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A Novel Method of Evidential Network Reasoning Based on the Logical Reasoning Rules and Conflict Measure

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ABSTRACT Evidential reasoning satisfies a weaker condition than probability theory and can deal with uncertain scenarios. However, there is still no complete and consistent theoretical system in the evidential reasoning now. In this paper, a novel method of evidential network reasoning based on the logical reasoning rules and conflict measure is proposed. The state of nodes is described through several basic probability assignments(BPAs). Two logical reasoning rules are defined to show that the occurrence of some antecedents in the parent nodes will lead to the sub-BPAs of the child node occurrence. Then the occurrence probability of sub-BPA with the child node is computed through antecedent probabilities in the parent nodes. Besides, the support degree of sub-BPAs is computed based on the evidential distance. The final BPA of the child node is obtained by combining weighted sub-BPAs. Two examples about fault diagnosis and threat assessment and prediction are given respectively to illustrate the effectiveness of the proposed method. Different from the previous work, this paper defines a new reasoning rule called OR rule, and the computation based on pure logic makes the reasoning process more intuitive. Besides, the weights of sub-BPAs are computed considering both the occurrence probability and conflict measure.

INDEX TERMS Evidential network, reasoning rules, occurrence probability, fault diagnosis, threat assessment.

I. INTRODUCTION

How to inference properly under the uncertain scenario is still an open issue. Bayesian network based on probabilistic reasoning is one of the effective methods. In recent years, it has been a hotspot and has also been applied in many fields [1]–[4]. Evidence theory, firstly proposed by Dempster [5], is an extension of Bayesian probabilistic theory and satisfies weaker conditions than probabilistic theory. It can distinguish the uncertain and unknown scenarios. Many researches have been conducted about it, especially in conflict management [6], [7] and uncertainty measure [8]–[10]. Evidential network, firstly proposed by Xu Hong and Smets in 1994 [11], extends Bayesian network by introducing the belief function instead of Bayesian probabilities. Due to the advantages of evidence theory, evidential network can inference properly under the more uncertain scenario. Moreover, evidential network inherits the graph structure of the Bayesian network, which makes the reasoning process intuitive.

Many researches about evidential network [12]–[17] and the evidential reasoning approach [18]–[21] have been conducted in the past few years. Simon and Weber combined Bayesian network and evidence theory to deal with the reliability problems [22]–[26]. An alternative approach to evidential network construction based on operator of composition of basic assignments was proposed [27]. A BeliefNet tool, which provided an effective tool for dealing with the algorithm of evidential network, was developed by Trabelsi and Yaghlane [28]. A dynamic evidential network that combined evidence theory and interval numbers was applied for fault diagnosis of complex systems [29]. A model of evidential network based on Dezert-Smarandache theory to improve target identification of multi-sensors was proposed [30]. Evidential network with fuzzy sets was applied as medical prognosis and diagnosis models [31].

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Besides, evidential network has also been preliminarily applied in many fields, such as threat assessment [32], [33], hypothesis resolution [34], intelligent control [35], [36], quality inspection [37], home-based care [38], detection [39] and so on. For example, Benavoli *et al.* modeled threat by a network of entities and relationships between them, while the uncertainties in the relationships were represented by belief functions as defined in the theory of evidence [32]. For a better system safety assessment, an innovative heuristic approach was developed to determine the prior belief masses based on the prior imprecise probabilities [33]. Jaunzemis *et al.* applied judicial evidential reasoning for hypothesis resolution [34]. In their method, Dempster-Shafer theory was applied to model hypothesis knowledge and quantify ambiguity, and an equal-effort heuristic was proposed to balance time-efficiency and impartiality. X.Hong *et al.* combined Dempster-Shafer theory of evidence and the Equally Weighted Sum operator, then evidential contextual information was represented, analysed and merged to achieve a consensus in automatically inferring activities of daily living for inhabitants in Smart Homes [35]. In order to identify deviations from normal operation of a cyber-physical systems, Friedberg *et al.* used novel approaches to integrate low-level sensors of different types, in particular those for cyber-attack detection, and reliability into evidential networks [36]. Lee *et al.* proposed a context-reasoning method with home-based care to process sensor data with an evidential form based on the Dezert-Smarandache theory (DSmT) [38].

Although many researches about evidential network have been conducted, there are still some open issues. Firstly, there is no consistent evidential reasoning rule, so that a method can often only deal with individual cases. Secondly, the computation complexity of reasoning and learning will increase with the increasing of the network complexity because the number of parameters is large. Thirdly, some counterintuitive fusion results will occur when evidences are in high conflict or disagreement. Fourthly, most methods of evidential reasoning cannot apply for dynamic systems varying with time.

In the previous study, an evidential network approach extended by belief rules and uncertainty measures was proposed. A novel evidential network was defined and a novel and effective framework for dependence assessment in human reliability analysis was presented. The maximum entropy principle was used to derive basic probability assignments in the reasoning process [40]. In this paper, based on the new evidential network, the authors further propose a novel method of evidential network reasoning based on the pignistic probability and evidence distance. Mainly, each parent node of the network is related to a BPA and the child node is related to several sub-BPAs. Two reasoning rules, called AND rule and OR rule respectively, are defined. The authors use pignistic probability transformation to get antecedent probabilities in parent nodes. Then occurrence probabilities of sub-BPAs are calculated from antecedent probabilities. Support degree of each sub-BPA is also calculated according to distance between evidences. The weights of sub-BPAs are obtained considering both occurrence probabilities and support degree, then the authors can get the BPA of the child node by combining sub-BPAs. In the end, two examples are given to illustrate the effectiveness of the proposed method. The first one is about fault diagnosis with a complicated fault tree. The second is threat assessment and prediction, which contributes to commanders making decision. The main contributions of the paper are as follows. The reasoning process based on pure logic is more intuitive. Besides, the method in [40] can be only used in AND rule. While in this paper, another reasoning rule called OR rule is proposed and the proposed method can be used to both AND rule and OR rule. Except for the reasoning rules, the conflict measure between sub-BPAs is also considered, which contributes to improving the accuracy of information fusion. Weighted fusion of BPAs is used before Dempster's rule to resolve the high conflict between evidences.

The rest of this paper is organized as follows. Section II gives some brief introductions about D-S evidence theory and evidential network approach. In Section III, the procedures of the proposed method are given in detail. In Section IV, an example based on the fault diagnosis is given to prove the effectiveness of the proposed method. In Section V, an example of threat assessment and prediction is given to further prove the effectiveness of the proposed method. At last, Section VI concludes this paper.

II. PRELIMINARIES

In this section, some basic theories about D-S evidence theory and evidential network are introduced.

A. D-S EVIDENCE THEORY

Evidence theory was firstly proposed by Dempster [5] and then developed by Shafer, which has several merits to deal with uncertainty information. There are many researches about evidence theory in recent years [41]–[45]. Evidence theory has been also applied in many fields, such as product engineering [46], decision making [47]–[50], zero-sum polymatrix games [51] and so on. And a belief entropy that can measure uncertainty of a BPA is attracting many researchers [52]–[54]. The basic definition of evidence theory and its combination rule are as below.

Definition 1: A frame of discernment (FOD) is a set of mutually exclusive and collectively exhaustive events denoted by $\Omega = \{\theta_1, \theta_2, \ldots, \theta_n\}$. *The power set of* Ω *is denoted as* 2^Ω. Φ *is the empty set, and A, B and C are subsets of* 2^Ω. *A basic probability assignment(BPA), which is also called a mass function, is a mapping* $m : 2^\Omega \rightarrow [0, 1]$ *that satisfies*

$$m(\Phi) = 0 \quad and \sum_{A\subseteq\Omega} m(A) = 1. \tag{1}$$

Given two independent BPAs m_1 and m_2, they can be combined by Dempster's rule. The rule is as below.

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{B \cap C = A} m_1(B)m_2(C) & A \neq \Phi, \\ 0 & A = \Phi. \end{cases} \tag{2}$$

with

$$K = \sum_{B \cap C = \Phi} m_1(B)m_2(C), \tag{3}$$

where K is a measure of conflict between two bodies of evidence. If $K = 1$, they are totally conflict.

B. PIGNISTIC PROBABILITY TRANSFORMATION

Two levels are classified to describe the beliefs: one is the credal level where belief is entertained, the other is the pignistic level where beliefs are feasible to make decisions. Pignistic probability is used for decision making and uses Principle of Insufficient Reason to derive from BPA.

