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Support Vector Regression-Based Active Subspace (SVR-AS) Modeling of High-Speed Links for Fast and Accurate Sensitivity Analysis

HANZHI MA[1], (Graduate Student Member, IEEE), ER-PING LI[1], (Fellow, IEEE), ANDREAS C. CANGELLARIS[2], (Life Fellow, IEEE), AND XU CHEN[2], (Member, IEEE)

[1]College of Information Science and Electronics Engineering, Zhejiang University–University of Illinois at Urbana–Champaign (ZJU–UIUC) Institute, Zhejiang University, Hangzhou 310027, China

[2]Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign, Urbana, IL 61820, USA

Corresponding author: Er-Ping Li (liep@zju.edu.cn)

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ABSTRACT A methodology based on the joint usage of support vector regression and active subspace is introduced in this paper for accelerated sensitivity analysis of high-speed links through parameter space dimensionality reduction. The proposed methodology uses the gradient directly obtained by support vector regression with Gaussian kernel to generate an active subspace with its application to the high-speed link model. Active subspace generated by this method is defined by the directions that are most influential on the desirable output measure. The resulting reduced-dimensional model is shown to perform well in sensitivity analysis of high-speed links including IBIS-AMI equalization, and is computationally more efficient than Sobol's method.

INDEX TERMS High-speed link, support vector regression, active subspace, sensitivity analysis, dimensionality reduction, eye diagram, surrogate modeling.

I. INTRODUCTION

Efficient methodologies for sensitivity analysis and design optimization of high-speed channels is a current topic of significant interest to the electronic design automation community [1]–[3]. More specifically, the task at hand is the advancement of reliable and computationally efficient methodologies for the investigation of the impact of the multitude of input design parameters on the performance of the link. These methodologies aims to reduce the expedient iteration toward an optimized design that meets design specs while mitigating or minimizing overdesign driven by worst-case assumptions. Toward this objective, the research community has been investigating approaches for fast sensitivity analysis and dimensionality reduction techniques aimed at identifying the subset of input design parameters most influential on the channel's performance. Once such a subset has been identified, reduced-order, surrogate models of the channel can be developed to accelerate design optimization.

The variance-based Sobol's method [4], [5] is a popular sensitivity analysis method widely used in many fields, such as environmental modeling [6], reliability engineering [7], and electronic engineering [2]. The objective of Sobol's method is to quantify the influence each input parameter has on the variance of the model output; thus, its results infer the most influential input parameters to be considered in the development of reduced space of input parameters to be used for design optimization studies. However, Sobol's method using Monte Carlo integral is computationally expensive, especially when the dimensionality of the space of input parameters is high.

Polynomial Chaos (PC) approaches have been proposed for the development of uncertainty quantification in the framework of high-speed links [8]–[10], reducing the computational cost of sensitivity analysis compared with Monte Carlo simulation [2]. Bayesian Active Learning has been proposed for the development of an accurate predictive

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probabilistic model to perform sensitivity analysis of an industrial high-speed channel [3]. Principal component analysis (PCA) method [11] and the Partial Least Squares (PLS) regression method [12] have been shown to be useful in identifying the principal components to be used in the development of a reduced input parameter space. Recently, active subspace method [13] have been proposed as an alternative way for dimensionality reduction and sensitivity analysis; however, the gradient information between input and output parameters required in this method is often difficult to obtain. This is the case in its application to high-speed links.

To resolve this problem, we propose a new algorithm, named Support Vector Regression based Active Subspace (SVR-AS). In the context of high-speed link design, rather than seeking the probability distribution of the output of interest [14], this methodology uses the support vector regression (SVR) [15] predictive model to construct a functional relationship between design parameters and eye-opening parameters [16], [17], the gradient of which can then be employed to calculate the active subspace and its corresponding active variables. Developed from the SVR predictive model, SVR-AS provides an alternative predictive function between active variables and model outputs including dimensionality reduction. The performance of this new methodology is examined through its application to the sensitivity analysis of a high-speed link including IBIS-AMI Tx/Rx models with equalization, and the performance and results are compared to Sobol's method. SVR-AS is found to exhibit good accuracy for sensitivity analysis with better computation efficiency compared with Sobol's method.

This paper is organized as follows. In Section II, the SVR-AS method is presented. Section III presents the details of the interconnect structure under consideration, namely, a high-speed link with IBIS-AMI model. In Section IV, the results from the application of SVR-AS to the sensitivity analysis of the high-speed link are presented. The performance of the proposed method and its comparison to other approaches are discussed in Section V. The paper concludes in Section VI with a summary of its contributions and an outlook on future developments.

II. METHODOLOGY

This work focuses on the development of a stable and accurate algorithm for sensitivity analysis and dimensionality reduction in a realistic high-speed link model with many design parameters. Inspired by active subspace [13], we propose a methodology that combines SVR and active subspace. We evaluate the accuracy and efficiency of our proposed method against Sobol's variance-based sensitivity indices and the predictive model generated by support vector machine (SVM) and SVR without dimensionality reduction.

Let $\boldsymbol{X} = \left\{x_1, \ldots, x_i, \ldots, x_p\right\}$ represent the input p-dimensional normalized design parameter space. Let Y denote the output of the model. The sampling data set is expressed as $D = \{(\boldsymbol{X_1}, Y_1), \ldots (\boldsymbol{X_n}, X_n)\}$.

A. SUPPORT VECTOR MACHINE AND SUPPORT VECTOR REGRESSION

SVM [18] is a kind of supervised learning algorithm for classification problem with generalized linear classifier, where Kernel method can be added for non-linear classification. When SVM is applied to solve the problem of regression prediction of a continuous function, it is also called SVR [15].

SVM is aimed at solving a convex optimization problem:

$$\begin{aligned} &\min_{\boldsymbol{w},b,\boldsymbol{\xi}} \frac{1}{2}\|\boldsymbol{w}\| + C\sum_{1\le i\le n}\xi_i, \\ &\text{s.t. } Y_i\left(\boldsymbol{w}^{\mathrm{T}}\varphi(\boldsymbol{X}_i)+b\right) \ge 1-\xi_i, \\ &\qquad \xi_i \ge 0,\ i = 1,2,\ldots,n, \end{aligned} \tag{1}$$

where $\boldsymbol{X}_i$ is a sampling of input design parameters, $\varphi(\boldsymbol{X}_i)$ is defined as the mapped $\boldsymbol{X}_i$ in higher-dimensional feature space, $\boldsymbol{w} = \left\{w_1, \ldots, w_p\right\}$ is a normal vector of hyperplane $\boldsymbol{w}^{\mathrm{T}}\varphi(\boldsymbol{X}_i) + b$, b represents displacement, $\boldsymbol{\xi} = \{\xi_1, \ldots, \xi_n\}$ is introduced as slack variable, and C is a positive constant.

