IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received March 30, 2020, accepted April 7, 2020, date of publication April 14, 2020, date of current version April 30, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2987821

Fixed-Time Cooperative Guidance Law With Angle Constraint for Multiple Missiles Against Maneuvering Target

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This work was supported in part by the National Natural Science Foundation of China under Grant 61403100, and in part by the Foundational Innovation Team Support Program under Grant 2017cx023.

ABSTRACT This paper addresses the problem of the fixed-time cooperative guidance (FxTCG) law for multi-missiles against a single maneuvering target with constraints of both the interception time and the line-of-sight (LOS) angle. Firstly, by utilizing the consensus protocol and the fixed-time consensus theory, a new FxTCG in the LOS direction is presented to guarantee that the multiple missiles intercept the incoming target synchronously. Moreover, the acceleration of the target causes disturbance, so a simple fixed-time disturbance observer (FxTDO) is introduced to compensate FxTCG with the disturbance estimation. Subsequently, the FxTCG in the normal direction of the LOS is designed based on an adaptive fixed-time convergence reaching law and the proposed FxTDO, which ensures the fixed-time convergence of the LOS angular velocity and LOS angle between each missile and the maneuvering target. Finally, numerical simulations and adequate analyses are carried out to illustrate the accuracy, effectiveness, and robustness of the proposed FxTCG scheme.

INDEX TERMS Cooperative guidance law, fixed-time convergence, LOS angle constraint, consensus protocol.

I. INTRODUCTION

As the complexity of the modern combat environment increases, the performance of highly maneuverable targets has been enhanced significantly. This constitutes a serious challenge to the modern guidance system, because the traditional single missile combat mode cannot satisfy the requirements of the complex battlefield. Therefore, with the support of information technology, multi-missile cooperation is becoming an important combat mode. Compared with the single missile, the multi-missile cooperation is more effective and more accurate. Based on the multi-missile cooperative guidance law, multiple missiles can share dynamic information to achieve higher interception precision and ensure the consistency of the impact time for all missiles in the terminal guidance stage. Hence the operational effectiveness of the missile defense system has been significantly improved [1]–[3].

As one of the key technologies of cooperative combat, the multi-missile cooperative interception guidance law has been a research focus for many scholars. To achieve simultaneous arrival, Jeon *et al.* [4] designed an impact time control guidance law (ITCG) by combining the time error feedback with proportional navigation guidance. In [5], Kumar and Ghose proposed an ITCG with angle constraints by switching between the ITCG and the impact angle constraints guidance (IACG) law. However, the singularity phenomenon was significant. To solve this problem, a positive continuous nonlinear function of the LOS angle was presented in [6]. Although extensive research has been carried out in [2]–[12] for multiple missiles against a single target, every missile is an independent individual and there is no information exchange among multiple missiles. Therefore, the impact time is prescribed, which cannot be adaptively adjusted.

The associate editor coordinating the review of this manuscript and approving it for publication was Shihong Ding.

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Considering the communication among multiple missiles, a leader-follower scheme was developed for the centralized cooperative guidance law in [13], which was designed with the PNG law and the feedback of the flight time error. The impact time is adaptively adjusted to achieve the simultaneous attack in the centralized cooperative guidance law. The followers can adjust their guidance commands according to the impact time of the leader. To reduce the requirements of communication, a distributed cooperative guidance law was proposed in [14], [15]. Based on the distributed cooperative guidance law, the missiles can attack the target simultaneously by exchanging the information with neighboring missiles. However, these cooperative guidance laws in [13]–[16] cannot be applied to maneuvering targets.

To further increase the communication efficiency of multi-missile systems, consensus protocols are applied to the multi-missile guidance law. By combining finite-time control [17] with consensus protocols [18]–[20], the finite-time consensus was promoted to obtain rapid convergence. The multi-missile guidance law usually has two parts: the LOS direction and the normal direction of LOS. The finite-time cooperative guidance (FTCG) law enjoyed rapid and precise convergence for the impact time of the multi-missiles in [21], [22]. An FTCG law with LOS angle constraint was proposed for an unknown maneuvering target in [23] by utilizing the finite-time consensus protocol, sliding mode control, and a non-homogeneous disturbance observer (NHDO). The acceleration of the target can be treated as the disturbance. And the disturbance estimation was obtained by the NHDO to compensate the proposed guidance law. Other disturbance observers or filters were also adopted to solve the disturbance compensate problem of guidance laws, which can be found in [24], [25]. According to [23], the cooperative guidance law was improved and extended to three-dimensional application in [26]. In another recent paper [27], Lv designed a distributed FTCG law with LOS angle constraints by the integral sliding mode control. Moreover, in [28], an FTCG law with LOS angle constraint was designed for maneuvering target interception based on the terminal sliding mode control. Then, the hyperbolic tangent function based adaptive algorithm is proposed to reduce chattering. Nevertheless, both the disturbance observer and the cooperative guidance law were finite-time convergent in [21]–[23], [26]–[28], which cannot guarantee the boundedness of the convergence time. Furthermore, the initial conditions have a serious impact on the convergence time which grows unboundedly with the initial errors of initial conditions. This seriously affects the guidance efficiency.

In recent years, fixed-time stability has received significant attention because the convergence time of a fixed-time stable system is irrelevant to initial conditions, and fixed-time stability is also regarded as an improvement of finite-time stability. With some modifications, fixed-time control schemes can be directly incorporated into the cooperative guidance law. This enhances the robustness, reduces the limitation of the initial conditions, and expands the application range of the cooperative guidance law. In [29], considering the LOS angle constraint, a distributed FxTCG law with FxTDO was developed. On this basis, an adaptive FxTCG law with constraints of LOS angles was proposed in [30]. Nevertheless, the negative power term hidden behind the FxTCG in the LOS direction was unresolved.

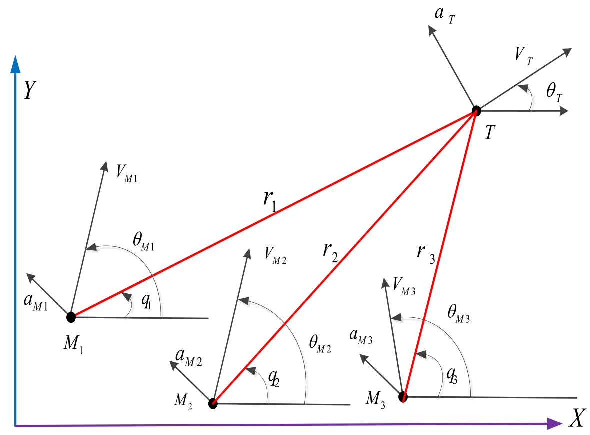

FIGURE 1. Homing engagement geometry.

Motivated by the problems mentioned above, we developed a novel adaptive FxTCG law with constraints of the LOS angle and the impact time. The main contributions of our paper are listed as follows:

1) Based on the fixed-time convergence theory and algebraic graph, this paper presents a new FxTCG law in the LOS direction, which guarantees that the time-to-go for each missile achieves the fixed-time consensus.