Definition 2: Let m be a BPA on Ω. Its associated pignistic probability function $BetP_m : \Omega \to [0, 1]$ is defined as [55]

$$BetP_m(\omega) = \sum_{A \subseteq \Omega, \omega \in A} \frac{1}{|A|} \frac{m(A)}{1 - m(\Phi)}, \quad m(\Phi) \neq 1 \tag{4}$$

where $|A|$ is the cardinality of subset A. This process is called the Pignistic probability transformation.

C. DISTANCE BETWEEN TWO BODIES OF EVIDENCE

Definition 3: Let m_1 and m_2 be two BPAs on the same frame of discernment. The distance between them is defined as [56]

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\vec{m_1} - \vec{m_2})^T D(\vec{m_1} - \vec{m_2})} \tag{5}$$

where D is a $(2^\Omega \times 2^\Omega)$-dimensional matrix whose elements are

$$D(A, B) = \frac{|A \cap B|}{|A \cup B|}, \quad A \in 2^\Omega, \ B \in 2^\Omega.$$

Furthermore, the similarity between BPAs is [57]

$$S_{ij} = 1 - d_{BPA}(m_i, m_j). \tag{6}$$

D. EVIDENTIAL NETWORK APPROACH

The evidential network, which was firstly proposed by Xu and Smets [11], uses conditional belief function to represent the relationship between network nodes. An evidential network is defined as a directed acyclic graph (DAG) $G = ((N, A), D)$, where $N = \{X_1, X_2, X\}$ represents a set of nodes, $A = \{(X_1, X), (X_2, X)\}$ represents a set of arcs between nodes and $D = \{\alpha_k | E(x_{1i}, x_{2j}) = x_k\}$ represents the set of belief probabilities that are associated with each node. The network propagates basic belief assignments as a priori belief mass on variables. A conditional belief table quantifies the dependency between a node and its parents and allows to compute its mass distribution according to other variables [24].

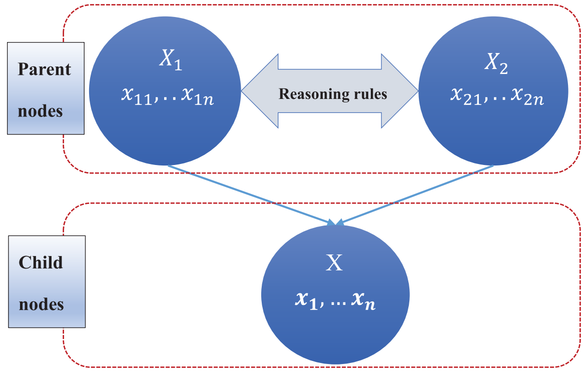

FIGURE 1. A basic evidential network mode.

TABLE 1. An example of conditional belief mass table in an evidential network.

$X_1 \| X_2$	$\{x_{21}\}$	$\{x_{22}\}$	$\{x_{21}, x_{22}\}$
$\{x_{11}\}$	$\{x_1\}$	$\{x_2\}$	$\{x_2\}$
$\{x_{12}\}$	$\{x_1\}$	$\{x_2\}$	$\{x_1, x_2\}$
$\{x_{11}, x_{12}\}$	$\{x_1, x_2\}$	$\{x_2\}$	$\{x_1\}$

TABLE 2. An example of the novel evidential network.

$X_1 \| X_2$	$\{x_{21}\}$	$\{x_{22}\}$
$\{x_{11}\}$	m_X^1	m_X^2
$\{x_{12}\}$	m_X^3	m_X^4

Evidential network mainly consists of two parts: network structure and network parameters. It is a directed acyclic graph, which includes node's set N and arc's set A. Each node in set N represents a variable. The variable is generally an abstract of problems, which represents phenomenon, states or attributes. The arc represents the causal relationship between nodes. The arrow of the arc represents the direction of the causal relationship, which is from parent nodes to child nodes. In Fig. 1, X_1, X_2 are parent nodes and X is a child node. The combined actions of X_1 and X_2 affect the state of X. Evidential network parameters D reflect the degree of correlation or influence between nodes. It can be quantified by a conditional belief mass table, such as Table 1, which shows the relationship between the state of parent nodes and child nodes. For example, there is a kind of relationship between $\{x_{11}\}$, $\{x_{21}\}$ and $\{x_1\}$.

Recently, a novel evidential network was proposed [40]. Table 2 is an example of it. In this novel evidential network, antecedents are only singletons so there is a small amount of computation. Besides, the new evidential network provides several sub-BPAs for the child node, so it can handle both the random uncertainty and epistemic uncertainty.

III. PROPOSED METHOD

The annotations of symbols with the proposed method are shown in Table 3. Assume there are $n + 1$ nodes

TABLE 3. Annotations of symbols.

Symbols	Annotations
X_i	node
m_{X_i}	the BPA of X_i
x_{ij}	jth singleton of ith node
$x_{i.}$	any singletons of ith node
m_X^y	yth sub-BPA of a child node
$\wedge$	intersection
$\vee$	union
$BetP_m$	pignistic probability of a BPA
$p_{x_{in_j}}$	pignistic probability of ith node jth singleton
$p_{x_{i.}}$	any pignistic probability of ith node
p_y	occurrence probability of yth sub-BPA
S_y	similarity of yth sub-BPA
w_y	weight of yth sub-BPA

$X_1, X_2, \ldots, X_n, X$ in an evidential network. Only one node X is the child node and the other n nodes are its parent nodes. There are also $n+1$ basic probability assignments $m_{X_1}, m_{X_2}, \ldots, m_{X_n}, m_X$. Here, each of the nodes corresponds to a basic probability assignment (BPA). For example, X_1 corresponds to m_{X_1}, X_2 corresponds to m_{X_2},...,X corresponds to m_X. Specially, the discernment frames of different parent nodes are different from each other and the BPAs of parent nodes are independent from each other. So using the Dempster's combination rule to fuse BPAs of parent nodes will have some difficulties. In the following part, the authors will propose a evidential reasoning method to show how to calculate the BPA of the child node on the basis of the BPAs of parent nodes.

In order to a easier expression, the following symbols are introduced. Take m_{X_1} as an example. Assume there are n_1 singletons in the frame of discernment of m_{X_1}. The symbols are $m_{X_1} : \{x_{11}, x_{12}, \ldots, x_{1n_1}\}$. Then it can be denoted as $x_{1.}$, which means an arbitrary singleton in $\{x_{11}, x_{12}, \ldots, x_{1n_1}\}$. All the symbols of parent nodes can be denoted as follows.

$$
\begin{aligned}
m_{X_1} &: \{x_{11}, x_{12}, \ldots, x_{1n_1}\} \to x_{1.};\\
m_{X_2} &: \{x_{21}, x_{22}, \ldots, x_{2n_2}\} \to x_{2.};\\
&\cdots\\
m_{X_n} &: \{x_{n1}, x_{n2}, \ldots, x_{nn_n}\} \to x_{n.};\\
m_X &: \{x_1, x_2, \ldots, x_n\} \to x_{.}.
\end{aligned}
$$

Example 1: *If m_{X_1} is defined on $\{\{x_{11}\}, \{x_{11}, x_{12}\}, \{x_{11}, x_{12}, x_{13}\}\}$, then the singletons are x_1,x_2 and x_3. The symbol is as below.*

$$m_{X_1} : \{x_{11}, x_{12}, x_{13}\} \to x_{1.}.$$

According to the conditional belief mass table introduced in Section II, the BPA of child node X is related to several sub-BPAs. Here, the authors denote them as $m_X^y, y = 1, 2, \ldots, n_1 \times n_2 \times \ldots \times n_n$.

TABLE 4. An example of conditional belief mass table for the AND rule.

