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A Hybrid Observability Analysis Method for Power System State Estimation

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ABSTRACT For observability analysis algorithm of large-scale power system state estimation, the traditional topological algorithm is complicated and may experience combinatorial explosion problems. Meanwhile, the computation speed and numerical stability of the numerical algorithm are greatly affected by the scale of power systems. In an effort to solve these problems, this study proposed a topological-numerical hybrid observability analysis algorithm using SCADA measurements. In this paper, a basic conclusion that there is at most one unknown complex voltage of an observable island is presented; the theoretical basis and the concept of recursive combination of observable islands are described. Then, three combinability rules are proposed and recursively applied to directly judge whether the local observable islands can be combined or not. Moreover, a minimum number of equivalent branches with power flow pseudo-measurements are added inside the remaining islands to form a final simplified connected network. Finally, the numerical observability analysis method based on the decoupled DC model is carried out to obtain the final observability analysis results. Tests show that the proposed hybrid algorithm has obvious advantages in regard to the execution speed for large-scale systems and also avoids the numerical instability in large-scale power grids.

INDEX TERMS Power flow solvability, recursive, state estimation, topological observability, hybrid observability.

NOMENCLATURE

Degree	Number of observable islands that a boundary bus connects to through un-observable branches
Degree x bus	A boundary bus whose *degree* equals *x*
Equivalent networks (ENs)	One or more networks after internal network equivalent containing only boundary buses and unobservable branches
Network to be combined (G_i)	Network joined by unobservable branches with at least one end having injection measurements
Final simplified network (FSN)	Network formed after topological observability analysis by connecting the equivalent networks with equivalent branches that have pseudo power flow measurements
Adjacent list of boundary buses with injection measurements (BIAList)	The adjacent list corresponds to a network connected by all unobservable branches with at least one end having injection measurements. A detailed definition is provided in Section V.

I. INTRODUCTION

Observability analysis methods can be classified into topological, numerical and hybrid methods. The topological methods determine the network observability by searching the full spanning tree of the entire network based on graph theory [1]. The existing topological algorithms [2]–[6] regard the bus injection measurement as a power flow measurement of one of the branches connected to it, which may cause combinatorial explosion in large scale networks. A majority of numerical methods use the decoupled DC model based on the gain matrix [7]–[10], the Jacobian matrix [11], [12] or the gram matrix [13]–[15]. Basic theories of the decoupled DC model and an iterative observability algorithm were first proposed in [7] and [8]. Then, [9], [16] presented a

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non-iterative method to reduce the execution time. Later, [17] and [18] verified that the non-iterative method may cause misjudgment of observable islands, so recent research has been focusing on changing the matrix operation and reducing the number of iterations to improve the execution speed. In fact, the speed of these improved algorithms is still slow in large systems because the computational effort of the numerical method itself increases on a high order with the system scale. Therefore, [20]–[21] proposed some hybrid methods, which used the topological methods to simplify the network scale before conducting the numerical observability analysis. However, almost all the topological parts of the existing hybrid methods only used branch flow measurements in their analysis processes, which cannot simplify the network sufficiently, especially when the proportion of injection measurements is large.

To further improve the execution speed of the hybrid algorithm, a topological observability analysis method [22] is introduced in this paper. This method makes full use of the bus injection measurements in the derived combinability rules. These rules are used recursively to merge observable islands to obtain a considerably simplified network, and then the numerical method in [9], [17] is applied to the simplified network to judge the final observability. After testing, the topological part of the proposed hybrid method shows a good performance for simplifying the network scale, and the time advantage of the hybrid algorithm over the numerical algorithm is significant in large-scale systems.

In Section II, the decoupled SE model for observability analysis and theoretical bases of the topological observability analysis method used in this paper are introduced. In Section III, the specific rules for combining the observable island are presented, and the use of these combinability rules is demonstrated by an IEEE 14-bus system. In section IV, the steps for applying numerical methods to the final simplified network are described. In Section V, the computer implementation of the topological part based on the operation of the *BIAList* is presented. Then the overall algorithm flow of the proposed hybrid method is given. In Section VI, a comparison of execution time between the proposed hybrid method and the pure numerical method is shown. Finally, Section VII concludes the paper.

II. THEORIES AND PRINCIPLES ABOUT TOPOLOGICAL OBSERVABILITY ANALYSIS

A. ANALYSIS BASES

For powers system state estimation (SE), the state variables are the bus complex voltages of all the buses of a power system. Strictly speaking, when considering the decoupled SE, one should apply the observability algorithm to both the $P-\theta$ model and the $Q-V$ model. On one hand, the development process and the results of the observability analysis are almost the same between the two models, On the other hand, real and reactive power measurements usually come in pairs in practice [8]. Therefore, we can use only the active part of the decoupled SE model ($P-\theta$ model) to explain our hybrid method for simplicity. That is, the voltage angle is used to represent the complex voltage of a bus as the state variable. The decoupled SE model is

$$z = \boldsymbol{H}x + \boldsymbol{e} \tag{1}$$

where $\boldsymbol{x}$ is the $n\times 1$ state vector of all bus voltage angles, $\boldsymbol{z}$ is the $m\times 1$ measurement vector of active power measurements, $\boldsymbol{H}$ is the $m\times n$ Jacobian matrix, m is the number of measurements and n is the number of buses including the slack bus.

Mathematically, the network is observable if, and only if, H have full-column rank

$$rank(\boldsymbol{H}) = n-1. \tag{2}$$

In this paper, only branch power flow measurements and power injection measurements are included in z during the observability analysis. Nevertheless, after the observability analysis, real SE can only be executed on those observable areas containing an actual voltage magnitude measurement.

