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Adaptive Dynamic Surface Control Based on Observer for Switched Non-Strict Feedback Systems With Full State Constraints

MIN WAN AND YANXIA YIN
School of Mechatronic Engineering, Southwest Petroleum University, Chengdu 610500, China
Corresponding author: Min Wan (18940103@qq.com)

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ABSTRACT As we all know, it is very difficult to design the controller and prove the stability for switched nonlinear systems. Therefore, the engineering application of switching system and the development of switching control theory are limited. In order to solve the control problem for constrained switched system, an adaptive output feedback control scheme based on backstepping technology is studied in this paper for switched non-strict feedback nonlinear systems with asymmetric time-varying full state constraints and unknown external disturbances. A switched state observer based on fuzzy logic system is designed to estimate the unmeasurable states of the uncertain switched system. Asymmetric time-varying barrier Lyapunov functions are adopted to keep the full states of the system satisfying their asymmetric time-varying constraints. A variable separation approach is used to address the algebraic loop problem of non-strict feedback structure. The stability of the closed-loop system and the semi-globally uniformly ultimately bounds of the signals are proved by the Lyapunov method and average dwell time theory. Finally, simulation results are given to show the effectiveness of the proposed control scheme. Different from the existing results, this paper is the first to investigate adaptive control for switched non-strict feedback systems with full state time-varying constraints, unmeasurable states and unknown disturbances, which is a more general case in real systems.

INDEX TERMS Dynamic surface control, full state constraints, fuzzy state observer, non-strict feedback, switched system.

I. INTRODUCTION

Switched systems are a class of important and typical hybrid systems that can be described by subsystems and the switching law between them [1]–[6]. In practice, many systems and processes have switching properties, such as mechanical systems, power systems, autonomous walking robot systems, missile trajectory controls, vehicle speed control systems, UAV controls and spacecraft control systems, and can thus be described as switching systems. Therefore, the switched system has attracted much attention, and some adaptive control methods have been proposed. Reference [1] proposed an adaptive tracking control method for uncertain switched nonlinear systems with arbitrary switching. In [2], a stabilizing controller was designed for switched systems under arbitrary switching. In [3], two kinds of state feedback controllers based on backstepping technology were designed for the global stabilization control of switched nonlinear systems. In [4], an adaptive neural control based on dwell time theory was investigated for switched systems with switching jumps and uncertainties. In [5], an adaptive fuzzy backstepping control was proposed for a pure feedback switched system, which can keep the tracking error in the neighbourhood of the origin. In [6], a finite time control method for nonlinear switched systems was designed based on a barrier power integrator.

The above control schemes require all states of the system to be measurable. However, in many cases, only the output signal of the system can be measured directly. To address the unmeasurable states problem, some output feedback control schemes for switched systems have been proposed. Reference [7] investigated the problem of adaptive neural output feedback tracking control for switched systems without

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the measurements of the system states. In [8], based on a switched fuzzy state observer, an adaptive output feedback control was designed for uncertain switched systems with dead zones. In [9], robust adaptive fuzzy control by using an observer was studied for uncertain nonlinear systems with arbitrary switching signals. In [10], for stochastic switched systems with unmeasured states and unmodeled dynamics, an adaptive fuzzy output feedback control was investigated. Reference [11] proposed an adaptive neural network control method for switched interconnected uncertain systems. However, the switched systems considered in [7]–[11] are all in strict feedback structures; thus, the above-mentioned methods cannot be applied to control non-strict feedback switched systems.

In practical applications, most systems are non-strict feedback structures; thus, it is necessary to study the control problem of non-strict feedback switched systems. In [12], an adaptive fuzzy output feedback control was proposed for switched non-strict feedback nonlinear systems with input nonlinearities. In [13], an adaptive fuzzy output feedback control was proposed for the switched non-triangular structure form with time-varying delays. A variable separation approach was introduced in [14] to solve the problem of non-strict feedback, and an adaptive fuzzy output feedback stabilization controller was set up based on backstepping technology. In [15], for MIMO-switched systems in non-strict feedback structure form, an adaptive output feedback control method was proposed based on a command filter. In [16], through the design of a linear state observer, adaptive fuzzy output feedback control was extended to address the problem of control for non-strict feedback switched large-scale systems with unmeasurable states. It should be noted that all the above-mentioned methods cannot solve the output or full state constraint problem of the system.

In practical engineering, constraints exist on the system output or states. Only by limiting the output or states to the allowable range of the process can the safety of the equipment and operators be ensured. To solve the problem of these constraints, many studies have used the barrier Lyapunov function (BLF) [17]–[22]. To address asymmetric time-varying constraints, some researchers have proposed asymmetric time-varying BLFs (ABLFs) [23]–[26]. However, the control methods in [23]–[26] were all aimed at unswitched systems in strict feedback structures and required that all states be measurable. In [27], an adaptive control for a full state constrained switched strict feedback system was designed, but these constraints are static, not time varying. Reference [28], [29] proposed a fuzzy adaptive tracking control approach by using ABLFs for switched non-strict feedback systems with full state constraints. Reference [30] extended the results in [28], [29] to the controller design for MIMO systems. However, the control approaches in [27]–[30] required all the states to be measurable directly. To the best of our knowledge, there are no results on adaptive control for switched non-strict feedback systems with unmeasured states and full state time varying constraints.

Motivated by the above discussion, this paper investigates adaptive control for switched uncertain nonlinear systems with asymmetric time-varying full state constraints, coupled with unmeasurable states and a non-strict feedback problem. This issue involves three obstacles:

1) The considered system in this paper is coupled with switching signals, full state time varying constraints, uncertain nonlinear functions, the algebraic loop problem, the unmeasurable states problem, and external disturbances. Compared with the existing results, the structure of the system is more complex, and more problems need to be solved simultaneously. Therefore, the controller design and stability proof are very difficult.

2) The non-strict feedback structure of the system will lead to the algebraic loop problem in the controller design. At the same time, because the states of the system are not measurable, it is difficult to design a state observer to integrate into the controller to effectively estimate the states of all the switching subsystems with a non-strict feedback structure.

3) The existing results need strict preconditions, such as n-order differentiable and bounded conditions of the input signals, and unknown functions must satisfy the monotonically increasing condition. These strict preconditions lead to the lack of practicability of control methods. Adaptive fuzzy backstepping technology has been widely used in the control of uncertain nonlinear systems such as manipulator, but there is the problem of "explosion of complexity" due to repeated derivation of virtual control law in backstepping [31], [32]. Therefore, it is necessary to reduce the computational burden and eliminate the strict preconditions. These requirements make the design of the control scheme more challenging.

This paper proposes a novel adaptive dynamic output feedback tracking control strategy for switched non-strict feedback systems with full state time-varying constraints and external disturbances. The main contributions and innovations are as follows:

1) Different from the existing results, this paper is the first to investigate adaptive control for switched non-strict feedback systems with full state time-varying constraints, unmeasurable states and unknown disturbances, which is a more general case in real systems. The control methods proposed in [17]–[22] cannot address the time-varying constraints problem. The control methods in [23]–[26] were all aimed at a non-switched system in strict feedback form. The results in [27]–[30] require full states of the system to be measurable directly. At present, no existing reference can comprehensively solve the above problems. It is a novel idea and a great challenge to integrate these problems into the controller design for switched nonlinear uncertain systems.

2) This proposed adaptive control scheme can address non-strict feedback systems, strict feedback systems and other mismatching uncertain systems. In addition, it can also address symmetric static constraints and asymmetric time-varying constraints, output constraints and full state constraints, as well as the output feedback system and state

feedback system. The designed control algorithm has strong applicability.

3) This proposed adaptive control scheme does not need n-order differentiable and bounded conditions of the input signals and a monotonically increasing condition of unknown functions. However, these strict assumptions are common in the existing references [33], [34]. Moreover, by adopting dynamic surface technology and first-order filters, this control scheme can avoid the problem of the ''explosion of complexity''. Therefore, this control scheme not only conforms to engineering practice but also has a simple algorithm and requires a small number of calculations.

II. SYSTEM DESCRIPTION AND ASSUMPTION

Consider the following switched non-strict feedback system [14]:

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma(t)}(x) + d_{1,\sigma(t)}(t) \\ \quad \vdots \\ \dot{x}_i = x_{i+1} + f_{i,\sigma(t)}(x) + d_{i,\sigma(t)}(t) \\ \quad \vdots \\ \dot{x}_n = u_{\sigma(t)} + f_{n,\sigma(t)}(x) + d_{n,\sigma(t)}(t) \\ y = x_1 \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n-1, x = [x_1, x_2, \ldots, x_n]^T \in R^n$ are the state vectors, and only x_1 can be directly measured. $u_{\sigma(t)} \in R$ is the control signal, and $y \in R$ is the output of the system. $\sigma(t) : [0, \infty) \to M = \{1, 2, \ldots, m\}$ is the switching signal, and it is a piecewise continuous function of time from the right. The kth subsystem is active when $\sigma(t) = k$. $u_k \in R$ is the control input of the kth subsystem. $f_{i,\sigma(t)}(x)$ $(i = 1, 2, \ldots, n)$ are unknown smooth functions of the nonlinear system. $d_{i,\sigma(t)}(t)$ $(i = 1, 2, \ldots, n)$ are unknown disturbances.

Assumption 1 [35]: The unknown disturbances $d_{i,\sigma(t)}(t)$ are bounded by unknown constants $\bar{d}_{i,\sigma(t)}$ such that $\left|d_{i,\sigma(t)}(t)\right| \le \bar{d}_{i,\sigma(t)}$, where $i = 1, 2, \cdots, n$.

Assumption 2 [23], [29]: There exist constants $\bar{K}_{ci}^j$ and $\underline{K}_{ci}^j$ $(i = 1, 2, \cdots, n, j = 0, 1, 2, \cdots, n)$ such that $\bar{k}_{ci}(t) \le \bar{K}_{ci}^0$, $\left|\bar{k}_{ci}^{(j)}(t)\right| \le \bar{K}_{ci}^j$ and $\left|\underline{k}_{ci}^{(j)}(t)\right| \le \underline{K}_{ci}^j$, with $\forall t \ge 0$.

Assumption 3 [25], [36]: There exist functions $\bar{Y}_0 : R_+ \to R_+$ and $\underline{Y}_0 : R_+ \to R_+$ satisfying $\bar{Y}_0 < \bar{k}_{c1}(t)$ and $\underline{Y}_0 > \underline{k}_{c1}(t)$, with $\forall t > 0$, and a positive constant Y_1 such that the desired trajectory $y_d(t)$ and its time derivative satisfy $\underline{Y}_0(t) \le y_d(t) \le \bar{Y}_0(t)$ and $|\dot{y}(t)| \le Y_1$, with $\forall t > 0$.

Assumption 4 [37], [38]: There exist constants $l_{i,\sigma(t)}$ such that $\left|f_{i,\sigma(t)}(x) - f_{i,\sigma(t)}(\hat{x})\right| \le l_{i,\sigma(t)} \left\|x - \hat{x}\right\|$, where $i = 1, 2, \cdots, n$, $\hat{x} = \left[\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n\right]^T$ is the estimate of $x = [x_1, x_2, \ldots, x_n]^T$, and $\|x\|$ is the 2-norm of the state vector x.

Lemma 1 [2], [14], [28]: A switched system (1) is defined to have a switching signal $\delta(t)$ with an average dwell time τ_a if there exist two positive constants $\tau_a > 0$ and $N_0 > 0$, such that $N_\delta(t, T) \le \frac{T-t}{\tau_a} + N_0$, with $\forall T \ge t \ge 0$, where $N_\delta(t, T)$ is the number of switches occurring in the interval $[t, T)$.

Control objective: The control objective is to design an adaptive output feedback controller to keep the output $y(t)$ tracking the desired trajectory $y_d(t)$, with the tracking error $z_1 = y - y_d$ kept as small as possible. Moreover, full states of the closed system must be kept in the prescribed bounds.

III. ADAPTIVE SWITCHED FUZZY STATE OBSERVER DESIGN

A fuzzy logic system can be written as:

$$\hat{f}(x|\theta) = \theta^T \xi(x) \tag{2}$$

where $\xi(x)$ is the fuzzy basis function vector and $\theta \in R^N$ is the adjustable weight parameter vector.

Lemma 2 [39]: If $f(x)$ is a continuous function defined on the compact set Ω, then for any given small constant $\varepsilon > 0$, there exists a fuzzy logic system such that $\sup_{x\in\Omega} |f(x) - \theta^{\mathbf{T}}\xi(x)| \le \varepsilon$.

