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A Modified Approach to Conflict Management From the Perspective of Non-Conflicting Element Set

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ABSTRACT In Dempster-Shafer (D-S) evidence theory, how to deal with conflict is an important and open topic. Two key strategies for resolving conflicts of evidence are considered, namely information averaging and information focus. How to balance this relationship is still a question worth considering. Recently, Ma *et al.* studied evidence conflicts from the perspective of complete conflict set and proposed a flexible rule for conflict evidence combination. The proposed combination rule seems to take into account the above two strategies. However, through analysis, we find that Ma *et al.*'s method tends to use the average method to solve conflicting propositions, while Dempster's combined rule has a weak focusing function. In this paper, based on the concept of non-conflict element set, a new conflict handling method is proposed. First, the similarity between the evidences is characterized by the correlation coefficient; based on this, a new weighting scheme of evidence is developed. In addition, the propositional support is reasonably allocated through discounts.Through numerical examples, the applicability and superiority of the method are compared and analyzed. The results show that the proposed method takes two strategies of averaging and focusing into consideration, and the information variance is small.

INDEX TERMS Dempster-Shafer (D-S) evidence theory, correlation coefficient, non-conflicting element set, conflict management.

I. INTRODUCTION

In the real-world, information is often cumbersome and uncertain. Therefore, the handling of uncertain information is particularly important. There are many theoretical studies on handling uncertain information, such as rough sets theory [1], [2], fuzzy set theory [3], [4], Z-number theory [5]–[7], D-number theory [8]–[11], evidence reasoning theory [12], [13], and other theories and research [14]–[17]. In particular, D-S evidence theory, as a method of uncertain information modeling, has the following three main advantages. First, it meets a weaker condition than Bayes' probability theory, that is, "it does not have to satisfy probability additivity". Secondly, knowledge and data from different experts or data sources can be synthesized. Thirdly, description of uncertainty is flexible and convenient. Because of its advantages in dealing with uncertain information methods, it has been widely used in fault diagnosis [18]–[20], transportation solution evaluation [21], information fusion [22], [23], stock investment selection [24], supplier selection decision [25], reliability analysis [26], etc [27]–[29]. However, when there is a high degree of conflict between the evidence, the results usually produced by using Dempster's combination rules don't reflect the actual distribution of beliefs [30]. To solve this problem, many scholars have studied the work related to conflict evidence management. Through the induction and analysis of many existing researches, it can be divided into the methods to modify Dempster's combination rules and to modify data model [31]–[34]. The main methods to modify Dempster's combination rules include Lefevre *et al.*'s unified reliability function combination method [35], Yager [36] and Smets *et al.*'s [37] the conflict evidence combination method proposed, and Smardndache *et al.*'s conflict proportional allocation rule PCR3 [38], etc. Currently, researchers are inclined to Haenni's [39] point of view, and papers

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that modify data model dominate [40]. The main methods to modify data model include Murphy's arithmetic average method [41], Deng *et al.*'s improved weighted average method [42] based on evidence distance [43], etc. Recently, based on the methods proposed by Deng *et al.*, some scholars have proposed improved conflict methods. Among them, Jiang's method based on correlation coefficient [44], Song *et al.*'s method based on divergence [45], Pan *et al.*'s method based on associationcoefficient [46], and so on [47], [48]. For the time being, research with modified data as the main point of view is dominant [49]. In particular, Liu [50] comprehensively considered the applicability of Dempster's combination rule in the case of conflict, proposed the method of using binary groups of k and *difBetP* to describe the conflict, and proposed the proposal of using Demspster's combination rule in the authoritative journal *Artificial Intelligence*. Liu's viewpoint provides a new idea for the development of D-S evidence theory. At present, the research on evidence theory is still developing [51]–[53].

However, Ma *et al.* [54] found that the methods proposed by scholars represented by Murphy does not seem reasonable. In their method, when the feature information is extracted from the evidence source, the information is focused using only the Dempster combination rule for the average information source after the merger or the average weight. This leads to a weak loss of information, which affects the judgment of the second information. Therefore, it is a meaningful work to distinguish and screen specific propositions in conflict evidence and then use Dempster's combination rules for information focusing. Recently, a flexible evidence combination rule [54] was proposed. The conflict elements in the conflict evidence were distinguished based on the complete conflict set and then combined using the Dempster's combination rule. Obviously, this method is more reasonable than the method of Murphy *et al.* However, through analysis, we find that Ma *et al.*'s method tends to use the average method to solve conflicting propositions, while Dempster's combined rule has a weak focusing function. To better balance the relationship between averaging and focusing, in this paper, we propose a new evidence combination rule to resolve conflicts. First, we propose the concept of a non-conflicting element set to distinguish non-conflicting elements. Then, we use the correlation coefficient to reasonably characterize the degree of similarity between the evidences. Based on this, we propose a more reasonable way to express the weight of the evidence proposition. Finally, in the proposed evidence combination rule, we assign the weight discount value to the elements in the non-conflicting element set to fuse the evidence. The numerical results show that this method is easy to achieve the average of conflicting information and the information variance of fusion results is small. Compared with the method proposed by Ma *et al.*, Dempster's combination rule's information focus feature is better maintained.

The structure of this paper is as follows. Second II introduces the background of the research. In section III, we propose a new conflict evidence combination rule and analyze and discuss it. In section IV, the comparison of numerical examples shows the effectiveness of the proposed method. Section V illustrates the applicability and superiority of the proposed method through an application example. The conclusion of this paper are given in section VI.

II. PRELIMINARIES

In this section, we briefly review some background knowledge, such as D-S evidence theory, correlation coefficient, Ma *et al.*'s conflict resolution method, and some other scholars' representative research in conflict management.

A. DEMPSTER-SHAFER EVIDENCE THEORY

D-S evidence theory originated in the 1960s, when A.P. Dempster used upper and lower probabilities to solve the problem of multi-value mapping. He published a series of papers successively from 1967, which marked the official birth of evidence theory [55]. Dempster's student G. shafer further developed evidence theory, introduced the concept of trust function, and formed a set of mathematical methods of "evidence" and "combination" to deal with uncertain reasoning [56]. D-S evidence theory is a generalization of bayesian reasoning method, which is mainly carried out by using bayesian conditional probability in probability theory, and prior probability should be known. However, D-S evidence theory does not need to know the prior probability, and can well represent "uncertainty". Due to its advantages in processing uncertain information, D-S evidence theory has been widely applied in many fields, such as multi-attribute decision making [57], data fusion [58], [59], evidence reliability assessment [60], fault diagnosis [61], [62], medical treatment [63], intelligent decision making [64] and so on.

1) FRAME OF DISCERNMENT

Definition 1: Let Θ be a set of mutually exclusive and collectively exhaustive events defined by

$$\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\} \tag{1}$$

where the set Θ is called the frame of discernment.

The 2^Θ is the Θ power set, which is expressed as

$$2^\Theta = \{\emptyset, \{\theta_1\}, \cdots \{\theta_N\}, \{\theta_1, \theta_2\}, \cdots \{\theta_1, \theta_2, \cdots, \theta_i\}, \cdots, \Theta\} \tag{2}$$

and $\emptyset$ is an empty set.

If $A \in 2^\Theta$, A is called a hypothesis or proposition.

2) MASS FUNCTION

Definition 2: For a frame of discernment Θ, a mass function is expressed as a mapping, i.e., from 2^Θ to [0, 1], formally defined by

$$m : 2^\Theta \rightarrow [0, 1] \tag{3}$$

which satisfies the following two attributes:

$$m(\emptyset) = 0 \quad and \quad \sum_{\theta \subseteq \Theta} m(\theta) = 1 \tag{4}$$

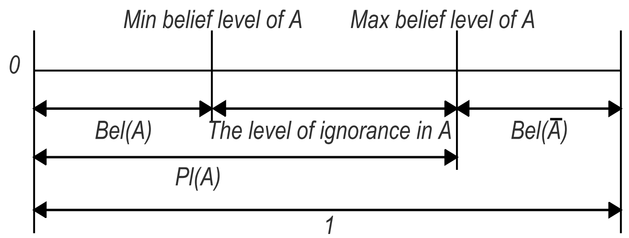

FIGURE 1. **The relationship between *Pl* and *Bel*.**

In D-S evidence theory, m is also called a Basic Probability Assignment (BPA). For exmaple, $m(A)$ is BPA of A, which accurately reflects the extent to which A is supported. If $m(A) > 0$, A is a focal element of the mass function.

3) BELIEF AND PLAUSIBILITY FUNCTIONS

Definition 3: From the BPA, a belief function Bel and a plausibility function Pl are defined, respectively, as

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{5}$$

and

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{A \bigcap B = \emptyset} m(B) \tag{6}$$

where $\bar{A} = \Theta - A$, $Bel : 2^{\Theta} \to [0, 1]$ and $Pl : 2^{\Theta} \to [0, 1]$.

The relationship between Pl function and Bel function is shown in Fig.1.