2) A novel adaptive fixed-time guidance law is developed by utilizing a fixed-time adaptive reaching law and the FxTDO, such that the LOS angle of all missiles can converge to the expected LOS angle rapidly. The FxTCG can also guarantee high accuracy of the proposed guidance system.

Our paper is organized as follows. Section 2 discusses the cooperative guidance model. Then, an adaptive FxTCG law is presented and proven in Section 3. Section 4 provides the numerical simulations. Lastly, Section 5 gives the conclusions.

II. PROBLEM FORMULATION

In this section, a brief description about the planar guidance geometry of multi-missiles and targets is provided as the preliminary. Then, the algebraic graph is formulated in detail.

A. DYNAMICAL MODEL

In this paper, multi-missile interception of a maneuvering target is studied. In Figure.1, the schematic diagram of the planar engagement geometry is shown. Each missile and target are treated as a point mass. The state variables of the target and missiles are defined with subscripts t and i, $(i = 1, 2 \cdots n)$, respectively. XOY denotes the inertial reference frame.

The corresponding relative kinematic equations can be described by

$$\dot{r}_i = V_T \cos(q_i - \theta_T) - V_{Mi} \cos(q_i - \theta_{Mi}) \quad (1)$$

$$r_i \dot{q}_i = -V_T \sin(q_i - \theta_T) + V_{Mi} \sin(q_i - \theta_{Mi}) \quad (2)$$

$$\dot{\theta}_{Mi} = \frac{a_{Mi}}{V_{Mi}} \quad \dot{\theta}_T = \frac{a_T}{V_T} \quad (3)$$

where V_T and V_{Mi} define the velocity of the target and missiles, respectively. And $V_{Mi} > V_T$. r_i is the relative distance between the missile and the target. The flight path angle, LOS angle, normal acceleration of the missile, and normal acceleration of the target are denoted by θ_{Mi}, q_i, a_{Mi} and a_T, respectively.

The derivatives of (1) and (2) can be obtained as

$$\ddot{r}_i = r_i \dot{q}_i^2 + u_{ri} - w_{ri} \quad (4)$$

$$\ddot{q}_i = -\frac{2\dot{r}_i \dot{q}_i}{r_i} - \frac{u_{qi}}{r_i} + \frac{w_{qi}}{r_i} \quad (5)$$

where u_{ri} and w_{ri} are the acceleration components of the missile and the target in the LOS direction. In this paper, w_{ri} is regarded as zero. u_{qi} and w_{qi} denote the acceleration components of the missile and the target in the normal direction of the LOS, respectively.

Define the state variables as $x_{1i} = r_i, x_{2i} = \dot{r}_i, x_{3i} = q_i - q_{fi}$, and $x_{4i} = \dot{x}_{3i} = \dot{q}_i$. Therefore, the cooperative guidance model can be described as

$$\begin{cases} \dot{x}_{1i} = x_{2i} \\ \dot{x}_{2i} = x_{1i} x_{4i}^2 - u_{ri} \\ \dot{x}_{3i} = x_{4i} \\ \dot{x}_{4i} = -\frac{2x_{2i}}{x_{1i}} x_{4i} - \frac{u_{qi}}{x_{1i}} + \frac{w_{qi}}{x_{1i}} \end{cases} \quad (6)$$

The change of the velocity $\dot{r}$ is relatively small in the actual guidance process. Thus, t_{go} can be estimated by

$$t_{goi} = -\frac{r_i}{\dot{r}_i} \quad (7)$$

Taking the derivative of (7) yields

$$\dot{t}_{goi} = -1 + \frac{x_{1i}^2 x_{4i}^2}{x_{2i}^2} - \frac{x_{1i}}{x_{2i}^2} u_{ri} \quad (8)$$

By treating t_{goi} as an extra state variable, we obtain the new dynamic equations as follows.

$$\begin{cases} \dot{t}_{goi} = -1 + \frac{x_{1i}^2 x_{4i}^2}{x_{2i}^2} - \frac{x_{1i}}{x_{2i}^2} u_{ri} \\ \dot{x}_{3i} = x_{4i} \\ \dot{x}_{4i} = -\frac{2x_{2i}}{x_{1i}} x_{4i} - \frac{u_{qi}}{x_{1i}} + \frac{w_{qi}}{x_{1i}} \end{cases} \quad (9)$$

Define a new state variable

$$\tilde{u}_{ri} = \frac{x_{1i}^2 x_{4i}^2}{x_{2i}^2} - \frac{x_{1i}}{x_{2i}^2} u_{ri} \quad (10)$$

Substituting (10) into (9) yields

$$\begin{cases} \dot{t}_{goi} = -1 + \tilde{u}_{ri} \\ \dot{x}_{3i} = x_{4i} \\ \dot{x}_{4i} = -\frac{2x_{2i}}{x_{1i}} x_{4i} - \frac{u_{qi}}{x_{1i}} + d_{qi} \end{cases} \quad (11)$$

where $d_{qi} = \frac{w_{qi}}{x_{1i}}$.

Assumption 1: d_{qi} is denoted as the unknown and bounded external disturbance, which is caused by the acceleration of the target. d_{qi} and $\dot{d}_{qi}$ satisfy $|d_{qi}| \leq d_0$ and $|\dot{d}_{qi}| \leq d_1$, where d_0 and d_1 are unknown and bounded.

The objective of our paper is presented as follows. In the LOS direction, u_{ri} is designed to guarantee the consistency of the interception time for every missile within a fixed time. u_{qi} is proposed to guarantee that the LOS angular velocity will approach zero, and the LOS angle will converge to the expected value in the normal direction of LOS.

B. ALGEBRAIC GRAPH

Suppose that there are n missiles in the cooperative attack mission, and the graph $G(M, E, A)$ denotes the information communication in the multi-missile system. Graph $G(M, E, A)$ comprises a node $M = \{M_1, M_2, \cdots M_n\}$, an edge $E \subseteq \{(M_i, M_j) : M_i, M_j \in M\}$, and a weighted adjacency matrix $A = [a_{ij}] \in R_{n\times n}$. When the information flows from M_j to M_i, the matrix element a_{ij} in A satisfies $a_{ij} \neq 0$. Otherwise, $a_{ij} = 0$. It is noticed that the graph G is undirected if $(M_i, M_j) \in E \Leftrightarrow (M_j, M_i) \in E$. The adjacency matrix A is symmetric in the undirected graph G. If the undirected graph is connected, a path exists between any two nodes. The set of neighbors of M_i is defined by

$$N_i = \{j \in M : a_{ij} \neq 0\} = \{j \in M : (M_i, M_j) \in E\} \quad (12)$$

C. PRELIMINARY LEMMAS

Definition 1 [31]–[33]: Consider a non-linear system

$$\dot{x}(t) = f(x(t)), x(0) = x_0 \quad (13)$$

where $x(t) = [x_1(t), x_2(t), \cdots x_n(t)]^T \in R^n$. Assume that the origin is an equilibrium point of system (13). The origin of system (13) is a finite-time stable equilibrium if the origin is Lyapunov stable. Then, there exists a settling time function $T : R^n \to R^+$, such that for every $x_0 \in R^n$, $x(t, x_0)$ is a solution of system (13) satisfying $\lim_{t\to T(x_0)} x(t, x_0) = 0$. If the origin of system (13) has global finite-time stability and is convergent to the origin within a bounded convergence time $T(x_0)$, it is said to be fixed-time stable. Therefore, there exists a bounded positive constant $T_{\max}$ such that $T(x_0) < T_{max}$.