Lagrange multiplier method is a common method to solve convex optimization problem with constraints. After deducing the Lagrange function, dual problem and Karush-Kuhn-Tucker (KKT) conditions of Eq. (1), it is easy to get the SVM model function:

$$\begin{aligned} g(\boldsymbol{X}) &= \operatorname{sgn}\left(\boldsymbol{w}^{\mathrm{T}}\varphi(\boldsymbol{X})+b\right) \\ &= \operatorname{sgn}\left(\sum_{1\le i\le n}\alpha_i Y_i \kappa(\boldsymbol{X},\boldsymbol{X}_i)+b\right), \end{aligned} \tag{2}$$

where $\kappa\left(\boldsymbol{X}_i, \boldsymbol{X}_j\right) = \varphi(\boldsymbol{X}_i)^{\mathrm{T}}\varphi\left(\boldsymbol{X}_j\right)$ is defined as the kernel function to calculate the inner product of $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ in the higher-dimensional feature space, and $\alpha_i \ge 0$ is Lagrange multiplier calculated by sampling data.

In regression prediction tasks, we want to find a regression model, expressed by $h(\boldsymbol{X}) = \boldsymbol{w}^{\mathrm{T}}\varphi(\boldsymbol{X}) + b$, such that $h(\boldsymbol{X})$ is as close as possible to Y. Thus, the SVR problem can be regarded as

$$\begin{aligned} &\min_{\boldsymbol{w},b,\boldsymbol{\xi},\boldsymbol{\xi}'} \frac{1}{2}\|\boldsymbol{w}\| + C\sum_{1\le i\le n}\left(\xi_i+\xi_i'\right), \\ &\text{s.t. } h(\boldsymbol{X}_i) - Y_i \le \varepsilon + \xi_i, \\ &\qquad Y_i - h(\boldsymbol{X}_i) \le \varepsilon + \xi_i', \\ &\qquad \xi_i \ge 0,\ \xi_i' \ge 0, \quad i = 1,2,\ldots,n, \end{aligned} \tag{3}$$

where ξ_i and ξ_i' are slack variables, and SVR allows a margin of tolerance ε. Similarly, the final SVR predictive function can be calculated via a Lagrange multiplier method:

$$h(\boldsymbol{X}) = \sum_{1\le i\le n}\left(\alpha_i' - \alpha_i\right)\kappa(\boldsymbol{X},\boldsymbol{X}_i) + b, \tag{4}$$

where $\alpha_i \ge 0$ and $\alpha_i' \ge 0$ are introduced as Lagrange multipliers.

B. ACTIVE SUBSPACE METHOD

An alternative approach, called active subspace, rather than determining a subset of the inputs as important, identifies a set of important directions in the space of all input parameters for sensitivity analysis and dimensionality reduction [13]. The model output, Y, is more influenced by the perturbation on input parameters along these directions rather than its orthogonal directions [19].

Consider a generic, multivariate function, $Y = F(\boldsymbol{X})$, where normalized design parameters are from a uniform density over the hypercube $\boldsymbol{X} \in [-1, 1]^p$ and F is an abstract representation of the map from normalized inputs to the predictions.

Denote the gradient of F as a column vector

$$\nabla_{\boldsymbol{X}} F(\boldsymbol{X}) = \left[\frac{\partial F}{\partial x_1}, \frac{\partial F}{\partial x_2}, \ldots, \frac{\partial F}{\partial x_p}\right]^{\mathrm{T}}. \tag{5}$$

A symmetric and positively semi-definite matrix $\boldsymbol{Z}$ is defined when draw sampling $\boldsymbol{X}_i$ independently,

$$\boldsymbol{Z} = \frac{1}{n} \sum_{1 \le i \le n} \nabla_{\boldsymbol{X}} F(\boldsymbol{X}_i) \left(\nabla_{\boldsymbol{X}} F(\boldsymbol{X}_i)\right)^{\mathrm{T}}, \tag{6}$$

which admits a real eigenvalue decomposition

$$\boldsymbol{Z} = \boldsymbol{W} \boldsymbol{\Lambda} \boldsymbol{W}^{\mathrm{T}}, \tag{7}$$

where $\boldsymbol{\Lambda} = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p)$, $\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_p \ge 0$. We can partition the eigenvectors as

$$\boldsymbol{W} = [\boldsymbol{W}_1 \ \boldsymbol{W}_2], \quad \boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_1 & \\ & \boldsymbol{\Lambda}_2 \end{bmatrix}, \tag{8}$$

where $\boldsymbol{W}_1$ contains the first q eigenvectors with the largest eigenvalues. The subspace defined by $\boldsymbol{W}_1$ is called active subspace and represents the important directions along which perturbation in the input have the largest impact on the output. The magnitude of the eigenvector components can be regarded as the relative sensitivity index of the input parameters [19]. Then $\boldsymbol{y} = \boldsymbol{W}_1^{\mathrm{T}} \boldsymbol{X}$ denotes the vector of the active variables, which provides a lower-dimensional representation of the input parameters. Active subspace is based on the premise that it is reasonable to approximate $F(\boldsymbol{X})$ by a function of $q < p$ combinations of $\boldsymbol{X}$, represented by the vector of the generated active variables, when the eigenvalues in $\boldsymbol{\Lambda}_2$ are much smaller than those in $\boldsymbol{\Lambda}_1$.

C. SUPPORT VECTOR REGRESSION BASED ACTIVE SUBSPACE

The gradient of function F plays an important and necessary role in the active subspace algorithm. In [13], active subspace is realized by either local or global linear approximation of the relationship between output and inputs, which allows the gradient ∇F to be estimated. This approach presents two challenges in its application to high-speed link models. First, additional EM modeling and channel simulations need to be performed in order to construct the linear estimator between design parameters and eye-opening parameters. Second, prior results show that the eye opening estimation problem is best solved using Gaussian kernel rather than linear regression [16]. Linear regression model performs relatively poor in eye opening prediction of high-speed link models, going against the accurate active subspace identification. Thus, we choose to use a predictive function from trained SVR with Gaussian kernel to replace the unknown relationship between input parameters and output and to implement active subspace.

