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Guaranteed Cost Control of Genetic Regulatory Networks With Multiple Time-Varying Discrete Delays and Multiple Constant Distributed Delays

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ABSTRACT This paper focus on the problem of guaranteed cost control for a class of genetic regulatory networks with multiple time-varying discrete delays and multiple constant distributed delays. Firstly, a novel method is proposed to establish a sufficient condition for the existence of guaranteed cost controller. The sufficient condition includes only several simple inequalities, which can easily solved by standard tool softwares, such as MATLAB. Secondly, the desired guaranteed cost controller is designed based on the solution of these inequalities. Thirdly, the proposed method is also available to the stabilization problem of genetic regulatory networks under consideration. Finally, the results of two numerical examples demonstrate the applicability of theoretical results. Compared with the existing results, this present paper has three merits: (i) Do not require to construct any Lyapunov–Krasovskii functional; (ii) the class of genetic regulatory networks under consideration is more general; (iii) the designed controller can be easily realized.

INDEX TERMS Genetic regulatory networks, guaranteed cost control, multiple discrete delays, multiple distributed delays.

I. INTRODUCTION

As one class of complex dynamic nonlinear systems, genetic regulation networks (GRNs) describe the interactions among mRNAs and proteins in gene expression [1], [2]. Now, the research on GRNs have been multi-disciplinary, such as mathematics, statistics, biology and medicine. Particularly, in recent years, one of hot topics in control theory at home and abroad is the analysis and design of GRNs. As a result, a great number of excellent works have been achieved. (see [3]–[13] and the references therein).

Generally, the aim of controlling a dynamic system is to seek a controller such that the resulting closed-loop system is asymptotically/exponentially stable and satisfies the requirement of certain performance index. When the performance index is taken as the quadratic cost function, the corresponding control problem is known as guaranteed cost

control (GCC). The GCC can give an upper bound of the quadratic cost function. The first work on GCC is presented by Chang and Peng in 1972 [14]. Then some representative works on this general topic are reported (see [8], [15]–[17] and the references therein). On the basis of Chang and Peng’s method, the GCC problem of uncertain linear systems is deeply studied [16]. Then the authors initially investigated the GCC problem of uncertain time-delay systems by employing the so-called Riccati equation method [18]. To decrease the computational complexity of the method, the linear matrix inequality (LMI) method is used to solve the GCC problem time-delay systems (see, e.g., [8], [15], [19]). Recently, the GCC problem has been further studied by many experts and scholars (see [20]–[25] and the references therein).

Due to the slow transcription and translation processes in gene expression, time delays are unavoidable. So, functional differential/difference equations have been employed to model GRNs. It is well known that time delays often result in poor performance and/or instability in dynamical

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systems [26]–[29]. Therefore, more effort has been devoted to the analysis and design of delayed GRNs based on the functional differential/difference equation models (see [3], [11], [13], [30]–[37] and the references therein). Up to now, the researches on delayed GRNs mainly focus on dynamic analysis and state estimation, while very few results are about control problems. To the best of authors' knowledge, the involved control problems include stabilization [30], [31], [38]–[42], H_∞ control [39], [40], [43], [44], sliding mode control [45], passivity control [46], gene circuit control [33], and GCC [8]. The so-called Lyapunov–Krasovskii functional (LKF) method is employed to design controllers in these literature except for [42]. The LKF method is mainly divided into two steps: (i) construct an appropriate LKF; and (ii) choose appropriate techniques to estimate the derivative/difference of LKF. However, there is no united approach to realize these two steps. Moreover, the LKF method may result in solving complex matrix inequalities, which requires a long compute time. An analysis technique is proposed in [42] to stabilize a class of delayed GRNs via periodically intermittent control. Therefore, it is necessary to find the other methods to design controllers for delayed GRNs.

Motivated by the above discussion, in this paper we will propose a new method to study the GCC problem for a class of GRNs with multiple time-varying discrete delays and multiple constant distributed delays. A sufficient condition for the existence of guaranteed cost controller is first investigated. The sufficient condition can easily checked by standard tool softwares, since it involves only several simple inequalities. From which, the desired guaranteed cost controller can be designed based on the solution of these inequalities. The applicability of the proposed method is presented by a pair of numerical examples. Compared with these mentioned literature on the control problems of delayed GRNs, this present paper has the following advantages: (i) No LKF requires to be constructed; (ii) the GRN model under consideration is more general; (iii) the proposed method to design controller can be easily realized.

The rest of this paper is organized as follows. In the next section, we will formulate the GCC problem to be addressed. The existence and design method of guaranteed cost controller will be presented in Section III. Two numerical examples will be demonstrated in Section IV, and the conclusions will be made in Section V.

Notations: Let $\mathbb{R}^{n \times m}$ be the set of all $n \times m$ matrices over the real number field \mathbb{R} , and set $\mathbb{R}^n = \mathbb{R}^{n \times 1}$. For given $X = [x_{ij}]$, $Y = [y_{ij}] \in \mathbb{R}^{n \times m}$, we say $X \leq Y$ ($X < Y$) if $x_{ij} \leq y_{ij}$ ($x_{ij} < y_{ij}$) for any $1 \leq i \leq n$ and $1 \leq j \leq m$. Particularly, if X and Y are symmetric, then the symbol $X < Y$ ($X \leq Y$) refers to the matrix $X - Y$ is negative definite (semi-definite). Let $\text{diag}(\cdot)$ and $\text{col}(\cdot)$ stand for a block-diagonal and -column matrix, respectively. Let $C([a, b], \mathbb{R}^n)$ be the \mathbb{R} -linear space consisting of all continuous functions $\theta : [a, b] \rightarrow \mathbb{R}^n$. The norm $\|\cdot\|_\tau$ on $C([-\tau, 0], \mathbb{R}^n)$ is defined by $\|\theta\|_\tau = \sup_{-\tau \leq s \leq 0} \|\theta(s)\|$, where $\|\cdot\|$ is the 2-norm on \mathbb{R}^n . Let $\langle n \rangle$ stand for the set $\{1, 2, \dots, n\}$.