Lemma 1 [34], [35]: For system (13), assume a Lyapunov function $V(x)$, such that $\dot{V}(x) \leq -(\alpha_1 V(x)^p + \beta_1 V(x)^q)^k + \Omega$, where α_1, β_1, p, q are parameters, $pk < 1, qk > 1, k \in R^+$ and $0 < \Omega < \infty$. Then, the system is fixed-time stable.

Moreover, the residual of the solution of system (13) is

$$\left\{ \lim_{t\to T} x \middle| V(x) \le \min \left\{ \alpha_1^{-1/p} \left(\frac{\Omega}{1-\Theta^k}\right)^{\frac{1}{kp}}, \beta_1^{-1/p} \left(\frac{\Omega}{1-\Theta^k}\right)^{\frac{1}{kq}} \right\} \right\} \tag{14}$$

where Θ satisfies $0 < \Theta \le 1$.The convergence time is bounded by

$$T \le \frac{1}{\alpha_1^k \Theta^k (1-pk)} + \frac{1}{\beta_1^k \Theta^k (qk-1)} \tag{15}$$

Lemma 2 [36]: For any $x_i \in R, i = 1, 2, \ldots, n$, $\left(\sum_{i=1}^n |x_i|\right)^v \le \sum_{i=1}^n |x_i|^v$, where $v \in R^+$ and $v \in (0, 1]$.

Lemma 3 [36]: For any $x_i \in R, i = 1, 2, \ldots, n$, $\left(\sum_{i=1}^n |x_i|\right)^v \le n^{v-1} \left(\sum_{i=1}^n |x_i|^v\right)$, where $v \in R^+$ and $v > 1$.

Lemma 4 [37]: For system (13), assume a Lyapunov function $V(x)$, such that $\dot V(x) \le -(\alpha_1 V(x)^p + \beta_1 V(x)^q)^k$, where α_1, β_1, m, n are parameters, $mk < 1, nk > 1, \gamma \in R^+$. Then, the system is fixed-time stable. The convergence time is upper bounded as

$$T < T_{\max} := \frac{1}{a_1^k(1-mk)} + \frac{1}{\beta_1^k(nk-1)} \tag{16}$$

Lemma 5 [37]: Consider a scalar system

$$\dot x = -\lceil h_1 \lceil x \rfloor^m + h_2 \lceil x \rfloor^n \rfloor^\gamma \tag{17}$$

where $h_1, h_2 > 0, n > m, m\gamma < 1$, and $n\gamma > 1$. For a given vector $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$, $\lceil x \rfloor^m = \mathrm{sign}(x)\,|x|^m$,where $\mathrm{sign}(*)$ defines the sign function. Then, the system is fixed-time stable. The upper bound of the convergence time is

$$T < T_{\max} := \frac{1}{h_1^\gamma(1-m\gamma)} + \frac{1}{h_2^\gamma(n\gamma-1)} \tag{18}$$

III. COOPERATIVE GUIDANCE SCHEME

This section presents the FxTCG in the LOS direction and the normal of the LOS direction, respectively. Then, the stability of the proposed guidance system is proven in detail.

A. GUIDANCE LAW IN THE LOS DIRECTION

The guidance model in the LOS direction is obtained

$$\begin{cases} \dot x_{1i} = x_{2i} \\ \dot x_{2i} = x_{1i} x_{4i}^2 - u_{ri} \end{cases} \tag{19}$$

A new variable t_{fi} is introduced, and we have

$$t_{fi} = t + t_{goi} \tag{20}$$

Suppose all missiles are launched simultaneously. t_{goi} for each missile will reach consensus when t_{fi} is convergent. It is obvious that

$$t_{fi} = \tilde u_{ri} \tag{21}$$

Based on the Definition 3, T_{fi} for every missile has uniform convergence performance within a fixed time with the protocol $\tilde u_{ri}$.

Theorem 1: Assume the undirected graph of system (13) is connected. Then, system (13) can achieve stability within a fixed time by the consensus protocol as follows

$$\tilde u_{ri} = \lceil h_1 \lceil \sum_{j\in N_i} a_{ij}(x_j - x_i) \rfloor^m + h_2 \lceil \sum_{j\in N_i} a_{ij}(x_j - x_i) \rfloor^n \rfloor^\gamma \tag{22}$$

where $h_1, h_2 > 0, n > m, m\gamma < 1$,and $n\gamma > 1$.

Proof: Considering the following Lyapunov function

$$V(x) = \frac{1}{4} \sum_{i=1}^n \sum_{j=1}^n a_{ij}(x_j - x_i)^2 \tag{23}$$

The time derivative of $V(x)$ is

$$\begin{aligned} \dot V_i &= \sum_{i=1}^n \frac{\partial V(x)}{\partial x_i} \dot x_i \\ &= -\sum_{i=1}^n a_{ij}(x_j - x_i) \lceil h_1 \lceil \sum_{j\in N_i}^{a_{ij}(x_j-x_i)} \rfloor^m \\ &\quad + h_2 \lceil \sum_{j\in N_i} a_{ij}(x_j - x_i) \rfloor^n \rfloor^\gamma \\ &= -\sum_{i=1}^n a_{ij}(x_j - x_i) \left| \begin{matrix} h_1 \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^m \\ + h_2 \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^n \end{matrix} \right|^\gamma \\ &\quad \mathrm{sign}\left(\sum_{i=1}^n a_{ij}(x_j - x_i) \right) \\ &= -\left| \sum_{i=1}^n a_{ij}(x_j - x_i) \right| \left| \begin{matrix} h_1 \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^m \\ + h_2 \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^n \end{matrix} \right|^\gamma \end{aligned} \tag{24}$$

Therefore, (24) can be simplified as

$$\begin{aligned} \dot V_i &\le -\left| h_1 \sum_{i=1}^n \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^{m+\frac{1}{\gamma}} \right. \\ &\quad \left. + h_2 \sum_{i=1}^n \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^{n+\frac{1}{\gamma}} \right|^\gamma \\ &\le -\left(\begin{matrix} h_1 \left(\sum_{i=1}^n \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^2 \right)^{\frac{m\gamma+1}{2\gamma}} \\ + h_2 n^{\frac{(n-2)\gamma+1}{2\gamma}} \left(\sum_{i=1}^n \left| \sum_{j\in N_i} a_{ij}(x_j - x_i) \right|^2 \right)^{\frac{n\gamma+1}{2\gamma}} \end{matrix} \right)^\gamma \end{aligned} \tag{25}$$

