IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received April 3, 2020, accepted April 11, 2020, date of publication April 20, 2020, date of current version May 7, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2988915

Antimonotonicity, Chaos and Multidirectional Scroll Attractor in Autonomous ODEs Chaotic System

YUE LIU [1], AND HERBERT HO-CHING IU [2], (Senior Member, IEEE)
[1]College of Electrical and Electronic Engineering, Changchun University of Technology, Changchun 130012, China
[2]School of Electrical, Electronic and Computer Engineering, The University of Western Australia, Perth, WA 6009, Australia

Corresponding author: Yue Liu (lycn81@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61703055.

ABSTRACT Three-dimensional autonomous ordinary differential equations (ODEs) are the simplest and most important chaotic systems in nonlinear dynamics. In fact, they have been applied in many fields. In this paper, a systematic methodology for analyzing complex behavior of the ODEs chaotic system, as one of the ODEs chaotic systems, the improved TCS which satisfies the condition $a_{12}a_{21} = 0$, is proposed. It is dissipative, chaos, symmetric, antimonotonicity and can generate multiple directional ($M \times N \times L$) scroll attractors. Then, bifurcation diagrams, Lyapunov exponents, time series, Poincare sections, and Hausdroff dimensions are analyzed by setting the parameters and initial value. More interestingly, antimonotonicity (named reverse period-doubling bifurcation) and coexisting bifurcations are also reported. Finally, the results of theoretical analyses may be verified by electric experimental.

INDEX TERMS Antimonotonicity, multidirectional coexistence attractors, ODEs chaotic system.

I. INTRODUCTION

Chaos, as one of the most important topics, has attracted many scholars to focus on the study and analysis in this field [1]. During the past decades, the research on chaos has progressed starting from theoretical research [2] and chaos control and anti-control [3]–[5], chaotic circuits [6] effectively. Chaotic system is utilized in various areas of nonlinear science [7]–[15], such as chaos synchronization [7], [15], information processing [8], secure communication [9], image encryption [10], [11], [39], [40], chaotic electronic circuits [6], [12], sonar sensors [13], [43] and neural network [14], etc. As we all know, because of the chaotic behavior and double scroll attractor, Chua circuit has obtained wide-spread attention all over the world [20]. Subsequently, many literatures were found the various types of scroll attractors are based on nonlinear function, such as the sinusoidal [16], trigonometric [17], transformations [18], piecewise [19], [23], saturation, sign [20]–[22], tangent, multi-fold surface, wings forms of scroll attractors [25], and so on [21]–[24]. However, the highly complex dynamics, such as antimonotonicity [26]–[28] and coexisting attractors [29], [30], [35], [44], as a new research direction, are still in its infancy [31]–[34]. Especially, the occurrence of two or more hidden attractors, asymptotically and attracting sets have been found in nonlinear systems. Also, a lot of work on coexisting multiple attractors has been carried out. For example, in 2017, J. Kengne, A. *et al*. proposed a chaotic oscillator derived from jerk circuit and investigated antimonotonicity, periodic windows and crises [26]. Then, Z. T. Njitacke and co-workers found a novel jerk circuit obtained and the complex dynamic behavior was reported [27]. The same year, Z.T. Njitacke, and his fellows also described similar characters in other novel jerk circuit. Then, the rich dynamic behaviors including antimonotonicity (i.e. concurrent creation and annihilation of periodic orbits) [26]–[28], hidden coexisting attractors [29], basins of various coexisting attractors [30]–[32] and multistability [33] are analyzed. In addition, butterfly attractors are emerged from Lorenz-like system and coexisting attractors are spotted. Ling Zhou and co-workers proposed a simple fourth order memristive Twin-T oscillator and coexisting attractors and antimonotonicity are also detected [31]. Furthermore, a polynomial function method for generating multiple chaotic attractors from the Sprott B system is proposed in [35]. In [36] reported about a novel chaotic system with three nonlinearities and an S-Box was developed for cryptographic operations. In [37] introduced an extended Lü system and

The associate editor coordinating the review of this manuscript and approving it for publication was Chao-Yang Chen.

studied the control problem of this system. An improved BFO was proposed, named Chaotic BFO in [38]. In [39], [40] proposed an alternative security model and analyzed a number of encryption algorithms based on chaotic systems in the field of medical image encryption. In [41], [42] generalized chaos systems, synchronization circuits and a secure communication scheme were proposed for the RGB digital image and 24-bit true color image, respectively. In [43] set up some tested experiments to evaluate the existing cryptographic algorithms for sensors. In [44] proposed an improved salp swarm-based optimizer. In [45] constructed a chaotic system with infinitely many coexisting chaotic attractors, just to name a few.

Motivated by the aforementioned works, this paper focuses on analyzing the dynamics of an improved TCS, such as sensitivity to initial conditions, parameters, and multidirectional scroll attractors (*X-Y-Z* coordinate axes) in the phase space. It is also highly symmetric and multiple coexisting attractors. This improved TCS, a new perspective, discusses the mechanism of chaos (i.e. scroll attractors, dynamic behavior, concurrent creation and destruction of periodic orbits and coexisting attractors).

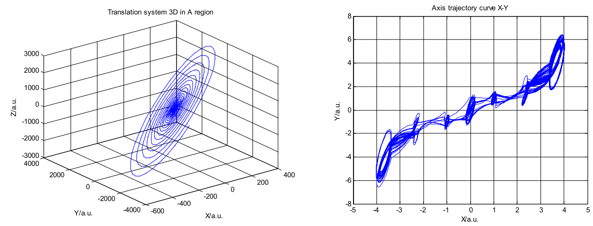
The layout of the paper is as follows. In Section II, the background of the translation chaotic system and motivation of this paper are described. In Section III, the mathematical model is depicted, and some basic information (the phase diagram and time domain) are obtained. In Sec. IV, some basic properties, such as dissipativity, equilibrium, and stability are analyzed. In Sec. V, simulation analyses of the dynamical system are investigated by bifurcation diagram, Lyapunov exponents and Hausdroff dimension. Many various periodic attractors, multidirectional scroll attractors, coexisting attractors and antimonotonicity are discussed. In Sec.VI, A voltage-controlled nonlinear memristor circuit and its fingerprints are exhibited. Also, theoretical analyses and numerical simulations and further verified by this circuit experiment. Finally some concluding remarks and proposal are drawn in Sec.V II.

II. BACKGROUND OF TRANSATION CHAOTIC SYSTEM

Recall that the translation chaotic system (named TCS) was proposed in 2016 [20]. as a type of canonical form satisfies the condition $a_{12}a_{21} = 0$ and includes other chaotic systems with the same condition. There are three main features that belong to TCS, as follows: (i) this new type of chaotic system is three-dimensional autonomous ODEs. (ii) compared with the Chua system ($a_{12}a_{21} > 0$), it should be different dynamic characteristics. Based on the mechanism of chaos, it would generate multiple scroll attractors.