II. PROBLEM FORMULATION

Consider the GRNs with multiple interval time-varying discrete delays and multiple constant distributed delays:

$$\begin{aligned} \dot{m}_i(t) = & -a_i m_i(t) + \sum_{j=1}^n b_{ij} f_j(p_j(t - \sigma_{ij}(t))) \\ & + \sum_{j=1}^n w_{ij} \int_{t-\delta_{ij}}^t f_j(p_j(s)) ds + u_i(t) + J_i, \end{aligned} \quad (1a)$$

$$\dot{p}_i(t) = -c_i p_i(t) + d_i m_i(t - \tau_i(t)) + v_i(t), \quad t \geq 0, \quad (1b)$$

$$m_i(s) = \varphi_i(s), \quad p_i(s) = \phi_i(s), \quad s \in [-\kappa, 0], \quad i \in \langle n \rangle, \quad (1c)$$

where the subscript i refers to the i -th node of GRN, $m_i : [-\kappa, +\infty) \rightarrow [0, +\infty)$ and $p_i : [-\kappa, +\infty) \rightarrow [0, +\infty)$ are the mRNA and protein concentrations at time t , respectively, $\kappa = \max\{\bar{\sigma}, \bar{\delta}, \bar{\tau}\}$ with $\bar{\sigma} = \max_{1 \leq i, j \leq n} \bar{\sigma}_{ij}$, $\bar{\delta} = \max_{1 \leq i, j \leq n} \delta_{ij}$ and $\bar{\tau} = \max_{1 \leq i \leq n} \bar{\tau}_i$, $\sigma_{ij} : [0, +\infty) \rightarrow [0, \bar{\sigma}_{ij}]$ and $\tau_i : [0, +\infty) \rightarrow [0, \bar{\tau}_i]$ are continuous functions representing the time-varying discrete delays, $\delta_{ij} \geq 0$ is the constant distributed delays, $a_i > 0$ and $c_i > 0$ are the degradation rates of mRNA and protein, respectively, $d_i > 0$ refers to the translation rate, J_i is the external input, $b_{ij}, w_{ij} \in \mathbb{R}$, $\varphi_i \in C([-\kappa, 0], [0, +\infty))$ and $\phi_i \in C([-\kappa, 0], [0, +\infty))$ are the initial functions, $u_i : [0, +\infty) \rightarrow \mathbb{R}$ and $v_i : [0, +\infty) \rightarrow \mathbb{R}$ are the control inputs, and $f_i : [0, +\infty) \rightarrow [0, 1)$ is the regulatory function in the Hill form, that is, $f_i(s) = s^{h_i}/(1 + s^{h_i})$ with the Hill constant $h_i \geq 1$.

When $\sigma_{ij}(t)$ and δ_{ij} are independent on the choice of i , the GRN model (1) with $u_i = v_i \equiv 0$ reduces into [3, (2.3)]. Clearly, f_i is a monotonically increasing function with saturation, and it satisfies

$$f_i(0) = 0, \quad |f_i(s_1) - f_i(s_2)| \leq \mu_i |s_1 - s_2| \quad (2)$$

for any different $s_1, s_2 \in [0, +\infty)$, where $\mu_i > 0$.

Definition 1: The pair $(\bar{m}, \bar{p}) \in \mathbb{R}^n \times \mathbb{R}^n$ with $\bar{m} = \text{col}(\bar{m}_1, \bar{m}_2, \dots, \bar{m}_n)$ and $\bar{p} = \text{col}(\bar{p}_1, \bar{p}_2, \dots, \bar{p}_n)$ is called a nonnegative equilibrium point of GRN (1) with $u_i = v_i \equiv 0$, if it satisfies $\bar{m} \geq 0, \bar{p} \geq 0$ and

$$\begin{aligned} -a_i \bar{m}_i + \sum_{j=1}^n (b_{ij} + w_{ij} \delta_{ij}) f_j(\bar{p}_j) + J_i &= 0, \\ -c_i \bar{p}_i + d_i \bar{m}_i &= 0, \quad i \in \langle n \rangle. \end{aligned}$$

Following the approach to prove [47, Theorem 1], one can easily show the following sufficient condition for the existence of nonnegative equilibrium of GRN (1) with $u_i = v_i \equiv 0$.

Proposition 1: For every $i \in \langle n \rangle$, let \mathbb{J}_i be the subset $\{j : b_{ij} + \delta_{ij} w_{ij} < 0\}$ of $\langle n \rangle$. Then GRN (1) with $u_i = v_i \equiv 0$ has at least one nonnegative equilibrium point, if one of the following (i) and (ii) holds:

- (i) $\mathbb{J}_i = \emptyset$ for all $i \in \langle n \rangle$.
- (ii) $\sum_{j \in \mathbb{J}_i} |b_{ij} + \delta_{ij} w_{ij}| \leq J_i$ for all $i \in \langle n \rangle$.

Let (m^*, p^*) be a nonnegative equilibrium of GRN (1) with $u_i = v_i \equiv 0$, and let $x_i(t) = m_i(t) - m_i^*$ and $y_i(t) = p_i(t) - p_i^*$ for any $i \in \langle n \rangle$ and $t \geq -\kappa$. Then

$$\begin{aligned} \dot{x}_i(t) &= -a_i x_i(t) + \sum_{j=1}^n b_{ij} g_j(y_j(t - \sigma_{ij}(t))) \\ &\quad + \sum_{j=1}^n w_{ij} \int_{t-\delta_{ij}}^t g_j(y_j(s)) ds + u_i(t), \end{aligned} \quad (3a)$$

$$\dot{y}_i(t) = -c_i y_i(t) + d_i x_i(t - \tau_i(t)) + v_i(t), \quad t \geq 0, \quad (3b)$$

$$x_i(s) = \vartheta_i(s), \quad y_i(s) = \theta_i(s), \quad s \in [-\kappa, 0], \quad i \in \langle n \rangle, \quad (3c)$$

where $g_j(y_j(\cdot)) = f_j(y_j(\cdot) + p_j^*) - f_j(p_j^*)$, $\vartheta_i(s) = \phi(s) - m_i^*$ and $\theta_i(s) = \phi(s) - p_i^*$.

