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Multi-Dimensional Taylor Network-Based Adaptive Output-Feedback Tracking Control for a Class of Nonlinear Systems

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ABSTRACT In this paper, the output feedback adaptive multi-dimensional Taylor network (MTN) tracking control for a class of nonlinear systems with unmeasurable states is investigated. Firstly, a nonlinear state observer is designed to estimate the unmeasurable states, and then an adaptive MTN-based output-feedback control approach is developed via backstepping technique. Secondly, in view of the simple structure of MTN, the controller based on MTN has the advantages of simple structure and fast calculation speed. Thirdly, in order to avoid the ‘‘differential explosion’’ problem inherited from the backstepping design, dynamic surface control (DSC) technique is introduced in the process of controller design. The results demonstrate that this scheme guarantees the stability and tracking performance of the closed-loop system. Finally, simulation examples are given to reveal the viability of the proposed method.

INDEX TERMS Multi-dimensional Taylor network, nonlinear systems, adaptive control, output-feedback, backstepping.

I. INTRODUCTION

In recent years, more and more scholars have begun to pay attention to the stability analysis and controller design of nonlinear systems, and many interesting results have been reported [1], [2]. Due to the output-feedback control is more suitable for practical engineering systems [3], significant progress has been made in the design of output-feedback controllers for nonlinear systems, such as uncertain nonlinear systems [4], input-delayed systems with time-varying uncertainties [5], Markovian jump systems [6], and large-scale stochastic nonlinear systems [7]. However, compared with full state feedback control, for example, strict-feedback [8], pure-feedback [9] and non-strict feedback [10], the design of output feedback control is more difficult and challenging, the results of controller design for nonlinear systems are relatively few. Consequently, it remains a significant and interesting task to put forward a state observer with good estimation performance and design an output feedback controller with good control performance for nonlinear systems.

In view of the excellent performance of neural networks (NNs) and fuzzy logic systems (FLSs), especially the traits of nonlinear, capacity of study and self adapting, the approximation-based adaptive neural or fuzzy control schemes have become a useful approach to deal with uncertain nonlinear systems [11]–[26]. Meanwhile, NNs-based or FLSs-based control approaches have been applied to uncertain discrete-time nonlinear systems [11], dynamic parameters adjustment nonlinear systems [12], dynamic uncertainties nonlinear systems [13], strict-feedback nonlinear systems [14]–[16], pure-feedback nonlinear systems [17], [18], switched nonlinear systems [19]–[22], MIMO nonlinear systems [23], [24] and stochastic nonlinear systems [25], [26]. Although the adaptive neural or fuzzy backstepping control has achieved great progress, three aspects can not be ignored: (i) the training time of most

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NNs or FLSs are usually too long and there also exists local minimum. (ii) Most of the NNs can not be applied to actual dynamic systems because their neurons have limited functions. (iii) The accuracy of fuzzy control is not high enough and oscillation may occur. This encourages us to investigate new approximation-based adaptive control approaches for the control of nonlinear systems to solve the above problems. In this context, the idea of multi-dimensional Taylor network (MTN) emerged.

MTN is a three-layer feedback network, and includes the input layer, middle layer and output layer. MTN-based approach was first proposed to solve the problem of prediction control. Later, it was successfully extended to the control of nonlinear systems, and significant results have been achieved, for instance, based on account of discrete MTN, Yan and Kang [27] studied the asymptotic tracking and dynamic regulation of SISO nonlinear systems. Kang and Yan [28] proposed a MTN controller to stabilize the nonlinear time-varying delay systems with an inaccurate model. Han and Yan [29] studied the problem of adaptive tracking control for SISO uncertain stochastic nonlinear systems based on MTN. Yan and Han [30] investigated the problem of adaptive MTN decentralized tracking control for a class of large-scale stochastic nonlinear systems. Yan *et al.* [31] proposed an optimal output-feedback tracking control approach for SISO stochastic nonlinear systems. However, to the best of the authors' knowledge, fewer efforts have been devoted to the MTN-approximation-based adaptive output-feedback tacking control for nonlinear systems [32], [33]. Therefore, the construction of adaptive MTN tracking control algorithm for nonlinear systems is still an interesting and challenging subject, which has some inspiration for our research.

For the above-mentioned observations, this paper tries to study the adaptive output-feedback tracking control design problem for a class of nonlinear systems with unmeasurable states, and proposes an output-feedback control scheme based on adaptive MTN. Firstly, using the method by references [34], [35], a nonlinear state observer is designed to estimate the unmeasurable states. Secondly, the backstepping technique and MTN are combined to construct an adaptive output-feedback control scheme. Meanwhile, in order to avoid the "differential explosion" problem inherited from the backstepping design, DSC technique is introduced in the process of controller design. Thirdly, the stability of the closed-loop control system, the boundedness of the tracking error and control signals are ensured by Lyapunov stability theory. Finally, simulation results are presented to demonstrate the effectiveness of the design approach. The contributions of this paper are highlighted as follows:

(i) A novel adaptive output feedback control method based on MTN is proposed for a class of nonlinear systems with unmeasurable states. The proposed method can obtain accurate tracking results with low computational cost, and has good real-time performance and convergence.

(ii) The computational complexity of the designed MTN-based controller is greatly minimizes through the following two aspects: a) Because of the simple structure of MTN, the controller based on MTN has the advantages of simple structure and fast calculation speed. b) At every step of backstepping, combining MTN method with DSC technique, the calculating amount is reduced as well as the problem of the nonlinear is effectively handled.

Throughout this paper, the following notations are used. $\mathbb{R}$ indicates the set of all real numbers, $\mathbb{R}^n$ denotes the real n dimensional space. In formula $\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{P}_{m_n}(s)$, n denotes the input number of MTN, m represents the highest power of the polynomials in the middle layer of MTN, $\boldsymbol{\theta}^T$ is the weight vector of MTN.

II. SYSTEM DESCRIPTIONS AND PRELIMINARY

A. PROBLEM DESCRIPTION

Consider the following nonlinear system with external disturbances:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + d_i(t) \\ \quad i = 1, \cdots, n-1 \\ \dot{x}_n = u + f_n(\bar{x}_n) + d_n(t) \\ \quad y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the system output. $d_i(t)$ is bounded interference, $\bar{\boldsymbol{x}}_i = [x_1, \cdots, x_i]^{\mathrm{T}} \in \mathbb{R}^i$, $i = 1, 2, \cdots, n$. $f(\cdot): \mathbb{R}^i \to \mathbb{R}$ is known smooth function with $f_i(\mathbf{0}) = \mathbf{0}$.

The objective of this paper is to design an adaptive controller ensuring that y tracks y_d, where y_d is a given continuous reference signal.

Rewriting the nonlinear system (1) into the following form

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + F(\boldsymbol{x}) + Bu + D(t) \\ y = C\boldsymbol{x} \end{cases} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, F(\boldsymbol{x}) = \begin{bmatrix} f_1(\bar{\boldsymbol{x}}_1) \\ f_2(\bar{\boldsymbol{x}}_2) \\ \vdots \\ f_n(\bar{\boldsymbol{x}}_n) \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix},$$

$$D(t) = [d_1(t), \cdots, d_n(t)]^T \quad \text{and } C = [1, 0, \cdots, 0]^{\mathrm{T}}.$$

The study of this paper is based on following assumptions.

Assumption 1: The given reference signal y_d and its time derivatives up to the n-th order are continuous and bounded.

Assumption 2: For each $i = 1, \ldots, n$, d_i satisfies $|d_i| \leq \bar{d}_i$, where $\bar{d}i$ is an unknown constant.

Assumption 3: [34], [35] There exist a matrix H and a function $h(\boldsymbol{x})$, such that $F(\boldsymbol{x}) = Hh(\boldsymbol{x})$, and $h(\boldsymbol{x})$ satisfies:

$$\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}} + \left(\frac{\partial h(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)^{\mathrm{T}} \geq 0, \quad \forall \boldsymbol{x} \in \mathrm{R}^n \tag{3}$$

where $h(x)$ and $F(\boldsymbol{x})$ are vector-valued function with $F(\mathbf{0}) = 0$.

