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Modeling and Analysis of Performance Degradation Data for Reliability Assessment: A Review

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ABSTRACT Reliability engineering plays an important role in the design, manufacture, maintenance, and replacement of industrial products. Over the last few decades, accelerated degradation testing (ADT) has been largely utilized to shorten test durations, reduce the samples needed, and provide sufficient degradation data to ensure the effective reliability assessment of the concerned products. Meanwhile, performance degradation modeling has been recognized as an essential approach to help researchers and producers understand the health conditions of the deteriorating systems. However, the diversity in reliability tests, degradation models, and statistical analysis techniques has increased the difficulty in selecting appropriate reliability assessment methods in specific scenarios. Besides, there are no systematic reviews focused on modeling and analysis of performance degradation data. Therefore, this paper aims to (1) present ADT fundamentals, including the basic theory, ADT methods, accelerated stress variables, type of acceleration models, as well as ADT optimization, (2) comprehensively review current states and future challenges in degradation modeling, (3) discuss the problem of model mis-specification and compare different approaches for parameter estimation, (4) highlight future opportunities and possible directions deserving further research.

INDEX TERMS Accelerated degradation testing, acceleration models, degradation modeling, parameter estimation, reliability assessment.

I. INTRODUCTION

The evolving industrial societies have been characterized by the fast pace of newly developed products appearing on the market. Meanwhile, today's customers gradually pay more attention to product performance characteristics (PCs), such as reliability, multifunction, safety, as well as the cost performance. Therefore, competitive commercial products are supposed to possess high-reliability characteristics at relatively low costs. At present, industrial systems have been improved with the development of material science and manufacturing levels, which have also led to an increasing number of long-lifetime products. For these items, it tends to be increasingly time-consuming and costly to collect sufficient degradation data or failure information for reliability assessment by using traditional reliability tests. To shorten the test time required and reduce costs, accelerated testing (AT) technology is adopted to stimulate the potential defects and accelerate the degradation process of the concerned products at high stress levels. When the applied stress loading does not change the failure mechanisms of the product, the degradation data collected can be extrapolated to normal stress levels through an appropriate degradation modeling and statistical analysis method, and thus reliability assessment for the product at a normal stress level can be completed. With the use of AT technology, accelerated degradation modeling has been recognized as an essential toolkit that can be used to assess product degradation levels and health conditions based on lifetime distributions and degradation data [1], as shown in Fig. 1.

Currently, the advancements in knowledge, approaches and simulation techniques, the increase in information gain and data availability, offer new opportunities of modeling, analysis, and assessment for reliability engineering. At the same time, the emergence of highly reliable products has also promoted the fast development of accelerated reliability

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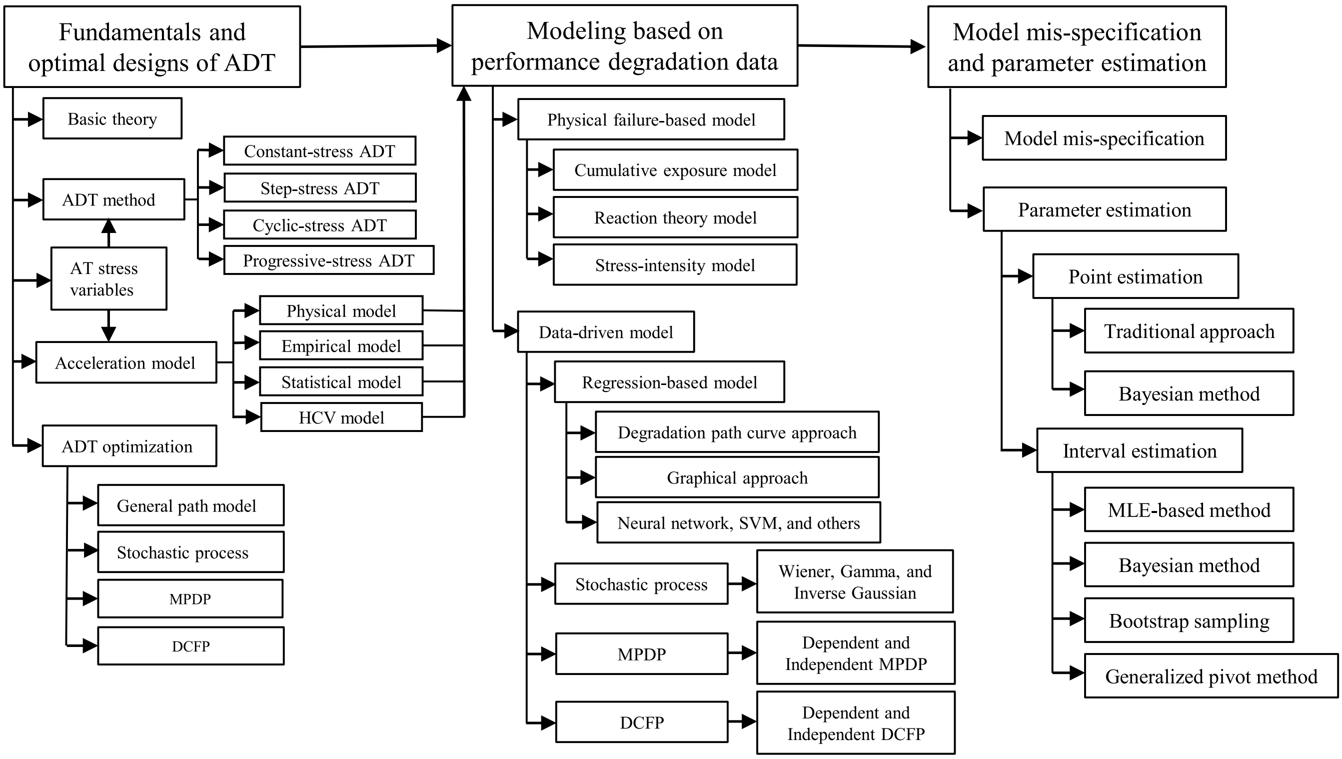

FIGURE 1. Ideas behind accelerated performance degradation modeling.

tests for reliability assessment. For instance, AT methods have evolved from constant-stress [2], [3] to step-up-stress [4], step-down-stress [5], [6], progressive-stress [7], [8], and cyclic-stress approaches [9], [10]. Being exposed to higher stress levels, modern engineering systems will generate more unknown failure mechanisms, random uncertainties, and interactions, especially in multi-component systems [11]. At this stage, the multi-source variability, such as the nonlinearity [12], [13], individual differences [14], [15], environmental stress factors [16], [17], measurement errors [18], [19], the temporal variability [20], model uncertainties [21], [22], and change points [23], has gradually been taken into account in performance degradation modeling. Besides, a few scholars have carried out research on multiple performance degradation processes (MPDPs) [24], dependent competing failure processes (DCFPs) [25], and the degradation analysis under dynamic environmental conditions [26].

In the literature, researchers have reviewed several critical issues in modeling and analysis of accelerated degradation data for reliability assessment, such as the optimal design of ADT, degradation modeling, statistical inference, as well as engineering applications. For instance, Nelson [1] presented a comprehensive review of degradation models, basic theories, statistical analysis methods, and test plans for employing ADT technology in reliability modeling. Besides, a large number of ADT plans and successful applications are presented in [27], [28], which motivated extensive practitioners and researchers to study ADT optimization under different objectives and predefined constraints. Some underlying ideas behind ADT planning are briefly summarized by Meeker and Escobar [29]. Recently, Limon *et al.* [30] comprehensively reviewed the optimal design of ADT and methods for statistical analysis in accelerated degradation modeling. Elsayed [31] provided an overview of ADT planning, resulting in degradation data for a limited test duration, as well as the use of performance degradation data for maintenance strategy-making and reliability assessment. In addition, Ye and Xie [32] have reviewed ADT planning for the general path models and the stochastic processes.

It is not very difficult to find from the reviews mentioned above that current studies focus more on ADT planning and optimization, and lack a comprehensive review on reliability modeling and statistical analysis of accelerated degradation data. Currently, degradation modeling is full of empirical experience and unrealistic assumptions, thus leading to low reliability of the assessment results. Therefore, this paper aims to fill this gap and concentrates on several essential aspects for accelerated degradation analysis, such as ADT fundamentals, ADT optimization, type of degradation modeling, model mis-specification, and parameter estimation. To this end, this paper is organized as follows. In section 2, we introduce ADT fundamentals, including the underlying theory, ADT methods, accelerated stress variables, and several types of commonly used acceleration models. The optimal design of ADT is also recalled in this section. Then, different types of degradation modeling methods are explored in section 3, which cover the physical failure-based methods and the data-driven methods. Because of the consideration of different applications, we paid great attention to reviewing several important modifications made to data-driven modeling approaches. Since the statistical analysis is an indispensable aspect of reliability assessment, section 4 is devoted to discuss the problem of model mis-specification and compare different methods for parameter estimation. Future opportunities and possible directions in modeling and analysis of accelerated degradation data are highlighted in section 5. Finally, section 6 closes this paper.

II. FUNDAMENTALS AND OPTIMAL DESIGNS OF ADT

A. FUNDAMENTALS OF ADT

In reliability engineering, the term ''acceleration'' usually means making ''time'' go more quickly so that degradation information can be obtained more rapidly [33]. Generally, an acceleration method is to increase the usage rate or employ higher stress levels during reliability tests. In order to develop an efficient acceleration method, scholars need a fundamental understanding of ADT technology, including the failure mechanism of a product, type of accelerated stresses applied, as well as the degradation process. The methods and ideas behind ADT are presented in the following subsections.

1) THE BASIC THEORY

In the last century, it would take a long time for a highly reliable product to fail, even under severe conditions. Nelson has exerted an example to show a costly effort made to remove a high-stress-induced failure mode that would never occur in normal use ([1], page 38). To overcome this challenge, accelerated life testing (ALT) is developed as a remedy to accelerate the failure process of the concerned product, thus reducing the time and costs to obtain sufficient degradation data. Unfortunately, some highly reliable products still have difficulties failing within a predefined test duration, even adopting ALT methods. Thus, little to no failure information is available. In the 1960s, Gertsbackh *et al.* [34] found that the degradation process could be accelerated by increasing stress levels applied to these items, and proposed that degradation data may be helpful in modeling the deteriorations of high-reliability products. Under Pieruschka's assumption [23], the lifetime of a product under use conditions and high stress levels are assumed to follow the same distribution with some stress-dependent model parameters, which opened up a new way to study reliability assessment for long-lifetime products.

Generally speaking, accelerated degradation information mainly includes three main elements, namely, the accelerated time, performance degradation, and the stress levels applied. Degradation modeling and statistical analysis are two indispensable aspects for the implementation of reliability assessmeny. An appropriate degradation model is a key point for modeling degradation data and representing the reliability of the concerned products. Meanwhile, a flexible parameter estimation method is a crucial point for the statistical analysis of degradation data. In most reliability assessment frameworks, degradation models are developed with an in-depth analysis of the failure mechanism or degradation data, in which stress-dependent parameters can also be determined. Within the incorporation of acceleration stress variables, unknown model parameters in accelerated degradation models are obtained by using the traditional statistical analysis methods or computer-based techniques. Then, a suitable lifetime distribution is utilized to fit product pseudo-lifetime. Finally, the assessment results are extrapolated to normal working conditions by combining environmental stresses with the failure threshold of concerned PCs, and thus the reliability assessment of the product at normal stress levels can be completed.

2) TYPES OF ADT METHOD AND VARIABLE

Among all the AT techniques, ALT and ADT are two essential categories that have been widely used in reliability engine-eering [30]. However, since ALT cannot always work well when accelerating the degradation process of long-lifetime products, it is necessary to either increase the sample size or prolong the test time. To deal with these challenges, scholars proposed the way of adopting accelerated degradation data for reliability assessment [34]. In ADT technology, stresses that exceed normal levels are employed to accelerate deteriorations of a product, obtain degradation information, and then model the degradation process for reliability modeling under normal stress levels. Generally speaking, the use of ADT overcomes many defects in ALT methods, e.g., only recording the failure time, neglecting specific failures or degradation changes. In addition, ADT makes up for a lack of failure data in ALT, thus significantly improving the evaluation efficiency. At this stage, researchers have paid a huge amount of attention to ADT technology, which has been a research hotspot in the field of reliability engineering.

After nearly 60 years' development, ADT methods can be classified as constant-stress ADT (CSADT) [35], [36], step-up-stress ADT (SUADT) [4], [37], step-down-stress ADT (SDADT) [5], [38], progressive-stress ADT (PSADT) [8], and cyclic-stress ADT [10], as shown in Fig. 2. Among these AT approaches, CSADT received the most applications for the convenience of stress application and statistical inference. However, lengthy testing durations are needed in this method when researchers cannot collect degradation data in a given test duration. To deal with this problem, step-stress loading can yield degradation relatively quicker than CSADT. Unfortunately, it is difficult to estimate unknown parameters and extrapolate reliability results to use conditions when using SSADT. As a practical alternative, progressive-stress loading is another accepted stress application where testing samples are subjected to a continually increasing stress over the test period [8]. Finally, cyclic-stress loading may be the best choice when test units are exposed to higher repetitive stress loading, e.g., sinusoidal voltage or fatigue stress [39].

The selection of accelerated stress variables is one of the most crucial steps for reasonably using ADT technology, which depends on a full understanding of the use conditions and the failure mechanisms of the concerned products. For example, temperature and current stresses are often utilized to accelerate failures of batteries, which tend to fail because of over-voltage or overheating at normal working conditions. Humidity, current, lighting, and vibration are usually applied to damage rubber items [40], whereas temperature, humidity, current, and voltage are used to accelerate the deteriorations of electronic equipment. At present, some commonly used accelerated stresses include temperature [41], [42], humidity [43], [44], mechanical stress [45], [46], current [47], [48], voltage [49], [50], ultraviolet radiation [51], as well as some combined stresses [52]–[54].

Furthermore, how to decide the number of accelerated stresses to be utilized in ADT is another critical problem. At present, single-stress ADT has received extensive applications in reliability engineering since it is easier to be implemented and verified. However, engineering products may experience several environmental stresses at use conditions, and therefore, some multi-stress ADT approaches have gradually attracted scholars' attention. Unfortunately, the interactions between different degradation processes and components are tough to be monitored. Despite these challenges, investigations on the influence of temperature and voltage [55], temperature and current [56], and humidity and temperature [57] on the products have been conducted.