In Fig.1, the quantity $Bel(A)$ can be interpreted as a measure of one's belief that hypothesis A is true. The plausibility $Pl(A)$ can be viewed as the total amount of belief that could be potentially placed in A. The $[Bel(A), Pl(A)]$ indicates the uncertain interval for A.

4) DEMPSTER's COMBINATION RULE

Definition 4: Let m_1 and m_2 are two mass functions on the discernment frame Θ, $\oplus$ represents the orthogonal summation operation, and the Dempster's combination rule is defined as follows:

$$[m_1 \oplus m_2](\theta) = \begin{cases} 0 & \theta = \emptyset \\ \dfrac{\sum_{A_1 \cap A_2 = \theta} m_1(A_1) m_2(A_2)}{1 - k} & \theta \neq \emptyset \end{cases} \tag{7}$$

where k is defined as follows:

$$k = \sum_{A_1 \cap A_2 = \emptyset} m_1(A_1) m_2(A_2) \tag{8}$$

In Eq.(3), k denotes a normalization constant, which is used to indicate the degree of collision between the evidences m_1 and m_2. When $k = 0$, it means that there is no conflict between the evidences m_1 and m_2. If $k = 1$, it means that the evidence m_1 and m_2 completely conflict. At this time, the equation cannot be used for evidence fusion, because the denominator is 0, and the eqation has lost its meaning.

When using the Dempster-Shafer combination rule to fuse multiple pieces of evidence, the Eq.(9) can be used.

$$[m_1 \oplus m_2 \cdots \oplus m_n] = \begin{cases} 0 \quad \theta = \emptyset \\ \dfrac{\sum_{A_1 \bigcap A_2 \cdots \cap A_n = \theta, A_1, A_2, \cdots, A_n \subseteq \Theta} m_1(A_1) m_2(A_2) \cdots m_n(A_n)}{\sum_{A_1 \bigcap A_2 \cdots \cap A_n \neq \emptyset, A_1, A_2, \cdots, A_n \subseteq \Theta} m_1(A_1) m_2(A_2) \cdots m_n(A_n)} \\ \theta \neq \emptyset \end{cases} \tag{9}$$

with the restriction is:

$$\sum_{\substack{A_1 \bigcap A_2 \cdots \cap A_n \neq \emptyset \\ A_1, A_2, \cdots, A_n \subseteq \Theta}} m_1(A_1) m_2(A_2) \cdots m_n(A_n) \neq 0 \tag{10}$$

B. EVIDENCE DISTANCE

Jousselme [43] proposed a distance measure for belief functions, the evidence distance is defined as follows.

Definition 5: Let m_1 and m_2 be two BPAs on the same discernment frame Θ, and the distance between m_1 and m_2 is defined as follows.

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\vec{m}_1 - \vec{m}_2)^T D (\vec{m}_1 - \vec{m}_2)} \tag{11}$$

where $\vec{m}_1$ and $\vec{m}_2$ are vectorized BPAs and D is an $2^N \times 2^N$ matrix whose elements are $D(A, B) = \frac{|A \cap B|}{|A \cup B|}$ with $A, B \in P(\Theta)$.

C. CORRELATION COEFFICIENT

Jiang [44] proposed a correlation coefficient for the belief function, which is defined as follows.

Definition 6: For a discernment frame Θ with N elements, suppose the mass of two pieces of evidence denoted by m_1, m_2. A correlation coefficient is defined as:

$$r_{BPA}(m_1, m_2) = \frac{c(m_1, m_2)}{\sqrt{c(m_1, m_1) \cdot c(m_2, m_2)}} \tag{12}$$

where $c(m_1, m_2)$ is the degree of correlation denoted as:

$$c(m_1, m_2) = \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{13}$$

where A_i and A_j is the focal element of mass and $i, j = 1, \ldots, 2^N$; $|\cdot|$ is the cardinality of a subset.

D. YAGER's CONFLICT MANAGEMENT APPROACH

Because the classic Dempster's combination rule cannot handle highly conflicting evidence, Yager [36] proposed a new rule for conflict evidence combination. The conflict in the proposed method is assigned to the complete set Θ. Yager's methods are defined as follows.

Definition 7:

$$\begin{aligned} m(\emptyset) &= 0 \\ m(\Theta) &= \sum_{\cap A_i=\Theta} \prod_{1\leqslant j\leqslant n} m_j(A_i) + k \\ m(A) &= \sum_{\cap A_i=A} \prod_{1\leqslant j\leqslant n} m_j(A_i) \quad (A \neq \emptyset, A \neq \Theta) \end{aligned} \tag{14}$$

where k is the conflict coefficient in Dempster's combination rule.

E. SMETS et al.'s CONFLICT MANAGEMENT METHOD

Compared to Yager's conflict management method, Smets and Kennes [37] proposed to assign conflicts to the empty set $\emptyset$. The method to conflict management proposed by Smets *et al.* is as follows.

Definition 8:

$$\begin{aligned} m(\Theta) &= 0 \\ m(\emptyset) &= \sum_{\cap A_i=\Theta} \prod_{1\leqslant j\leqslant n} m_j(A_i) + k \\ m(A) &= \sum_{\cap A_i=A} \prod_{1\leqslant j\leqslant n} m_j(A_i) \quad (A \neq \emptyset, A \neq \Theta) \end{aligned} \tag{15}$$

where k is the conflict coefficient in Dempster's combination rule.

F. SUN et al.'s CONFLICT MANAGEMENT METHOD

Due to the conflicting evidence combination rule proposed by Yager, when the number of evidence sources exceeds two, the fusion result is not reasonable [65]. Sun and Ye [65] proposed an improved rule of conflict evidence combination.

Definition 9: Let $m_1, m_2, \cdots, m_n$ correspond to the evidence set: $F_1, F_2, \cdots, F_n$ and let the conflict between the evidence sets i and j to be k_{ij}. The k_{ij} is defined as follows:

$$k_{ij} = \sum_{\substack{A_i\cap A_j=\emptyset \\ A_i\in F_i, A_j\in F_j}} m_i(A_i)m_j(A_j) \tag{16}$$

Next, let ε is the credibility of evidence, $\varepsilon = e^{-\kappa}$, in which $\kappa = \frac{1}{n(n-1)/2}\sum_{i<j} \kappa_{ij}$. The evidence synthesis rule is defined as follows:

$$\begin{aligned} m(\emptyset) &= 0 \\ m(A) &= p(A) + k * \varepsilon * q(A), A \neq \emptyset, X \\ m(X) &= p(X) + k * \varepsilon * q(X) + k(1-\varepsilon) \end{aligned} \tag{17}$$

where

$$p(A) = \sum_{\substack{A_i\in Fi \\ \cap_{i=1}^{n} A_i=A}} m_1(A_1)m_2(A_2)\cdots m_n(A_n),\ q(A) = \frac{1}{n}\sum_{i=1}^{n} m_i(A) \tag{18}$$

Finally, Eq.(17) can be written as follows:

$$m(A) = (1-k)\frac{p(A)}{1-k} + k * \varepsilon * q(A) \tag{19}$$

G. MURPHY's CONFLICT MANAGEMENT METHOD

Based on the idea of revising data, Murphy [41] proposed an arithmetic average method of combining conflicting evidence. That is, all evidence is averaged before fusing. In other words, if the system contains n pieces of evidence, firstly, the BPAs are averaged, and then the Dempster's combination rule is used for $n-1$ fusion.

H. DENG et al.'s CONFLICT MANAGEMENT METHOD

Yong *et al.* [42] believed that there is a flaw in the Murphy's method, where all evidence is equally important and does not well consider the correlation between evidence collected from multiple sources. Therefore, based on evidence distance [43], Deng *et al.* improved Murphy's method, and his method is defined as follows.

Definition 10: Suppose the distance between two bodies of evidence $(\Re_i, m_i)$ and $(\Re_j, m_j)$ can be calculated by the evidence distance (see from Definition 5) and is denoted as $d(m_i, m_j)$. The similarity measure function $Sim(\cdot)$ between the two bodies of evidence and $(\Re_j, m_j)$ defined as:

$$Sim(m_i, m_j) = 1 - d(m_i, m_j) \tag{20}$$

A similarity measure matrix (SMM) constructed by means of all similarity degrees between evidence bodies is defined as follows:

$$SMM = \begin{bmatrix} 1 & \cdots & s_{12} & \cdots & s_{1j} & \cdots & s_{1k} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{i1} & \cdots & s_{i2} & \cdots & s_{ij} & \cdots & s_{in} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_{k1} & \cdots & s_{n2} & \cdots & s_{nj} & \cdots & 1 \end{bmatrix} \tag{21}$$

The support degree of the body of evidence $(\Re_i, m_i)(i = 1, 2, \cdots, k)$ is defined as:

$$Sup(m_i) = \sum_{j=1, j\neq i}^{k} Sim(m_i, m_j) \tag{22}$$

The credibility degree Crd_i of the body of evidence $(\Re_i, m_i)(i = 1, 2, \cdots, k)$ is defined as:

$$Crd_i = \frac{Sup(m_i)}{\sum_{i=1}^{k} Sup(m_i)} \tag{23}$$

Finally, the modified average (or the weight average) of the evidence $MAE(m)$ is given as:

$$MAE(m) = \sum_{i=1}^{n}(Crd_i \times m_i) \tag{24}$$

If there are n pieces of evidence, as Murphy's method, the classical Dempster's combination rule is used to combine the weighted average of the mass $n-1$ times.