B. BASIC THEORIES OF TOPOLOGICAL OBSERVABILITY ANALYSIS

Topological observability is defined as the existence of a spanning measurement tree T of full rank in a n-bus network N. The spanning tree connects all the nodes through branches with a metered or a calculated power flow [19]. If a network is not overall observable, it will be divided into several spanning trees T_i of subnetworks with n_i buses such that $n = \sum_{i=1}^{r} n_i$, where r is the number of spanning trees. These subnetworks are defined as *observable island*s Isl_i. Observable islands are connected by *unobservable branches* k-l when $k \in Isl_i, l \in Isl_j (i \neq j)$ and the bus k and l are called *boundary buses*. Apart from the boundary buses, other buses in an observable island are called *internal buses*. The measurement model of each island Isl_i can be written as

$$z_i = \boldsymbol{H}_i\boldsymbol{x}_i + \boldsymbol{e}_i (i = 1, 2, \cdots, r), \tag{3}$$

where $\boldsymbol{x_i}$ is the state vector of buses on Isl_i; $\boldsymbol{z_i}$ contains power flow measurements of the internal branches and the power injection measurements of internal buses. Since the islands are all observable, we have

$$rank(\boldsymbol{H_i}) = n_i - 1 \tag{4}$$

Then if bus $k (k \in Isl_i)$ are chosen as the reference bus, state variables x_m of the other buses $m (m \in Isl_i; m \neq k)$ can be represented by x_k, i.e.$x_m = f(x_k)$, by solving the measurement equations (3). Therefore, a property of the observable island can be derived, that is

Property 1: There is at most one unknown state variable in an observable island.

C. COMBINABILITY PRINCIPLES FOR OBSERVABLE ISLANDS

According to the definition of observable islands, it is easy to think of using branch power flow measurements to form the initial observable islands of a power grid. The isolated bus without branch flow measurements is also regarded as an initial observable island which only has a boundary bus. These initial islands are not the final observable islands because we can use the injection measurements on boundary buses to further combine the initial islands. Besides, the injection measurements on internal buses are redundant measurements because branch power flow measurements of $\boldsymbol{z_i}$ can already make the $rank(\boldsymbol{H_i})$ equal n_i-1.

In this paper, a combinability principle for combining the observable islands by boundary injection measurements are proposed as follows:

Combinability Principle: There are r ($r \geq 2$) connected observable islands in a subnetwork L. Take one boundary bus of one of these islands as a reference bus arbitrarily, if the voltage angle of at least one bus on each of the other r-1 observable islands can be solved or determined by boundary injection measurements equations, then these r observable islands can be combined into one observable island.

Proof: It is assumed that the number of buses contained in a subnetwork L is n_L. According to property 1, state variables in each island Isl_i can be represented as

$$\boldsymbol{x_i} = \boldsymbol{f_i}(x_{k_i})(x_{k_i} \in \boldsymbol{x_i};\ k_i \in Isl_i;\ i = 1, 2, \cdots, r), \tag{5}$$

where k_i is the reference boundary bus of Isl_i. The equations of boundary injection measurements are written as

$$z_b = \boldsymbol{H}_b \boldsymbol{x}_b + \boldsymbol{e}_b, \tag{6}$$

where $\boldsymbol{x_b}$ is the state vector of boundary buses on the islands of L and $x_{k_i} \in \boldsymbol{x_b}(i = 1, 2, \cdots, r)$. Setting the bus k_i of Isl_i as the reference bus of L, if L satisfies the combinability principle, all the $x_{k_j}(j = 1, 2, \cdots, r; j \neq i)$ can be represented by x_{k_i} when solving equations (6). Then by (5), we can find that all the state variables of L can be represented by x_{k_i}, which means the Jacobian matrix $\boldsymbol{H_L}$ of all measurement equations of L has $rank(\boldsymbol{H_L}) = n_L - 1$. Therefore, the r observable islands can be combined as a whole observable island.

End of proof.

It can be noted from the proof that there must be a new full rank spanning measurement tree in the new observable island after the combination. Also, the new island will belong to another subnetwork if the whole network is still unobservable. Therefore, the combination process can start recursively from parts of a network to the whole network until there are no more islands can be combined through this topological method.

D. INTERNAL NETWORK EQUIVALENT

For $P - \theta$ model, θ_m of one end of an internal branch k-m in Fig. 1 can be represented by θ_k according to Property 1. So logically speaking, the power flow P_{km} of the internal branch k-m can also be represented by θ_k, i.e. $P_{km} = P_{km}(\theta_k)$. Hence, we can logically regard the sum of the power flows on the internal branches connected to boundary bus k as an equivalent internal injection $P_{eq}(\theta_k)$. By doing so,

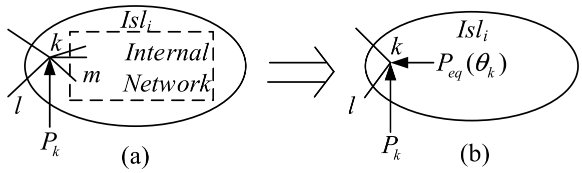

FIGURE 1. Internal network equivalent of an observable island.

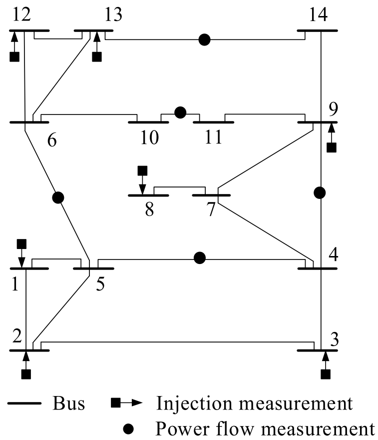

FIGURE 2. Illustration of the IEEE 14-bus system and its measurement configuration.

internal details of Isl_i can be neglected so that only *equivalent networks (ENs)* containing boundary buses and unobservable branches are reserved when combining observable islands. This procedure is called *internal network equivalent*. It should be noted that the internal network equivalent mentioned here is not a real network equivalent calculation, but only a logical equivalent. In Fig. 1, (a) can be reduced to (b) according to the above internal equivalent.

