h}\right]$, if the Estimator 1 reaches an acceptable exactness for the variables and perturbations estimation in the nonlinear plant $\dot{h} = Ah + f(h, v) + Bu + B\varkappa$, $y = Ch$, then the algorithm finishes, otherwise, we must return to step 6.

The Figures 7 and 8 show the variables and perturbations estimation of the Estimator 1 and Estimator 2 applied to the gasification plant. The Table 4 shows the MSE of (19).

From Figures 7 and 8, since the Estimator 1 reaches better variables and perturbations estimation of the plant than the Estimator 2, it is seen that the Estimator 1 reaches better performance. In addition, the Figures 7 and 8 show that in the variables and perturbations estimation of Estimator 1, the unwanted chattering is not presented. In the Table 4, since the MSE is smaller for the Estimator 1 than for the Estimator 2, it is seen that the Estimator 1 reaches better exactness for the variables and perturbations estimation.

Remark 5: In the past two sections, it would be very tedious and expensive to have the sensors to measure all plant variables. Consequently, it highlights one of the major contributions of this article, it is that with the outlets measurement, our estimator can roughly estimate the variables and perturbations.

V. CONCLUSION

In this article, an estimator was studied for the variables and perturbations estimation in nonlinear plants. The variables error convergence was analyzed with the Lyapunov approach. Our estimator was compared to a previous estimator in the gas turbine and gasification plants concluding that our estimator reached a better performance than the previous estimator for the variables and perturbations estimation. In addition, in our estimator, the unwanted chattering is not presented. Our estimator can be applied to many types of nonlinear plants such as electric, mechanical, hydraulic or thermal. In the future work, we will seek to use another alternative map that allows us to reduce the unwanted chattering for this type of estimators, we will explore other types of strategies for the perturbations estimation, for the perturbations attenuation, or for trajectories reaching in perturbed plants.

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JOSÉ DE JESÚS RUBIO (Member, IEEE) is currently a Full Time Professor with the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional. He has published over 136 international journal articles with 2071 cites from Scopus. He has been the Tutor of 4 P.Ph.D. students, 19 Ph.D. students, 42 M.S. students, 4 S. students, and 17 B.S. students. He serves in the Editorial Board for the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, the IEEE TRANSACTIONS ON FUZZY SYSTEMS, *Neural Computing and Applications*, *Frontiers in Neurorobotics*, the *Journal of Intelligent and Fuzzy Systems*, *Mathematical Problems in Engineering*, the *International Journal of Advanced Robotic Systems*, the IEEE LATIN AMERICA TRANSACTIONS, *Evolving Systems*, and the *International Journal of Business Intelligence and Data Mining*. He is a Guest Editor of *Neurocomputing*, in 2019, *Applied Soft Computing*, in 2019, the *International Journal of Applied Mathematics and Computer Science*, in 2019, the *Journal of Computational and Applied Mathematics*, in 2019, the *Journal of Supercomputing*, in 2019, and *Computational Intelligence and Neuroscience*, from 2015 to 2016.

DANY IVAN MARTINEZ is currently pursuing the Ph.D. degree with the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional. He has published three articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.

VICTOR GARCIA is currently pursuing the Ph.D. degree with the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional. He has published two articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.

GUADALUPE JULIANA GUTIERREZ is currently a Full Time Professor with the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional. She has published ten articles in international journals. Her fields of interest are robotic systems, modeling, and intelligent systems.

TOMAS MIGUEL VARGAS is currently pursuing the Ph.D. degree with the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional. He has published two articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.

GENARO OCHOA received the Ph.D. degree from the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional, in 2017. He has published nine articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.

RICARDO BALCAZAR received the Ph.D. degree from the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional, in 2017. He has published eight articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.

JAIME PACHECO is currently a Full Time Professor with the Sección de Estudios de Posgrado e Investigación, ESIME Azcapotzalco, Instituto Politécnico Nacional. He has published 25 articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.

JESUS ALBERTO MEDA-CAMPAÑA (Member, IEEE) is currently a Full Time Professor with the Sección de Estudios de Posgrado e Investigación, ESIME Zacatenco, Instituto Politécnico Nacional. He has published 30 articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.

DANTE MUJICA-VARGAS is currently a Full Time Professor with the Tecnológico Nacional de México/CENIDET. He has published 15 articles in international journals. His fields of interest are robotic systems, modeling, and intelligent systems.