In Ref [20], the conventional TCS was proposed as following:

$$\begin{cases} \dot{x} = -\alpha x + z + (\alpha - 1) f\,(x + y + z) \\ \dot{y} = -\beta z + \beta f\,(x + y + z) \\ \dot{z} = \gamma y + \delta z - (\gamma + \delta) f\,(x + y + z) \end{cases} \tag{1a}$$



(a) (b)

FIGURE 1. The 2N+1-scroll attractors: (a)N=1;(b)N=3.

where x, y, z are state variables, and α, β, γ, δ are real parameters. Also *f(x+y+z)* is the nonlinear *sign* function, which could generate *2N+1*-scroll attractors, as below:

$$\begin{aligned} f\,(x+y+z) &= \frac{1}{2}\sum_{k=1}^{N}\begin{bmatrix} \operatorname{sign}(x+y+z+1) \\ +\operatorname{sign}(x+y+z-1) \end{bmatrix} \\ &\quad +\frac{1}{2}\sum_{k=2}^{N}\left\{\frac{2k}{2k-1}\begin{bmatrix} \operatorname{sign}((x+y+z)+2k) \\ +\operatorname{sign}((x+y+z)-2k) \end{bmatrix}\right\} \end{aligned} \tag{1b}$$

where N, an integer, is the sum of the upper bound. When $\alpha = 2, \beta = 0.2, \gamma = \delta = -5, N = 0$, the single chaotic behavior is obtained as Fig.1(a); $\alpha = 2, \beta = 0.44, \gamma = \delta = -5, N = 3$, the chaotic behavior is obtained (7 scroll attractors) as Fig.1(b). Similar *2N+1*-scroll attractors can be obtained by changing the parameters N in the *f(x+y+z)* function.

However, system (1a), as one example of this type that satisfies the condition $a_{12}a_{21} = 0$, has some insufficient, such as: (i) the 4 parameters are not enough to reflect the complex dynamic characteristics of this type of chaotic systems, for example antimonotonicity, multi-stability, coexisting attractors and so on, and requires more details; (ii) nonlinear function is the most important factor and necessary condition to influence the distribution (location and quantity) of scroll attractors. By substituting (1b) into (1a), the multiple directional scroll attractor cannot be generated, but only one direction, that is $x + y + z = 1$; (iii) as a type of canonical form of ODEs chaotic systems, the distribution of its scroll should be multidirectional. For this purpose, the conventional TCS should be further extended (i.e. improved TCS). The motivation of this paper is to complement and improve the characteristics of them and provide a theory for its potential engineering research.

As we all know, there are lots of chaotic systems could generate multiple quality and directional scroll attractors and belong to ODEs chaotic systems, such as Chua system ($a_{12}a_{21} > 0$), Jerk system, Sprott B system, *et al.* However, they satisfy the different dynamic characteristics and are different from TCS. In addition, there are some chaotic systems could generate the different attractors, like butterfly attractors, such as Chen system ($a_{12}a_{21} < 0$), Lü system ($a_{12}a_{21} = 0$), Lorenz system ($a_{12}a_{21} > 0$) and Lorenz-like

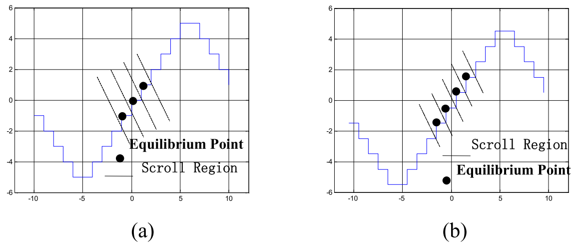


FIGURE 2. **Principle of multi-scroll attractors: (a) 2N+1; (b) 2(N+1).**

systems ($a_{12}a_{21} > 0$), and so on. However, these chaotic systems are three-dimensional quadratic ODEs. Obviously, they are quite different from TCS. Furthermore, the related research of coexisting attractors in scroll chaotic system has not been found in other literature before.

III. THE MODEL OF NOVEL CHAOTIC SYSTEM

Here, the improved TCS as following

$$\begin{cases} \dfrac{dx}{dt} = a\left(x - f\left(x\right)\right) + r\left(y - f\left(y\right)\right) \\ \dfrac{dy}{dt} = b\left(z - f\left(z\right)\right) \\ \dfrac{dz}{dt} = p\left(x - f\left(x\right)\right) + q\left(y - f\left(y\right)\right) + c\left(z - f\left(z\right)\right) \end{cases} \tag{2}$$

where x, y, z are state variables; a, b, c, r, p, q are real parameters and $f(x)$, $f(y)$, $f(z)$ are the nonlinear *sign* functions. Firstly, it can be noticed that *sign* function and initial value are key points and necessary conditions for the location and quantity of multidirectional scroll attractors. Also, all the state variables are real. In system (2), there are six parameters and each of them will be considered as the main bifurcation control parameter. The numerical simulation is carried out with the following dimensionless parameters.

When $M\epsilon R$, $N\epsilon R$, $L\epsilon R$ are integers, multidirectional scrolls attractors can be obtained by the nonlinear *sign* function, which can be given, as follow:

(1) $f_1(y) = f_1(z) = f_1(x)$ could generate $2N+1$- scroll along one direction (x-axis or y-axis or z-axis):

$$f_1\left(x\right) = A\text{sign}(x) + A\sum_{n=1}^{N}\left[\text{sign}\left(x + 2nA\right) + \text{sign}\left(x - 2nA\right)\right] \tag{3a}$$

(2) $f_2(y) = f_2(z) = f_2(x)$ could generate $N+1$- double scroll attractors along one direction (x-axis or y-axis or z-axis):

$$f_2(x) = A\sum_{n=1}^{N}\left\{\text{sign}\left[x + (2n-1)A\right] + \left[x - (2n-1)A\right]\right\} \tag{3b}$$

In one fixed direction, the principle of multi-scroll attractors could be described as follows, in Fig. 2.

When $A = 0.5$, $a = 0.6$, $p = 2$, $q = 1.5$, $r = 2$, $b = 0.4$, $c = 0.6$, $4 \times 5 \times 6$-scrolls can be shown in Fig. 3.