Remark 1: It should be noted that there are some physical systems satisfy Assumption 3, such as single link flexible joint robot systems [36] and omnidirectional intelligent navigation systems [37].

Assumption 4: [34], [35] Matrices A, C and H defined in (2) and (3) satisfy the following linear matrix inequality (LMIs):

$$\begin{bmatrix} (A+LC)^{\mathrm{T}}Q_1+Q_1(A+LC)+Q_2 & Q_1H+(I+KC)^T \\ H^{\mathrm{T}}Qn_1+(I+KC) & 0 \end{bmatrix} \leq 0$$

where $Q_1 = Q_1^{\mathrm{T}} > 0$, $Q_2 = Q_2^{\mathrm{T}} > 0$, $K = [k_1, \cdots, k_n]^{\mathrm{T}}$, and $L = [l_1, \cdots, l_n]^{\mathrm{T}}$.

B. MULTI-DIMENSIONAL TAYLOR NETWORK

Figure 1 shows the structure of MTN with n inputs and the highest power of the polynomials in the middle layer is m, where $s_1, \cdots, s_n$ are the input vector of the MTN, $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_n]^{\mathrm{T}}$ is the weight vector of the MTN.

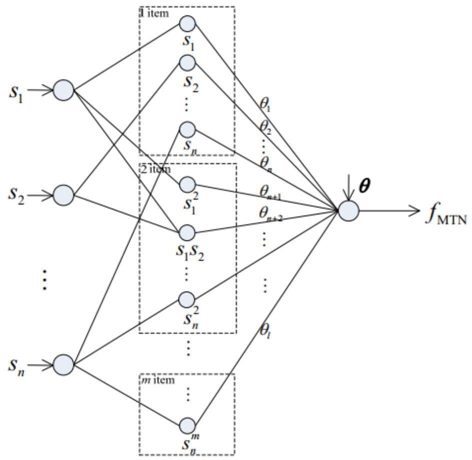

FIGURE 1. The topological structure of MTN.

In this paper, the unknown nonlinear functions in the system will be approximated by the MTN. In particular, suppose $f(\boldsymbol{s})$ is defined on a compact set $\Omega_S \in R^n$, then we have

$$f(\boldsymbol{s}) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{P}_{m_n}(\boldsymbol{s})$$

where

$$\boldsymbol{P}_{m_n}(\boldsymbol{s}) = [\underbrace{s_1, \cdots, s_n}_{1\ term}, \underbrace{s_1^2, s_1s_2, \cdots s_n^2}_{2\ term}, \cdots, \underbrace{s_1^m, \cdots, s_n^m}_{m\ term}]^{\mathrm{T}},$$
$$\boldsymbol{\theta}^{\mathrm{T}} = [\theta_1, \cdots, \theta_n]^{\mathrm{T}} \quad \text{and} \quad \boldsymbol{s}^{\mathrm{T}} = [s_1, \cdots, s_n]^{\mathrm{T}}.$$

Lemma 1: [29] Assume that $\phi(\boldsymbol{s})$ is a continuous function defined on a compact set $\Omega_{\boldsymbol{s}}$. Then, for any given desired level of accuracy $\varepsilon > 0$, there exists a MTN, such that

$$\varphi(\mathbf{s}) = \boldsymbol{\theta}^{*\mathrm{T}}\boldsymbol{P}_{m_n}(\mathbf{s}) + \delta(\boldsymbol{s}) \tag{4}$$

where $\boldsymbol{\theta}^*$ is the ideal weight vector and defined as

$$\boldsymbol{\theta}^* := \arg\min_{\boldsymbol{\theta}\in R^l}\left\{\sup_{s\in\Omega_s}\left|\phi(s) - \boldsymbol{\theta}^{\mathrm{T}}P_{m_n}(s)\right|\right\}$$

and $\delta(\boldsymbol{s})$ denotes the approximation error and satisfies $|\delta(\boldsymbol{s})| \leq \varepsilon$.

III. MTN-BASED ADAPTIVE OUTPUT-FEEDBACK CONTROLLER DESIGN

A. NONLINEAR OBSERVER DESIGN

First of all, the following observer [34], [35] is used to estimate the unmeasured states

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}}+L(C\hat{\boldsymbol{x}} - y) + F[\hat{\boldsymbol{x}} + K(C\hat{\boldsymbol{x}} - y)] + Bu \tag{5}$$

where $\hat{\boldsymbol{x}} = [\hat{x}_1, \cdots, \hat{x}_n]^{\mathrm{T}}$ is the observer state vector and matrices K and L satisfy Assumption 4.

Define the observer error as $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$, from (2), we have

$$\dot{\tilde{\boldsymbol{x}}} = (A + LC)\tilde{x} + F(x) - F(v)$$

where $v = \hat{x} + K(C\hat{x} - y)$.

Consider the following Lyapunov function

$$V_0 = \frac{1}{2}\tilde{\boldsymbol{x}}^{\mathrm{T}}Q_1\tilde{\boldsymbol{x}} \tag{6}$$

then, the time derivative of V_0 is

$$\begin{aligned}\dot{V}_0 = &\frac{1}{2}\tilde{\boldsymbol{x}}^{\mathrm{T}}\left(Q_1(A + LC) + (A + LC)^{\mathrm{T}}Q_1\right)\tilde{\boldsymbol{x}} \\ &+\tilde{\boldsymbol{x}}^{\mathrm{T}}Q_1H\varphi + \tilde{\boldsymbol{x}}^{\mathrm{T}}Q_1D\end{aligned}$$

where $\phi(\boldsymbol{x}, \boldsymbol{\mu}) = h(\boldsymbol{x}) - h(\boldsymbol{x} - \boldsymbol{\mu})$, and by taking into consideration of Assumption 3, Lemma 1 and formula $F(\boldsymbol{x}) = Hh(\boldsymbol{x})$. Similar to the literature [35], we have

$$\dot{V}_0 \leq -\frac{1}{2}\tilde{\boldsymbol{x}}^{\mathrm{T}}Q_2\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^{\mathrm{T}}Q_1D \tag{7}$$

From Assumption 2, there exist constant matrix $\bar{D} = [\bar{d}_1, \cdots, \bar{d}_n]^{\mathrm{T}}$ such that

$$D \leq \bar{D} \tag{8}$$

Then, by the Young's inequality, we have

$$\tilde{\boldsymbol{x}}^{\mathrm{T}}Q_1D \leq \frac{1}{2}\|\tilde{\boldsymbol{x}}\|^2 + \frac{1}{2}\left\|Q_1\bar{D}\right\|^2 \tag{9}$$

By (7), (8) and (9), we have

$$\dot{V}_0 \leq -\frac{1}{2}\|\tilde{\boldsymbol{x}}\|^2(\lambda - 1) + \frac{1}{2}\left\|Q_1\bar{D}\right\|^2 \tag{10}$$

where $\lambda = \lambda_{\min}(Q_1)\lambda_{\min}(Q_2)$.

B. MTN-BASED CONTROLLER DESIGN

According to (1) and (5), we have following entire system

$$\begin{cases} \dot{y} = \hat{x}_2 + \tilde{x}_2 + \phi_1(x_1) + d_1 \\ \dot{\hat{x}}_2 = \hat{x}_3 - l_2\tilde{x}_1 + \phi_2(\bar{\hat{x}}_2, y) \\ \vdots \\ \dot{\hat{x}}_n = u - l_n\tilde{x}_1 + \phi_n(\bar{\hat{x}}_n, y) \end{cases} \tag{11}$$

where $\phi_i(\bar{\hat{\boldsymbol{x}}}_i, y) = F(\hat{x}_1 + k_1(\hat{x}_1 - y), \cdots, \hat{x}_i + k_i(\hat{x}_1 - y))$, $i = 2, \cdots, n$ and $\phi_1(x_1) = f_1(x_1)$.