3) ACCELERATION MODEL

In order to capture accelerated effects in degradation analysis, acceleration models are developed to incorporate accelerated stress variables into degradation models. Generally speaking, acceleration modeling is also based on an in-depth analysis of the stress loading applied, working conditions, and the failure mechanism of the products. It has been recognized as the basis for extrapolation of reliability indicators, which will directly impact the accuracy of reliability assessment results. In the past, Meeker and Escobar [33] have reviewed physical failure-based relationships, e.g., the Arrhenius model and the Eyring model. However, there are some other models that have been successfully utilized in accelerated degradation modeling. Therefore, we decide to summarize acceleration models from a different perspective and divide them into three categories, namely, the physical acceleration model, the empirical acceleration model, and the statistical acceleration model.

a: PHYSICAL ACCELERATION MODEL

In this type of acceleration modeling, if the failure mechanism of a product is thoroughly understood, then the relationships between model parameters and the accelerated stress can be derived based on the physical process or chemical reaction laws. Specifically, the Arrhenius model and the Eyring model are the most commonly used physical acceleration models, which have gained considerable acceptance because of many successful applications in addressing engineering problems.

Dating back to the 1880s, the Swedish scientist Arrhenius first proposed the Arrhenius relationship when he found that chemical reaction rates were approximately proportional to the applied temperature after conducting massive experiments. Since then, this physics-based acceleration model has received wide applications for incorporating the temperature stress in accelerated degradation modeling, and a large number of products are designed to damage by thermal stress, e.g., rubber [9], batteries [42], carbon film resistors [58], light-emitting diodes (LED) [59], and electrical connectors [35], [60]. In these cases, the Arrhenius relationship or its logarithmic-linear forms are the best choice for acceleration modeling [56], [61].

Another physical acceleration relationship is the Eyring model, which is not limited to describing chemical reaction rates under temperature-accelerated conditions [33]. It is worth noting that, compared with the Arrhenius relationship, the Eyring model further considers the effects of material properties, working conditions, activation energy, and reaction dynamics on the degradation rate of testing samples. Within the term of the reaction rate, this model can be written as:

$$R(T, X) = \varphi \times T^m \times \exp\left(-\frac{k_1}{k \times T}\right) \times \exp\left(k_2 X + \frac{k_3 X}{kT}\right) \tag{1}$$

where T denotes temperature, and X is an additional non-thermal accelerating variable. k_1, k_2, k_3 are characteristics of a certain chemical reaction or physical process, and R denotes the reaction rate.

The Eyring model is mainly used to describe accelerating effects that thermal and non-thermal stresses have on the rate of a simple chemical reaction [42], [62]. Over the last few decades, considerable attention has been paid to extending this model for acceleration degradation modeling. For example, Mejdoubi and his co-authors [63] employed voltage and temperature to accelerate the degradation process of super-capacitors, and then incorporated the aging effects using the generalized Eyring relationship. Besides, the current is another environmental stress that has been combined with other stress factors to accelerate chemical reaction processes. Applications of the combined stresses can be seen in the reliability analysis of electrical connectors [52], lumen [56], and memory devices [64]. In the literature, Elsayed and Liao [65] simply multiplied the Arrhenius model representing temperature with the inverse power law model describing current to obtain a temperature-current acceleration model. However, the interactions between these chemical reactions are neglected, thus leading inaccurate predictions. For improvement, this over-simplified model is replaced with the generalized Eyring model in Chiang's work [56]. Moreover, Peck and Zierdt [66], Luvalle *et al.* [67], and Klinger [68] employed a modified Eyring model to study the aging effects of temperature and humidity on products. Notably, Klinger

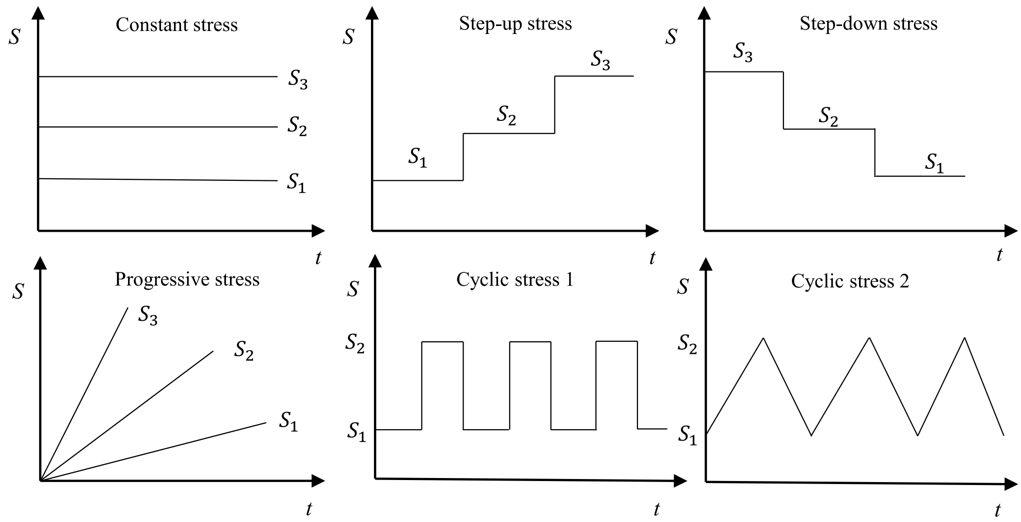

FIGURE 2. Different types of ADT loading method [30].

[68] replaced the term of relative humidity in the temperature-humidity Eyring model using a logistic transformation. More applications of the generalized Eyring model can be seen in the characterization of other stress variables, such as vibration and current [52], as well as temperature and lighting [69].

The extrapolation results of reliability assessment based on an understanding of product failure mechanisms are generally more reliable. Moreover, some complex reaction processes can be easily described by using these physics-based models. Unfortunately, it is impossible to understand all the failure details of the concerned items when the degradation process is disturbed by environmental stresses. Also, each product may possess several activation energy values, which are closely associated with particular failure patterns. When thermal stress exceeds a certain threshold, it will definitely lead to non-Arrhenius behavior [70]. As a result, the statistical analysis process tends to be extremely complicated. Since activation energy is related to functional materials of the product, the Arrhenius model and the Eyring model may not be suitable to describe all the temperature-accelerated degradation processes.

b: EMPIRICAL ACCELERATION MODEL

Apart from the items damaged by temperature, more products are accelerated to weaken by non-thermal stresses in practice. For example, heavy machine tools are usually subject to mechanical stresses, voltage, current, corrosion, and vibration in operation conditions. However, the interactions are tough to be monitored, thus leading difficulties in understanding the underlying chemical reactions or physical processes. As a practical alternative, the empirical acceleration model is developed to describe the aging processes that lack physical or chemical explanations. Specifically, this type of acceleration modeling methods includes the inverse power law model, the exponential model, the inverse-log model, the inverse linear model [58], as well as the Coffin-Manson model [71].

In the last century, when researchers studied degradation processes under electrical stresses, the inverse power law relationship was derived to describe the life-stress relationship for reliability assessment [72], [73]. Up to now, it has been the most popular empirical acceleration model, which can be written as follows:

$$\eta = aS^{\upsilon} \tag{2}$$

where S denotes the stress level applied, and v represents a constant parameter related to failure modes or other factors. η denotes the lifetime indicators of the product under the accelerated stress S, such as the p-th quantile lifetime.

In view of the degradation of carbon film resistors and fatigue cracks, Park and Padgett [58] incorporated accelerated stress variables into the geometric Brownian motion and the Gamma process by using the above-mentioned empirical models, respectively. Outputs of the simulation showed that the inverse power law relationship led to the best prediction accuracy [58]. Besides, Erto and Giorgio [74] assumed that the scale parameter of the Weibull distribution decreased as a power of the imposed variables, whereas Srivastava and Gupta [75] utilized the inverse power law model to establish the lifetime-stress relationship of solar lighting equipment.

The Coffin-Manson model is a great alternative to calculate the number of cycles to failure when a product experiences cyclic mechanical or thermal stress at normal working conditions. Under this circumstance, the relationship between cyclic stresses and the product lifetime cannot be characterized by the physical or the empirical models mentioned above [46]. In the literature, Hillman [76] used the Coffin-Manson model to compare the isothermal fatigue behavior of Sn-Ag-Cu and Sn-Pb solder joints during the power cycles. Besides, Zhang and Bang [77] proposed a wire bonding lifetime prediction model based on the Coffin-Manson relationship and then assessed the degradation levels of LED packages by a numerical analysis. Recently, empirical evidence has proved that the maximal amplitude and the frequency of the stress applied have a significant impact on the reliability assessment of the products under cyclic-stress loading. Therefore, this model fails to describe aging effects when products are under high thermal stresses. For details about the Coffin-Manson model, see [71].

In the 1960s, Box and Cox [78] introduced the Box-Cox transformation into acceleration modeling to generalize the above-mentioned empirical acceleration models. Under their framework, all the power law relationships are special cases of the Box-Cox model. For more details about the Box-Cox transformation, see Escobar and Meeker's excellent review [33]. Compared with physical acceleration models, these empirical ones can perform better when incorporating stresses into degradation modeling where the failure mechanism of the product is not evident. More importantly, the goodness of fitting of empirical acceleration models tends to be better than that of the physics-based methods. However, the challenge is that it is tough to check the validity of extrapolation results without a good understanding of degradation mechanisms.

c: STATISTICAL ACCELERATION MODEL

The statistical acceleration model is developed based on statistical analysis methods, which can work well when the degradation data are difficult to interpret using physical or chemical methods. Sometimes, misleading interpretations can be obtained when components in multi-unit systems possess different failure patterns [26]. To handle this problem, Nelson [1] derived a simple linear acceleration relationship through a regression analysis. However, this model is too simplified to address complicated engineering problems. At present, the statistical acceleration relationships can be classified as the generalized logarithmic-linear model, the multinomial model, as well as the proportional hazard model (also called the nonparametric model) [79], [80].

In the literature, Cox [81] proposed the proportional hazard model, which assumes that environmental stress factors have multiplier effects on the baseline risk rate function of the products. Besides, Tian and Liao [79] designed a condition-based maintenance policy for multi-component systems based on the proportional hazards model. Considering the effects of time-varying coefficients and time-scale transformations, Elsayed *et al.* [82] developed an extended linear hazard regression model with time-dependent parameters for reliability analysis under normal working conditions by employing failure data obtained during accelerated conditions. However, since these relationships are obtained by purely fitting degradation data without any physical explanations, statistical models may be unsuitable for characterizing life-stress relationships outside the range of the concerned data, thus less being used for reliability modeling [83].

Furthermore, it is not difficult to find that many products are subject to more than two environmental stress variables in practice. For instance, rubber items will weaken with the joint efforts of temperature, current, lighting, mechanical stress, and humidity [40]. However, all the above-mentioned models are unsuitable for describing the complicated aging relationships under multiple or non-constant stress loaning. To address this problem, Park and Padgett [84] proposed the Hyper-Cuboidal Volume (HCV) model based on the cumulative damage theory. Within the terms of the reaction rate, the generalized HCV model is presented as:

$$R\left(X_1, X_2, \ldots, X_n\right) = l_0 \prod_{i=1}^{n} \left(H\left(X_i\right)\right)^{l_i} \tag{3}$$

where l_i are some unknown parameters to be estimated and $H(\cdot)$ is one of any monotone functions. It is worth noting that all the power law models, such as the inverse-log power law, exponential, inverse-linear, and Arrhenius relationship, can be transformed from the HCV model when there is only one accelerating variable. At present, this model has been used in the degradation analysis of carbon fiber composites [84], film resistors, and metal fatigue crack propagation [85].

At present, several types of acceleration models have been developed to capture the aging effect of accelerated stress variables for ensuring a more reliable reliability assessment. Many outstanding researchers, e.g., Nelson, Meeker, and Escobar, have devoted vast efforts to the research on how to design more efficient AT methods and how to choose a proper acceleration model based on the failure mechanism and the working condition of a product. The challenge is that interactions between each unit and failure modes in multi-component systems are hard to be identified. If acceleration models to be used for degradation modeling are simplified, predictions may be far away from the actual values under use conditions. Moreover, some physical processes and chemical reactions are too costly and time-consuming to be identified, even with advanced test equipment. How to characterize the time-varying dependency between different failure modes is still robust, especially in multi-component systems. In the future, deep learning-based methods, e.g., the convolutional neural network [86], as well as the long short-term memory neural network [87], may be helpful in extracting useful information and identifying different failure processes from performance degradation data.

B. THE OPTIMAL DESIGN OF ADT

Although ADT has already been quite efficient in collecting lifetime data, it is still costly to conduct. For instance, scholars need to design four or five levels of stress with over 12 rubber items at each level, and therefore, at least 48 test units are needed. Sometimes, it would be impossible to have so many samples on hand if products are newly developed or expensive. Therefore, designing a suitable AT method and an appropriate test plan is essential for the effective use of available resources for ADT technology. In fact,

TABLE 1. Acceleration models and some applications.