For the kth subsystem, Equation (1) can be rewritten in matrix form:

$$\begin{aligned} \dot{x} &= A_k x + K_k y + \sum_{i=1}^{n} B_i \left[f_{i,k}(x) + d_{i,k}\right] + B u_k \\ y &= Cx \end{aligned} \tag{3}$$

where

$$A_k = \begin{bmatrix} -k_{1,k} & & \\ \vdots & I & \\ -k_{n,k} & 0 & 0 \end{bmatrix}, \quad K_k = \begin{bmatrix} k_{1,k} \\ \vdots \\ k_{n,k} \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 & \ldots 1 & \ldots 0 \end{bmatrix}^T, \quad B = \begin{bmatrix} 0 & \ldots & 0 \ldots 1 \end{bmatrix}^T,$$

and $C = \begin{bmatrix} 1 \ldots 0 \ldots 0 \end{bmatrix}$. By choosing K_k to make A_k is a strict Hurwitz matrix such that the following equation exists:

$$A_k^T P_k + P_k A_k = -Q_k \tag{4}$$

where $Q_k > 0$ is any given positive definite diagonal matrix and $P_k > 0$ is a positive definite symmetric matrix.

Because the states $x_2, \ldots, x_n$ of the system (1) cannot be directly measured, a state observer should be designed to estimate the unmeasurable states. By defining the estimation of $x = [x_1, x_2, \ldots, x_n]^T$ as $\hat{x} = \left[\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n\right]^T$, we can have:

$$f_{i,k}(x) = f_{i,k}(\hat{x}) + \Delta f_{i,k} \tag{5}$$

where $\Delta f_{i,k} = f_{i,k}(x) - f_{i,k}(\hat{x})$, $i = 1, 2, \ldots, n$, and $k \in \{1, 2, \ldots, m\}$.

According to Lemma 2, $f_{i,k}(\hat{x})$ $(i = 1, 2, \ldots, n)$ can be rewritten as follows:

$$f_{i,k}(\hat{x}) = \theta_{i,k}^T \xi_{i,k}(\hat{x}) + \varepsilon'_{i,k} \tag{6}$$

where $\xi_{i,k}(\hat{x})$ is the fuzzy basic function vector, $\theta_{i,k}$ is the adaptive parameter vector, and $\varepsilon'_{i,k}$ is the error of the approximation.

Define $\theta^*_{i,k}$ as the optimal parameter vector of $\theta_{i,k}$ such that:

$$\theta^*_{i,k} = \arg\min_{\boldsymbol{\theta}_{i,k}\in\Omega_{i,k}} \left[\sup_{\hat{x}\in U}\left|f_{i,k}(\hat{x}) - \boldsymbol{\theta}^T_{i,k}\boldsymbol{\xi}_{i,k}(\hat{x})\right|\right] \tag{7}$$

where $\Omega_{i,k}$ and U are the compact sets of $\theta_{i,k}$ and $\hat{x}$, respectively.

Now, the minimal approximation error can be defined as:

$$\varepsilon_{i,k}(\hat{x}) = f_{i,k}(\hat{x}) - \theta^{*T}_{i,k}\xi_{i,k}(\hat{x}) \tag{8}$$

where $|\varepsilon_{i,k}(\hat{x})| \leq \varepsilon^*_{i,k}$ and $\varepsilon^*_{i,k}$ are unknown constants. The adaptive parameter error vector can be defined as $\tilde{\theta}_{i,k} = \theta^*_{i,k} - \theta_{i,k}$.

Substituting Equations (5) and (8) into (4) results in

$$\begin{aligned}\dot{x} &= A_kx + K_ky + \sum_{i=1}^{n} B_i\left[\theta^{*T}_{i,k}\xi_{i,k}(\hat{x}) + \varepsilon_{i,k}(\hat{x}) + \Delta f_{i,k} + d_{i,k}\right] \\ &\quad + Bu_k \\ y &= Cx.\end{aligned} \tag{9}$$

Then for the kth subsystem, the fuzzy state observer can be designed as:

$$\begin{aligned}\dot{\hat{x}} &= A_k\hat{x} + K_ky + \sum_{i=1}^{n} B_i\left[\theta^T_{i,k}\xi_{i,k}(\hat{x})\right] + Bu_k \\ \hat{y} &= C\hat{x}.\end{aligned} \tag{10}$$

Define the observer error vector as $\tilde{x} = x - \hat{x}$; according to Equations (3) and (10), we can obtain the observer error equation:

$$\begin{aligned}\dot{\tilde{x}} &= A\tilde{x} + \sum_{i=1}^{n} B_i\left[\tilde{\theta}^T_{i,k}\xi_{i,k}(\hat{x}) + \varepsilon_{i,k}(\hat{x}) + \Delta f_{i,k} + d_{i,k}\right] \\ &= A\tilde{x} + \sum_{i=1}^{n} B_i\left[\tilde{\theta}^T_{i,k}\xi_{i,k}(\hat{x})\right] + \varepsilon_k + \Delta F_k + D_k\end{aligned} \tag{11}$$

where $\varepsilon_k = [\varepsilon_{1,k}, \varepsilon_{2,k}, \ldots, \varepsilon_{n,k}]^T$, $\Delta F_k = [\Delta f_{1,k}, \Delta f_{2,k}, \ldots, \Delta f_{n,k}]^T$ and $D_k = [d_{1,k}, d_{2,k}, \ldots, d_{n,k}]^T$.

IV. ADAPTIVE CONTROL LAW DESIGN

Define the tracking error z_1, virtual error z_i, virtual control law $\alpha_{i-1,k}$, and first-order filters as:

$$\begin{cases} z_1 = y - y_d \\ z_i = \hat{x}_i - \omega_{i-1,k} \\ \upsilon_{i-1,k}\dot{\omega}_{i-1,k} + \omega_{i-1,k} = \alpha_{i-1,k} \\ \omega_{i-1,k}(0) = \alpha_{i-1,k}(0) \end{cases} \tag{12}$$

where $i = 2, \ldots, n$ and $k \in \{1, 2, \ldots, m\}$. $\upsilon_{i-1,k}$ is the time constant of the filter, that is, by letting $\alpha_{i-1,k}$ pass through a filter that has the time constant $\upsilon_{i-1,k}$, we can obtain $\omega_{i-1,k}$.

The filter output error can be defined as $e_{i-1,k} = \omega_{i-1,k} - \alpha_{i-1,k}$, and we can obtain $\dot{\omega}_{i-1,k} = -\frac{e_{i-1,k}}{\upsilon_{i-1,k}}$.

Step 1: The time derivative of z_1 can be obtained as:

$$\begin{aligned}\dot{z}_1 &= x_2 + f_{1,k} + d_{1,k} - \dot{y}_d \\ &= z_2 + e_{1,k} + \alpha_{1,k} + \tilde{x}_2 + \theta^{*T}_{1,k}\xi_{1,k}(\hat{x}) \\ &\quad + \varepsilon_{1,k} + d_{1,k} - \dot{y}_d + \Delta f_{1,k} \\ &= z_2 + e_{1,k} + \alpha_{1,k} + \tilde{x}_2 + \theta^{*T}_{1,k}\xi_{1,k}(\hat{x}) \\ &\quad + \varepsilon_{1,k} + d_{1,k} - \dot{y}_d + \Delta f_{1,k} \\ &\quad - \theta^{*T}_{1,k}\xi_{1,k}(\hat{x}_1) + \tilde{\theta}^T_{1,k}\xi_{1,k}(\hat{x}_1) + \theta^T_{1,k}\xi_{1,k}(\hat{x}_1).\end{aligned} \tag{13}$$

Define the following functions:

$$k_{a1}(t) = y_d(t) - \underline{k}_{c1}(t) \tag{14}$$

$$k_{b1}(t) = \bar{k}_{c1}(t) - y_d(t) \tag{15}$$

$$q_1(z_1) = \begin{cases} 1 & z_1 > 0 \\ 0 & z_1 \leq 0. \end{cases} \tag{16}$$

Now, the Lyapunov function can be chosen as:

$$\begin{aligned}V_{0,k} &= \tilde{x}^TP_k\tilde{x} + \frac{q_1(z_1)}{2}\log\frac{k^2_{b1}(t)}{k^2_{b1}(t) - z^2_1} \\ &\quad + \frac{1 - q_1(z_1)}{2}\log\frac{k^2_{a1}(t)}{k^2_{a1}(t) - z^2_1} + \frac{1}{2\gamma_{1,k}}\tilde{\theta}^T_{1,k}\tilde{\theta}_{1,k}\end{aligned} \tag{17}$$

where $\gamma_{1,k} > 0$ is the design constant.

With $\varsigma_{a1} = \frac{z_1(t)}{k_{a1}(t)}$, $\varsigma_{b1} = \frac{z_1(t)}{k_{b1}(t)}$, and $\varsigma_1 = q_1\varsigma_{b1} + (1 - q_1)\varsigma_{a1}$, Equation (17) can be rearranged as:

$$V_{0,k} = \tilde{x}^TP_k\tilde{x} + \frac{1}{2}\log\frac{1}{1 - \varsigma^2_1} + \frac{1}{2\gamma_{1,k}}\tilde{\theta}^T_{1,k}\tilde{\theta}_{1,k}. \tag{18}$$

The time derivative of $V_{0,k}$ can be obtained as:

$$\begin{aligned}\dot{V}_{0,k} &= \dot{\tilde{x}}^TP_k\tilde{x} + \tilde{x}^TP_k\dot{\tilde{x}} + \frac{\varsigma_1\dot{\varsigma}_1}{1 - \varsigma^2_1} - \frac{1}{\gamma_{1,k}}\tilde{\theta}^T_{1,k}\dot{\theta}_{1,k} \\ &= \tilde{x}^T\left[P_kA^T_k + A_kP_k\right]\tilde{x} + \frac{\varsigma_1\dot{\varsigma}_1}{1 - \varsigma^2_1} - \frac{1}{\gamma_{1,k}}\tilde{\theta}^T_{1,k}\dot{\theta}_{1,k} \\ &\quad + 2\tilde{x}^TP_k\left(\sum_{i=1}^{n} B_i\tilde{\theta}^T_{i,k}\xi_{i,k}(\hat{x}) + \varepsilon_k + \Delta F_k + D_k\right).\end{aligned} \tag{19}$$

Because

$$\frac{\varsigma_1\dot{\varsigma}_1}{1 - \varsigma^2_1} = \frac{q_1\varsigma_{b1} + (1 - q_1)\varsigma_{a1}}{1 - \varsigma^2_1}(q_1\dot{\varsigma}_{b1} + (1 - q_1)\dot{\varsigma}_{a1}) \tag{20}$$

$$\dot{\varsigma}_{b1} = \frac{\dot{z}_1k_{b1}(t) - z_1\dot{k}_{b1}(t)}{k^2_{b1}(t)} \tag{21}$$

$$\dot{\varsigma}_{a1} = \frac{\dot{z}_1k_{a1}(t) - z_1\dot{k}_{a1}(t)}{k^2_{a1}(t)} \tag{22}$$

we obtain

$$\begin{aligned}&\dot{V}_{0,k} \\ &= -\tilde{x}^TQ_k\tilde{x} + 2\tilde{x}^TP_k\left(\sum_{i=1}^{n} B_i\tilde{\theta}^T_{i,k}\xi_{i,k}(\hat{x}) + \varepsilon_k + \Delta F_k + D_k\right) \\ &\quad + \frac{q_1\varsigma_{b1} + (1 - q_1)\,\varsigma_{a1}}{1 - \varsigma^2_1}(q_1\dot{\varsigma}_{b1} + (1 - q_1)\dot{\varsigma}_{a1}) - \frac{1}{\gamma_{1,k}}\tilde{\theta}^T_{1,k}\dot{\theta}_{1,k} \\ &= -\tilde{x}^TQ_k\tilde{x} + 2\tilde{x}^TP_k\left(\sum_{i=1}^{n} B_i\tilde{\theta}^T_{i,k}\xi_{i,k}(\hat{x}) + \varepsilon_k + \Delta F_k + D_k\right)\end{aligned}$$

$$+\frac{q_1\varsigma_{b1}}{1-\varsigma_1^2}\dot{\varsigma}_{b1}+\frac{(1-q_1)\varsigma_{a1}}{1-\varsigma_1^2}\dot{\varsigma}_{a1}-\frac{1}{\gamma_{1,k}}\tilde{\theta}_{1,k}^T\dot{\theta}_{1,k}$$
$$= -\tilde{x}^TQ_k\tilde{x}+2\tilde{x}^TP_k\left(\sum_{i=1}^n B_i\tilde{\theta}_{i,k}^T\xi_{i,k}(\hat{x})+\varepsilon_k+\Delta F_k+D_k\right)$$
$$+\frac{q_1\varsigma_{b1}}{k_{b1}(1-\varsigma_{b1}^2)}\left(\dot{z}_1-\frac{z_1\dot{k}_{b1}}{k_{b1}}\right)$$
$$+\frac{(1-q_1)\varsigma_{a1}}{k_{a1}(1-\varsigma_{a1}^2)}\left(\dot{z}_1-\frac{z_1\dot{k}_{a1}}{k_{a1}}\right)$$
$$-\frac{1}{\gamma_{1,k}}\tilde{\theta}_{1,k}^T\dot{\theta}_{1,k}. \tag{23}$$