With the Gaussian kernel given by

$$\kappa(\boldsymbol{X}, \boldsymbol{X}_i) = \exp\left(-\frac{1}{2\sigma^2} \|\boldsymbol{X} - \boldsymbol{X}_i\|^2\right), \tag{9}$$

the SVR predictive model shown in Eq. (4) becomes

$$h(\boldsymbol{X}) = \sum_{1 \le i \le n} \left(\alpha_i' - \alpha_i\right) \exp\left(-\frac{1}{2\sigma^2} \|\boldsymbol{X} - \boldsymbol{X}_i\|^2\right) + b, \tag{10}$$

where $\sigma > 0$ is the width of Gaussian kernel. The partial derivatives of $h(\boldsymbol{X})$ with respect to each input parameter x_k, $k = 1, \ldots, p$ can be deduced as

$$\frac{\partial h(\boldsymbol{X})}{\partial x_k} = \sum_{1 \le i \le n} \left(\alpha_i' - \alpha_i\right) \left(-\frac{1}{\sigma^2}(x_k - x_{ik})\right) \exp\left(-\frac{1}{2\sigma^2} \sum_{1 \le j \le p} \left(x_j - x_{ij}\right)^2\right) \tag{11}$$

Thus, Eq. (11) can be further used in Eq. (5) to calculate the gradient of function $F(\boldsymbol{X})$. With that in mind, it is easy to calculate matrix $\boldsymbol{Z}$ and obtain the eigenvectors and active variables from SVR-AS. Fig.1 illustrates the workflow of SVR-AS method. In this way, the surrogate model and active subspace identification come from the same data sets. Specifically, the active subspace is identified for free after we have constructed the SVR model with Gaussian kernel. It is worth mentioning that the proposed SVR-AS method can be applied to not only Gaussian kernel but also any other kernels that are differentiable in closed-form.

III. APPLICATION EXAMPLE

We consider the chip-to-chip, high-speed serial link model depicted in Fig. 2. It consists of the transmitter block (Tx, IBIS-AMI model and package included), a microstrip line, a land grid array (LGA) model [20], a via model, a strip line, the cascade of the aforementioned via model, LGA, and microstrip line forming a path to the receiver, and, finally, the receiver block (Rx, IBIS-AMI model and package included).

A. GEOMETRY AND DESIGN PARAMETERS

In the following we describe the seven blocks cascaded in series and forming the entire interconnect channel between Tx and Rx. This work focuses on uncertainties in the design parameters of the Tx microstrip line, the strip line and the Rx microstrip line. The via and LGA models have fixed design parameters.

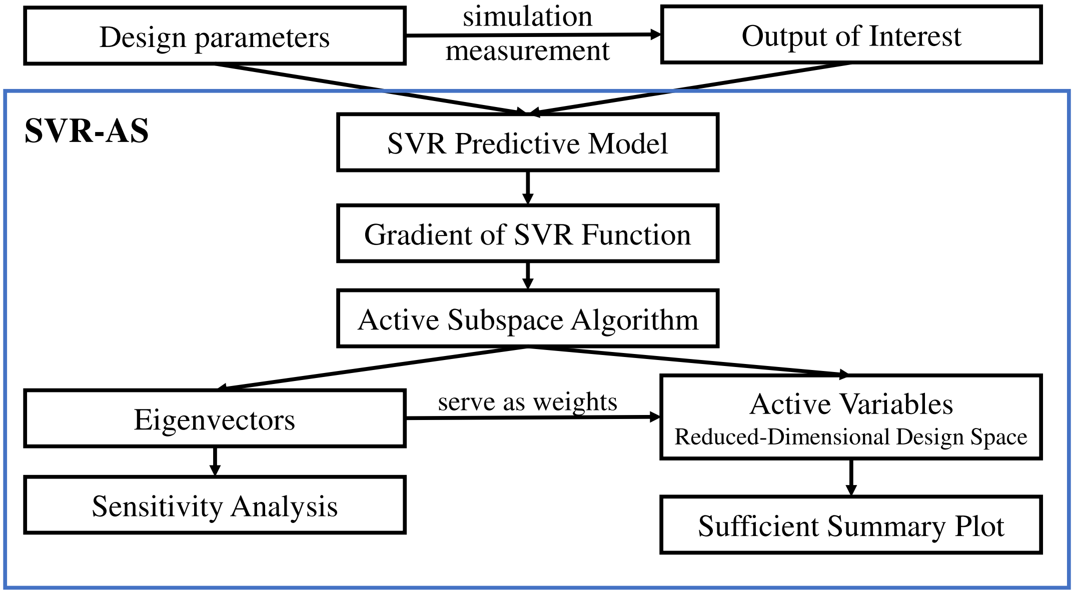

FIGURE 1. The workflow of SVR-AS method.

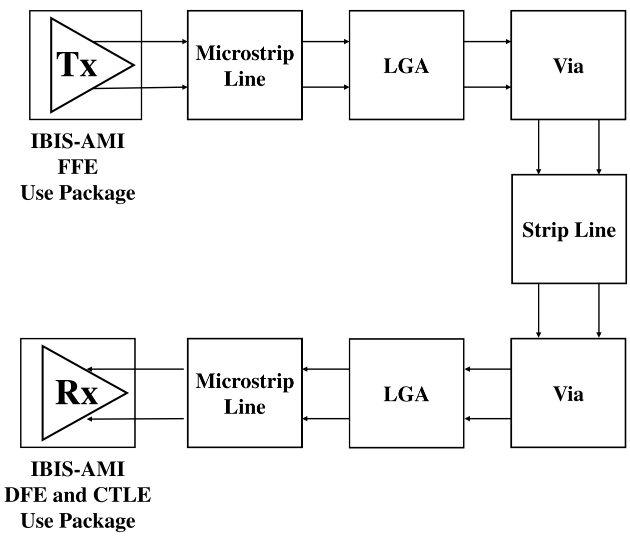

FIGURE 2. The structure of the entire high-speed link.

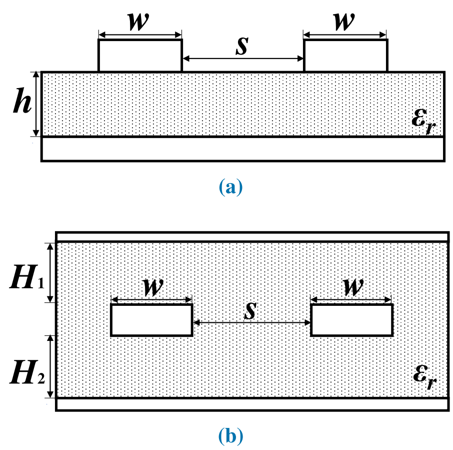

FIGURE 3. The cross-sectional geometries of the microstrip line and strip line: (a) microstrip line; (b) strip line.