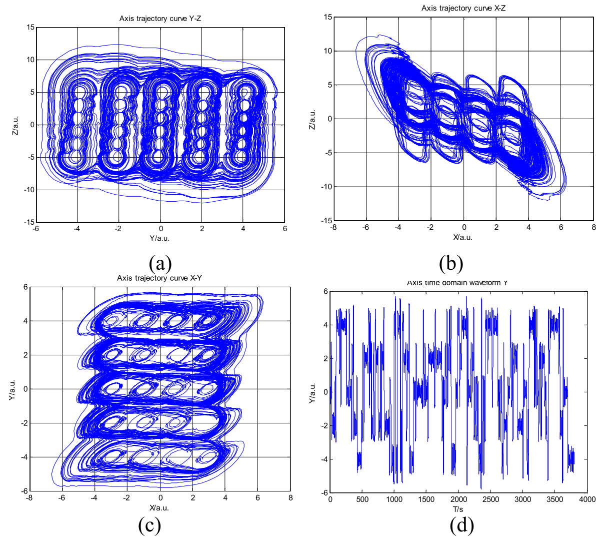


FIGURE 3. **4 × 5×6-scroll attractors: (a) y-z; (b)x-z; (c) x-y; (d) y(t).**

IV. THE BASIC CHAOS INFORMATION

It can be observed that improved TCS with equation (3) is symmetrical about the origin $P_0(0,0,0)$ and invariant under $(x, y, z) \rightarrow (-x,-y,-z)$. Also system (2) can generate multi-equilibriums including single zero equilibrium P_0 (a trivial symmetric static solution), and many symmetric nonzero equilibriums $P_{\pm}$. When we fix all the parameter values, if (x,y,z) is a solution, the $(-x,-y,-z)$ is also a solution. Consequently, attractors in phase space is symmetric about $P_{0.}$ Also, to satisfy the exact symmetry of the model equations, they must occur in pairs. This exact symmetry may be used to explain the appearance of multiple coexisting attractors in state space.

To evaluate the dissipativity, the mathematical expression of exponential constrain rate is deduced as

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = a + c \tag{4}$$

When $a < -c$, equation (4) is negative, $\nabla V < 0$, implying that system (2) is dissipative, which means that asymptotic motion settles onto an attractor and each volume containing trajectory shrinks to zero at an exponential rate as $t \rightarrow \pm\infty$. The linearization of system (2) at zero equilibrium P_0 can be obtained by the following equations:

$$\begin{cases} 0 = a\left(x - f\left(x\right)\right) + r\left(y - f\left(y\right)\right) \\ 0 = b\left(z - f\left(z\right)\right) \\ 0 = p\left(x - f\left(x\right)\right) + q\left(y - f\left(y\right)\right) + c\left(z - f\left(z\right)\right) \end{cases} \tag{5}$$

Also the Jacobian matrix as following.

$$J = \begin{bmatrix} a & r & 0 \\ 0 & 0 & b \\ p & q & c \end{bmatrix} \tag{6}$$

$$\begin{aligned} \det\left(\lambda I - J\right) &= \left(\lambda - a\right)\left[\lambda\left(\lambda - c\right) - b\left(rp + q\right)\right] \\ &= \left(\lambda - a\right)\left[\lambda^2 - c\lambda - b\left(rp + q\right)\right] \\ \lambda_1 &= a; \end{aligned}$$

TABLE 1. The location and Quantity of scroll attractors, equilibrium, eigenvalues and corresponding eigenvectors.

M/N/L	*sign*(*x*)	Attractors	Equilibrium	Eigenvalues and corresponding eigenvectors
0/0/0	$f_1(y)$、$f_1(z)$、$f_1(x)$	1	(0,0,0)	λ_1= -1.2931, λ_2=-0.0535 + 1.3338i, λ_3=-0.0535 - 1.3338i; ξ_1=[0.6175, -0.5350, 0.5765]^T, ξ_2=[-0.3269-0.1189i, -0.025 - 0.6263i, 0.6972]^T, ξ_3=[-0.3269+0.1189i, - 0.0251+0.6263i, 0.6972]^T
0/0/0	$f_2(y)$、$f_2(z)$、$f_2(x)$	2×2×2	(-1,-1,-1), (1,1,1), (-2,-2,-2), (2,2,2)	
1/2/3	$f_2(y)$、$f_2(z)$、$f_2(x)$	4×6×8	(-2,-3,-4), (-2,-3,-3), (-2,-3,-2),…, (2,3,2), (2,3,3), (2,3,4)	λ_1= -1.3471, λ_2=0.0735 + 1.0736i, λ_3=0.0735 - 1.0736i; ξ_1=[0.6061, -0.2264, 0.7625]^T, ξ_2=[0.3897+0.2833i, -0.0209+0.3046i, -0.8214]^T, ξ_3=[0.3897- 0.2833i, -0.0209 - 0.3046, -0.8214]^T
1/2/2	$f_1(y)$、$f_2(z)$、$f_2(x)$	4×5×6	(-2,-2,-3), (-2,-2,-2), (-2,-2,-1),…, (2,2,1), (2,2,2), (2,2,3)	
…		…	…	…

$$\lambda_{2,3} = \frac{1}{2}\left[c \pm \sqrt{c^2 + 4b\,(rp + q)}\right] \tag{7}$$

The coefficients are all nonzero. The location and quantity of the equilibriums are different according to $M/N/L$. Thus, the attractors, equilibriums, the eigenvalues and corresponding eigenvectors are shown in Table.1.

It is found that when the pairs of nonzero equilibrium points are a hyperbolic saddle focus (or simple, saddle focus), the relationship between the parameters are $a > b > 0$ & $cr > 0$ & $b < 2\sqrt{cr}$.

V. DYNIMICS ANALYSIS

In order to investigate the rich variety of bifurcation modes. System (2) will be simulated for a sufficiently long time, and the transient state is cancelled. Two indicators are exploited to define the type of scenario giving rise to chaos, namely the bifurcation diagram and the graph of Lyapunov Exponents (*LEs*). For $LE_{max} > 0$, small perturbations grow exponentially, and the system evolves chaotically within the folded space of a strange attractor [26].

A. THE OCCURRENCE OF CHAOS

To investigate the sensitivity and chaos, a single parameter is changed within a certain interval, and other five parameters are fixed. we can obtain bifurcation diagrams in Fig. 4. Let $M = N = L = 2$.Case 1 : $A = 1, p = 1.5, q = 1.2, r = 1, b = 1, c = 0.6$ when $a\epsilon[0, 2]$, the bifurcation diagram of x is given in Fig. 4(a). Case 2: $A = 1, p = 1.5, q = 1.2, a = 0.6, b = 1, c = 0.6$, when $r\epsilon[0, 1]$, the bifurcation diagram of x is given in Fig. 4(b). Case 3: $A = 1$, $p = 1.5$, $q = 1.2$, $r = 1$, $a = 0.6, c = 0.6$, when $b\epsilon[0, 2.8]$ the bifurcation diagram of x is given in Fig. 4(c). Case 4: $A = 1$, $a = 0.6$, $q = 1.2$, $r = 0.8$, $b = 1.2$, $c = 0.6$, when $p\epsilon[0, 18]$ the bifurcation diagram of x is given in Fig. 4(d). Case 5: $A = 1$, $p = 1.5$, $a = 0.6$, $r = 1$, $b = 1$, $c = 0.6$, when $q\epsilon[0, 8]$ the bifurcation diagram of x is given in Fig. 3(e). Case 6: $A = 1$, $p = 1.5$, $q = 1.2$, $r = 1$, $b = 1$, $a = 0.6$, when $c\epsilon[0, 3]$ the bifurcation diagram of x is given in Fig. 4(f). We can see the improved TCS shows abundant and sophisticated dynamical behaviors.