By defining $\mu_1=\frac{q_1}{k_{b1}^2-z_1^2}+\frac{(1-q_1)}{k_{a1}^2-z_1^2}$, we have

$$\dot{V}_{0,k}$$
$$= -\tilde{x}^TQ_k\tilde{x}+2\tilde{x}^TP_k\left(\sum_{i=1}^n B_i\tilde{\theta}_{i,k}^T\xi_{i,k}(\hat{x})+\varepsilon_k+\Delta F_k+D_k\right)$$
$$+\mu_1z_1\left(\dot{z}_1-q_1\frac{z_1\dot{k}_{b1}}{k_{b1}}-(1-q_1)\frac{z_1\dot{k}_{a1}}{k_{a1}}\right)-\frac{1}{\gamma_{1,k}}\tilde{\theta}_{1,k}^T\dot{\theta}_{1,k}$$
$$= -\tilde{x}^TQ_k\tilde{x}+2\tilde{x}^TP_k\left(\sum_{i=1}^n B_i\tilde{\theta}_{i,k}^T\xi_{i,k}(\hat{x})+\varepsilon_k+\Delta F_k+D_k\right)$$
$$+\mu_1z_1\Big(\tilde{x}_2+\theta_{1,k}^{*T}\xi_{1,k}(\hat{x})-\theta_{1,k}^{*T}\xi_{1,k}(\hat{x}_1)$$
$$+\varepsilon_{1,k}+d_{1,k}+\Delta f_{1,k}\Big)$$
$$+\mu_1z_1\begin{pmatrix} z_2+\alpha_{1,k}+e_{1,k}+\tilde{\theta}_{1,k}^T\xi_{1,k}(\hat{x}_1)+\theta_{1,k}^T\xi_{1,k}(\hat{x}_1)\\ -\dot{y}_d-q_1\frac{z_1\dot{k}_{b1}}{k_{b1}}-(1-q_1)\frac{z_1\dot{k}_{a1}}{k_{a1}}\end{pmatrix}$$
$$-\frac{1}{\gamma_{1,k}}\tilde{\theta}_{1,k}^T\dot{\theta}_{1,k}. \tag{24}$$

According to Assumption 1, Assumption 4, Young's inequalities, and $\xi_{1,k}^T(\cdot)\xi_{1,k}(\cdot)\le 1$, the following inequalities exist:

$$2\tilde{x}^TP_k(\varepsilon_k+D_k)$$
$$\le 2\|\tilde{x}\|^2+\|P_k\|^2\left\|\varepsilon_k^*\right\|^2+\|P_k\|^2\sum_{i=1}^n\bar{d}_{i,k} \tag{25}$$
$$2\tilde{x}^TP_k\Delta F_k$$
$$\le \|\tilde{x}\|^2+\|P_k\|^2\|\Delta F_k\|^2$$
$$\le \|\tilde{x}\|^2+\|P_k\|^2\left(|\Delta f_1|^2+|\Delta f_2|^2+\cdots+|\Delta f_n|^2\right)$$
$$\le \|\tilde{x}\|^2+\|P_k\|^2\left(\sum_{j=1}^n l_{j,k}^2\|\tilde{x}\|^2\right) \tag{26}$$
$$2\tilde{x}^TP_k\left(\sum_{i=1}^n B_i\tilde{\theta}_{i,k}^T\xi_{i,k}(\hat{x})\right)$$
$$\le n\|\tilde{x}\|^2+\|P_k\|^2\sum_{i=1}^n\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k} \tag{27}$$
$$\mu_1z_1\left(\tilde{x}_2+\varepsilon_{1,k}+d_{1,k}+\Delta f_{1,k}\right)$$
$$\le 2(\mu_1z_1)^2+\frac{1}{2}\|\tilde{x}\|^2+\frac{1}{2}\varepsilon_{1,k}^{*\,2}+\frac{1}{2}\bar{d}_{1,k}^2+\frac{1}{2}l_{1,k}^2\|\tilde{x}\|^2 \tag{28}$$

$$\mu_1z_1\left(\theta_{1,k}^{*T}\xi_{1,k}(\hat{x})-\theta_{1,k}^{*T}\xi_{1,k}(\hat{x}_1)\right)\le(\mu_1z_1)^2+\left\|\theta_{1,k}^*\right\|^2. \tag{29}$$

Substituting Equations (25)-(29) into (24) results in

$$\dot{V}_{0,k}$$
$$\le -\lambda_{1,k}\|\tilde{x}\|^2+\|P_k\|^2\sum_{i=1}^n\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}$$
$$+\mu_1z_1\begin{bmatrix} z_2+3\mu_1z_1+\alpha_{1,k}+\theta_{1,k}^T\xi_{1,k}(\hat{x}_1)\\ -\dot{y}_d+q_1\frac{z_1\dot{k}_{b1}}{k_{b1}}+(1-q_1)\frac{z_1\dot{k}_{a1}}{k_{a1}}\end{bmatrix}$$
$$+\frac{1}{\gamma_{1,k}}\tilde{\theta}_{1,k}^T\left(\gamma_{1,k}\mu_1z_1\xi_{1,k}(\hat{x}_1)-\dot{\theta}_{1,k}\right)+\mu_1z_1e_{1,k}+M_{1,k} \tag{30}$$

where $\lambda_{1,k}=\lambda_{\min}(Q_k)-n-\frac{7}{2}-\frac{1}{2}l_{1,k}^2-\|P_k\|^2\left(\sum_{j=1}^n l_{j,k}^2\right)$ and $M_{1,k}=\|P_k\|^2\left\|\varepsilon_k^*\right\|^2+\|P_k\|^2\sum_{i=1}^n\bar{d}_{i,k}+\frac{1}{2}\varepsilon_{1,k}^{*\,2}+\frac{1}{2}\bar{d}_{1,k}^2+\left\|\theta_{1,k}^*\right\|^2$.

The virtual controller $\alpha_{1,k}$ and the parameter adaptive law $\theta_{1,k}$ are designed as follows:

$$\alpha_{1,k} = -c_{1,k}z_1-c_1z_1-3\mu_1z_1-\theta_{1,k}^T\xi_{1,k}(\hat{x}_1)+\dot{y}_d \tag{31}$$
$$\dot{\theta}_{1,k} = \gamma_{1,k}\mu_1z_1\xi_{1,k}(\hat{x}_1)-2\sigma_{1,k}\theta_{1,k} \tag{32}$$

where $c_{1,k}>0$ and $\sigma_{1,k}>0$ are the design parameters. $c_1=\sqrt{\left(\frac{\dot{k}_{b1}}{k_{b1}}\right)^2+\left(\frac{\dot{k}_{a1}}{k_{a1}}\right)^2+\beta}$, where β is a positive design constant.

Substituting Equations (31) and (32) into (30) results in

$$\dot{V}_0 < -(\lambda_{\min}(Q)-1)\|\tilde{x}\|^2-\lambda_1\mu z_1^2+\mu z_1z_2$$
$$+\mu z_1e_1+\frac{1}{2}\|P\delta\|^2+\frac{1}{2}D_1^2+\left\|\theta_1^*\right\|^2+\frac{2\sigma_1}{\gamma_1}\tilde{\theta}_1^T\theta_1. \tag{33}$$

There are Young's inequalities are used in (34) and (35):

$$\frac{2\sigma_{1,k}}{\gamma_{1,k}}\tilde{\theta}_{1,k}^T\theta_{1,k}\le-\frac{\sigma_{1,k}}{\gamma_{1,k}}\theta_{1,k}^T\theta_{1,k}+\frac{\sigma_{1,k}}{\gamma_{1,k}}\theta_{1,k}^{*T}\theta_{1,k}^* \tag{34}$$
$$z_1e_{1,k}\le z_1^2+\frac{1}{4}e_{1,k}^2. \tag{35}$$

Substituting (34) and (35) into (33) leads to

$$\dot{V}_{0,k}$$
$$\le -\lambda_{1,k}\|\tilde{x}\|^2+\|P_k\|^2\sum_{i=1}^n\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}-c_{1,k}\mu_1z_1^2+\mu_1z_1z_2$$
$$+\mu_1z_1e_{1,k}+\frac{2\sigma_{1,k}}{\gamma_{1,k}}\tilde{\theta}_{1,k}^T\theta_{1,k}+M_{1,k}$$
$$\le -\lambda_{1,k}\|\tilde{x}\|^2+\|P_k\|^2\sum_{i=1}^n\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}-\left(c_{1,k}-1\right)\mu_1z_1^2+\mu_1z_1z_2$$
$$+\frac{1}{4}\mu_1e_{1,k}^2-\frac{\sigma_{1,k}}{\gamma_{1,k}}\theta_{1,k}^T\theta_{1,k}+\frac{\sigma_{1,k}}{\gamma_{1,k}}\theta_{1,k}^{*T}\theta_{1,k}^*+M_{1,k}. \tag{36}$$

Substituting the following inequality Equation (37)

$$-\frac{1}{2}\tilde{\theta}_{1,k}^T\tilde{\theta}_{1,k} \geq -\theta_{1,k}^{*}{}^T\theta_{1,k}^* - \theta_{1,k}^T\theta_{1,k} \tag{37}$$

into Equation (36) results in

$$\begin{aligned}\dot{V}_{0,k} \leq & -\lambda_{1,k}\|\tilde{x}\|^2 + \|P_k\|^2 \\ & \times\sum_{i=1}^{n}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k} - (c_{1,k}-1)\,\mu_1 z_1^2 + \mu_1 z_1 z_2 \\ & +\frac{1}{4}\mu_1 e_{1,k}^2 - \frac{\sigma_{1,k}}{2\gamma_{1,k}}\tilde{\theta}_{1,k}^T\tilde{\theta}_{1,k} + \frac{2\sigma_{1,k}}{\gamma_{1,k}}\left\|\theta_{1,k}^*\right\|^2 + M_{1,k}.\end{aligned} \tag{38}$$

By choosing the following Lyapunov function

$$V_{1,k} = V_{0,k} + \frac{1}{2}e_{1,k}^2 \tag{39}$$

we obtain:

$$\begin{aligned}&\dot{V}_{1,k} \\ &= \dot{V}_{0,k} + e_{1,k}\left(-\frac{e_{1,k}}{\upsilon_{1,k}} - \dot{\alpha}_{1,k}\right) \\ &\leq \dot{V}_{0,k} - \frac{e_{1,k}^2}{\upsilon_{1,k}} + e_{1,k}^2 + \frac{1}{4}\psi_{1,k}^2 \\ &\leq -\lambda_{1,k}\|\tilde{x}\|^2 + \|P_k\|^2\sum_{i=1}^{n}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k} - (c_{1,k}-1)\,\mu_1 z_1^2 \\ &\quad +\mu_1 z_1 z_2 - \left(\frac{1}{\upsilon_{1,k}} - 1 - \frac{1}{4}\mu_1\right)e_{1,k}^2 - \frac{\sigma_{1,k}}{2\gamma_{1,k}}\tilde{\theta}_{1,k}^T\tilde{\theta}_{1,k} \\ &\quad +\frac{2\sigma_{1,k}}{\gamma_{1,k}}\left\|\theta_{1,k}^*\right\|^2 + \frac{1}{4}\psi_{1,k}^2 + M_{1,k}\end{aligned} \tag{40}$$

where $\psi_{1,k}$ is the maximum absolute value of $\dot{\alpha}_{1,k}$.