Fig. 3a and 3b depict the cross-sectional geometries of the differential microstrip line and strip line, respectively. Table 1 summarizes the information of 16 geometric design parameters associated with the lines. Substrate relative permittivity, ε_r, channel width, w, channel spacing, s, substrate thickness, h, and channel length, l, are considered in the Tx microstrip line, the strip line, and the Rx microstrip line. For the strip line, substrate thickness is characterized by two parameters, H_1 and H_2, as shown in Fig. 3b. The ranges of values of these 16 design parameters is shown in Table 1. These 16 parameters constitute an input parameter space of relatively high dimensionality such that a brute-force parameter sweep for design optimization is intractable. For a more expedient analysis, identification of the most influential design parameters and, leveraging dimensionality reduction, the associated development of a reduced-order model is the approach considered in this paper.

B. SIMULATION WITH EQUALIZATION

The output of interest is the eye opening of the channel. It is obtained through a statistical high-speed link analysis using commercial IBIS-AMI behavioral I/O models for the transmitter and the receiver, including effects of transmitter feed forward equalization (FFE), receiver continuous time linear equalization (CTLE), and receiver decision-feedback equalization (DFE).

We use ANSYS Q3D Extractor [21] and Keysight ADS [22] for the high-speed link simulation needs of this paper.

TABLE 1. Design parameters of the high-speed link.

	Parameter	Nominal value	Range
Tx Microstrip Line	ε_r	4.4	3.6-5.2
	w	5 mil	4.5-5.5 mil
	s	8 mil	7.2-8.8 mil
	h	4 mil	3.7-4.3 mil
	l	10 mm	2-20 mm
Strip Line	ε_r	4.4	3.6-5.2
	w	5 mil	4.5-5.5 mil
	s	8 mil	7.2-8.8 mil
	H_1	8 mil	7.2-8.8 mil
	H_2	8 mil	7.2-8.8 mil
	l	200 mm	50-500 mm
Rx Microstrip Line	ε_r	4.4	3.6-5.2
	w	5 mil	4.5-5.5 mil
	s	8 mil	7.2-8.8 mil
	h	4 mil	3.7-4.3 mil
	l	10 mm	2-20 mm

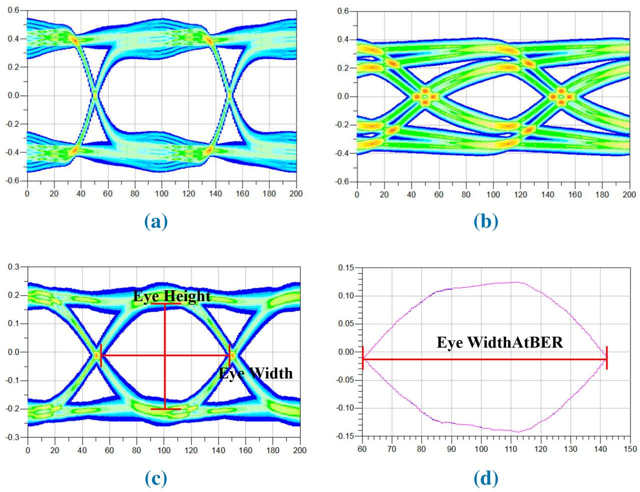

FIGURE 4. Eye openings of the high-speed link: (a) Tx output signal; (b) Rx input signal; (c) Rx output signal; (d) Rx output signal at 10^{-12} BER.

ANSYS Q3D Extractor is used to calculate the S-parameters for the Tx microstrip line, the strip line, and the Rx microstrip line. The calculated S-parameters, combined with the fixed S-parameters of the via model and the LGA, are used in Keysight ADS to calculate the eye opening. On the computer used for our simulations, it takes approximately 75 seconds for each Q3D extraction and 4 seconds for each ADS simulation using Intel(R) Xeon(R) CPU X5650 @ 2.67 GHz with 12 cores.

Fig. 4 illustrates the effects of equalization when the microstrip lines and strip line are designed using the nominal values in Table 1. Eye diagrams of Tx output signal, Rx input signal and Rx output signal are depicted in Fig. 4a, 4b and 4c, respectively. We consider three eye opening measures of the Rx output signal, namely, Eye Height, Eye Width, and Eye WidthAtBER, as shown in Fig. 4c and 4d. Eye Height is the distance between the 3σ points of the logic-1 and logic-0 histograms, measured across the eye level boundary. Eye Width is the distance between the 3-sigma points of the crossing time histograms. Eye WidthAtBER is the maximum width of contour at 10^{-12} bit error rate (BER).

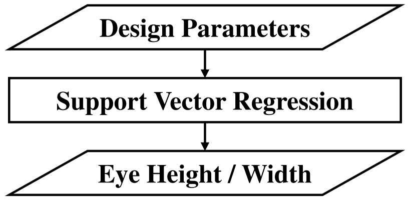

FIGURE 5. The workflow of eye height and eye width prediction.

IV. NUMERICAL RESULTS

A. PREDICTIVE MODEL BY SVM AND SVR

SVM and SVR are utilized as a surrogate predictive model to find the relationship between the 16 design parameters and the Eye Height, Eye Width, Eye WidthAtBER of Rx output signal, respectively. Fig. 5 depicts the workflow of Eye Height and Eye Width prediction, in which only SVR method is used.

We note that some of the designs resulting from values of the design parameters within the ranges indicated in Table 1 result in extremely poor output with some of them having no eye opening at 10^{-12} BER. These cases are not included in the data sets used for the development of the SVR predictive model. A more complex workflow is established, depicted in Fig. 6, for Eye WidthAtBER prediction as follows. SVM model is first used to examine whether an eye opening exists at 10^{-12} BER. If there is no eye opening for the specific set of design parameters, Eye WidthAtBER is judged to be zero. Otherwise, the design parameters are fed into another SVR model to find the exact value of Eye WidthAtBER.

We use 2000 data sets generated by ANSYS Q3D and Keysight ADS to explore the SVM and SVR model (both Fig. 5 and Fig. 6) for eye opening prediction. Predictive models are implemented by the MATLAB machine learning toolbox with Bayesian hyperparameter optimization. These 2000 data sets are randomly generated through sampling the uniform distributions of the design parameters. They are divided into one group of 1500 sets used for training and another group of 500 sets to be used for testing. Among these 1500 training sets, 1437 include Eye WidthAtBER data. Thus, the SVR model in Fig. 6 uses these 1437 sets for model training.

