# Improved H-Infinity Hybrid Model Predictive Fault-Tolerant Control for Time-Delayed Batch Processes Against Disturbances

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**ABSTRACT** An H-infinity model predictive fault-tolerant control strategy is proposed for multi-phase batch processes with interval delay and actuator failures. First, state variables, state errors and output tracking errors are introduced to establish an extended-state-space switched system model. Then, based on this model, a predictive fault-tolerant control law is designed for tracking the set point by the output that satisfies the requirements of the optimal performance index under input and output constraints. The feasibility conditions for the solvability of the control law are presented in the form of linear matrix inequalities. In addition, the designed switching law is constructed, and the gain of the control law is obtained via the optimization algorithm. This design has several advantages: the output tracking is faster, the tracking performance is superior, and the trace is smoother at the switching time. Finally, through a comparison with traditional methods, the effectiveness and feasibility of this method are demonstrated via injection molding simulation.

**INDEX TERMS** Batch production systems, added delay, fault tolerant control, linear matrix inequalities, Lyapunov methods, predictive models, switching systems, predictive control.

## I. INTRODUCTION

As industrial production models exhibit a variety of characteristics, such as small scale, diversity, high added value and technological intensiveness, batch production technology has attracted increasing attention and has begun to play an important role in many fields. Although many studies have considered batch processes, the high-precision control of modern industrial processes remains a challenge. One reason for such challenges is the occurrence of time delays and perturbations, which may deteriorate the tracking performance and result in reduced production efficiency [1]–[3]. Furthermore, both the requirement for a high automation level and the complex process conditions increase the likelihood of system faults, including actuator faults, internal faults, and sensor faults. Actuator faults, which are the most frequently occurring faults, substantially impact the system. If these faults are not detected and corrected in a timely manner,

then the production performance will deteriorate, and equipment and personnel safety issues may occur. Once a fault has been detected, the corresponding fault-tolerant control strategy must be quickly implemented to mitigate the impact of the fault on the control performance of the system. Fault-tolerant control refers to the tolerance of the system to faults such that, after a fault occurs, the control performance is not substantially affected, and the system can still operate stably for a period of time in the current state to ensure the quality of the products produced during this period.

In recent years, there have been many achievements in the research on fault-tolerant control for batch processes [4]–[8]. These results mainly occurred in two areas. In one area, the batch process is regarded as a one-dimensional (1D) system that is related only to time, as in [4] and the references therein. In the other area, the batch process is regarded as a two-dimensional (2D) system that is related to time and the batch direction. In [7], the author transformed a batch production process with unknown perturbations and actuator faults into a two-dimensional Fornasini-Marchesini (2D-FM)

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model and designed a controller that ensures the closed-loop convergence of the system along the directions of time and batch. Regarding uncertainty, state delays and actuator failures, Wang *et al.* [8] proposed H-infinity learning fault-tolerant guaranteed cost control based on an equivalent 2D system description of these processes.

A series of research results have been presented for the fault-tolerant control of batch processes. The most popular methods for controlling batch processes are 2D iterative learning control (ILC) methods based on the repetitiveness of production processes [9]–[14], including single-phase batch processes [12], [13] and multi-phase batch processes [14]. However, as such methods adopt the same control law, if the deviation of the system output from the set value exceeds a threshold, then the deviation will continuously increase with time. Moreover, for safety, system constraints and restrictions must be considered during the design of the control system. It is imperative to develop new control methods. Model predictive control (MPC) with feedback correction and rolling optimization at every moment is widely used. In references [15]–[23], a set of control strategies that combine ILC with MPC are proposed. Recently, MPC was combined with fault-tolerant control (FTC) and applied [24]. Most research results focus on 2D systems theory.

As batch processes are processes that vary slowly with time, no perfect repetitiveness occurs; i.e., if system information cannot be repeated between batches, then the ILC method is no longer applicable. At this time, it is more suitable for the production process to be regarded as a 1D system [25]–[30]. Among these results, various design methods are combined with model predictive control, such as the neural network method and the extended-state control model method. The design strategy of the extended-state control model method is to introduce the state error and the output error along the time direction, expand the original model into a new error model, and design a control law under this model for the realization of system tracking control. This method has been widely applied since it has produced satisfactory control results. Another advantage of this design is that a time-delayed system can be transformed into a non-time-delayed system via dimension expansion. Another design method is available for time-delayed systems, in which the controller design depends on the upper and lower bounds of the time delay. Although it is difficult to design the Lyapunov function for the latter solution, its dimensionality and computing load are smaller than those of the former solution. If the control process involves uncertainties and external perturbations, then the stability of MPC will be negatively affected, while the H-infinity model predictive control strategy [31], [32] can analyze the stability of uncertain closed-loop systems, reduce the impact of perturbations on the system's control performance, and enable the controlled objects to remain robust to external perturbations. Therefore, such control methods are widely used.

The above research on batch processes focused mainly on single-phase processes. However, most batch processes are multi-phase processes, and the phases influence one another.

Multi-phase batch process control has become a hot topic in research [29], [33]–[36]. In [34], a switching system model was used to study a multi-phase batch production process, and a related predictive control strategy was proposed. In [35], the average dwell time method was used to study a multi-phase system, and an optimal control strategy is proposed. According to the research results, these models were all based on two-dimensional systems. As we have explained, the information of batch processes is not always repeated in the batch direction. In addition, few studies consider time delays, external disturbances and other issues. Therefore, the available research results cannot satisfy the market demand, and further research on multi-phase batch processes is urgently needed, especially for cases in which the batch information is not repeated and time delays occur.

This paper proposes an improved H-infinity hybrid model predictive fault-tolerant control method for time-delayed batch processes against disturbances. In contrast to control approaches for single-phase batch processes from the literature discussed above, for multi-phase batch processes, we have introduced not only the above variables but also new state variables that are related to output errors to form an extended switching system model. The main contributions of this work are as follows: (1) A new type of model is constructed, and the suitable switching conditions and running times in various phases are specified based on this model. (2) The solvability condition of model predictive control is specified in terms of linear matrix inequalities (LMIs), and the upper bound for the system's optimal performance index is identified to improve the efficiency of the production process. (3) The designed controller is robust against disturbances and time delays, and it realizes superior tracking performance. Finally, to evaluate the performance of the proposed method, the proposed method is compared with the traditional method. The results demonstrate that the proposed method realizes a short running time and satisfactory tracking performance.