After the internal equivalent, the boundary injection measurement P_k is only concerned with the voltage angles of boundary buses.

$$P_k = \sum_{l \in \boldsymbol{a1}(k)} P_{kl} + \sum_{m \in \boldsymbol{a2}(k)} P_{km} = \sum_{l \in \boldsymbol{a1}(k)} P_{kl}(\theta_k, \theta_l) + P_{eq}(\theta_k) \tag{7}$$

where $\boldsymbol{a1}(k)$ and $\boldsymbol{a2}(k)$ are the set of buses connected with bus k through unobservable branches and internal branches respectively. Mathematically, the internal equivalent is equal to the elimination of internal variables of the measurement equation (3). Therefore, the internal measurement details will not affect the results of the following observability analysis.

III. COMBINABILITY RULES

In this section, three combinability rules for the combination of observable islands are proposed based on the theories in section II. An illustrative IEEE 14-bus system is used to better demonstrate the proposed rules and the configuration of the system is shown in Fig. 2.

A. SIMPLE COMBINABILITY RULES

First and foremost, two major premises about the independency of the injection measurements is presented. One is that

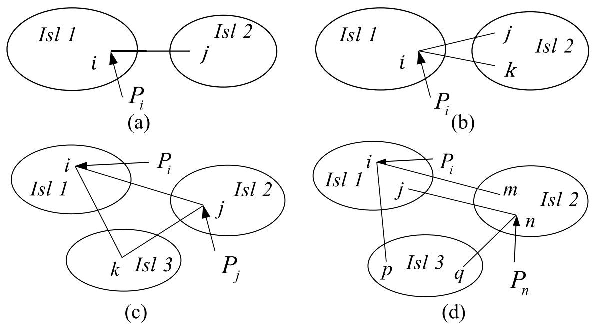

FIGURE 3. Some topologies conforming to Rule 1 and Rule 2.

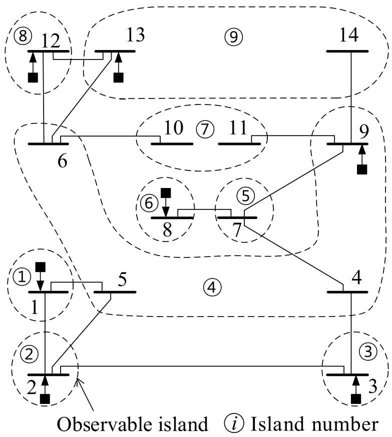

FIGURE 4. Initial observable islands of the IEEE 14-bus system.

during the topological observability analysis, when there are multiple injections on a bus, only when all injections have measurements can the bus be considered as having injection measurements, and multiple injections on the bus can be combined into one injection measurement. No distinction is made between injection measurements and zero-injection measurements. The other is actual power flows exists in an operating power grid so that the injection power flow equations have a unique solution. That means all the real-time injection measurement equations can be considered as independent ones.

Besides, once there are observable islands formed or combined, the internal network equivalent should be carried out for the next combination. So the internal buses and internal branches will not be shown in Fig. 3 to Fig. 6.

Initially, two simple combinability rules are proposed to combine the observable islands connected by *degree 1* (defined in the NOMENCLATURE) and *degree 2 buses* after internal network equivalent.

Rule 1: If a *degree 1 bus* on an observable island has an injection measurement, then the two observable islands, which are connected by the unobservable branches sending from the *degree 1* bus, can be combined as one.

Rule 2: If three observable islands related by a *degree 2 bus* are the same as those islands related by another *degree 2 bus*, and the two *degree 2 buses* both have injection measurements, then the three islands can be combined as one.

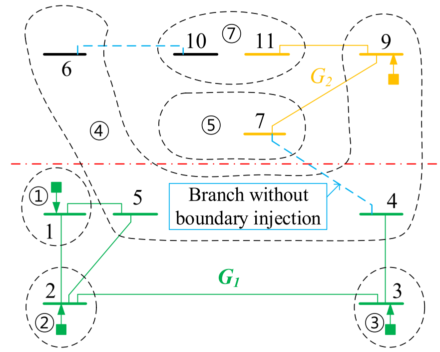

FIGURE 5. Observable islands after Rules 1 and 2 and the topology of G_i.

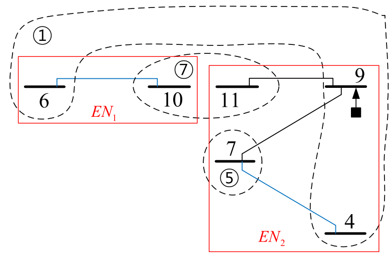

FIGURE 6. IEEE 14-bus system after topological observability.

Proof: Some of the subnetworks conforming to Rule 1 are shown in Fig. 3 (a-b). Based on Property 1, the two islands in Rule 1 can provide at most two unknown state variables. So if bus i of Isl_1 is set as the reference, the unknown state variable of the Isl_2 can be solved by the injection measurement equation of the *degree 1 bus i*.

As for Rule 2, some of the topologies conforming to Rule 2 are shown in Fig. 3 (c-d). The two injection measurement equations provided by the two *degree 2 buses* contain three unknown state variables of the three islands. After setting one boundary bus of the three islands as a reference bus, the two state variables of the other islands can be calculated by the two equations. Therefore, a subnetwork conforming to the Rule 1 and Rule 2 satisfies the combinability principle and can be combined as one observable island.