Step $ith(i = 2, 3, \ldots, n-1)$: The time derivative of z_i is obtained as:

$$\begin{aligned}\dot{z}_i &= \dot{\hat{x}}_{i+1} + k_{i,k}\tilde{x}_1 + \hat{f}_{i,k} - \dot{\omega}_{i-1,k} \\ &= z_{i+1} + e_{i,k} + \alpha_{i,k} + k_{i,k}\tilde{x}_1 + \theta_{i,k}^T\xi_{i,k}(\hat{x}) - \dot{\omega}_{i-1,k} \\ &= z_{i+1} + e_{i,k} + \alpha_{i,k} + k_{i,k}\tilde{x}_1 - \tilde{\theta}_{i,k}^T\xi_{i,k}(\hat{x}) + \theta_{i,k}^{*}{}^T\xi_{i,k}(\hat{x}) \\ &\quad -\theta_{i,k}^{*}{}^T\xi_{i,k}(\underline{\hat{x}}_i) + \tilde{\theta}_{i,k}^T\xi_{i,k}(\underline{\hat{x}}_i) + \theta_{i,k}^T\xi_{i,k}(\underline{\hat{x}}_i) - \dot{\omega}_{i-1,k}\end{aligned} \tag{41}$$

where $\underline{\hat{x}}_i = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_i]^T$ and $i = 2, 3, \ldots, n$.

Define the following function:

$$k_{ai}(t) = \psi_{i-1,k} - \underline{k}_{ci}(t) \tag{42}$$

$$k_{bi}(t) = \bar{k}_{ci}(t) - \psi_{i-1,k} \tag{43}$$

$$q_i(z_i) = \begin{cases}1 & z_i > 0 \\ 0 & z_i \leq 0.\end{cases} \tag{44}$$

The Lyapunov function can be chosen as:

$$\begin{aligned}V_{i,k} &= V_{i-1,k} + \frac{q_i(z_i)}{2}\log\frac{k_{bi}^2(t)}{k_{bi}^2(t) - z_i^2} \\ &\quad +\frac{1-q_i(z_i)}{2}\log\frac{k_{ai}^2(t)}{k_{ai}^2(t) - z_i^2} + \frac{1}{2}e_{i,k}^2 + \frac{1}{2\gamma_{i,k}}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}\end{aligned} \tag{45}$$

where $\gamma_{i,k} > 0$ is the design parameter.

By defining $\varsigma_{ai} = \frac{z_i(t)}{k_{ai}(t)}$, $\varsigma_{bi} = \frac{z_i(t)}{k_{bi}(t)}$, and $\varsigma_i = q_i\varsigma_{bi} + (1-q_i)\varsigma_{ai}$, Equation (45) can be rearranged as:

$$V_{i,k} = V_{i-1,k} + \frac{1}{2}\log\frac{1}{1-\varsigma_i^2} + \frac{1}{2}e_{i,k}^2 + \frac{1}{2\gamma_{i,k}}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}. \tag{46}$$

The time derivative of $V_{i,k}$ can be obtained as:

$$\dot{V}_{i,k} = \dot{V}_{i-1,k} + \frac{\varsigma_i\dot{\varsigma}_i}{1-\varsigma_i^2} + e_{i,k}\dot{e}_{i,k} - \frac{1}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\dot{\theta}_{i,k}. \tag{47}$$

Because

$$\frac{\varsigma_i\dot{\varsigma}_i}{1-\varsigma_i^2} = \frac{q_i\varsigma_{bi} + (1-q_i)\,\varsigma_{ai}}{1-\varsigma_i^2}\left(q_i\dot{\varsigma}_{bi} + (1-q_i)\dot{\varsigma}_{ai}\right) \tag{48}$$

$$\dot{\varsigma}_{bi} = \frac{\dot{z}_i k_{bi}(t) - z_i\dot{k}_{bi}(t)}{k_{bi}^2(t)} \tag{49}$$

$$\dot{\varsigma}_{ai} = \frac{\dot{z}_i k_{ai}(t) - z_i\dot{k}_{ai}(t)}{k_{ai}^2(t)} \tag{50}$$

we obtain:

$$\begin{aligned}&\dot{V}_{i,k} \\ &= \dot{V}_{i-1,k} + \frac{q_i\varsigma_{bi} + (1-q_i)\,\varsigma_{ai}}{1-\varsigma_i^2}\left(q_i\dot{\varsigma}_{bi} + (1-q_i)\,\dot{\varsigma}_{ai}\right) \\ &\quad +e_{i,k}\dot{e}_{i,k} - \frac{1}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\dot{\theta}_{i,k} \\ &= \dot{V}_{i-1,k} + \frac{q_i\varsigma_{bi}}{1-\varsigma_i^2}\dot{\varsigma}_{bi} + \frac{(1-q_i)\,\varsigma_{ai}}{1-\varsigma_i^2}\dot{\varsigma}_{ai} + e_{i,k}\dot{e}_{i,k} - \frac{1}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\dot{\theta}_{i,k} \\ &= \dot{V}_{i-1,k} + \frac{q_i\varsigma_{bi}}{k_{bi}\left(1-\varsigma_{bi}^2\right)}\left(\dot{z}_i - \frac{z_i\dot{k}_{bi}}{k_{bi}}\right) \\ &\quad +\frac{(1-q_i)\,\varsigma_{ai}}{k_{ai}\left(1-\varsigma_{ai}^2\right)}\left(\dot{z}_i - \frac{z_i\dot{k}_{ai}}{k_{ai}}\right) \\ &\quad +e_{i,k}\dot{e}_{i,k} - \frac{1}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\dot{\theta}_{i,k}.\end{aligned} \tag{51}$$

By defining $\mu_i = \frac{q_i}{k_{bi}^2 - z_i^2} + \frac{(1-q_i)}{k_{ai}^2 - z_i^2}$, we have

$$\begin{aligned}&\dot{V}_{i,k} \\ &= \dot{V}_{i-1,k} + \mu_i z_i\left(\dot{z}_i - q_i\frac{z_i\dot{k}_{bi}}{k_{bi}} - (1-q_i)\frac{z_i\dot{k}_{ai}}{k_{ai}}\right) \\ &\quad +e_{i,k}\dot{e}_{i,k} - \frac{1}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\dot{\theta}_{i,k} \\ &= \dot{V}_{i-1,k} + \mu_i z_i\left(k_{i,k}\tilde{x}_1 + \theta_{i,k}^{*}{}^T\xi_{i,k}(\hat{x}) - \theta_{i,k}^{*}{}^T\xi_{i,k}(\underline{\hat{x}}_i)\right) \\ &\quad +\mu_i z_i\begin{pmatrix} z_{i+1} + \alpha_{i,k} + e_{i,k} - \tilde{\theta}_{i,k}^T\xi_{i,k}(\hat{x}) \\ +\tilde{\theta}_{i,k}^T\xi_{i,k}(\underline{\hat{x}}_i) + \theta_{i,k}^T\xi_{i,k}(\underline{\hat{x}}_i) \\ -\dot{\omega}_{i-1,k} - q_i\frac{z_i\dot{k}_{bi}}{k_{bi}} - (1-q_i)\frac{z_i\dot{k}_{ai}}{k_{ai}}\end{pmatrix} \\ &\quad +e_{i,k}\dot{e}_{i,k} - \frac{1}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\dot{\theta}_{i,k}.\end{aligned} \tag{52}$$

According to Assumption 1, Assumption 4, Young's inequalities, and $\xi_{i,k}^T(\cdot)\xi_{i,k}(\cdot) \leq 1$, the following inequalities exist:

$$\mu_i z_i(k_{i,k}\tilde{x}_1) \leq \frac{1}{2}(\mu_i z_i)^2 + \frac{1}{2}k_{i,k}^2\|\tilde{x}\|^2 \tag{53}$$

$$\mu_i z_i\left(\theta_{i,k}^{*T}\xi_{i,k}(\hat{x})-\theta_{i,k}^{*T}\xi_{i,k}(\hat{\underline{x}}_i)\right)\leq(\mu_i z_i)^2+\left\|\theta_{i,k}^*\right\|^2 \quad (54)$$

$$-\mu_i z_i\left(\tilde{\theta}_{i,k}^T\xi_{i,k}(\hat{x})\right)\leq\frac{1}{2}(\mu_i z_i)^2+\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}. \quad (55)$$

Substituting Equations (53)-(55) into (52) results in

$$\begin{aligned}\dot{V}_{i,k}\leq&\ \dot{V}_{i-1,k}+\mu_i z_i\\&\times\left[\begin{array}{l}z_{i+1}+\frac{3}{2}\mu_i z_i+k_{i,k}\tilde{x}_1+\alpha_{i,k}+\theta_{i,k}^T\xi_{i,k}(\hat{\underline{x}}_i)\\-\dot{\omega}_{i-1,k}-q_i\frac{z_i\dot{k}_{bi}}{k_{bi}}-(1-q_i)\frac{z_i\dot{k}_{ai}}{k_{ai}}\end{array}\right]\\&+\frac{1}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\left(\gamma_{i,k}\mu_i z_i\xi_{i,k}(\hat{\underline{x}}_i)-\dot{\theta}_{1,k}\right)+\mu_i z_i e_{i,k}\\&+e_{i,k}\dot{e}_{i,k}+\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}+\left\|\theta_{i,k}^*\right\|^2. \end{aligned} \quad (56)$$

$\alpha_{i,k}$ and $\theta_{i,k}$ are designed as follows:

$$\begin{aligned}\alpha_{i,k}=&-c_{i,k}z_i-\frac{\mu_{i-1}}{\mu_i}z_{i-1}-c_i z_i-\frac{3}{2}\mu_i z_i\\&-k_{i,k}\tilde{x}_1-\theta_{i,k}^T\xi_{i,k}(\hat{\underline{x}}_i)-\frac{\omega_{i-1,k}-\alpha_{i-1,k}}{\upsilon_{i-1,k}}\end{aligned} \quad (57)$$

$$\dot{\theta}_{i,k}=\gamma_{i,k}\mu_i z_i\xi_{i,k}(\hat{\underline{x}}_i)-2\sigma_{i,k}\theta_{i,k} \quad (58)$$

where $c_{i,k}>0$ and $\sigma_{i,k}>0$ are the design parameters. $c_i=\sqrt{\left(\frac{\dot{k}_{bi}}{k_{bi}}\right)^2+\left(\frac{\dot{k}_{ai}}{k_{ai}}\right)^2+\beta}$, where $\beta>0$ is a positive design constant.

Substituting Equations (57) and (58) into (56) results in

$$\begin{aligned}\dot{V}_{i,k}\leq&\ \dot{V}_{i-1,k}-c_{i,k}\mu_i z_i^2+\mu_i z_i z_{i+1}+\mu_i z_i e_{i,k}\\&+\frac{2\sigma_{i,k}}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\theta_{i,k}+e_{i,k}\dot{e}_{i,k}+\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}+\left\|\theta_{i,k}^*\right\|^2.\end{aligned} \quad (59)$$

Young's inequalities appear in (60) and (61):

$$\frac{2\sigma_{i,k}}{\gamma_{i,k}}\tilde{\theta}_{i,k}^T\theta_{i,k}\leq-\frac{\sigma_{i,k}}{\gamma_{i,k}}\theta_{i,k}^T\theta_{i,k}+\frac{\sigma_{i,k}}{\gamma_{i,k}}\theta_{i,k}^{*T}\theta_{i,k}^* \quad (60)$$

$$z_i e_{i,k}\leq z_i^2+\frac{1}{4}e_{i,k}^2. \quad (61)$$

Substituting (60) and (61) into (59) leads to

$$\begin{aligned}&\dot{V}_{i,k}\\&\leq\dot{V}_{i-1,k}-\left(c_{i,k}-1\right)\mu_i z_i^2+\mu_i z_i z_{i+1}+\frac{1}{4}\mu_i e_{i,k}+\left\|\theta_{i,k}^*\right\|^2\\&\quad-\frac{\sigma_{i,k}}{\gamma_{i,k}}\theta_{i,k}^T\theta_{i,k}+\frac{\sigma_{i,k}}{\gamma_{i,k}}\theta_{i,k}^{*T}\theta_{i,k}^*+e_{i,k}\dot{e}_{i,k}+\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}.\end{aligned} \quad (62)$$

Substituting the following inequality Equation (63)

$$-\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}\geq-\theta_{i,k}^{*T}\theta_{i,k}^*-\theta_{i,k}^T\theta_{i,k} \quad (63)$$

into Equation (62) results in

$$\begin{aligned}&\dot{V}_{i,k}\\&\leq\dot{V}_{i-1,k}-\left(c_{i,k}-1\right)\mu_i z_i^2+\mu_i z_i z_{i+1}+\frac{1}{4}\mu_i e_{i,k}^2\\&\quad-\frac{\sigma_{i,k}}{2\gamma_{i,k}}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}+\frac{2\sigma_{i,k}}{\gamma_{i,k}}\left\|\theta_{i,k}^*\right\|^2+e_{i,k}\left(-\frac{e_{i,k}}{\upsilon_{i,k}}-\dot{\alpha}_{i,k}\right)\\&\quad+\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}+\left\|\theta_{i,k}^*\right\|^2\leq\dot{V}_{i-1,k}-\left(c_{i,k}-1\right)\mu_i z_i^2\\&\quad+\mu_i z_i z_{i+1}+\frac{1}{4}\mu_i e_{i,k}^2-\frac{\sigma_{i,k}}{2\gamma_{i,k}}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}+\frac{2\sigma_{i,k}}{\gamma_{i,k}}\left\|\theta_{i,k}^*\right\|^2\\&\quad-\frac{e_{i,k}^2}{\upsilon_{i,k}}+e_{i,k}^2+\frac{1}{4}\psi_{i,k}^2+\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}+\left\|\theta_{i,k}^*\right\|^2\\&\leq\dot{V}_{i-1,k}-\left(c_{i,k}-1\right)\mu_i z_i^2+\mu_i z_i z_{i+1}\\&\quad-\left(\frac{1}{\upsilon_{i,k}}-1-\frac{1}{4}\mu_i\right)e_{i,k}^2-\frac{\sigma_{i,k}}{2\gamma_{i,k}}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}\\&\quad+\frac{2\sigma_{i,k}}{\gamma_{i,k}}\left\|\theta_{i,k}^*\right\|^2+\frac{1}{4}\psi_{i,k}^2+\frac{1}{2}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}+\left\|\theta_{i,k}^*\right\|^2\end{aligned} \quad (64)$$

where $\psi_{i,k}$ is the maximum absolute value of $\dot{\alpha}_{i,k}$.