The prediction results with respect to Eye Height, Eye Width and Eye WidthAtBER are presented in Fig. 7. The root mean square errors (RMSE) between simulation results and prediction results are 0.01 V, 1.09×10^{-12} sec and 3.84×10^{-12} sec with respect to Eye Height, Eye Width and Eye WidthAtBER. The mean absolute errors are 0.0085 V, 8×10^{-13} sec and 1.87×10^{-12} sec, respectively. It can be inferred that these three SVR model and one SVM model are trained well and can successfully help predict eye opening based on the design parameters within the design range.

B. EIGENVECTOR AND ACTIVE VARIABLE BY SVR-AS

A well-trained SVR provides a certain relationship between design parameters and eye opening. As mentioned in

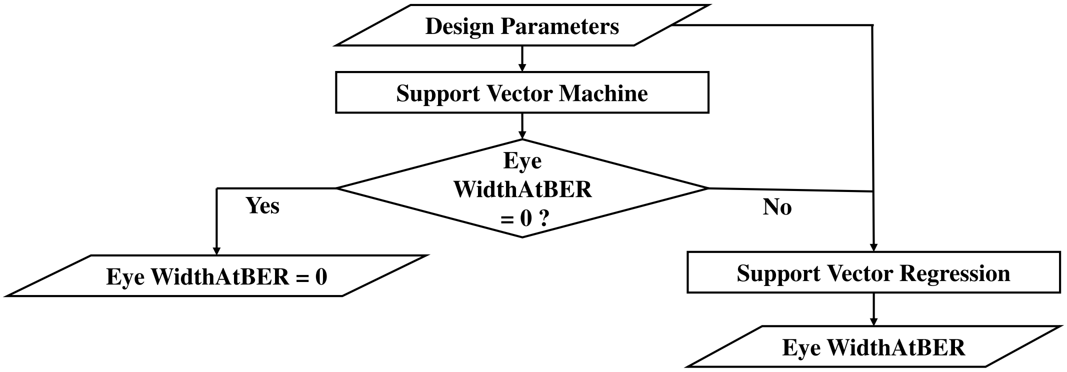

FIGURE 6. The workflow of eye WidthAtBER prediction.

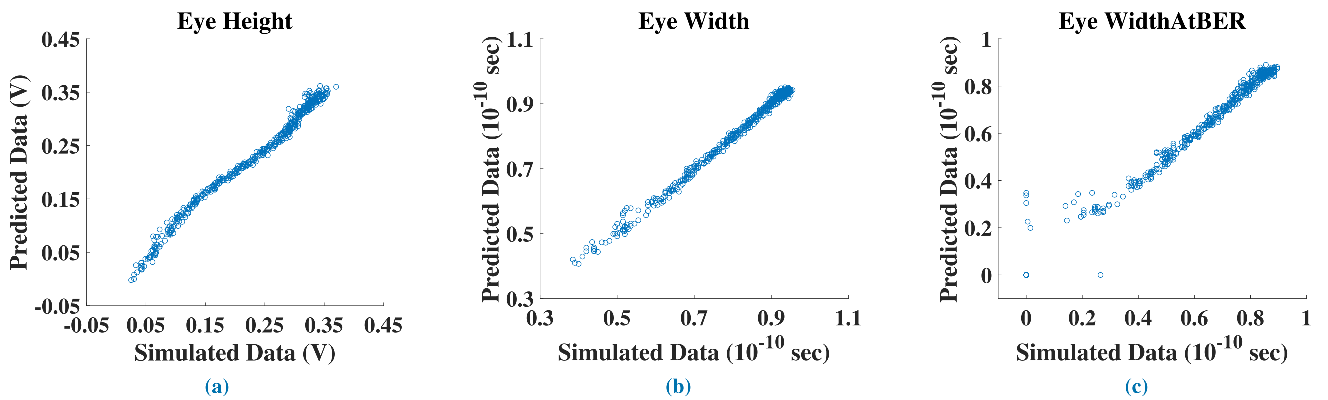

FIGURE 7. The prediction results of eye opening: (a) eye height; (b) eye width; (c) eye WidthAtBER.

Section II, this relationship can be expressed by a specific function, Eq. (4), and its gradient is induced as the combination of Eq. (5) and Eq. (11). This gradient is then utilized to calculate active subspace and active variables for sensitivity analysis and dimensionality reduction.

SVR-AS uses the same data sets used for SVR model training. In other words, no extra training data or additional simulations are needed to compute the active subspace. Table 2 depicts the eigenvalues calculated by SVR-AS with respect to Eye Height, Eye Width and Eye WidthAtBER. These eigenvalues are listed in order of decreasing value. The order-of-magnitude gap between the first and the second eigenvalues for all three cases suggests a dominant one-dimensional active subspace. Consequently, direction $\boldsymbol{W}_1$ includes only the first eigenvector and the active variable $y = \boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{X}$ is a one-dimensional linear combination of the corresponding input parameters.

The weights of the linear combination are the components of the first eigenvector $\boldsymbol{W}_1$. These weights also provide an intuitive sensitivity analysis of the 16 design parameters, where components with larger magnitude indicate that the corresponding parameters have a larger influence on the output. Results in Table 3 indicate the design parameters with top-7 largest magnitude of $\boldsymbol{W}_1$. Clearly, "the length of the strip line" is the most influential parameter, and its corresponding $\boldsymbol{W}_1$ value is the largest, greater than 0.98, in all three cases. This result is consistent with our expectations based on past experiences. Additionally, results in Table 3 indicate that the normalized changes in "the dielectric constant of the strip line", "the length of the Tx microstrip line" and "the length of the Rx microstrip line" also have a significant effect on eye diagram.

TABLE 2. Eigenvalues of eye height, eye width and eye WidthAtBER.