First of all, a change of coordinates is introduced as follows

$$\begin{cases} z_1 = y - y_d \\ z_i = \hat{x}_i - \alpha_{i,f}\ (i = 2, \cdots, n) \end{cases} \tag{12}$$

where $\alpha_{i,f}$ is the output of the first-order filter with α_{i-1} as the input.

Step 1: According to (12), we have

$$\dot{z}_1 = \hat{x}_2 + \tilde{x}_2 + \phi_1 + d_1 - \dot{y}_d \tag{13}$$

Consider the following Lyapunov function

$$V_1 = V_0 + \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\tilde{\boldsymbol{\theta}}_1 \tag{14}$$

where $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\theta}_1 - \hat{\boldsymbol{\theta}}_1$ is the parameter error and $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ is any constant matrix.

The time derivative of V_1 is

$$\dot{V}_1 = \dot{V}_0 + z_1(\hat{x}_2 + \tilde{x}_2 + \phi_1 + d_1 - \dot{y}_d) - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 \tag{15}$$

By the Young's inequality, we have

$$z_1\tilde{x}_2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}\|\tilde{\boldsymbol{x}}\|^2 \tag{16}$$

$$z_1 d_1 \leq \frac{1}{2}z_1^2 + \frac{1}{2}d_1^2 \tag{17}$$

substituting (16) and (17) into (15) gives

$$\dot{V}_1 \leq \dot{V}_0 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 + \frac{1}{2}\|\tilde{\boldsymbol{x}}\|^2 + \frac{1}{2}d_1^2 + z_1(\hat{x}_2 + \tilde{f}_1) \tag{18}$$

where $\tilde{f}_1 = \phi_1 - \dot{y}_d + z_1$.

According to Lemma 1, for any $\varepsilon_1 > 0$, there exists a MTN $\boldsymbol{\theta}_1^{\mathrm{T}}S(z_1)$, such that

$$\tilde{f}_1 = \boldsymbol{\theta}_1^{\mathrm{T}}S_1 + \sigma_1(z_1), |\sigma_1(z_1)| \leq \varepsilon_1 \tag{19}$$

Based on (18) and (19), taking the virtual control signal α_1 as

$$\alpha_1 = -k_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}}S_1(z_1) \tag{20}$$

where $k_1 > 0$ is a design parameter.

Form (19) and (20), and by the Young's inequality, we have

$$z_1(\hat{x}_2 + \tilde{f}_1) \leq z_1(\hat{x}_2 - \alpha_1) + z_1\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}S_1 - k_1 z_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2 \tag{21}$$

Substituting (21) into (18) gives

$$\dot{V}_1 \leq -\frac{1}{2}(\lambda - 2)\|\tilde{\boldsymbol{x}}\|^2 + \frac{1}{2}\left\|Q_1\bar{D}\right\|^2 - \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1 + z_1\tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}S_1 + \frac{1}{2}d_1^2 + z_1(\hat{x}_2 - \alpha_1) - k_1 z_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2 \tag{22}$$

In order to avoid the repetitive differential of α_1 , a new variable $\alpha_{2,f}$ is introduced and let α_1 pass through a first-order filter whose time constant is τ_2, and $\alpha_{2,f}$ is

$$\tau_2\dot{\alpha}_{2,f} + \alpha_{2,f} = \alpha_1, \alpha_{2,f}(0) = \alpha_1(0) \tag{23}$$

where $\tau_2 > 0$ is time constant.

Define the output error of the filter as

$$\chi_2 = \alpha_{2,f} - \alpha_1 \tag{24}$$

Due to $z_2 = \hat{x}_2 - \alpha_{2,f}$, and by (23) and (24), we have

$$z_1(\hat{x}_2 - \alpha_1) = z_1 z_2 + z_1\chi_2 \tag{25}$$

substituting (25) into (22) gives

$$\dot{V}_1 \leq -\frac{1}{2}(\lambda - 2)\|\tilde{\boldsymbol{x}}\|^2 + \frac{1}{2}\left\|Q_1\bar{D}\right\|^2 + \tilde{\boldsymbol{\theta}}_1^{\mathrm{T}}(z_1 S_1 - \Gamma_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_1) + \frac{1}{2}d_1^2 - k_1 z_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2 + z_1 z_2 + z_1\chi_2. \tag{26}$$

Step 2: A new state variable $\alpha_{3,f}$ is introduced, and α_2 is input into a first-order low-pass filter with a time constant of τ_3 to obtain a new variable $\alpha_{3,f}$ as

$$\tau_3\dot{\alpha}_{3,f} + \alpha_{3,f} = \alpha_2, \alpha_{3,f}(0) = \alpha_3(0) \tag{27}$$

where $\tau_3 > 0$ is time constant.

Define the output error of the filter as

$$\chi_3 = \alpha_{3,f} - \alpha_2 \tag{28}$$

The time-derivative of χ_3 is

$$\dot{\chi}_3 = -\frac{\chi_3}{\tau_3} + B_3(\bar{z}_2, \tilde{x}, \bar{\hat{\boldsymbol{\theta}}}_2, \bar{\chi}_3) \tag{29}$$

where $\bar{z}_2 = [z_1, z_2]^{\mathrm{T}}$, $\bar{\chi}_2 = [\chi_1, \chi_2]^{\mathrm{T}}$, $\bar{\hat{\boldsymbol{\theta}}}_2 = [\hat{\theta}_1, \hat{\theta}_2]^{\mathrm{T}}$ and

$$B_3(\bar{z}_2, \tilde{x}, \bar{\hat{\boldsymbol{\theta}}}_2, \bar{\chi}_3) = -\frac{\partial\alpha_2}{\partial y}\left(\hat{x}_2 + \tilde{x}_2 + \phi_1(x_1)\right) - \frac{\partial\alpha_2}{\partial\hat{\boldsymbol{\theta}}_1}\dot{\hat{\boldsymbol{\theta}}}_1 - \frac{\partial\alpha_2}{\partial\hat{\boldsymbol{\theta}}_2}\dot{\hat{\boldsymbol{\theta}}}_2 - \frac{\partial\alpha_2}{\partial\hat{x}_2}\dot{\hat{x}}_2 - \frac{\partial\alpha_1}{\partial\alpha_{2,f}}\dot{\alpha}_{2,f}$$

Consider the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}\chi_2^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\Gamma_2^{-1}\tilde{\boldsymbol{\theta}}_2 \tag{30}$$

where $\tilde{\boldsymbol{\theta}}_2 = \boldsymbol{\theta}_2 - \hat{\boldsymbol{\theta}}_2$ is the parameter error, and $\Gamma_2 = \Gamma_2^{\mathrm{T}} > 0$ is any constant matrix.

The time-derivative of V_2 is

$$\dot{V}_2 = \dot{V}_1 - \tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}\Gamma_2^{-1}\dot{\hat{\boldsymbol{\theta}}}_2 + \chi_2\left(-\frac{\chi_2}{\tau_2} + B_2\right) + z_2(\hat{x}_3 + \tilde{f}_2) \tag{31}$$

where $\tilde{f}_2 = \phi_2 - l_2\tilde{x}_1 - \dot{\alpha}_{2,f}$.

Similar to Step 1, a new MTN $\boldsymbol{\theta}_2^{\mathrm{T}}S_2(\mathbf{z}_2)$ is employed to approximate the unknown function $\tilde{f}_2$, for any given $\varepsilon_2 > 0$, we have

$$\tilde{f}_2 = \boldsymbol{\theta}_2^{\mathrm{T}}S_2 + \sigma_2(\mathbf{z}_2), |\sigma_2(\mathbf{z}_2)| \leq \varepsilon_2 \tag{32}$$

where $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$, and $\sigma_2(\mathbf{z}_2)$ is approximation error.