From the relation between f_j and g_j , it follows from (2) that

$$g_j(0) = 0, \quad |g_j(s)| \leq \mu_j |s|, \quad \forall s \in (-\infty, +\infty). \quad (4)$$

By applying the state feedback controller

$$u_i(t) = k_i x_i(t), \quad v_i(t) = l_i y_i(t), \quad i \in \langle n \rangle, \quad t \geq 0 \quad (5)$$

to GRN (3), we derive the following closed-loop system:

$$\begin{aligned} \dot{x}_i(t) &= -\hat{a}_i x_i(t) + \sum_{j=1}^n b_{ij} g_j(y_j(t - \sigma_{ij}(t))) \\ &\quad + \sum_{j=1}^n w_{ij} \int_{t-\delta_{ij}}^t g_j(y_j(s)) ds, \end{aligned} \quad (6a)$$

$$\dot{y}_i(t) = -\hat{c}_i y_i(t) + d_i x_i(t - \tau_i(t)), \quad t \geq 0, \quad (6b)$$

$$x_i(s) = \vartheta_i(s), \quad y_i(s) = \theta_i(s), \quad s \in [-\kappa, 0], \quad i \in \langle n \rangle, \quad (6c)$$

where k_i and l_i are state feedback gains to be designed, $\hat{a}_i = a_i - k_i$ and $\hat{c}_i = c_i - l_i$.

Define a performance index J_c associated with the closed-loop system (6) by

$$J_c = \int_0^\infty \mathcal{X}^T(t) \mathcal{F} \mathcal{X}(t) dt \quad (7)$$

with

$$\begin{aligned} \mathcal{X} &= \text{col}(x(t), u(t), y(t), v(t)), \quad \mathcal{F} = \text{diag}(\mathcal{Q}_1, \mathcal{R}_1, \mathcal{Q}_2, \mathcal{R}_2), \\ x(t) &= \text{col}(x_1(t), \dots, x_n(t)), \quad y(t) = \text{col}(y_1(t), \dots, y_n(t)), \\ u(t) &= \text{col}(u_1(t), \dots, u_n(t)), \quad v(t) = \text{col}(v_1(t), \dots, v_n(t)), \end{aligned}$$

where $\mathcal{Q}_k^T = \mathcal{Q}_k > 0$ and $\mathcal{R}_k^T = \mathcal{R}_k > 0$ ($k = 1, 2$) are known real matrices.

Definition 2: We say that a state feedback controller in the form of (5) globally exponentially stabilizes GRN (3), if the zero equilibrium of the resulting close-loop system (6) is globally exponentially stable, that is, there exist positive constants α and γ such that

$$\|(x(t), y(t))\| \leq \alpha \|(\theta, \vartheta)\|_\kappa e^{-\gamma t}$$

for any $t \geq 0$ and $\theta, \vartheta \in C([-\kappa, 0], \mathbb{R}^n)$, where

$$\begin{aligned} \|(x(t), y(t))\| &= (\|x(t)\|^2 + \|y(t)\|^2)^{1/2}, \\ \|(\theta, \vartheta)\|_\kappa &= \max\{\|\theta(s)\|_\kappa, \|\vartheta(s)\|_\kappa\}. \end{aligned}$$

Definition 3: We say that a state feedback controller in the form of (5) is a guaranteed cost controller for GRN (3), if it globally exponentially stabilizes GRN (3) and satisfies $J_c \leq \chi \|(\theta, \vartheta)\|_\kappa$ for some scalar $\chi > 0$.

The aim of this paper is to design a guaranteed cost controller (5) for GRN (3).

III. GUARANTEED COST CONTROLLERS

In this section we will present a novel method to design guaranteed cost controllers for GRN (3).

Theorem 1: For given positive scalars $\gamma, \bar{\sigma}_{ij}, \delta_{ij}, \bar{\tau}_j$ and μ_j ($i, j \in \langle n \rangle$) and matrices $\mathcal{Q}_k^T = \mathcal{Q}_k > 0$ and $\mathcal{R}_k^T = \mathcal{R}_k > 0$ ($k = 1, 2$), the state feedback controller (5) with $k_i = \bar{x}_i^{-1} \hat{k}_i$ and $l_i = \bar{\eta}_i^{-1} \hat{l}_i$ is a guaranteed cost controller for GRN (3), if there exist positive scalars ξ_1 and ξ_2 , and vectors $\bar{x} := \text{col}(\bar{x}_1, \dots, \bar{x}_n) > 0$, $\bar{\eta} := \text{col}(\bar{\eta}_1, \dots, \bar{\eta}_n) > 0$, $\hat{k} := \text{col}(\hat{k}_1, \dots, \hat{k}_n)$ and $\hat{l} := \text{col}(\hat{l}_1, \dots, \hat{l}_n)$ such that

$$-A\bar{x} + \hat{k} + (\hat{B} + \hat{W})\mu\bar{\eta} < -\gamma\bar{x}, \quad (8)$$

$$-C\bar{\eta} + \hat{l} + \hat{D}\bar{x} < -\gamma\bar{\eta}, \quad (9)$$

$$\begin{bmatrix} -\xi_1 I_n & \Lambda_{\bar{x}} & \Lambda_{\hat{k}} \\ \Lambda_{\bar{x}} & -\mathcal{Q}_1^{-1} & 0 \\ \Lambda_{\hat{k}} & 0 & -\mathcal{R}_1^{-1} \end{bmatrix} \leq 0, \quad (10)$$

$$\begin{bmatrix} -\xi_2 I_n & \Lambda_{\bar{\eta}} & \Lambda_{\hat{l}} \\ \Lambda_{\bar{\eta}} & -\mathcal{Q}_2^{-1} & 0 \\ \Lambda_{\hat{l}} & 0 & -\mathcal{R}_2^{-1} \end{bmatrix} \leq 0, \quad (11)$$

where

$$\begin{aligned} A &= \text{diag}(a_1, \dots, a_n), \quad C = \text{diag}(c_1, \dots, c_n), \\ \hat{B} &= [|b_{ij}| e^{\gamma \bar{\sigma}_{ij}}]_{n \times n}, \quad \hat{W} = \left[\frac{|w_{ij}| e^{\gamma \delta_{ij}} - 1}{\gamma} \right]_{n \times n}, \\ \mu &= \text{diag}(\mu_1, \dots, \mu_n), \quad \hat{D} = \text{diag}(d_1 e^{\gamma \bar{\tau}_1}, \dots, d_n e^{\gamma \bar{\tau}_n}), \\ \Lambda_{\bar{x}} &= \text{diag}(\bar{x}_1, \dots, \bar{x}_n), \quad \Lambda_{\hat{k}} = \text{diag}(\hat{k}_1, \dots, \hat{k}_n), \\ \Lambda_{\bar{\eta}} &= \text{diag}(\bar{\eta}_1, \dots, \bar{\eta}_n), \quad \Lambda_{\hat{l}} = \text{diag}(\hat{l}_1, \dots, \hat{l}_n). \end{aligned}$$

Proof: First, we show that the state feedback controller (5) with $k_i = \bar{x}_i^{-1} \hat{k}_i$ and $l_i = \bar{\eta}_i^{-1} \hat{l}_i$ globally exponentially stabilizes GRN (3).