Acceleration relationship	Accelerated stress(es)	Applications
Arrhenius	Temperature	rubber items [9], batteries [42], carbon film resistors [58], LED [59], electrical connectors [35]
Power law	Mechanical stress, current, voltage, pressure, temperature, corrosion, etc.	insulating fluids [72], [88], train components [89], steel bridges [90], carbon film resistors [58]
Coffin-Manson	Cyclic mechanical or thermal stress	solder joint [46], steel components [71], medical devices [91]
Hyper-Cuboidal Volume	Common accelerated stress variables	carbon film resistors [85], metal fatigue cracks [85], carbon fiber composites [84]
Statistical model	Common accelerated stress variables	insulators [92], LED [65], thermal oxides [82]
	Temperature & voltage	supercapacitors [63], mobility transistors [55]
	Temperature & current	lumen [56], LED [93]
Generalized Eyring	Temperature & humidity	electronic devices [66], [68], [57]
	Vibration & current	electrical connectors [52]
	Temperature & lighting	phosphor plates [69]

several design variables, such as type of accelerated stress variables, stress levels, the number of samples allocated to each stress level, and inspection time, have been considered in ADT planning under predefined constraints, e.g., limited test durations, the given budget, and availability of resources required for designing ADT. In order to obtain better and more degradation information under restrictions, it is necessary to optimize ADT methods under different conditions. In the literature, the results of ADT planning mainly depend on optimization objectives, which are summarized as follows:

1) the D-optimality [94]: maximizing the determinant value of the posterior information matrix.
2) the A-optimality [95]: minimizing the sum of all the parameter estimation values of accelerated degradation models.
3) the V-optimality [59], [61]: minimizing the expected variance of the q-th percentile of the lifetime distribution of the product under normal working conditions.
4) the M-optimality [35]: minimizing the asymptotic variance of the acceleration factor and concentrating on the degradation mechanism equivalence rather than the evaluation precision or prediction accuracy.
5) Pareto-optimality [6]: maximizing the Kullback-Leibler (KL) divergence, minimizing the total cost of tests, and the quadratic variance of a certain percentile of the lifetime distribution at usage condition simultaneously.

Many existing studies on the optimal design of ADT have proven that reliability assessment results can be improved by properly designing the sample size, inspection time, as well as detection intervals allocated to each stress level [4], [96], [97]. However, some of the objectives mentioned above focus on evaluation precision or prediction accuracy, whereas others pay more attention to information gain or the equivalence of degradation mechanisms. Consequently, the concern of the given budget and different optimal criterions may confuse engineers and producers: how to choose the right optimization principle for ADT? Inspired by these problems, we decide to review current research on the optimal design of ADT governed by different degradation models.

1) THE OPTIMAL DESIGN OF CSADT

Among all the ADT methods, CSADT has been widely used for reliability assessment since it is easy to be carried out with low costs. In the laboratory, the constant current is usually used to accelerate the degradation process of LED [2], [98], and film capacitors [3]. More applications of constant-stress loading can be seen in degradation analysis of sealing rubber rings [9], electronic connectors [61], and crack growth [99]. However, the variability of degradation models (e.g., the regression-based approach [100]–[102] and stochastic process [37], [103]) makes ADT planning complicated, especially when environmental stress factors, individual differences, and measurement errors, are considered.

Recently, Chen *et al.* [59] incorporated random effects and the measurement variability into a nonlinear Wiener process, and then proposed an optimization algorithm to decide ideal stress levels, the number of test units allocated to each stress level, the inspection frequency, as well as the total measurement time for minimizing the asymptotic variance of maximum likelihood estimation (MLE) of unknown parameters under normal working conditions with the constraints of sample sizes, test durations, and test costs. Then, Sun *et al.* [16] further investigated the impact of environmental covariates when planning ADT, in which a transformed Eyring model, the inverse power law model, and the Arrhenius relationship are employed to describe the life-stress relationships, respectively. Besides, Pan *et al.* [104] proposed a CSADT optimization scheme under the V-optimality, in which the degradation process is characterized by a modified Wiener process.

As for ADT governed by the Gamma process, Tsai *et al.* [97] developed a multi-stress AT method by determining the sample size and termination time with a fixed measurement frequency under a predefined budget where the expected variance of the q-th percentile of the lifetime distribution under normal working conditions is minimized. Besides, Duan and Wang [103] considered the optimal design of CSADT characterized by a fixed-effect and a random-effect Gamma process, in which the V-, D-, and the A-optimality are adopted to optimize the test plans, respectively.

Furthermore, Ye *et al.* [61] studied how to optimize CSADT under a random-effect Inverse Gaussian (IG) process by determining the stress level and sample size for each stress level by using the V-optimality. Wu *et al.* [36] proposed a multi-objective plan to optimize CSADT governed by the IG process, in which the maximum determinant of the posterior information matrix and the minimum asymptotic variance of the q-th percentile of the lifetime distribution are all considered. Unlike the studies mentioned above, Wang *et al.* [35] adopted the M-optimality to plan CSADT under the IG process with covariates and random effects where the equivalence of the failure mechanism gains more attention than the evaluation precision or prediction accuracy. Inspired by their research, Ye *et al.* [61] adopted a similar method to study ADT planning under the IG process under the constraints of sample size, termination time, and stress regions.

2) THE OPTIMAL DESIGN OF SSADT

Step-stress is another accepted stress application where samples are subjected to higher stresses than use conditions. And then, the stress is increased to a higher one after a testing interval, but it should also be lower than the stress that could induce different failure mechanisms [27]. Due to this feature, SSADT has the advantage of shortening test durations and decreasing the needed number of samples [105]. However, the drawback of this ADT method is that the evaluation accuracy will decrease with the accelerating process. Besides, it is hard to extrapolate reliability assessment results obtained to normal working conditions.

Despite the defects, SSADT methods have still attracted researchers' attention and been employed in accelerated degradation analysis of electronic products and mechanical structures [38], [60], [106]. Compared with the optimal design of CSADT, there are a relatively smaller number of studies focused on SSADT. In the literature, Sung *et al.* [107] characterized the degradation process using the Wiener process, in which the asymptotic variance of the expected parameters of the q-th quantile of the lifetime distribution at use conditions is minimized by determining stress levels and stress change times. Then, this method has also been used to optimize an SSADT method governed by a multivariate Gamma process by determining the sample size, inspection intervals, and measurement frequency under the restriction of experimental costs [108]. Besides, Duan *et al.* [37] investigated an exponential-IG degradation model for SSADT by determining the sample size, inspection frequency, and measurement times under the V-optimality.

Recently, the Bayesian-based methods have been adopted to optimize testing plans in reliability engineering. Under the Bayesian framework, model parameters are treated as random variables to capture uncertainties, and therefore, the prior information of model parameters can be used in ADT design to obtain more accurate assessment results. In contrast, model parameters in traditional ADT optimization methods are set as crisp values, and this is why these methods are called local optimal schemes. Inspired by the Bayesian theory, Zhao *et al.* [109] determined the best stress levels and measurement frequency by using the Large-sample approximation to derive asymptotic Bayesian functions under the D-, A-, and V-optimality, respectively [109]. Besides, Liu and Tang [110] characterized the degradation process of transistors with a general path model, in which a Bayesian-Monte Carlo (MC) simulation is developed to estimate unknown parameters of the optimization function. Li *et al.* [111] further studied how to plan SSADT under the IG process using a Bayesian method under three optimal objectives, including the relative entropy, the quadratic error function, and the D-optimality. Finally, the Markov Chain Monte Carlo (MCMC) simulation is utilized for parameter estimation in his study. Inspired by the Pareto approach, Li *et al.* [6] proposed another multi-objective optimization by maximizing the Kullback-Leibler (KL) divergence, minimizing the total cost of tests and the quadratic variance of a certain percentile of the lifetime distribution at usage condition under the Bayesian framework. It should be mentioned that multi-source uncertainties and environmental stress factors are neglected in the optimal design of ADT at this stage, which will lead to unrealistic testing plans, occasionally. More importantly, statistical analysis in the ADT governed by complicated models can be overwhelmed, even with computer-aided methods. In the future, flexible methods like deep learning algorithms should be developed to help compute the values of optimization objectives more efficiently.

3) THE OPTIMAL DESIGN OF OTHER ADT

Apart from constant-stress and step-stress, progressive-stress loading is another accepted stress in ADT technology and reliability assessment [112], [113]. When the applied loading increases linearly over time, this type of stress is also known as ramp-stress [30]. In the 1980s, Yin and Sheng [114] firstly studied PSALT to derive lifetime distributions of a product. Sung [113] assumed that the degradation process fellow the Wiener process, in which the ramp-stress ADT is optimized by minimizing the expected variance of the q-th percentile

TABLE 2. Literature on the optimal design of ADT.

Authors	Loading stress(es)	Loading method	Degradation process	Parameter-stress relationship	Optimization principle(s)
Wang *et al.* [35]	Temperature	Constant	IG	Modified Arrhenius	M
Wu *et al.* [36]	Temperature	Constant	IG	Arrhenius	V and D
Tseng *et al.* [4]	Temperature	Constant	Gamma	Arrhenius	V
Sung & Yum [107]	Temperature	Step	Wiener	Linear	V
Li *et al.* [6]	Temperature	Step	IG	Linear	the Pareto
Li *et al.* [112]	Temperature	Progressive	Wiener	Power law	V
Li *et al.* [94]	Temperature	Progressive	IG	Modified Arrhenius	D
Hu, Lee & Tang [95]	Temperature	Step	Wiener	Arrhenius	V, D, and A
Ye, Chen & Peng [61]	Temperature	Constant	IG	Arrhenius	V
Tsai *et al.* [96]	Current	Constant	Gamma	None	V
Chen, Li & Pan [59]	Temperature	Constant	Wiener	Arrhenius	V
Duan & Wang [103]	Temperature	Constant	Gamma	Exponential*	V, D, and A
Duan & Wang [37]	Weight	Step	IG	Exponential	V
Zhan, Pan & Xie [109]	Temperature	Step	Wiener	Arrhenius	V, D, and A
Li *et al.* [111]	Temperature	Step	IG	Modified Arrhenius	three objectives*
Tsai *et al.* [97]	Temperature, current	Constant	Gamma	Generalized Eyring	D
Tsai, Lio & Jiang [119]	Temperature, current	Constant	Wiener	Generalized Eyring	V
Li & Jiang [116]	Temperature	Step	DCFP	Arrhenius	D
Pan & Sun [108]	Mechanical stress	Step	MPDP	Power law	V
Haghighi & Bae [117]	Mechanical stress	Step	DCFP	None	D
Zhao, Xu & Liu [118]	Temperature	Constant	DCFP	Arrhenius	V, D, and A
Sung [113]	Not Given	Ramp	Wiener	Exponential	V

Remark:
(1) Exponential*: the link function is transformed as an exponential form from the proportional degradation rate model.
(2) three objectives*: the optimization objectives include the relative entropy, the quadratic loss function, and the D-optimality.

of the lifetime distribution of the product at use conditions. Recently, Peng and Tseng [8] developed an optimal PSADT plan where a nonlinear general path curve model is used to characterize the degradation process. However, the proposed approach has uncritical prediction errors, thus leading to a wider confidence interval.

Furthermore, engineering systems are usually characterized as multifunctional, which may deteriorate with several PCs or competing failures. For instance, the degradation of lithium-ion batteries in electric vehicles can be approximated by the reduction of the capacity or the increase of the impedance [115]. Therefore, it may be of help to consider several vital performance indicators, as well as competing risks in ADT planning. In the literature, Li and Jiang [116] employed the drift Brownian motion to model the degradation process with competing failures, in which the sample size and inspection time under each stress level are optimized by minimizing the expected variance of the q-th percentile of the lifetime distribution of the products under normal working conditions. Besides, Haghighi and Bae [117] proposed another method to jointly analyze linear degradation data and competing risks in an SSADT plan where the stress levels, failure times, and the amount of degradation damage at the moment of failures are determined. In addition, Zhao *et al.* [118] proposed an optimal plan for a CSADT method under the D-, A-, and V-optimality, respectively, in which degradation damage and random shocks are all taken into account. Finally, Table 2 shows the optimal design of ADT under different degradation models and optimization objectives.

In this section, we mainly summarize some essential aspects of ADT technology, e.g., the basic theory, acceleration models, ADT methods and variables, and ADT optimization. Though scholars have investigated many efforts to plan more efficient reliability tests to improve the prediction accuracy, increase the evaluation precision, and reduce total costs, there are still many problems deserving further research in this field. Firstly, current studies on ADT plans are mainly based on simple degradation models, in which multi-source uncertainties and environmental factors have not been considered. As a result, testing plans tend to be too simple to deal with engineering problems. Though it is easy to consider random factors into reliability modeling, the estimation of model parameters would be complicated. To overcome this weakness, neural networks and deep learning- based methods may be better alternatives to the Bayesian approach to produce a more effective solver to easily compute the values of optimization objectives. For the sake of simplicity consideration, many proposed ADT plans are developed by considering only one accelerated stress variable, and therefore, multiple-stress AT methods should be studied in the future. In addition, there are some other realistic factors, such as the energy consumption and precision of testing equipment, that should be considered in ADT planning. Finally, apart from CSADT and SSADT, the optimal planning on PSADT and cyclic-stress ADT are also worthy of future investigations.

III. PERFORMANCE DEGRADATION MODELING

Having obtained sufficient degradation data of the concerned products through ALT or ADT, performance degradation modeling is a promising approach to make full use of the precious data. In reliability engineering, the lifetime of a product refers to the time when the accumulated degradation reaches a predefined threshold [33]. Accelerated degradation models are then developed to link product reliability indicators under high stress levels and use conditions. The ultimate goal of accelerated degradation modeling is to develop time-to-failure distributions. In order to simplify the modeling process, there are some underlying assumptions in what follows [120]:

1) the degradation process is irreversible; that is, the product performance is monotonous over time.
2) one accelerated degradation model corresponds to a particular degradation process, a failure mechanism, or a certain failure mode. If there are several failure modes, degradation models should be modified during the corresponding process.
3) the initial degradation quantity can be ignored in degradation modeling.
4) the failure mechanisms at high and normal stress levels should stay consistent.

The key to reliability assessment is to establish a suitable degradation model based on an in-depth analysis of failure physics and degradation data. At present, the physical failure-based models and the data-driven models are the most popular methods to characterize degradation processes for reliability assessment. The difference is that physics-based approaches are much easier to be implemented for reliability modeling, whereas the data-driven methods, e.g., the graphical approach, the degradation path curve approach, and stochastic processes, are developed through a statistical analysis by using data processing techniques. As mentioned, modern systems usually possess multiple PCs. Meanwhile, degradation processes may also be disturbed by random external shocks, thus resulting in competing failure risks. If any of failure modes or essential PCs is neglected in reliability modeling, it may greatly impact the optimal design of ADT and the accuracy of reliability assessment. Therefore, it is essential to consider MPDPs and DCFPs in modeling accelerated performance degradation data.