The remainder of this paper is organized as follows. Section II describes the system. In Section III, the design of model predictive fault-tolerant control is considered. In this part, the equivalent model is established, the sufficient conditions for the feasibility of model predictive control are established, and the optimization algorithm is constructed. An injection molding simulation example is presented to illustrate the performance of the proposed methods in Section IV. Finally, the conclusions of this study are presented in Section V.

## II. PROBLEM DESCRIPTION

The following discrete-time faulty switched system with uncertain parameter perturbation and an interval time delay is considered:

$$\begin{cases} x(k+1) = (A^{\sigma(k)} + \Delta_a^{\sigma(k)}(k))x(k) + A_d^{\sigma(k)}x(k-d(k)) \\ \qquad\quad + B^{\sigma(k)}u^F(k) + w^{\sigma(k)}(k) \\ y(k) = C^{\sigma(k)}x(k) \end{cases} \tag{1}$$

where $k$ is the finite discrete-time; $x(k)$, $y(k)$ and $u^F(k)$ represent the system state, system output and system control output under a fault condition, respectively; $w(k)$ is the unknown external perturbation; $\sigma(k) = [0, \infty) \rightarrow \bar{p} = \{1, 2, \cdots, N\}$ is the time- or state-dependent piecewise constant switching signal; $\sigma(k) = i$ denotes the activation of the $i^{th}$ system; $A^i$, $B^i$, $A_d^i$, $C^i$ are the coefficient matrices for the $i^{th}$ subsystem with suitable dimensions; $\Delta_a^i$ is the perturbation matrix of unknown parameters, which satisfies the condition $\Delta_a^i = D^i F^i(k) E^i$; $D^i$, $E^i$ are known real matrices with suitable dimensions; and $F^i(k)$ is an unknown matrix that satisfies the condition $F^i(k) F^{iT}(k) = I$.

The interval time delay $d(k)$ satisfies the condition

$$\underline{d} \leq d(k) \leq \overline{d}$$

where $\underline{d}$ and $\overline{d}$ denote the upper and lower bounds, respectively, of the interval time delay.

*Remark 1:* Some real systems can be expressed in the form of system (1). For example, the process of a continuous stirred tank reactor (CSTR) in [37] can be expressed in this form; in [38], when the update law with an input delay is designed, it can be converted into model (1).

In practical production operation, due to the long-term overload operation of the equipment, actuator faults in the system are unavoidable; hence, it is difficult for the input $u^i(k)$ of the system to track its set value. Actuator faults mainly include partial failure, complete failure and stuck failure. The first two faults are represented by defining the value of $\alpha^i$. If $\alpha^i > 0$ and $\alpha^i \neq 1$, then partial failure occurs; if $\alpha^i = 0$, then complete failure occurs. For stuck failure, the input through the actuator is constant. In general, $\alpha^i = 1$ means that the system is behaving normally. In case of complete failure or stuck failure, the controller will no longer function. Herein, we will discuss only partial failure. Thus, the faulty model can be expressed as:

$$\begin{cases} x(k+1) = (A^{\sigma(k)} + \Delta_a^{\sigma(k)}(k))x(k) + A_d^{\sigma(k)} x(k-d(k)) \\ \qquad\quad + B^{\sigma(k)} \alpha u(k) + w^{\sigma(k)}(k) \\ y(k) = C^{\sigma(k)} x(k) \end{cases} \quad (2)$$

In real production processes, the model dimensions for two adjacent phases may differ. When the system is switched to the next phase, the form of switching can be expressed as:

$$x^{i+1}(T^i) = J^i x^i(T^i)$$

where $J^i$ is the state transition function. If $J^i = I^i$, then the system states of two adjacent phases have the same physical meaning. If the system state is known, then it is critical to determine the system switching time. Assuming that the $i^t h$ phase is within the time range $[T_s^{i-1}, T_s^i]$ and that the switching time is $T_s^i(s \in (i, 2, 3, \ldots))$,

$$T_s^i = \min \left\{ t > T_s^{i-1} | G^i(x(k)) < 0 \right\}, \quad T_s^0 = 0,$$

where $G^i(x(k)) < 0$ is the switching condition that is related to the system state. For multi-phase batch processes,

the switching sequence for the entire operation phase is expressed as:

$$\Sigma = \left\{ T_1^1, \sigma\left(T_1^1\right), T_1^2, \sigma\left(T_1^2\right), \ldots, T_1^p, \sigma\left(T_1^p\right), \\ T_2^1, \sigma\left(T_2^1\right), \ldots T_2^p, \sigma\left(T_2^p\right), \ldots, T_s^i, \sigma\left(T_s^i\right), \ldots \right\}$$

## III. DESIGN OF MODEL PREDICTIVE FAULT-TOLERANT CONTROL
### A. ESTABLISHMENT OF THE EQUIVALENT MODEL
Since the information between batches is not repeated, here, we regard the batch process as a 1D system, and we use 1D variable dimension switched system theory to study the conditions that are required for the system to operate smoothly even after the occurrence of a fault. In addition, the proposed strategy is a passive FTC that handles the fault occurrence as parameter uncertainties. Based on the conditions of activation at various phases, phase $i$ control input $u^{iF}(k)$ represents the input signal under an actuator fault, which can be expressed as:

$$u^{iF}(k) = \alpha^i u^i(k) \quad with \ 0 < \underline{\alpha}^i < \alpha^i < \overline{\alpha}^i \quad (3)$$

where $\underline{\alpha}^i < 1, \overline{\alpha}^i > 1$.