$X_1 \wedge X_2$	$\{x_{21}\}$	$\{x_{22}\}$
$\{x_{11}\}$	m_X^1	m_X^2
$\{x_{12}\}$	m_X^3	m_X^4

A. REASONING RULES

Definition 4 (Reasoning Rule 1): *The reasoning rule 1 between parent nodes and child node is defined as follows:*

$$x_{1.} \wedge x_{2.} \wedge \ldots \wedge x_{n.} \to m_X^y (y = 1, 2, \ldots, n_1 \times n_2 \times \ldots \times n_n).$$

*In the reasoning rule 1, $x_{1.}, x_{2.} \ldots x_{n.}$ are the antecedents of sub-BPAs of the child node. If each of the parent nodes has an arbitrary singleton occurrence simultaneously, then a specified sub-BPA in the child node will occur. The reasoning rule 1 gives a one-to-one correspondence relationship (**bijection**) between antecedents and sub-BPAs. The authors also call it the AND rule in this paper.*

Example 2: *A conditional belief mass table is given to illustrate the reasoning rule 1. In Table 4, parent nodes X_1 and X_2 have two singletons respectively. The child node X has four sub-BPAs. There are four antecedents $\{x_{11}\}$, $\{x_{12}\}$, $\{x_{21}\}$ and $\{x_{22}\}$ in the conditional belief mass table. If $\{x_{11}\}$ and $\{x_{21}\}$ occur, then m_X^1 will occur; if $\{x_{11}\}$ and $\{x_{22}\}$ occur, then m_X^2 will occur; if $\{x_{12}\}$ and $\{x_{21}\}$ occur, then m_X^3 will occur; if $\{x_{12}\}$ and $\{x_{22}\}$ occur, then m_X^4 will occur. This IF-THEN rule can be illustrated by the reasoning rule 1 as follows:*

$$
\begin{aligned}
x_{11} \wedge x_{21} &\to m_X^1, \quad x_{11} \wedge x_{22} \to m_X^2,\\
x_{12} \wedge x_{21} &\to m_X^3, \quad x_{12} \wedge x_{22} \to m_X^4.
\end{aligned}
$$

Definition 5 (Reasoning Rule 2): *The reasoning rule 2 between parent nodes and child node is defined as follows:*

$$x_{1.} \vee x_{2.} \vee \ldots \vee x_{n.} \to m_X^y (y = 1, 2, \ldots, n_1 \times n_2 \times \ldots \times n_n).$$

*In the reasoning rule 2, $x_{1.}, x_{2.} \ldots x_{n.}$ are the antecedents of sub-BPAs of the child node. If at least one parent node has an arbitrary singleton occurrence among all the parent nodes, then a specified sub-BPA in the child node will occur. That means as long as one parent node has an arbitrary singleton occurrence, a specified sub-BPA in the child node will occur. The reasoning rule 2 also gives a one-to-one correspondence relationship (**bijection**) between antecedents and sub-BPAs. The authors also call it the OR rule in this manuscript.*

Example 3: *Reasoning rule 2 can be also illustrated by the the conditional belief mass table. In Table 5, parent nodes X_1 and X_2 have two singletons respectively. The child node X has four sub-BPAs. There are four antecedents $\{x_{11}\}$, $\{x_{12}\}$, $\{x_{21}\}$ and $\{x_{22}\}$ in the conditional belief mass table. If $\{x_{11}\}$ or $\{x_{21}\}$ occurs, then m_X^1 will occur; if $\{x_{11}\}$ or $\{x_{22}\}$ occurs, then m_X^2 will occur; if $\{x_{12}\}$ or $\{x_{21}\}$ occurs, then m_X^3 will occur; if $\{x_{12}\}$ or $\{x_{22}\}$ occurs, then m_X^4 will occur. This IF-THEN rule can*

TABLE 5. An example of conditional belief mass table for the OR rule.

$X_1 \vee X_2$	$\{x_{21}\}$	$\{x_{22}\}$
$\{x_{11}\}$	m_X^1	m_X^2
$\{x_{12}\}$	m_X^3	m_X^4

FIGURE 2. AND rule.

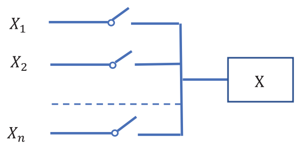

FIGURE 3. OR rule.

be illustrated by the reasoning rule 2 *as follows*:

$$x_{11} \vee x_{21} \rightarrow m_X^1, \quad x_{11} \vee x_{22} \rightarrow m_X^2,$$
$$x_{12} \vee x_{21} \rightarrow m_X^3, \quad x_{12} \vee x_{22} \rightarrow m_X^4.$$

In fact, the AND rule corresponds to the logical conjunction while the OR rule corresponds to the logical disjunction. In order to illustrate the rationality of the defined rules, Fig. 2 and 3 are given. Fig. 2 shows a series circle. If and only if all the switches close, then the terminal works. It reflects a logical relation that the occurrence of all the preconditions leads to a specified result, which corresponds to the AND rule. Similarly, Fig. 3 shows a parallel circle. As long as one of the switches closes, then the terminal works. It reflects another logical relation that the occurrence of any precondition leads to a specified result, which corresponds to the OR rule.

In this manuscript, the reasoning rules and the sub-BPAs of the child node are known conditions, which is derived from expert experience and domain knowledge. The weighted average BPA of the child node can be calculated by the following formula. Then the authors can use the classical Dempster's rule to combine the weighted average of the masses $n-1$ times.

$$m_X(A) = \sum_{y=1}^{n} w_y m_X^y(A),$$
$$A \subseteq \Theta_X, \quad n = n_1 \times n_2 \times \ldots \times n_n. \tag{7}$$

The following part will show how the weights $w_y(y = 1, 2, \ldots, n)$ are calculated.

B. ANTECEDENT PROBABILITY AND OCCURRENCE PROBABILITY

In Section II, the pignistic transformation has been introduced. The pignistic transformation can transform a BPA into a pignistic probability function. In this paper, the antecedents of the sub-BPAs are singletons in parent nodes. The authors apply the pignistic transformation to the BPAs of parent nodes. Then the pignistic probability of each singleton in a BPA can be derived.

Corollary 1: For a BPA, if w is a singleton of it, then the sum of the BetP(w) is one.

Example 4: Assume there is a power set of a discernment frame $A = \{\{a_1\}, \{a_1, a_2\}, \ldots, \{a_1, a_2, \ldots, a_n\}\}$. *A BPA is defined on A.*

$$m(\{a_1\}) = m(\{a_1, a_2\}) = \ldots = m(\{a_1, a_2, \ldots, a_n\}) = 1/n.$$

The pignistic probabilities of singletons of this BPA are as follows.

$$\begin{aligned}
BetP_m(\{a_1\}) &= 1/n + 1/2n + \ldots + 1/n^2;\\
BetP_m(\{a_2\}) &= 1/2n + 1/3n + \ldots + 1/n^2;\\
BetP_m(\{a_3\}) &= 1/3n + 1/4n + \ldots + 1/n^2;\\
&\ldots\\
BetP_m(\{a_n\}) &= 1/n^2.
\end{aligned}$$

The sum of them is

$$\sum_{i=1}^{n} BetP_m(\{a_i\}) = 1/n + 2/2n + 3/3n + \ldots + n/n^2 = 1.$$

Definition 6: Based on the Corollary 1, the authors know that the sum of pignistic probabilities of all singletons in a BPA is one and each of them is non-negative. This satisfies the property of a general probability distribution. So the authors define the collection of pignistic probabilities of all singletons in a parent node as a probability distribution. In this paper, the singletons are also the antecedents of the sub-BPAs. So each of the components of this probability distribution represents the occurrence probability of a antecedent in the reasoning rule. The authors also call it an ***antecedent probability distribution****.*

In order to simplify the denotation, the authors introduce some symbols. Take m_{X_1} as an example. Let $BetP_{m_{X_1}}(x_{11}) = p_{x_{11}}, \ldots, BetP_{m_{X_1}}(x_{1n_1}) = p_{x_{1n_1}}$. So $\{p_{x_{11}}, \ldots, p_{x_{1n_1}}\}$ is the antecedent probability distribution. Let $p_{x_{1.}}$ denote an arbitrary component in $\{p_{x_{11}}, \ldots, p_{x_{1n_1}}\}$. The relationship of symbols are as follows.

$$\begin{aligned}
&\{BetP_{m_{X_1}}(x_{11}), \ldots, BetP_{m_{X_1}}(x_{1n_1})\} \rightarrow \{p_{x_{11}}, \ldots, p_{x_{1n_1}}\} \rightarrow p_{x_{1.}};\\
&\{BetP_{m_{X_2}}(x_{21}), \ldots, BetP_{m_{X_2}}(x_{2n_2})\} \rightarrow \{p_{x_{21}}, \ldots, p_{x_{2n_2}}\} \rightarrow p_{x_{2.}};\\
&\ldots\\
&\{BetP_{m_{X_n}}(x_{n1}), \ldots, BetP_{m_{X_n}}(x_{nn_n})\} \rightarrow \{p_{x_{n1}}, \ldots, p_{x_{nn_n}}\} \rightarrow p_{x_{n.}}.
\end{aligned}$$