End of proof.

It should be noted that Rule 1 and Rule 2 are designed for a local structure L of a network. Observable islands of L that satisfies the rules can be combined whether they are connected with other islands $Isl_i (Isl_i \notin L)$ or not.

Now, we use the IEEE 14-bus system to show the usage of these simple combinability rules. Fig. 4 shows the initial observable islands formed by searching the spanning tree of branch power flow measurements in Fig. 2. The internal network equivalent is also performed after that. By examining Fig. 4, it can be found that *degree 1 bus* 8 satisfies Rule 1, and *degree 2 buses* 12 and 13 satisfy Rule 2. Thus, observable islands (⑤, ⑥) and (④, ⑧, ⑨) can be combined respectively. When carrying out combinations, all islands are combined into the one with the minimum island number. The newly formed observable islands after the internal network equivalent are shown in Fig. 5.

B. COMBINABILITY RULE FOR NETWORK TO BE COMBINED G_I

In equivalent networks (*ENs*) formed by the internal network equivalent, there may be some boundary buses that do not have injection measurements, and their incident boundary buses may also not have injection measurements. The voltage angles of these buses will not be included in any equation of boundary injection measurements, so it is difficult to directly use the *combinability principle* to judge the combinability of these *ENs*. Therefore, the unobservable branch without injection measurements on its two ends should be removed logically from the *ENs* to form the *network to be combined* (G_i) temporarily. By doing so, it can be ensured that the voltage angles of all the boundary buses on G_i can be included in the injection measurement equations *of* G_i. Considering the IEEE 14-bus system in Fig. 5, branches 4-7 and 6-10 should be removed from the *EN*s and then G_1 containing buses (1, 2, 3, 4, 5) and G_2 containing buses (7, 9, 11) are formed.

Then, a combinability rule for G_i is proposed as follows.

Rule 3: Suppose the number of observable islands connected by G_i is n; if at least n-1 of them have a boundary injection measurement on G_i, these n islands can be combined into one.

Proof: The buses on G_i in Rule 3 all belong to the n observable islands connected by G_i. Thus, the measurement model of G_i has at most n unknown state variables based on Property 1. Also, the n-1 boundary injection equations of the model are independent of each other as shown in part A of section III. Therefore, setting the bus of one island on G_i as the reference, the n-1 unknown state variables of the other islands can be calculated from the n-1 independent equations. Thus, these n observable islands satisfy the *Combinability Principle* and can be combined.

End of proof.

Because the internal network equivalent may affect the *degree* of boundary buses, the three combinability rules need to be applied recursively until there is no new combining operation.

Rule 3 can be used to determine the observability in Fig. 5. Since four observable islands (①, ②, ③, ④) are related by G_1 and three of them have injection measurements on their boundary buses connected by G_1. It means G_1 satisfies Rule 3, the four islands can be combined to island ①. Since there are no more topologies that satisfy Rules 1, 2 and 3, islands (①, ⑤, ⑦) cannot be combined in the topological process.

It should be noted that the removed unobservable branches without injection measurements on their ends should be restored when the combination process is finished, as shown in Fig. 6.

Since the three combinability rules are **sufficient but not necessary conditions** for the combination process, some observable islands that can actually be combined may be missed if only using these rules. Thus, it is necessary to use a numerical method for the simplified network to determine the final observability.

C. THE CONNECTED FINAL SIMPLIFIED NETWORK

After the topological observability analysis, there may still be some *ENs* that are not directly connected by unobservable branches, like the EN_1(containing bus 6 and 10) and EN_2 (containing bus 4, 7, 9, 11) in Fig. 6. This kind of disconnected networks cannot be used directly in numerical methods. To avoid this problem, the existing hybrid methods, such as that proposed in [20], regard an observable island as a supernode. They set the coefficients of the injection measurement equations of different boundary buses to random integer values to achieve the independence of these equations. However, these random integer values may transform some actual independent injection equations into dependent ones when the system scale becomes large. Thus, this paper proposes a new approach to join these *ENs* into a whole connected network, which is called *the final simplified network (FSN)*, by adding "virtual" equivalent branches with power flow pseudo measurements. Here, "virtual" means that there is no need to calculate the actual impedance values of the equivalent branches, because all branch reactance values will be set to 1 in the next numerical observability analysis.

Because an equivalent branch is inside an observable island, according to Property 1, the branch power flow of the equivalent branch can be represented by the only unknown voltage state variable of this island. Therefore, it can be considered that there are pseudo power flow measurements on these equivalent branches.

As shown in section II, the internal measurement details will not affect the result of the subsequent observability analysis. Therefore, to simplify the computation, there is no need to add equivalent branches between every boundary bus of an island. Instead, only a few virtual equivalent branches are added by the following method, which can ensure that all the *ENs* are connected.

The concrete implementation is shown in step A1∼A2.

A1: Generate the set of the existing boundary buses $\boldsymbol{R}$, $\boldsymbol{R}$ records the original serial number of the boundary node, and the number of elements in $\boldsymbol{R}$ is n_R.

A2: Traverse $\boldsymbol{R}$ in the order of serial number from small to large, and compare one by one with the bus whose serial number is arranged at the back. If two buses in $\boldsymbol{R}$ are in the same observable island but belong to two different numbered *ENs*, then add an equivalent branch with pseudo power flow measurements, at the same time, revise the bigger serial number of the two *ENs* to the smaller one.

The pseudo codes of A1∼A2 are shown in Algorithm 1.

By using the above method, after adding equivalent branch 4-6 in observable island ①, the two *ENs* are connected in Fig. 6. The final simplified network (*FSN*) is shown in Fig. 7.