According to the proof in [33, Th. 5.1], we obtain

$$\dot{V}_i \leq -\left(h_1 \left(2\lambda_2 \left(L_A\right) V\right)^{\frac{m\gamma+1}{2\gamma}} + h_2 n^{\frac{(n-2)\gamma+1}{2\gamma}} \left(2\lambda_2 \left(L_A\right) V\right)^{\frac{n\gamma+1}{2\gamma}}\right)^{\gamma} \tag{26}$$

where $\lambda_2(L_A)$ is the second smallest characteristic value of L_A. If $V \neq 0$, then suppose $y = \sqrt{2\lambda_2(L_A)V}$ is the solution of

$$\dot{y} = -\lceil h_1 \lceil \lambda_2 \left(L_A\right) y \rfloor^{\frac{m\gamma+1}{2\gamma}} + h_2 n^{\frac{(n-2)\gamma+1}{2\gamma}} \lfloor \lambda_2 \left(L_A\right) y \rfloor^{\frac{n\gamma+1}{2\gamma}} \rfloor^{\gamma} \tag{27}$$

According to Lemma 4, we have

$$\lim_{t \to T} \left| x_j(t) - x_i(t) \right| = 0 \tag{28}$$

The settling time T is upper bounded by

$$T_{\max} = \frac{n^{\frac{(n-2)\gamma+1}{2\gamma}}}{\lambda_2 \left(L_A\right)} \left(\frac{1}{h_1^{\gamma}(1 - \frac{m\gamma+1}{2})} + \frac{1}{h_2^{\gamma}(\frac{n\gamma+1}{2} - 1)} \right) \tag{29}$$

Theorem 2: If the undirected graph of the multi-missile system is connected, the impact time of all missiles will converge to the same value within a fixed time with the protocol

$$u_{ri} = x_{1i} x_{4i}^2 - \frac{x_{2i}^2}{x_{1i}} \tilde{u}_{ri} \tag{30}$$

where

$$\tilde{u}_{ri} = \lceil h_{1i} \lceil \sum_{j \in N_i} a_{ij} (\frac{x_{1j}}{x_{2j}} - \frac{x_{1i}}{x_{2i}}) \rfloor^m + h_{2i} \lceil \sum_{j \in N_i} a_{ij} (\frac{x_{1j}}{x_{2j}} - \frac{x_{1i}}{x_{2i}}) \rfloor^n \rfloor^{\gamma} \tag{31}$$

Proof: Taking the derivative of (20), and combining it with (6), we can get

$$\begin{aligned} \dot{t}_{fi} &= \frac{\dot{x}_{4i}^2}{\dot{x}_{2i}^2} - \frac{x_{1i}}{\dot{x}_{2i}^2} u_{ri} \\ &= \lceil h_{1i} \lceil \sum_{j \in N_i} a_{ij} (\frac{x_{1j}}{x_{2j}} - \frac{x_{1i}}{x_{2i}}) \rfloor^m + h_{2i} \lceil \sum_{j \in N_i} a_{ij} (\frac{x_{1j}}{x_{2j}} - \frac{x_{1i}}{x_{2i}}) \rfloor^n \rfloor^{\gamma} \end{aligned} \tag{32}$$

By utilizing Theorem 1, t_{fi} can realize uniform convergence if $t > t_{fi}$. This indicates that the target can be simultaneously intercepted by all the missiles.

B. FIXED-TIME DISTURBANCE OBSERVER

The cooperative guidance model in the normal direction of LOS is described by

$$\begin{cases} \dot{x}_{3i} = x_{4i} \\ \dot{x}_{4i} = -\dfrac{2x_{2i}}{x_{1i}} x_{4i} - \dfrac{u_{qi}}{x_{1i}} + d_{qi} \end{cases} \tag{33}$$

where the target acceleration is treated as the bounded external disturbance and it can be defined as

$$d_{qi} = \frac{w_{qi}}{x_{1i}} \tag{34}$$

The unknown target acceleration is a crucial factor for the design of the interception guidance law. However, the external disturbance d_{qi} cannot be measured in the real flight. To address this issue, an FxTDO is proposed, which can estimate the external disturbance d_{qi}. Then, the estimated information can be used to compensate the normal guidance law. Here, the FxTDO is given by [38]

$$\begin{cases} \dot{z}_1 = z_2 + \theta \eta_1 \nu \left(x_2 - z_1, \varepsilon_1, \varepsilon_2\right) - \dfrac{2x_{2i}}{x_{1i}} x_{4i} - \dfrac{u_{qi}}{x_{1i}} \\ \dot{z}_2 = \theta^2 \eta_2 \nu \left(x_2 - z_1, \varepsilon_1, \varepsilon_2\right) \end{cases} \tag{35}$$

where z_1 estimates the value of x_2, and z_2 estimates the value of the disturbance d_{qi}. $\eta_1, \eta_2 > 0, \eta_1 \geq 2\sqrt{\eta_2}, \theta \geq 0$, $\varepsilon_1 \in (0.5, 1)$, and $\varepsilon_2 \in (1, 1.5)$. $\nu(\cdot)$ is the correction term and there is

$$\nu(x, \varepsilon_1, \varepsilon_2) = \begin{cases} \lceil x \rfloor^{\varepsilon_1}, |x| < 1 \\ \lceil x \rfloor^{\varepsilon_2}, |x| \geq 1 \end{cases} \tag{36}$$

The stability of the proposed FxTDO was proven in [38]. Hence the target's acceleration estimation can converge to the actual value within a fixed time.

C. GUIDANCE LAW IN THE NORMAL DIRECTION OF LOS

In the normal direction of LOS, a novel adaptive sliding mode guidance law is proposed to guarantee all missiles tend to the desired LOS angle. According to the guidance model in (33), a nonsingular fast terminal sliding mode surface is designed

$$s_i = x_{3i} + \delta_1 \lceil x_{3i} \rfloor^{\lambda_1} + \delta_2 \lceil x_{4i} \rfloor^{\lambda_2} \tag{37}$$

where $\delta_1 > 0, \delta_2 > 0, \lambda_1, \lambda_2, 2 > \lambda_2 > 1$ and $\lambda_1 > \lambda_2$.