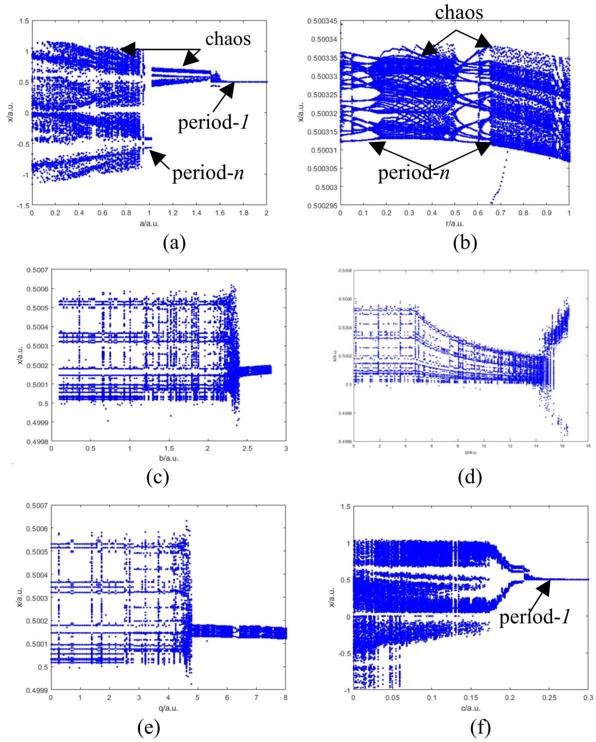


FIGURE 4. Bifurcation diagrams: (a) aϵ [0,2]; (b) rϵ[0,1]; (c) bϵ [0,2.8]; (d) p ϵ[0,18];(e) qϵ[0,8]; (f) cϵ[0,3].

From the bifurcation diagrams, we can observe clearly that system (2) has complex dynamic behaviors. And various $M{\times}N{\times}L$ multi-directional scroll attractors can be obtained by changing the value of parameters and *sign* function. Two parameters spaces will be considered in the following:

(1) $a\epsilon[0,2]$

The bifurcation diagrams of x are depicted in Fig.4(a). Along with increase of a, the orbits transform from chaos to coexisting period-n to chaos and to period-1 successively. Furthermore, two main periodic windows with period-n, and period-1 appear in the intervals *[0.96, 1]* and *[1.6, 2]*.

(2) $r\epsilon[0,1]$

In Fig.4 (b), along with increase of r, the orbits transform from chaos to coexisting period-14 to chaos and to

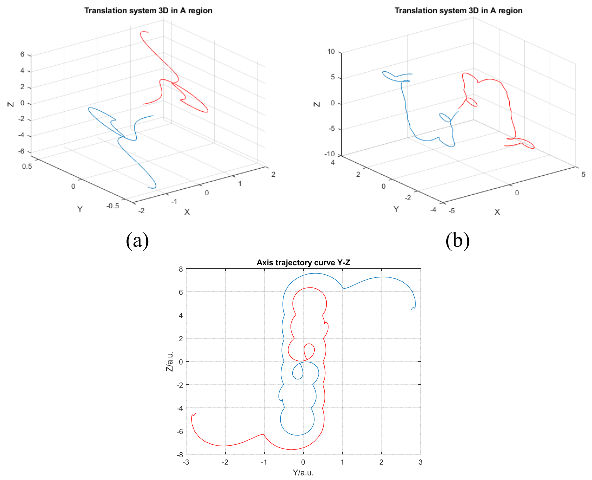


FIGURE 5. The coexisting attractors, $a = c = 0.6, p = r = 2, q = 1.5, b = 0.4$, and initial conditions (x_0, y_0, z_0) are $(\pm 0.1, \pm 0.1, \pm 0.1)$: (a) period- n in3D; (b) scroll period- n in3D; (c); scroll period- n in Y-Z.

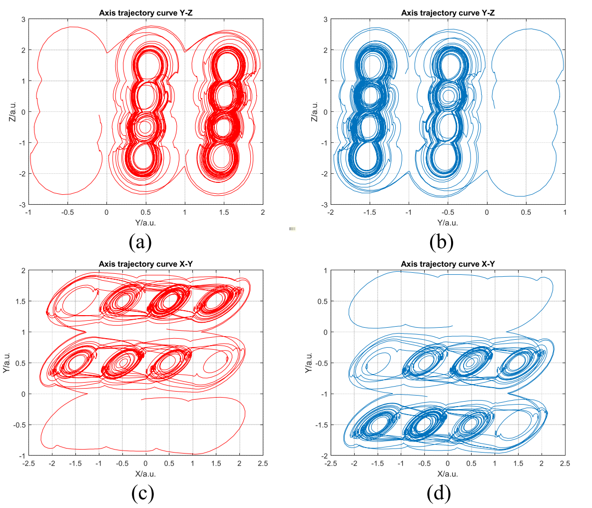


FIGURE 6. The chaotic attractor, and initial conditions (x_0, y_0, z_0) are $(\pm 0.1, \pm 0.1, \pm 0.1)$: (a)~(b) Y-Z; (c)~(d) X-Y.

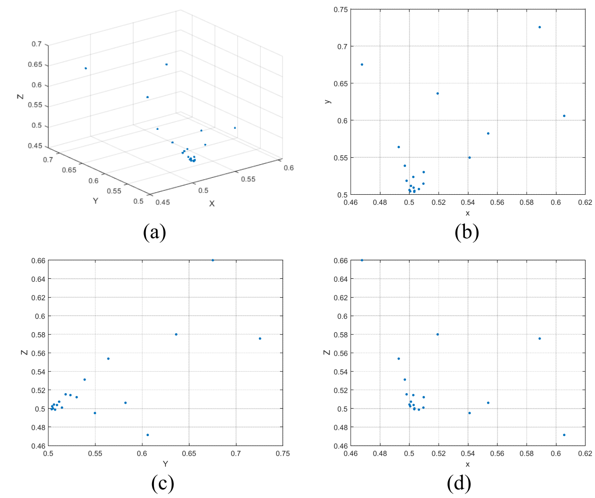


FIGURE 7. Poincaré Mapping: (a) x+y+z=1; (b) z=0; (c) x=0; (d) y=0.

period-16 to chaos successively. Furthermore, two main periodic windows with period-14, and period-16 appear in the intervals *[0, 0.15]* and *[0.5,0.66]*.