I. JIANG's CONFLICT MANAGEMENT METHOD

Based on the correlation coefficient [44], Jiang improved the method of Jiang *et al.* [44].

In Jiang's method, she used the correlation coefficient to characterize the similarity between the two evidence bodies rather than the evidence distance. Where, the correlation coefficient matrix (CCM) is expressed as follows (25), as shown at the bottom of the next page:

The other parts are the same as Deng *et al.*'s method.

J. MA et al.'s CONFLICT MANAGEMENT METHOD

Recently, based on the concept of the complete conflict set and evidence distance [43], Ma *et al.* [54] proposed a flexible rule for the combination of conflicting evidence. The proposed combination rule is defined as follows.

Definition 11: For a mass function set $M = \{m_k \mid k = 1, \cdots, n\}$ over a frame Θ, let $m_{\oplus M}$ be the value of combining all the mass functions in M by Dempster's combination rule, $m_{\oplus M}(A)$ be the weight value of a subset $A \subseteq \Theta$ over M, and γ_M be the complete conflict set of M. Then the mass function m_M that global-completely combines a mass functions set M is given by:

$$m_M(X) = \begin{cases} 0 & X = \emptyset \\ (1-\delta_M)m_{\oplus M}(X) & X \subseteq \Theta, X \notin \gamma_M, X \neq \emptyset \\ \sum\limits_{X \subseteq A} \omega_M(A)\frac{|X|}{|A|} & X \subseteq \Theta, X \in \gamma_M, X \neq \emptyset \end{cases} \tag{26}$$

where δ_M is the compatible redistribution value given by:

$$\delta_M = \begin{cases} 0 & \gamma_M = \emptyset \\ \sum\limits_{X \in \gamma_M} \sum\limits_{X \subseteq A} \omega_M(A)\frac{|X|}{|A|} & otherwise \end{cases} \tag{27}$$

III. PROPOSED A NEW EVIDENCE COMBINATION RULE

In this section, a novel combination of evidence rule is proposed. As demonstrated by Ma *et al.* [54], we also believe that the counter-intuitive behavior of Dempster's combination rule is caused by some conflicting elements of the original mass function. First, based on the set of non-conflicting elements, we distinguish the focus elements of propositions that do not conflict. Then, we consider the combination process as a two-stage process. The first step, use Dempster's combination rule to combine all existing evidence. The second step, based on the correlation coefficient [44], obtain the weight value of the focus element, and redistribute the combination quality value between non-conflicting elements. If the given elements are in the set of non-conflicting elements, that is, there are no conflicting elements, the proposed method and Dempster's combination rule fusion results is the same.

A. THE PROPOSED METHOD

1) NON-CONFLICTING ELEMENT SET

Definition 12: Let $M = \{m_k \mid k = 1, \cdots, n\}$ be the set of functions on the discernment frame Θ. F_k is the set of focus elements of all $m_k \in M$, and $\kappa_M \subseteq 2^{\Theta}$ is the non-conflicting element set of the mass function set M if and only if any $A \in \kappa_M$, there are two focus element sets F_i and F_j for both mass functions $m_i \in M$ and $m_j \in M$, there are $A \in F_i$ and any $B \in F_j$, $A \cap B \neq \emptyset$. For any $C \subset A$, there are $C \cap B \neq \emptyset$ and $\sum_{p \cap q = C} m_1(p)m_2(q) \neq 0$.

2) ELEMENT WEIGHT

Definition 13: For a mass function set $M = \{m_k \mid k = 1, \cdots, n\}$ on the discernment frame Θ, let $r_{BPA}(m_i, m_j)$ denotes the degree of similarity between any two pieces of evidence. Then for the subset A on the discernment frame Θ and $A \in M$. The weight of the given element is as follows:

$$\omega'_M(A) = \sum_{i=1}^{n} m_i(A)\varepsilon(m_i) \tag{28}$$

where $\varepsilon(m_i)$ indicates the degree of credibility of m_i, which is defined as follows:

$$\varepsilon(m_i) = \begin{cases} 0 & m_i(\Theta) = 1 \\ \dfrac{\sum\limits_{j=1, i\neq j}^{n} r_{BPA}(m_i, m_j)}{\sum\limits_{i=1}^{n}\sum\limits_{j=1, i\neq j}^{n} r_{BPA}(m_i, m_j)} & otherwise \end{cases} \tag{29}$$

The element weight reflect the reliability of each BPA, which is based on the correlation coefficient. Here we demonstrate a few important properties that the element totality satisfies.

Property 1: **The empty set has a weight of 0.**

Proof: By Eq.(28), we can get $\omega'_M(\emptyset) = \sum\limits_{i=1}^{n} m_i(\emptyset)\varepsilon(m_i)$. From Eq.(4), we know that $m(\emptyset) = 0$. Therefore, we can obtain that $\omega'_M(\emptyset) = \sum\limits_{i=1}^{n} 0 \times \varepsilon(m_i) = 0$.

Property 2: **The sum of the weights of the elements on the set M (M on the discernment frame Θ) is 1.**

Proof: By Eq.(8), Eq.(9) and $\sum\limits_{A\subseteq\Theta} m_i(A) = 1$ (see Eq.(4)), we can obtain:

$$\begin{aligned} \sum_{A\subseteq\Theta} \omega'_M(A) &= \sum_{A\subseteq\Theta}\sum_{i=1}^{n} m_i(A)\frac{\sum\limits_{j=1,j\neq i}^{n} r_{BPA}(m_i, m_j)}{\sum\limits_{i=1}^{n}\sum\limits_{j=1,j\neq i}^{n} r_{BPA}(m_i, m_j)} \\ &= \sum_{i=1}^{n}\left(\frac{\sum\limits_{j=1,j\neq i}^{n} r_{BPA}(m_i, m_j)}{\sum\limits_{i=1}^{n}\sum\limits_{j=1,j\neq i}^{n} r_{BPA}(m_i, m_j)}\sum_{A\subseteq\Theta} m_i(A)\right) \\ &= \sum_{i=1}^{n}\left(\frac{\sum\limits_{j=1,j\neq i}^{n} r_{BPA}(m_i, m_j)}{\sum\limits_{i=1}^{n}\sum\limits_{j=1,j\neq i}^{n} r_{BPA}(m_i, m_j)} \times 1\right) = 1 \end{aligned}$$

3) A NEW RULE OF EVIDENCE COMBINATION

Definition 14: For a mass function set $M = \{m_k \mid k = 1, \cdots, n\}$ on a discernment frame Θ. A is a subset of the discernment frame Θ. $m_{\oplus}(A)$ denotes the fused value of the subset A of the discernment frame on M. κ_M denotes a non-conflicting element set of M. $m_M(\theta)$ indicates that the mass function on the set M is defined as

$$m_M(\theta) = \begin{cases} 0 & \theta = \emptyset \\ \upsilon_M \, m_{\oplus M}(\theta) & \theta \subseteq \Theta, \theta \in \kappa_M, \theta \neq \emptyset \\ \omega'_M(\theta) & \theta \subseteq \Theta, \theta \notin \kappa_M, \theta \neq \emptyset \end{cases} \quad (30)$$

where υ_M is the discount distribution value, which is given below:

$$\upsilon_M = \begin{cases} 1 & \kappa_M = \partial \\ \dfrac{\omega'_M(\theta)}{\sum\limits_{\theta \notin \kappa_M} \sum\limits_{i=1}^{n} \omega'(\theta_i)} & \text{otherwise} \end{cases} \quad (31)$$

in which ∂ represents the collection of all propositional categories in the given BPAs.

An algorithm flowchart of the proposed method is shown in Fig.2.

B. SOME BASIC PROPERTIES

In the flexible combination rule [54], it is proved that this combination rule satisfies some unique properties. In this subsection, based on these important properties, we will highlight that the proposed method also satisfies these basic properties, so as to show the applicability of the proposed method.