Therefore, the batch process with an interval time delay, a perturbation and an actuator fault can be expressed as:

$$\begin{cases} x^i(k+1) = (A^i + \Delta_a^i(k))x^i(k) + A_d^i x^i(k-d(k)) \\ \qquad\qquad + B^i \alpha^i u^i(k) + w^i(k) \\ y^i(k) = C^i x^i(k) \end{cases} \quad (4)$$

The following notations are introduced:

$$\beta^i = diag[\beta_1^i, \beta_2^i, \ldots, \beta_m^i], \quad \beta_0^i = diag[\beta_{10}^i, \beta_{20}^i, \ldots, \beta_{m0}^i],$$
$$\beta^i = \frac{\overline{\alpha}^i + \underline{\alpha}^i}{2}, \quad \beta_0^i = \frac{\overline{\alpha}^i - \underline{\alpha}^i}{\overline{\alpha}^i + \underline{\alpha}^i}.$$

Therefore, $\alpha_0^i$ exists; hence,

$$\alpha^i = \left(I^i + \alpha_0^i\right)\beta^i, \left|\alpha_0^i\right| \leq \beta_0^i \leq I^i,$$

where

$$\alpha_0^i = diag[\alpha_{01}^i, \alpha_{02}^i, \ldots, \alpha_{0m}^i],$$
$$\left|\alpha_0^i\right| = diag[\left|\alpha_{01}^i\right|, \left|\alpha_{02}^i\right|, \ldots, \left|\alpha_{0m}^i\right|].$$

For system (4), the controller will be designed using the novel model predictive fault-tolerant control method to ensure satisfactory control performance. The main steps are as follows: the error model and new state variables are introduced and used to transform the model into an equivalent model, and the novel controller and the switching law are designed on this basis. The main strategy is as follows: The difference operator $\Delta$ is introduced, and $\Delta x(k+1) = x(k+1) - x(k)$ is defined. Then, the following equation can be obtained from model (4):

$$\Delta x^i(k+1) = A^i(k)\Delta x^i(k) + A_d^i \Delta x^i(k-d(k)) \\ \qquad\qquad + B^i \alpha^i \Delta u^i(k) + \bar{w}^i(k) \quad (5)$$

where

$$\bar{w}^i(k) = [\Delta_a^i(k) - \Delta_a^i(k-1)]x^i(k-1) + \Delta w^i(k)$$

$A^i(k) = (A^i + \Delta_a^i(k))$, and $e^i(k)$ is the error between the system output and the expected output at the $i^{th}$ phase:

$$e^i(k) = y(k) - y_r(k) \tag{6}$$

The following equation can be obtained from (5) and (6):

$$e^i(k+1) = C^i A^i(k)\Delta x^i(k) + C^i A_d^i \Delta x_i(k-d(k))$$
$$+ C^i B^i \alpha^i \Delta u^i(k) + C^i \bar{w}^i(k) + e^i(k) \tag{7}$$

A new state variable is introduced:

$$\hat{x}^i(k+1) = \hat{x}^i(k) + e^i(k) \tag{8}$$

where $\hat{x}^i(k)$ is selected based on the state of the extended information that is determined by $e^i(k)$. Let

$$\bar{x}^i(k) = \begin{bmatrix} \Delta x_i^T(k) & \hat{x}^{iT}(k) & e^{iT}(k) \end{bmatrix}^T,$$

then the state model of the dimension-extended system with extended information at the $i^{th}$ phase is obtained:

$$\begin{cases} \bar{x}^i(k+1) = \bar{A}^i(k)\bar{x}^i(k) + \bar{A}_d^i\bar{x}^i(k-d(k)) \\ \qquad + \bar{B}^i \alpha^i \Delta u^i(k) + \bar{G}^i\bar{w}^i(k) \\ \bar{y}^i(k) = \bar{C}^i\bar{x}^i(k) \end{cases} \tag{9}$$

where

$$\bar{B}^i = \begin{bmatrix} B^i \\ 0 \\ C^i B^i \end{bmatrix}, \quad \bar{G}^i = \begin{bmatrix} I \\ 0 \\ C^i \end{bmatrix}, \bar{A}_d^i = \begin{bmatrix} A_d^i & 0 & 0 \\ 0 & 0 & 0 \\ C^i A_d^i & 0 & 0 \end{bmatrix},$$

$$\bar{C}^i = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}^T, \quad \bar{A}^i(k) = \bar{A}^i + \bar{\Delta}_a^i(k)$$

$$= \begin{bmatrix} A^i & 0 & 0 \\ 0 & I & I \\ C^i A^i & 0 & I \end{bmatrix} + \begin{bmatrix} \Delta_a^i(k) & 0 & 0 \\ 0 & 0 & 0 \\ C^i \Delta_a^i(k) & 0 & 0 \end{bmatrix}.$$

When the $i^{th}$ phase is switched over to the $i+1^{th}$ phase, the phase transition can be expressed as:

$$\bar{x}^{i+1}(k)$$
$$= \begin{bmatrix} J^i \Delta x^i(k) \\ \hat{x}^i(k-1) + C^{i+1}J^i\Delta x^i(k-1) + C^{i+1}J^i x^i(k-2) - y_r^{i+1} \\ C^{i+1}J^i\Delta x^i(k) + C^{i+1}J^i x^i(k-1) - y_r^{i+1} \end{bmatrix}.$$

## B. DESIGN OF THE PREDICTIVE FAULT-TOLERANT CONTROLLER

The objective of robust predictive control is to design a predictive controller such that the output can track the set point under faulty conditions and satisfy the performance index at each moment. Let $\bar{x}^i(k+j|k)$, $\Delta u^i(k+j|k)$ and $\bar{y}^i(k+j|k)$ be the predicted state value, predicted input value and predicted output value, respectively, for $k+j$ at moment $k$, and let $\bar{x}^i(k|k) = \bar{x}(k)$ and $\Delta u^i(k|k) = \Delta u^i(k)$. The state feedback control law is designed as follows:

$$\Delta u^i(k+j|k) = \bar{K}^i\bar{x}^i(k+j|k) \tag{10}$$

where $\bar{K}^i$ denotes the gain of the proposed controller. At this time, model (9) is transformed into:

$$\begin{cases} \bar{x}^i(k+j+1|k) = (\bar{A}^i(k) + \bar{B}^i\alpha^i\bar{K}^i)\bar{x}^i(k+j|k) \\ \qquad + \bar{A}_d^i\bar{x}^i(k-d(k)+j|k) + \bar{G}^i\bar{w}^i(k+j|k) \\ \bar{y}^i(k+j|k) = \bar{C}^i\bar{x}^i(k+j|k) \end{cases} \tag{11}$$