Now, we obtain

$$\begin{aligned}&\dot{V}_{i,k}\\&\leq-\lambda_{1,k}\left\|\tilde{x}\right\|^2+\left\|P_k\right\|^2\sum_{i=1}^{n}\tilde{\theta}_{i,k}^T\tilde{\theta}_{i,k}-\sum_{j=1}^{i}\left(c_{j,k}-1\right)\mu_j z_j^2\\&\quad+\mu_j z_j z_{j+1}-\sum_{j=1}^{i}\left(\frac{1}{\upsilon_{j,k}}-1-\frac{1}{4}\mu_j\right)e_{j,k}^2-\sum_{j=1}^{i}\frac{\sigma_{j,k}}{2\gamma_{j,k}}\tilde{\theta}_{j,k}^T\tilde{\theta}_{j,k}\\&\quad+\sum_{j=1}^{i}\left(\frac{2\sigma_{j,k}}{\gamma_{j,k}}+1\right)\left\|\theta_{j,k}^*\right\|^2+\frac{1}{4}\sum_{j=1}^{i}\psi_{j,k}^2\\&\quad+\frac{1}{2}\sum_{j=2}^{i}\tilde{\theta}_{j,k}^T\tilde{\theta}_{j,k}+M_{1,k}.\end{aligned} \quad (65)$$

Step n: The time derivative of z_n is obtained as

$$\dot{z}_n=u_k+k_{n,k}\tilde{x}_1+\theta_{n,k}^T\xi_{n,k}(\hat{x})-\dot{\omega}_{n-1,k}. \quad (66)$$

Define the following functions:

$$k_{an}(t)=\psi_{n-1,k}-\underline{k}_{cn}(t) \quad (67)$$

$$k_{bn}(t)=\bar{k}_{cn}(t)-\psi_{n-1,k} \quad (68)$$

$$q_n(z_n)=\begin{cases}1 & z_n>0\\0 & z_n\leq 0.\end{cases} \quad (69)$$

The Lyapunov function can be chosen as

$$\begin{aligned}V_{n,k}=&\ V_{n-1,k}+\frac{q_n(z_n)}{2}\log\frac{k_{bn}^2(t)}{k_{bn}^2(t)-z_n^2}\\&+\frac{1-q_n(z_n)}{2}\log\frac{k_{an}^2(t)}{k_{an}^2(t)-z_n^2}+\frac{1}{2\gamma_{n,k}}\tilde{\theta}_{n,k}^T\tilde{\theta}_{n,k}\end{aligned} \quad (70)$$

where $\gamma_{n,k}>0$ is the design parameter.

By defining $\varsigma_{an}=\frac{z_n(t)}{k_{an}(t)}$, $\varsigma_{bn}=\frac{z_n(t)}{k_{bn}(t)}$, and $\varsigma_n=q_n\varsigma_{bn}+(1-q_n)\varsigma_{an}$, Equation (70) can be rearranged as:

$$V_{n,k}=V_{n-1,k}+\frac{1}{2}\log\frac{1}{1-\varsigma_n^2}+\frac{1}{2\gamma_{n,k}}\tilde{\theta}_{n,k}^T\tilde{\theta}_{n,k}. \quad (71)$$

The time derivative of $V_{n,k}$ can be obtained as

$$\dot{V}_{n,k}=\dot{V}_{n-1,k}+\frac{\varsigma_n\dot{\varsigma}_n}{1-\varsigma_n^2}-\frac{1}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T\dot{\theta}_{n,k}. \quad (72)$$

Because

$$\frac{\varsigma_n\dot{\varsigma}_n}{1-\varsigma_n^2}=\frac{q_n\varsigma_{bn}+(1-q_n)\varsigma_{an}}{1-\varsigma_n^2}\left(q_n\dot{\varsigma}_{bn}+(1-q_n)\dot{\varsigma}_{an}\right) \quad (73)$$

$$\dot{\varsigma}_{bn} = \frac{\dot{z}_n k_{bn}(t) - z_n \dot{k}_{bn}(t)}{k_{bn}^2(t)} \quad (74)$$

$$\dot{\varsigma}_{an} = \frac{\dot{z}_n k_{an}(t) - z_n \dot{k}_{an}(t)}{k_{an}^2(t)} \quad (75)$$

we obtain

$$\begin{aligned}
&\dot{V}_{n,k} \\
&= \dot{V}_{n-1,k} + \frac{q_n \varsigma_{bn} + (1-q_n)\varsigma_{an}}{1-\varsigma_n^2}\left(q_n \dot{\varsigma}_{bn} + (1-q_n)\dot{\varsigma}_{an}\right) \\
&\quad - \frac{1}{\gamma_{n,k}} \tilde{\theta}_{n,k}^T \dot{\theta}_{n,k} \\
&= \dot{V}_{n-1,k} + \frac{q_n \varsigma_{bn}}{1-\varsigma_n^2}\dot{\varsigma}_{bn} + \frac{(1-q_n)\varsigma_{an}}{1-\varsigma_n^2}\dot{\varsigma}_{an} - \frac{1}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T \dot{\theta}_{n,k} \\
&= \dot{V}_{n-1,k} + \frac{q_n \varsigma_{bn}}{k_{bn}(1-\varsigma_{bn}^2)}\left(\dot{z}_n - \frac{z_n \dot{k}_{bn}}{k_{bn}}\right) \\
&\quad + \frac{(1-q_n)\varsigma_{an}}{k_{an}(1-\varsigma_{an}^2)}\left(\dot{z}_n - \frac{z_n \dot{k}_{an}}{k_{an}}\right) - \frac{1}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T \dot{\theta}_{n,k}. \quad (76)
\end{aligned}$$

By defining $\mu_n = \frac{q_n}{k_{bn}^2 - z_n^2} + \frac{(1-q_n)}{k_{an}^2 - z_n^2}$, we have

$$\begin{aligned}
\dot{V}_{n,k} &= \dot{V}_{n-1,k} + \mu_n z_n \begin{pmatrix} \dot{z}_n - q_n \frac{z_n \dot{k}_{bn}}{k_{bn}} \\ -(1-q_n)\frac{z_n \dot{k}_{an}}{k_{an}} \end{pmatrix} - \frac{1}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T \dot{\theta}_{n,k} \\
&= \dot{V}_{n-1,k} + \mu_n z_n \left(k_{n,k}\tilde{x}_1 - \tilde{\theta}_{n,k}^T \xi_{n,k}(\hat{x})\right) \\
&\quad + \mu_n z_n \begin{pmatrix} u_k + \theta_{n,k}^T \xi_{n,k}(\hat{x}) + \tilde{\theta}_{n,k}^T \xi_{n,k}(\hat{x}) \\ -\dot{\omega}_{n-1,k} - q_n \frac{z_n \dot{k}_{bn}}{k_{bn}} - (1-q_n)\frac{z_n \dot{k}_{an}}{k_{an}} \end{pmatrix} \\
&\quad - \frac{1}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T \dot{\theta}_{n,k}. \quad (77)
\end{aligned}$$

According to Assumption 1, Assumption 4, Young's inequalities, and $\xi_{n,k}^T(\cdot)\xi_{n,k}(\cdot) \le 1$, the following inequalities exist:

$$-\mu_n z_n \left(\tilde{\theta}_{n,k}^T \xi_{n,k}(\hat{x})\right) \le \frac{1}{2}(\mu_n z_n)^2 + \frac{1}{2}\tilde{\theta}_{n,k}^T \tilde{\theta}_{n,k}. \quad (78)$$

Substituting Equation (78) into (77) results in

$$\begin{aligned}
\dot{V}_{n,k} &\le \dot{V}_{n-1,k} + \mu_n z_n \\
&\quad \times \begin{bmatrix} u_k + \frac{1}{2}\mu_n z_n + k_{n,k}\tilde{x}_1 + \theta_{n,k}^T \xi_{n,k}(\hat{x}) \\ -\dot{\omega}_{n-1,k} - q_n \frac{z_n \dot{k}_{bn}}{k_{bn}} - (1-q_n)\frac{z_n \dot{k}_{an}}{k_{an}} \end{bmatrix} \\
&\quad + \frac{1}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T \left(\gamma_{n,k}\mu_n z_n \xi_{n,k}(\hat{x}) - \dot{\theta}_{n,k}\right) + \frac{1}{2}\tilde{\theta}_{n,k}^T \tilde{\theta}_{n,k}. \quad (79)
\end{aligned}$$

u_k and $\theta_{n,k}$ are designed as follows:

$$\begin{aligned}
u_k &= -c_{n,k} z_n - \frac{\mu_{n-1}}{\mu_n} z_{n-1} - c_n z_n - \frac{1}{2}\mu_n z_n \\
&\quad - k_{n,k}\tilde{x}_1 - \theta_{n,k}^T \xi_{n,k}(\hat{x}) - \frac{\omega_{n-1,k} - \alpha_{n-1,k}}{\upsilon_{n-1,k}} \quad (80) \\
\dot{\theta}_{n,k} &= \gamma_{n,k}\mu_n z_n \xi_{n,k}(\hat{x}) - 2\sigma_{n,k}\theta_{n,k} \quad (81)
\end{aligned}$$

where $c_{n,k} > 0$ and $\sigma_{n,k} > 0$ are the design parameters. $c_n = \sqrt{\left(\frac{\dot{k}_{bn}}{k_{bn}}\right)^2 + \left(\frac{\dot{k}_{an}}{k_{an}}\right)^2 + \beta}$, where $\beta > 0$ is a positive design constant.