Taking the virtual control signal α_2 as

$$\alpha_2 = -k_2 z_2 - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}}S_2(\mathbf{z}_2), (k_2 > 0) \tag{33}$$

By (32) and (33), we have

$$z_2(\hat{x}_3 + \tilde{f}_2) \leq z_2(\hat{x}_3 - \alpha_2) + z_2\tilde{\boldsymbol{\theta}}_2^{\mathrm{T}}S_2 - k_2 z_2^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\varepsilon_2^2 \tag{34}$$

Due to $z_3 = \hat{x}_3 - \alpha_{3,f}$, and by (34),we have

$$\dot{V}_2 \le -\frac{1}{2}(\lambda-2)\|\tilde{\boldsymbol{x}}\|^2 + \frac{1}{2}\left\|Q_1\bar{D}\right\|^2 + \sum_{i=1}^{2} z_i z_{i+1} + \sum_{i=1}^{2} z_i\chi_{i+1} + \frac{1}{2}\sum_{i=1}^{2}\varepsilon_i^2 + \sum_{i=1}^{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_iS_i - \Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i) - \sum_{i=1}^{2}k_iz_i^2 + \frac{1}{2}\sum_{i=1}^{2}z_i^2 + \chi_2\left(-\frac{\chi_3}{\tau_3}+B_3\right) + \frac{1}{2}d_1^2 \quad (35)$$

Step $i(3 \le i \le n-1)$. A new state variable $\alpha_{i+1,f}$ is introduced, and $\alpha_{i,f}$ is input into a first-order low-pass filter with a time constant of τ_{i+1} to obtain a new variable $\alpha_{i+1,f}$ as

$$\tau_{i+1}\dot{\alpha}_{i+1,f} + \alpha_{i+1,f} = \alpha_i, \alpha_{i+1,f}(0) = \alpha_i(0) \quad (36)$$

where $\tau_{i+1} > 0$ is time constant.

Due to $z_{i+1} = \hat{x}_{i+1} - \alpha_{i+1,f}$, define the output error of the first-order low-pass filter as

$$\chi_{i+1} = \alpha_{i+1,f} - \alpha_i \quad (37)$$

The time-derivative of χ_{i+1} is

$$\dot{\chi}_{i+1} = -\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1}(\bar{z}_i, \tilde{x}, \bar{\hat{\boldsymbol{\theta}}}_i, \bar{\chi}_{i+1}) \quad (38)$$

where $\bar{z}_i = [z_1, \cdots, z_i]^{\mathrm{T}}$, $\bar{\chi}_i = [\chi_1, \cdots, \chi_i]^{\mathrm{T}}$, $\bar{\hat{\boldsymbol{\theta}}}_i = [\hat{\theta}_1, \cdots, \hat{\theta}_i]^{\mathrm{T}}$ and

$$B_{i+1}(\bar{z}_i, \tilde{x}, \bar{\hat{\boldsymbol{\theta}}}_i, \bar{\chi}_{i+1}) = -\frac{\partial\alpha_i}{\partial y}(\hat{x}_2 + \tilde{x}_2 + \phi_1(x_1)) - \sum_{j=1}^{i}\frac{\partial\alpha_i}{\partial\hat{\boldsymbol{\theta}}_j}\dot{\hat{\boldsymbol{\theta}}}_j - \sum_{j=2}^{i}\frac{\partial\alpha_i}{\partial\hat{x}_j}\dot{\hat{x}}_j - \sum_{j=1}^{i-1}\frac{\partial\alpha_i}{\partial\alpha_{j+1,f}}\dot{\alpha}_{j+1,f}.$$

Consider the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\chi_i^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i \quad (39)$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ is the parameter error, and $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ is any constant matrix.

The time-derivative of V_i is

$$\dot{V}_i = \dot{V}_{i-1} - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i + z_i(\hat{x}_{i+1} + \tilde{f}_i) + \chi_i\left(-\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1}\right) \quad (40)$$

where $\tilde{f}_i = \phi_i - l_i\tilde{x}_1 - \dot{\alpha}_{i,f}$ and $l_i > 0$.

Similar to Step 2, for any given $\varepsilon_i > 0$, we have

$$\tilde{f}_i = \boldsymbol{\theta}_i^{\mathrm{T}}S_i(\mathbf{z}_i) + \sigma_i(\mathbf{z}_i), |\sigma_i(\mathbf{z}_i)| \le \varepsilon_i \quad (41)$$

where $\mathbf{z}_i = [z_1, \cdots, z_i]^{\mathrm{T}}$, and $\sigma_i(\mathbf{z}_i)$ is approximation error.

Take the virtual control signal α_i as

$$\alpha_i = -k_iz_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}S_i(\mathbf{z}_i), (k_i > 0) \quad (42)$$

By (41) and (42), we have

$$z_i(\hat{x}_{i+1} + \tilde{f}_i) \le z_i(\hat{x}_{i+1} - \alpha_i) + z_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}S_i - k_iz_i^2 + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_i^2 \quad (43)$$

Due to $z_{i+1} = \hat{x}_{i+1} - \alpha_{i+1,f}$ and (43), we have

$$\dot{V}_i \le -\frac{1}{2}(\lambda-2)\|\tilde{\boldsymbol{x}}\|^2 + \frac{1}{2}\left\|Q_1\bar{D}\right\|^2 + \sum_{i=1}^{i} z_i z_{i+1} + \sum_{i=1}^{i} z_i\chi_{i+1} + \sum_{i=1}^{i-1}\chi_i\left(-\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1}\right) + \sum_{i=1}^{i}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_iS_i - \Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i) + \frac{1}{2}\sum_{i=1}^{i}\varepsilon_i^2 + \frac{1}{2}d_1^2 - \sum_{i=1}^{i}k_iz_i^2 + \frac{1}{2}\sum_{i=1}^{i}z_i^2. \quad (44)$$

Step n: Consider the following Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\chi_n^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\Gamma_n^{-1}\tilde{\boldsymbol{\theta}}_n \quad (45)$$

where $\tilde{\boldsymbol{\theta}}_n = \boldsymbol{\theta}_n - \hat{\boldsymbol{\theta}}_n$ is the parameter error.

According to (45) with $i = n$, we have

$$\dot{V}_n = \dot{V}_{n-1} - \tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}\Gamma_n^{-1}\dot{\hat{\boldsymbol{\theta}}}_n + \chi_n\left(-\frac{\chi_{n+1}}{\tau_{n+1}} + B_{n+1}\right) + z_n(u + \tilde{f}_n) \quad (46)$$

where $\tilde{f}_n = \phi_n - l_n\tilde{x}_1 - \dot{\alpha}_{n,f}$ and $l_n > 0$.

Similarly, by the Lemma 2.1, for any given $\varepsilon_n > 0$, we have

$$\tilde{f}_n = \boldsymbol{\theta}_n^{\mathrm{T}}S_n + \sigma_n(\mathbf{z}_n), \ |\sigma_n(\mathbf{z}_n)| \le \varepsilon_n \quad (47)$$

where $\mathbf{z}_n = [z_1, \cdots, z_n]^{\mathrm{T}}$, and $\sigma_n(\mathbf{z}_n)$ is approximation error.