Eigenvalue Number	Eye Height	Eye Width	Eye WidthAtBER
1	**0.0306**	**0.0800**	**0.1357**
2	5.04E-05	9.50E-05	1.06E-04
3	2.15E-05	9.43E-05	5.91E-05
4	1.44E-05	8.77E-05	4.42E-05
5	1.07E-05	5.28E-05	3.59E-05
6	8.95E-06	4.09E-05	3.16E-05
7	7.28E-06	3.82E-05	2.74E-05
8	6.80E-06	3.09E-05	2.26E-05
9	5.65E-06	2.32E-05	1.67E-05
10	4.29E-06	2.26E-05	1.39E-05
11	3.98E-06	1.80E-05	9.61E-06
12	1.98E-06	1.45E-05	8.58E-06
13	1.55E-06	1.32E-05	4.80E-06
14	7.96E-07	1.08E-05	3.61E-06
15	3.65E-07	9.58E-06	2.71E-06
16	1.17E-07	6.77E-06	1.17E-06

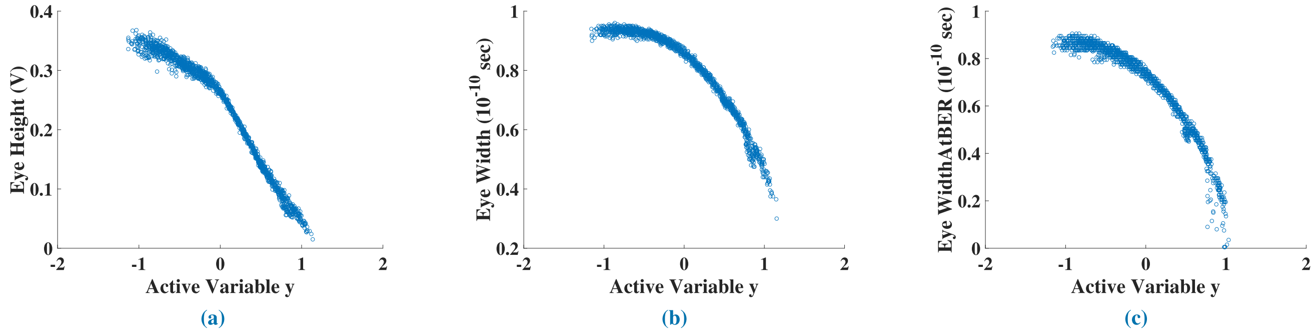

FIGURE 8. Sufficient summary plots of eye opening verse active variables: (a) eye height; (b) eye width; (c) eye WidthAtBER.

TABLE 3. Parameters with top-7 magnitude of W_1.

	Eye Height		Eye Width		Eye WidthAtBER	
	Parameter	$\|\boldsymbol{W}_1\|$	Parameter	$\|\boldsymbol{W}_1\|$	Parameter	$\|\boldsymbol{W}_1\|$
1	Strip line l	0.9891	Strip line l	0.9852	Strip line l	0.9853
2	Strip line ε_r	0.1326	Strip line ε_r	0.1603	Strip line ε_r	0.1585
3	Rx Microstrip line l	0.0403	Tx Microstrip line l	0.0418	Tx Microstrip line l	0.0394
4	Tx Microstrip line l	0.0398	Rx Microstrip line l	0.0339	Rx Microstrip line l	0.0352
5	Strip line H_2	0.0186	Strip line H_2	0.0193	Strip line H_2	0.0266
6	Strip line w	0.0181	Strip line w	0.0161	Strip line w	0.0189
7	Strip line s	0.0116	Strip line s	0.0138	Strip line s	0.0150

Based on the one-dimensional active subspace defined by $\boldsymbol{W}_1$, the 16 design parameters can be combined into a one-dimensional active variable $y = \boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{X}$. We can create sufficient summary plots based on the active variables y and corresponding outputs with the respect to Eye Height, Eye Width and Eye WidthAtBER as illustrated in Fig. 8. It can be inferred from the plots that active variables keep a clear relationship with outputs; hence their importance in effective design parameter space dimensionality reduction. Again, as expected, the eye tends to close when the length of the strip line and/or the microstrip line is increases. The relationship presented in sufficient summary plots can be used to predict eye openings with new active variables and also infer the design of active variables for desired eye openings.

C. SENSITIVITY ANALYSIS RESULTS BY SOBOL's METHOD

Sobol's method based on variance decomposition is a global and model-independent sensitivity analysis method [4], [5]. For the purposes of this work, it is used as the reference method to evaluate the performance of SVR-AS. The total-order index, ST, introduced by [23] is one of the most important outputs in Sobol's method. It accounts the contribution to the output variance caused by an input parameter, x_i, including both its own effect and all interactions with other input parameters. Considering a model represented by a function $Y = f(\boldsymbol{X}) = f(x_1, \ldots x_p)$, the total-order index can be naturally deduced as $ST_i = \frac{E_{X_{-i}}\left(V_{x_i}(Y|\boldsymbol{X}_{-i})\right)}{V(Y)}$. Here, $\boldsymbol{X}_{-i}$ is defined as a $(p-1)$-dimensional parameter space, which consists all input parameters expect x_i.

The total-order sensitivity indices ST of 16 design parameters with respect to Eye Height, Eye Width, and Eye WidthAtBER are calculated by sensitivity analysis library in Python [24] using 54000 high-speed link simulations of designs generated through the sampling of the space of the input design parameters. Table 4 lists the Top-7 influential parameters. Same with results from SVR-AS, "the length of the strip line" is the most influential parameter. "The dielectric constant of the strip line", "the length of the Tx microstrip line" and "the length of the Rx microstrip line" also have a more pronounced effect on eye opening compared to the rest of the design parameters.

Fig. 9 shows the evolution of ST among 16 parameters for Eye Height prediction. It can be inferred that, after about 27000 high-speed link simulations, the value of ST among these parameters has converged. The ST of Eye Width and Eye WidthAtBER exhibit similar convergence tendencies.

V. DISCUSSION

A. SVR-AS VS ACTIVE SUBSPACE METHOD

Active subspace method provides an appealing approach for high-dimensional parameter study. Eigenvectors and active variables provided by active subspace method can be used for sensitivity analysis and input space dimensionality reduction, respectively. However, the gradient information of the relationship between input parameters and output required in active subspace method is difficult to obtain in high-speed link analysis.

SVR-AS provides the feasibility of its application to eye opening estimation. After we have constructed the SVR model, SVR function provides an accurate gradient

TABLE 4. Parameters with Top-7 total-order sensitivity indices.