The optimal performance indices for system (11) are designed as follows:

$$\min_{\Delta u(k+j|k)} \max_{\Delta a^i(k+j|k),\bar{w}(k+j|k)} J_\infty^i(k)$$

$$J_\infty^i(k) = \sum_{j=0}^{\infty} \begin{bmatrix} \bar{x}^{iT}(k+j|k)\bar{Q}_1^i\bar{x}^i(k+j|k) + \Delta u^{iT}(k+j|k)\bar{R}_1^i \\ \Delta u^i(k+j|k) - \eta^i\bar{w}^{iT}(k+j|k)\bar{w}^i(k+j|k) \end{bmatrix}$$

$$\tag{12}$$

where the following condition must be satisfied:

$$\begin{cases} \|\Delta u^i(k+j|k)\| \le \Delta u_m^i \\ \|\bar{y}^i(k+j|k)\| \le \bar{y}^i \end{cases} \tag{13}$$

The minimum upper-bound value of the objective function (performance index $J_\infty(k)$) is obtained under maximum perturbation and minimum control input. In the formulas above, $\bar{Q}_1^i$ and $\bar{R}_1^i$ are the process state weight matrix and the input weight matrix, respectively, and $\Delta u_m^i$ and $\Delta y_m^i$ are the upper-bound values of $\Delta u^i$ and $\bar{y}^i$, respectively. The following notation is introduced to simplify the expression:

$$V^i\left(\bar{x}^i(k+j|k)\right)$$
$$= V_j^i, V_1^i(\bar{x}^i(k+j|k)) = V_{1j}^i,$$
$$\bar{x}_j^i = \bar{x}^i(k+j|k), \quad \bar{x}_{rj}^i = \bar{x}^i(r+j|k),$$
$$\bar{x}_{d+j}^i = \bar{x}^i(k+j-d(k)|k), \quad \bar{x}_{\tilde{d}+j}^i = \bar{x}^i\left(k+j-\tilde{d}|k\right),$$
$$\bar{x}_{\tilde{d}+rj}^i = \bar{x}^i\left(r+j-\tilde{d}|k\right),$$
$$\bar{\varepsilon}_j^{iT} = [\bar{x}^{iT}(k+j|k) \ \bar{x}_d^{iT}(k+j|k) \ \bar{x}_{\tilde{d}}^{iT}(k+j|k)],$$
$$\bar{\delta}_j = \bar{x}^i(k+j+1|k) - \bar{x}^i(k+j|k),$$
$$\bar{\delta}_{rj} = \bar{x}^i(r+j+1|k) - \bar{x}^i(r+j|k).$$

Using the Lyapunov stability theorem, the controller design problem is transformed into the equivalent model stability problem. To demonstrate the system stability, the Lyapunov-Krasuski function (LKF) is defined as:

$$V_j^i = \sum_{l=1}^{5} V_l^i(k+j|k)$$

$$V_{1j}^i = \bar{x}_j^{iT}\bar{P}_1^i\bar{x}_j^i = \bar{x}_j^{iT}\theta^i\bar{P}_2^{-i}\bar{x}_j^i$$

$$V_{2j}^i = \sum_{r=k-d(k)}^{k-1} \bar{x}_{rj}^{iT}\bar{\alpha}_i^{k-1-r}\theta^i\bar{T}_2^{-i}\bar{x}_{rj}^i$$

$$V_{3j}^i = \sum_{r=k-\tilde{d}}^{k-1} \bar{x}_{rj}^{iT}\bar{\alpha}_i^{k-1-r}\theta^i\bar{M}_2^{-i}\bar{x}_{rj}^i$$

$$V_{4j}^i = \sum_{s=-\bar{d}}^{-\underline{d}} \sum_{r=k+s}^{k-1} \bar{x}_{rj}^{iT} \bar{\alpha}_i^{k-1-r} \theta^i \bar{T}_2^{-i} \bar{x}_{rj}^i$$

$$V_{5j}^i = \bar{d} \sum_{s=-\bar{d}}^{-1} \sum_{r=k+s}^{k-1} \bar{\delta}_{rj}^{iT} \bar{\alpha}_i^{k-1-r} \theta^i \bar{G}_2^{-i} \bar{\delta}_{rj}^i \quad (14)$$

where $\bar{P}_1^i, \bar{T}_1^i, \bar{M}_1^i, \bar{G}_1^i$ are positive-definite symmetric matrices, $0 < \bar{\alpha}_i < 1$, and $\theta^i$ is a positive number. To ensure system stability, it is necessary to satisfy the following Lyapunov inequality constraints:

$$J_j^i = \bar{x}_j^{iT} \bar{Q}_1^i \bar{x}_j^i + \Delta u_j^{iT} \bar{R}_1^i \Delta u_j^i - \eta^i \bar{w}_j^{iT} \bar{w}_j^i \quad (15)$$

$$V_{j+1}^i - V_j^i \le V_{j+1}^i - \alpha_i V_j^i \le -J_j^i \quad (16)$$

Moreover, if $V^i(\bar{x}^i(\infty)) = 0$, $\bar{x}^i(\infty) = 0$, and the upper bound of $J_\infty^i(k)$ exists and satisfies $\theta^i > 0$, then:

$$J_\infty^i(k) \le V^i(k) \le \theta^i \quad (17)$$

*Definition 1:* For any $t > t_0$ and any switching signal $\sigma(k)$, where $t_0 \le k < t$, $N^i(t_0, t)$ denotes the number of switches of the $i^{th}$ subsystem within the time interval $(t_0, t)$. $\tilde{T}^i(t_0, t)$ is called the total operating time of the $i^{th}$ subsystem. For any specified $\tau_i > 0$, the following formula is established: $N^i(t_0, t) \le \frac{\tilde{T}^i(t_0, t)}{\tau_i}$, where $\tau_i > 0$ is the average dwell time.