To attenuate the chattering and ensure the fixed-time convergence of the guidance system, a fixed-time adaptive reaching law [39] can be designed by

$$\dot{s}_i = -k_1 \lceil s_i \rfloor^{p_i} - k_2 \lceil s_i \rfloor^{q_i} - \left(\hat{\sigma} - 1\right) (\hat{\xi} + \hat{\tau} \left|x_{4i}\right|) \text{sign}(s_i) \tag{38}$$

where $0 < p_i < 1, q_i > 1, k_1 > 0$, and $k_2 > 0$. $\hat{\sigma}, \hat{\xi}$, and $\hat{\tau}$ are adaptive gains. As indicated in (38), the adaptive gains $\hat{\sigma}, \hat{\xi}$, and $\hat{\tau}$ as well as the state variable x_{4i} are used to adjust the magnitude of the switching term $\left(\hat{\sigma} - 1\right)(\hat{\varsigma} + \hat{\tau} \left|x_{4i}\right|)$. Furthermore, to mitigate the chattering phenomenon caused by the large control gain, a new adaptive reaching law is proposed. Adaptive gains are adjusted by

$$\begin{aligned} \dot{\hat{\xi}} &= \frac{1}{2\theta_0 c_1} \left(\left|s_i\right| - \mu_1 \hat{\xi}\right) \\ \dot{\hat{\tau}} &= \frac{1}{2\theta_0 c_2} \left(\left|s_i\right| \left|x_{4i}\right| - \mu_2 \hat{\tau}\right) \\ \dot{\hat{\sigma}} &= \frac{1}{2\theta_0 c_3} \left(\frac{\hat{\sigma}^2}{1 - \hat{\sigma}^{-1}} \left|s_i\right| \left(\hat{\xi} + \hat{\tau} \left|x_{4i}\right| + \mu_3 \hat{\sigma}^{-1} \right) \right) \hat{\sigma} \end{aligned} \tag{39}$$

where $\theta_0 \in (0, 1)$ and $\theta_0^{(p+1/2)} + \theta_0 - 1 = 0$. The adaptive gains $\hat{\sigma}, \hat{\xi}$, and $\hat{\tau}$ will diminish gradually and the initial values of these adaptive gains satisfy $\hat{\sigma} \in (0, 1)$, $\hat{\tau} > 0$ and $\hat{\varsigma} > 0$, respectively. Then, c_1, c_2, and c_3 satisfy

$$\begin{aligned} c_1 &= \mu_1(\theta_1 - 0.5)/\theta_1 \\ c_2 &= \mu_2(\theta_2 - 0.5)/\theta_2 \\ c_3 &= \mu_3(\theta_3 - 0.5)/\theta_3 \end{aligned} \tag{40}$$

where $\theta_1 > 1/2, \theta_2 > 1/2$, and $\theta_3 > 1/2$. Controller parameters μ_1, μ_2 and μ_3 are coordinated by the simulation results. θ_i, μ_i need to be larger than the initial value of $|s_i|$ such that $\left(|s_i| - \mu_1\hat{\xi}\right) < 0$ and $\left(|s_i|\,|x_{4i}| - \mu_2\hat{\tau}\right) < 0$. With S approaching zero, $\lim\limits_{t>T} \hat{\xi}(t) = 0$ and $\lim\limits_{t>T} \hat{\tau}(t) = 0$ hold. Moreover, c_1, c_2, and c_3 are designed as (40),which makes the selection of parameters simple.

Based on (37) and (38), the cooperative guidance law can be designed as

$$\begin{aligned} u_{qi} &= x_{1i}\left[\frac{1}{\delta_2\lambda_2}\lceil x_{4i}\rfloor^{2-\lambda_2}\left(1+\delta_1\lambda_1|x_{3i}|^{\lambda_1-1}\right) - \frac{2x_{2i}x_{4i}}{x_{1i}}\right. \\ &\quad \left. + z_{2i} + \frac{k_1\lceil s_i\rfloor^{p_i} + k_2\lceil s_i\rfloor^{q_i} + (\hat{\sigma}-1)(\hat{\xi}+\hat{\tau}|x_{4i}|)\text{sign}(s_i)}{x_{1i}}\right] \end{aligned} \tag{41}$$

where z_2 is the estimation value of the disturbance obtained by the proposed FxTDO. Moreover, we use a hyperbolic tangent function $\tanh(x) = \left(e^x - e^{-x}\right)/\left(e^x + e^{-x}\right)$ as a substitute for the switching function sign(x) to reduce the chattering of the guidance law.

Theorem 3: If the FxTCG law is devised by (41) and the adaptive law is proposed by (38), then the LOS angle error x_{3i} and the LOS angular velocity x_{4i} of multi-missiles are fixed-time convergent.

Proof: Bringing (33) into the derivative of s_i, we have

$$\begin{aligned} \dot{s}_i &= \dot{x}_{3i} + \delta_1\lambda_1|x_{4i}|^{\lambda_1-1}\dot{x}_{3i} + \delta_2\lambda_2|x_{4i}|^{\lambda_2-1}\dot{x}_{4i} \\ &= x_{4i} + \delta_1\lambda_1|x_{3i}|^{\lambda_1-1}x_{4i} + \delta_2\lambda_2|x_{4i}|^{\lambda_2-1}\left(a_i + b_i u_{qi} + d_{qi}\right) \end{aligned} \tag{42}$$

where $a_i(x) = -2x_{2i}x_{4i}/x_{1i}$, and $b_i(x) = -1/x_{1i}$.

Substituting (41) into (42) yields

$$\begin{aligned} \dot{s}_i &= x_{4i} + \delta_1\lambda_1|x_{3i}|^{\lambda_1-1}x_{4i} + \delta_2\lambda_2|x_{4i}|^{\lambda_2-1}\left(a_i + b_i u_{qi} + d_{qi}\right) \\ &= \delta_2\lambda_2|x_{4i}|^{\lambda_2-1}\left(d_{qi} - z_{2i} + \frac{k_1\lceil s_i\rfloor^{p_i} + k_2\lceil s_i\rfloor^{q_i} + (\hat{\sigma}-1)(\hat{\xi}+\hat{\tau}|x_{4i}|)\text{sign}(s_i)}{x_{1i}}\right) \\ &= \rho(x_{4i})\left[\begin{array}{l}\left(d_{qi} - z_{2i}\right)x_{1i} + k_1\lceil s_i\lceil^{p_i} + k_2\lceil s_i\rfloor^{q_i} \\ +\left(\hat{\sigma}-1\right)(\hat{\xi}+\hat{\tau}|x_{4i}|)\text{sign}(s_i)\end{array}\right] \end{aligned} \tag{43}$$

where $\rho(x_{4i}) = \delta_2\lambda_2|x_{4i}|^{\lambda_2-1}/x_{1i}$, $\rho(x_{4i}) > 0$, and $x_{4i} \neq 0$.