Take controller u as

$$u = -k_nz_n - \hat{\boldsymbol{\theta}}_n^{\mathrm{T}}S_n(\mathbf{z}_n), (k_n > 0) \quad (48)$$

By the Young's inequality, we have

$$z_n(u + \tilde{f}_n) \le z_n\tilde{\boldsymbol{\theta}}_n^{\mathrm{T}}S_n - k_nz_n^2 + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_n^2 \quad (49)$$

Substituting (49) into (46) gives

$$\dot{V}_n \le -\frac{1}{2}(\lambda-2)\|\tilde{\boldsymbol{x}}\|^2 + \frac{1}{2}\left\|Q_1\bar{D}\right\|^2 + \sum_{i=1}^{n-1} z_i z_{i+1} + \sum_{i=1}^{n-1} z_i\chi_{i+1} + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 + \sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_iS_i - \Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i) + \sum_{i=1}^{n-1}\chi_i\left(-\frac{\chi_{i+1}}{\tau_{i+1}} + B_{i+1}\right) - \sum_{i=1}^{n}k_iz_i^2 + \frac{1}{2}\sum_{i=1}^{n}z_i^2 + \frac{1}{2}d_1^2. \quad (50)$$

By the Young's inequality, we have

$$\sum_{i=1}^{n-1} z_iz_{i+1} \le \sum_{i=1}^{n-1}\left(\frac{1}{2}z_i^2 + \frac{1}{2}z_{i+1}^2\right) \le \sum_{i=1}^{n} z_i^2 \quad (51)$$

$$\sum_{i=1}^{n-1} z_i\chi_{i+1} \le \sum_{i=1}^{n-1}\left(\frac{1}{2}z_i^2 + \frac{1}{2}\chi_{i+1}^2\right) \le \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\sum_{i=1}^{n-1}\chi_i^2 \quad (52)$$

$$\sum_{i=1}^{n-1}\chi_i\left(-\frac{\chi_{i+1}}{\tau_{i+1}}+B_{i+1}\right)\leq-\sum_{i=1}^{n-1}\frac{\chi_{i+1}^2}{\tau_{i+1}}+\frac{1}{2}\sum_{i=1}^{n-1}\xi_{i+1}^2\lambda_{i+1}^2\chi_{i+1}^2+\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2} \quad (53)$$

where $\xi_i, \lambda_i (i = 1, \cdots, n-1)$ is any constant greater than zero. Substituting (51), (52) and (53) into (50), we have

$$\dot{V}_n \leq -\frac{1}{2}(\lambda-2)\|\tilde{\boldsymbol{x}}\|^2+\frac{1}{2}\left\|Q_1\bar{D}\right\|^2-(k_i-4)\sum_{i=1}^{n}z_i^2 -\sum_{i=1}^{n-1}\left(-\frac{1}{2}+\frac{1}{\tau_{i+1}}-\frac{1}{2}\xi_{i+1}^2\lambda_{i+1}^2\right)\chi_i^2+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 +\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2}+\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}(z_iS_i-\Gamma_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i)+\frac{1}{2}d_1^2. \quad (54)$$

In summary, the design procedure of the MTN-based controller is shown in Figure 2.

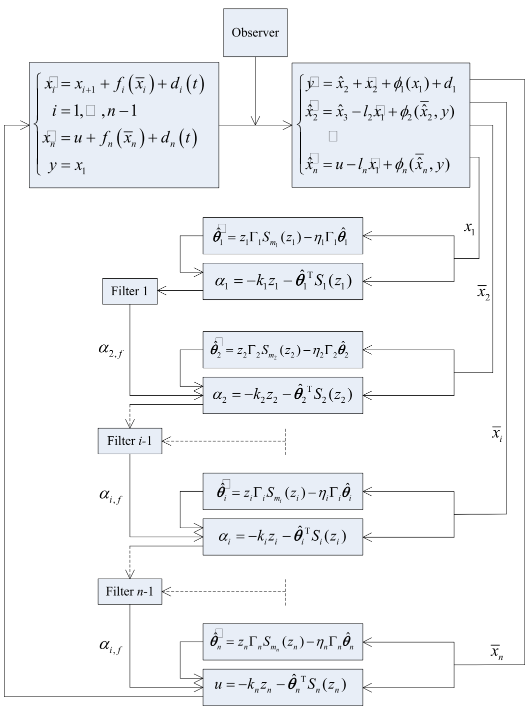

FIGURE 2. Block diagram of control system.

C. STABILITY ANALYSIS

Theorem 1: Considering the nonlinear system (1), if design the observer in the form of (5), design the control law u in the form of (48), the intermediate virtual control signals $\alpha_i (i = 1, \cdots, n-1)$ described as (42), and the adaptive laws $\dot{\hat{\boldsymbol{\theta}}}_i (i = 1, \cdots, n-1)$ defined as

$$\dot{\hat{\boldsymbol{\theta}}}_i = z_i\Gamma_iS_{m_i}(z_i)-\eta_i\Gamma_i\hat{\boldsymbol{\theta}}_i \quad (55)$$

where constants $k_i > 0$ and $\eta_i > 0$ are designed parameters, and constants matrices $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$. Then, under bounded initial conditions, all the signals in the closed-loop system are bounded, and the tracking error converges to a small neighborhood of the origin.

Proof: For the stability analysis of the closed-loop system, we choose the following Lyapunov equation:

$$V=\frac{1}{2}\tilde{\boldsymbol{x}}^{\mathrm{T}}Q_1\tilde{\boldsymbol{x}}+\frac{1}{2}\sum_{i=1}^{n}z_i^2+\frac{1}{2}\sum_{i=1}^{n}\chi_i^2+\frac{1}{2}\sum_{i=1}^{n}\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i \quad (56)$$

By (54) and (56), we have

$$\dot{V} \leq -\frac{1}{2}(\lambda-2)\|\tilde{\boldsymbol{x}}\|^2+\frac{1}{2}\left\|Q_1\bar{D}\right\|^2-(k_i-4)\sum_{i=1}^{n}z_i^2 -\sum_{i=1}^{n-1}\left(-\frac{1}{2}+\frac{1}{\tau_{i+1}}-\frac{1}{2}\xi_{i+1}^2\lambda_{i+1}^2\right)\chi_i^2+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 +\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2}+\sum_{i=1}^{n}\eta_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i+\frac{1}{2}d_1^2. \quad (57)$$

By the Lemma 1, we have

$$\eta_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i \leq -\bar{\eta}_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i+\frac{\eta_i}{2}\|\boldsymbol{\theta}_i\|^2 \quad (58)$$

where $\bar{\eta}_i = \eta_i \big/ 2\lambda_{\max}(\Gamma_i^{-1})$.

Substituting (58) into (57) gives

$$\dot{V} \leq -\frac{1}{2}(\lambda-2)\|\tilde{\boldsymbol{x}}\|^2+\frac{1}{2}\left\|Q_1\bar{D}\right\|^2-(k_i-4)\sum_{i=1}^{n}z_i^2 -\sum_{i=1}^{n-1}\left(-\frac{1}{2}+\frac{1}{\tau_{i+1}}-\frac{1}{2}\xi_{i+1}^2\lambda_{i+1}^2\right)\chi_i^2+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 +\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2}-\sum_{i=1}^{n}\left(\bar{\eta}_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\boldsymbol{\theta}}_i\right) +\frac{1}{2}\sum_{i=1}^{n}\left(\eta_i\|\boldsymbol{\theta}_i\|^2\right)+\frac{1}{2}d_1^2. \quad (59)$$

Let

$$a_i = \min\left\{\frac{\lambda-2}{\lambda_{\max}(Q_i)}, 2(k_i-4), 2\left(-\frac{1}{2}+\frac{1}{\tau_{i+1}}-\frac{1}{2}\xi_{i+1}^2\lambda_{i+1}^2\right), \bar{\eta}_i\right\}$$

$$a_0 = \min\{a_1, \cdots, a_n\}$$

$$b_0 = \frac{1}{2}\left\|Q_1\bar{D}\right\|^2+\frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2+\frac{1}{2}\sum_{i=1}^{n-1}\frac{1}{\xi_{i+1}^2}+\frac{1}{2}\sum_{i=1}^{n}\left(\eta_i\|\boldsymbol{\theta}_i\|^2\right)+\frac{1}{2}d_1^2.$$

then inequality (59) can be rewritten in the following form

$$\dot{V} \leq -a_0 V + b_0 \tag{60}$$

Using the similar arguments in [32], it is easy concluded that the conclusions of Theorem 1 is valid.