When $a = c = 0.6, p = r = 2, q = 1.5, b = 0.4$, coexisting bifurcation modes appear, and the coexisting attractors are shown in Fig. 5 and Fig. 6.

Note that: Because that the multidirectional scroll chaotic system is essentially different from other chaotic systems based on the mechanism of the scroll attractors. Therefore, it is difficult to observe of the period orbits in the phase trajectory curve, the value of period-n is not discussed here.

LEs are $LE_1 = 0.52$, $LE_2 = 0$, $LE_3 = -1.02$, respectively. Based on the obtained LEs, the Hausdroff dimension $D_L = 2.49$.

Another method for observing and analyzing complex dynamic behavior is Poincaré section, see in Fig. 7. It can be seen that it is consists by a series of isolated points, which means that the system is manifestly chaotic.

B. ANTIMONOTONICITY

It is well known that antimonotonicity could be observed in many nonlinear systems. When one bifurcation control parameter is monitored, we can found that periodic orbits could be created and then annihilated via reverse period doubling bifurcation scenarios [26]–[28]. This phenomenon has been reported in many literatures, such as Duffing oscillator, Chua circuit and Jerk circuit. Also, following the work of [28], reverse period-doubling scenarios can occur when some conditions are satisfied.

Now, TCS is experiencing this phenomenon. That means system (2) appears to be forward and reversed period-doubling bifurcations and the chaotic transition ‘‘stable-critical-unstable’’, as shown in Fig.3. The mechanism is that a stable symmetric periodic orbit, which around each equilibrium $P+$ or $P-$ of origin symmetry and named ‘‘periodic-n window’’. It could persist in some (rather small) parameter interval. Therefore, the phenomena of reverse period doubling and antimonotonicity do occur in this paper.

C. MULTIDITECTIONAL SCROLL ATTRACTOES

The bifurcation analysis shown that TCS has multiple stability, that is yielded windows of hysteretic dynamics. For instance, a hysteretic window can be identified in the range $r \in [0.15, 0.5] \cup [0.6, 1]$ (see Fig.4(b)). For this range, the long term behavior of the system depends crucially on initial conditions (x_0, y_0, z_0), thus giving rise to the interesting and striking behavior of coexisting multiple attractors. Many multiple scroll attractors (see Fig.5 and Fig.6) can be obtained. It ought to be mentioned that, the phenomenon of multiple stability involving not only four disconnected coexisting attractors previously observed [28], [29] in the Newton–Leipnik system and jerk system models but also multiple coexisting butterfly chaotic attractors could be observed in new 4D chaotic system [30]. However, the related research of coexisting attractors in scroll chaotic systems has not been found in other literature before. And, the intriguing situation involving the coexistence of infinitely many attractors, also called multiple stability [33], [34]. It is known that the occurrence of multiple attractors represents an additional form of randomness especially among multiple directional scroll attractors; Also some obvious potential applications including chaos-based secret communication, image encryption as well as neutral network, and so on. However, detailed development on this direction will be our next research work.

VI. EXPERIMENTAL CIRCUIT STUDY

The aim of this section is to verify the results of theoretical analysis. And, we choice *AD844* and saturation of the output signals ($\pm 12V$). It is also important to rescale the model by a factor of 5 for X, Y, Z.

The dynamics of system (2) could be realization base on voltage-controlled chaos circuit. It is converted back to the state equation and improved TCS as follows:

$$\begin{cases} \frac{dv_1}{dt} = \frac{R_{51}}{R_{59}}\left(v_1 - \frac{R_{59}}{R_{49}} f(v_1)\right) + \frac{R_{51}}{R_{60}}\left(v_2 - \frac{R_{60}}{R_{50}} f(v_2)\right) \\ \frac{dv_2}{dt} = \frac{R_{55}}{R_{54}}\left(v_3 - \frac{R_{54}}{R_{53}} f(v_3)\right) \frac{dv_3}{dt} \\ \frac{dv_2}{dt} = \frac{R_{57}}{R_{61}}\left(v_1 - \frac{R_{61}}{R_{64}} f(v_1)\right) \frac{R_{57}}{R_{62}}\left(v_2 - \frac{R_{62}}{R_{65}} f(v_2)\right) \\ \quad + \frac{R_{57}}{R_{63}}\left(v_3 - \frac{R_{63}}{R_{66}} f(v_3)\right) \end{cases} \tag{8}$$

where $v_1 = x, v_2 = y, v_3 = z, f(v)$ is the nonlinear *sign* function, $A = 1$:

(1) $f_1(v)$ could generate *2(N+1)*-scroll attractors along one direction:

$$f_1(v) = \text{sgn}(v) + \sum_{n=1}^{N}\left[\text{sign}(v+2n) + \text{sgn}(v-2n)\right] \tag{9a}$$

(2)$f_2(v)$ could generate $2N+1$-scroll attractors:

$$f_2(v) = \sum_{\text{n}=1}^{\text{N}}\{\text{sign}\,[v+(2n-1)] + [v-(2n-1)]\} \tag{9b}$$

The block diagram is shown in Fig.8, the "*Main*" section as shown in Fig.8(a), the "*sign*" section as shown in Fig.8 (b), which is utilized to obtain the nonlinearity needed for generating attractors in the circuit. It consists of three channels to conduct the integration of the three state variables v_1, v_2 and v_3, respectively. Here, all notations are defined in the circuitry as follows: $R_1 \sim R_{24} = 20k\Omega$; $R_{34} \sim R_{42} = 13.5k\Omega$; $R_{25} \sim R_{33} = R_{46} \sim R_{58} = 13.5k\Omega$; $R_{43} \sim R_{45} = 1k\Omega$; $R_{59} = R_{60} = 8.5k\Omega$; $R_{61} \sim R_{66} = 13.8k\Omega$; $V_{CC} = 12V$; $V_{EE} = -12V$; $C_1 \sim C_3 = 33nF$; Switch are $S_1 \sim S_N$ and $K_1 \sim K_N$. When $S_2 \sim S_N$ and $K_2 \sim K_N$. and closed in turn, $f(x) = f(y) = f(z) = f(v) = f_1(v)$ could generate $2(N+1)-$scroll attractors; When $S_1 \sim S_N$ and $K_1 \sim K_N$ are closed in turn, $f(x) = f(y) = f(z) = f(v) = f_2(v)$ could generate $2N+1-$scroll attractors. i.e., when S_1 and K_1 are closed, as shown in Fig.9, $2 \times 2 \times 2$-scroll attractors are obtained; when S_2, S_3, K_2, K_3 are closed, as shown in Fig.10, $3 \times 3 \times 2$-scroll attractors are obtained.