According to the definition of the FxTDO in (35), the error of the observer $\left|d_{qi} - z_{2i}\right|$ tends to zero within a fixed time. Therefore, there exists a bounded positive constant t_f as

$$\left|d_{qi} - z_{2i}\right| = 0, \quad t > t_{fi} \tag{44}$$

Considering the Lyapunov function

$$V_i = \frac{1}{2}s_i^2 + \theta_0\left(c_1\tilde{\xi}^2 + c_2\tilde{\tau}^2 + c_3\tilde{\sigma}^2\right) \tag{45}$$

Define the error function

$$\begin{aligned} \tilde{\xi} &= 0 - \hat{\xi} \\ \tilde{\tau} &= 0 - \hat{\tau} \\ \tilde{\sigma} &= 1 - \hat{\sigma}^{-1} \end{aligned} \tag{46}$$

Taking the derivative of V_i yields

$$\begin{aligned} \dot{V}_i &= s_i\dot{s}_i + 2c_1\theta_0\tilde{\xi}\dot{\tilde{\xi}} + 2c_2\theta_0\tilde{\tau}\dot{\tilde{\tau}} + 2c_3\theta_0\tilde{\sigma}\dot{\tilde{\sigma}} \\ &= -s_i\left(\rho(x_{4i})\left[\begin{array}{l}\left(d_{qi} - z_{2i}\right)x_{1i} + k_1\lceil s_i\rfloor^{p_i} + k_2\lceil s_i\rfloor^{q_i} + \\ \left(\hat{\sigma}-1\right)(\hat{\xi}+\hat{\tau}|x_{4i}|)\text{sign}(s_i)\end{array}\right]\right) \\ &\quad -2\theta_0 c_1\tilde{\chi}\dot{\hat{\chi}} - 2\theta_0 c_2\tilde{\Phi}\dot{\hat{\Phi}} + 2\theta_0 c_3\tilde{\mu}\hat{\mu}^{-2}\dot{\hat{\mu}} \\ &= -s_i\left(\rho(x_{4i})\left[\begin{array}{l}\left(d_i - z_{2i}\right)x_{1i} + k_1\lceil s_i\rfloor^{p_i} + k_2\lceil s_i\rfloor^{q_i} + \\ \left(\hat{\sigma}-1\right)(\hat{\xi}+\hat{\tau}|x_{4i}|)\text{sign}(s_i)\end{array}\right]\right) \\ &\quad -\tilde{\xi}\left(|s_i| - \mu_1\hat{\xi}\right) - \tilde{\tau}\left(|s_i|\,|x_{4i}| - \mu_2\hat{\tau}\right) \\ &\quad +\tilde{\sigma}\hat{\sigma}^{-1}\left(\frac{\hat{\sigma}^2}{1-\hat{\sigma}^{-1}}|s_i|\left(\begin{array}{l}\hat{\xi}+\hat{\tau}|x_4| \\ +\mu_3\hat{\sigma}^{-1}\end{array}\right)\right) \\ &= -s_i\left(\rho(x_{4i})\left[\begin{array}{l}k_1\lceil s_i\rfloor^{p_i} + k_2\lceil s_i\rfloor^{q_i} + \left(\hat{\sigma}-1\right) \\ (\hat{\xi}+\hat{\tau}|x_{4i}|)\text{sign}(s_i)\end{array}\right]\right) \\ &\quad -\tilde{\xi}|s_i| - \tilde{\tau}|s_i|\,|x_{4i}| + \mu_1\tilde{\xi}\hat{\xi} + \mu_2\tilde{\tau}\hat{\tau} \\ &\quad +\mu_3|s_i| + \hat{\sigma}|s_i|\left(\hat{\xi}+\hat{\tau}|x_{4i}|\right) \\ &\leq -k_1\rho(x_{4i})|s_i|^{p_i+1} - k_2\rho(x_{4i})|s_i|^{q_i+1} + \mu_1\tilde{\xi}\hat{\xi} \\ &\quad +\mu_2\tilde{\tau}\hat{\tau} + \mu_3|s_i| \end{aligned} \tag{47}$$

Moreover, the following expressions are obtained

$$\begin{aligned} \mu_1\tilde{\xi}\hat{\xi} &= -\mu_1\tilde{\xi}\left(\tilde{\xi} - 0\right) \leq -\mu_1\tilde{\xi}^2 \\ \mu_2\tilde{\tau}\hat{\tau} &= -\mu_2\tilde{\tau}\left(\tilde{\tau} - 0\right) \leq -\mu_2\tilde{\tau}^2 \end{aligned} \tag{48}$$

Then, (47) can be simplified as

$$\begin{aligned} \dot{V}_i &\leq -k_1|s_i|^{p_i+1} - k_2|s_i|^{q_i+1} - \mu_1\tilde{\xi}^2 - \mu_2\tilde{\tau}^2 + \mu_3|s_i| \\ &\leq -k_1|s_i|^{p_i+1} - k_2|s_i|^{q_i+1} + \mu_3|s_i| \end{aligned} \tag{49}$$

Considering $V_i^{(q_i+1)/2}$, Lemma 2 yields

$$\begin{aligned} V_i^{(p_i+1)/2} &\leq \left(\frac{1}{2}|s_i|^2 + \theta_0\left(c_1\tilde{\xi}^2 + c_2\tilde{\tau}^2 + c_3\tilde{\sigma}^2\right)\right)^{(p_i+1)/2} \\ &\leq \left(\frac{1}{2}\right)^{(p_i+1)/2}|s_i|^{p+1} + \theta_0\left(c_1\tilde{\xi}^2 + c_2\tilde{\tau}^2 + c_3\tilde{\sigma}^2\right) \end{aligned} \tag{50}$$

$$\begin{aligned} 2^{(1-q_i)/2}V_i^{(q_i+1)/2} &\leq 2^{-(q_i+1)/2}|s_i|^{q_i+1} + \left(\theta_0 c_1\tilde{\xi}^2\right)^{(q_i+1)/2} \\ &\quad + \left(\theta_0 c_2\tilde{\tau}^2\right)^{(q_i+1)/2} + \left(\theta_0 c_3\tilde{\sigma}^2\right)^{(q_i+1)/2} \end{aligned} \tag{51}$$

$$\begin{aligned} -2^{-(q_i+1)/2}|s_i|^{q_i+1} &\leq -2^{(1-q_i)/2}V_i^{(q_i+1)/2} + \left(\theta_0 c_1\tilde{\xi}^2\right)^{(q_i+1)/2} \\ &\quad + \left(\theta_0 c_2\tilde{\tau}^2\right)^{(q_i+1)/2} + \left(\theta_0 c_3\tilde{\sigma}^2\right)^{(q_i+1)/2} \end{aligned} \tag{52}$$

Likewise, considering $V_i^{(q_i+1)/2}$ and Lemma 3 yields

$$V_i^{(p_i+1)/2} \le \left(\frac{1}{2}|s_i|^2 + \theta_0\left(c_1\tilde{\xi}^2 + c_2\tilde{\tau}^2 + c_3\tilde{\sigma}^2\right)\right)^{(p_i+1)/2}$$
$$\le \left(\frac{1}{2}\right)^{(p_i+1)/2} |s_i|^{p_i+1} + \theta_0\left(c_1\tilde{\xi}^2 + c_2\tilde{\tau}^2 + c_3\tilde{\sigma}^2\right) \tag{53}$$

$$-2^{-(p_i+1)/2}|s_i|^{p_i+1} \le -V_i^{(p_i+1)/2} + \theta_0\left(c_1\tilde{\xi}^2 + c_2\tilde{\tau}^2 + c_3\tilde{\sigma}^2\right) \tag{54}$$