*Definition 2:* System (11) is said to be robustly exponentially stable under the switching signal if there exist positive constants $a, b$, and $0 < v_i < 1$ such that the following formula holds:

$$\left\| \bar{x}_j^T \right\|_t = \sqrt{\frac{b}{a} v_i^{t-t_0}} \left\| \bar{x}_j^T \right\|_{t_0}$$

*Lemma 1 (Schur Complement Lemma):* Let $W, L$ and $V$ be matrices of suitable dimensions in which $W, V$ are real matrices. Then,

$$L^T V L - W < 0$$

if and only if

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0.$$

*Lemma 2:* If $D, F, E$ and $M$ are positive-definite matrices with suitable dimensions, where $M = M^T$ and $F^T F < I$, then the following inequality is established, and if there exists $\varepsilon > 0$, then the sufficient and necessary condition for the establishment of the inequality is

$$M + \varepsilon^{-1} D D^T + \varepsilon E^T E < 0.$$

*Lemma 3:* For any vector $\delta(t) \in R^n$, positive numbers $k1$ and $k2$, and matrix $R \in R^{n \times n}$, the following matrix inequality is established:

$$-(k_1 - k_2 + 1) \sum_{t=k_1}^{t=k_2} \delta^T(t) R \delta(t) \le - \sum_{t=k_1}^{t=k_2} \delta^T(t) R \sum_{t=k_1}^{t=k_2} \delta(t)$$

*Theorem 1:* If $\bar{w}^i(k) \ne 0$, for specified constants $\underline{d} \le d(k) \le \bar{d}$, $\theta^i > 0$, and $0 < \bar{\alpha}_i < 1$, if there exist positive

definite symmetric matrices $\bar{P}_2^i, \bar{M}_2^i, \bar{T}_2^i, \bar{G}_2^i, \bar{T}_3^i, \bar{M}_3^i, \tilde{M}_3^i, \bar{G}_3^i$, and $\bar{Y}_1^i$ and positive real numbers $\eta^i, \varepsilon_a$, and $\varepsilon_b$ such that the following matrix inequalities are feasible for the switching signal with an average dwell time that satisfies the following inequality (23)

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} & \Upsilon_{15} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & 0 \\ * & * & \Upsilon_{33} & 0 & 0 \\ * & * & * & \Upsilon_{44} & 0 \\ * & * & * & * & \Upsilon_{55} \end{bmatrix} < 0 \quad (18)$$

$$\Upsilon_{11} = \begin{bmatrix} \bar{\varphi}_1^i & 0 & \bar{\alpha}_i^{\tilde{d}} \bar{G}_3^i & 0 \\ 0 & -\bar{\alpha}_i^{\tilde{d}} \bar{T}_3^i & 0 & 0 \\ \bar{\alpha}_i^{\tilde{d}} \bar{G}_3^i & 0 & \bar{\alpha}_i^{\tilde{d}} \tilde{M}_3^i - \bar{\alpha}_i^{\tilde{d}} \bar{G}_2^i & 0 \\ 0 & 0 & 0 & -\theta^i \eta^i \end{bmatrix}$$

$$\Upsilon_{22} = \begin{bmatrix} -\bar{P}_2^i & 0 \\ 0 & -\bar{d}^{-2} \bar{G}_2^i \end{bmatrix}, \quad \Upsilon_{23} = \begin{bmatrix} 0 & \bar{\varepsilon}_a^i \bar{D}^i \\ 0 & \bar{\varepsilon}_a^i \bar{D}^i \end{bmatrix},$$

$$\Upsilon_{24} = \begin{bmatrix} 0 & \bar{\varepsilon}_b^i \bar{B}^i \beta_0^i \\ 0 & \bar{\varepsilon}_b^i \bar{B}^i \beta_0^i \end{bmatrix}$$

$$\Upsilon_{12} = \begin{bmatrix} \bar{P}_2^i \bar{A}^{iT} + \bar{Y}_1^{iT} \beta^i \bar{B}^{iT} & \bar{P}_2^i \bar{A}^{iT} + \bar{Y}_1^{iT} \beta^i \bar{B}^{iT} - \bar{P}_2^i \\ \bar{T}_2^i \bar{A}_d^{iT} & \bar{T}_2^i \bar{A}_d^{iT} \\ 0 & 0 \\ \theta^i \bar{G}^{iT} & \theta^i \bar{G}^{iT} \end{bmatrix}$$

$$\Upsilon_{33} = \begin{bmatrix} -\varepsilon_a^i & 0 \\ 0 & -\varepsilon_b^i \end{bmatrix}, \quad \Upsilon_{44} = \begin{bmatrix} -\varepsilon_a^i & 0 \\ 0 & -\varepsilon_b^i \end{bmatrix},$$

$$\Upsilon_{13} = \begin{bmatrix} \bar{P}_2^i \bar{E}^{iT} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Upsilon_{14} = \begin{bmatrix} \bar{Y}_1^{iT} \beta^{iT} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Upsilon_{15} = \begin{bmatrix} \bar{Y}_1^{iT} \bar{R}^{\frac{i}{2}T} & \bar{P}_2^i \bar{Q}_1^{\frac{i}{2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Upsilon_{55} = \begin{bmatrix} -\theta^i I & 0 \\ 0 & -\theta^i I \end{bmatrix},$$

$$V^i < \mu^i V^{i-1} \quad (19)$$

$$\begin{bmatrix} -1 & \bar{x}_l^i \\ \bar{x}_1^i & -\Phi_l^i \end{bmatrix} < 0 \quad (20)$$

$$\begin{bmatrix} -\Delta u_m^2 & \bar{Y}_1^i \\ \bar{Y}_1^i & -\Phi_l^i \end{bmatrix} < 0 \quad (21)$$

$$\begin{bmatrix} -\Delta y_m^2 \Phi_l^i & \Phi_l^i \bar{C}^{iT} \\ \bar{C}^i \Phi_l^i & -I \end{bmatrix} < 0 \quad (22)$$

$$\tau_p > (\tau_p)^* = -\frac{\ln \mu_i}{\ln \alpha_i} \quad (23)$$

$$\bar{P}_2^i \bar{G}_2^{-i} \bar{P}_2^i = \bar{G}_3^i, \bar{P}_2^i \bar{T}_2^{-i} \bar{P}_2^i = \bar{T}_3^i, \bar{P}_2^i \bar{M}_2^{-i} \bar{P}_2^i = \bar{M}_3^i,$$
$$\bar{G}_2^{iT} \bar{M}_2^{-i} \bar{G}_2^i = \tilde{M}_3^i, \bar{G}_2^i \bar{P}_2^{-i} \bar{G}_2^i = \bar{P}_3^i, \tilde{K}^i = \bar{Y}_1^i \bar{P}_2^i,$$
$$\bar{\varphi}_1^i = -\bar{\alpha}_i \bar{P}_2^i + \bar{T}_3^i + \bar{M}_3^i + (\tilde{d} + \underline{d} + 1) \bar{T}_3^i - \bar{\alpha}_i^{\tilde{d}} \bar{G}_3^i,$$

where $*$ represents the transposed element in the symmetric position; then, the robust model predictive fault-tolerant control problem is solvable, and the fault-tolerant controller gain in the system is $\bar{K}^i = \bar{Y}_1^i \bar{P}_2^{-i}$. The LMI (18) is the sufficient condition. If the LMI (18) holds, then the system is stable.