1) SOME PROPERTIES OF THE FLEXIBLE COMBINATION RULE

Definition 15: Let Z be a set of all possible mass functions over a frame of discernment $\Theta = \{s_1, \cdots, s_n\}$, where $\{s_1, \cdots, s_n\}$ is a set of exhaustive and mutually exclusive elements, $M \subset Z$ be a set of mass functions over the frame of discernment Θ, and m_M be the combined result of all mass functions on set M. Then an operator $\odot : Z \times Z \to Z$ is a combination operator if it satisfies:

(i) ***Local Computation Availability***. *(a) Commutativity*: $m_1 \odot m_2 = m_2 \odot m_1$. (b) *Quasiassociativity*: $m_1 \cdots m_n = T(m_1 \odot \cdots \odot m_n) = T(m_1 \circ \cdots \circ m_n)$, *where $\circ$ is an associativity operator over Z and T is a mapping from Z to Z.*

(ii) ***Neutral Element Commitment***. *If $m_i(\Theta) = 1$, then $m_M \odot m_i = m_M$ for any mass function set M.*

(iii) ***Possibility Reservation***. *If $\exists m_i \in M$ and $a \subset \Theta$ such that $m_i(\{a\}) > 0$, then $m_M(\{a\}) > 0$.*

(iv) ***Convergence toward Certainty***. *If $\exists m_i \in M$ and $m_i(\{a\}) > m_i(X)$ for any $X \subset \Theta \setminus \{a\}$, then*

$$m_i^n(\{a\}) > m_i^{n-1}(\{a\})$$

where m_n^i means using combination operator $\odot$ to combine the mass function m_i n times.

(v) ***Invariance of Iterated Indifference Evidence***. *If $m_k(\{s_i\}) = \frac{1}{n}$ for any $s_i \in \Theta$ and any $m_k \in M$, then $m_M = m_k$.*

(vi) ***Weak Specialisation***. *If $m_M(A) > 0$, then there $\exists m_i \in M$ such that $m_i(B) > 0$ and $A \subseteq B$.*

2) SOME PROPERTIES AND PROOFS OF THE PROPOSED METHOD

In order to illustrate that the method satisfies the basic properties proposed by the Ma et al's method, we will prove the properties listed in previous subsubsection one by one, as follows:

(i) ***Local Computation Availability***

Proof: Since Dempster's combination rule satisfies exchangeability (see Eq.(7)), it can be seen from subsubsection III-A.3, which is clear that our combination rule also satisfies exchangeability. In addition, because our rule based on the D-S evidence theory and by Eq.(7) implementation of BPA redistribution, compatible with the redistribution of value to Eq.(30) has been given, as a result, our rule to meet Quasiassociativity. It follows that the new rule we proposed satisfies the first property in subsubsection III-B.1. ■

(ii) ***Neutral Element Commitment***

Proof: First, let $\odot$ be the operator given in subsubsection III-A.3 and $\oplus$ be the operator given in subsubsection III-A.3. Then, by subsection II-A, we know that for any mass function $m_i \in M$, if $m_i(\Theta) = 1$, we have $m_{\oplus M} \oplus m_i$. Therefore, throgh subsubsection III-A.1, we have $A_M = A_{M \cup \{m_i\}}$. Hence, through subsubsection III-A.1 and III-A.2, we have $\omega_M(B) = \omega_{M \cup \{m_i\}}(B)$ and $A_M = A_{M \cup \{m_i\}}$. Further, by subsubsection III-A.3 we have $m_M \odot m_i = m_M$. Therefore, the new rule we propose satisfies the second property in subsubsection III-B.1. ■

$$CCM = \begin{bmatrix} 1 & r_{BPA}(m_1, m_2) & \cdots & r_{BPA}(m_1, m_j) & \cdots & r_{BPA}(m_1, m_k) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{BPA}(m_i, m_1) & r_{BPA}(m_i, m_2) & \cdots & r_{BPA}(m_i, m_j) & \cdots & r_{BPA}(m_i, m_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{BPA}(m_k, m_1) & r_{BPA}(m_n, m_2) & \cdots & r_{BPA}(m_n, m_j) & \cdots & 1 \end{bmatrix} \quad (25)$$

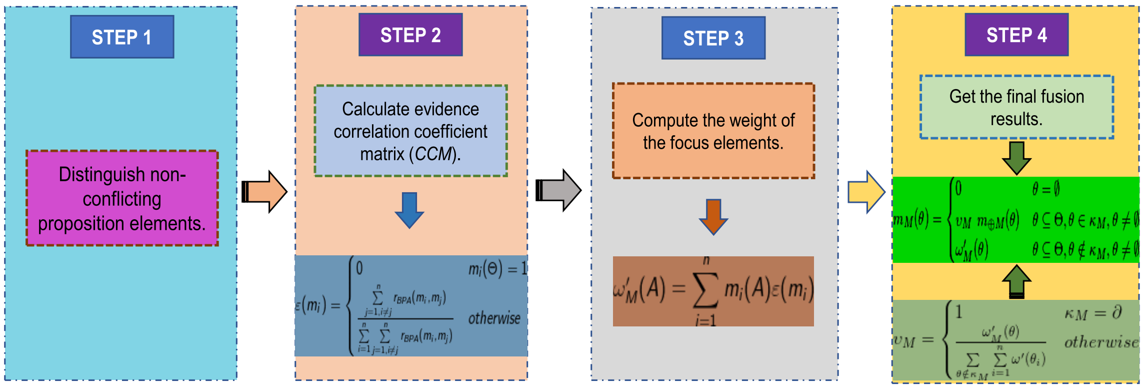

FIGURE 2. The flowchart of the proposed method.

(iii) ***Possibility Reservation***

Proof: Assume that there exist $m_i \in M$ and $a \in \Theta$ such that $m_i(\{a\}) > 0$ and $m_{\oplus M}(\{a\}) = 0$. Then, by subsubsection III-A.3 we have $\{a\} \in \gamma_M$ and $\sum_{X \subseteq A} \omega_M(A)\frac{|X|}{|A|} = 0$, or $\{a\} \notin \gamma_M$ and $(1 - \delta_M)m_{\oplus M}(X) = 0$. In the first case, for any $\{a\} \subseteq A$, since $\frac{|\{a\}|}{A} > 0$, $\omega_M(A) = 0$. Moreover, since $\{a\} \subseteq \{a\}$, by subsection II-J we have $\omega_M(\{a\}) = \sum_{k=1}^{n} m_k(\{a\})C(m_k) = 0$, Thus, we have $m_i(\{a\})\eta(m_i) = 0$. By II-J, we have $\eta(m_i) > 0$. So, $m_i(\{a\}) = 0$, which is contrary to the assumption of $m_i(\{a\}) > 0$. In the second case, since $\{a\} \notin \gamma_M$, by complete conflict set [54] and $m_i(\{a\}) > 0$, we have that for any set of focal elements F_j of mass function $m_j \in M$, there exists B $\in F_j$ such that $\{a\} \cap B \neq \emptyset$. So, by Eq.(7) we have $m_{\oplus M}(\{a\}) > 0$. Therefore, we have $\delta_M = 1$, which means that all the focal elements of each mass function in M belong to γ_M. That is, $\{a\} \in \gamma_M$. However, $\{a\} \notin \gamma_M$ and $\{a\} \in \gamma_M$ are self-contradiction. Thus, since both the cases of our assumption cause contradiction, we can conclude that if there exist $m_i \in M$ and $a \in \Theta$ such that $m_i(\{a\}) > 0$ then $m_M(\{a\}) > 0$. Thus, The new rule we propose satisfies the third property in subsubsection III-B.1. ■

(iv) ***Convergence toward Certainty***

Proof: When the mass function m_1 is combined n times using our proposed new rule, through complete conflict set [54] we know that the resulting conflict set γ_M is completely empty. Therefore, as can be seen from the subsubsection III-A.3, our rule combination results are the same as the results of the Dempster's combination rule. In addition, since the Dempster's combination rule has convergence toward certainty and our combination rule is based on the Dempster combination rule, so the new rule we proposed satisfies the fourth property in subsubsection III-B.1. ■

(v) ***Invariance of Iterated Indifference Evidence***

Proof: Similar to the proof of the attribute convergence toward certainty, when the non-conflicting element set contains all focus elements, namely $\kappa_M = \partial$ (see Eq.(31)), the combined result of our new rule is the same as that of Dempster. Therefore, by using Dempster's one, the iteration of indifferent evidence outputs the same result as each piece of evidence. So the new rule we proposed satisfies the fifth property in subsubsection III-B.1. ■

(vi) ***Weak Specialisation***

Proof: As can be seen from the subsubsection III-A.3, the combination rule consists of the following two parts:
(1) Obtaining by the discount value of the result of the Dempster's combination rule.
(2) Obtaining by the weight value.
Therefore, our proposed combination rule satisfies the sixth property in subsubsection III-B.1. ■

In summary, our proposed evidence combination rule satisfies all the properties in subsubsection III-B.1.

C. AVAILABILITY ANALYSIS

The proposed method in this paper use this correlation coefficient [44] to measure the similarity between two bodies of evidence in the process of element weight allocation. Although this evidence distance [43] can also be used, in some special cases, the correlation coefficient has a wider range of use.

In practical applications, the evidence we encounter is usually high latitude and multidimensional. We can find that if the dimensions of two pieces of evidence are different, the evidence distance cannot be processed. Therefore, in order to apply the evidence distance formula, we must not only guarantee the same size of evidence, but also make the frame of discernment of each evidence consistent. However, through comparison, we can clearly see that the correlation coefficient neatly avoids this problem. Therefore, in comparison, the correlation coefficient is more applicable.