D. APPLY NUMERICAL METHOD TO JUDGE THE FINAL OBSERVABILITY

In [9], a non-iterative numerical observability analysis method based on the decoupled DC model was used to reduce the execution time. However, [17] proved that the

Algorithm 1 Add equivalent branches to form the *FSN*

Input: The original serial number of the existing boundary buses, $\boldsymbol{R}$; the number of buses in $\boldsymbol{R}$, n_R; the existing adjacent list of boundary buses in $\boldsymbol{R}$, ***B_List***; the serial number of the equivalent network that a boundary bus belongs to, ***EN***; the serial number of the observable island that a bus belongs to, ***n_Isl***;

Output: the adjacent list of the *FSN*, which is the ***B_List*** after adding the equivalent branches

```
for i ← 1: n_R
    for j ← i+1: n_R
        if EN[R[i]] = EN[R[j]] then continue
        if n_Isl [R[i]] = n_Isl [R[j]] then
            add branch R[i] - R[j] with pseudo power flow measurement to the B_List
            if EN[R[i]] > EN[R[j]] then
                EN[R[j]] ← EN[R[i]]
            else
                EN[R[i]] ← EN[R[j]]
return B_List
```

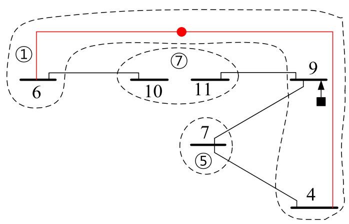

FIGURE 7. ***FSN* of the IEEE 14-bus system after adding equivalent branches.**

non-iterative manner may cause misjudgment of the observable islands. To ensure the validity of the observable analysis results, an iterative numerical method based on [9] and [17] is used in this paper.

At the beginning, all branch reactance values of the equations in decoupled SE model are set to 1 to form the decoupled DC model [9]. Then the numerical observability analysis for the *FSN* is carried out in the following steps, which are described in detail in section V of [17].

H1: Form the measurement Jacobian matrix $\boldsymbol{H}$ and the gain matrix $\boldsymbol{G}$, and perform the ***LDU*** triangular factorization.

H2: Check if $\boldsymbol{D}$ has only one zero pivot. If yes, stop. If not, take out the rows of matrix $\boldsymbol{L}^{-1}$ corresponding to the zero pivots to form the test matrix $\boldsymbol{W}$.

H3: Compute the matrix $\boldsymbol{C} = \boldsymbol{A}\boldsymbol{W}^T$. If at least one entry in a row of $\boldsymbol{C}$ is not zero, then the corresponding branch will be unobservable.

H4: Remove all the unobservable branches to obtain the observable islands. Then, remove all the irrelevant injection measurements and go to Step H1. If there are no irrelevant measurements, then stop and output the final observable islands.

The matrix $\boldsymbol{A}$ in step H3 is the branch-node incidence matrix, and the irrelevant injection measurement in step H4 is defined as the injection measurement of the boundary bus connecting more than one observable islands in [17], [18].

The measurement Jacobian matrix of the *FSN* of the IEEE 14-bus system is formed as follows:

$$\begin{array}{cc} Bus & \begin{array}{cccccc} 4 & 6 & 7 & 9 & 10 & 11 \end{array} \\ \boldsymbol{H} = & \begin{bmatrix} 0 & 0 & -1 & 2 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} P_9 \\ P_{46}^{eq} \end{array} \end{array} \tag{8}$$

After the above steps, branches 4-7, 6-10, 7-9 and 9-11 are identified as the unobservable branches. Removing all these branches from the origin network, the resulting observable islands are island ① (1, 2, 3, 4, 5, 6, 9, 12, 13, 14), island ② (7,8) and island ③ (10, 11), which is the same with the result of employing the numerical method directly.

IV. COMPUTER PROCESS AND ALGORITHM FLOW

A. DFS SEARCH ALGORITHM IN THE HYBRID METHOD

In graph theory, there are many search algorithms to find the spanning trees in a network, among which the depth first search (DFS) and the breadth first search (BFS) are the most commonly used ones. The implementation of the two methods would have an execution time that is asymptotically proportional to the number of branches in the graph being searched. When traversing the same graph, the two method have the same time complexities. Since the execution time is linear in the problem size, the topological algorithm can be used for large networks in real time compared with the numerical ones [2].

In computer programming, the adjacent list can directly reflect the topological structure of a network. So in the topological part of our hybrid method, the observability analysis can be carried out directly by searching the adjacent list. Here, we apply the DFS directly to the adjacent lists, which contains buses connected by the branches with power flow measurements, to generate the initial observable islands. When the numerical part has finished, all the unobservable branches are removed from the adjacent list of the whole network. By using the DFS, we can get the final observable islands. Furthermore, by using DFS to search *BIAList*, we also get the *network to be combined* G_i before using Rule 3.

B. THE OPERATIONS ON THE BIALIST

The combination process of observable islands can be completed by recursive operations on the *BIAList* based on the three combinability rules in section III.

In the *BIAList*, a single-linked list that has boundary bus i as the head bus is shown in Fig. 8(a). Where, N_i is the serial number of bus i; Isl_i refers to the observable island number to which boundary bus i belongs; and D_i is the previously defined bus *degree* which can be obtained easily by counting the number of different attribute values *Isl* of adjacent buses on its single-linked list. The value of Isl_i and D_i may change and should be updated when the combination of observable islands occurs. D_i =0 means bus i turns into an internal bus after one combination process, and its single-linked list

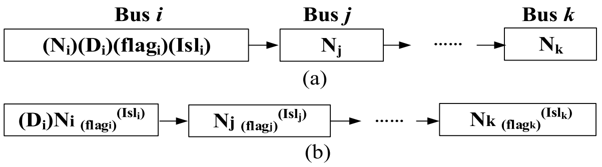

FIGURE 8. Structure of a single-linked adjacent list.

should be deleted from the *BIAList*. Besides, $flag_i = 1$ means boundary bus i has an injection measurement; otherwise, $flag_i = 0$.