Choose $\alpha_1 > 0$ such that

$$\alpha_1 \bar{x} \geq \text{col}(1, 1, \dots, 1), \quad \alpha_1 \bar{\eta} \geq \text{col}(1, 1, \dots, 1). \quad (12)$$

For any but fixed $\theta, \vartheta \in C([-\kappa, 0], \mathbb{R}^n)$, define

$$u(t) = \alpha_1 \|(\theta, \vartheta)\|_\kappa e^{-\gamma t} \bar{\eta}, \quad t \in [-\kappa, \infty) \quad (13)$$

and

$$v(t) = \alpha_1 \|(\theta, \vartheta)\|_\kappa e^{-\gamma t} \bar{x}, \quad t \in [-\kappa, \infty). \quad (14)$$

Then

$$|y(s)| \leq u(s), \quad |x(s)| \leq v(s), \quad \forall s \in [-\kappa, 0]. \quad (15)$$

We claim that $|y(t)| \leq u(t)$ and $|x(t)| \leq v(t)$ for any $t > 0$. Assume on the contrary that there exists $t_0 > 0$ such that either $|y(t_0)| \not\leq u(t_0)$ or $|x(t_0)| \not\leq v(t_0)$. Set $t_1 = \inf\{t > 0 : |y(t)| \not\leq u(t) \text{ or } |x(t)| \not\leq v(t)\}$. By continuity of functions x and y , it is obtained that $t_1 > 0$ and one of the following statements (C1) and (C2) holds:

(C1) $|y(t)| \leq u(t)$, $t \in [0, t_1]$, $|x(t)| \leq v(t)$, $t \in [0, t_1]$, and there is $i_0 \in \langle n \rangle$ and $\varepsilon > 0$ such that

$$|x_{i_0}(t_1)| = v_{i_0}(t_1), \quad |x_{i_0}(t)| > v_{i_0}(t), \quad \forall t \in (t_1, t_1 + \varepsilon).$$

(C2) $|x(t)| \leq v(t)$, $t \in [0, t_1]$, $|y(t)| \leq u(t)$, $t \in [0, t_1]$, and there is $j_0 \in \langle n \rangle$ and $\varepsilon > 0$ such that

$$|y_{j_0}(t_1)| = u_{j_0}(t_1), \quad |y_{j_0}(t)| > u_{j_0}(t), \quad \forall t \in (t_1, t_1 + \varepsilon).$$

When (C1) holds, it follows from (4) and (6) that

$$\begin{aligned} D^+ |x_{i_0}(t)| \Big|_{t=t_1} &= \text{sgn}(x_{i_0}(t_1)) \dot{x}_{i_0}(t_1) \\ &\leq -\hat{a}_{i_0} |x_{i_0}(t_1)| + \sum_{j=1}^n |b_{i_0j}| |g_j(y_j(t_1 - \sigma_{i_0j}(t_1)))| \\ &\quad + \sum_{j=1}^n |w_{i_0j}| \int_{t_1 - \delta_{i_0j}}^{t_1} |g_j(y_j(s))| ds \\ &\leq -\hat{a}_{i_0} |x_{i_0}(t_1)| + \sum_{j=1}^n |b_{i_0j}| \mu_j |y_j(t_1 - \sigma_{i_0j}(t_1))| \\ &\quad + \sum_{j=1}^n |w_{i_0j}| \mu_j \int_{t_1 - \delta_{i_0j}}^{t_1} |y_j(s)| ds, \end{aligned}$$

where D^+ denotes the Dini upper-right derivative. Then one can easily derive from (C1) and (13)–(15) that

$$\begin{aligned} D^+ |x_{i_0}(t)| \Big|_{t=t_1} &\leq -\hat{a}_{i_0} \alpha_1 e^{-\gamma t_1} \|(\theta, \vartheta)\|_\kappa \mathfrak{F}_{i_0} \\ &\quad + \sum_{j=1}^n |b_{i_0j}| \mu_j \alpha_1 e^{-\gamma t_1} e^{\gamma \sigma_{i_0j}(t_1)} \|(\theta, \vartheta)\|_\kappa \eta_j \\ &\quad + \sum_{j=1}^n |w_{i_0j}| \mu_j \frac{e^{\gamma \delta_{i_0j}} - 1}{\gamma} \alpha_1 e^{-\gamma t_1} \|(\theta, \vartheta)\|_\kappa \eta_j \\ &\leq \alpha_1 e^{-\gamma t_1} \|(\theta, \vartheta)\|_\kappa \\ &\quad \times \left[-\hat{a}_{i_0} \mathfrak{F}_{i_0} + \sum_{j=1}^n \left(|b_{i_0j}| e^{\gamma \sigma_{i_0j}} + |w_{i_0j}| \frac{e^{\gamma \delta_{i_0j}} - 1}{\gamma} \right) \mu_j \eta_j \right]. \end{aligned}$$

This, together with $k_{i_0} = \mathfrak{r}_{i_0}^{-1} \hat{k}_{i_0}$, (8) and (14), implies that

$$D^+ |x_{i_0}(t_1)| < -\gamma \alpha_1 \|(\theta, \vartheta)\|_\kappa e^{-\gamma t_1} \mathfrak{r}_{i_0} = \dot{v}_{i_0}(t_1),$$

and hence $D^+ (|x_{i_0}(t)| - v_{i_0}(t)) \Big|_{t=t_1} < 0$. So, the function $|x_{i_0}(t)| - v_{i_0}(t)$ is decreasing within some neighbourhood of t_1 , which contradicts to (C1).