In the following, we will further analyze the potential superiority of the correlation coefficient through two numerical examples.

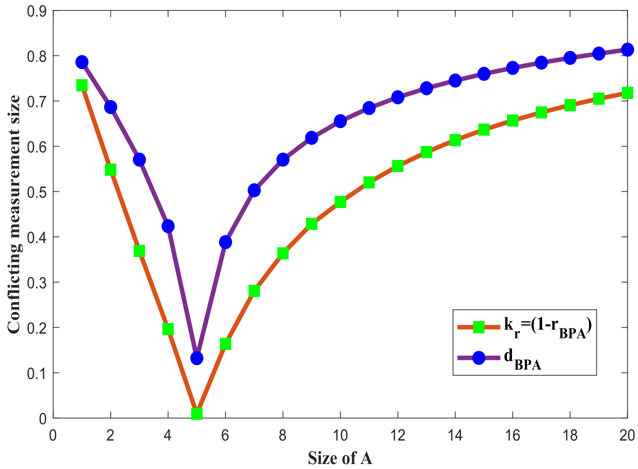

FIGURE 3. Comparison of correlation coefficient and evidence distance.

1) COUNTER-EXAMPLE 1

Example 1: Assuming that the number of discernment frame is 20, such as $\Theta = \{1, 2, \cdots, 20\}$, the BPAs of the two evidence bodies are defined as follows

$$m_1 : m_1(2, 3, 4) = 0.05, m_1(7) = 0.05, m_1(\Theta) = 0.1, m_1(A) = 0.8$$
$$m_2 : m_2(1, 2, 3, 4, 5) = 1$$

where set A adds one additional element at a time, starting from case 1, with $A = 1$, and ending with case 20.

Fig.3 clearly shows the comparison of two methods for measuring the degree of conflict of evidence. Through analysis, we can know that when the size of set A is close to 5, the conflict between the two evidence bodies should become smaller and smaller. When the size of the set A is exactly 5, the conflict should reach a minimum. When the size of set A is greater than 5, the conflict should become larger. We can find that in these three cases, the evidence distance and correlation coefficient can objectively reflect the changing trend of conflict. However, through comparison, it can be clearly seen that the correlation coefficient in this example varies more widely than the evidence distance. That is to say, for the set A of the same size, using the correlation coefficient can better reflect the depth of the conflict. Therefore, in summary, through this example, we can show that the correlation coefficient can more objectively reflect the conflict inconsistency caused by the change in the discernment frame of the evidence body.

2) COUNTER-EXAMPLE 2

Example 2: Assuming the discernment frame is $\Theta = \{A, B, C, D\}$, the BPAs of the two evidence bodies are as follows

$$m_1 : m_1(A) = 0.5, m_1(B) = 0.5, m_1(C) = 0, m_1(D) = 0$$
$$m_2 : m_2(A) = 0, m_2(B) = 0, m_2(C) = 0.5, m_2(D) = 0.5$$

We can see that, obviously, the focus elements supported by the evidence m_1 and m_2 are not the same, and they do not even intersect at all. Then, we use the formula of the correlation coefficient to calculate the similarity between them, and the calculation result is 0. Next, using the evidence distance calculation, unexpectedly, the result is not 0, but 0.7071. Distinctly, the calculation result using the evidence distance is unreasonable. Therefore, through comparison, the correlation coefficient is more superior.

3) SUMMARIZE THE ABOVE RESULTS

From the above discussion and the two counter-examples, as we can see, the evidence distance does not well explain the fact that there is a huge difference between the two evidences, which may pose a hidden danger to the allocation of the weight of the assigned focus element. From a comparative perspective, this also shows the potential superiority of using the correlation coefficient to measure conflicts of evidence in the article and assigning elements to focus elements.

D. ZADEH's COUNTER-EXAMPLE

Zadeh's counter-example [30] is a classic example, in D-S evidence theory, it states that when evidence conflicts, using Dempster's combination rule will produce results that violate human intuition. Subsequent studies by many scholars are trying their best to overcome this problem. In this section, we try to solve this counter-example using the proposed method, demonstrating the effectiveness of the proposed method to solve this problem. In addition, detailed calculation steps are used to explain the proposed method in more detail.

1) EXAMPLE STATEMENT

Example 3: Let the discernment frame be $\Theta = \{A_1, A_2, A_3\}$, m_1 and m_2 represent two pieces of evidence defined on the discernment frame Θ, their BPAs are as follows:

$$m_1 : m_1(\{A_1\}) = 0.99, m_1(\{A_2\}) = 0.01, m_1(\{A_3\}) = 0$$
$$m_2 : m_2(\{A_1\}) = 0, m_2(\{A_2\}) = 0.01, m_2(\{A_3\}) = 0.99$$

As can be seen from the two BPAs given above, the first one m_1 supports the proposition A_1 almost completely and the second piece of evidence m_2 also has almost complete support for proposition A_3. However, using Dempster's combination rule of evidence, the post-fusion proposition A_2 is fully supported (the calculation result is shown in the following formula), so fusion results are counter-intuitive the equation can be derived, as shown at the bottom of next page.

2) CALCULATE FUSION RESULTS BASED ON THE PROPOSED METHOD

Here, we now use the proposed method to solve Zadeh's counter-example. Detailed calculation steps are shown below.

Step 1: Distinguish non-conflicting proposition elements. According to the definition of non-conflicting element set, we can get that for $\{A_2\} \in F_1$ (F_1 represents the set of focus elements of m_1 and $m_1 \in M$), and for any $B \in F_2$ (F_2 represents the set of focus elements of m_2 and $m_2 \in M$), there are $\{A_2\} \cap B \neq \emptyset$. Therefore, we can get $\{A_2\} \in \kappa_M$. Therefore, we can obtain that $\kappa_M = \{\{A_2\}\}$.

Step 2: Calculate evidence correlation coefficient matrix (CCM).

$$CCM = \begin{bmatrix} r_{BPA}(m_1, m_1) & r_{BPA}(m_1, m_2) \\ r_{BPA}(m_2, m_1) & r_{BPA}(m_2, m_2) \end{bmatrix} = \begin{bmatrix} 1 & 1.0202 \times 10^{-4} \\ 1.0202 \times 10^{-4} & 1 \end{bmatrix}$$

Step 3: Compute the weight of the focus elements. By using Eq.(28) and Eq.(29), the detailed calculation process of the element A_1, A_2 and A_3 weight is shown below.

$$\begin{aligned}
&\omega'_M(\{A_1\}) \\
&= \sum_{i=1}^{2} m_i(\{A_1\}) \frac{\sum_{j=1,j\neq i}^{2} r_{BPA}(m_i, m_j)}{\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2} r_{BPA}(m_i, m_j)} \\
&= m_1(\{A_1\}) \frac{r_{BPA}(m_1, m_2)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} + m_2(\{A_1\}) \\
&\quad \times \frac{r_{BPA}(m_2, m_1)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} \\
&= 0.99 \times \frac{r_{BPA}(m_1, m_2)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} + 0 \\
&\quad \times \frac{r_{BPA}(m_2, m_1)}{r_{BPA}(m_2, m_1) + r_{BPA}(m_1, m_2)} \\
&= 0.99 \times \frac{r_{BPA}(m_1, m_2)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} \\
&= 0.99 \times \frac{1.0202 \times 10^{-4}}{1.0202 \times 10^{-4} + 1.0202 \times 10^{-4}} \\
&= 0.495
\end{aligned}$$

$$\begin{aligned}
&\omega'_M(\{A_2\}) \\
&= \sum_{i=1}^{2} m_i(\{A_2\}) \frac{\sum_{j=1,j\neq i}^{2} r_{BPA}(m_i, m_j)}{\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2} r_{BPA}(m_i, m_j)} \\
&= m_1(\{A_2\}) \frac{r_{BPA}(m_1, m_2)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} + m_2(\{A_2\}) \\
&\quad \times \frac{r_{BPA}(m_2, m_1)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} \\
&= 0.01 \times \frac{r_{BPA}(m_1, m_2)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} + 0.01 \\
&\quad \times \frac{r_{BPA}(m_2, m_1)}{r_{BPA}(m_2, m_1) + r_{BPA}(m_1, m_2)} \\
&= 0.01 \times \frac{1.0202 \times 10^{-4}}{1.0202 \times 10^{-4} + 1.0202 \times 10^{-4}} + 0.01 \\
&\quad \times \frac{1.0202 \times 10^{-4}}{1.0202 \times 10^{-4} + 1.0202 \times 10^{-4}} \\
&= 0.01
\end{aligned}$$

$$\begin{aligned}
&\omega'_M(\{A_3\}) \\
&= \sum_{i=1}^{2} m_i(\{A_3\}) \frac{\sum_{j=1,j\neq i}^{2} r_{BPA}(m_i, m_j)}{\sum_{i=1}^{2}\sum_{j=1,j\neq i}^{2} r_{BPA}(m_i, m_j)} \\
&= m_1(\{A_3\}) \frac{r_{BPA}(m_1, m_2)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} + m_2(\{A_3\}) \\
&\quad \times \frac{r_{BPA}(m_2, m_1)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} \\
&= 0 \times \frac{r_{BPA}(m_1, m_2)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} + 0.99 \\
&\quad \times \frac{r_{BPA}(m_2, m_1)}{r_{BPA}(m_2, m_1) + r_{BPA}(m_1, m_2)} \\
&= 0.99 \times \frac{r_{BPA}(m_2, m_1)}{r_{BPA}(m_1, m_2) + r_{BPA}(m_2, m_1)} \\
&= 0.99 \times \frac{1.0202 \times 10^{-4}}{1.0202 \times 10^{-4} + 1.0202 \times 10^{-4}} \\
&= 0.495
\end{aligned}$$

Then, we get that $\omega'_M(\{A_1\}) = 0.495$, $\omega'_M(\{A_2\}) = 0.01$, and $\omega'_M(\{A_3\}) = 0.495$.