Substituting (52) and (54) into (49) and making further simplification yields

$$\dot{V}_i \le -\chi_1 V_i^{(p_i+1)/2} - \chi_2 2^{1-q_i} V_i^{(q_i+1)/2} + \mu_3|s_i| + \left(\theta_0 c_1\tilde{\xi}^2\right)^{(q_i+1)/2} + \left(\theta_0 c_2\tilde{\tau}^2\right)^{(q_i+1)/2} + \left(\theta_0 c_3\tilde{\sigma}^2\right)^{(q_i+1)/2} + \theta_0 c_1\tilde{\xi}^2 + \theta_0 c_2\tilde{\tau}^2 + \theta_0 c_3\tilde{\sigma}^2 \tag{55}$$

Finally, equation (55) can be simplified as

$$\dot{V}_i \le -\chi_1 \dot{V}_i^{(p+1)/2} - \chi_2 2^{1-q} \dot{V}_i^{(q+1)/2} + \Omega \tag{56}$$

where

$$\chi_1 = \min\left\{k_1\rho(x_{4i})/\left(2^{-(p_i+1)/2}\right), 1\right\},$$
$$\chi_2 = \min\left\{k_2\rho(x_{4i})/\left(2^{-(q_i+1)/2}\right), 1\right\}.$$

And

$$\Omega = \mu_3|s_i| + \theta_0 c_1\tilde{\xi}^2 + \theta_0 c_2\tilde{\tau}^2 + \theta_0 c_3\tilde{\sigma}^2 + \left(\theta_0 c_1\tilde{\xi}^2\right)^{(q_i+1)/2} + \left(\theta_0 c_2\tilde{\tau}^2\right)^{(q_i+1)/2} + \left(\theta_0 c_3\tilde{\sigma}^2\right)^{(q_i+1)/2} > 0 \tag{57}$$

From Lemma 1, the guidance system is fixed-time convergent to the origin.

IV. SIMULATIONS AND RESULTS

In this section, various simulations are carried out for three missiles intercepting a target. The speed of the target is set to be $300 m/s$. The speed direction of the target is only changed by normal acceleration $a_T = 5g\cos(t)\ m/s^2$ during the whole flight, where g is the gravitational acceleration and $g = 9.8N/m$. The position of the target is (0m, 0m), and the initial heading angle of the target is 60°. The weighted adjacency matrix of these three missiles' communication topology is $A = [0\ 1\ 0;\ 1\ 0\ 1;\ 0\ 1\ 0]$. In addition, the max acceleration of three missiles is set to be $a_{i\max} = 20g$. Other parameters in the numerical experiment are listed in Table 1.

The parameters of the guidance law in the LOS direction are set as $h_1 = 1.5$, $h_2 = 1.5$, $m = 0.4$, $n = 1.6$, and $\gamma = 2$. Furthermore, the parameters of the guidance law in the normal direction of LOS are given by $\delta_1 = 1$, $\delta_2 = 8$, $\lambda_1 = 3$, $\lambda_2 = 1.5$, $k_1 = 4500$, $k_2 = 4500$, $p_i = 1.8$, $q_i = 0.8$, $\hat{\xi}_0 = \hat{\tau}_0 = 1000$, $\hat{\sigma}_0 = 0.4$, $c_1 = c_2 = c_3 = 1$, $d_1 = d_2 = d_3 = 5$ and $\theta_0 = 0.545$. For a fixed-time disturbance observer, the parameters are set to be $\eta_1 = 10$, $\eta_2 = 50$, $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 1.4$.

To validate the effectiveness of the designed FxTCG, simulations with the FTCG in [17] under the same initial conditions are carried out for comparison. Considering the disturbance observation problem, we investigate the original FxTCG law without FxTDO. The comparative simulation results are as shown in Figures. 2-10 and Table 2.

As shown in Fig. 2, it is obvious that all the three cooperative guidance laws can intercept the maneuvering target. Besides, Table 2 shows that the miss distances of these guidance laws in the scenarios are less than 0.67 m and the LOS angle errors are less than 0.23°. It is also revealed in Table 2 that the proposed FxTDO-FxTCG has higher precision in interception missions.

As can be seen in Fig. 3 and Fig. 4, the LOS angles q_i can reach the desired values q_{di} and the LOS angular velocities

TABLE 1. Initial conditions for three missiles.

Missile	Initial position (m, m)	Heading angle (°)	Desired angle (°)	Initial velocity (m/s)
M1	(5950,-1050)	15	5	590
M2	(4950,-850)	10	10	600
M3	(4100,1000)	-10	0	610
Target	(0,0)	60	-	300

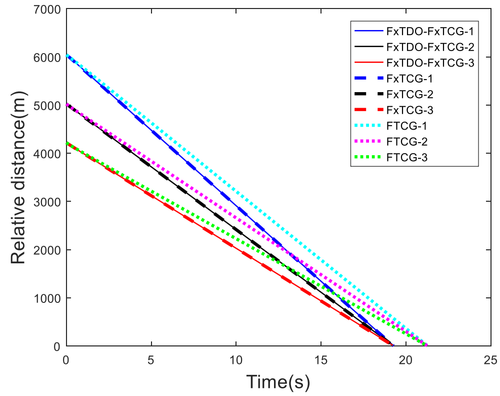

FIGURE 2. Relative distance between three missiles and the target.

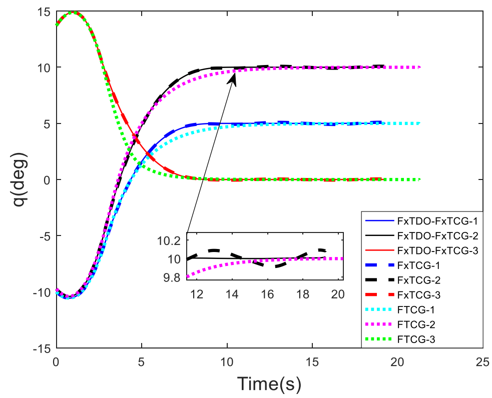

FIGURE 3. LOS angle.

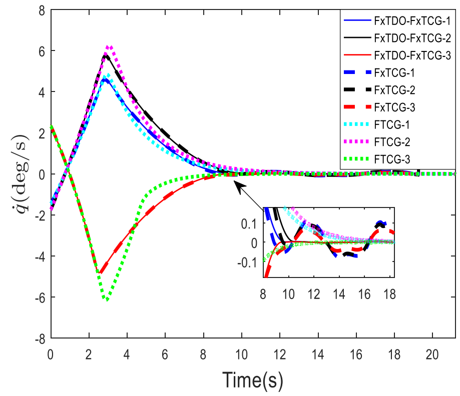

FIGURE 4. LOS angular velocity.

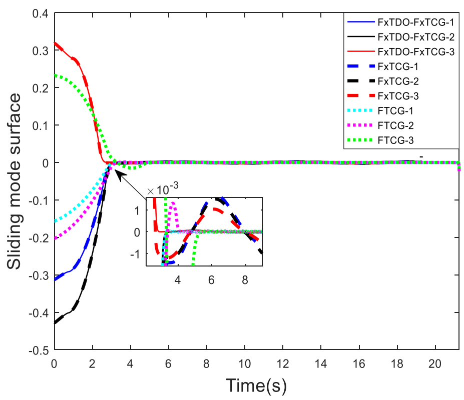

FIGURE 5. Sliding mode surface.