Definition 7 (AND): For the reasoning rule 1, if $p_y = p_{x_{1.}} p_{x_{2.}} \ldots p_{x_{n.}}$, *then the authors call the probability distribution* $P = \{p_1, p_2, \ldots, p_n\}, (n = n_1 \times n_2 \times \ldots \times n_n)$ *an* ***occurrence probability distribution****. Each of the component of the occurrence probability distribution indicates the occurrence probability of a sub-BPA in the child node.*

The equation below shows that the occurrence probability distribution satisfies the property of a general probability distribution. The BPAs of parent nodes are independent from each other, so $p_{x_{1.}}p_{x_{2.}}\dots p_{x_{n.}}$ indicates the probability of when each of the parent nodes has an antecedent occurrence simultaneously. Based on the reasoning rule 1, if each of the parent nodes has an antecedent occurrence simultaneously, then a specified sub-BPA in the child node will occur. So this is the occurrence probability distribution of the sub-BPAs in the child node.

$$\begin{aligned}\sum_{i=1}^{n} p_i &= \sum_{1}^{n_1}\sum_{1}^{n_2}\dots\sum_{1}^{n_n} p_{x_{1.}}p_{x_{2.}}\dots p_{x_{n.}} \\ &= \sum_{1}^{n_1}\sum_{1}^{n_2}\dots\sum_{1}^{n_{n-1}} p_{x_{1.}}p_{x_{2.}}\dots p_{x_{n-1.}}(\sum_{i=1}^{n_n} p_{x_{ni}}) \\ &= \dots \\ &= (\sum_{i=1}^{n_1} p_{x_{ni}})(\sum_{i=1}^{n_2} p_{x_{ni}})\dots(\sum_{i=1}^{n_n} p_{x_{ni}}) \\ &= 1\end{aligned}$$

Definition 8 (OR): For the reasoning rule 2, the ***occurrence probability distribution*** *is also denoted as* $P = \{p_1, p_2, \dots, p_n\}, (n = n_1 \times n_2 \times \dots \times n_n)$, *where*

$$p_y = \frac{1-(1-p_{x_{1.}})(1-p_{x_{2.}})\dots(1-p_{x_{n.}})}{n_1\times n_2\times\dots\times n_n-(n_1-1)\times(n_2-1)\times\dots\times(n_n-1)}.$$

The equation below shows that the occurrence probability distribution satisfies the property of a general probability distribution. The BPAs of parent nodes are independent from each other, so $1-(1-p_{x_{1.}})(1-p_{x_{2.}})\dots(1-p_{x_{n.}})$ indicates the probability of when at least one parent node has an antecedent occurrence among all the parent nodes. When $n_1\times n_2\times\dots\times n_n-(n_1-1)\times(n_2-1)\times\dots\times(n_n-1)$ is a normalization factor. Based on the reasoning rule 2, if at least one parent node has an arbitrary singleton occurrence among all the parent nodes, then a specified sub-BPA in the child node will occur. So this is the occurrence probability distribution of the sub-BPAs in the child node.

$$\begin{aligned}&\sum_{1}^{n_1}\sum_{1}^{n_2}\dots\sum_{1}^{n_n}[1-(1-p_{x_{1.}})(1-p_{x_{2.}})\dots(1-p_{x_{n.}})] \\ &= \sum_{1}^{n_1}\sum_{1}^{n_2}\dots\sum_{1}^{n_n} - \sum_{1}^{n_1}\sum_{1}^{n_2} \\ &\quad \dots\sum_{1}^{n_n}(1-p_{x_{1.}})(1-p_{x_{2.}})\dots(1-p_{x_{n.}}) \\ &= n_1\times n_2\times\dots\times n_n - \sum_{1}^{n_1}\sum_{1}^{n_2}\dots\sum_{1}^{n_{n-1}}(1-p_{x_{1.}})(1-p_{x_{2.}}) \\ &\quad \dots(1-p_{x_{n-1.}})\sum_{i=1}^{n_n}(1-p_{x_{ni}}) \\ &= \dots \\ &= n_1\times n_2\times\dots\times n_n - \sum_{i=1}^{n_1}(1-p_{x_{ni}})\sum_{i=1}^{n_2}(1-p_{x_{ni}}) \\ &\quad \dots\sum_{i=1}^{n_n}(1-p_{x_{ni}}) \\ &= n_1\times n_2\times\dots\times n_n-(n_1-1)\times(n_2-1)\times\dots\times(n_n-1)\end{aligned}$$

C. CONFLICT MEASURE

Another factor considered in the process of calculating weighs is the support degree of each sub-BPA. The authors consider this factor because it is an effective way to measure the conflict of the combination process [57]. Formula 8 is the similarity matrix of sub-BPAs. It gives the similarity degree between every two sub-BPAs. The support degree of each sub-BPA is derived by Formula 9. The more support one derives from other sub-BPAs, the less conflict between it and other sub-BPAs.

$$\begin{array}{cc} & \begin{array}{cccc} m_X^1 & m_X^2 & \cdots & m_X^n \end{array} \\ \begin{array}{c} m_X^1 \\ m_X^2 \\ \vdots \\ m_X^n \end{array} & \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{pmatrix} \end{array} \tag{8}$$

$$S_y = \sum_{j=1, j\neq y}^{n} S_{yj} \tag{9}$$

D. WEIGHT CALCULATION

Finally, the weights of sub-BPAs are derived by Formula 10. Then Formula 7 can be applied to calculate the weighted average BPA of the child node. There are two factors considered in calculating the weight of a sub-BPA. They are the occurrence probability of the sub-BPA and the support degree derived from other sub-BPAs. The former represents the reasoning process, the latter represents the conflict calculation. Here, the authors believe that the reasoning process precedes the conflict measure, because it is only necessary to consider the conflict if it is certain that the sub-BPA can occur. Besides, the larger the occurrence probability and the support degree of a sub-BPA, the larger weight it should derive. Formula 10 can satisfy the above conditions. First, if the larger the numerical values of p_y and S_y of a sub-BPA, then the larger the value of w_y. Second, if $p_y = 0$, which means that this sub-BPA is no occurrence, then $w_y = 0$. But if $S_y = 0$, then the occurrence probability is still considered in calculating the weight and $w_y \neq 0$.

$$w_y = \frac{p_y(1+S_y)}{\sum_{y=1}^{n} p_y(1+S_y)}, \quad n = n_1\times n_2\times\dots\times n_n. \tag{10}$$

So far, the evidential reasoning method has been introduced. Fig. 4 gives the flow chart of the proposed method. An example is given to illustrate the whole computation process.

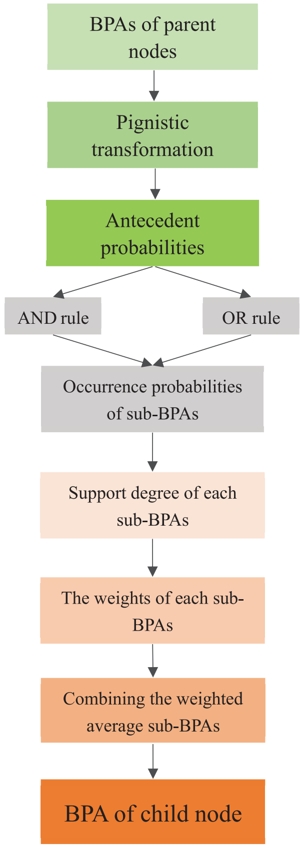

FIGURE 4. The flow chart of the proposed method.

TABLE 6. Conditional belief mass table for node X in the reasoning rule 1.