When (C2) holds, we have from (6) and (13)–(15) that

$$\begin{aligned} D^+ |y_{j_0}(t)| \Big|_{t=t_1} &= \text{sgn}(y_{j_0}(t_1)) \dot{y}_{j_0}(t_1) \\ &\leq -\hat{c}_{j_0} |y_{j_0}(t_1)| + d_{j_0} |x_{j_0}(t_1 - \tau_{j_0}(t_1))| \\ &\leq -\hat{c}_{j_0} \alpha_1 e^{-\gamma t_1} \|(\theta, \vartheta)\|_\kappa \eta_{j_0} \\ &\quad + d_{j_0} \alpha_1 e^{-\gamma t_1} e^{\gamma \tau_{j_0}(t_1)} \|(\theta, \vartheta)\|_\kappa \mathfrak{F}_{j_0} \\ &\leq \alpha_1 e^{-\gamma t_1} \|(\theta, \vartheta)\|_\kappa (d_{j_0} e^{\gamma \tau_{j_0}} \mathfrak{F}_{j_0} - \hat{c}_{j_0} \eta_{j_0}). \end{aligned}$$

Using $l_{j_0} = \eta_{j_0}^{-1} \hat{l}_{j_0}$, (9) and (13), we obtain

$$D^+ |y_{j_0}(t_1)| < -\gamma \alpha_1 e^{-\gamma t_1} \|(\theta, \vartheta)\|_\kappa \eta_{j_0} = \dot{u}_{j_0}(t_1).$$

This contradicts to (C2).

In summary, we have $|y(t)| \leq u(t)$ and $|x(t)| \leq v(t)$ for any $t \geq -\kappa$. Thus,

$$\begin{aligned} \|(x(t), y(t))\| &\leq \|(\mathbf{v}(t), \mathbf{u}(t))\| \\ &= (\|\mathbf{v}(t)\|^2 + \|\mathbf{u}(t)\|^2)^{\frac{1}{2}} \\ &= \alpha_1 e^{-\gamma t} \|(\theta, \vartheta)\|_\kappa (\|\mathfrak{r}\|^2 + \|\eta\|^2)^{\frac{1}{2}}, \quad \forall t \geq 0. \end{aligned}$$

Let $\alpha = \alpha_1 (\|\mathfrak{r}\|^2 + \|\eta\|^2)^{\frac{1}{2}}$. Then

$$\|(x(t), y(t))\| \leq \alpha e^{-\gamma t} \|(\theta, \vartheta)\|_\kappa \quad (16)$$

for any $t \geq 0$ and $\theta, \vartheta \in C([-\kappa, 0], \mathbb{R}^n)$, i.e., the zero equilibrium of the resulting close-loop system (6) is globally exponentially stable. By Definition 2, it is concluded that the state feedback controller (5) with $k_i = \mathfrak{r}_i^{-1} \hat{k}_i$ and $l_i = \eta_i^{-1} \hat{l}_i$ globally exponentially stabilizes GRN (3).

Second, we prove that $J_c \leq \chi(\|(\theta, \vartheta)\|_\kappa)$ for some nondecreasing function $\chi : [0, +\infty) \rightarrow [0, +\infty)$.

It follows from (10) and (11) that $\Lambda_\mathfrak{r} \mathcal{Q}_1 \Lambda_\mathfrak{r} + \Lambda_{\hat{k}} \mathcal{R}_1 \Lambda_{\hat{k}} \preceq \xi_1 I_n$ and $\Lambda_\eta \mathcal{Q}_2 \Lambda_\eta + \Lambda_{\hat{l}} \mathcal{R}_2 \Lambda_{\hat{l}} \preceq \xi_2 I_n$, and hence

$$\begin{aligned} \mathcal{Q}_1 + \Lambda_\mathfrak{r}^{-1} \Lambda_{\hat{k}} \mathcal{R}_1 \Lambda_{\hat{k}} \Lambda_\mathfrak{r}^{-1} &\preceq \xi_1 \Lambda_\mathfrak{r}^{-2}, \\ \mathcal{Q}_2 + \Lambda_\eta^{-1} \Lambda_{\hat{l}} \mathcal{R}_2 \Lambda_{\hat{l}} \Lambda_\eta^{-1} &\preceq \xi_2 \Lambda_\eta^{-2}. \end{aligned}$$

This, together with

$$\begin{aligned} \mathcal{X}^T(t) \mathcal{F} \mathcal{X}(t) &= x^T(t) (\mathcal{Q}_1 + \Lambda_\mathfrak{r}^{-1} \Lambda_{\hat{k}} \mathcal{R}_1 \Lambda_{\hat{k}} \Lambda_\mathfrak{r}^{-1}) x(t) \\ &\quad + y^T(t) (\mathcal{Q}_2 + \Lambda_\eta^{-1} \Lambda_{\hat{l}} \mathcal{R}_2 \Lambda_{\hat{l}} \Lambda_\eta^{-1}) y(t) \end{aligned}$$

for any $t \geq 0$, implies that

$$\begin{aligned} \mathcal{X}^T(t) \mathcal{F} \mathcal{X}(t) &\leq \xi_1 \max_{1 \leq i \leq n} \mathfrak{r}_i^{-2} \|x(t)\|^2 \\ &\quad + \xi_2 \max_{1 \leq i \leq n} \eta_i^{-2} \|y(t)\|^2 \\ &\leq \xi \| (x(t), y(t)) \|^2, \quad \forall t \geq 0, \end{aligned}$$

where $\xi = \max\{\xi_1 \max_{1 \leq i \leq n} \mathfrak{r}_i^{-2}, \xi_2 \max_{1 \leq i \leq n} \eta_i^{-2}\}$. Using (16), we drive that $\mathcal{X}^T(t) \mathcal{F} \mathcal{X}(t) \leq \xi \alpha^2 \|(\theta, \vartheta)\|_\kappa^2 e^{-2\gamma t}$ for any $t \geq 0$, and hence

$$J_c \leq \chi \|(\theta, \vartheta)\|_\kappa^2, \quad \chi = \frac{\xi \alpha^2}{2\gamma}. \quad (17)$$