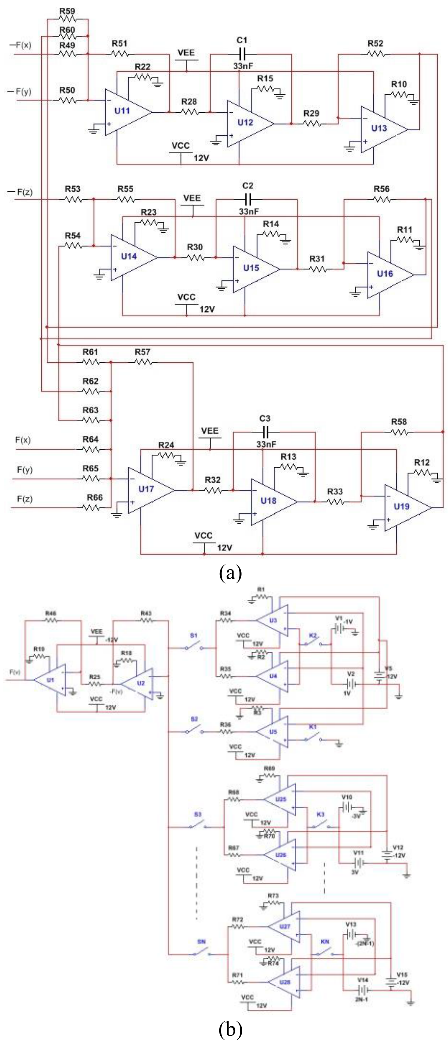


FIGURE 8. Experimental Circuit (a) "Main" in system's circuit: (b) "*sign*" in system's circuit.

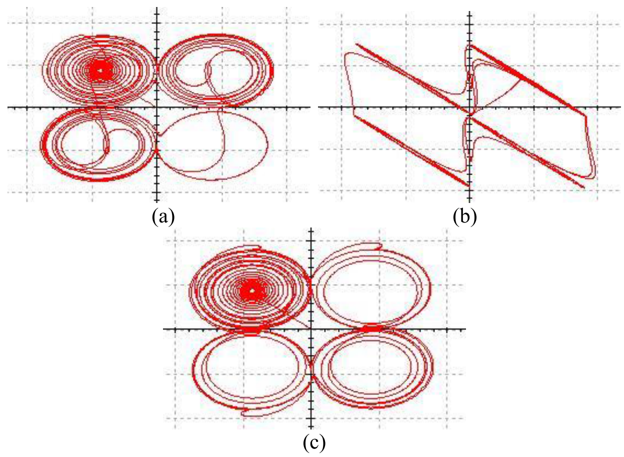


FIGURE 9. $2 \times 2 \times 2$ scroll attractors: (a) *x-y*; (b) *x-z*; (c) *y-z*.

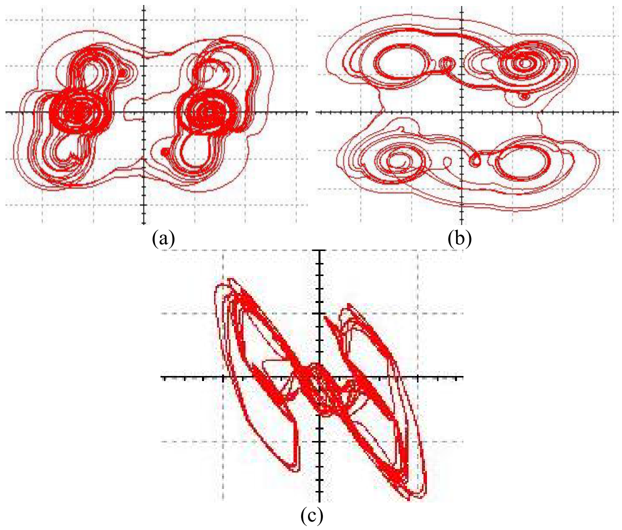


FIGURE 10. **3 × 3 × 2 scroll attractors: (a)*x-y*; (b)*x-z*; (c)*y-z*.**

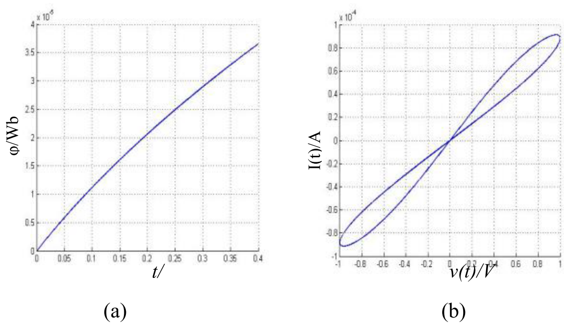


FIGURE 11. **Memristive dynamics: (a)$\varphi - t$; (b)v(t)−i(t).**

Note that: simulation results are not included ''period window'' for the sake of brevity, and only the trajectories of the attractor phase are obtained.

Since 2008, the solid state fabrication of physical nanoscale memristor (MR) has attracted immense interests worldwide from both industry and academia to production and manufacturing. However, they are still not popularized, and many based-MRs simulators are built with simple electronic circuits. Initial condition of the excitation voltages $f = 50kHz$, $u_1(t) = Usin(2\pi ft)$, $U = 1V$, $\varphi(0) = 0Wb$, and S_1 and K_1 are closed. And the circuit enclosed shown in Fig. 11.

VII. CONCLUSION

In summary, the dynamics of improved TCS, which is described by a continuous ODEs chaotic system and could generate multiple directional ($M\times N\times L$) scroll attractors with *sign* function. This system with a symmetric nonlinearity capable of rich and interesting varieties of nonlinear phenomena such as period-doubling bifurcation, chaos, antimonotonicity and coexisting multiple attractors. By exploiting classical nonlinear analysis tools such as bifurcation diagrams, graph of *LEs*, Poincare sections, equilibriums and stability, and phase space trajectories, the dynamics of this system has been characterized with respect to its parameters. As a major result of this work, it is shown that improved TCS and its circuit exhibits the unusual feature of multidirectional scroll attractors (i.e. coexistence scroll attractors depending only on initial sates). The MR emulator circuit makes use of the off-the-shelf electronic components and may be re-scaled over a fixed of frequencies. The scroll chaotic system introduced in this paper represents one of the important system/circuits reported to date and has not been found in other literature before. It capable of exhibiting such form of multi-stability, furthermore, antimonotonicity and coexisting attractors in scroll chaotic system. At the same time, a very good agreement is observed between theoretical and experimental results by the circuit is designed and implemented.