Remark 2: The inequality (60) implies that

$$V(t) \leq V(0) e^{-a_0 t} + \frac{b_0}{a_0}, \quad \forall t \geq 0 \tag{61}$$

According to (61), we know that $V(t)$, $\tilde{x}_i$, z_i, $\left\|\tilde{\boldsymbol{\theta}}_i\right\|$ are bounded. Thus, to guarantee that the tracking error converges to a small residual set around the origin in the sense of mean quartic value, we can properly adjust the parameters a_0 and b_0.

Remark 3: Recalling (56) and (61), we have

$$\sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{\boldsymbol{\theta}}_i \leq 2V(0) e^{-a_0 t} + 2\frac{b_0}{a_0} \tag{62}$$

Thus, for given $\varpi > 2b_0/a_0$, there exists a time T, for all $t \geq T$, such that

$$\sum_{i=1}^{n} \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}} \Gamma_i^{-1} \tilde{\boldsymbol{\theta}}_i \leq \varpi \tag{63}$$

which means that $\left\|\tilde{\boldsymbol{\theta}}_i\right\|$ converge to zero by properly adjusting the parameters, such as k_i, τ_i, ξ_i, η_i.

Remark 4: Theoretically speaking, based on Theorem 1, choosing appropriately the design parameters, such as k_i, η_i and Γ_i, can make the tracking error arbitrarily small. In practical application, however, these parameters should be selected appropriately to meet specific requirements.

IV. SIMULATION RESEARCH

In this section, we will demonstrate the effectiveness of the proposed adaptive MTN control method through two simulation examples.

Example 1: Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 - x_1^3 + 0.1 \sin t \\ \dot{x}_2 = u + x_1^3 - x_2^5 + 0.2 \cos t \\ y = x_1 \end{cases} \tag{64}$$

with the initial states $x_1(0) = 0$ and $x_2(0) = 0$.

According to (2) and (62), we have

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, F = \begin{bmatrix} x_1^3 \\ x_1^3 - x_2^5 \end{bmatrix}, D = \begin{bmatrix} 0.1 \sin t \\ 0.2 \cos t \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Let $h(\boldsymbol{x}) = \begin{bmatrix} x_1^3 \\ x_2^5 \end{bmatrix}$, $H = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$, then $F = Hh(\boldsymbol{x})$, and it is easy to verify that, when $Q_1 = \begin{bmatrix} 1.5 & 1 \\ 1 & 1 \end{bmatrix}$, $K = [-0.5, 1]^T$ and $L = [-1, -2]^T$, Assumptions 3-4 hold.

Design the following state observer

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - (\hat{x}_1 - x_1) - (\hat{x}_1 - 0.5(\hat{x}_1 - x_1))^3 \\ \dot{\hat{x}}_2 &= u - 2(\hat{x}_1 - x_1) + (\hat{x}_1 - 0.5(\hat{x}_1 - x_1))^3 \\ &\quad - (\hat{x}_2 + (\hat{x}_1 - x_1))^5 \end{aligned}$$

According to Theorem 1, the virtual control laws, the actual control law and the adaptive control laws are designed as

$$\begin{aligned} \alpha_1 &= -k_1 z_1 - \hat{\boldsymbol{\theta}}_1^{\mathrm{T}} S_1(z_1) \\ u &= -k_2 z_2 - \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} S_2(\mathbf{z}_2) \\ \dot{\hat{\boldsymbol{\theta}}}_i &= z_i \Gamma_i S_{m_i}(z_n) - \eta_i \Gamma_i \hat{\boldsymbol{\theta}}_i, \quad i = 1, 2 \end{aligned}$$

where $z_1 = x_1 - y_d$, $z_2 = \hat{x}_2 - \alpha_{2f}$, $\mathbf{z}_1 = z_1$, $\mathbf{z}_2 = [z_1, z_2]^{\mathrm{T}}$.

In the simulation, the parameters are chosen as follows: $k_1 = 15$, $k_2 = 10$, $\eta_1 = 0.5$, $\eta_2 = 1.5$, $\Gamma_1 = 20I_4$, $\Gamma_2 = 5I_9$, $\tau_2 = 0.005$. The reference signal $y_d = \sin t$. The simulation results are shown in Figures 3-8.

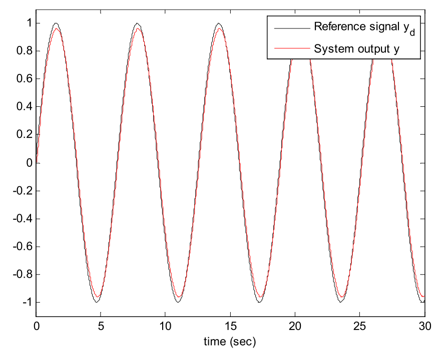

FIGURE 3. The system output and the reference signal of example 1.

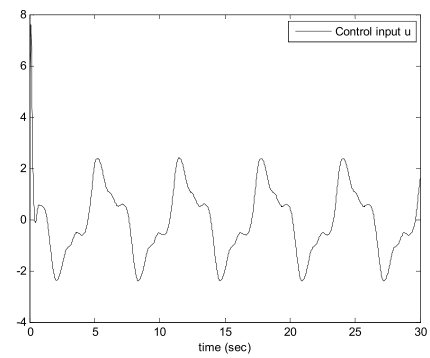

FIGURE 4. The control input of example 1.

The simulation results indicate that a good tracking control performance has been achieved. Figure 5 indicates the tracking error converges to a small neighbourhood around the origin. Figures 6-7 show that all signals of the closed-loop system, such as state x_1, x_2 and their estimation $\hat{x}_1, \hat{x}_2$ are bounded. Figure 8 depicts that the adaptive parameters $\left\|\hat{\boldsymbol{\theta}}_1\right\|$

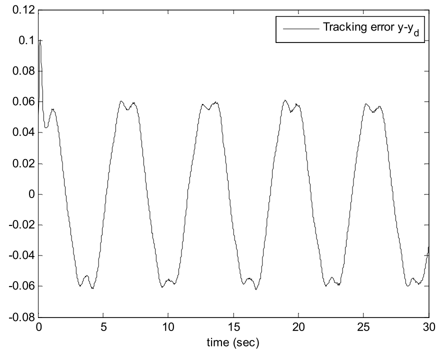

FIGURE 5. **The tracking error $y - y_d$ of example 1.**

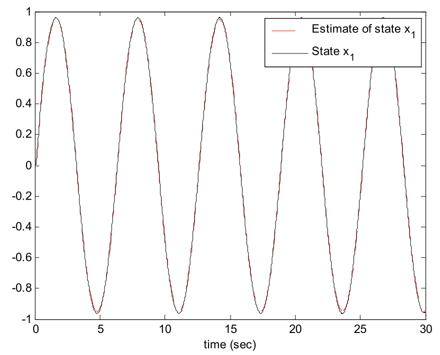

FIGURE 6. **State x_1 and its estimation $\hat{x}_1$ of example 1.**

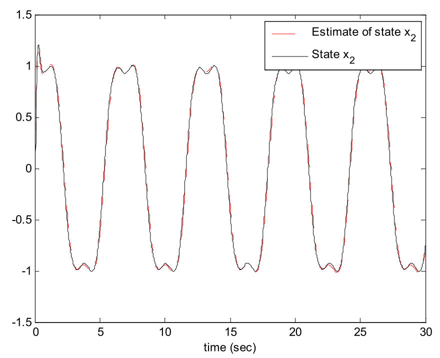

FIGURE 7. **State x_2 and its estimation $\hat{x}_2$ of example 1.**

and $\left\|\hat{\theta}_2\right\|$ are bounded. The presented simulation results verify the effectiveness of the proposed control approach.