It should be noted that the adjacent part of an actual single-linked list in Fig. 8 (a) only contains the serial number N_j to N_k of the adjacent buses j to k connected to the head bus i. However, for the convenience of analysis, the *Isl* and *flag* of the adjacent buses are also given in the *BIAList* during the following discussion, as shown in Fig. 8(b).

When combining observable islands on the *BIAList*, the *degree 1* or *degree 2* head buses are firstly found and judged to determine if they satisfy Rule 1 and Rule 2. If satisfied, the corresponding observable islands are combined by the operations on the *BIAList*. Then, the *BIAList* are searched by DFS to form *the network to be combined* G_i. After that, we can count the number of different observable islands (*Isl*) related by G_i as well as the number of different islands whose boundary bus on G_i have injection measurements ($flag = 1$). Based on Rule3, by comparing the two numbers, it could be determined if the observable islands related by G_i can be combined together. The above operations are recursively performed until no modification of the *BIAList* is made. At this time, if the *BIAList* is empty, the entire network is observable; otherwise, numerical observability analysis is needed.

The following operations are performed in sequence when islands combination is carried out on the *BIAList*.

B1: For head buses of a subnetwork satisfying a specific combinability rules, revise all *Isl* of these head buses to the smallest *Isl* among them. the *Isl* of other head buses that belong to the same islands as these head buses should also be revised.

B2: Searching the *BIAList*, remove the adjacent buses that have the same *Isl* as the head bus, that is, eliminate the unobservable branches that are transformed into internal (observable) branches after island combination. Then, revise the *degree* of each head bus.

B3: When a head bus's $D = 0$, there are no adjacent buses in this single-linked list, which means the head bus has been transformed into an internal bus. Then, the corresponding single-linked list is removed from the *BIAList*.

The pseudo codes of B1∼B3 are shown in Algorithm 2.

The IEEE14-bus system is used to illustrate the above operations. After forming the initial islands, the *BIAList* of the network in Fig. 4 is shown in Fig. 9(a). To save space, we do not put the single-linked lists of the head buses without injection measurements ($flag = 0$) in Fig. 9.

In Fig. 9(a), head bus 8 is a *degree 1 bus* which satisfies the Rule 1. In its single-linked list, Isl_8 is 6 and Isl_7 is 5, so Isl_8 is modified to 5 to combine island ⑤ to island ⑥. Since $D_8 = 0$, the singled-linked list of head bus 8 can be removed. Then, buses 12 and 13 are *degree 2 buses* that satisfy the Rule 2, so the following steps are conducted to combine islands ④, ⑧ and ⑨: 1) The minimal island number of the head buses in their lists is 4 ($Isl_6 = 4$), so Isl_{12} and Isl_{13} is revised to 4. Isl_{14} is also revised to 4 since bus 14 belongs

Algorithm 2 The combination of observable islands on *BIAList*

Input: The original serial number of head buses in a subnetwork which satisfies a specific combinability rules, ***ori_c***; the total number *nc* of buses in ***ori_c***; the serial number of the original branch power flow observable island that a bus belong to, ***Isl_bs***; the New-Old serial number indicator, ***n_io,*** which contains the new serial number of the original observable islands after combination, which should have been initialized after forming the initial observable islands with elements from 1 to the maximal island number; the *BIAList* of the network after forming the initial observable islands, including ***im_ls***[i][j] (containing the head bus order of the adjacent buses of head bus i in *BIAList*) and ***id_ls***[i] (the number of the adjacent buses of head bus i); the attribute values $flag_i$ and D_i of head bus i in *BIAList*, ***flag_ls*** and ***D_ls***; the original serial number of the head buses in *BIAList*, ***ori_ls***; the total number of head buses in *BIAList*, *nls*.

Output:the *BIAList* and its attribute arrays

//Function 1: revise the serial number of observable islands of head buses that satisfy the combinability rules. (B1)
min_isl ←*n_io*[*Isl_bs*[*ori_c*[1]]];
for i ← 2: *nc*
  if *n_io*[*Isl_bs*[*ori_c*[i]]] < *min_isl* **then**
    min_isl ← *n_io*[*Isl_bs* [*ori_c* [i]]];
for i ← 1: *nc*
  if *n_io*[*Isl_bs* [*ori_c* [i]]]≠*min_isl* **then**
    n_io[*Isl_bs* [*ori_c* [i]]] ← *min_isl;*
return
//Function 2: update the *BIAList* after island combination (B2-B3)
for i ← 1: *nls*
  if *flag_ls*[i] = 0 or *id_ls*[i] = 0 **then**
    continue;
  m ← *id_ls*[i];
  for j ← 1: m
    if*n_io* [*Isl_bs* [*ori_ls* [i]]] = *n_io* [*Isl_bs* [*ori_ls* [*im* [i][j]]]] **then**
      id_ls[i]–;
      record j that should be removed from *im_ls*[i][];
  if *id_ls*[i] = 0 **then continue;**
  remove the recorded adjacent buses from *im_ls*[i][];
  count the number of different values of *n_io*[*Isl_bs*[*ori_ls*[*im*[i][j]]]], j from 1 to *id_ls*[i], record it as n;
  D_ls[i] ← n;
return

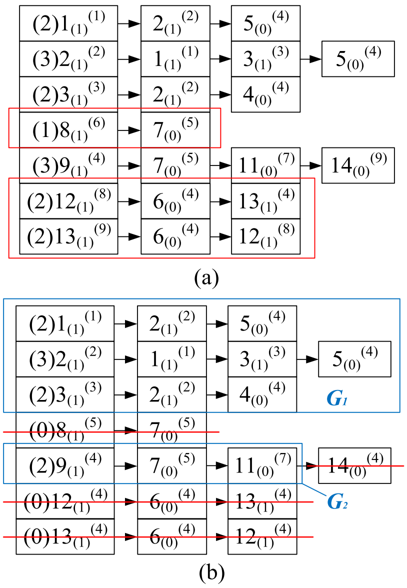

FIGURE 9. Operations on the ***BIAList*** of the IEEE 14-bus system.