ACKNOWLEDGMENT

The authors thank the support of the China Scholarship Council (CSC) for this paper.

REFERENCES

[1] T. Hikihara, P. Holmes, T. Kambe, and G. Rega, ''Introduction to the focus issue: Fifty years of chaos: Applied and theoretical,'' *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 22, no. 4, Dec. 2012, Art. no. 047501.

[2] D. Younesian and H. Norouzi, ''Chaos prediction in nonlinear viscoelastic plates subjected to subsonic flow and external load using extended Melnikov's method,'' *Nonlinear Dyn.*, vol. 84, no. 3, pp. 1163–1179, May 2016.

[3] D. Pazó, J. M. López, R. Gallego, and M. A. Rodríguez, ''Synchronizing spatio-temporal chaos with imperfect models: A stochastic surface growth picture,'' *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 24, no. 4, Dec. 2014, Art. no. 043115.

[4] L. Minati, ''Experimental synchronization of chaos in a large ring of mutually coupled single-transistor oscillators: Phase, amplitude, and clustering effects,'' *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 24, no. 4, Dec. 2014, Art. no. 043108.

[5] Y. I. Boev, T. E. Vadivasova, and V. S. Anishchenko, ''Poincaré recurrence statistics as an indicator of chaos synchronization,'' *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 24, no. 2, Jun. 2014, Art. no. 023110.

[6] C. Stegemann, H. A. Albuquerque, R. M. Rubinger, and P. C. Rech, ''Lyapunov exponent diagrams of a 4-dimensional chua system,'' *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 21, no. 3, Sep. 2011, Art. no. 033105.

[7] G. Wang and Y. Shen, ''Exponential synchronization of coupled memristive neural networks with time delays,'' *Neural Comput. Appl.*, vol. 24, no. 6, pp. 1421–1430, May 2014.

[8] B. Kumari and N. Gupta, ''Multifrequency oscillator-type active printed antenna using chaotic colpitts oscillator,'' *Int. J. Microw. Sci. Technol.*, vol. 2014, no. 14, pp. 1–10, Nov. 2014.

[9] R. Sakthivel, R. Anbuvithya, K. Mathiyalagan, Y.-K. Ma, and P. Prakash, ''Reliable anti-synchronization conditions for BAM memristive neural networks with different memductance functions,'' *Appl. Math. Comput.*, vol. 275, pp. 213–228, Feb. 2016.

[10] C. K. Volos, I. M. Kyprianidis, and I. N. Stouboulos, ''Image encryption process based on chaotic synchronization phenomena,'' *Signal Process.*, vol. 93, no. 5, pp. 1328–1340, May 2013.

[11] Y. Liu, ''Analysis and improvement of image encryption algorithm based translation chaotic system,'' in *Proc. CCNE*, Beijing, China, 2018, pp. 113–117.

[12] R. M. Nguimdo, R. Tchitnga, and P. Woafo, ''Dynamics of coupled simplest chaotic two-component electronic circuits and its potential application to random bit generation,'' *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 23, no. 4, Dec. 2013, Art. no. 043122.

[13] L. Fortuna, M. Frasca, and A. Rizzo, ''Chaotic pulse position modulation to improve the efficiency of sonar sensors,'' *IEEE Trans. Instrum. Meas.*, vol. 52, no. 6, pp. 1809–1814, Dec. 2003.

[14] L. M. Alonso, ''Nonlinear resonances and multi-stability in simple neural circuits,'' *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 27, no. 1, Jan. 2017, Art. no. 013118.

[15] V. Sharma, B. B. Sharma, and R. Nath, "Nonlinear unknown input sliding mode observer based chaotic system synchronization and message recovery scheme with uncertainty," *Chaos, Solitons Fractals*, vol. 96, pp. 51–58, Mar. 2017.

[16] W. K. S. Tang, G. Q. Zhong, G. Chen, and K. F. Man, "Generation of n-scroll attractors via sine function," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 48, no. 11, pp. 1369–1372, Nov. 2001.

[17] D. Yu, C. Zheng, H. H.-C. Iu, T. Fernando, and L. O. Chua, "A new circuit for emulating memristors using inductive coupling," *IEEE Access*, vol. 5, pp. 1284–1295, 2017.

[18] X. Ai, K. Sun, S. He, and H. Wang, "Design of grid multiscroll chaotic attractors via transformations," *Int. J. Bifurcation Chaos*, vol. 25, no. 10, Sep. 2015, Art. no. 1530027.

[19] S. He, K. Sun, H. Wang, X. Ai, and Y. Xu, "Design of N-dimensional multi-scroll Jerk chaotic system and its performances," *J. Appl. Anal. Comput.*, vol. 6, no. 4, pp. 1180–1194, Nov. 2016.

[20] Y. Liu, J. Guan, C. Ma, and S. Guo, "Generation of $2N + 1$-scroll existence in new three-dimensional chaos systems," *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 26, no. 8, Aug. 2016, Art. no. 084307.

[21] Y. Liu and S. Guo, "Generation and dynamics analysis of N-scrolls existence in new translation-type chaotic systems," *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 26, no. 11, Nov. 2016, Art. no. 113114.

[22] Y. Liu, "Generation and dynamics analysis of N-scroll existence via four different functions in translation chaotic system," in *Proc. 3rd Int. Conf. Modelling, Simulation Appl. Math. (MSAM)*, 2018, pp. 237–242.

[23] B. Wang, "Results on a novel piecewise-linear memristor-based chaotic system," *Complexity*, vol. 2019, no. 1, pp. 1–6, Jan. 2019.

[24] L. Chen, W. Pan, R. Wu, K. Wang, and Y. He, "Generation and circuit implementation of fractional-order multi-scroll attractors," *Chaos, Solitons Fractals*, vol. 85, no. 1, pp. 22–31, Apr. 2016.

[25] L. Zhou, C. Wang, and L. Zhou, "A novel no-equilibrium hyperchaotic multi-wing system via introducing memristor," *Int. J. Circuit Theory Appl.*, vol. 46, no. 1, pp. 84–98, Jan. 2018.

[26] J. Kengne, A. N. Negou, and D. Tchiotsop, "Antimonotonicity, chaos and multiple attractors in a novel autonomous memristor-based jerk circuit," *Nonlinear Dyn.*, vol. 88, no. 4, pp. 2589–2608, Jun. 2017.

[27] J. Kengne, A. N. Negou, and Z. T. Njitacke, "Antimonotonicity, chaos and multiple attractors in a novel autonomous jerk circuit," *Int. J. Bifurcation Chaos*, vol. 27, no. 7, Jun. 2017, Art. no. 1750100.