Example 2: On a similar method to [39], [40], a type of closed, continuously stirred tank, chemical reactor with one mode of feed stream and disturbances can be described as follows:

$$\begin{cases} \dot{x}_1 = x_2 + 0.5x_1 + 0.1 \sin t \\ \dot{x}_2 = u + 0.1 \cos t \\ y = x_1 \end{cases} \tag{65}$$

Using the same process of Example 1, design the following state observer

$$\dot{\hat{x}}_1 = \hat{x}_2 - (\hat{x}_1 - x_1) + 0.5 \left(\hat{x}_1 + (\hat{x}_1 - x_1)\right)$$
$$\dot{\hat{x}}_2 = u - (\hat{x}_1 - x_1)$$

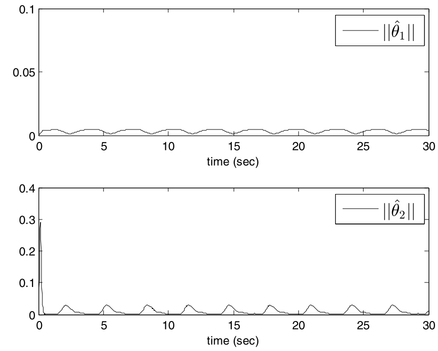

FIGURE 8. **The adaptive parameters $\left\|\hat{\theta}_1\right\|$ and $\left\|\hat{\theta}_2\right\|$ of example 1.**

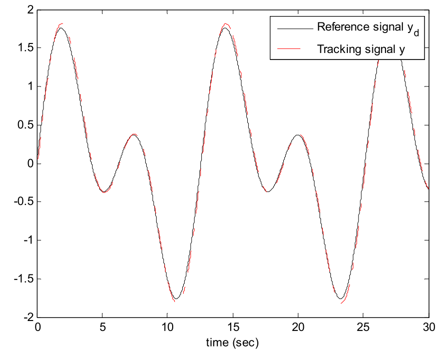

FIGURE 9. **The system output and reference signal of example 2.**

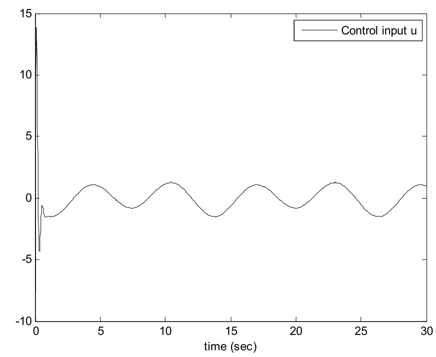

FIGURE 10. **The control input of example 2.**

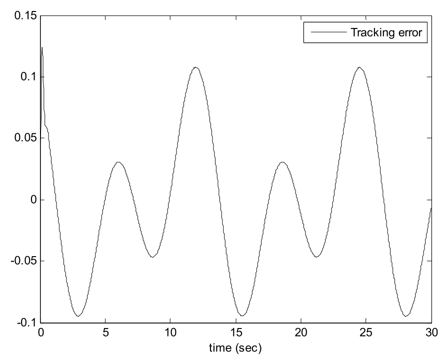

FIGURE 11. **The tracking error $y - y_d$ of example 2.**

The simulation results are shown in Figures 9-14. The simulation results further verify the effectiveness of the control method proposed in this paper.

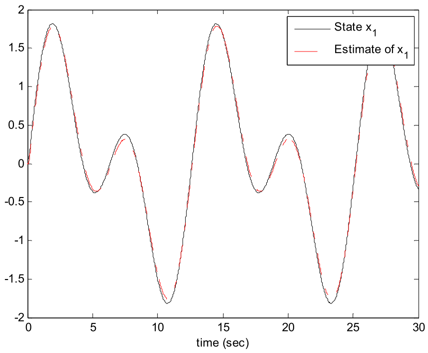

FIGURE 12. State x_1 and its estimation $\hat{x}_1$ of example 2.

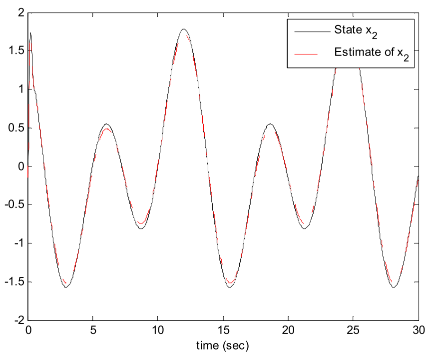

FIGURE 13. State x_2 and its estimation $\hat{x}_2$ of example 2.

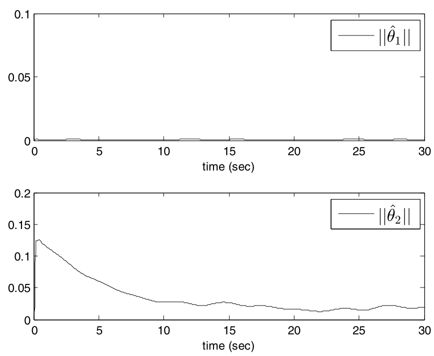

FIGURE 14. The adaptive parameters $\left\|\hat{\theta}_1\right\|$ and $\left\|\hat{\theta}_2\right\|$ of example 2.

V. CONCLUSION

In this paper, the problem of adaptive output-feedback tracking control has been investigated for a class of nonlinear systems with unmeasurable states based on multi-dimensional Taylor network approach. A nonlinear observer is designed to estimate the unmeasurable states of the system. By combining backstepping approach and dynamic surface control technique, a novel MTN-based adaptive output-feedback control scheme has been proposed. The designed MTN-based controller in this paper has the advantages of simple structure and fast computation speed, and the proposed approach can overcome the problem of "explosion of complexity". The simulation results show that the proposed control scheme can keep all signals of the closed-loop system bounded and the tracking error converges to any small neighborhood around the origin.

Our future work will be directed at further extending the proposed methodology to switching nonlinear systems and MIMO nonlinear systems.

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(Shanliang Zhu and Lei Chu are co-first authors.)

REFERENCES

[1] J. Song and S. He, "Finite-time H_∞ control for quasi-one-sided Lipschitz nonlinear systems," *Neurocomputing*, vol. 149, pp. 1433–1439, Feb. 2015.

[2] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. Autom. Control*, vol. 56, no. 7, pp. 1672–1678, Jul. 2011.

[3] L.-C. Guo, X. Zuo, J.-W. Liu, and H.-Q. Liang, "Output-feedback control of a class of stochastic nonlinear systems with power growth conditions," *Int. J. Control, Autom. Syst.*, vol. 12, no. 2, pp. 274–282, Apr. 2014.

[4] J. Back and H. Shim, "Adding robustness to nominal output-feedback controllers for uncertain nonlinear systems: A nonlinear version of disturbance observer," *Automatica*, vol. 44, no. 10, pp. 2528–2537, Oct. 2008.

[5] S. Hao, T. Liu, and B. Zhou, "Output feedback anti-disturbance control of input-delayed systems with time-varying uncertainties," *Automatica*, vol. 104, pp. 8–16, Jun. 2019.

[6] L. Wu, X. Su, and P. Shi, "Output feedback control of Markovian jump repeated scalar nonlinear systems," *IEEE Trans. Autom. Control*, vol. 59, no. 1, pp. 199–204, Jan. 2014.

[7] S.-J. Liu, J.-F. Zhang, and Z.-P. Jiang, "Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems," *Automatica*, vol. 43, no. 2, pp. 238–251, Feb. 2007.

[8] L. Liu, Y.-J. Liu, and S. Tong, "Neural networks-based adaptive finite-time fault-tolerant control for a class of strict-feedback switched nonlinear systems," *IEEE Trans. Cybern.*, vol. 49, no. 7, pp. 2536–2545, Jul. 2019.

[9] B. Jiang, Q. Shen, and P. Shi, "Neural-networked adaptive tracking control for switched nonlinear pure-feedback systems under arbitrary switching," *Automatica*, vol. 61, pp. 119–125, Nov. 2015.

[10] S. Tong, Y. Li, and S. Sui, "Adaptive fuzzy tracking control design for SISO uncertain nonstrict feedback nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 6, pp. 1441–1454, Dec. 2016.