	Eye Height		Eye Width		Eye WidthAtBER	
	Parameter	ST	Parameter	ST	Parameter	ST
1	Strip line l	0.9744	Strip line l	0.9677	Strip line l	0.9600
2	Strip line ε_r	0.0231	Strip line ε_r	0.0433	Strip line ε_r	0.0969
3	Tx Microstrip line l	0.0046	Tx Microstrip line l	0.0054	Tx Microstrip line l	0.0238
4	Rx Microstrip line l	0.0037	Rx Microstrip line l	0.0044	Rx Microstrip line l	0.0218
5	Strip line H_2	0.0009	Strip line w	0.0019	Strip line H_2	0.0043
6	Strip line w	0.0008	Strip line H_2	0.0019	Strip line w	0.0037
7	Strip line s	0.0004	Strip line s	0.0010	Strip line s	0.0025

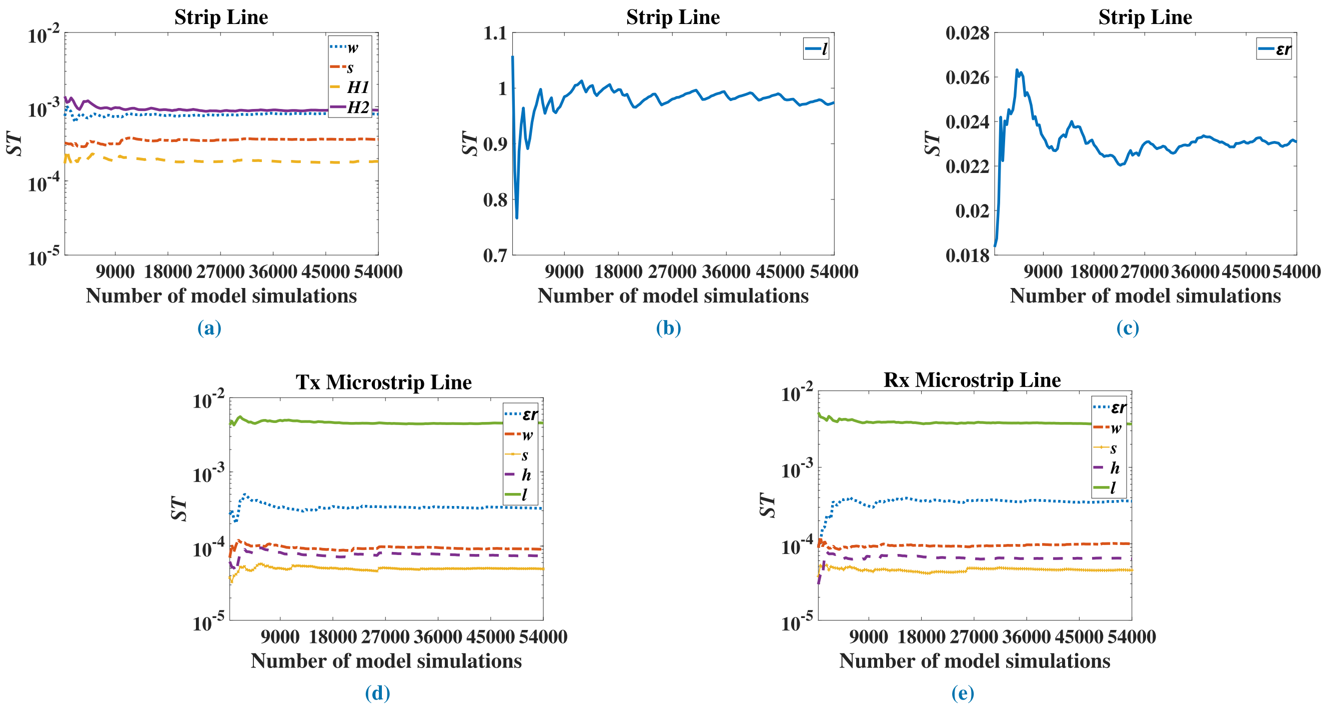

FIGURE 9. The evolution of ST among 16 parameters for eye height prediction: (a) w, s, H_1 and H_2 of the strip line; (b) l of the strip line; (c) ε_r of the strip line; (d) ε_r,w, s, h and l of the Tx microstrip line; (e)ε_r, w, s, h and l of the Rx microstrip line.

information and the active subspace can be identified using the same data sets. SVR-AS maintains the advantage of active subspace method: the magnitude of the eigenvector components presents the relative sensitivity analysis for the input parameters; active variables provide a lower-dimensional representation of the input parameters.

B. SVR-AS VS SOBOL's METHOD

SVR-AS uses the components of $\boldsymbol{W}_1$ to examine the influence of each of the input design parameters on the outputs, while Sobol's method calculates total-order sensitivity indices. The numerical values of $\boldsymbol{W}_1$ and ST are difficult to compare because of different normalization. However, the results for the ranking of the 16 design parameters in these two methods present the same top-7 important parameters.

Considering the loss and dispersion effect of the transmission line, quantified through the product loss $= \gamma \times l = (\alpha + j\beta) \times l$, where γ is the propagation constant, l is the length of the transmission line, α is the attenuation constant and β is the phase constant. Thus, the length of the transmission line is expected to be a very influential parameter as verified by the results of SVR-AS and Sobol's method. Also, because the range of values of the strip line is much wider than that of the other design parameters, the dielectric permittivity of the strip line is also ranking in the top-4 parameters.

Although Sobol's method and SVR-AS both provide an accurate sensitivity analysis of the design parameters, SVR-AS has a significant advantage with respect to computation efficiency, especially for applications where SVR is an appropriate choice for surrogate modeling. Sobol's method is computationally demanding, requiring about simulations of 27,000 designs for convergence. These data require more than 24-days (27000 simulations x 79 seconds/simulation = 2133000 seconds) simulation time. SVR-AS is computationally more efficient, since active subspace calculation shares the same data sets with SVR model. For the specific study, SVR required about 32.9 hours to simulate the

1500 training designs. The SVR-AS method for dimensionality reduction and sensitivity analysis is calculated from the SVR model parameters directly, requiring no additional simulations beyond the training set.

C. SVR-AS VS SVR

SVR-AS is developed based on the predictive function of SVR. The sufficient summary plots shown in Fig. 8 indicate a certain relationship between active variables and outputs. These plots can also be used as a predictive method. Different from the 16 design parameters needed in SVR predictive model, the function defined by the sufficient summary plot only need a one-dimensional input.

Eigenvector components calculated by SVR-AS help identify the most important design parameters, which can be further used to reduce the dimension of the input parameter space in SVR model. In other words, SVR-AS can provide the subset of design parameters that are needed to develop an adequate reduced input parameter space for the high-speed link predictive model.

Moreover, SVR provides a forward predictive function that can be used in high-speed link post-design procedure. Sufficient summary plots in SVR-AS not only provide this ability but also give an inverse function from output Y to active variable y. This inverse relationship is promising in providing guidance for high-speed link pre-design and optimization.