By Definition 3, the proof is completed by combining the above two aspects. ■

Remark 1: From (17), it is observed that the performance index J_c can be viewed as a function of variables $\gamma, \xi_1, \xi_2, \alpha_1, \mathfrak{r}, \eta, \hat{k}$ and \hat{l} , and these variables are coupled by the LMIs (8)–(11). Due to the complexity of $\chi(\cdot)$, it is difficult to obtain the optimal (i.e., minimal) value of J_c . Now, for given $\gamma > 0$, we presents an approach to guarantee that the upper bound of J_c given by (17) is as small as possible. In other words, solve the following optimal problem:

$$\begin{aligned} \min & (\xi_1 + \xi_2) \\ \text{subject to} & \gamma, \xi_1 > 0, \xi_2 > 0, x > 0, y > 0, \hat{k}, \hat{l}, \\ & (8)–(11), (12). \end{aligned} \quad (18)$$

Remark 2: It is seen from (16) that γ is the decay rate of the resulting closed-loop system (6). So, it is desired that

γ is as large as possible. However, with the increase of γ , the solvability of inequalities (8) and (9) will decrease. So, one can use a testing method to obtain the maximal value of γ that guarantees the feasibility of those inequalities in Theorem 1.

Remark 3: There are three differences between [8] and the present paper: (i) The LKF method is used in [8], while no LKF is required in this paper; (ii) The GRN model in this paper is more general than one in [8]; and (iii) The guaranteed cost controller designed in this paper can ensure the global exponential stability of the resulting closed-loop system, but only asymptotic stability can be guaranteed in [8].

Remark 4: If the LMIs (10) and (11) are omitted, then the controller obtained from Theorem 1 can globally exponentially stabilize GRN (3).

IV. AN ILLUSTRATIVE EXAMPLE

In this section, we will demonstrate the effectiveness and the proposed method by two numerical examples.

Example 1: Consider GRN (1) with parameters: $n = 3$, $a_1 = 4$, $a_2 = 3.5$, $a_3 = 3$, $b_{11} = -0.1$, $b_{12} = -0.3$, $b_{22} = -0.4$, $b_{23} = -0.6$, $b_{31} = -0.8$, $b_{33} = -0.2$, $b_{13} = b_{21} = b_{32} = 0$, $w_{11} = 0.3$, $w_{12} = w_{22} = 0.5$, $w_{23} = w_{31} = 0.1$, $w_{33} = 0.7$, $w_{13} = w_{21} = w_{32} = 0$, $J_1 = 0.4$, $J_2 = J_3 = 1$, $c_1 = 4$, $c_2 = 2$, $c_3 = 3.7$, $d_1 = 0.7$, $d_2 = 0.9$, $d_3 = 1.3$, $\delta_{11} = 1.2$, $\delta_{12} = 0.2$, $\delta_{13} = 0.3$, $\delta_{21} = 1$, $\delta_{22} = 0.4$, $\delta_{23} = 3.1$, $\delta_{31} = \delta_{33} = 1.5$, $\delta_{32} = 2$, $f_i(s) = s^2/(1 + s^2)$, $\tau_i(s) = (\sin s + 1)/4$ and

$$\sigma_{ij}(s) = \begin{cases} (\cos s + 1)/8, & \text{if } (i, j) \in \{(1, 3), (2, 2), (3, 2)\}, \\ (\sin s + 1)/4, & \text{if } (i, j) \in \{(1, 2), (2, 1), (2, 3)\}, \\ 1, & \text{if } (i, j) \in \{(1, 1), (3, 1), (3, 3)\} \end{cases}$$

for any $s \geq 0$ and $i \in \langle 3 \rangle$.

It is clear that $\mu_1 = \mu_2 = \mu_3 = 3\sqrt{3}/8$, $\bar{\tau}_1 = \bar{\tau}_2 = \bar{\tau}_3 = 1/2$, $\bar{\sigma}_{13} = \bar{\sigma}_{22} = \bar{\sigma}_{32} = 1/4$, $\bar{\sigma}_{12} = \bar{\sigma}_{21} = \bar{\sigma}_{23} = 1/2$, $\bar{\sigma}_{11} = \bar{\sigma}_{31} = \bar{\sigma}_{33} = 1$, $\mathbb{J}_1 = \{2\}$, $\mathbb{J}_2 = \{2, 3\}$ and $\mathbb{J}_3 = \{1\}$. Thus, it is derived that $\sum_{j \in \mathbb{J}_i} |b_{ij} + \delta_{ij}w_{ij}| \leq J_i$ for all $i \in \langle 3 \rangle$, and hence, by Proposition 1, the GRN under consideration has at least one nonnegative equilibrium. Furthermore, by direct computation, it is obtained that (m^*, p^*) with $m^* = \text{col}(0.0992, 0.2837, 0.3372)$ and $p^* = \text{col}(0.0174, 0.1276, 0.1185)$ is a nonnegative equilibrium of the considered GRN. By moving the nonnegative equilibrium (m^*, p^*) to the origin, we can transform the considered GRN into the form (3).

The performance index J_c associated with the considered GRN is given by (7) with

$$\mathcal{Q}_1 = \text{diag}(2, 2, 2), \quad \mathcal{R}_1 = \text{diag}(0.2, 0.2, 0.2), \\ \mathcal{Q}_2 = \text{diag}(3, 3, 3), \quad \mathcal{R}_2 = \text{diag}(0.1, 0.1, 0.1).$$

Take $\gamma = 0.5$. Then $\hat{D} = \text{diag}(0.8988, 1.1556, 1.6692)$, and

$$\hat{B} = \begin{bmatrix} 0.1649 & 0.3852 & 0 \\ 0 & 0.4533 & 0.7704 \\ 1.3190 & 0 & 0.3297 \end{bmatrix},$$

$$\hat{W} = \begin{bmatrix} 0.4933 & 0.1052 & 0 \\ 0 & 0.2214 & 0.7423 \\ 0.2234 & 0 & 1.5638 \end{bmatrix}.$$