$X_1 \wedge X_2$	$\{x_{21}\}$	$\{x_{22}\}$
$\{x_{11}\}$	$m_X^1(\{x_1, x_2\}) = 1$	$m_X^2(\{x_1\}) = 0.5$, $m_X^2(\{x_2\}) = 0.1$, $m_X^2(\{x_1, x_2\}) = 0.4$
$\{x_{12}\}$	$m_X^3(\{x_1\}) = 0.7$, $m_X^3(\{x_2\}) = 0.2$, $m_X^3(\{x_1, x_2\}) = 0.1$	$m_X^4(\{x_1\}) = 0.2$, $m_X^4(\{x_1, x_2\}) = 0.8$

Example 5: This example illustrates the computation process of the proposed method. Two BPAs and a conditional belief mass table are given.

$$m_{X_1}(\{x_{11}\}) = 0.6, \quad m_{X_1}(\{x_{12}\}) = 0.3, \quad m_{X_1}(\{x_{11}, x_{12}\}) = 0.1;$$
$$m_{X_2}(\{x_{21}\}) = 0.5, \quad m_{X_2}(\{x_{22}\}) = 0.1, \quad m_{X_2}(\{x_{21}, x_{22}\}) = 0.4.$$

- *Antecedent probability*

$$p_{x_{11}} = BetP_{m_{X_1}}(x_{11}) = 0.6 + \frac{0.1}{2} = 0.65,$$
$$p_{x_{12}} = BetP_{m_{X_1}}(x_{12}) = 0.3 + \frac{0.1}{2} = 0.35;$$

TABLE 7. Conditional belief mass table for node X in the reasoning rule 2.

$X_1 \vee X_2$	$\{x_{21}\}$	$\{x_{22}\}$
$\{x_{11}\}$	$m_X^1(\{x_1, x_2\}) = 1$	$m_X^2(\{x_1\}) = 0.5$, $m_X^2(\{x_2\}) = 0.1$, $m_X^2(\{x_1, x_2\}) = 0.4$
$\{x_{12}\}$	$m_X^3(\{x_1\}) = 0.7$, $m_X^3(\{x_2\}) = 0.2$, $m_X^3(\{x_1, x_2\}) = 0.1$	$m_X^4(\{x_1\}) = 0.2$, $m_X^4(\{x_1, x_2\}) = 0.8$

$$p_{x_{21}} = BetP_{m_{X_2}}(x_{21}) = 0.5 + \frac{0.4}{2} = 0.70,$$
$$p_{x_{22}} = BetP_{m_{X_2}}(x_{22}) = 0.1 + \frac{0.4}{2} = 0.30.$$

- *Occurrence probability*
  AND rule:
  According to Definition 7, the occurrence probabilities of sub-BPAs with the AND rule can be calculated, which means the joint occurrence of antecedents in different parent nodes can lead to a specified sub-BPA occurrence.

$$p_1 = p_{x_{11}} \times p_{x_{21}} = 0.455, \quad p_2 = p_{x_{11}} \times p_{x_{22}} = 0.195,$$
$$p_3 = p_{x_{12}} \times p_{x_{21}} = 0.245, \quad p_4 = p_{x_{12}} \times p_{x_{22}} = 0.105.$$

  OR rule:
  According to Definition 8, the occurrence probabilities of sub-BPAs with the OR rule can be calculated, which means at least one antecedent occurring can lead to a specified sub-BPA occurrence.

$$p_1 = \frac{1-(1-p_{x_{11}})(1-p_{x_{21}})}{3} = 0.2983,$$
$$p_2 = \frac{1-(1-p_{x_{11}})(1-p_{x_{22}})}{3} = 0.2683,$$
$$p_3 = \frac{1-(1-p_{x_{12}})(1-p_{x_{21}})}{3} = 0.2517,$$
$$p_4 = \frac{1-(1-p_{x_{12}})(1-p_{x_{22}})}{3} = 0.1817.$$

- *Support degree*

$$S_{12} = 0.6394, \quad S_{13} = 0.4852, \quad S_{14} = 0.8586,$$
$$S_{23} = 0.8419, \quad S_{24} = 0.7764, \quad S_{34} = 0.6192.$$
$$S_1 = 1.9832, \quad S_2 = 2.2577, \quad S_3 = 1.9463,$$
$$S_4 = 2.2542.$$

- *Weight calculation*
  AND rule:

$$w_1 = 0.4441, \quad w_2 = 0.2079,$$
$$w_3 = 0.2362, \quad w_4 = 0.1118.$$

  OR rule:

$$w_1 = 0.2874, \quad w_2 = 0.2822,$$
$$w_3 = 0.2395, \quad w_4 = 0.1909.$$

- *Results*
  AND rule:

$$m_X(\{x_1\}) = \sum_{y=1}^{4} w_y m_X^y(\{x_1\}) = 0.2917,$$

$$m_X(\{x_2\}) = \sum_{y=1}^{4} w_y m_X^y(\{x_2\}) = 0.0680,$$

$$m_X(\{x_1, x_2\}) = \sum_{y=1}^{4} w_y m_X^y(\{x_1, x_2\}) = 0.6403.$$

OR rule:

$$m_X(\{x_1\}) = \sum_{y=1}^{4} w_y m_X^y(\{x_1\}) = 0.3469,$$

$$m_X(\{x_2\}) = \sum_{y=1}^{4} w_y m_X^y(\{x_2\}) = 0.0761,$$

$$m_X(\{x_1, x_2\}) = \sum_{y=1}^{4} w_y m_X^y(\{x_1, x_2\}) = 0.5770.$$

Using Dempster's rule 3 *times. The results are as below.*
AND rule:

$$m_X(\{x_1\}) = 0.6997, \quad m_X(\{x_2\}) = 0.0998,$$
$$m_X(\{x_1, x_2\}) = 0.2006.$$

OR rule:

$$m_X(\{x_1\}) = 0.7725, \quad m_X(\{x_2\}) = 0.0889,$$
$$m_X(\{x_1, x_2\}) = 0.1386.$$

IV. EXAMPLE ONE: FAULT DIAGNOSIS

In this section, an example based on fault diagnosis is given to illustrate the application of the proposed method. The fault tree is shown in Fig. 5 and the symbols are introduced in Table 8. Electric motor is a device that transforms electric energy to mechanical energy. There are three kinds of signals indicating work condition of an electric motor, respectively voltage, frequency and revolving speed. The internal structure is divided into mechanical and electronic parts. Voltage and frequency can reflect the work condition of electronic part while mechanical part can be reflected by frequency and revolving speed. The electronic part and mechanical part are controlled by two subsystems respectively. In the case of an electric motor fault diagnosis, three kinds of sensors are used to collect different kinds of fault information, respectively voltage, frequency and revolving speed. Fault layer 1 consists of two kinds of faults, circuit failure and machinery failure. Whether they will occur or not depends on the signals of the sensor layer. Fault layer 2 is the motor subsystem fault. Which subsystem will fail depends on the type of circuit failure and mechanical failure. Table 9, 10 and 11 give the occurrence probability of each circumstance, which can be derived from domain knowledge and experts experience. Knowledge in special fields can provide a foundation for them, and experts can summarize and conclude the relationship between elements then they can be given. Then the reasoning rules are described as below.

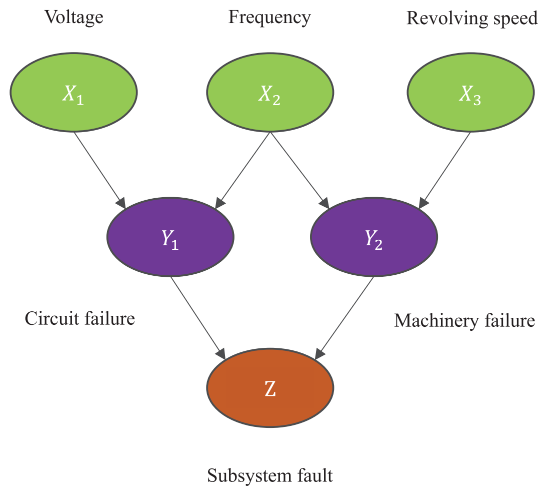

FIGURE 5. The fault tree diagram of the example.

TABLE 8. Symbols in the example of fault diagnosis.