A. PHYSICAL FAILURE-BASED APPROACH

The physical failure-based degradation models are developed based on a thorough understanding of the failure mechanism of a product and an in-depth analysis of the chemical reaction or physical process. Up to now, the most representative ones are the cumulative exposure model [121], the reaction theory model [122], and the stress-intensity model [123], which were developed several decades ago for different usages.

The cumulative exposure model is derived based on the level of the accumulated damage caused by material stress. Therefore, this model is more suitable for studying material fatigue of metallic products. For instance, Walker [124] utilized this model to study the effect of the strain ratio on fatigue crack propagation processes, whereas Forman [125] considered the impact of fracture toughness and crack growth rates based on Walker's model. Then, the reaction theory model is employed to fit lifetime data of the products through chemical reactions that cause failures. In the 1990s, Meeker and Luvalle [122] characterized the degradation path of a printed circuit board based on the corresponding chemical reaction law. Then, Cary and Koenig [126] employed a similar approach to model the deterioration of submarine cables, whereas Salcedo *et al.* [127] found the voltage threshold of long-channel MOSFETs based on the underlying reaction process. The stress-intensity model is developed by deriving the relationship between stress loading and material strains [123]. In the past, Place *et al.* [128] employed this model to describe the degradation of a helicopter transmission system where the effects of the degradation accumulation on system failures are obtained based on strain-stress functions (S-N curve).

Over the last century, physical failure-based models have attracted certain attention since the reliability assessment and lifetime prediction through these methods are quite accurate and reliable. However, the problem is that the failure process of a modern industrial product is complicated and costly to accurately grasp, even with the implementation of high-technology equipment. In addition, each of these models has limited application domains. To be specific, the cumulative exposure model and the stress-intensity model are suitable for characterizing metal fatigue, whereas the reaction theory model performs better to describe reaction processes based on chemical reaction laws. With the development of statistical analysis methods and computer-based software packages, this type of modeling approach goes beyond the scope of the present study.

B. DATA-DRIVEN APPROACH

Compared with the physics-based approaches, the data-driven modeling methods have received much more applications for reliability assessment due to excellent statistical properties. Generally speaking, this type of modeling methods can be divided into two large categories: regression-based models and stochastic processes, as well as some important extensions. In the past, regression-based models have been employed in addressing engineering problems for its simple modeling and parameter estimating process. However, a large number of testing samples and degradation data are needed when using this method for degradation modeling.

Besides, reliability extrapolation through simple a regression analysis is not as accurate as that of the stochastic process since random dynamics can be easily incorporated into these models. More importantly, it is very convenient to characterize MPDPs and DCFPs using stochastic process-based approaches. Therefore, random processes, as well as their extensions, are becoming a research hotspot in the field of reliability engineering.

1) REGRESSION-BASED MODEL

This type of modeling method is developed by a proper regression analysis of the degradation data, which can be based on the following three types: artificial intelligence algorithms, the degradation path curve approach, as well as the graphical approach [129]. At this stage, commonly used intelligence algorithms, e.g., neural networks [130], [131] and support vector machines (SVMs) [132], can directly fit online and offline degradation data without strict restrictions, thus having great flexibility in reliability modeling. However, the problem is that a large number of samples and degradation data are also needed in neural network-based approaches for ensuring the prediction accuracy. In addition, an inappropriate number of hidden layers in neural networks may lead to the problem of over-fitting or under-fitting. As a practical alternative, SVM can work well even with multi-dimensional data or a small number of samples. Unfortunately, if the degradation data are contaminated by background noise, it will increase difficulties in extracting useful features from the monitored data. Recently, some other useful techniques, e.g., wavelet analysis, particle filtering, Kalman filtering, and the Bayesian method, have been successfully incorporated into these intelligence algorithms to enhance modeling abilities. For more details and applications of these algorithms for reliability analysis, see the excellent reviews [133], [134].

a: THE DEGRADATION PATH CURVE APPROACH

The degradation path curve approach is established based on the degradation trajectory versus time, in which some parameters are set as random variables to capture individual differences while others stay constant. For the fact that samples used are under different stress levels, the underlying assumption under this approach is that each product follows the same degradation model with some stress-dependent parameters. After incorporating accelerated stress variables, unknown parameters in the degradation path curve under normal use conditions can be deduced. Generally speaking, the random-effect degradation path curve approach was firstly proposed by Meeker and Lu [135]. Up to now, this method has kept accessible for a long period since stochastic dynamics in degradation processes can be well characterized. According to the shape of degradation trajectories, the degradation path curves can further be divided into nonlinear and linear ones. The general form of this degradation model can be written as follows:

$$Y(t) = D(t;\theta) + \varepsilon(t) \tag{4}$$

where $Y(t)$ denotes the degradation quantity over time, and $D(t:\theta)$ represents the actual degradation process.θ are the random parameters characterizing unit-to-unit differences. $\varepsilon(t)$ is a normally-distributed term representing measurement errors. In the 2000s, Wang [136] extended the work by Meeker and Lu [135], and enumerated the underlying assumptions of the random-coefficient degradation model as follows:

1) the degradation process with the operating time and degradation level can be observed at any time.
2) the products being monitored comes from a population, each of which exhibits the same degradation path.
3) a product will fail when the degradation damage exceeds a predefined threshold.
4) the distribution model of random parameters is known.

Inspired by the works by Meeker and Lu [135], and Wang [136], extensive scholars have made great efforts to study the random-coefficient degradation path curve approach for health management, reliability assessment, and remaining useful lifetime (RUL) estimation. For instance, Joseph and Yu [137] proposed a differential form of (4) to describe random perturbation, whereas Lu *et al.* [138] proposed a linear regression model with a random slope and intercept to fit degradation data. It should be mentioned that confidence intervals of unknown model parameters in Lu's work [138] are obtained through Bootstrap sampling, the maximum likelihood ratio-based method, and the asymptotic normal approximation, respectively. Besides, Yang and Jeang [139] developed a random-coefficient model to describe the aging process of cutting tools. Later, Gebreel *et al.* [140] improved this method by fusing different sources of degradation information through the Bayesian framework. Unlike the assumptions in [135], random coefficients in their work just follow a Brownian motion error process. For more applications of the nonlinear degradation path cure approach in reliability engineering, see [141]–[143].

The degradation path curve approach has a relatively simple modeling process with few difficulties in parameter estimation. It is also flexible to incorporate random effects by setting some parameters randomly-distributed, whereas this operation is limited in stochastic process-based methods. However, since this method is a significant simplification of reality, the RUL and reliability estimation may be far away from the actual values. Besides, a large number of test samples are needed to obtain sufficient degradation data for ensuring the validity of assessment results. At this stage, researchers have not considered all the random uncertainties when using this model for reliability analysis, which may also lead to inaccurate predictions. Therefore, more flexible methods to incorporate multi-source variability should be developed in the near future.

b: THE GRAPHICAL APPROACH

When the degradation trend does not change obviously, the degradation path curve approach is replaced with the graphical method to assess product reliability [144]. In this approach, a probability distribution function (PDF) is firstly selected to illustrate the degradation data collected from all the samples at each measurement time. Since the degradation process of each sample varies with the accelerated stress and the amount of time, some parameters in lifetime PDFs are set as functions of the stress level and test time. Therefore, the graphical approach is also called a time-dependent distribution model. Finally, lifetime distributions are obtained by a statistical analysis of the degradation data. However, it is difficult to ensure that all the samples are simultaneously measured in practice. For improvement, Zuo *et al.* [145] extended the basic model to deal with cases where test units are not measured simultaneously. Compared with the degradation path curve model, this method does not require a large number of degradation data from each product or make any assumptions on the degradation path, thus having also received some applications for reliability assessment.

Among the research on this method, the normal, Weibull, exponential, and the log-normal distribution are the most commonly used models to characterize degradation data. In the 1980s, Nelson [146] firstly studied the graphical approach for reliability assessment, in which the log-normal distribution is employed to fit the accelerated degradation data. To be specific, the mean parameter is assumed to be a function of the measurement time, whereas the variance keeps independent of time. Moreover, Zuo *et al.* [145] assumed that degradation data of a product fellow the Weibull distribution with time-changing parameters by using a rank regression analysis method for parameter estimation. Inspired by Zuo's work, Jiang and Jardine [144] studied the segment graphical model based on the Weibull distribution, whereas Xue and Yang [147] adopted the normal distribution to characterize the degradation data, and then obtained time-varying values of the mean and variance by the nonlinear least square method. For more studies on the graphical approach, see [148], [149].

The graphical approach can be adopted for reliability assessment when the degradation state of the product does not have an apparent regularity over time. It is worth noting that reliability predictions under this approach tend to be more accurate than that of the general path model. It also shows advantages in the following cases:

1) the degradation data cannot be measured repeatedly, or each product degradation process cannot be modeled by other approaches.

2) the degradation path of each product varies greatly, thus leading to the difficulty in parameter estimation when using other methods.

However, individual differences are neglected in this model, which may not always be realistic when dealing with practical engineering problems. Meanwhile, performance degradation data are supposed to be measured simultaneously for ensuring the accuracy of extrapolation. Then, uncertainties in the two-step statistical analysis method can lead to the accumulation of computational errors. Compared with other methods, this method receives fewer applications in reliability engineering. For future investigations, researchers should consider how to incorporate multi-source uncertainties into the time-dependent parameter distribution models for obtaining more reliable and accurate predictions.

2) STOCHASTIC PROCESS

After introducing regression-based models, this section will present another critical reliability analysis method, namely, the stochastic process, including the Wiener process [18], [23], [150], the Gamma process [17], [24], [97], and the IG process [36], [151]. There are also some extensions, such as the Levy degradation model [152]. Recently, Tseng proposed a more general form, namely, the exponential diffusion model, which includes the above-mentioned stochastic processes as special cases [153]. Due to the multi-source variability, the use of stochastic process-based approaches to characterize product degradation processes has certain advantages. For instance, environmental stress factors can be incorporated into these models by treating parameters as functions of the applied stresses. On the other hand, there are too many assumptions when using stochastic processes to model degradation data, which may lead to unrealistic extrapolation at use conditions.

a: WIENER PROCESS

In the 1970s, the Wiener process was first introduced into reliability engineering by Ghhikara and Folks because of its excellent statistical properties [154]. The basic Wiener process can be denoted as follows:

$$Y_W(t) = x_0 + \lambda\Lambda(t) + \sigma B(\Lambda(t)) \tag{5}$$

where $Y_W(t)$ denotes the degradation process, and x_0 is the initial degradation state that can be ignored in degradation analysis.λ is the drift coefficient representing the degradation rate, andσ is the diffusion coefficient. $B(\cdot)$ is the standard Brownian movement capturing the dynamics in degradation processes. $\Lambda(t)$ is a time-scale transformation function, which can be determined by the prior knowledge of the concerned system. From a physical perspective, the degradation increment in an infinitesimal duration can be regarded as an additive superposition of many small external efforts, which can be characterized by the normal distribution because of the central limit theorem. Therefore, the independent increment $\Delta Y_W(t)$ is assumed to follow the normal distribution:

$$\Delta Y_W(t) \sim N\left(\lambda\Lambda(t), \sigma^2\Lambda(t)\right) \tag{6}$$

In this formula, $\Delta\Lambda(t) = \Lambda(t+\Delta t) - \Lambda(t)$ is non-negative and monotone increasing. When the Wiener process $Y_W(t)$ firstly arrives at the failure threshold D, the lifetime T follows a modified IG distribution [155], that is, $IG(y; a, b)$ with $a = D/\lambda$ and $b = (D/\sigma)^2$. The PDF of the basic Wiener process can be written as:

$$f_W(y; a, b) = \left(\frac{b}{2\pi y^3}\right)^{1/2} \exp\left(-\frac{b(y-a)^2}{2a^2 y}\right), \quad y > 0 \quad (7)$$

At present, the Wiener process and its modified forms have received full applications for maintenance strategy-making, reliability assessment, and RUL estimation. In the literature, Jin *et al.* [156] utilized a modified Wiener process to calculate the RUL of secondary batteries, whereas Tsai [2] employed the Wiener diffusion model to analyze lumen degradation data. Besides, Wiener process-based methods have been adopted for reliability assessment of mechanical structures, e.g., metal fatigue [157], [158], head wears [159], momentum wheels [160], and pumps [161].

Unfortunately, the conventional Wiener process only concentrates on the current degradation data, while other available information during the entire sequence of observations is neglected. Meanwhile, it cannot be used to deal with online degradation data. For improvement, a simple Wiener process with an adaptive drift is derived, in which the recursive filter is employed to update the drift coefficient, and unknown model parameters are deduced by the expectation-maximization (EM) algorithm [162]. Currently, the recursive filter has been replaced with the Kalman filter [12] and the particle filter [156] to enhance its online modeling ability. Another problem is that the basic Wiener process can only be applied to describe degradation data with linear trends, which may not always be realistic, This is because degradation processes of many engineering systems are usually disturbed by random external factors, e.g., environmental covariates, individual differences, errors in measurement, and so on, thus showing nonlinear behavior. To better characterize the deterioration of these systems, the above-mentioned uncertainties and external factors should be considered in degradation modeling. Recently, reliability assessment based on the Wiener process has been paid much attention, and massive extensions have appeared.

Wiener Process With Nonlinearity: To overcome the weakness of the traditional Wiener process, some state and time-scale transformation techniques are introduced to nonlinear degradation modeling. In the literature, Park and Padgett changed a nonlinear geometric Brownian motion process into a linear one by using a log-transformation technique [58], [157]. In their works, other sources of useful degradation information, e.g., historical data and subjective experience, are not included. For improvement, Gebraeel [140] proposed a Bayesian method that can use real-time monitoring data to update unknown parameters. Unfortunately, it is tough to determine a proper prior distribution and estimate model parameters in the posterior distribution of the Bayesian framework. Another famous state transformation function is the Box-Cox transformation, which includes the log-transformation as a special case [78]. For more characteristics about this technique, see Sakia's excellent work [163]. Up to now, this method has only been employed in the linear regression-based approach and the Wiener process, thus needing to be extended in the Gamma process and the IG process [164]. Although many investigations have been conducted on the transformation techniques, it remains challenging to determine more general state transformation functions.