Substituting Equations (80) and (81) into (79) results in

$$\dot{V}_{n,k} \le \dot{V}_{n-1,k} - c_{n,k}\mu_n z_n^2 + \frac{2\sigma_{n,k}}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T \theta_{n,k} + \frac{1}{2}\tilde{\theta}_{n,k}^T \tilde{\theta}_{n,k}. \quad (82)$$

Young's inequality is presented in (83):

$$\frac{2\sigma_{n,k}}{\gamma_{n,k}}\tilde{\theta}_{n,k}^T \theta_{n,k} \le -\frac{\sigma_{n,k}}{\gamma_{n,k}}\theta_{n,k}^T \theta_{n,k} + \frac{\sigma_{n,k}}{\gamma_{n,k}}\theta_{n,k}^{*\,T}\theta_{n,k}^*. \quad (83)$$

Substituting (83) into (82) leads to

$$\begin{aligned}
\dot{V}_{n,k} &\le \dot{V}_{n-1,k} - c_{n,k}\mu_n z_n^2 - \frac{\sigma_{n,k}}{\gamma_{n,k}}\theta_{n,k}^T \theta_{n,k} \\
&\quad + \frac{\sigma_{n,k}}{\gamma_{n,k}}\theta_{n,k}^{*\,T}\theta_{n,k}^* + \frac{1}{2}\tilde{\theta}_{n,k}^T \tilde{\theta}_{n,k}. \quad (84)
\end{aligned}$$

Substituting the following inequality Equation (85)

$$-\frac{1}{2}\tilde{\theta}_{n,k}^T \tilde{\theta}_{n,k} \ge -\theta_{n,k}^{*\,T}\theta_{n,k}^* - \theta_{n,k}^T \theta_{n,k} \quad (85)$$

into Equation (84) results in

$$\begin{aligned}
\dot{V}_{n,k} &\le \dot{V}_{n-1,k} - c_{n,k}\mu_n z_n^2 - \frac{\sigma_{n,k}}{2\gamma_{n,k}}\tilde{\theta}_{n,k}^T \tilde{\theta}_{n,k} \\
&\quad + \frac{2\sigma_{n,k}}{\gamma_{n,k}}\left\|\theta_{n,k}^*\right\|^2 + \frac{1}{2}\tilde{\theta}_{n,k}^T \tilde{\theta}_{n,k}. \quad (86)
\end{aligned}$$

Now, we obtain

$$\begin{aligned}
&\dot{V}_{n,k} \\
&\le -\lambda_{1,k}\|\tilde{x}\|^2 + \sum_{i=2}^{n}\left(\|P_k\|^2 - \frac{\sigma_{i,k}}{2\gamma_{i,k}} + \frac{1}{2}\right)\tilde{\theta}_{i,k}^T \tilde{\theta}_{i,k} \\
&\quad + \left(\|P_k\|^2 - \frac{\sigma_{1,k}}{2\gamma_{1,k}} + 1\right)\tilde{\theta}_{1,k}^T \tilde{\theta}_{1,k} \\
&\quad - \sum_{i=1}^{n-1}(c_{i,k} - 1)\mu_i z_i^2 - c_{n,k}\mu_n z_n^2 \\
&\quad - \sum_{i=1}^{n-1}\left(\frac{1}{\upsilon_{i,k}} - 1 - \frac{1}{4}\mu_i\right)e_{i,k}^2 + \sum_{i=1}^{n-1}\left(\frac{2\sigma_{i,k}}{\gamma_{i,k}} + 1\right)\left\|\theta_{i,k}^*\right\|^2 \\
&\quad + \frac{2\sigma_{n,k}}{\gamma_{n,k}}\left\|\theta_{n,k}^*\right\|^2 + \frac{1}{4}\sum_{i=1}^{n-1}\psi_{i,k}^2 + M_{1,k}. \quad (87)
\end{aligned}$$

V. STABILITY ANALYSIS

By defining the Lyapunov function of the closed-loop system as $V_k = V_{n,k}$, we obtain the derivation of V_k as in (88).

$$\begin{aligned}
&\dot{V}_{n,k} \\
&\le -\lambda_{1,k}\|\tilde{x}\|^2 - \frac{1}{2}\sum_{i=2}^{n}\left(\frac{\sigma_{i,k}}{\gamma_{i,k}} - 2\|P_k\|^2 - 1\right)\tilde{\theta}_{i,k}^T \tilde{\theta}_{i,k} \\
&\quad - \frac{1}{2}\left(\frac{\sigma_{1,k}}{\gamma_{1,k}} - 2\|P_k\|^2 - 2\right)\tilde{\theta}_{1,k}^T \tilde{\theta}_{1,k} - \sum_{i=1}^{n-1}(c_{i,k} - 1)\mu_i z_i^2
\end{aligned}$$

$$-c_{n,k}\mu_n z_n^2-\sum_{i=1}^{n-1}\left(\frac{1}{\upsilon_{i,k}}-1-\frac{1}{4}\mu_i\right)e_{i,k}^2$$
$$+\sum_{i=1}^{n-1}\left(\frac{2\sigma_{i,k}}{\gamma_{i,k}}+1\right)\left\|\theta_{i,k}^*\right\|^2+\frac{2\sigma_{n,k}}{\gamma_{n,k}}\left\|\theta_{n,k}^*\right\|^2$$
$$+\frac{1}{4}\sum_{i=1}^{n-1}\psi_{i,k}^2+M_{1,k}. \tag{88}$$

By choosing Q_k, c_i, c_n, and υ_i such that

$$\lambda_{1,k}>0 \tag{89}$$
$$c_{i,k}-1>0,\quad i=1,2,\ldots,n-1 \tag{90}$$
$$c_{n,k}>0 \tag{91}$$
$$\frac{1}{\upsilon_{i,k}}-1-\frac{1}{4}\mu_i>0,\quad i=1,2,\ldots,n-1. \tag{92}$$

We now obtain

$$\begin{aligned}\dot V_{n,k}&\\ \le&-\frac{\lambda_{1,k}}{\lambda_{\max}(P_k)}\left(\frac{1}{2}\tilde x^TP_k\tilde x\right)-\sum_{j=1}^{n-1}2(c_{i,k}-1)\frac{1}{2}\log\frac{1}{1-\varsigma_i^2}\\ &-2c_{n,k}\frac{1}{2}\log\frac{1}{1-\varsigma_n^2}-\frac{1}{2}\sum_{i=2}^{n}\left(\frac{\sigma_{i,k}}{\gamma_{i,k}}-2\left\|P_k\right\|^2-1\right)\tilde\theta_{i,k}^T\tilde\theta_{i,k}\\ &-\frac{1}{2}\left(\frac{\sigma_{1,k}}{\gamma_{1,k}}-2\left\|P_k\right\|^2-2\right)\tilde\theta_{1,k}^T\tilde\theta_{1,k}\\ &-\sum_{j=1}^{n-1}2\left(\frac{1}{\upsilon_{i,k}}-1-\frac{1}{4}\mu_i\right)\frac{1}{2}e_{i,k}^2+M_k\end{aligned} \tag{93}$$

where

$$M_k=\sum_{j=1}^{n-1}\left(\frac{2\sigma_{i,k}}{\gamma_{i,k}}+1\right)\left\|\theta_{i,k}^*\right\|^2+\frac{2\sigma_{n,k}}{\gamma_{n,k}}\left\|\theta_{n,k}^*\right\|^2+\frac{1}{4}\sum_{j=1}^{n}\psi_{i,k}^2+M_{1,k}.$$

Define as (94), shown at the bottom of this page.

Define $C=\min C_k$ and $D=\max M_k$; then, (93) can be written as

$$\dot V_k\le -CV_k+D. \tag{95}$$

Theorem 1: For an uncertain switched non-strict feedback nonlinear system (1), under Assumptions 1-4, if the switching signal $\sigma(t)$ satisfies the condition $\tau_a>\frac{\ln\mu}{C}$ and all the initial states are in the prescribed limits, then the proposed adaptive fuzzy output feedback control scheme can guarantee that all the variables in the closed-loop system are bounded and that the time-varying constraints are not transgressed. Moreover, the observer and the tracking error can be kept in a small neighbourhood of zero.

Proof: $W(t)=e^{Ct}V_{\sigma(t)}(x(t))$ is piecewise differentiable along the solutions of system (1). According to (95), on each interval $[t_j,t_{j+1})$ we can obtain

$$\dot W(t)=Ce^{Ct}V_{\sigma(t)}(x(t))+e^{Ct}\dot V_{\sigma(t)}(x(t))\le De^{Ct}. \tag{96}$$

With the same proof in [40], we can have

$$V_k(x(t))\le\mu V_l(x(t)) \tag{97}$$

where $\mu>1$, and $k,l\in M$. Then, we can imply

$$\begin{aligned}W(t_{j+1})&=e^{Ct_{j+1}}V_{\sigma(t_{j+1})}\left(x(t_{j+1})\right)\\ &\le\mu e^{Ct_{j+1}}V_{\sigma(t_j)}\left(x(t_{j+1})\right)\\ &=\mu W(t_{j+1}^-)\\ &\le\mu\left[W(t_j)+\int_{t_j}^{t_{j+1}}De^{Ct}dt\right].\end{aligned} \tag{98}$$

For any $T>t_0=0$, iterating the inequality (98) from $j=0$ to $j=N_\sigma(T,0)-1$, we can obtain

$$\begin{aligned}W(T^-)&\le W(t_{N_\sigma(T,0)})+\int_{t_{N_\sigma(T,0)}}^{T}De^{Ct}dt\\ &\le\mu\left[\begin{array}{l}W(t_{N_\sigma(T,0)-1})+\int_{t_{N_\sigma(T,0)-1}}^{t_{N_\sigma(T,0)}}De^{Ct}dt\\ +\mu^{-1}\int_{t_{N_\sigma(T,0)}}^{T}De^{Ct}dt\end{array}\right]\\ &\le\cdots\\ &\le\mu^{N_\sigma(T,0)}\left[\begin{array}{l}W(0)+\sum_{j=0}^{N_\sigma(T,0)-1}\mu^{-j}\int_{t_j}^{t_{j+1}}De^{Ct}dt\\ +\mu^{-N_\sigma(T,0)}\int_{t_{N_\sigma(T,0)}}^{T}De^{Ct}dt\end{array}\right].\end{aligned} \tag{99}$$

Since $\tau_a>\frac{\ln\mu}{C}$, for any $\delta\in(0,C-\ln\mu/\tau_a)$, we have $\tau_a>\ln\mu/(C-\delta)$. We can now have

$$N_\sigma(T,t)\le N_0+\frac{(C-\delta)(T-t)}{\ln\mu},\quad\forall T\ge t\ge 0. \tag{100}$$

Since $N_\sigma(T,0)-j\le 1+N_\sigma(T,t_{j+1})$ $(j=0,1,\ldots,N_\sigma(T,0))$, we have

$$\mu^{N_\sigma(T,0)-j}\le\mu^{1+N_0}e^{(C-\delta)(T-t_{j+1})}. \tag{101}$$

Since $\delta<C$, and

$$\int_{t_j}^{t_{j+1}}De^{Ct}dt\le e^{(C-\delta)t_{j+1}}\int_{t_j}^{t_{j+1}}De^{\delta t}dt \tag{102}$$

$$C_k=\min\left\{\begin{array}{l}\frac{\lambda_{1,k}}{\lambda_{\max}(P_k)},2(c_{i,k}-1),2c_n,\sigma_{1,k},-2\frac{\left\|P_k\right\|^2}{\gamma_{1,k}},-\frac{2}{\gamma_{1,k}},\\ \sigma_{i,k},-2\frac{\left\|P_k\right\|^2}{\gamma_{i,k}},-\frac{1}{\gamma_{i,k}},2\left(\frac{1}{\upsilon_{i,k}}-1-\frac{1}{4}\mu_i\right),\sigma_{i,k}\end{array}\right\}. \tag{94}$$

we have

$$W(T^{-}) \leq \mu^{N_\sigma(T,0)} W(0) + \mu^{1+N_0} e^{(C-\delta)T} \int_0^T D e^{\delta t} dt. \quad (103)$$

According to [4], there exist two κ functions $\underline{\alpha}(|x|)$ and $\bar{\alpha}(|x|)$, which satisfy $\underline{\alpha}(|x|) \leq V_k(x) \leq \bar{\alpha}(|x|)$. Then, for $\forall T > 0$, we can obtain

$$\begin{aligned} &\underline{\alpha}(\|x(T)\|) \\ &\leq V_{\sigma(T^-)}\left(x(T^-)\right) \\ &\leq e^{N_0 \ln \mu} e^{\left(\frac{\ln \mu}{\tau_a} - C\right)T} \bar{\alpha}(\|x(0)\|) + \mu^{1+N_0} \frac{D}{\delta}\left(1 - e^{-\delta T}\right) \\ &\leq e^{N_0 \ln \mu} e^{\left(\frac{\ln \mu}{\tau_a} - C\right)T} \bar{\alpha}(\|x(0)\|) + \mu^{1+N_0} \frac{D}{\delta}. \end{aligned} \quad (104)$$

From (104), we have

$$\begin{aligned} \frac{1}{2} \log \frac{1}{1 - \varsigma_i^2}(T) \leq e^{N_0 \ln \mu} e^{\left(\frac{\ln \mu}{\tau_a} - C\right)T} \bar{\alpha}(\|x(0)\|) \\ + \mu^{1+N_0} \frac{D}{\delta}\left(1 - e^{-\delta T}\right), \quad \forall T > 0. \end{aligned} \quad (105)$$

Since $\tau_a > \frac{\ln \mu}{C}$, and by selecting suitable $Q_k, l_{i,k}, c_{i,k}, r_{i,k}$ and $\sigma_{i,k}$ values, then for any given $\varsigma > 0$, we can obtain

$$\begin{aligned} &\lim_{t\to\infty} \log \frac{1}{1 - \varsigma_i^2}(t) \\ &\quad \leq 2 e^{N_0 \ln \mu} e^{\left(\frac{\ln \mu}{\tau_a} - C\right)T} \bar{\alpha}(\|x(0)\|) + 2\mu^{1+N_0} \frac{D}{\delta} \leq \varsigma^2. \end{aligned} \quad (106)$$

Moreover, we can have

$$\|\tilde{x}\| \leq \max_{k \in M} \sqrt{2D/\left(C \lambda_{\min}(P_k)\right)}. \quad (107)$$

The inequality (107) implies that the error of the state observer can be reduced by choosing the appropriate design parameters [12], [14].