Step 4: Get the final fusion results. Through Eq.(30) and Eq.(31), the final fusion results of the two bodies of evidence are

$$\begin{aligned}
m_M(\{A_1\}) &= \omega'_M(\{A_1\}) = 0.495 \\
m_M(\{A_3\}) &= \omega'_M(\{A_3\}) = 0.495 \\
m_M(\{A_2\}) &= \upsilon_M \; m_{\oplus M}(\{A_2\}) = 0.01 \times 1 = 0.01
\end{aligned}$$

3) SUMMARY AND DISCUSSION

It can be seen from the calculation results of the above steps that the proposed method can resolve this conflict problem. In order to further verify the rationality of the proposed method, we calculated the fusion results of some existing methods for Zadeh's counter-example, as shown in Table 1.

$$\begin{aligned}
m_{\oplus\{m_1,m_2\}}(\{A_2\}) &= \frac{\sum_{A_i\cap A_j=A_2} m_1(A_i)m_2(A_j)}{1-\sum_{A_i\cap A_j=\emptyset} m_1(A_i)m_2(A_j)} \\
&= \frac{m_1(\{A_2\})\, m_2(\{A_2\})}{1-(m_1(\{A_1\}))\, m_2(\{A_2\} + m_1(\{A_1\})\, m_2(\{A_3\}) + m_1(\{A_2\})\, m_2(\{A_3\}))} \\
&= \frac{0.01 \times 0.01}{1-(0.99 \times 0.99 + 0.99 \times 0.01 + 0.01 \times 0.99)} = 1
\end{aligned}$$

TABLE 1. Comparison of fusion results of the Zadeh's counter-example.

Method	$m(A)$	$m(B)$	$m(C)$	Θ	$\emptyset$
Dempster [55]	0	1	0	-	-
Yager [36]	0	0.0001	0	0.9999	-
Smets et al. [37]	0	0.0001	0	-	0.9999
Sun et al. [65]	0.1821	0.0038	0.1821	0.6320	-
Murphy [41]	0.4950	0.0100	0.4590	-	-
Deng et al. [42]	0.4950	0.0100	0.4950	-	-
Ma et al. [54]	0.4950	0.0100	0.4950	-	-
Proposed method	0.4950	0.0100	0.4950	-	-

In the above results, the methods of Dempster, Yager, and Smets *et al.* obviously cannot effectively solve the conflict problem. The Sun *et al.*'s method allocated most of the support to the Θ, which did not help the final decision. We find that the proposed method is the same as that of Murphy, Deng *et al.*, and Ma *et al.*, which shows the rationality of the proposed method.

IV. NUMERICAL EXAMPLE

In this paper, by comparing with some existing methods, the superiority of the proposed method in conflict handling is illustrated.

A. EXAMPLE STATEMENT

Example 4: Suppose the discernment frame is $\Theta = \{A_1, A_2, A_3\}$. m_1, m_2, $\cdots$, m_5 represent five pieces of evidence defined on the discernment frame Θ, their BPAs are as follows:

$$m_1 : m_1(\{A_1\}) = 0.7, m_1(\{A_2\}) = 0.15, m_1(\{A_3\}) = 0.15$$
$$m_2 : m_2(\{A_2\}) = 0.5, m_2(\{A_3\}) = 0.5$$
$$m_3 : m_3(\{A_1\}) = 0.7, m_3(\{A_2\}) = 0.15, m_3(\{A_3\}) = 0.15$$
$$m_4 : m_4(\{A_1\}) = 0.7, m_4(\{A_2\}) = 0.15, m_4(\{A_3\}) = 0.15$$
$$m_5 : m_5(\{A_1, A_3\}) = 0.8, m_5(\{A_2\}) = 0.2$$

B. CALCULATE FUSION RESULTS BASED ON THE PROPOSED METHOD

We use the proposed rule of evidence combination to calculate the results of the fusion of five pieces of evidence. The specific calculation steps are shown below.

Step 1: Distinguish non-conflicting proposition elements. According to the definition of non-conflicting element set, we know that for $\{A_2\} \in F_1$ (F_1 represents the set of focus elements of m_1 and $m_1 \in M$), and for any $B \in F_i$ $(i = 2, \cdots, 5)$ and F_i represents the set of focus elements of m_i and $m_i \in M$), there are $\{A_2\} \cap B \neq \emptyset$. Therefore, we can get $\{A_2\} \in \kappa_M$. Similarly, we can get $\{A_3\}$ and $\{A_1, A_3\}$ also in κ_M. Finally, we can obtain that $\kappa_M = \{\{A_2\}, \{A_3\}, \{A_1, A_3\}\}$.

Step 2: Calculate evidence correlation coefficient matrix (CCM) the equation can be derived, as shown at the bottom of next page.

Step 3: Compute the weight of the focus elements. By using Eq.(28) and Eq.(29), the detailed calculation process of the element A_1 weight is shown below.

$$\begin{aligned}
\omega'_M(\{A_1\}) &= \sum_{i=1}^{5} m_i(\{A_1\}) \frac{\sum_{j=1,j\neq i}^{5} r_{BPA}(m_i, m_j)}{\sum_{i=1}^{5}\sum_{j=1,j\neq i}^{5} r_{BPA}(m_i, m_j)} \\
&= \frac{1}{\sum_{i=1}^{5}\sum_{j=1,j\neq i}^{5} r_{BPA}(m_i, m_j)} \\
&\quad \times \sum_{i=1}^{5} m_i(\{A_1\}) \sum_{j=1,j\neq i}^{5} r_{BPA}(m_i, m_j) \\
&= \frac{1}{12.4497}(m_1(\{A_1\}) \sum_{j=1,j\neq i}^{5} r_{BPA}(m_1, m_j) + m_2(\{A_1\}) \\
&\quad \times \sum_{j=1,j\neq i}^{5} r_{BPA}(m_2, m_j) \\
&\quad + m_3(\{A_1\}) \sum_{j=1,j\neq i}^{5} r_{BPA}(m_3, m_j) + m_4(\{A_1\}) \\
&\quad \times \sum_{j=1,j\neq i}^{5} r_{BPA}(m_4, m_j) \\
&\quad + m_5(\{A_1\}) \sum_{j=1,j\neq i}^{5} r_{BPA}(m_5, m_j)) \\
&= \frac{1}{12.4497} \times (0.7 \sum_{j=1,j\neq i}^{5} r_{BPA}(m_1, m_j) + 0 \\
&\quad \times \sum_{j=1,j\neq i}^{5} r_{BPA}(m_2, m_j) + 0.7 \\
&\quad \times \sum_{j=1,j\neq i}^{5} r_{BPA}(m_3, m_j) + 0.7 \\
&\quad \times \sum_{j=1,j\neq i}^{5} r_{BPA}(m_4, m_j) + 0 \\
&\quad \times \sum_{j=1,j\neq i}^{5} r_{BPA}(m_5, m_j)) \\
&= \frac{1}{12.4497} \times (0.7 \sum_{j=1,j\neq i}^{5} r_{BPA}(m_1, m_j) + 0.7
\end{aligned}$$

TABLE 2. Comparison results of some existing methods.