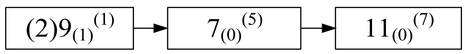

FIGURE 10. ***BIAList*** **of the IEEE 14-bus system after all combination processes.**

to the same island with bus 13. 2) Bus 14 is deleted from the list of head bus 9 and all adjacent buses of head bus 12 and 13 are deleted. D_9 =2 and D_{12} = D_{13} =0. 3) For D_{12} and D_{13} are both equal to zero, the two single-linked list of head bus 12 and 13 are also removed from *BIAList*. The above operations are illustrated in Fig. 9(b).

After that, by using DFS, two *networks to be combined* G_1 (1, 2, 3, 4, 5) and G_2(7, 9, 11) are generated and the single-linked lists of them are shown in the two blue frames in Fig. 9(b). Because the number of observable islands (*Isl*) related by G_1 is 4, and three head buses on G_1 have injection measurements (*flag*=1), so islands (①, ②, ③, ④) related by G_1 can be combined. Thus, Isl_2, Isl_3, Isl_4 and Isl_5 are revised to 1 in turn and the three single-linked list corresponding to G_1 are deleted. After the combination operations, the new *BIAList* shown in Fig. 10 still has a head bus with *flag* = 1, so numerical observability analysis is still needed.

C. ALGORITHM FLOW

The overall algorithm flow of the proposed hybrid method is presented as follows:

S1: Form an adjacent list of buses that connected by branch flow measurements. Then, conduct DFS to this adjacent list to form the initial observable islands.

S2: Considering all the unobservable branches with at least one end having injection measurements to form the *BIAList*.

S3: Search the *BIAList* to find *degree 1* or *degree 2* head buses of a subnetwork that satisfies the Rule 1 or Rule 2. Then, search their lists to find the buses connected to them and combine the corresponding islands by step **B1** as shown in part A.

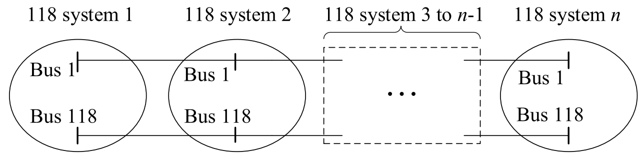

FIGURE 11. **Structure of the testing system containing *n* 118-bus subsystems.**

S4: Process the *BIAList* by splitting the boundary buses without injection measurements into the leaf nodes. Then, do DFS to the *BIAList* to form the *networks to be combined* G_i. Search the single-linked lists of each G_i to find the number of observable islands and the head buses whose attribute *flag* equal to 1 and attribute *Isl* have different values. Comparing the two number, if the G_i satisfies the Rule 3, conduct step **B1** on the *BIAList* to combine the observable islands related to it.

S5: If there are newly combined observable islands in step **S3** or **S4**, conduct step **B2** and **B3** to revise the *BIAList* and go to **S3**. Otherwise, if there is no head bus with *flag* = 1 in the *BIAList*, go to **S7**.

S6: Add equivalent branches with pseudo flow measurements by steps **A1**∼**A2** to form the *FSN*. Then, apply the numerical method **H1 to H4** to the *FSN* and determine the final observable islands.

S7: Find the observable islands containing voltage magnitude measurements where the SE can be executed.

V. EXECUTION TIME COMPARISON AND DISCUSSION

In this section, the performance of the proposed method is discussed by comparing it with existing methods. For testing convenience, all branch power flow measurements in our tests are set as single-ended measurements, which can be considered as an extreme case of measurement configuration. Compared with the case that power flow measurements are set to both ends of a branch, this configuration is more advantageous to the calculation speed of numerical method.

All the tests were conducted using a 2.40 GHz Intel(R) Core (TM) i5-4210U CPU with 8.0 GB of RAM.

A. CASE TESTS AND TIME COMPARISON

The test examples are constructed by connecting bus 1 and bus 118 of one IEEE 118-bus system to the corresponding bus of the adjacent 118-bus system in series, as shown in Fig. 11. In order to articulate the substantial benefits of the proposed method, we use three different systems which contain 10, 50 and 100 118-bus subnetworks respectively.

As shown in Table 1, the measurement configuration of each case are generated arbitrarily according to the given number. Some intermediate results of observability analysis and their CPU calculation time are listed in the table. These intermediate processes contain forming the initial observable islands, combining observable islands by topological rules, and numerical observability analysis. The existing hybrid method [21] and iterative numerical method [9] for comparison only contain some of the intermediate processes.

TABLE 1. Comparison of the computation results among the proposed method and the existing methods.