*Proof:* Multiplying the left and right sides of inequality (18) by $diag\left[\bar{P}_2^{-i}\ \bar{T}_2^{-i}\ \bar{G}_2^{-i}\ \theta^{-i}\ I\ I\ I\ I\ I\ I\ I\right]$, by using Schur complement Lemma 1 and Lemma 2, due to the occurrence of a perturbation and a fault in the control system, and letting

$$\tilde{A}^i = \bar{A}^i(k) + \bar{B}^i\alpha^i\bar{K}^i = A^i + \Delta a^i + \bar{B}^i\left(\beta^i + \alpha_0^i\beta^i\right)\bar{K}^i$$
$$= A^i + D^iF^i(k)E^i + \bar{B}^i\beta^i\bar{K}^i + \bar{B}^i\alpha_0^i\beta^i\bar{K}^i,$$

we obtain:

$$\begin{bmatrix} \varphi_1^i & 0 & \bar{\alpha}_i^{\tilde{d}}\bar{G}_2^{-i} & 0 \\ 0 & -\bar{\alpha}_i^{\tilde{d}}\bar{T}_2^{-i} & 0 & 0 \\ \bar{\alpha}_i^{\tilde{d}}\bar{G}_2^{-i} & 0 & -\bar{\alpha}_i^{\tilde{d}}\bar{M}_2^{-i} - \bar{\alpha}_i^{\tilde{d}}\bar{G}_2^{-i} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} \bar{\Lambda}_1^{iT} \\ \bar{G}^{iT} \end{bmatrix}\bar{P}_2^{-i}\begin{bmatrix} \bar{\Lambda}_1^i & \bar{G}^i \end{bmatrix} + \begin{bmatrix} \bar{\Lambda}_2^{iT} \\ \bar{G}^{iT} \end{bmatrix}\tilde{d}^2\bar{G}_2^{-i}\begin{bmatrix} \bar{\Lambda}_2^i & \bar{G}^i \end{bmatrix}$$
$$< -\begin{bmatrix} 0 & 0 & 0 & \sqrt{\eta^i} \end{bmatrix}^T\theta^{-i}\begin{bmatrix} 0 & 0 & 0 & \sqrt{\eta^i} \end{bmatrix}$$
$$+ \begin{bmatrix} \bar{\lambda}_1^{iT} \\ 0 \end{bmatrix}\theta^{-i}\begin{bmatrix} \bar{\lambda}_1^{iT} & 0 \end{bmatrix} + \begin{bmatrix} \bar{\lambda}_2^{iT} \\ 0 \end{bmatrix}\theta^{-i}\begin{bmatrix} \bar{\lambda}_2^{iT} & 0 \end{bmatrix} \quad (24)$$

namely,

$$\Phi_2 < \Phi_3 \quad (25)$$

where

$$\varphi_1^i = -\bar{\alpha}_i\bar{P}_2^{-i} + \bar{T}_2^{-i} + \bar{M}_2^{-i} + \left(\tilde{d} + \underline{d} + 1\right)\bar{T}_2^{-i} - \bar{\alpha}_i^{\tilde{d}}\bar{G}_2^{-i},$$
$$\bar{\Lambda}_1^i = \begin{bmatrix} \tilde{A}^i & \bar{A}_d^i & 0 \end{bmatrix}, \quad \bar{\Lambda}_2^i = \begin{bmatrix} \tilde{A}^i - I & \bar{A}_d^i & 0 \end{bmatrix},$$
$$\bar{\lambda}_1^i = \begin{bmatrix} \bar{Q}^{\frac{i}{2}} & 0 & 0 & 0 \end{bmatrix}, \quad \bar{\lambda}_2^i = \begin{bmatrix} \bar{R}^{\frac{i}{2}}\bar{K}^i & 0 & 0 & 0 \end{bmatrix}.$$

The increment function is defined as $\Delta V^i = V_{j+1}^i - \alpha_iV_j^i$, and the following formulas are obtained from the definition of the Lyapunov function and Lemma 3:

$$\begin{cases} \Delta V_{1j}^i \le \bar{x}_{j+1}^{iT}\theta^i\bar{P}_2^{-i}\bar{x}_{j+1}^i - \bar{\alpha}_i\bar{x}_j^{iT}\theta^i\bar{P}_2^{-i}\bar{x}_j^i \\ \Delta V_{2j}^i \le \bar{x}_j^{iT}\theta^i\bar{T}_2^{-i}\bar{x}_j^i - \bar{x}_{d+j}^{iT}\bar{\alpha}_i^{\tilde{d}}\theta^i\bar{T}_2^{-i}\bar{x}_{d+j}^i \\ \qquad + \sum_{s=k-\tilde{d}}^{k-\underline{d}} \bar{x}_{rj}^{iT}\bar{\alpha}_i^{k-r}\theta^i\bar{T}_2^{-i}\bar{x}_{rj}^i \\ \Delta V_{3j}^i \le \bar{x}_j^{iT}\theta^i\bar{M}_2^{-i}\bar{x}_j^i - \bar{x}_{d+rj}^{iT}\bar{\alpha}_i^{\tilde{d}}\theta^i\bar{M}_2^{-i}\bar{x}_{d+rj}^i \\ \Delta V_{4j}^i \le (\tilde{d} - \underline{d} + 1)\bar{x}_j^{iT}\theta^i\bar{T}_2^{-i}\bar{x}_j^i - \sum_{s=k-\tilde{d}}^{k-\underline{d}} \bar{x}_{rj}^{iT}\bar{\alpha}_i^{k-r}\theta^i\bar{T}_2^{-i}\bar{x}_{rj}^i \\ \Delta V_{5j}^i \le \tilde{d}^2\bar{\delta}_j^{iT}\theta^i\bar{G}_2^{-i}\bar{\delta}_j^i - \tilde{d}\sum_{r=k-\tilde{d}}^{k-1} \bar{\alpha}_i^{k-r}\bar{\delta}_{rj}^{iT}\theta^i\bar{G}_2^{-i}\bar{\delta}_{rj}^i \\ \qquad \le \tilde{d}^2[\bar{x}_{j+1}^i - \bar{x}_j^i]\theta^i\bar{G}_2^{-i}[\bar{x}_{j+1}^i - \bar{x}_j^i] \\ \qquad -\bar{\alpha}_i^{\tilde{d}}[\bar{x}_{j+1}^i - \bar{x}_{d+j}^i]\theta^i\bar{G}_2^{-i}[\bar{x}_{j+1}^i - \bar{x}_{d+j}^i] \end{cases} \quad (26)$$

By summing the inequalities (26) and multiplying both sides by $\theta^{-i}$, we obtain:

$$\begin{cases} \Delta V_j^i = \sum_{r=1}^5 \Delta V_r^i(k+j|k) \\ \theta^{-i}\Delta V_j^i \le \begin{bmatrix} \bar{\varepsilon}_j^{iT} & \bar{w}^{iT} \end{bmatrix}\Phi_2\begin{bmatrix} \bar{\varepsilon}_j^i \\ \bar{w}^i \end{bmatrix} < -\theta^{-i}J_j^i(k) \quad (27) \\ \theta^{-i}J_j^i = \begin{bmatrix} \bar{\varepsilon}_j^{iT} & \bar{w}^{iT} \end{bmatrix}\Phi_3\begin{bmatrix} \bar{\varepsilon}_j^i \\ \bar{w}^i \end{bmatrix} \end{cases}$$

Hence,

$$\Delta V_j^i < -J_j^i(k) \quad (28)$$

The system is proved to comply with the Lyapunov inequality constraints. In addition, according to the performance function (15), $J_j^i(k) > 0$; therefore, $\Delta V_j^i < 0$, namely, $V_{j+1}^i < \alpha_iV_j^i$. By setting $V_j^i = V^i(k)$, the following holds for $t_0 < k < t$:

$$V^{\sigma(t)}(t) \le \alpha_iV^{\sigma(t-1)}(t-1) \le \alpha^{t-T_s^{i-1}}V^{i-1}\left(T_s^{i-1}\right) \quad (29)$$

where $T_s^{i-1}$ denotes the switching time at the $i^{th}$ phase. From (19), we obtain

$$V^{\sigma(t)}(t) \le \alpha_i^{t-T_s^{i-1}}\mu_iV^{i-1}\left(T_s^{i-1}\right)$$
$$\le \prod_{i=1}^p (\alpha_i)^{\tilde{T}(t_0,t)}\prod_{i=1}^p (u_i)^{\frac{\tilde{T}(t_0,t)}{\tau_i}}V^{\sigma t_0}(t_0) \quad (30)$$

and considering the definition of the Lyapunov function, we obtain:

$$a\left\|\bar{x}_l^i(t+j|t)\right\|^2 \le V^{\sigma(t)}(t) \le \nu^{t-t_0}V^{\sigma t_0}(t_0)$$
$$\le b\nu^{t-t_0}\|\bar{x}_l(t_0+j|t_0)\|^2 \quad (31)$$

where

$$\|\bar{x}_1(t+j|t)\| \le \sqrt{\frac{b}{a}}\nu^{t-t_0}\|\bar{x}_1(t_0+j|t)\|,$$
$$a = \min\lambda_{\min}\left(\bar{P}_1^i\right) \quad (\forall i \in N),$$
$$b = \lambda_{\max}(\bar{P}_1^i) + \tilde{d}(\lambda_{\max}(\bar{T}_1^i) + \lambda_{\max}(\bar{M}_1^i)$$
$$+ \frac{d+\tilde{d}}{2}(\tilde{d} - \underline{d} + 1)\lambda_{\max}(\bar{T}_1^i)$$
$$+ \frac{\tilde{d}^2(1+\tilde{d})}{2}\lambda_{\max}(\bar{G}_1^i)).$$

According to Definition 2, if the switching signal satisfies the condition $\tau^i \ge -\frac{\ln\mu_i}{\ln\alpha_i}$, then the time-delayed switched system is robustly stable.

In the following paragraphs, it will be proven that the upper bound $\theta^i > 0$ of performance function $J_\infty^i(k)$ exists, so that (17) can be established. Since $0 < \bar{\alpha}^i < 1$, in combination with formula (28), the following inequality is obtained:

$$V_{j+1}^i - V_j^i < -J_j^i \quad (32)$$

The sum of the inequality above is calculated from $j = 0$ to $j = \infty$:

$$V^i(k+1|k) - V^i(k|k) + V^i(k+2|k) - V^i(k+1|k)$$
$$+ \ldots + V^i(k+1+\infty|k) - V^i(k+\infty|k)$$
$$= V^i(k+\infty|k) - V^i(k|k) < -\sum_{j=0}^\infty J_j^i(k) \quad (33)$$

namely,

$$V^i(k + \infty|k) - V^i(k|k) < -J^i_\infty(k) \quad (34)$$

If $V^i\left(\bar{x}^i_1(\infty)\right) = 0$ and $\bar{x}^i_1(\infty) = 0$, then:

$$J^i_\infty(k) < V^i(k|k) \quad (35)$$

Taking $\bar{x}^i_l(k) = max\left(\bar{x}^i_1(r), \bar{\delta}^i_1(r)\right)$ and $r \in \left(k - \tilde{d}, k\right)$, we obtain

$$V^i\left((\bar{x}_1(k)\right) \le \bar{x}^i_l(k)\bar{\Psi}_l\bar{x}^{iT}_l(k) \quad (36)$$