Another nonlinearity-to-linearity transformation method is proposed by Whitmore, who has converted the nonlinear degradation path of cables into linear ones by using a time-scale transformation function [165]. Since then, a large number of scholars have been motivated to adopt this method for nonlinear degradation modeling. In the literature, two popular forms of time-scale transformations are written as $\Lambda(t) = 1 - \exp(-\upsilon t^{\gamma})$ [166] and $\Lambda(t) = t^{\gamma}$ [167]. It is worth noting that the latter one is much more commonly used in the modeling and analysis of performance degradation data for its excellent statistical properties.

However, the problem is that only a small number of nonlinear degradation paths can be converted to linear ones through these transformation techniques. Besides, it is robust to get closed forms of lifetime PDFs of the products when using the nonlinear Wiener process. To overcome this challenge, Si and his co-authors [13] assumed that the drift coefficient of the Wiener process is time-dependent without using any time-scale transformation, and then a closed-form PDF is obtained by changing the constant threshold to an arbitrary boundary. Recently, the validity of this modeling method has been proved by several scholars using numerical analyses [168], [169]. At current stage, the most popular link functions to characterize the relationship between the accelerated time and the drift coefficient are the power law model and the exponential model. For more research on nonlinear modeling with the Wiener process, see [170]–[172].

Wiener Process With Individual Differences: Random factors, which are usually neglected in the traditional Wiener process, are almost inevitable in engineering problems. The multi-source variability, e.g., individual differences, errors in measurement, model uncertainties, and the temporal variability, can greatly impact the accuracy of degradation modeling. To enhance the abilities to incorporate these factors, some parameters in the Wiener process are addressed as random variables or functions of the applied stresses. For example, the diffusion coefficient is usually assumed to represent the temporal variability, which indicates inherent dynamics in degradation processes. And the drift coefficient is assumed to represent the degradation rate that is one of the most vital factors in degradation modeling. Compared with the research on the drift coefficient, few researchers focus on the diffusion coefficient, except for Wang [15], who assumed that the diffusion coefficient fellow the Gamma distribution, and Ye and Xie [32], who connected the drift and the diffusion coefficient by setting them proportional.

In ADT, the observed degradation for samples from the same batch may be different because of several unobservable factors, e.g., the initial defect sizes variations in raw materials, and unobservable usage patterns. Under this circumstance, random-effect Wiener processes are proposed to deal with these unobserved heterogeneities. A general way of capturing individual differences in the Wiener process is assuming the drift or diffusion coefficient to follow a certain distribution [159], [167], [173]. In the literature, Ye *et al.* [167] compared the goodness-of-fitting of four different types of Wiener process-based models, namely, (1) the simple Wiener process; (2) the RD-Wiener model, in which the drift coefficient is randomly-distributed; (3) the RV-Wiener model where the diffusion coefficient is treated as a random variable; and (4) the RDV-Wiener model, in which both the drift and diffusion coefficient are normally-distributed variables. More recently, the normal distribution is replaced with the skew-normal distribution since the former one can bring biases in parameter estimation [171], [174], [175]. Though the skew-normal-distributed drift coefficient can improve the accuracy of assessment results, it can also lead to the difficulty in other aspects of the statistical analysis of degradation data [174]. Besides, the model parameters to be randomly-distributed and the distributions are determined based on empirical experience, which may lead to the problem of model mis-specification.

Wiener Process With Covariates: According to engineering experience, the deteriorations of engineering systems are usually influenced by environmental stress factors. For example, temperature, humidity, ultraviolet light, and pressure can significantly accelerate the aging process of rubber items [40]. Other influential factors include current, vibration, use rate, and specimen size, which are called covariates, or markers in reliability engineering. To incorporate the effects of environmental covariates into the Wiener process, the drift coefficient and the diffusion coefficient are set as functions of accelerated stress variables. Generally, the functional relation between the drift parameter and the stress is called a link function. In order to choose a suitable link function, scholars need to understand how acceleration factors have affected the degradation process by analyzing the data obtained by ADT. Some commonly used link functions are the physics-based models or statistical relationships, e.g., linear functions, the Arrhenius model, the exponential model, and the inverse power law model [176].

Currently, there are two main assumptions when incorporating environmental stress variables into the Wiener process-based degradation models. That is, (1) the drift coefficient is a function of covariates, whereas the diffusion coefficient stays constant [58], [84]; (2) both the drift and diffusion coefficient change with the accelerated stress [65], [166]. In the literature, Bian *et al.* [177] and Bian and Gebraeel [178] have conducted research on how to incorporate covariates into the Wiener process using some link functions under dynamic environments. However, unlike other scholars, Flory *et al.* [179] studied how to incorporate accelerated stress factors by assuming that both the drift coefficient and the diffusion coefficient are functions of environmental stresses. For more studies on the Wiener process with covariates, see [65], [137].

At present, there are still many challenges when considering environmental stress factors in the Wiener process. Firstly, the majority of the studies on the Wiener process with covariates at this stage assess the reliability of products under constant stress loading, which is uncommon in engineering practice. There are also some products that work under non-constant-stress conditions. However, it tends to be tough to construct environmental loading PDFs under random loading. Besides, there are not equivalent stress transformation methods other than the temperature stress. Meanwhile, link functions are usually determined by subjective experience, which may also lead to the problem of model mis-specification. For the multi-component systems and these products possessing several degradation phases, it remains challenging to determine interactions between different components and degradation paths. Releasing these restrictions may improve the prediction accuracy in the future.

Wiener Process With Measurement Errors: Imperfect measurements are also inevitable, especially when degradation data are not measured directly. Generally, contaminated data are supposed to be addressed before being used for reliability modeling. Though measurement errors will lead to difficulties in the statistical analysis of degradation data [18], [180], scholars still considered this factor in degradation modeling. Specifically, this variability is treated as an additional and normally-distributed part of the basic Wiener process. In the literature, only a few scholars, e.g., Zhai and Ye [180], Peng and Hsu [181], solely consider measurement errors in reliability modeling. In contrast, a large number of researchers have been attracted to study the joint effects of random effects, temporal variability, and measurement errors [173], [182], [183]. For example, Peng and Tseng considered unit-to-unit variations and measurement errors when studying the problem of model mis-specification [173]. Besides, Si *et al.* [182] developed a linear Wiener process considering all the above-mentioned variability, in which the Kalman filtering is employed for estimating the degradation state and unknown parameters. Recently, this work has been extended into a nonlinear Wiener process [183].

The general method to incorporate individual differences, measurement errors, and environmental stress covariates in degradation modeling is to treat some parameters as random variables. Up to now, how to deal with interactions between different random factors and how to choose proper random distributions are still challenging. Though considering all the variability in the Wiener process can improve the prediction accuracy, it can also increase the complexity of degradation models and lifetime PDFs. Due to the mixed factors, e.g., random shocks, maintenance, and calibrations, abrupt change points are likely to exist in the degradation process of some products [184]. Unfortunately, current studies on the Wiener process with two or more degradation phases are insufficient. Meanwhile, the linear drift function is over-used at this stage, which significantly reduces the modeling ability of the multi-phase Wiener process. Therefore, the nonlinear drift coeffi-cient should also be taken into consideration in the future.

b: GAMMA PROCESS

Compared with the Wiener process, the Gamma process has higher requirements for the degradation data with independent and non-negative increments [185]. In the 1970s, the Gamma process was introduced into reliability modeling by Abdel-Hameed [186]. And currently, this model has been used in modeling and analysis of degradation data of some industrial products, e.g., LED [20], [97], [187], lasers [21], carbon film resistors [58], [188], and nuclear power plants [189]. The basic Gamma process $Y_{Ga}(t)$ has non-negative and independent increments $\Delta Y_{Ga}(t)$ that follows the Gamma distribution as:

$$\Delta Y_{Ga}(t) \sim Ga(\alpha \Lambda(t), \beta) \tag{8}$$

where β denotes the scale parameter and α represents the shape parameter, both of which should be positive. According to the additivity of the Gamma distribution, the non-negative and independent increments at any given interval should follow a transformed Gamma distribution $Ga(y; m, n)$ with $m = 1/\sqrt{D\beta}$ and $n = D/\alpha$. The PDF of the Gamma process can be written as:

$$f_{Ga}(y; m, n) = \frac{1}{2\sqrt{2}mn}\left[\left(\frac{n}{y}\right)^{1/2} + \left(\frac{n}{y}\right)^{3/2}\right] \times \exp\left[-\frac{(n-y)^2}{2m^2ny}\right] \tag{9}$$

Lawless and Crowder [17] have proved that the Gamma process can be regarded as a compound Poisson process when the Poisson rate tends to be infinite, and the size of increments tends to zero proportionally. Recently, van Noortwijk [190] presented an excellent overview of the Gamma process, including its definitions, applications, and simulation methods. Because of the consideration of different applications, modifi-cations have been made to the basic Gamma process so that it can work for different problems. Therefore, we will focus more on the extensions of the basic model with environmental covariates, random effects, and measurement errors.

Gamma Process With Covariates: Like the modeling approaches in the Wiener process-based methods, covariates can also be incorporated into the Gamma process by setting the scale or the shape parameter as functions of the accelerated stress. In the 1990s, Singpurwalla [191] found that environmental stresses in degradation modeling were assumed to keep constant, which may not always be realistic. Then, the Gamma process within the consideration of failure mechanisms of a product and dynamic characteristics of operating conditions is developed. Based on Singpurwalla's work, Lawless and Crowder [17] proposed a modified Gamma process, in which the scale parameter follows the Gamma distribution to capture unit-to-unit differences. Furthermore, the scale parameter is assumed as a function of covariates to incorporate environmental stress factors. In the literature, three methods to incorporate covariates into the Gamma process are presented as follows:

1) the scale parameter is a function of the accelerated stress, whereas the shape parameter stays constant [17], [192].
2) the shape parameter is a function of the accelerated stress, whereas the scale parameter is independent of covariates [4].
3) both the scale parameter and the shape parameter are functions of environmental covariates [193].

Recently, the validity of these assumptions has been proved through the constant acceleration factor hypothesis [188]. To the best of our knowledge, the most popular link functions to describe covariates in the Gamma process are the power law relationship and some physics-based models. However, the parameter estimation in modified Gamma processes may be more robust than that of the basic model, and therefore, more efficient approaches should be proposed to deal with this problem. Finally, researchers are supposed to develop flexible functions to link model parameters, and the accelerated stress since the stress-dependent relationships are determined based on subjective knowledge or empirical experience at this stage.

Gamma Process With Individual Differences: Since the scale parameter of the Gamma process can present degradation variations of a product, this coefficient is usually treated as random variables following the Gamma or normal distribution to capture individual differences. In the literature, Wang [188] studied two types of random-effect Gamma process with a time-scale transformation, in which both the conjugate prior distribution considering a Gamma-distributed scale parameter, and the non-conjugate prior distribution with random scale and shape parameter are developed using the Bayesian method. Besides, Hao *et al.* [20] characterized the degradation data of LED with a modified Gamma process considering individual differences. Yang *et al.* [194] described the degradation of drive electrohydraulic servo valves based on a random-effect Gamma process where the scale coefficient is assumed to follow the Gamma distribution. Compared with research on random-effect Wiener processes, it is clear that there are fewer studies on how to describe individual differences when using the Gamma process for reliability assessment. This is partly because the optimization of resulting likelihood functions tends to be complicated with excessive parameters in Gamma process-based approaches.

Gamma Process With Measurement Errors: It is quite likely that degradation data are contaminated by imperfect measurements. Most often, errors in measurement are white noises and do not accumulate over time. Recently, it has also been proven that imperfect measures can lead to inaccurate assessment results and high-dimensional integrals in parameter estimation [18]. In related studies, Lu *et al.* [195] assumed that the measurement variability follows a Gamma distribution and estimated model parameters using the Genz-transform and quasi-MC method. Outputs of the simulation show that the proposed estimators are more efficient than the traditional MLE in terms of the bias and standard deviations. To study the influence of measurement errors on reliability assessment, Zhou *et al.* [196] proposed a Gamma process with the particle filtering using the EM algorithm to filter measurement errors. Finally, his study proved that measurement noises can significantly complicate modeling and analysis of accelerated degradation data. Besides, Zhang *et al.* [197] adopted a multivariate normal distribution to characterize measurement noises since the potential biases of degradation data are neglected when adopting the normal distribution. Furthermore, Wei and Xu [198] modeled real-time monitoring data with the Gamma process where errors are assumed to follow the Gaussian distribution. Unfortunately, the measurement variability has only been described by constant-parameter distributions, which means that errors from the measured data are independent of the degradation level. To overcome this weakness, Pulcini [19] proposed a perturbed Gamma process with a time-dependent distribution to model the dependence between random errors and degradation states. For similar research, see [199].

It is not difficult to find that there are a small number of scholars focusing on the Gamma process since this model is only suitable for describing monotonic degradation data. Although a large amount of attention has been paid to handling multi-source uncertainties in the Gamma process for ensuring the accuracy of reliability assessment, only few scholars have considered all the external random factors in one degradation analysis simultaneously. Although the MCMC simulation and the Bayesian method have been introduced to overcome the difficulty in parameter estimation, these methods are far more adequate to address complicated engineering problems. In addition, change points are likely to exist in the degradation process of a product due to maintenance tasks or overloading. Currently, it is tough to determine the number and the location of the change points, and therefore, the multi-phase Gamma process is worthy of further investigations.

c: IG PROCESS

Although the Wiener process and the Gamma process have been largely employed for reliability assessment, these models cannot always characterize degradation processes perfectly, such as the degradation data of lasers [200]. Then, the IG process is proposed as an essential alternative, which can work better than the Wiener and Gamma process in special cases. A simple IG process with a time transformation function $\Lambda(t)$, the mean parameter μ, and the scale parameter η can be defined as follows:

$$Y_{IG}(t) \sim IG\left(\mu\Lambda(t), \eta\Lambda(t)^2\right) \tag{10}$$

with $Y_{IG}(0) \equiv 0$. It has independent increments following the IG distribution as:

$$\Delta IG(t) \sim IG\left(\mu\Delta\Lambda(t), \eta\Delta\Lambda(t)^2\right) \tag{11}$$

The PDF of the IG process $Y_{IG}(t) \sim IG(y; \mu, \eta)$ is denoted as follows:

$$f_{IG}(y; \mu, \eta) = \sqrt{\frac{\eta\Lambda^2(t)}{2\pi y^3}} \exp\left(-\frac{\eta(y - \mu\Lambda(t))^2}{2\mu^2 y}\right), \quad y > 0 \tag{12}$$

The IG process is less used in reliability modeling because of the lack of an intuitive physical background interpretation. Currently, Ye and Chen [201] proved that the IG process is a limit of compound Poisson processes, which gives a physical explanation for this model. It is worth noting that, compared with the Gamma process, it is easier to incorporate random effects and covariates in the IG process due to the flexible inverse relation between the IG and Wiener process.