Voltage sensor X_1	Voltage signal 1	x_{11}
	Voltage signal 2	x_{12}
Frequency sensor X_2	Frequency signal 1	x_{21}
	Frequency signal 2	x_{22}
Revolving speed sensor X_3	Revolving speed signal 1	x_{31}
	Revolving speed signal 2	x_{32}
Circuit failure Y_1	Control circuit 1	y_{11}
	Control circuit 2	y_{12}
Mechanical failure Y_2	Abnormal rotor	y_{21}
	Misalignment	y_{22}
Motor subsystem Z	Failure of subsystem 1	z_1
	Failure of subsystem 2	z_2

The AND rules between X_1, X_2 and Y_1:

- If voltage signal 1 and frequency signal 1 are detected simultaneously, then the probability of the fault occurring in the control circuit 1 is 0.8 and uncertainty is 0.2.
- If voltage signal 1 and frequency signal 2 are detected simultaneously, then the fault occurs in the control circuit 1.
- If voltage signal 2 and frequency signal 1 are detected simultaneously, then the probability of the fault occurring in the control circuit 1 is 0.4 and in the control circuit 2 is 0.6.
- If voltage signal 2 and frequency signal 2 are detected simultaneously, then the probability of the fault occurring in the control circuit 1 is 0.3, in the control circuit 2 is 0.3 and uncertainty is 0.4.

The OR rules between X_2, X_3 and Y_2:

TABLE 9. Conditional belief mass table for node Y_1.

$X_1 \wedge X_2$	$\{x_{21}\}$	$\{x_{22}\}$
$\{x_{11}\}$	$m^1_{Y_1}(\{y_{11}\}) = 0.8$, $m^1_{Y_1}(\{y_{11}, y_{12}\}) = 0.2$	$m^2_{Y_1}(\{y_{11}\}) = 1$
$\{x_{12}\}$	$m^3_{Y_1}(\{y_{11}\}) = 0.4$, $m^3_{Y_1}(\{y_{12}\}) = 0.6$	$m^4_{Y_1}(\{y_{11}\}) = 0.3$, $m^4_{Y_1}(\{y_{12}\}) = 0.3$, $m^4_{Y_1}(\{y_{11}, y_{12}\}) = 0.4$

TABLE 10. Conditional belief mass table for node Y_2.

$X_2 \vee X_3$	$\{x_{31}\}$	$\{x_{32}\}$
$\{x_{21}\}$	$m^1_{Y_2}(\{y_{21}\}) = 0.1$, $m^1_{Y_2}(\{y_{22}\}) = 0.9$	$m^2_{Y_2}(\{y_{21}\}) = 0.5$, $m^2_{Y_2}(\{y_{22}\}) = 0.3$, $m^2_{Y_2}(\{y_{21}, y_{22}\}) = 0.2$
$\{x_{22}\}$	$m^3_{Y_2}(\{y_{22}\}) = 0.2$, $m^3_{Y_2}(\{y_{21}, y_{22}\}) = 0.8$	$m^4_{Y_2}(\{y_{21}\}) = 0.3$, $m^4_{Y_2}(\{y_{22}\}) = 0.4$, $m^4_{Y_2}(\{y_{21}, y_{22}\}) = 0.3$

- If frequency signal 1 or revolving speed signal 1 is detected, then the probability of abnormal rotor is 0.1 and misalignment is 0.9.
- If frequency signal 1 or revolving speed signal 2 is detected, then the probability of abnormal rotor is 0.5, misalignment is 0.3 and uncertainty is 0.2.
- If frequency signal 2 or revolving speed signal 1 is detected, then the probability of abnormal rotor is 0.2 and misalignment is 0.8.
- If frequency signal 2 or revolving speed signal 2 is detected, then the probability of abnormal rotor is 0.3, misalignment is 0.4 and uncertainty is 0.3.

The AND rules between Y_1, Y_2 and Z:

- If control circuit 1 failure and abnormal rotor occur simultaneously, then the failure probability of subsystem 1 is 0.6, subsystem 2 is 0.3 and uncertainty is 0.1.
- If control circuit 1 failure and misalignment of rotor occur simultaneously, then the failure probability of subsystem 1 is 0.5 and subsystem 2 is 0.5.
- If control circuit 2 failure and abnormal rotor occur simultaneously, then the failure probability of subsystem 1 is 0.3, subsystem 2 is 0.5 and uncertainty is 0.2.
- If control circuit 2 failure and misalignment of rotor occur simultaneously, then the failure probability of subsystem 1 is 0.5, subsystem 2 is 0.2 and uncertainty is 0.3.

Sensors can detect the signals from the motor. Then the authors can turn the data collected from the sensors into BPAs, which represents the probability of the corresponding signals detected. For example, $m_{X_1}(\{x_{11}\}) = 0.3$ denotes the probability of voltage signal 1 detected is 0.3.

$$m_{X_1}(\{x_{11}\}) = 0.3, \quad m_{X_1}(\{x_{12}\}) = 0.5,$$
$$m_{X_1}(\{x_{11}, x_{12}\}) = 0.2$$
$$m_{X_2}(\{x_{21}\}) = 0.6, \quad m_{X_2}(\{x_{22}\}) = 0.4,$$
$$m_{X_2}(\{x_{21}, x_{22}\}) = 0.0$$
$$m_{X_3}(\{x_{31}\}) = 0.7, \quad m_{X_3}(\{x_{32}\}) = 0.2,$$
$$m_{X_3}(\{x_{31}, x_{32}\}) = 0.1$$

Based on the proposed method, the BPAs of Y_1 and Y_2 are computed.

$$m_{Y_1}(\{y_{11}\}) = 0.8876, \quad m_{Y_1}(\{y_{12}\}) = 0.1108,$$
$$m_{Y_1}(\{y_{11}, y_{12}\}) = 0.0016$$
$$m_{Y_2}(\{y_{21}\}) = 0.1613, \quad m_{Y_2}(\{y_{22}\}) = 0.8114,$$
$$m_{Y_2}(\{y_{21}, y_{22}\}) = 0.0273$$

The results show that control circuit 1 is most likely to be in failure and the machinery failure is most likely to be misalignment. From Table 9, three of the sub-BPAs indicate that it is more likely to have failure of control circle 1 and they get a higher occurrence probabilities. Also, Table 9 shows that three sub-BPAs tend to give more likelihood to be misalignment. And the antecedent probabilities of frequent signal 1 and revolving speed signal 1 are larger, which can make a larger occurrence probability of $m^1_{Y_2}$ showing misalignment is more likely. It shows that the result derived from the proposed method is logical. Finally, the authors can compute the occurrence probabilities of the subsystem failure based on the proposed method.

$$m_Z(\{z_1\}) = 0.6313, \quad m_Z(\{z_2\}) = 0.3687.$$

The fusion result shows the subsystem 1 is most likely to be in failure. From the sub-BPAs in Table 11, three of them indicate that failure of subsystem 1 is more likely to occur. Also, the antecedent probabilities of control circuit 2 and abnormal rotor are smaller, which leads to a smallest occurrence

TABLE 11. Conditional belief mass table for node Z.

$Y_1 \wedge Y_2$	$\{y_{21}\}$	$\{y_{22}\}$
$\{y_{11}\}$	$m^1_Z(\{z_1\}) = 0.6$, $m^1_Z(\{z_2\}) = 0.3$, $m^1_Z(\{z_1, z_2\}) = 0.1$	$m^2_Z(\{z_1\}) = 0.5$, $m^2_Z(\{z_2\}) = 0.5$
$\{y_{12}\}$	$m^3_Z(\{z_1\}) = 0.3$, $m^3_Z(\{z_2\}) = 0.5$, $m^3_Z(\{z_1, z_2\}) = 0.2$	$m^4_Z(\{z_1\}) = 0.5$, $m^4_Z(\{z_2\}) = 0.2$, $m^4_Z(\{z_1, z_2\}) = 0.3$

TABLE 12. Nodes and state descriptions of TAP.

Node	Description	State field
TL	Threat level	{High, Medium, Low}
HI	Attack intention	{High, Medium, Low}
C	Attack ability	{Good, Medium, Bad}
M	Flight attitude	{Yes, No}
FCR	Fire-control radar	{ON, OFF}
NF	Enemy plane platform	{True, False}
IFF	Friend or foe identification	{Yes, No}
WE	Attack extent	{Long, Medium, Small}
I	Attack preparation	{High, Medium, Low}

TABLE 13. Nodes and state descriptions of TAP.