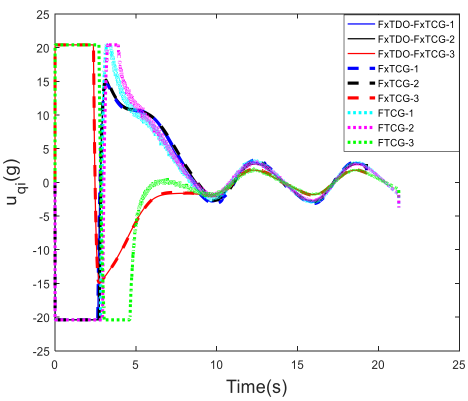

FIGURE 6. Acceleration command in the direction of LOS.

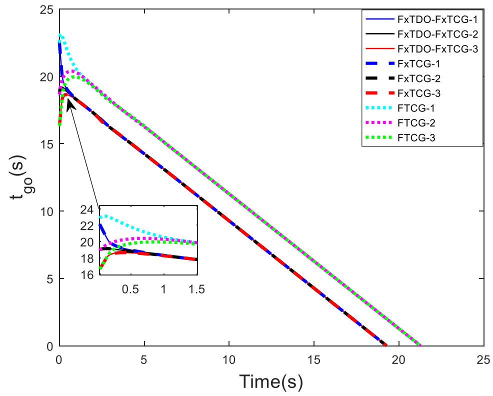

FIGURE 7. Time-to-go for three missiles.

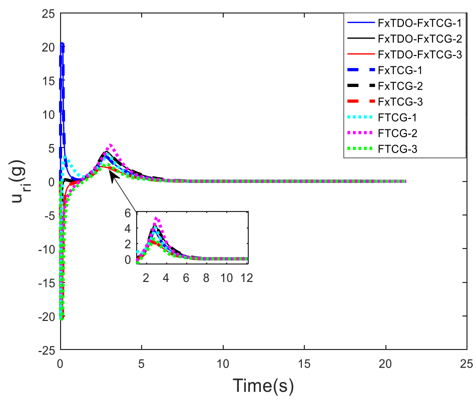

FIGURE 8. Acceleration command in the normal direction of LOS.

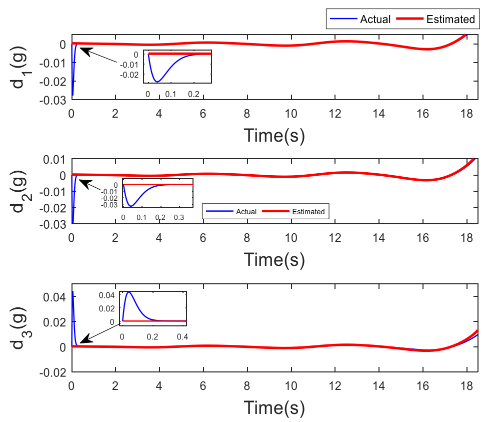

FIGURE 9. Actual and estimated values of external disturbances.

can also tend to zero rapidly for all the three cooperative guidance laws. Fig. 5 further demonstrates that the sliding mode surface of the FxTDO-FxTCG can converge to zero within the expected time. Moreover, curves of FxTDO-FxTCG in Fig. 2-Fig. 5 are relatively smooth. From the foregoing, the performance of the designed guidance law is better than those of the other two cooperative guidance laws.

In Fig. 6, the acceleration commands u_{qi} for the three missiles are relatively large at the beginning of the guidance. The reason is that a larger acceleration command enables $\dot{q}$ to approach zero rapidly and thus converging q to the desired value. In addition, it is worth noting that the acceleration command is chattering at the beginning of the guidance by FTCG. In contrast, the chattering is suppressed by the proposed guidance law.

As shown in Fig. 7, it indicates that t_{goi} of each missile has consistent convergence, which demonstrates the effectiveness of the proposed acceleration command u_{ri}. Furthermore, with the use of the FTCG, it takes about 1.4s for t_{goi} to reach consensus. By contrast, with the use of the FxTCG, it only takes around 0.5s for t_{goi} to reach the same value. Therefore, it is obvious that the proposed FxTCG enjoys a faster convergence rate. Fig. 8 gives the acceleration command u_{ri} during the

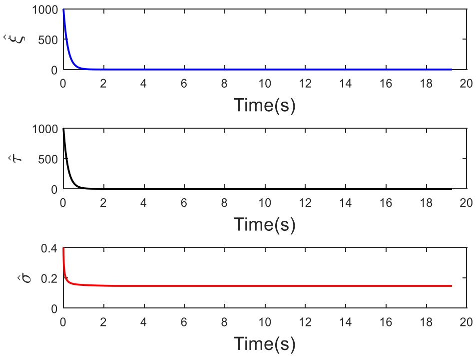

FIGURE 10. Adaptive parameters $\hat{\xi}_i$, $\hat{\tau}_i$ and $\hat{\sigma}_i$.

TABLE 2. Miss distances, impact LOS angles and interception times with different cooperative guidance laws.

	Missile	M1	M2	M3
FxTDO-FxTCG	Miss distance	0.12	0.16	0.26
	Impact LOS angle	0.001	0.001	0.001
	Interception time	19.27	19.27	19.27
FxTCG	Miss distance	0.55	0.62	0.51
	Impact LOS angle	0.15	0.23	0.12
	Interception time	19.45	19.45	19.45
FTCG	Miss distance	0.53	0.67	0.55
	Impact LOS angle	0.12	0.16	0.18
	Interception time	21.24	21.24	21.24

engagement. u_{ri} is used to adjust t_{goi} of multi-missiles such that t_{goi} achieves consistent convergence in the fixed time. Thus, u_{ri} is relatively large in the initial phase of the process.

It can be seen from Fig. 9 that the FxTDO has excellent estimation performance for the unknown target acceleration. The variation curves of the adaptive gains are described in Fig. 10, which illuminates the strong convergence and adaptability. Given the above discussion, the proposed FxTCG based on the FxTDO enjoys satisfactory convergence such that the guidance system has much less chattering and higher precision.

V. CONCLUSION

This paper presents the FxTDO based FxTCG for multiple missiles intercepting the maneuvering target. Based on the consensus protocol, we design a fixed-time guidance law in the LOS direction, which drives the time-to-go of the multi-missile system to achieve consensus within a fixed time. According to the target's acceleration estimation by FxTDO, we design the adaptive fixed-time guidance law in the normal of the LOS direction so that the desired LOS angles can be achieved within a fixed time. Meanwhile, stability analysis of our guidance law is conducted by adopting the Lyapunov methodology. Simulation results show that the designed FxTDO-FxTCG law has better convergence performance than the FTCG. Also, the FxTCG law with LOS angle and impact time constraints has strong robustness and adaptability. In our follow-up work, we will extend the proposed algorithm to the three-dimensional guidance law.

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