As for the applications of the IG process for reliability assessment, Zhang *et al.* [202] used this model to describe the corrosion of energy pipelines with the Bayesian inference and the MCMC simulation for parameter estimation. In addition, Peng *et al.* [203] modified the simple IG process considering a time-varying degradation rate to characterize the degradation process of a heavy machine tool's spindle system. Since the next increment may be dependent on the current degradation state in practical problems, Guo *et al.* [204] improved the basic IG process to predict the next damage increment in the following time interval. Besides, Ma *et al.* [205] proposed an IG process with a time-dependent degradation rate to describe the degradation process of hydraulic piston pumps based on an in-depth analysis of the failure mechanism of the pumps.

At present, there are only a few studies on the IG process with the consideration of individual differences, measurement errors, and environmental stress factors. Like the methods employed in the Wiener and Gamma process, the commonly used approach to incorporate random effects into the IG process is assuming some model parameters to follow random distributions. In the literature, three typical types of random-effect IG processes are obtained by referring to the Wiener process [201]. That is:

1) the random drifts model (RD-IG), in which the mean follows a truncated normal distribution.
2) the random volatility model (RV-IG) where the shape parameter follows a Gamma distribution.
3) the random drift-volatility model (RDV-IG) where both the mean and shape parameter are randomly distributed.

However, the underlying problem is that the statistical analysis of performance degradation data in these models tend to be robust. In recent years, this difficulty eases to some extent with the use of the Bayesian framework [206]. Considering that the degradation rate of some products in the same population tends to be asymmetric and non-normal, the normal distribution is replaced with the skew-normal one to capture individual differences [174]. Inspired by this idea, a random-effect IG process with measurement variability is proposed to characterize the degradation data of GaAs laser and fatigue cracks [151]. In the work by Wang and Xu [200], an efficient parameter estimation method based on the EM algorithm is developed to calculate the MLE of unknown parameters of the IG process, in which covariate information and unit-to-unit heterogeneity are all considered. Furthermore, both Zhang *et al.* [202] and Qin *et al.* [207] assumed that random errors fellow a zero-mean normal distribution, and utilized the modified IG process to model the imperfect inspection data of energy pipelines. However, a common feature of the products, which leads to difficulties in adopting the above-mentioned IG process models, is to have self-accelerating degradation. This means the degradation of a product at the moment depends not only on the current stage of the product but also on the degradation level itself at the very moment. For improvement, Peng *et al.* [208] proposed a transformed IG process to characterize an age- and state-dependent degradation process by linking the degradation state and the degradation increment.

The IG process is still new in reliability engineering since it has strict requirements for degradation data. In the future, there are several aspects of the IG process deserving more explorations. Firstly, the physical explanation and statistical analysis of this model need to be further investigated to widen its applications in reliability engineering. Then, there are only a few scholars focusing on the IG process with multi-source uncertainties that may significantly reduce the accuracy of assessment results. Some other vital factors, e.g., dynamic degradation rates, time-varying measurement variability, and competing failures, are also worthy of more investigations. Since industrial products can have age- and state-dependent degradation processes with different features, more types of age- and state-dependent models are supposed to be developed with different modeling capabilities. Finally, much more attention should be paid to IG process modeling with change points because of self-recovery or maintenance tasks.

3) MULTI-PERFORMANCE DEGRADATION PROCESS

In the last century, due to the limitations of accelerated testing equipment, only the most important PC that can reflect the health conditions of the concerned product is considered for reducing the difficulty in reliability modeling. However, this operation may impact the accuracy of product reliability assessment results. Two typical examples for this point, which motivate research on the reliability analysis considering two or more product PCs, are the degradation process of vehicle batteries and LED. To be specific, the degradation level of lithium-ion batteries in electric vehicles can be approximated by the reduction of the capacity or the increase of the impedance [115]. Then, LED will fail when the amount of attenuation or the light intensity exceeds a predefined threshold [20]. The differences between these performance indicators and measurement methods result in different types of degradation observations, which can be characterized as MPDPs. Multi-performance degradation modeling methods at this stage can be based on the following two categories: independent MPDPs and dependent MPDPs.

In the first modeling method, the degradation process of each PC is assumed to be independent of other indicators, thus neglecting the interactions between different PCs. For instance, Crk [209] modeled the degradation process of a product, in which the failure mechanism of each PC is assumed to be independent. Besides, Barker and Newby [210] established an inspection and maintenance strategy for a system using an independent stochastic model to describe the multivariate degradation process. Li and Pham [14] modeled a MPDP through two stochastic processes and a shock model. For more modeling details about independent MPDPs, see [211].

However, the assumption in independent MPDPs may not always be realistic in practice since many engineering systems may possess several functions and experience random shocks. In the literature, both Shen *et al.* [212] and Wang [213] have proved that the accuracy of reliability evaluations will become lower if dependent multivariate degradation processes are addressed as independent ones. In order to describe the interactions between different PCs of the concerned product, the multi-normal distribution is introduced into degradation modeling, in which performance degradation data at different measurement times are assumed to be normally distributed. However, this method is only helpful in dealing with some simple degradation processes. Meanwhile, constructing multi-normal distributions to model MPDPs is also unrealistic in many cases since the linear dependency in this model is too simple to characterize complicated degradation interactions.

For improvement, copula functions, which are developed by Sklar [214] and have received full applications in the finance industry, are introduced into reliability engineering by Nelson [215]. The copulas (also called "connection functions") are employed to describe the joint distribution of multi-dimensional variables with marginal distributions, which can significantly reduce the complexity of reliability models and difficulties in parameter estimation. At present, commonly used copula functions include the Gaussian, Gumbel, Clayton, and Frank copula (see Table 1 in [216]) with the covariance, Spearman rank correlation coefficient, and Kendall's tau to describe the level of dependency. Generally, a bivariate copula function is a joint PDF of two random variables on the interval [0,1] as

$$C(p, q) = P(P \leq p, Q \leq q) = F_{PQ}(p, q) \tag{13}$$

where the $F_{PQ}(p, q)$ is the joint PDF of random variables P and Q. If $F_1(y_1)$ and $F_2(y_2)$ denote the marginal PDFs of two random variables Y_1 and Y_2, a bivariate distribution function. $F(y_1, y_2)$ can be established based on the Sklar's theory as follows:

$$F(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2) = C(F_1(y_1), F_2(y_2)) \tag{14}$$

In the literature, Sari [217] constructed a bivariate linear degradation path for reliability assessment with the Frank copula to describe the dependency between two performance indicators, whereas Pan *et al.* [218] utilized a time-scale transformed Wiener process to characterize a bivariate degradation process, which is later replaced with a nonlinear one to model the fatigue crack growth by Wang *et al.* [219]. Besides, Zhang [220] described the degradation process of satellite rechargeable lithium batteries based on lifetime data and bivariate degradation fused by the Bayesian analytical framework. Then, Pan and Balakrishnan [24] proposed a Gamma process-based modeling method with the Birnbaum–Saunders distribution to describe the dependence. In order to overcome challenges in parameter estimation of the bivariate Gamma process, the Bayesian-MCMC simulation is proposed to avoid calculating high dimensional integrals by resampling techniques [221]. Following the logic of degradation modeling with the Wiener and Gamma process, Peng *et al.* [222] further studied how to model MPDPs with incomplete measurement data based on the IG process, in which a two-stage estimation method is proposed with higher flexibility and efficiency. Inspired by Ye's work [201], Duan [223] *et al.* developed three types of bivariate random-effect IG process to characterize individual differences, which have motivated many researchers to study MPDPs with IG process-based modeling approaches.

However, there are also some limitations since most of the copula functions used in current studies are only suitable to describe bivariate dependent degradation processes. Moreover, the dependence structure between each PC is assumed to be constant, thus neglecting individual differences versus time [216]. However, this is not always realistic since the dependency may become weaker or more robust with the continuous change of external environments. Under this circumstance, time-varying copulas are developed as practical alternatives to handle this problem [224]. Recently, Zhang *et al.* [225] utilized time-dependent copulas to induce a stress-strength correlation for a structural reliability analysis. For more applications of time-varying copulas, see [226].

Apart from the bivariate copulas, there are also some other multivariant copulas, e.g., the Gaussian copula, the t-Copula, and the Vine copula. Among them, the Vine copula is the most commonly used connecting function to describe different dependent relationships between three or more performance indicators [227]. The key to constructing this copula is to decompose its joint distribution to some marginal distributions and bivariate copulas. In the literature, Bedford and Cooke have compre-hensively reviewed Vine copulas [227] and showed how to decompose marginal PDFs [228]. For applications of the Vine copula, see [229], [230].

Due to changeable environmental factors and complex structures, it is necessary to consider several vital performance indicators of the product in reliability assessment. Independent degradation modeling methods and multinormal distributions may be helpful in the characterization of MPDPs. However, the complicated dependency between each PC tends to be over-simplified, which may lead to inaccurate predictions. To enhance modeling capabilities, some time-varying copulas, as well as multivariate copulas, are proposed to describe the complicated dependence between different

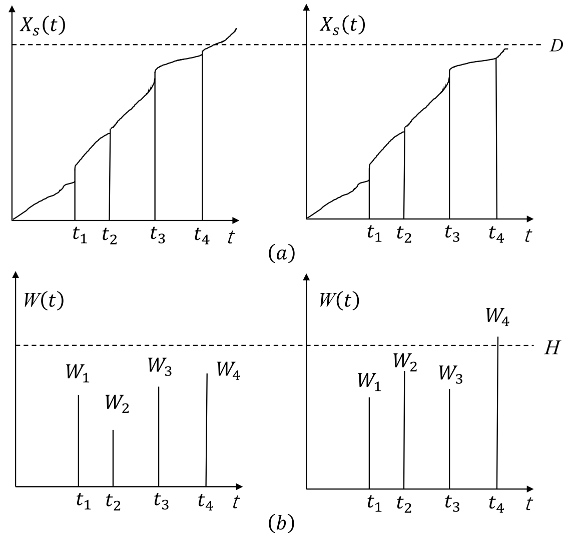

FIGURE 3. (a) Soft failure process, and (b) Hard failure process [234], where W_i denotes the size of the i_{th} shock load, and $X_s(t)$ is the total degradation at t.

PCs. In the future, this field is still worthy of further investigations. Firstly, the proposed ways of modeling MPDPs are limited to simple degradation models, e.g., the general path model and the stochastic process without considering the multi-source variability. Besides, the optimal design of ADT governed by MPDPs is still new in reliability engineering, which will become increasingly urgent with the fast development of sensor technology. Moreover, the accuracy of reliability assessment is significantly affected by the degradation model and PCs selected. Therefore, how to extract effective features from the massive amount of monitoring data, as well as how to choose suitable bivariate copulas when decomposing joint distributions of multivariate copulas are two urgent research directions.

4) COMPETING FAILURE PROCESS

Being exposed to random external shocks can create a crucial part of the damage, which can lead to the failure of products. Generally, engineering systems possess several failure modes that compete against each other. For example, power units that supply electrical energy by chemical reactions weaken during usage. Then, it can also suddenly fail because of overvoltage or overheating [231]. In reliability engineering, there are two major failure modes, namely, the "soft failure" that refers to the failure caused by wear, erosion, fatigue, and aging, whereas the "hard failure" indicates the sudden failure caused by random shocks and other overloads, see Fig. 3.

In the past, random shocks cannot always be measured due to a lack of inspection techniques [232]. The use of sensor technology has attracted increased interest in the analysis of degradation data considering competing risks. Following the logic of modeling MPDPs, it is natural to consider the failure interaction when a system is subject to natural degradation and random shocks. In the literature, Lehmann [233] firstly

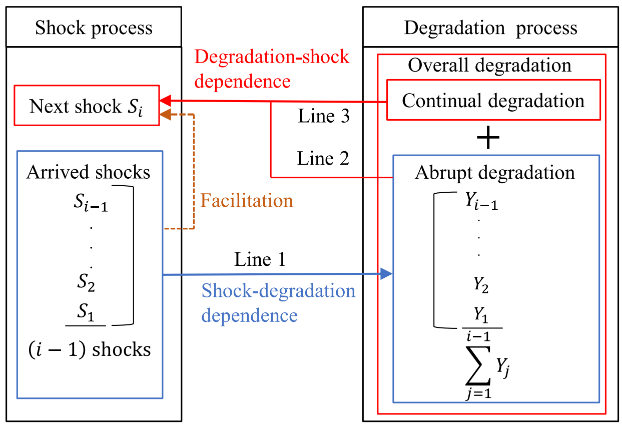

FIGURE 4. The dependence between the degradation process and the shock process, where S_i represents the i_{th} shock [234].

proposed the famous degradation-threshold-shock (DTS) model to characterize degradation processes with competing risks. Some assumptions under this model are summarized as follows [11]:

1) The component will experience a hard failure when the shock load exceeds the pre-set maximum load.

2) The soft failure occurs when the total degradation of one component is more significant than the failure threshold.

3) The Poisson process is used to model random shocks.

Up to now, DCFPs have been classified into two categories based on the above assumptions: the shock-degradation model and the degradation-shock model [234], as depicted in Fig. 4, In this figure, line 1 shows that arrival shocks will result in abrupt degradations, and then increase the degradation rate, whereas line 2 and line 3 show that the overall degradation will increase the intensity of the shock process.