Method	$m(\{A_1\})$	$m(\{A_2\})$	$m(\{A_3\})$	$m(\{A_1, A_3\})$	Θ	$\emptyset$
Dempster [55]	0	0.2000	0.8000	-	-	-
Yager [36]	0	0.0003	0.0014	-	0.9983	-
Smets et al. [37]	0	0.0003	0.0014	-	-	0.9983
Jiang [44]	0.9708	0.0028	0.0257	0.0007	-	-
Murphy [41]	0.9175	0.0090	0.0721	0.0015	-	-
Deng et al. [42]	0.6830	0.0293	0.2797	0.0081	-	-
Ma et al. [54]	0.6107	0.0917	0.3667	-	-	-
Sun et al. [65]	0.4036	0.1329	0.1369	-	0.3266	-
Proposed method	0.4898	0.1020	0.4082	-	-	-

$$\times \sum_{j=1, j\neq i}^{5} r_{BPA}(m_3, m_j) + 0.7 \times \sum_{j=1, j\neq i}^{5} r_{BPA}(m_4, m_j))$$
$$= \frac{1}{12.4497} \times (0.7\times 2.9035 + 0.7 \times 2.9035 + 0.7\times 2.9035) = 0.4898$$

Similarly, by calculating, we get that $\omega'_M(\{A_2\}) = 0.1984$, $\omega'_M(\{A_3\}) = 0.1606$ and $\omega'_M(\{A_1, A_3\}) = 0.1512$.

Step 4: Get the final fusion results. Through Eq.(30) and Eq.(31), the final fusion results of the two bodies of evidence are

$$m_M(\{A_1\}) = \omega'_M(\{A_1\}) = 0.4898$$
$$m_M(\{A_2\}) = \upsilon_M \, m_{\oplus M}(\{A_2\}) = 0.5102 \times 0.2 = 0.1020$$
$$m_M(\{A_3\}) = \upsilon_M \, m_{\oplus M}(\{A_3\}) = 0.5102 \times 0.8 = 0.4082$$

C. ANALYSIS AND DISCUSSION

Comparison results of some existing methods are shown in Table 2. A closer comparison is shown in Fig.4.

Obviously, as can be seen from Table 2, the methods of Dempster, Yager and Smets *et al.* cannot handle conflict evidence. Hence, we mainly discuss the methods of Jiang, Murphy, Deng *et al.*, Sun *et al.*, and Ma *et al.*

As can be seen from Fig.4, the method of Sun *et al.* assigns most of the support to the unknown Θ, which does not help the final decision. Murphy's method and Jiang's method almost all the support is allocated to the proposition A_1. Among them, Murphy's support for the conflicting proposition A_1 is 0.9175; Jiang's support for the proposition is 0.9708. Compared with other scholars' methods, Murphy's method and Jiang's method have low support for non-conflicting set propositions A_2 and A_3. In other words,

$$CCM = \begin{bmatrix} r_{BPA}(m_1,m_1) & \cdots & r_{BPA}(m_1,m_3) & \cdots & r_{BPA}(m_1,m_5) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{BPA}(m_3,m_1) & \cdots & r_{BPA}(m_3,m_3) & \cdots & r_{BPA}(m_3,m_5) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{BPA}(m_5,m_1) & \cdots & r_{BPA}(m_5,m_3) & \cdots & r_{BPA}(m_5,m_5) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0.2900 & 1 & 1 & 0.2487 \\ 1 & 1 & 0.2900 & 0.2900 & 0.8575 \\ 1 & 0.2900 & 1 & 1 & 0.2487 \\ 1 & 0.2900 & 1 & 1 & 0.2487 \\ 0.2487 & 0.8575 & 0.2487 & 0.2487 & 1 \end{bmatrix}$$

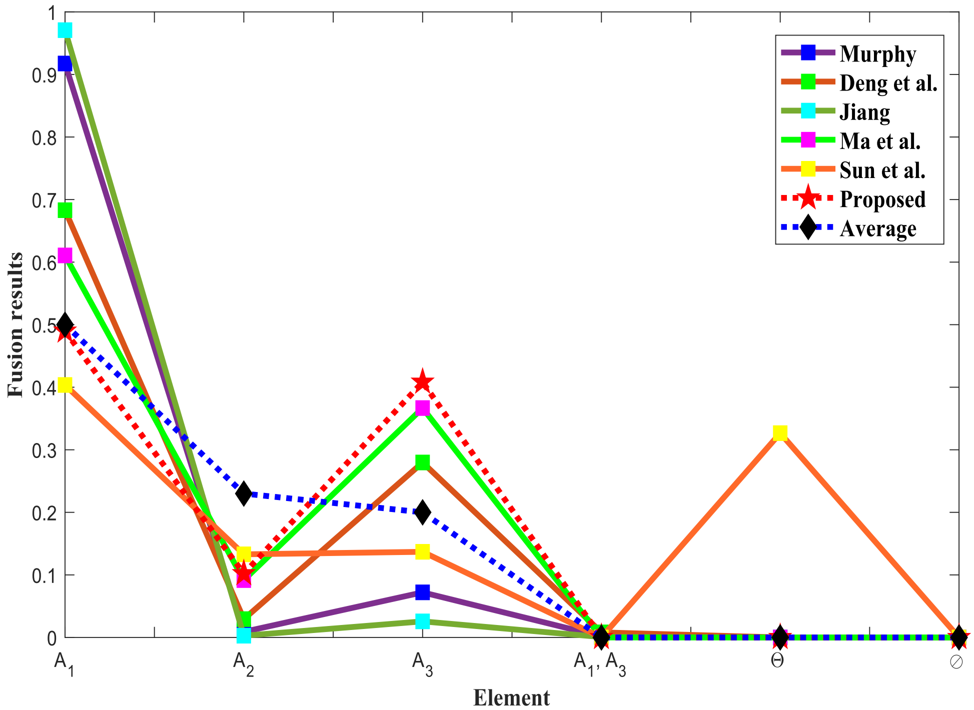

FIGURE 4. Comparison results with some methods.

TABLE 3. Comparison of variance.

Method	Variance
Sun et al. [65]	0.0358
Murphy [41]	0.0656
Deng et al. [42]	0.0185
Jiang [44]	0.0832
Ma et al. [54]	0.0136
Proposed method	0.0119

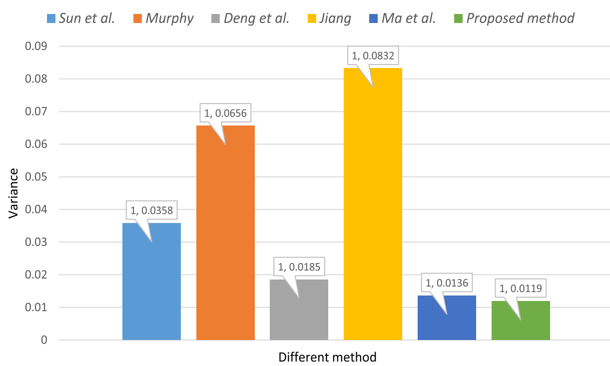

FIGURE 5. Comparison of variances of some methods.

there is very low support for the proposed non-conflicting set of elements in the proposed combination rule. This is not consistent with our intent to add focus to the fusion results. Deng *et al.*'s method weakened the support of the proposition A_1 and slightly strengthened the support of the proposition A_3, but the support of the non-conflicting proposition A_2 is still very low. Compared with other methods, Ma's method improves the support of the A_2 and A_3 propositions in non-conflict sets, which seems to be consistent with our purpose of enhancing the focus function.

To illustrate the problem, we calculate the variance between these methods and the mean, as shown in Table 3 and Fig.5. As can be seen from Fig.5, the variances of Jiang's and Murphy's methods are relatively large, and the proposed method has the smallest value. In other words, the proposed method has significantly increased support for elements in non-conflicting sets. This also shows that the proposed method is enhanced, after the conflict proposition is subdivided, the Dempster's combination rule part is used. That is, the focusing function is enhanced.

To sum up, when dealing with conflicts caused by multiple subset propositions, our method variance is more stable, which well reflects the focus of Dempster's combination rule. Therefore, our method is superior to the Ma *et al.*'s method.

V. AN APPLICATION EXAMPLE ON PATTERN RECOGNITION

In this section, an application example is used to further illustrate the superiority of the proposed method in conflict management.

Algorithm 1 Algorithm for Generating BPA Based on Interval Number

Input: Data corresponding to the three types of iris data sets

Output: Normalized BPA

1 Based on the maximum and minimum values of the set samples, the interval number model is constructed;

2 Use Eq.(33) to compute the distance between unidentified sample attribute value and interval number;

3 Use Eq.(32) to calculate the similarity between undetermined sample attribute values and the number of intervals;

4 Normalize similarity and generate BPA.

A. APPLICATION BACKGROUND

First, a brief introduction to the iris data set is necessary. The iris data set is a classic dataset that is often used as an example in the fields of statistical learning and machine learning. The data set contains 150 records in three categories, each with 50 pieces of data. Each record has four attribute characteristics: Sepal Length (SL), Sepal Width (SW), Petal Length (PL), and Petal Width (PW). These four attribute characteristics can be used to predict whether the iris flower belongs to Iris-Setosa, Iris-Versicolour and Iris-Virginica.

How to use the iris data to generate a usable BPA is an important topic. In order to solve this problem, many methods have been discussed in current research [66]. In this paper, we use the interval model proposed by Kang *et al.* [67] to obtain BPAs, and further discuss the role of the proposed method. Some brief introductions to interval models are as follows.