[11] Y.-J. Liu, C. L. P. Chen, G.-X. Wen, and S. Tong, "Adaptive neural output feedback tracking control for a class of uncertain discrete-time nonlinear systems," *IEEE Trans. Neural Netw.*, vol. 22, no. 7, pp. 1162–1167, Jul. 2011.

[12] X. H. Chang, C. Yang, and J. Xiong, "Quantized fuzzy output feedback H_∞ control for nonlinear systems with adjustment of dynamic parameters," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 10, pp. 2005–2015, Sep. 2019.

[13] Y.-Q. Han, "Adaptive tracking control of nonlinear systems with dynamic uncertainties using neural network," *Int. J. Syst. Sci.*, vol. 49, no. 7, pp. 1391–1402, May 2018.

[14] Y. Pan, Y. Liu, and H. Yu, "Simplified adaptive neural control of strict-feedback nonlinear systems," *Neurocomputing*, vol. 159, pp. 251–256, Jul. 2015.

[15] B. Chen, X. Liu, K. Liu, and C. Lin, "Direct adaptive fuzzy control of nonlinear strict-feedback systems," *Automatica*, vol. 45, no. 6, pp. 1530–1535, Jun. 2009.

[16] D. Wang and J. Huang, "Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form," *IEEE Trans. Neural Netw.*, vol. 16, no. 1, pp. 195–202, Jan. 2005.

[17] M. Wang, X. Liu, and P. Shi, "Adaptive neural control of pure-feedback nonlinear time-delay systems via dynamic surface technique," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 6, pp. 1681–1692, Dec. 2011.

[18] S. S. Ge and C. Wang, "Adaptive NN control of uncertain nonlinear pure-feedback systems," *Automatica*, vol. 38, no. 4, pp. 671–682, Apr. 2002.

[19] L. Ma, X. Huo, X. Zhao, and G. Zong, "Adaptive fuzzy tracking control for a class of uncertain switched nonlinear systems with multiple constraints: A small-gain approach," *Int. J. Fuzzy Syst.*, vol. 21, no. 8, pp. 2609–2624, Nov. 2019.

[20] X. Huo, L. Ma, X. Zhao, and G. Zong, "Event-triggered adaptive fuzzy output feedback control of MIMO switched nonlinear systems with average dwell time," *Appl. Math. Comput.*, vol. 365, Jan. 2020, Art. no. 124665.

[21] Y. Yin, X. Zhao, and X. Zheng, "New stability and stabilization conditions of switched systems with mode-dependent average dwell time," *Circuits, Syst., Signal Process.*, vol. 36, no. 1, pp. 82–98, Jan. 2017.

[22] Y. Chang, Y. Wang, F. E. Alsaadi, and G. Zong, "Adaptive fuzzy output-feedback tracking control for switched stochastic pure-feedback nonlinear systems," *Int. J. Adapt. Control Signal Process.*, vol. 33, pp. 1567–1582, Oct. 2019.

[23] T. P. Zhang and S. S. Ge, "Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs," *Automatica*, vol. 43, no. 6, pp. 1021–1033, Jun. 2007.

[24] M. Chen, S. Sam Ge, and B. How, "Robust adaptive neural network control for a class of uncertain MIMO nonlinear systems with input nonlinearities," *IEEE Trans. Neural Netw.*, vol. 21, no. 5, pp. 796–812, May 2010.

[25] Q. Zhou, P. Shi, H. Liu, and S. Xu, "Neural-network-based decentralized adaptive output-feedback control for large-scale stochastic nonlinear systems," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 42, no. 6, pp. 1608–1619, Dec. 2012.

[26] Z. Yu, S. Li, Z. Yu, and F. Li, "Adaptive neural output feedback control for nonstrict-feedback stochastic nonlinear systems with unknown backlash-like hysteresis and unknown control directions," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 1147–1160, Apr. 2018.

[27] H.-S. Yan and A.-M. Kang, "Asymptotic tracking and dynamic regulation of SISO non-linear system based on discrete multi-dimensional Taylor network," *IET Control Theory Appl.*, vol. 11, no. 10, pp. 1619–1626, Jun. 2017.

[28] A.-M. Kang and H.-S. Yan, "Stability analysis and dynamic regulation of multi-dimensional Taylor network controller for SISO nonlinear systems with time-varying delay," *ISA Trans.*, vol. 73, pp. 31–39, Feb. 2018.

[29] Y.-Q. Han and H.-S. Yan, "Adaptive multi-dimensional Taylor network tracking control for SISO uncertain stochastic non-linear systems," *IET Control Theory Appl.*, vol. 12, no. 8, pp. 1107–1115, May 2018.

[30] H. Yan and Y. Han, "Decentralized adaptive multi-dimensional Taylor network tracking control for a class of large-scale stochastic nonlinear systems," *Int. J. Adapt. Control Signal Process.*, vol. 33, no. 4, pp. 664–683, Apr. 2019.

[31] H.-S. Yan, Y.-Q. Han, and Q.-M. Sun, "Optimal output-feedback tracking of SISO stochastic nonlinear systems using multi-dimensional Taylor network," *Trans. Inst. Meas. Control*, vol. 40, no. 10, pp. 3049–3058, Jun. 2018.

[32] Y.-Q. Han, "Output-feedback adaptive tracking control of stochastic nonlinear systems using multi-dimensional Taylor network," *Int. J. Adapt. Control Signal Process.*, vol. 32, no. 3, pp. 494–510, Mar. 2018.

[33] Y. Q. Han and H. S. Yan, "Observer-based multi-dimensional Taylor network decentralised adaptive tracking control of large-scale stochastic nonlinear systems," *Int. J. Control*, Sep. 2018, doi: 10.1080/00207179.2018.1521994.

[34] X. Fan and M. Arcak, "Observer design for systems with multivariable monotone nonlinearities," *Syst. Control Lett.*, vol. 50, no. 4, pp. 319–330, Nov. 2003.

[35] Y. Zhang, Y. Liu, and Y. Ding, "A new nonlinear output tracking controller via output-feedback," *J. Control Theory Appl.*, vol. 4, no. 4, pp. 372–378, Nov. 2006.

[36] S. Raghavan and J. K. Hedrick, "Observer design for a class of nonlinear systems," *Int. J. Control*, vol. 59, no. 2, pp. 515–528, 1994.

[37] O. M. Aamo, M. Arcak, T. I. Fossen, and P. V. Kokotovic, "Global output tracking control of a class of euler-Lagrange systems with monotonic non-linearities in the velocities," *Int. J. Control*, vol. 74, no. 7, pp. 649–658, Jan. 2001.

[38] S. J. Yoo, J. B. Park, and Y. H. Choi, "Adaptive dynamic surface control for stabilization of parametric strict-feedback nonlinear systems with unknown time delays," *IEEE Trans. Autom. Control*, vol. 52, no. 12, pp. 2360–2365, Dec. 2007.

[39] M. Barkhordari Yazdi and M. R. Jahed-Motlagh, "Stabilization of a CSTR with two arbitrarily switching modes using modal state feedback linearization," *Chem. Eng. J.*, vol. 155, no. 3, pp. 838–843, Dec. 2009.

[40] B. Niu, D. Wang, H. Li, X. Xie, N. D. Alotaibi, and F. E. Alsaadi, "A novel neural-network-based adaptive control scheme for output-constrained stochastic switched nonlinear systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 2, pp. 418–432, Feb. 2019.

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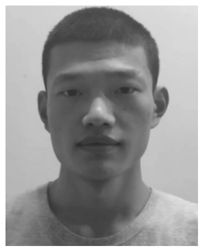

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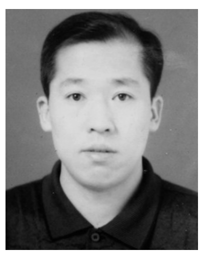

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