[28] Z. T. Njitacke, J. Kengne, and L. K. Kengne, "Antimonotonicity, chaos and multiple coexisting attractors in a simple hybrid diode-based jerk circuit," *Chaos, Solitons Fractals*, vol. 105, no. 1, pp. 77–91, Dec. 2017.

[29] W. Zhou, G. Wang, Y. Shen, F. Yuan, and S. Yu, "Hidden coexisting attractors in a chaotic system without equilibrium point," *Int. J. Bifurcation Chaos*, vol. 28, no. 10, Sep. 2018, Art. no. 1830033.

[30] Q. Lai, T. Nestor, J. Kengne, and X.-W. Zhao, "Coexisting attractors and circuit implementation of a new 4D chaotic system with two equilibria," *Chaos, Solitons Fractals*, vol. 107, no. 1, pp. 92–102, Feb. 2018.

[31] L. Zhou, C. Wang, X. Zhang, and W. Yao, "Various attractors, coexisting attractors and antimonotonicity in a simple fourth-order memristive twin-T oscillator," *Int. J. Bifurcation Chaos*, vol. 28, no. 4, Apr. 2018, Art. no. 1850050.

[32] S. Zhang and Y. Zeng, "A simple jerk-like system without equilibrium: Asymmetric coexisting hidden attractors, bursting oscillation and double full Feigenbaum remerging trees," *Chaos, Solitons Fractals*, vol. 120, no. 1, pp. 25–40, Mar. 2019.

[33] M. Chen, M. Sun, B. Bao, H. Wu, Q. Xu, and J. Wang, "Controlling extreme multistability of memristor emulator-based dynamical circuit in flux-charge domain," *Nonlinear Dyn.*, vol. 91, no. 2, pp. 1395–1412, Jan. 2018.

[34] J. Ma, P. Zhou, B. Ahmad, G. Ren, and C. Wang, "Chaos and multi-scroll attractors in RCL-shunted junction coupled jerk circuit connected by memristor," *PLoS ONE*, vol. 13, no. 1, 2018, Art. no. e0191120.

[35] Q. Lai and S. Chen, "Generating multiple chaotic attractors from Sprott B system," *Int. J. Bifurcation Chaos*, vol. 26, no. 11, Oct. 2016, Art. no. 1650177.

[36] Q. Lai, A. Akgul, C. Li, G. Xu, and Ü. Çavuşoğlu, "A new chaotic system with multiple attractors: Dynamic analysis, circuit realization and S-Box design," *Entropy*, vol. 20, no. 1, pp. 1–15, Dec. 2017.

[37] Q. Lai, B. Norouzi, and F. Liu, "Dynamic analysis, circuit realization, control design and image encryption application of an extended Lü system with coexisting attractors," *Chaos, Solitons Fractals*, vol. 114, pp. 230–245, Sep. 2018.

[38] Q. Zhang, H. Chen, J. Luo, Y. Xu, C. Wu, and C. Li, "Chaos enhanced bacterial foraging optimization for global optimization," *IEEE Access*, vol. 6, pp. 64905–64919, 2018.

[39] K. Shankar, M. Elhoseny, E. D. Chelvi, S. K. Lakshmanaprabu, and W. Wu, "An efficient optimal key based chaos function for medical image security," *IEEE Access*, vol. 6, pp. 77145–77154, 2018.

[40] C. Li, B. Feng, S. Li, J. Kurths, and G. Chen, "Dynamic analysis of digital chaotic maps via state-mapping networks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 66, no. 6, pp. 2322–2335, Jun. 2019.

[41] Z. Wu, X. Zhang, and X. Zhong, "Generalized chaos synchronization circuit simulation and asymmetric image encryption," *IEEE Access*, vol. 7, pp. 37989–38008, 2019.

[42] A. Belazi, M. Talha, S. Kharbech, and W. Xiang, "Novel medical image encryption scheme based on chaos and DNA encoding," *IEEE Access*, vol. 7, pp. 36667–36681, 2019.

[43] M. Sharafi, F. Fotouhi-Ghazvini, M. Shirali, and M. Ghassemian, "A low power cryptography solution based on chaos theory in wireless sensor nodes," *IEEE Access*, vol. 7, pp. 8737–8753, 2019.

[44] Q. Zhang, H. Chen, A. A. Heidari, X. Zhao, Y. Xu, P. Wang, Y. Li, and C. Li, "Chaos-induced and mutation-driven schemes boosting salp chains-inspired optimizers," *IEEE Access*, vol. 7, pp. 31243–31261, 2019.

[45] Q. Lai, P. D. K. Kuate, F. Liu, and H. H.-C. Iu, "An extremely simple chaotic system with infinitely many coexisting attractors," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, early access, Jul. 9, 2019, doi: 10.1109/TCSII.2019.2927371.

YUE LIU received the bachelor's degree from Liaoning Shiyou University, Liaoning, China, in 2005, the master's degree from the School of Automation Engineering, Northeast Electric Power University, Jilin, China, in 2009, and the Ph.D. degree from Jilin University, Changchun, China, in 2018.

She joined the School of Electrical and Electronic Engineering, Changchun University of Technology, as a Lecturer, in 2018. From 2019 to 2020, she was a Visiting Scholar with The University of Western Australia, Australia. She has published one book and over ten articles in these areas. Her research interests include nonlinear dynamics, memristive systems, chaos theory, and chaos control and anti-control.

HERBERT HO-CHING IU (Senior Member, IEEE) received the B.Eng. (Hons.) degree in electrical and electronic engineering from The University of Hong Kong, Hong Kong, in 1997, and the Ph.D. degree from The Hong Kong Polytechnic University, Hong Kong, in 2000.

He joined the School of Electrical, Electronic and Computer Engineering, The University of Western Australia, as a Lecturer, in 2002, where he is currently a Professor. He has authored over 100 articles in these areas. He is the coauthor of *Development of Memristor Based Circuits* (World Scientific, 2013). His research interests include power electronics, renewable energy, nonlinear dynamics, current sensing techniques, and memristive systems. He has received two IET Premium Awards, in 2012 and 2014, the Vice-Chancellor's Mid-Career Research Award, in 2014, and the IEEE PES Chapter Outstanding Engineer, in 2015. He currently serves as an Associate Editor for the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS II, the IEEE TRANSACTIONS ON POWER ELECTRONICS, IEEE ACCESS, the *IEEE Circuits and Systems Magazine*, *IET Power Electronics*, and the *International Journal of Bifurcation and Chaos*. He is the Co-Editor of *Control of Chaos in Nonlinear Circuits and Systems* (Singapore: World Scienti_c, 2009). He also serves an Editorial Board Member for the IEEE JOURNAL OF EMERGING AND SELECTED TOPICS IN CIRCUITS AND SYSTEMS and the *International Journal of Circuit Theory and Applications*.