Network Scale and Meas. Configuration		Method	Obs. Analysis Result (Number of Obs. Islands)	CPU time (sec)	
Buses / Branches	Flow / Inj.		Initial Islands / Islands after Comb. / Final Islands	Form Initial Islands / Islands Comb. / Numerical Analysis	Total
1180/1808	1250/885	A	117 / 4 / 4	0.012 / 0.017 / 0.016	0.045
		B	117/ -- / 4	0.012 / -- / 0.025	0.037
		C	-- / -- / 4	-- / -- / 0.054	0.054
5900/9048	7240/4410	A	244 / 10 / 10	0.057 / 0.054 / 0.036	0.147
		B	244 / -- / 10	0.057 / -- / 0.156	0.213
		C	-- / -- / 10	-- / -- / 2.069	2.069
	6250/4440	A	599 / 19 / 19	0.065 / 0.096 / 0.043	0.204
		B	599 / -- / 19	0.065 / -- / 0.297	0.362
		C	-- / -- / 19	-- / -- / 2.047	2.047
	4760/4460	A	1458 / 41 / 38	0.076 / 0.149 / 0.045	0.270
		B	1458/ -- / 38	0.076 / -- / 0.786	0.862
		C	-- / -- / 38	-- / -- / 1.312	1.312
11800/18098	14500/8800	A	522 / 17 / 17	0.111 / 0.124 / 0.074	0.309
		B	522 / -- / 17	0.111 / -- / 0.348	0.459
		C	-- / -- / 17	-- / -- / 7.995	7.995
	12500/8850	A	1157 / 30 / 30	0.121 / 0.235 / 0.076	0.432
		B	1157 / -- / 30	0.121 / -- / 0.818	0.939
		C	-- / -- / 30	-- / -- / 8.116	8.116
	9500/8900	A	3076 / 81 / 75	0.173 / 0.403 / 0.085	0.661
		B	3076 / -- / 75	0.173 / -- / 3.036	3.209
		C	-- / -- / 75	-- / -- / 8.822	8.822

Notes: Method A is the proposed method in this paper; Method B is the hybrid method [21]; Method C is the improved iterative numerical method based on [9] (presented in part D, section III)

Their intermediate results and CPU time are also listed in the table.

The following conclusions can be drawn from Table 1:

1) In most cases, the two hybrid method are faster than the numerical method, and this advantage will extremely increase when the network scale goes large. The execution time of the numerical method is even a dozen times more than the proposed method in the 11800-bus system.
2) Compared with the existing hybrid method [21], the speed advantage of the proposed method is not remarkable when the network is small and the number of branch flow measurements is large. However, our hybrid method is more suitable for the large scale system without enough branch flow measurements. Since the execution time of the numerical part in method [21] will increase a lot with the decrease of branch flow measurements in large scale systems. While the proposed method can avoid this problem by further reducing the network scale through the measurement islands combination process.

Moreover, in addition to the cases shown in Table 1, extensive cases have been tested and most of them have shown that the observable islands gained after the topological part of the proposed method are exactly the final observable islands. That means the injection measurements have been fully utilized through the proposed topological approach.

B. FURTHER DISCUSSION ABOUT THE ALGORITHM PERFORMANCE

Apart from the speed advantage, the proposed hybrid method is helpful to reduce the numerical instability. It is well known that floating-point calculations in triangular decomposition may cause round-off errors. Zero pivots may be misclassified more easily due to the great disparity of the gain matrix values when the network scale becomes larger [21]. In our tests, some incorrect results occurred when testing the numerical method in the 11800-bus system. For example, when the number of total measurements is about 20000 and the bus injection measurements are more than the branch power flow measurements, there will be some abnormal results of the final observable islands obtained by the numerical method. But under the same configuration, the result of our hybrid method is correct. Because the smaller the network scale used in the numerical part is, the smaller the possibility of the misjudgment of zero pivots is. Thus, reducing the network size by the topological part can improve the stability of the numerical part.

Although an integer decomposition method is used to avoid the round-off errors that has been proposed in [10], this method cannot be applied to large-scale system. Because we find that there is a cyclic multiplication of integers during the iterative process of this algorithm, and the integer overflow error is easily to occur even in some small-scale system. In our tests, we found that the 64-bit integer data type is not enough even in the IEEE 118-bus system.

Moreover, the topological method used in this paper can solve the ill-conditioned cases demonstrated in [18]. Take the case in Fig. 12 as an example which traditional numerical methods like [10], [17] and [22] cannot handle.

When using these numerical methods, the following candidate observable islands are identified: {1, 2, 3, 4, 5, 7, 8, 10, 13}, {6}, {9}, {11} and {12}. However, when using the topological part of our hybrid method, it is obviously

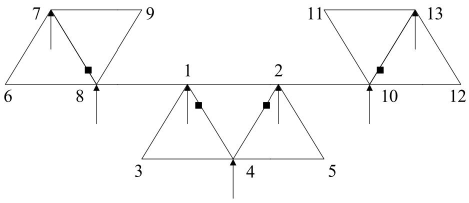

FIGURE 12. An ill-conditioned case given in [18].

that there are not enough measurements to solve the voltage phasors of bus 3, 5, 6, 9, 11 and 12. So the final identified observable islands are {1, 2, 4}, {3}, {5}, {6}, {7, 8}, {9}, {10, 13}, {11}, {12}, which is consistent with the correct result derived in [18]. Therefore, the topological part of the hybrid method can reduce misjudgment when these types of ill-conditioned cases occur. Moreover, the numerical algorithm we use in our hybrid method can also avoid the ill-conditioned problems by removing the irrelevant injection measurement and iterating, as shown in step H4. Therefore, the hybrid algorithm proposed in this paper can deal with this kind of pathological problem well.

VI. CONCLUSION

In this paper, a hybrid observability analysis method is proposed. Compared with current hybrid methods, the topological part of this method improves the effect of the network simplification, reduces the computational effort of the numerical part, and avoids numerical instability in large-scale systems. The testing results show that the execution speed of the proposed hybrid method is much faster than the purely numerical method and existing hybrid method in large-scale systems under normal measurement configurations, which exactly meets the requirements of modern large-scale power systems.

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