Num	Antecedent	Consequence	Num	Antecedent	Consequence
1	(IFF is Yes)	NF is {(T,0.05),(F,0.95)}	15	(WE is M)∧(I is M)	C is {(M,1)}
2	(IFF is No)	NF is {(T,0.7),(F,0.3)}	16	(WE is M)∧(I is L)	C is {(M,0.3),(B,0.7)}
3	(M is Yes)∧(FCR is ON)∧(NF is T)	HI is {(H,1)}	17	(WE is S)∧(I is H)	C is {(M,0.9),(B,0.1)}
4	(M is Yes)∧(FCR is ON)∧(NF is F)	HI is {(H,0.3),(M,0.7)}	18	(WE is S)∧(I is M)	C is {(M,0.2),(B,0.8)}
5	(M is Yes)∧(FCR is OFF)∧(NF is T)	HI is {(H,0.1),(M,0.9)}	19	(WE is S)∧(I is L)	C is {(B,1)}
6	(M is Yes)∧(FCR is OFF)∧(NF is F)	HI is {(M,0.3),(L,0.7)}	20	(HI is H)∧(C is G)	TL is {(H,1)}
7	(M is No)∧(FCR is ON)∧(NF is T)	HI is {(H,0.6),(M,0.4)}	21	(HI is H)∧(C is M)	TL is {(H,0.7),(M,0.3)}
8	(M is No)∧(FCR is ON)∧(NF is F)	HI is {(M,0.8),(L,0.2)}	22	(HI is H)∧(C is B)	TL is {(H,0.2),(M,0.8)}
9	(M is No)∧(FCR is OFF)∧(NF is T)	HI is {(M,0.6),(L,0.4)}	23	(HI is M)∧(C is G)	TL is {(H,0.4),(M,0.6)}
10	(M is No)∧(FCR is OFF)∧(NF is F)	HI is {(L,1)}	24	(HI is M)∧(C is M)	TL is {(M,1)}
11	(WE is L)∧(I is H)	C is {(G,1)}	25	(HI is M)∧(C is B)	TL is {(M,0.6),(L,0.4)}
12	(WE is L)∧(I is M)	C is {(G,0.5),(M,0.5)}	26	(HI is L)∧(C is G)	TL is {(M,0.8),(L,0.2)}
13	(WE is L)∧(I is L)	C is {(M,0.6),(B,0.4)}	27	(HI is L)∧(C is M)	TL is {(M,0.4),(L,0.6)}
14	(WE is M)∧(I is H)	C is {(G,0.4),(M,0.6)}	28	(HI is L)∧(C is B)	TL is {(L,1)}

probability of m_Z^3 indicating subsystem 2 is more likely than subsystem 1. Therefore, the result of evidential reasoning is intuitively consist with the data in Table 11.

From the above calculation, the method can be applied to multilayer fault diagnosis of complex devices. Compared with the traditional fault tree, this method has a wider range because each node in the fault tree can represent several faults or sensor signals. Moreover, this method can deal with the uncertainty information better because the evidence theory is introduced. Evidential network combining the evidence theory and graph theory ensures the effective of the reasoning process. The reasoning rules ensure the reasoning process is logical and reasonable. In order to further prove the effectiveness of the proposed method, another example is given below.

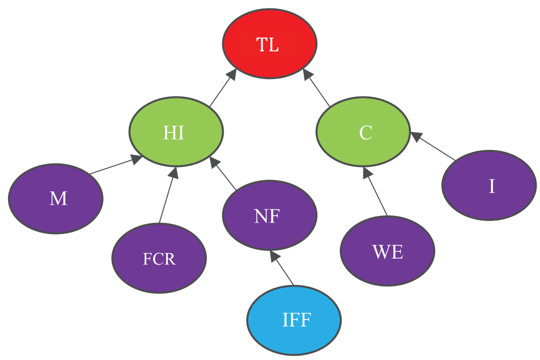

FIGURE 6. The topology graph of TAP.

V. EXAMPLE TWO: THREAT ASSESSMENT AND PREDICTION

Threat assessment and prediction (TAP) can assist military commanders to quickly perceive the scenario in the complex military battlefield environment, more effectively make the right decision. There are a lot of uncertain information and expert knowledge in the real TAP scenario, which can be effectively dealt with evidential reasoning. Here, an example of threat assessment and prediction in an air-to-air confrontation is given to illustrate the effectiveness of the proposed evidential reasoning approach [58]. The evidential network is constructed in Fig 6 according to technology analysis and expert knowledge. There are total nine nodes and the purpose is to identify the threat level. The meanings and condition levels of each nodes are shown in Table 12. According to expert knowledge and technology specification, the relationship between nodes conforms to the AND rule proposed in this paper. The conditional belief table of evidential network is given in Table 13. For example, rule 3 to 10 reflect the relationship between node M, FCR, NF and HI. The condition of flight attitude, fire-control radar and enemy plane can determine the level of enemy plane. For rule 4, if flight attitude is high, fire-control radar is on and enemy plane is false, then the belief value of high attack intention is 0.3 and medium attack intention is 0.7.

Assume there are four groups of hypothetical observation data shown in Table 14. According to the proposed evidential reasoning method, the occurrence probabilities and weights are calculated and shown in Table 15, and the reasoning results are calculated and shown in Table 16. For example, IFF of T2 is $\{(Y, 0.4), (N, 0.6)\}$, which means observation data show the reliability of that IFF is 'Yes' is 0.4 and 'No' is 0.6. Based on the AND rule and algorithm, the reliability of that enemy plane is true is 0.44 while false is 0.56. Also, the BPA of HI is calculated based on the BPAs of M, FCR and NF, which indicates attack intention is more likely to be medium. C is computed based on WE and I, and the result shows C tends to be bad. Finally, the TL is derived from HI and C, which shows that a greater likelihood is that the threat level is medium. Similar to T2, TL of T1, T3 and T4 can be also reasoning through the proposed approach. Here, the results are further analyzed. The reasoning results are consistent with intuitive analysis.

TABLE 14. Four groups of hypothetical observation data.

Hypoth	M	FCR	IFF	WE	I
T1	{(Y,1),(N,0)}	{(ON,1),(OFF,0)}	{(Y,0),(N,1)}	{(L,1),(M,0),(S,0)}	{(H,1),(M,0),(L,0)}
T2	{(Y,0.5),(N,0.5)}	{(ON,0.8),(OFF,0.2)}	{(Y,0.4),(N,0.6)}	{(L,0),(M,0.3),(S,0.7)}	{(H,0),(M,0.6),(L,0.4)}
T3	{(Y,0.3),(N,0.7)}	{(ON,0.2),(OFF,0.8)}	{(Y,0.3),(N,0.7)}	{(L,0),(M,1),(S,0)}	{(H,0.9),(M,0.1),(L,0)}
T4	{(Y,0.4),(N,0.6)}	{(ON,0.6),(OFF,0.4)}	{(Y,0.9),(N,0.1)}	{(L,0),(M,0.8),(S,0.2)}	{(H,0),(M,0.3),(L,0.7)}

TABLE 15. Occurrence probability (OP), support degree (S) and weight (W).