Previously, the degradation process and the shock process are assumed to be independent in some cases. For instance, Li [14] described an independent competing failure process of a multi-state degraded system subject to external shocks and degradation damage. In addition, Huang and Askin [211] described the degradation process of electronic devices with independent competing risks and utilized the Weibull distribution to model the soft failure and sudden damage. Moreover, Keedy and Feng [235] developed a maintenance framework considering two independent failure modes: the soft failure due to cyclic stresses and the hard failure caused by abrupt overloads. For more research on independent competing failure processes, see [236], [237]. However, according to practical engineering experience, external shocks can impact the degradation process because of the change of material properties and reliability structures. Therefore, the dependency between different failure modes should be considered, especially when systems are under multiple or non-constant stress loading.

As for the shock-degradation models, scholars assumed that external shocks would lead to an increase in the accumulated damage or degradation rates. For instance, Wang et al. [238] characterized the crack growth process where the external shock increased the degradation rate. Besides, Rafiee et al. [239] proposed three methods to model the degradation of a micro-electro-mechanical system considering different shock patterns, in which soft failures due to natural degradation, hard failures caused by the external shocks, and another increasing degradation due to the same shocks are considered. Recently, Peng et al. [240] employed the cumulative shock model to characterize external random loading, in which shocks will lead to abrupt degradation damage when the size of shocks reaches the given level. Besides, Bocchetti et al. [241] considered the deterioration of cylinder liners, in which the degradation rate changes because of the exposure to a particular shock. Moreover, external shocks can also increase degradation rates and damage simultaneously. For instance, Hao and Yang [242] have considered the increase of degradation rate and damage caused by external shocks in the degradation process of pier columns of sea bridges. Zhou et al. [243] assumed that random shocks would lead to hard failures easily, and each reaching shock would also increase the degradation rate. Recently, this method is adopted to design hybrid preventive maintenance of competing failures under a random environment by Yang et al. [244].

Compared with the research on shock-degradation models, fewer scholars have considered the degradation-shock modeling methods. In the literature, Fan et al. [245] assumed that the amplitudes of external shocks are closely associated with the degradation process where the shock is described by a non-homogeneous Poisson process, and the degradation process decides the shock intensity. Besides, Lin et al. [246] assumed that current degradation levels could determine the damage caused by random shocks, whereas Kong et al. [232] considered two patterns of shock magnitudes: the interarrival and arrival patterns based on Rafiee's work [239].

Currently, researchers have devoted considerable efforts to develop new approaches for characterizing external shocks and dynamic interactions between different failure modes. There have also been a large number of published papers related to reliability assessment for engineering systems experiencing several types of failure modes. However, some assumptions in these studies are supposed to be relaxed for future investigations. Firstly, the degradation rate is assumed to be constant, and the shock intensity is a linear function to the degradation level, both of which may not be realistic assumptions [247]. Meanwhile, interactions between different shock loading are usually neglected, thus leading to unreliable predictions when addressing practical engineering problems [248]. In addition, degradation models used in DCFPs are limited to some simple models, e.g., the basic Winer process and the degradation path curve model, in which external random factors, such as individual differences, environmental covariates, and measurement errors, are neglected. Releasing these restrictions may produce more accurate predictions. Finally, more attention should be paid to reliability modeling for complex systems, e.g., k out of n systems and load-sharing systems, as well as the optimal design of ADT governed by DCFPs.

IV. MODEL MIS-SPECIFICATION AND PARAMETER ESTIMATION

Following performance degradation modeling, the selection of lifetime distributions and parameter estimation are two closely related parts in reliability assessment. This is mainly because a suitable lifetime distribution and a statistical analysis method can not only improve the evaluation accuracy but also reduce the difficulty in parameter estimation. Among existing studies, the Weibull, exponential, normal, and log-normal distribution have received wide applications in the analysis of performance degradation data because of excellent statistical characteristics. Besides, there are some methods for model selection, such as the total mean square error (TMSE), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and some other statistics [103], [249]. However, these methods may not always be valid to determine the most suitable lifetime distribution to characterize the lifetime of a product. For instance, both the Gamma and Weibull distribution can well fit the recovery time of a complex system [250]. The following is a comprehensive summary of model mis-specification and parameter estimation.

A. MODEL MIS-SPECIFICATION

In reliability engineering, probability distributions are adopted to quantify product lifetime at normal working conditions. For this reason, the selection of a suitable distribution model is quite essential. Among existing studies, the Weibull [9], [251], exponential [252], normal [151], [171], log-normal [253], skew-normal [151], [249], extreme-value [254], and Gamma distribution [255] have been widely used in the statistical analysis of performance degradation data. Moreover, some of these models can be transformed from others through pow law transformations. For instance, the generalized exponential distribution can be transformed to the log-normal, two-parameter Gamma, and the Weibull distribution [252], [256]. Under this circumstance, degradation data in the same population can be perfectly fitted by different distributions [257]. For instance, Kwon and Frangopol [253] modeled the degradation data of two bridges using the Weibull, log-normal, and Gamma distribution, respectively. In addition, Barker and Baroud [250] predicted the recovery time of an engineering system using the Gamma and Weibull distribution. Although performance degradation data in these studies are well fitted with several distributions, reliability assessment results may be significantly different. In reliability modeling, this problem is called model mis-specification.

In the 1960s, Cox [258] firstly recognized the influence of mis-specified models, and then Pascual and Montepiedra [259] proved Cox's conclusion through computer simulation-based methods. Recently, Kundu and his co-authors have conducted extensive research on how to select the most suitable lifetime distribution for a given data set. For example, the differences between the Weibull and log-normal distribution have been studied by deriving the asymptotic distribution of likelihood ratio (LR) estimators where the minimal sample size to distinguish these distributions is also deduced [260]. Besides, they further study how to discriminate other distributions, such as the generalized exponential and Weibull distribution [261], and the log-normal and generalized exponential distribution [262]. For similar works by other researchers, see [263], [264].

At present, studies on model mis-specification are far more adequate to deal with an increasing number of engineering problems since existing methods for discriminating different distributions are limited to MLE- and LR-based techniques, both of which are not suitable to deal with online monitored data. Besides, only few scholars, except for Kundu [265] and Marshall [266], have considered how to simultaneously differentiate three or more distributions based on the same batch of performance degradation data. The sample size has a significant impact on the selection of a suitable lifetime model. However, scholars mainly focus on offline data at this stage. For future investigations, online degradation processes should also be taken into consideration.

B. PARAMETER ESTIMATION

1) POINT ESTIMATION

A model parameter estimation aims at obtaining point values or confidence intervals of unknown model parameters. Generally speaking, narrow confidence intervals often mean higher credibility of the estimated values. At this stage, some commonly used parameter estimation approaches include the traditional approach, the Bayesian framework, as well as some combined methods. During the last few decades, traditional approaches and the EM algorithm have been fully adopted to estimate unknown parameters in acceleration and degradation models. However, traditional methods cannot always work well when the statistical analysis is complicated because of high-dimension integrals or hyperparameters. Then, the Bayesian statistical method and the MCMC simulation are introduced to deal with this challenge by avoiding complex mathematical calculations. Recently, significant progress has been made in this field of research with the development of computer-based techniques.

a: THE TRADITIONAL APPROACH

The classical methods for point estimation mainly include the graph estimation, the moment estimation, the least squares estimation (LSE), MLE, and some linear estimation methods, e.g., the best linear unbiased estimation (BLUE), the good linear unbiased estimation (GLUE), and the best linear invariable estimation (BLIE) [267]. The graphical estimation can be used to easily extrapolate estimation values and reliability assessment results under normal working stress levels. However, its estimation accuracy is not so high as that of other approaches. LSE and MLE have kept popular for parameter estimation during the last few decades [19], [187]. Unfortunately, these methods have poor performances when degradation data are contaminated and then depart from the correct results. In this case, the robust estimator, e.g., Huber's M-estimator, may be a possible remedy for the deficiencies of LSE- and MLE-based approaches [268]. In order to compare the performance of these traditional estimators, Elmahdy and Aboutahoun [269] utilized MLE, the graph estimation, the nonlinear median rank regression analysis, and the Bayesian method for parameter estimation in two types of Weibull distributions. Outputs of the simulation show that the Bayesian method has the best goodness of fitting with smaller biases. Despite the fact that traditional estimators can be easily used in the analysis of degradation data, they cannot yield the best extrapolation when degradation data are measured imperfectly, or testing samples are inadequate, thus leading to unreliable predictions.

As an excellent alternative, the EM algorithm is introduced to find the MLE of model parameters when the degradation data are implied or imperfect. Ruud and Paul [270] presented detailed steps of this algorithm for parameter estimation, in which the expectation step (E-step) needs to calculate the conditional expectation of MLE with incomplete data, and the maximization step (M-step) is supposed to find maximal values of the expected likelihood functions. In the 1970s, the EM algorithm was introduced by Dempster and his co-authors [271] to overcome difficulties in classical methods. Due to its excellent statistical characteristics, this estimator has gradually been utilized for parameter estimation in stochastic processes [23], [160], [180], DCFPs [268], and MPDPs [272]. Moreover, a two-stage EM estimation method is developed for the statistical analysis in multivariate degradation processes [273]. Specifically, unknown parameters in the marginal degradation functions are estimated in the first stage, and the second stage involves the estimation of copulas based on the output of the first stage [273]. Recently, the Kalman [12] and particle filter [274] are incorporated into the EM algorithm to improve its real-time estimation ability.

Generally speaking, classical estimation methods have high requirements for the simple size since it can largely influence the selection of lifetime distributions. If the data volume does not meet the minimal requirement, these approaches may have inaccurate predictions or no closed-form PDFs. Meanwhile, most of these approaches are unsuitable for dealing with online data, which has been solved with the use of the EM algorithm. However, this method has an overwhelmed and long iterative period that may put pressure on computing systems. At the same time, the uncertainties in the two-step estimation may lead to the accumulation of computational errors. With the increased complexity in degradation models, scholars need to conduct more research on how to improve this algorithm and then implement it in the statistical inference of MPDPs, DCFPs, as well as multi-phase degradation processes.

b: THE BAYESIAN METHOD

In modern industrial societies, it is almost impossible to construct reliability databases by only collecting degradation data from reliability tests. Therefore, an effective reliability

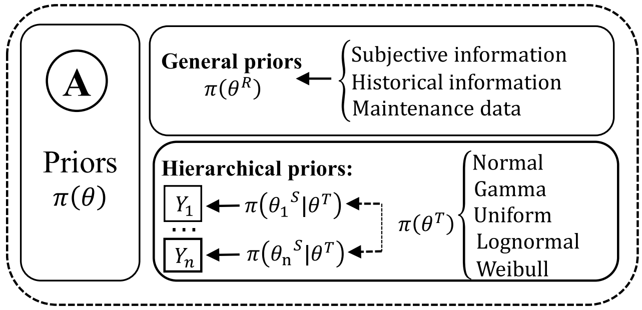

FIGURE 5. A general Bayesian framework for the IG process models: construction of prior distributions [206]. Where θ^F, θ^R, θ^H represent the fixed parameters, the random-effect parameters, and the hyper-parameters in probability distributions. Y_i denotes the degradation path, and $\pi\left(\theta^T\right)$ is the hierarchical prior distribution.

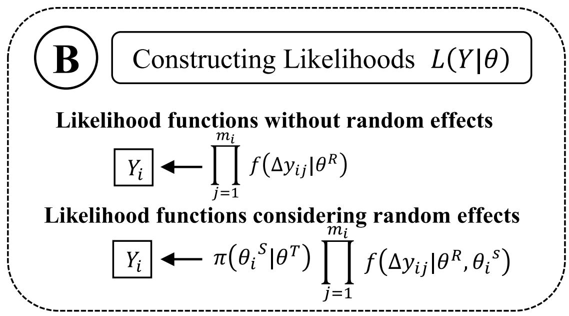

FIGURE 6. A general Bayesian framework for the IG process models: construction of the likelihood function [206]. Where $L\,(Y\mid\theta)$ is the likelihood function and $f\left(\Delta y_{ij}\mid\theta^R\right)$ denotes the PDF of independent degradation increments Δy_{ij} under the IG process without random effects, whereas $f\left(\Delta y_{ij}\mid\theta^R, \theta_i^S\right)$ is the PDF of the random-effect IG process.

analysis needs to use different types of information throughout the whole service life of the products. Generally speaking, the available information for reliability modeling includes failure reporting, subjective experience gained from outputs of computational simulation, data analysis, and the historical data of existing similar products [275]. However, these types of degradation data are ignored in traditional reliability analysis methods. Then, the Bayesian framework is developed to fuse multi-source information for more efficient reliability assess-ment. Besides, this method can also update model parameters to be estimated in real-time [276]. It is worth noting that the most significant advantage of the Bayesian statistical analysis method over the traditional ones is that model uncertainties can be taken into consideration, which can help characterize degradation processes disturbed by external random factors [277]. For these reasons, Bayesian-based methods have been adopted in ADT optimization [6], [94], stochastic process modeling [203], [207], [223], and multivariate degradation modeling for complex systems [26].