For two arbitrary interval numbers, $M = [m_1, m_2]$ and $N = [n_1, n_2]$. The degree of similarity $Sim(M, N)$ between them is denoted as follows

$$Sim(M, N) = \frac{1}{1 + \alpha D(M, N)} \tag{32}$$

where α is the support coefficient, which is greater than 0. $D(M, N)$ is the distance between interval number M and N [68]. The definition of this distance function $D(\cdot)$ is as follows (33), as shown at the bottom of the next page.

Based on the above interval number, the algorithm flow for forming BPA is described below.

B. EXPERIMENT PROCEDURE

The general idea of experimental design can be divided into two stages. Stage one, based on the basic probability assignments generated by the iris data set, we calculate the recognition rate of the proposed method for iris. Stage two, based on stage one, considering the interference environment, that is, the basic probability allocation of the conflict, the recognition results of this method are obtained.

1) STAGE ONE: FOR NORMAL DATA SAMPLES

Step 1: Randomly select 120 samples from the iris data set, of which 40 are selected for each species, and use the minimum and maximum values of the obtained samples to construct interval number models, as shown in Table 4.

Step 2: For the remaining 30 samples, the remaining number of each category is 10, treat them as test samples of unknown category (here suppose the selected sample data is $\langle 4.5, 2.3, 1.3, 0.3\rangle$).

Step 3: Based on the application background mentioned in this article, the similarity of each interval number is obtained by using Eq.(32) and Eq.(33), where the support factor α is set to 5. Then, we can get the BPAs of each attribute, as shown in Table 5.

Step 4: For the four attributes, construct four evidences, and then fuse through the proposed combination formula (i.e., Eq.(30)), and finally obtain the final fusion result in Table 6.

Step 5: The category of unknown samples is determined by the result of fusion. That is, which BPA value is the largest, then its corresponding category is the category of the unknown sample.

In consideration of scale of samples, we test all 150 data sets according to similar steps as above. Through experimental statistics, the overall recognition rate is 96%, of which the recognition rate of Setosa iris is 100%, the recognition rate of Versicolour is 98%, and the recognition rate of Virginica is 90%. Under the same conditions, the comparison results between this method and Kang *et al.* are shown in Table 7.

TABLE 4. Number of intervals counted by the sample.

Category	Attribute characteristic			
	SL	SW	PL	PW
Setosa (Se)	[4.4, 5.8]	[2.3, 4.4]	[1.0, 1.9]	[0.1, 0.6]
Versicolour (Ve)	[4.9, 7.0]	[2.0, 3.4]	[3.0, 5.1]	[1.0, 1.7]
Virginica (Vi)	[4.9, 7.9]	[2.2, 3.8]	[4.5, 6.9]	[1.4, 2.5]

TABLE 5. BPA for each iris category.

BPA	Attribute characteristic			
	SL	SW	PL	PW
$m(Se)$	0.200	0.094	0.710	0.585
$m(Ve)$	0.104	0.171	0.118	0.163
$m(Vi)$	0.081	0.128	0.076	0.110
$m(Se, Ve)$	0.170	0.159	0	0
$m(Se, Vi)$	0.170	0.125	0	0
$m(Ve, Vi)$	0.104	0.164	0.096	0.142
$m(Se, Ve, Vi)$	0.170	0.159	0	0

TABLE 6. The final fusion result of BPAs.

$m(Se)$	$m(Ve)$	$m(Vi)$	$m(Se, Ve)$	$m(Se, Vi)$	$m(Ve, Vi)$	$m(Se, Ve, Vi)$
0.818	0.115	0.062	0	0	0	0.006

TABLE 7. Final fusion result of BPAs (Recognition rate of iris).

Method	Setosa	Versicolor	Virginica	Global
Kang et al. [67]	100%	98%	90%	96%
Proposed	100%	98%	90%	96%

TABLE 8. Fusion results of BPAs in extreme cases (Recognition rate of iris).

Method	Setosa	Versicolor	Virginica	Global
Kang et al. [67]	87.5%	99%	97%	94.5%
Proposed	100%	99%	98%	99%

We can find that the proposed method and Kang *et al.*'s method have the same recognition rate for the three categories of iris. In other words, both methods can correctly identify the corresponding category of iris. It can be seen from Eq.(30) and Eq.(31) that when the elements in the non-conflicting element set are all power set elements, the combination rule at this time degenerates into the Dempster combination rule. This also illustrates the generality of the proposed method when dealing with non-conflicting evidence.

2) STAGE TWO: FOR CONFLICTING DATA SAMPLES

Based on the example in stage one, we consider the following scenario. The information collected by the sensor responsible for the Petal Length (PL) of the iris is as follows.

$$PL : m(Se) = x, m(Ve) = 0.828 - x, m(Vi) = 0.076,$$
$$m(Ve, Vi) = 0.096$$

where x is an unknown term.

It is assumed that under certain extreme conditions, due to the effects of severe weather, the sensor does not support the iris of the Setosa (Se) category, which is $x = 0$. Then, for 40 iris samples in this category, randomly generate 5 $x = 0$ experimental data for PL. Based on this, we use the proposed method and the Kang *et al.*'s method to calculate the recognition rate of the iris according to the method steps mentioned in stage one. Finally, through experimental statistics, the results are shown in the Table 8.

We found that, overall, the Kang *et al.*'s method has a 94.5% recognition rate for the three categories of iris, and our method is 99%. More specifically, we look for the data with large differences in the recognition rate of the iris categories mentioned above, and we found that the Kang *et al.*'s method has a recognition rate of only 87.5% for iris in the Se category, and our method is 100%. Below we analyze the reasons for this results. Consider one iris in 5 $x = 0$ iris samples, where the support for the Se by the attribute PL (i.e., BPA) is 0. Compared with the other three attributes (i.e., SL, SW, and PW), this attribute PL obviously conflicts with the support of the Se, because these attributes have higher support for the iris of the Se. After using the Dempster combination rule to fuse these attribute data (i.e., BPA of SL, SW, PL, and PW), the obtained results support 0 for iris in the Se category. The reason for this phenomenon is that the Dempster combination rule in this method cannot handle conflicting data. Therefore, for these five interference sample data, the recognition rate of Se is $(40 - 5)/40 = 87.5\%$. In contrast, the proposed method avoids the above problems well. In the specific calculation process, this non-conflicting element set is used to distinguish conflicting propositions. In this application, Se is distinguished. Based on this, the evidence is effectively fused by combining element weights and then using the proposed combination rule. Obviously, by comparison, the proposed method is more superior under extreme conditions.

C. SUMMARY

In summary, the experiments in stage one show that when the proposed method is merging non-conflicting evidence, the proposed combination formula of evidence is degenerate to the Dempster combination rule. For a given iris sample data set, the recognition rate is high. Based on experimental stage one, the experiment in stage two further considered the influence of extreme environment on the identification of iris recognition rate. Under simulated harsh environment, that is to say, if there is conflict on the given iris sample data. At this time, the proposed combination formula can obtain effective fusion results by modifying the BPA method, and the experimental results further verify the effectiveness of the proposed method. Therefore, the above experiments show that the proposed method has strong robustness and superiority in practical applications.

VI. CONCLUSION

D-S evidence theory is an effective tool for the extraction and modeling of uncertain information. However, when evidence conflicts, how to effectively fuse multi-source data is a topic worthy of more and more scholars' extensive discussion.

Through analysis, most of the existing researches on conflicts of evidence stay on the idea of modifying data sources and combination formulas. However, how to distinguish conflicts more accurately has received little attention from

$$\begin{aligned} D(M,N) &= \sqrt{\int_{-1/2}^{1/2}\left\{\left[\frac{(m_1+m_2)}{2}+x(m_2-m_1)\right]-\left[\frac{(n_1+n_2)}{2}+x(n_2-n_1)\right]\right\}^2 dx} \\ &= \left[\frac{(m_1+m_2)}{2}-\frac{(n_1+n_2)}{2}\right]^2+\frac{[(m_2-m_1)+(n_2-n_1)]^2}{12} \end{aligned} \tag{33}$$

scholars. The idea of a complete conflict set proposed by Ma *et al.* greatly enriches this theoretical flaw. However, this paper finds the shortcomings of Ma *et al.*'s method through analysis. Based on the set of non-conflicting elements, a new conflict evidence management method is proposed. In summary, the main contributions of this paper are not as follows. First, the proposed modified conflict evidence combination method improves the deficiencies in Ma *et al.*'s method, further strengthens the support of the weak proposition fusion, and numerical examples also illustrate the feasibility of the proposed method. Second, from the perspective of non-conflicting element sets, this method provides a theoretical approach to the management of conflicts in D-S evidence theory. Third, the validity and superiority of the proposed method are further verified through the application of pattern recognition in the real world. In addition, although this paper proposes a theoretical method for conflict resolution, the reliability and usability of the method require further experiments and applications to verify.

In further research, we will continue to enrich the theoretical connotation of this method. In addition, we hope to extend the proposed method to suit more and more complex applications, such as fault diagnosis, data classification, and so on.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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