By (104) and $\delta > 0$, we can imply that: if the average dwell time of the switched signal $\sigma(t)$ satisfies $\tau_a > \frac{\ln \mu}{C}$, then for the bounded initial conditions, $e_{i,k}$, $\tilde{x}$, $\tilde{\theta}_{i,k}$ and z_i are bounded. Moreover, $-k_{ai}(t) < z_i(t) < k_{bi}(t)$ holds. By $-k_{a1}(t) < z_1(t) < k_{b1}(t)$ and $z_1 = x_1 - y_d$, we can imply $\underline{k}_{c1}(t) \leq x_1(t) \leq \bar{k}_{c1}(t)$. Moreover, $\hat{x}_1$ is bounded. $\alpha_{1,k}$ is a function of z_1, $\hat{x}_1$ and $\dot{y}_d$; thus, $\alpha_{1,k}$ and $\omega_{1,k}$ are bounded. By $-k_{a2}(t) < z_2(t) < k_{b2}(t)$ and $z_2 = \hat{x}_2 - \omega_{1,k}$, we can imply $\underline{k}_{c2}(t) \leq \hat{x}_2(t) \leq \bar{k}_{c2}(t)$. $\alpha_{2,k}$ is a function of $z_2, z_1, \hat{x}_1, \hat{x}_2, \alpha_{1,k}$ and $\omega_{1,k}$; thus $\alpha_{2,k}$ and $\omega_{2,k}$ are bounded. As implied above, we can have $\underline{k}_{ci}(t) \leq \hat{x}_i(t) \leq \bar{k}_{ci}(t)$. Moreover, we can infer that all the states satisfy $\underline{k}_{ci}(t) \leq x_i \leq \bar{k}_{ci}(t)$.

The configuration of the control scheme proposed above for is shown in Fig. 1.

VI. COMPARISONS WITH SOME PREVIOUS RESULTS

To further illustrate the contributions of this method, some comparisons with previous results in [7]–[16], [23]–[39] will be given in this section.

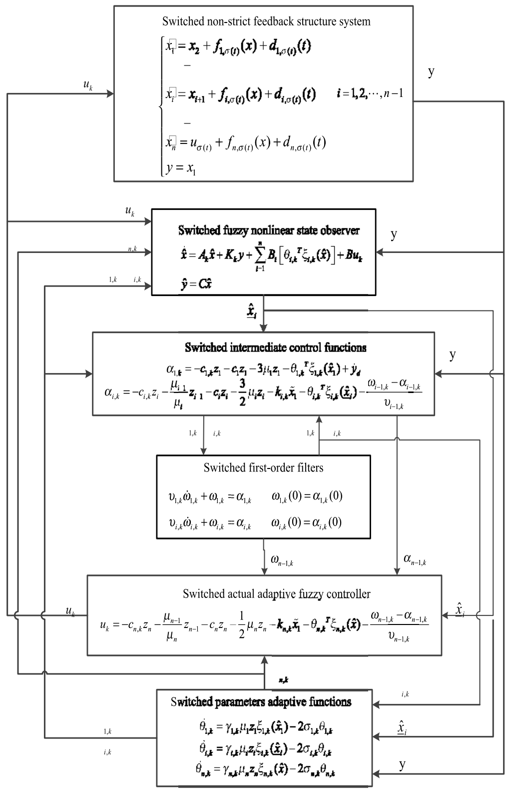


FIGURE 1. Block diagram of the control scheme.

1) The control methods in [23]–[26], [31]–[39] can address only the nonlinear system (108) without switching signals:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x) \quad (i = 1, 2, \ldots, n-1) \\ \dot{x}_n = u(t) + f_n(x) \\ y = x_1 \end{cases} \quad (108)$$

The control methods in [7]–[11], [27] address strict feedback system (109):

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i) x_{i+1} \quad (i = 1, 2, \ldots, n-1) \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n) u \\ y = x_1 \end{cases} \quad (109)$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$, $f_i(\bar{x}_i)$ and $g_i(\bar{x}_i)$ are the functions of partial state variables.

While the control method in this paper is designed for the nonlinear system (110) with switching signals and non-strict feedback structure:

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma(t)}(x) + d_{1,\sigma(t)}(t) \\ \quad \vdots \\ \dot{x}_i = x_{i+1} + f_{i,\sigma(t)}(x) + d_{i,\sigma(t)}(t) \\ \quad \vdots \\ \dot{x}_n = u_{\sigma(t)} + f_{n,\sigma(t)}(x) + d_{n,\sigma(t)}(t) \\ y = x_1 \end{cases} \quad (110)$$

where $x = [x_1, x_2, \ldots, x_n]^T$, $f_i(x)$ is the functions of whole state variables, and $\sigma(t)$ is the switching signal.

Whether it's controller design or stability proof, switched system is far more difficult than non-switched system. We unable to use the control methods in [23]–[26], [31]–[39] to control switched system due to the switching problem. Moreover, when the controller of the non-strict feedback switched system is designed by the control methods in [7]–[11], [27], the virtual control signal and adaptive law of each subsystem are the functions of whole state variables. Consequently, the algebraic loop problem arises, which makes the controller design of a non-strict feedback switched system very difficult. Therefore, the controller design method of the non-strict feedback switched system (60) considered in this paper is quite different from that of the controller design methods in [7]–[11], [23]–[27], [31]–[39].

2) References [12]–[16] proposed adaptive control methods for the non-strict feedback switched system, but the state constraints were not considered. Though [28]–[30] presented the control schemes for constrained systems, but all states of the systems in [28]–[30] should be measurable. The strict limitation makes these control methods difficult to realize in practical applications. Therefore, control methods in [12]–[16], [28]–[30] cannot be used to control a non-strict feedback nonlinear system with unmeasurable state variables that is discussed in this paper.

3) This proposed adaptive control scheme does not need n-order differentiable and bounded conditions of the input signals and a monotonically increasing condition of unknown functions. However, these strict assumptions are common in the existing references [33], [34]. Moreover, by adopting dynamic surface technology and first-order filters, this control scheme can avoid the problem of the ''explosion of complexity''. Therefore, this control scheme not only conforms to engineering practice but also has a simple algorithm and requires a small number of calculations.

VII. SIMULATIONS

Consider the following switched non-strict feedback system [12]:

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma(t)}(x_1, x_2) + d_{1,\sigma(t)}(t) \\ \dot{x}_2 = u_{\sigma(t)} + f_{2,\sigma(t)}(x_1, x_2) + d_{2,\sigma(t)}(t) \\ y = x_1 \end{cases} \tag{111}$$

where $f_{1,1}(x_1, x_2) = x_1 \sin\left(x_2^2\right)$, $f_{2,1}(x_1, x_2) = x_2/\left(1 + x_1^2\right)$, $d_{1,1}(t) = 0.1\sin(t)$, $d_{2,1}(t) = 0.1\cos(t)$, $f_{1,2}(x_1, x_2) = x_2 \sin(x_1 x_2)$, $f_{2,2}(x_1, x_2) = x_1/\left(10 + 2x_2^2\right)$, $d_{1,2}(t) = 0.1\sin(t)$, and $d_{2,2}(t) = 0.1\cos(t)$. The reference signal is given as $y_d = \sin(t)$. The constraints are given as $\underline{k}_{c1} = -1.0 + 0.8\sin(t)$, $\bar{k}_{c1} = 1.0 + 0.4\sin(t)$, $\underline{k}_{c2} = -2.0 + 0.5\sin(t)$, and $\bar{k}_{c2} = 1.6 + 0.4\sin(t)$.

The fuzzy membership function is designed as

$$\mu_{F_{i,k}^l}(\hat{x}_i) = \exp\left[-\frac{(\hat{x}_i - 3 + l)^2}{2}\right] \tag{112}$$

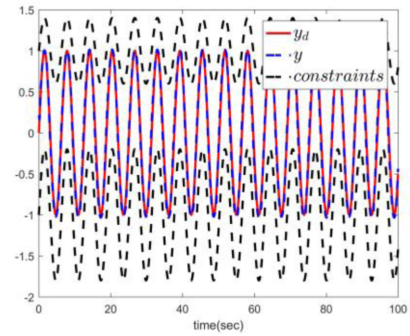


FIGURE 2. Trajectories of y, y_d and the output constraints.

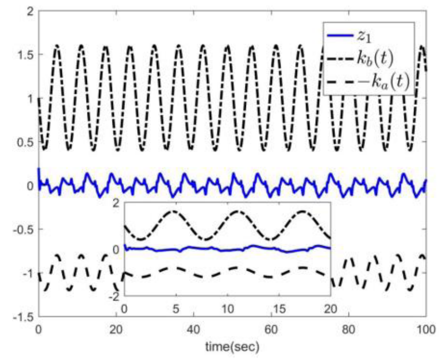


FIGURE 3. The tracking error z_1 and the error bounds.

where $i = 1, 2$, $l = 1, 2, \ldots, 5$ and $k = 1, 2$.

Then fuzzy logic systems can be written as

$$\hat{f}_{i,k}\left(\hat{x}_1, \hat{x}_2 \left| \theta_{i,k}\right.\right) = \theta_{i,k}^T \xi_{i,k}\left(\hat{x}_1, \hat{x}_2\right) \tag{113}$$

where $i = 1, 2$ and $k = 1, 2$.

Switched fuzzy state observers are designed as

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \theta_{1,k}^T \xi_{1,k}\left(\hat{x}_1, \hat{x}_2\right) + k_{1,k}\left(x_1 - \hat{x}_1\right) \\ \dot{\hat{x}}_2 = u_k + \theta_{2,k}^T \xi_{2,k}\left(\hat{x}_1, \hat{x}_2\right) + k_{2,k}\left(x_1 - \hat{x}_1\right) \end{cases} \tag{114}$$

where $k = 1, 2$.

Choosing the parameters as $c_{i,1} = 6$, $c_{i,2} = 7$, $k_{1,1} = k_{2,1} = 8$, $k_{1,2} = k_{2,2} = 10$, $\gamma_{i,1} = \gamma_{i,2} = 0.01$, $\sigma_{i,1} = \sigma_{i,2} = 0.05$, $\beta = 0.01$, and $\tau = 0.2$, and choosing $\tau_a = 10$ and $\mu = 1.1$, then $\tau_a = 10 > \frac{\ln 1.1}{0.01}$.

The initial conditions of the system and observer are chosen as to be $\mathbf{x}(0) = [0.2, 0.2]^T$ and $\hat{\mathbf{x}}(0) = [-0.1, 0.2]^T$. All the initial values of the adaptive parameters are chosen to be zero.

The simulation results are shown in Figs. 2-7. Fig. 2 shows the trajectories of $y(t)$, $y_d(t)$, $\underline{k}_{c1}(t)$ and $\bar{k}_{c1}(t)$. We can see that $y(t)$ tracks $y_d(t)$ with good performance and does not transgress its constraints. Fig. 3 shows that the tracking error $z_1(t)$ does not transgress the constraints $-k_a(t) < z_1(t) < k_b(t)$, $\forall t \geq 0$. Fig. 3 shows the trajectories of x_1 and $\hat{x}_1$. Fig. 4 shows the trajectories of x_2, $\hat{x}_2$, $\underline{k}_{c2}(t)$ and $\bar{k}_{c2}(t)$. We can see that x_2 does not violate its constraint bounds. Fig. 5 shows the control signal $u(t)$. Fig. 6 shows the switched signal $\sigma(t)$. From the simulation results, we can determine that the proposed control scheme in this paper can guarantee that all

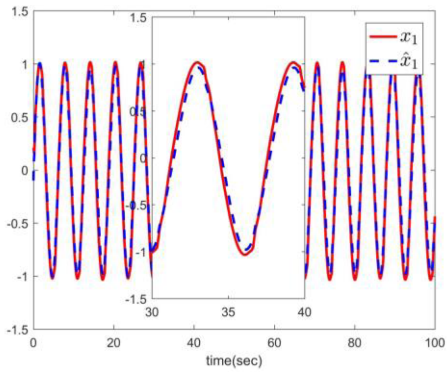


FIGURE 4. The trajectories of x_1 and $\hat{x}_1$.