In the literature, Peng *et al.* [206] and Peng *et al.* [275] have studied the IG process in degradation modeling from a Bayesian perspective. In this framework, the first step

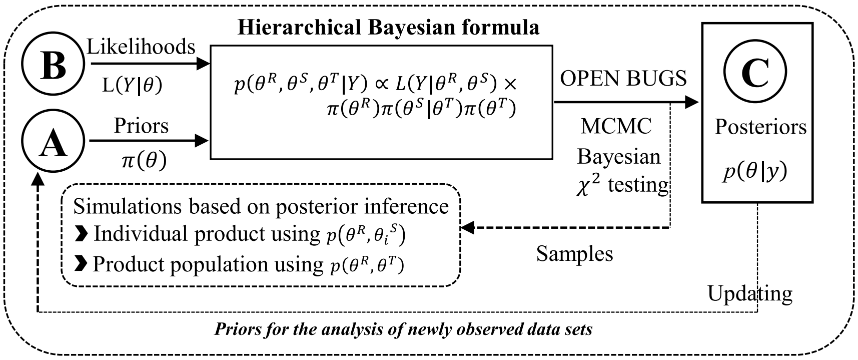

FIGURE 7. A general Bayesian framework for the IG process models: construction and analysis of the posterior distribution [206].

is to construct the prior distributions of unknown model parameters based on available information, such as subjective information, historical data, and maintenance data, as shown in Fig. 5. Then, the framework moves to Fig. 6, in which the likelihood function is established by multiplying likelihood contributions of each degradation path. Having constructed the prior distribution and the likelihood function, the joint posterior distribution of unknown parameters is formulated by the hierarchical Bayesian formula, see Fig. 7. It is worth noting that the key to establishing the Bayesian framework is to determine the prior and posterior distribution of unknown model parameters based on sample data and statistical distributions. At present, non-informative distributions are more popular than information models for constructing the prior distribution [278]. The challenge of using non-information models is how to deal with high dimensional integrals when calculating the posterior distribution. This challenge has not been overcome until the MCMC simulation is introduced for parameter estimation [279]. In this technique, the Markov Chain is adopted to produce samples following a random distribution, and the posterior distribution is derived when the resampling process remains steady. Then, the MC simulation is employed to obtain marginal distributions of the posterior distributions by Gibbs or Metropolis sampling that can be implemented through software packages, e.g., Open BUGS and Win BUGS [206].

Inspired by the two-step EM estimation method and the Bayesian theory, Peng *et al.* [222] proposed a two-stage Bayesian method for the statistical analysis of a multivariate IG process. In this framework, unknown model parameters in marginal degradation processes are estimated in the first stage, and the second stage involves the estimation of copulas based on the outputs of the first stage. For more applications of the two-stage estimation method, see [280], [281].

2) CONFIDENCE INTERVAL ESTIMATION

In addition to the point estimation of model parameters, the confidence intervals of the above parameters are also desired in the analysis of performance degradation data. At present, confidence intervals are usually obtained by the asymptotic distribution of MLE, the Bayesian probability method [282], the pivotal variable method [283], as well as the Bootstrap sampling method [284].

In the traditional statistical analysis approaches, MLE-based methods are the most popular ones for constructing confidence intervals in different lifetime distributions, e.g., the bivariate exponential [285], normal [286], Gamma [287], Burr-type [288], and the generalized exponential distribution [289]. When estimating model parameters for a two-parameter exponential distribution, Krishnamoorthy [290] found that it is nearly impossible to obtain a closed-form interval based on the asymptotic distribution of MLE, even with the change of the sample size. Then, the generalized pivot method is introduced to handle this problem, and outputs of the MC simulation show that this approach performs better than MLE. In addition, the accuracy of confidence intervals obtained by the method of generalized pivotal quantities can be improved with an increasing number of samples [283]. Since an assumed initial value is essential to start the iterative process when using some MLE-based estimation approaches, Kundu and Gupta [284] proposed another estimator, namely the approximate maximum likelihood estimation (AMLE), to deal with this challenge. Unfortunately, both MLE and AMLE have poor statistical properties when the sample size is not insufficiently large.

As a practical alternative, the Bayesian probability method is proposed to obtain confidence intervals with incomplete data [283]. At this stage, the most commonly used Bayesian probability intervals include the highest posterior density interval (HPD) with the probability $(1-\theta)$, and the equal tailed probability interval with the probability $(\theta/2)$ on each side [291]. For the difference between these intervals, the reader is referred to Kundu and Gupta [284]. Compared with the intervals obtained by traditional approaches, these Bayesian-based intervals have a weaker sensitivity to the sample size. However, it is tough to get closed-form endpoints when constructing the HPD interval.

Finally, Bootstrap sampling is an efficient computer-based technique to assess the accuracy of reliability predictions with limited degradation data where the traditional methods are not valid [292]. This method firstly fits distribution characteristics of samples using the resampling technique of observed sample data, avoiding the construction of complex statistics. Besides, it permits the estimation of confidence intervals when the sampling distribution of the estimators cannot be determined beforehand. More importantly, the Bootstrap intervals have shorter lengths with higher convergence percentages.

The sample size can significantly impact the convergence probability and the length of intervals when adopting the traditional methods for parameter estimation. In contrast, the Bayesian method and the Bootstrap sampling method can produce narrower confidence intervals with higher convergence percentages, even with small sample sizes or incomplete data. Though increasing the number of units may improve the intervals obtained, this is not always realistic in engineering. Therefore, to determine the suitable sample size for different statistical analysis methods is worthy of further research. Then, degradation data in modern engineering systems are usually measured online. Therefore, future researchers are supposed to consider how to construct confidence intervals based on online degradation data.

V. FUTURE OPPORTUNITIES AND DIRECTIONS

The fourth industrial revolution, smart manufacturing, 5G, artificial intelligence, big data, and the Internet of things are gradually changing the way we design, manufacture, and provide products and services. The rapid iteration of industrial products also promotes the development of ADT technology, performance degradation modeling, and statistical analysis methods with higher requirements. Currently, abrupt change points, the dependency, interactions, MPDPs, DCFPs, and the multi-source variability have been gradually considered in performance degradation modeling for reliability assessment. Extensive related studies and reports can provide scientific guidance for the development of new commercial products. However, with the increasing requirements from consumers and the fast iteration of products, degradation modeling and statistical analysis methods at this stage are far more adequate to deal with complicated engineering problems. And there are still many challenges when adopting performance degradation data for reliability assessment. The following is a summary of future opportunities and possible directions.

A. BIG DATA

In the past, it is costly and tough to collect adequate failure information or degradation data for degradation modeling, especially from highly reliable products. With the use of AT technology and efficient statistical methods, this challenge eases to some extent. Advanced monitoring techniques, e.g., sensor technology and computer-aided software packages, can be utilized to obtain a huge amount of real-time information regarding health status, system loading, and environmental parameters. Moreover, other related degradation data, e.g., physical failures, subjective experience gained from outputs of computational simulation, field lifetime data, degradation data in AT, and the historical data of similar products, can also be fused through the Bayesian framework.

However, the underlying problem is that a vast amount of degradation and environmental information and will post high pressure on the analysis of degradation data in several ways. Firstly, if the dimension of the degradation data to be used for reliability assessment is not reduced with some intelligence algorithms beforehand, large data sets will challenge computing and operating systems [83]. Though the Bayesian framework performs well in data fusion, it is still hard to incorporate this analytical method into complicated models, such as DCFPs and MPDPs. Then, Gibbs and Metropolis sampling techniques are quite essential for Bayesian-based methods to generate random-distributed samples. However, the prior and posterior distribution of the Bayesian framework cannot be determined easily. Meanwhile, how to construct proper density functions of these sampling techniques remains challenging. Condition degradation data are usually measured multi-dimensionally, and therefore, more efficient techniques, such as the principal component analysis, should be developed to extract as many of the useful features as possible from a large amount of degradation data of the products, and combine the advantages of data-driven approaches to obtain a more efficient reliability assessment framework.

B. MULTI-PHASE DEGRADATION MODELING

Apart from continuous degradation processes, engineering products may also possess multiple degradation phases during the whole service life because of self-recovery or maintenance tasks. At present, discrete processes are usually described by the shock models and Markov Chain-based methods. To be specific, the shock models are employed to characterize the effects of random loading on the degradation process of a product. Unfortunately, researchers have only considered the dependence between the degradation process and the random shock in the same direction at this stage [293]. There may be opposite cases that should be modeled in the future. Then, the next degradation state in Markov Chain-based approaches is assumed to only depend on current damage states, whereas degradation levels in stochastic processes are solely associated with system ages, both of which are not realistic in practice. In fact, one of the most common features of industrial products is to have self-mitigating degradation, which means that the performance degradation level at the current time depends not only on the present age but also on the degradation itself. Therefore, it would be more reasonable to model degradation processes within the consideration of system ages, as well as degradation states [168].

Another problem is that some products, such as the plasma display panels [281] and liquid coupling devices [294], may experience two or more degradation phases with change points as life cycle stages move on. Under this circumstance, single-phase models cannot capture degradation direction perfectly. Currently, several scholars have recognized this problem, and proposed more suitable modeling methods. For instance, Kong [23] found a sharply abrupt increase in the bearing degradation data and then developed a two-phase Winer process to characterize this degradation process.

In addition, Prakash [295] proposed a two-phase Gamma process, whereas Bae [281] employed a hierarchical Bayesian change-point regression approach to characterize two-phase degradation processes. However, only few researchers, except for Wen *et al.* [184], have noticed that modern engineering products may also possess multiple change points, thus showing unusual degradation behavior during the whole service life.

Currently, the degradation process before and after change points are assumed to follow the same model, which greatly simplifies two-phase degradation modeling. Meanwhile, few scholars have explicitly considered both the dependency between random shocks and degradation processes, and among the degradation processes themselves. Moreover, change points may exist at any location between inspection epochs, which are assumed to be fixed in offline estimation and online updating at this stage. Capturing random points in degradation paths is quite difficult, let alone taking the impact that this stochastic factor has on degradation modeling and reliability assessment into consideration [293]. Finally, current studies on multi-phase modeling are limited to the Wiener process. Therefore, the Gamma process, the IG process, and the Bayesian-based models should also be taken into account for future investigations.

C. MODELING WITH RANDOM FAILURE THRESHOLDS

In reliability engineering, the change of failure thresholds will directly impact the reliability assessment of a product (see (7), (9), and (12)). Therefore, it is regarded as a vital indicator of the concerned product in degradation modeling. At this stage, the failure threshold is generally assumed to stay constant, which indicates that the ability of a product to resist failures will not decrease as life cycle stages move on. However, as mentioned in DCFPs, random shocks can increase degradation damage or rates and then reduce product resistance to failures. In other words, engineering systems are deteriorating with weakening resistance to failures when withstanding random shocks, and these shocks will ultimately result in the change of failure thresholds. Though there have been a large number of studies related to shock models, few scholars have considered the relationship between failure boundaries and random shocks. In the literature, Jiang and his co-authors [296] studied the dependency between three different cases of shock patterns and the failure threshold, and then developed a DCFP with a time-varying failure boundary to characterize the degradation process of rotating gears. For future investigations, scholars need to consider how to adopt stochastic processes and MPDPs with shifting failure thresholds for reliability assessment.

D. MULTI-PERFORMANCE DEGRDATON MODELING

In the last century, for reducing the complexity in degradation models and the statistical analysis for reliability assessment, only the most crucial PC that can reflect the degradation state of a product, and the accelerated stress that has the greatest impact on product lifetime, are considered in degradation modeling. However, many engineering products are subjected to changeable environmental stress factors and possess several important PCs, which are usually neglected in traditional modeling methods. To take these factors into consideration, copula functions are introduced into reliability engineering, which opened up a new way to study MPDPs. Up to now, significant progress has been made in this field of research, and a large number of technical papers have been published.

For future investigations, there are still many challenges to be addressed. For instance, researchers pay more attention to the multivariate Winer process at this stage, whereas the Gamma and IG process have not been sufficiently studied. Meanwhile, the statistical analysis in MPDPs and CFDPs tend to be robust, even with the Bayesian framework and intelligent algorithms. Therefore, flexible estimation methods should be developed to overcome these challenges. Besides, interactions between different components may accelerate the degradation rate of engineering products, especially in multi-component systems. However, this time-dependent factor is hard to be monitored, thus causing difficulties in accelerated degradation modeling. Currently, few researchers have considered this factor in modeling and analysis of MPDPs and DCFPs. In addition, most of the copulas used in current studies are only suitable to describe the dependency between two performance indicators, which are far more adequate to characterize the degradation process of multifunctional products. In the future, it is urgent to develop more flexible copula functions that can be used to describe complicated relationships between three or more PCs for accelerated degradation modeling.

E. ADT OPTIMIZATION

The optimal design of ADT has become one of the most vital research branches in the field of reliability engineering. At this stage, the vast majority of ADT studies focus on constant-stress and step-stress AT methods, whereas progressive-stress and cyclic-stress have not been paid enough attention. Besides, there are some products that are under non-constant stress loading. In response to such problems, it is essential to develop methods and models capable of characterizing the inevitable evolving environment. Besides, the application of a compre-hensive stress in ADT based on a full understanding of the impact of environmental stresses on the concerned items under normal working conditions may be helpful in reducing the test time needed and improving the test efficiency. Another problem is that few researchers have considered the optimal design of ADT governed by MPDPs, DCFPs, as well as multi-phase degradation models. Meanwhile, how to plan ADT under different objectives and constraints is a good topic for future investigations. Then, a series of underlying problems should also be addressed, e.g., lifetime distribution selection, goodness-of-fit testing, and parameter estimation.

VI. CONCLUSION

Inspired by the research on reliability modeling and analysis of accelerated degradation data, we present a comprehensive overview of the ADT technology, degradation modeling, and parameter estimation for reliability assessment. To be specific, the fundamentals of ADT, including its basic theory, types of AT method, acceleration models, and variables of accelerated stress, are all reviewed. Since ADT optimization is an essential research branch of reliability engineering, we move to discuss the optimal design of ADT governed by different degradation models under several optimization objectives and predefined constraints. For future investigations, researchers still have a large room to study ADT optimization under MPDPs, DCFPs, as well as multi-phase models. Some other realistic factors, e.g., the accuracy and energy consumption of testing equipment, are also supposed to be considered.

Degradation modeling and statistical analysis are two indispensable aspects of reliability assessment, which have been given particular attention in current years. Among these approaches, physics-based models are less commonly used for reliability analysis, whereas data-driven models, such as the degradation path curve approach, the graphical approach, stochastic processes, as well as modified degradation models, are more prevalent at this stage. Rather than focusing on the applications of different lifetime distributions, we pay more attention to the problem of model mis-specification, which can significantly influence the accuracy of reliability assessment results. Unfortunately, distinguishing methods are only limited to MLE- and LR-based approaches. Then, we move to compare the characteristics of different parameter estimation methods, including traditional approaches, the Bayesian method, and Bootstrap sampling techniques. Among them, the Bayesian statistical analysis method keeps popular in the point estimation of unknown parameters, whereas the Bootstrap sampling techniques perform better in constructing confidence intervals. Finally, future opportunities and possible directions in accelerated degradation modeling based on performance degradation data are highlighted.

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