			T1		T2		T3		T4	
Num	BPA	S	OP	W	OP	W	OP	W	OP	W
1	NF is {(T,0.05),(F,0.95)}	0.3500	0.0000	0.0000	0.4000	0.4000	0.3000	0.3000	0.9000	0.9000
2	NF is {(T,0.7),(F,0.3)}	0.3500	1.0000	1.0000	0.6000	0.6000	0.7000	0.7000	0.1000	0.1000
3	HI is {(H,1)}	1.3229	0.7000	0.5368	0.1760	0.1021	0.0303	0.0181	0.0276	0.0159
4	HI is {(H,0.3),(M,0.7)}	3.6778	0.3000	0,4632	0.2240	0.2617	0.0297	0.0357	0.2124	0.2458
5	HI is {(H,0.1),(M,0.9)}	3.2566	0.0000	0.0000	0.0440	0.0468	0.1212	0.1324	0.0184	0.0194
6	HI is {(M,0.3),(L,0.7)}	3.0914	0.0000	0.0000	0.0560	0.0572	0.1188	0.1247	0.1416	0.1433
7	HI is {(H,0.6),(M,0.4)}	3.2142	0.0000	0.0000	0.1760	0.1852	0.0707	0.0765	0.0414	0.0432
8	HI is {(M,0.8),(L,0.2)}	3.6166	0.0000	0.0000	0.2240	0.2583	0.0693	0.0821	0.3186	0.3639
9	HI is {(M,0.6),(L,0.4)}	3.7780	0.0000	0.0000	0.0440	0.0525	0.2828	0.3467	0.0276	0.0326
10	HI is {(L,1)}	1.5855	0.0000	0.0000	0.0560	0.0362	0.2772	0.1839	0.2124	0.1359
11	C is {(G,1)}	1.2689	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	C is {(G,0.5),(M,0.5)}	3.7930	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	C is {(M,0.6),(B,0.4)}	4.2700	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14	C is {(G,0.4),(M,0.6)}	3.9666	0.0000	0.0000	0.0000	0.0000	0.9000	0.9160	0.0000	0.0000
15	C is {(M,1)}	3.1000	0.0000	0.0000	0.1800	0.1790	0.1000	0.0840	0.2400	0.2215
16	C is {(M,0.3),(B,0.7)}	3.8784	0.0000	0.0000	0.1200	0.1420	0.0000	0.0000	0.5600	0.6150
17	C is {(M,0.9),(B,0.1)}	3.6272	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18	C is {(M,0.2),(B,0.8)}	3.4907	0.0000	0.0000	0.4200	0.4575	0.0000	0.0000	0.0600	0.0607
19	C is {(B,1)}	2.2622	0.0000	0.0000	0.2800	0.2215	0.0000	0.0000	0.1400	0.1028
20	TL is {(H,1)}	1.6399	0.6757	0.5068	0.0000	0.0000	0.0322	0.0177	0.0000	0.0000
21	TL is {(H,0.7),(M,0.3)}	3.4227	0.0000	0.0000	0.0928	0.0855	0.0557	0.0513	0.0491	0.0482
22	TL is {(H,0.2),(M,0.8)}	4.3079	0.0000	0.0000	0.2036	0.2250	0.0000	0.0000	0.0683	0.0804
23	TL is {(H,0.4),(M,0.6)}	4.3527	0.3243	0.4932	0.0000	0.0000	0.1780	0.1985	0.0000	0.0000
24	TL is {(M,1)}	3.5000	0.0000	0.0000	0.1737	0.1627	0.3078	0.2886	0.2344	0.2340
25	TL is {(M,0.6),(L,0.4)}	4.3735	0.0000	0.0000	0.3810	0.4263	0.0000	0.0000	0.3261	0.3887
26	TL is {(M,0.8),(L,0.2)}	4.3126	0.0000	0.0000	0.0000	0.0000	0.1562	0.1729	0.0000	0.0000
27	TL is {(M,0.4),(L,0.6)}	3.8142	0.0000	0.0000	0.0466	0.0467	0.2701	0.2709	0.1347	0.1438
28	TL is {(L,1)}	1.5229	0.0000	0.0000	0.1023	0.0537	0.0000	0.0000	0.1873	0.1049

1) For hypothesis T1, flight attitude is true, fire-control radar is on and enemy is highly true, so that the attack intention is high. Attack intent is long and attack preparation is high, thus the attack ability is good. A high attack intention and ability lead to a high threat level.
2) For hypothesis T2, flight attitude is highly uncertain, fire-control radar is more likely to be on and enemy is also uncertain, so that the attack intention is more likely to be medium. Attack intent is more likely to be small and attack preparation tends to be medium, thus the attack ability is bad. A more apparent attack intention with a bad attack ability leads to a medium threat level.
3) For hypothesis T3, flight attitude is more tending to be no, fire-control radar is more likely to be off and enemy is uncertain, so that the attack intention is not apparent. Attack intent is medium and attack preparation is high, so the attack ability is medium. A uncertain attack intention and a medium attack ability lead to a medium threat level.
4) For hypothesis T4, flight attitude is a little tending to be no, fire-control radar is likely to be on and enemy is more likely to be false, so the attack intention is medium. Attack intent is more likely to be medium and attack preparation is tending to be low, thus the attack ability is bad. A medium attack intention with a bad attack ability causes a medium threat level.

TABLE 16. Calculation results of each nodes.

Node	T1	T2	T3	T4
NF	{(T,0.7),(F,0.3)}	{(T,0.44),(F,0.56)}	{(T,0.5050),(F,0.4950)}	{(T,0.1150),(F,0.8850)}
HI	{(H,0.6757), (M,0.3243),(L,0)}	{(H,0.2964),(M,0.5547), (L,0.1489)}	{(H,0.0879),(M,0.4858), (L,0.4263)}	{(H,0.1174),(M,0.5605), (L,0.3220)}
C	{(G,1),(M,0),(B,0)}	{(G,0),(M,0.3131), (B,0.6869)}	{(G,0.3664),(M,0.6336), (B,0)}	{(G,0),(M,0.4182), (B,0.5818)}
TL	{(H,0.7041), (M,0.2959),(L,0)}	{(H,0.1048),(M,0.6429), (L,0.2523)}	{(H,0.1330),(M,0.6698), (L,0.1971)}	{(H,0.0498),(M,0.6036), (L,0.3466)}
TL (after combining n-1 times)	{(H,0.8499), (M,0.1501),(L,0)}	{(H,0.0001),(M,0.9963), (L,0.0036)}	{(H,0.0001),(M,0.9993), (L,0.0006)}	{(H,0),(M,0.9654), (L,0.0346)}

TABLE 17. Calculation results of each nodes in [58].

Node	T1	T2	T3	T4
NF	{(T,0.7),(F,0.3)}	{(T,0.4219),(F,0.5781)}	{(T,0.5279),(F,0.4721)}	{(T,0.0581),(F,0.9419)}
HI	{(H,0.9176), (M,0.0824),(L,0)}	{(H,0.3396),(M,0.5277), (L,0.1327)}	{(H,0.0762),(M,0.4186), (L,0.5052)}	{(H,0.0724),(M,0.5361), (L,0.3915)}
C	{(G,1),(M,0),(B,0)}	{(G,0),(M,0.2435), (B,0.7565)}	{(G,0.3707),(M,0.6293), (B,0)}	{(G,0),(M,0.4145), (B,0.5855)}
TL	{(H,0.9970), (M,0.003),(L,0)}	{(H,0.0831),(M,0.6612), (L,0.2557)}	{(H,0.0861),(M,0.7311), (L,0.1828)}	{(H,0.0202),(M,0.5320), (L,0.4477)}

Furthermore, Table 17 shows the computation results in [58]. The authors compare the computation results in Table 16 with Table 17. It shows that the assessment results are nearly same, which further proves that the proposed method can effectively deal with scenarios with uncertain information.

From above computation and analysis, evidential reasoning is an alternative approach to deal with TAP under uncertain information. Each node is represented by a BPA, which contributes to representing a uncertain condition. The relationship between nodes is given through reasoning rules and the decision results are computed through evidential reasoning and fusion. Finally, the threat level is derived after combining all the uncertain conditions. The whole process is beneficial for commanders to make more accurate decision.

VI. CONCLUSION

In this paper, a novel method of evidential network reasoning based on the logical reasoning rules and conflict measure is proposed. Two reasoning rules called the AND rule and OR rule are defined firstly. Antecedent probability is derived from pignistic transformation. The occurrence probabilities of the sub-BPAs in the child nodes are derived according to the antecedent probabilities and reasoning rules. The support degree of each sub-BPAs is derived based on the distance between bodies of evidence. Then the weight of each sub-BPA is obtained from the occurrence probability and the support degree. The BPA of child node is derived by combining the weighted average sub-BPAs. There are several advantages of the proposed method. Firstly, both the occurrence probability and the support degree are considered when calculating the weights, which improves the accuracy of information fusion. Secondly, the process to obtain occurrence probabilities logically conforms to the reasoning rules, so it is intuitive. Thirdly, weighted fusion of BPAs is used to resolve the high conflict between evidences. Two examples are given to prove that the proposed method can deal with scenarios with uncertain and multi-source information effectively. The first one is fault diagnosis with three senors, which contributes to detecting fault of a complicated system. The second one is threat assessment and prediction, which can offer an assistant decision basis for commanders. This method may still have a few disadvantages. For example, BPAs in the conditional belief table are partially based on the experience of experts, so the application can be limited without expert's support. Other complicated logical relationship may exist except for interaction and union. Parent nodes are independent from each other, which cannot be met in some scenarios. In the future research, the authors will further improve the evidential reasoning method.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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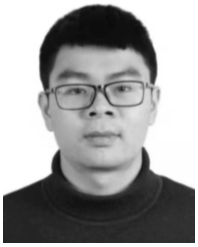

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