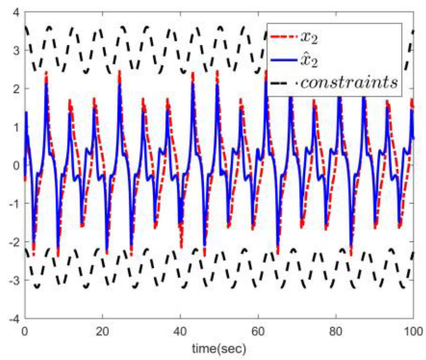


FIGURE 5. The trajectories of x_2, $\hat{x}_2$ and the constraints.

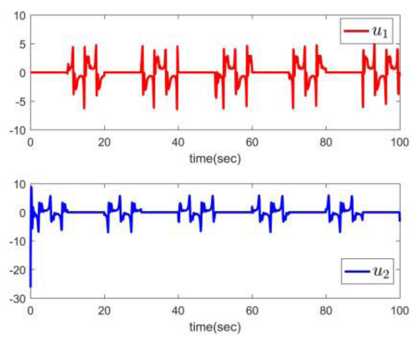


FIGURE 6. The control input $u(t)$.

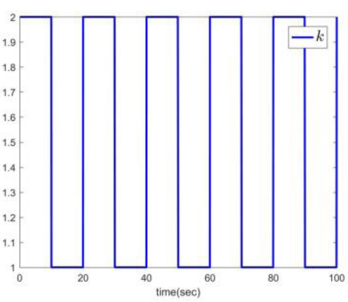


FIGURE 7. The switched signal $\sigma(t)$.

the signals in the closed system are SGUUB and that all the states do not violate the constraint bounds.

VIII. CONCLUSION

This paper proposes an adaptive control scheme based on a fuzzy state observer for switched uncertain non-strict feedback nonlinear systems with asymmetric time-varying full state constraints. To estimate the unmeasurable states in the system, a switched adaptive fuzzy state observer is designed. To satisfy the asymmetric time-varying constraints, ABLFs are adopted. To address the "explosion of complexity" problem, DSC technology is employed in the backstepping control. Finally, the Lyapunov method and average dwell time theory are used to prove the stability of the closed-loop system and the SGUUB of signals in the system. Moreover, the tracking error and the observer error can be kept within a sufficiently small neighbourhood of zero by choosing the appropriate parameters. Our future work is to extend the results to large-scale switched non-strict feedback nonlinear systems and stochastic switched nonlinear systems.

REFERENCES

[1] X. Zhao, X. Zheng, B. Niu, and L. Liu, "Adaptive tracking control for a class of uncertain switched nonlinear systems," *Automatica*, vol. 52, pp. 185–191, Feb. 2015.

[2] J.-L. Wu, "Stabilizing controllers design for switched nonlinear systems in strict-feedback form," *Automatica*, vol. 45, no. 4, pp. 1092–1096, Apr. 2009.

[3] R. Ma and J. Zhao, "Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings," *Automatica*, vol. 46, no. 11, pp. 1819–1823, Nov. 2010.

[4] T.-T. Han, S. S. Ge, and T. H. Lee, "Adaptive neural control for a class of switched nonlinear systems," *Syst. Control Lett.*, vol. 58, no. 2, pp. 109–118, Feb. 2009.

[5] Y. Li, S. Tong, and T. Li, "Adaptive fuzzy backstepping control design for a class of pure-feedback switched nonlinear systems," *Nonlinear Anal., Hybrid Syst.*, vol. 16, pp. 72–80, May 2015.

[6] S. Zheng and W. Li, "Fuzzy finite time control for switched systems via adding a barrier power integrator," *IEEE Trans. Cybern.*, vol. 49, no. 7, pp. 2693–2706, Jul. 2019.

[7] L. Long and J. Zhao, "Adaptive output-feedback neural control of switched uncertain nonlinear systems with average dwell time," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 7, pp. 1350–1362, Jul. 2015.

[8] S. Tong, S. Sui, and Y. Li, "Observed-based adaptive fuzzy tracking control for switched nonlinear systems with dead-zone," *IEEE Trans. Cybern.*, vol. 45, no. 12, pp. 2816–2826, Dec. 2015.

[9] S. Tong and Y. Li, "Adaptive fuzzy output feedback control for switched nonlinear systems with unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 47, no. 2, pp. 295–305, Feb. 2017.

[10] Y. Li, S. Sui, and S. Tong, "Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 47, no. 2, pp. 403–414, Feb. 2017.

[11] Y. Li and S. Tong, "Adaptive neural networks prescribed performance control design for switched interconnected uncertain nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 7, pp. 3059–3068, Jul. 2018.

[12] S. Tong, Y. Li, and S. Sui, "Adaptive fuzzy output feedback control for switched nonstrict-feedback nonlinear systems with input nonlinearities," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 6, pp. 1426–1440, Dec. 2016.

[13] L. Huang, Y. Li, and S. Tong, "Fuzzy adaptive output feedback control for a class of switched non-triangular structure nonlinear systems with time-varying delays," *Int. J. Syst. Sci.*, vol. 49, no. 1, pp. 1132–1146, Nov. 2018.

[14] Y. Li and S. Tong, "Adaptive fuzzy output-feedback stabilization control for a class of switched nonstrict-feedback nonlinear systems," *IEEE Trans. Cybern.*, vol. 47, no. 4, pp. 1007–1016, Apr. 2017.

[15] Y. Li and S. Tong, "Command-filtered-based fuzzy adaptive control design for MIMO-switched nonstrict-feedback nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 3, pp. 668–681, Jun. 2017.

[16] Y. Li and S. Tong, "Fuzzy adaptive control design strategy of nonlinear switched large-scale systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 12, pp. 2209–2218, Dec. 2018.

[17] Y.-J. Liu, S. Lu, S. Tong, X. Chen, C. L. P. Chen, and D.-J. Li, "Adaptive control-based barrier Lyapunov functions for a class of stochastic nonlinear systems with full state constraints," *Automatica*, vol. 87, pp. 83–93, Jan. 2018.

[18] D.-P. Li, Y.-J. Liu, S. Tong, C. L. P. Chen, and D.-J. Li, "Neural networks-based adaptive control for nonlinear state constrained systems with input delay," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1249–1258, Apr. 2019.

[19] T. Jiang, J. Liu, and W. He, "Adaptive boundary control for a flexible manipulator with state constraints using a barrier Lyapunov function," *J. Dyn. Syst., Meas., Control*, vol. 140, no. 8, pp. 081018-1–081018-7, Aug. 2018.

[20] J. Yi, J. Li, and J. Li, "Adaptive fuzzy output feedback control for nonlinear nonstrict-feedback time-delay systems with full state constraints," *Int. J. Fuzzy Syst.*, vol. 20, no. 6, pp. 1730–1744, Aug. 2018.

[21] W. He, Y. Dong, and C. Sun, "Adaptive neural network control of unknown nonlinear affine systems with input deadzone and output constraint," *ISA Trans.*, vol. 58, pp. 96–104, Sep. 2015.

[22] K. Sun, S. Mou, J. Qiu, T. Wang, and H. Gao, "Adaptive fuzzy control for nontriangular structural stochastic switched nonlinear systems with full state constraints," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 8, pp. 1587–1601, Aug. 2019, doi: 10.1109/TFUZZ.2018.2883374.

[23] Y.-J. Liu, S. Lu, D. Li, and S. Tong, "Adaptive controller design-based ABLF for a class of nonlinear time-varying state constraint systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 7, pp. 1546–1553, Jul. 2017.

[24] L. Edalati, A. K. Sedigh, M. A. Shooredeli, and A. Moarefianpour, "Adaptive fuzzy dynamic surface control of nonlinear systems with input saturation and time-varying output constraints," *Mech. Syst. Signal Process.*, vol. 100, pp. 311–329, Feb. 2018.

[25] M. Wan, Q. Liu, J. Zheng, and J. Song, "Fuzzy state observer-based adaptive dynamic surface control of nonlinear systems with time-varying output constraints," *Math. Problems Eng.*, vol. 2019, Apr. 2019, Art. no. 3683581.

[26] K. P. Tee, B. Ren, and S. S. Ge, "Control of nonlinear systems with time-varying output constraints," *Automatica*, vol. 47, no. 11, pp. 2511–2516, Nov. 2011.

[27] L.-B. Wu, J. H. Park, X.-P. Xie, and N.-N. Zhao, "Adaptive fuzzy tracking control for a class of uncertain switched nonlinear systems with full-state constraints and input saturations," *IEEE Trans. Cybern.*, early access, Jan. 30, 2020, doi: 10.1109/TCYB.2020.2965800.

[28] W. Wu, S. Tong, and Y. Li, "Fuzzy adaptive tracking control for switched nonlinear systems with full time-varying state constraints," *Neurocomputing*, vol. 352, pp. 1–11, Aug. 2019.

[29] S. Zheng and W. Li, "Adaptive control for switched nonlinear systems with coupled input nonlinearities and state constraints," *Inf. Sci.*, vol. 462, pp. 331–356, Sep. 2018.

[30] A. Chen, L. Liu, and Y. Liu, "Adaptive control design for MIMO switched nonlinear systems with full state constraints," *Int. J. Adapt. Control Signal Process.*, vol. 33, no. 10, pp. 1583–1600, Oct. 2019.

[31] Z. Chen, F. Huang, C. Yang, and B. Yao, "Adaptive fuzzy backstepping control for stable nonlinear bilateral teleoperation manipulators with enhanced transparency performance," *IEEE Trans. Ind. Electron.*, vol. 67, no. 1, pp. 746–756, Jan. 2020.

[32] W. Liu, C.-C. Lim, P. Shi, and S. Xu, "Backstepping fuzzy adaptive control for a class of quantized nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 5, pp. 1090–1101, Oct. 2017.

[33] B. Chen, C. Lin, X. Liu, and K. Liu, "Adaptive fuzzy tracking control for a class of MIMO nonlinear systems in nonstrict-feedback form," *IEEE Trans. Cybern.*, vol. 45, no. 12, pp. 2744–2755, Dec. 2015.

[34] H. Wang, X. Liu, K. Liu, and H. R. Karimi, "Approximation-based adaptive fuzzy tracking control for a class of nonstrict-feedback stochastic nonlinear time-delay systems," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 5, pp. 1746–1760, Oct. 2015.

[35] W. Min and Q. Liu, "An improved adaptive fuzzy backstepping control for nonlinear mechanical systems with mismatched uncertainties," *Automatika*, vol. 60, no. 1, pp. 1–10, Jan. 2019.

[36] M. Wan and Q. Liu, "Adaptive fuzzy backstepping control for uncertain nonlinear systems with tracking error constraints," *Adv. Mech. Eng.*, vol. 11, no. 5, pp. 1–11, Apr. 2019.

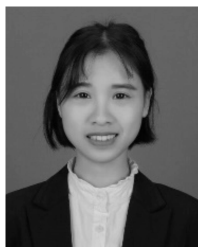
[37] J. Cao and J. Wang, "Global asymptotic and robust stability of recurrent neural networks with time delays," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 2, pp. 417–426, Feb. 2005.

[38] C.-C. Hua, Q.-G. Wang, and X.-P. Guan, "Adaptive fuzzy output-feedback controller design for nonlinear time-delay systems with unknown control direction," *IEEE Trans. Syst., Man, Cybern. B. Cybern.*, vol. 39, no. 2, pp. 363–374, Apr. 2009.

[39] S. Tong, Y. Li, and S. Sui, "Adaptive fuzzy tracking control design for SISO uncertain nonstrict feedback nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 6, pp. 1441–1454, Dec. 2016.

[40] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *Proc. 38th IEEE Conf. Decis. Control*, Phoenix, AZ, USA, Dec. 1999, pp. 2655–2660.

MIN WAN received the B.S. degree in automation from Dalian Maritime University, Dalian, China, in 2000, the M.S. degree in detection technology and automation device from the University of Electronic Science and Technology, Chengdu, China, in 2006, and the Ph.D. degree in mechanical engineering from Southwest Petroleum University, Chengdu, in 2019. She is currently an Associate Professor with the School of Mechatronic Engineering, Southwest Petroleum University. Her current research interests include fuzzy logic systems and adaptive control.



YANXIA YIN received the B.S. degree in electronic information science and technology from Southwest Petroleum University (SWPU), Chengdu, China, in 2019, where she is currently pursuing the M.S. degree in instrumentation engineering with the School of Mechatronic Engineering. Her research interests include adaptive control, intelligent control, and fuzzy control for nonlinear systems.