By employing the toolbox YALMIP of MATLAB, one can obtain a feasible solution of the LMIs (8)–(11) as follows:

$$\xi_1 = 19.7290, \quad \varkappa = \text{col}(1.8659, 1.8134, 1.7478), \\ \xi_2 = 18.6677, \quad \eta = \text{col}(1.4511, 1.2968, 1.3841), \\ \hat{k} = \text{col}(-1.5530, -1.8256, -2.2249), \\ \hat{l} = \text{col}(-2.8990, -5.6692, -4.1993).$$

By Theorem 1, the state feedback controller (5) with

$$k_1 = -1.7663, \quad k_2 = -2.5050, \quad k_3 = -3.4160, \\ l_1 = -3.5904, \quad l_2 = -5.6428, \quad l_3 = -4.7735$$

is a guaranteed cost controller for the GRN under consideration. Moreover, choose $\alpha_1 = 0.7712$. Then $\alpha_1 \varkappa \geq \text{col}(1, 1, 1)$ and $\alpha_1 \eta \geq \text{col}(1, 1, 1)$. Following the proof of Theorem 1, we can derive $\alpha = 3.0391$ and $\xi = 11.1004$, and hence $J_c \leq 102.5235 \|\theta, \vartheta\|_{\kappa}^2$ with $\kappa = 3.1$. If the initial functions of the resulting closed-loop system are taken as $\vartheta_i(t) \equiv \vartheta_i$ and $\theta_i(t) \equiv \theta_i$ for all $i \in \langle 3 \rangle$ and $t \in [-3.1, 0]$, where $|\vartheta_i| \leq 1$ and $|\theta_i| \leq 1$, then $J_c \leq 102.5235$. In addition, we take 100 values of such initial functions by using the function rand in MATLAB, the simulation results present that all of the state trajectories of the resulting closed-loop system converge to the origin. Partial simulation results are shown in Figures 1–4. From which, it is seen that the closed-loop state trajectories converge quicker than the open-loop ones, which shows the effectiveness of the designed controller.

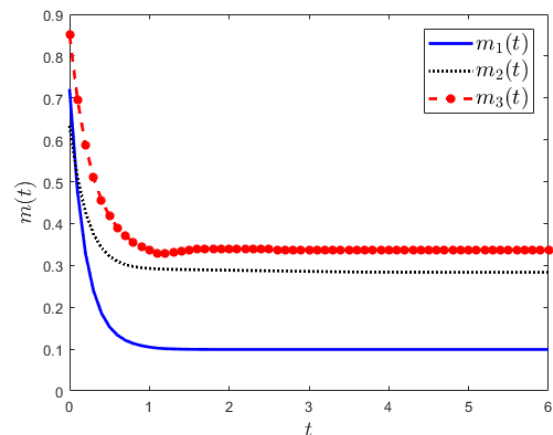


FIGURE 1. State trajectories of the open-loop system (Example 1).

For the GRN under consideration, the maximal value of γ that guarantees the feasibility of those inequalities in Theorem 1 is 4.2006. In this case, it is obtained that $J_c \leq 4.0633 \times 10^{12} \|\theta, \vartheta\|_{\kappa}^2$ with $\kappa = 3.1$, which provides a very rough bound of J_c . Therefore, in the practical problems, it is necessary to consider a balance between the decay rate γ and J_c 's bound. For this end, for several different values of $\gamma > 0$, the bounds of J_c obtained by solving the optimal problem (18) are given in Table 1.

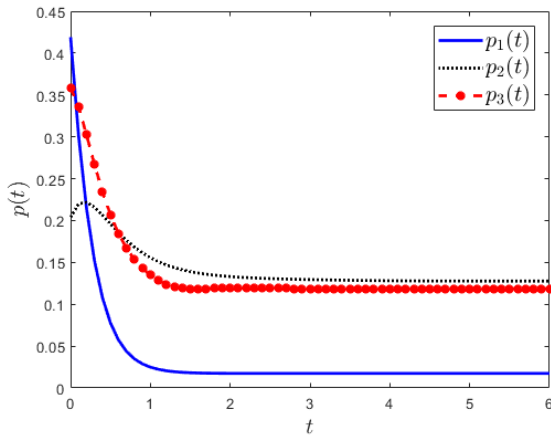


FIGURE 2. State trajectories of the open-loop system (Example 1).

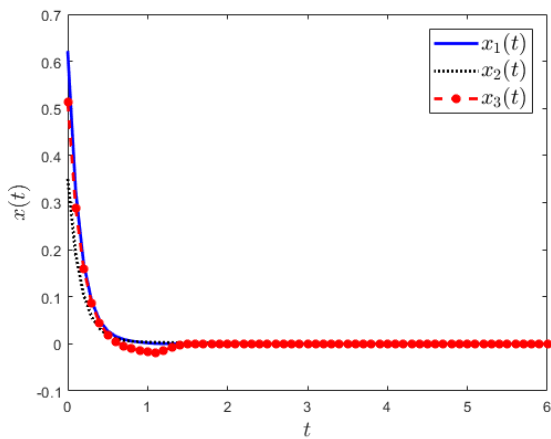


FIGURE 3. State trajectories of the closed-loop system (Example 1).

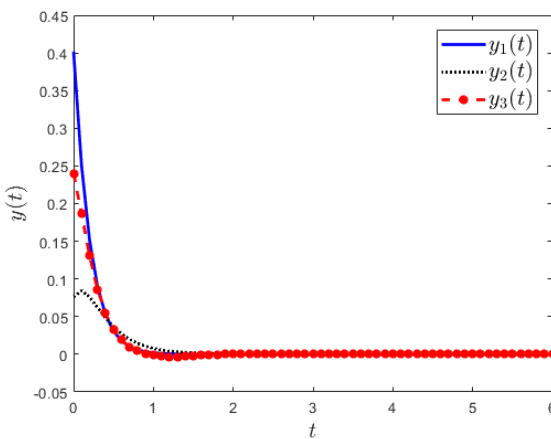


FIGURE 4. State trajectories of the closed-loop system (Example 1).

TABLE 1. The upper bounds of J_c for different γ .