VI. CONCLUSION

In this paper, an efficient method, SVR-AS, is proposed for sensitivity analysis and dimensionality reduction of complex high-speed link model. The proposed SVR-AS method uses SVR predictive model to replace the unknown relationship between input design parameters and eye-opening characteristics, and the gradient of SVR function is employed to calculate the active subspace and its corresponding active variables. This methodology performs well in sensitivity analysis and is computationally more efficient than Sobol's method. In addition to its efficiency, SVR-AS also achieves the goal of the reduction of the dimensionality of the input design space with the generated active variable serving as a weighted linear combination of the input design parameters, with the weights serving as measures of the influence of each design parameter on the output. The proposed SVR-AS method can be applied to other microwave structures when the relationship between design parameters and a scalar quantity of interest can be described by SVR function. Ongoing research explores the usage of SVR-AS for input optimization of SVR predictive model and PCB pre-design strategy.

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HANZHI MA (Graduate Student Member, IEEE) received the B.S. degree in engineering from Zhejiang University, Hangzhou, China, in 2017, where she is currently pursuing the Ph.D. degree with the College of Information Science and Electronics Engineering, Zhejiang University–University of Illinois at Urbana–Champaign Institute.

Her current research interest includes machine learning techniques for EMI/SI/PI analysis.

ER-PING LI (Fellow, IEEE) is currently a Changjiang-Distinguished Professor with the Department of Information Science and Electronics Engineering, Zhejiang University, China, where he is also the Dean of the Zhejiang University–University of Illinois at Urbana–Champaign Institute. Since 1989, he has been serving as a Research Fellow, a Principal Research Engineer, an Associate Professor, and the Technical Director of the Singapore Research Institute and University. In 2000, he joined the Singapore A*STAR Research Institute of High Performance Computing, as a Principal Scientist and the Director. He has authored or coauthored over 300 articles published in refereed international journals and authored two books published by John–Wiley–IEEE Press and Cambridge University Press. He holds and has filed a number of patents at the U.S. patent office. His research interests include electrical modeling and the design of micro/nano-scale integrated circuits, 3D electronic package integration, and nano-plasmonic technology.

Dr. Li is a Fellow of MIT Electromagnetics Academy, USA. He was the President of the 2006 International Zurich Symposium on EMC, the Founding General Chair of the Asia–Pacific EMC Symposium, and the General Chair of APEMC, in 2008, 2010, 2012, and 2016, and the 2010 IEEE Symposium on Electrical Design for Advanced Packaging Systems. He is the Founding Member of the IEEE MTT-RF Nanotechnology Committee. He was recipient of the 2015 IEEE Richard Stoddard Award on EMC, the IEEE EMC Technical Achievement Award, the Singapore IES Prestigious Engineering Achievement Award, the Changjiang Chair Professorship Award from the Ministry of Education in China, and a number of best paper awards. He was elected as an IEEE EMC Distinguished Lecturer, in 2007. He has been a general chair and technical chair of many international conferences. He has been invited to give numerous invited talks and plenary speeches at various international conferences and forums. He served as an Associate Editor for the IEEE Microwave and Wireless Components Letters, from 2006 to 2008, and a Guest Editor for the IEEE Transactions on Electromagnetic Compatibility, in 2006 and 2010, and the IEEE Transactions on Microwave Theory and Techniques APMC Special Issue, in 2010. He is currently an Associate Editor of the IEEE Transactions on Electromagnetic Compatibility and the IEEE Transactions on Components, Packaging, and Manufacturing Technology.

ANDREAS C. CANGELLARIS (Life Fellow, IEEE) received the Diploma degree in electrical engineering from the Aristotle University of Thessaloniki, Thessaloniki, Greece, in 1981, and the M.S. and Ph.D. degrees in electrical engineering from the University of California at Berkeley, Berkeley, CA, USA, in 1983 and 1985, respectively.

After a two-year tenure with the Department of Electronics, General Motors Research Laboratories, he was with the Department of Electrical and Computer Engineering (ECE), University of Arizona, as an Assistant Professor, from 1987 to 1992, and then as an Associate Professor, from 1992 to 1997. In 1997, he joined the Department of ECE, University of Illinois at Urbana–Champaign (UIUC), Urbana, IL, USA, as a Full Professor, in 1997, where he is currently the Vice Chancellor for Academic Affairs, a Provost, and an M. E. Van Valkenburg Professor. He has authored or coauthored over 250 articles in peer-reviewed journals and conference proceedings in the topics of finite methods for electromagnetic field modeling, multiconductor transmission line modeling, and signal and power integrity of integrated electronic systems. His expertise and research interests are in applied and computational electromagnetics and its applications to the advancement of modeling methodologies and computer-aided design (CAD) tools in the support of electrical performance analysis and noise-aware design of integrated electronic systems. His research group has conducted pioneering work in the advancement of modeling methodologies and computer tools for predictive electromagnetic response performance of the signal and the power distribution networks of integrated electronics at the board, package, and chip levels. Several of the prototype tools developed by his group have been transferred to the microelectronics industry and have contributed to the development of several EDA signal integrity tools.

Prof. Cangellaris was the Cofounder of the IEEE Topical Meeting on Electrical Performance of Electronic Packaging, in 1991. From 2010 to 2012, he was a Distinguished Lecturer of the IEEE MTT-S. He is an Active Member of the IEEE Microwave Theory and Techniques Society (IEEE MTT-S) and the IEEE Components Packaging and Manufacturing Technology Society, contributing to numerous professional activities, conferences, and symposia organized and sponsored by them. He was the recipient of the 2000 Outstanding Faculty Teaching Award from the Department of ECE, UIUC, and the 2005 Alexander von Humboldt Research Award for outstanding contributions to electromagnetic theory. He is currently an Editor of the IEEE Press Series on Electromagnetic Wave Theory.

XU CHEN (Member, IEEE) received the B.S., M.S., and Ph.D. degrees from the Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign (UIUC), in 2005, 2014, and 2018, respectively.

From 2005 to 2012, he was a Hardware Engineer with IBM Systems Group, Poughkeepsie, NY, USA. In 2013 and 2015, he worked at Apple, Cupertino, CA, USA. Since 2018, he has been a Teaching Assistant Professor with the Department of Electrical and Computer Engineering, UIUC. His primary research interests are in stochastic modeling, uncertainty quantification, and machine learning techniques, especially as they apply to electromagnetics and circuit problems. He is currently a Faculty Member of the Center for Advanced Electronics through Machine Learning (CAEML). He was the recipient of the Best Conference Paper award at the IEEE Electrical Design of Advanced Packaging and Systems, in 2017. He was also the recipient of the Raj Mittra Outstanding Research Award, the Mavis Future Faculty Fellowship, the USNC–URSI Fellowship Grant Award, and others.