γ	0.1	0.5	1	2
bounds of J_c	136.0357	102.5235	81.1388	48435

Example 2: Consider GRN (1) with parameters: $n = 2$, $a_i = 2.5$, $c_i = 4$, $d_i = 1.8$, $J_i = 1.64$, $b_{ij} = 0$, $w_{ii} = 1$, $w_{ik} = -1$, $\delta_{ij} = 1$, $\tau_i(s) \equiv 6$ and $f_i(s) = s^2/(1+s^2)$ for $s \geq 0$ and $i, j, k = 1, 2$ with $i \neq k$.

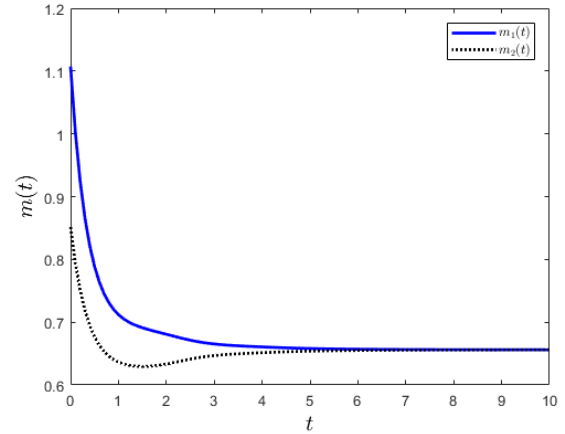


FIGURE 5. State trajectories of the open-loop system in Example 2.

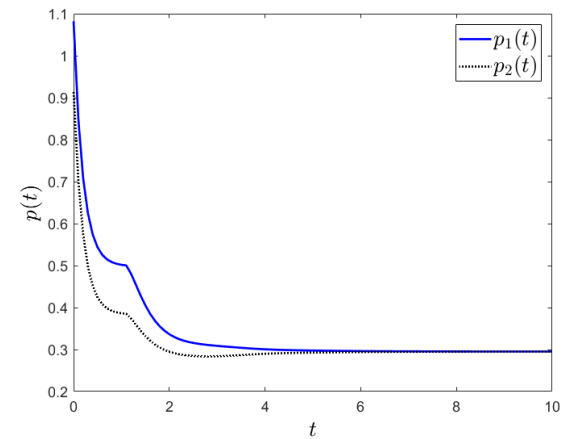


FIGURE 6. State trajectories of the open-loop system in Example 2.

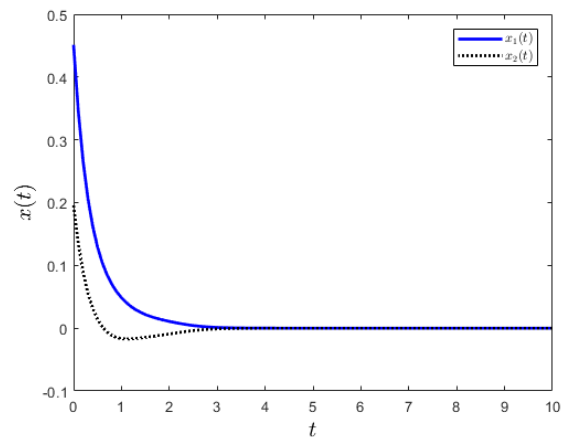


FIGURE 7. State trajectories of the closed-loop system in Example 2.

Clearly, $\mu_i = 0.65$, $\bar{\tau}_i = 1$, $\mathbb{J}_1 = \{2\}$ and $\mathbb{J}_2 = \{1\}$, and hence $\sum_{j \in \mathbb{J}_i} |b_{ij} + \delta_{ij} w_{ij}| = 1 < 1.64 = J_i$ for $i = 1, 2$. Applying Proposition 1, we obtain that the considered GRN has at least one nonnegative equilibrium. Furthermore, (m^*, p^*) with $m^* = \text{col}(0.6560, 0.6560)$ and $p^* = \text{col}(0.2952, 0.2952)$ is a nonnegative equilibrium. By moving (m^*, p^*) to the origin, one can transform the GRN under consideration into the form (3).

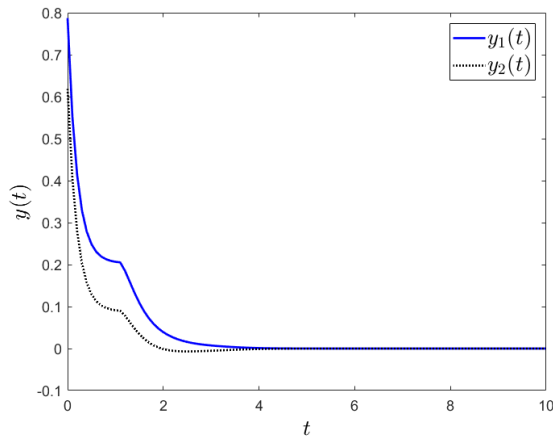


FIGURE 8. State trajectories of the closed-loop system in Example 2.

Take $\gamma = 0.45$, $\tau = \text{col}(1, 1)$, $\eta = \text{col}(0.7952, 0.7952)$, $\hat{k} = \text{col}(-0.1, -0.1)$ and $\hat{l} = \text{col}(0, 0)$. Then the LMIs (8) and (9) are feasible. By Remark 4, the state feedback controller (5) with $k_1 = k_2 = -0.1$ and $l_1 = l_2 = 0$ globally exponentially stabilizes the considered GRN, which claims that (m^*, p^*) is the unique nonnegative equilibrium. The simulation results of the open-loop and closed-loop state trajectories are presented in Figures 5–8, respectively, which shows the effectiveness of the obtained stabilization controller.

For the GRN under consideration, the stabilization conditions given in [42, Remark 4] are not satisfied. Therefore, the control method proposed in [42, Remark 4] is not available to the example.

V. CONCLUSION

This paper has dealt with the GCC problem of GRNs with multiple time-varying discrete delays and multiple constant distributed delays. A novel method is first proposed to investigate a sufficient condition for the existence of guaranteed cost controller. The sufficient condition is to solve only several simple inequalities, which can be proceeded by employing the MATLAB. Furthermore, the expired guaranteed cost controller is designed based on the solution of these inequalities. It should be mentioned that the proposed method can be also applied to design stabilization controller for the class of GRNs under consideration. The obtained theoretical results are explained by a pair of numerical examples.

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