As $0 < \alpha_i < 1$, from the definition of the Lyapunov function, it follows that:

$$\bar{\Psi}^i_l = \bar{P}^i_1 + \tilde{d}\left(\bar{T}^i_1 + \bar{M}^i_1\right) + \frac{d + \tilde{d}}{2}\left(\tilde{d} - \underline{d} + 1\right)\bar{T}^i_1$$
$$+ \frac{\tilde{d}^2\left(1 + \tilde{d}\right)}{2}\bar{G}^i_1 \quad (37)$$

The following inequality is obtained from the matrix inequality (20) in combination with the Schur complement lemma:

$$\bar{x}^i_l(k)\Phi^{-1}_l\bar{x}^{iT}_l(k) < 1 \le \prod^p_{i=1}\left(\alpha_i\mu^{\frac{1}{\tau_i}}_i\right)^{\tilde{T}(t_0,t)}V^{\sigma(t_0)}(t_0) \quad (38)$$

Letting $\nu = max\left(\alpha_i\mu^{\frac{1}{\tau_i}}_i\right)$, $\bar{x}^i_l(k) = max\left(\bar{x}^i_1(r), \bar{\delta}^i_1(r)\right)$, $r \in \left(k - \tilde{d}, k\right)$, $\bar{\Psi}^{-i}_l\theta = \Phi_l$. Then, the following inequality is obtained:

$$\bar{x}^i_l(k)\left(\bar{\Psi}^{-i}_l\theta\right)^{-1}\bar{x}^{iT}_l(k) < 1 \quad (39)$$

namely,

$$\bar{x}^i_l(k)\bar{\Psi}^i_l\bar{x}^{iT}_l(k) \le \theta^i \quad (40)$$

Combining (35) and (36), there exists an upper bound $\theta^i$ of $J^i_\infty(k)$ such that:

$$J^i_\infty(k) \le V^i\left(\bar{x}_1(k)\right) \le \theta^i \quad (41)$$

In the following section, the system constraints will be discussed.

For constraint (13), we obtain:

$$\left\|\Delta u^i_j\right\|^2 = \left\|\bar{K}^i\bar{x}^i_j\right\|^2 = \left\|\bar{Y}^i_1\theta^{-1}\bar{P}^i_1\bar{x}^i_j\right\|^2 \le \left\|\bar{Y}^i_1\theta^{-i}\bar{\Psi}_l\bar{x}^i_j\right\|^2$$
$$= \bar{Y}^i_1\bar{x}^i_j\Phi^{-1}_l\bar{x}^i_j\Phi^{-T}_l\bar{Y}^{iT}_1 \le \bar{Y}^i_1\Phi^{-T}_l\bar{Y}^{iT}_1 \quad (42)$$

Formula (21) is obtained from Lemma 1.

For output constraint (13), we obtain:

$$\left\|\Delta y^i_j\right\|^2 = \left\|\bar{C}^i\bar{x}^i_j\right\|^2 \le \bar{x}^i_j\bar{C}^{iT}\bar{C}^i\bar{x}^{iT}_j < \Delta y^{i2}_{mj} \quad (43)$$

and we further obtain:

$$\bar{C}^{iT}\bar{C}^i \le \bar{x}^{-iT}_j\Delta y^{i2}_m\bar{x}^{-i}_j \le \Delta y^{i2}_m\Phi^{-i}_l \quad (44)$$

Output constraint (22) is obtained from Lemma 1.

Theorem 1 provides a sufficient condition for the solvability of the model predictive fault-tolerant control problem for batch processes with interval delays. If the lower bound of the interval delays is zero, then this control problem becomes the corresponding control problem with constant delays. As a special case of interval time-varying delay systems, the following corollary can be easily obtained from Theorem 1.

Set

$$\begin{cases} V^i_j = \sum^3_{l=1}V^i_l(k + j|k) \\ V^i_{1j} = \bar{x}^{iT}_j\bar{P}^i_1\bar{x}^i_j = \bar{x}^{iT}_j\theta^i\bar{P}^{-i}_2\bar{x}^i_j \\ V^i_{2j} = \sum^{k-1}_{r=k-d}\bar{x}^{iT}_{rj}\bar{\alpha}^{k-1-r}_i\bar{T}^i_1\bar{x}^i_{rj} = \sum^{k-1}_{r=k-d}\bar{x}^{iT}_{rj}\bar{\alpha}^{k-1-r}_i\theta^i\bar{T}^{-i}_2\bar{x}^i_{rj} \\ V^i_{3j} = \tilde{d}\sum^{-1}_{s=-\tilde{d}}\sum^{k-1}_{r=k+s}\bar{\delta}^{iT}_{rj}\bar{\alpha}^{k-1-r}_i\bar{G}^i_1\bar{\delta}^i_{rj} \\ \quad = \tilde{d}\sum^{-1}_{s=-\tilde{d}}\sum^{k-1}_{r=k+s}\bar{\delta}^{iT}_{rj}\bar{\alpha}^{k-1-r}_i\theta^i\bar{G}^{-i}_2\bar{\delta}^i_{rj} \end{cases} \quad (45)$$

Similar to the proof of Theorem 1, Corollary 1 can be obtained. The content is represented as follows:

*Corollary 1:* If $\bar{w}^i(k) \ne 0$, then for specified constants $0 \le \underline{d} \le \tilde{d}$, $\theta^i > 0$, and $0 < \alpha_i < 1$, if there exist positive-definite symmetric matrices $\bar{P}^i_2, \bar{T}^i_2, \bar{G}^i_2, \bar{T}^i_3, \bar{M}^i_3, \bar{G}^i_3$, and $\bar{Y}^i_1$ and positive real numbers $\eta^i, \varepsilon_a$, and $\varepsilon_b$ such that the following matrix inequalities are feasible for the switching signal with an average dwell time that satisfies inequality (23),

$$\begin{bmatrix} \tilde{\Upsilon}_{11} & \tilde{\Upsilon}_{12} & \tilde{\Upsilon}_{13} & \tilde{\Upsilon}_{14} & \tilde{\Upsilon}_{15} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & 0 \\ * & * & \Upsilon_{33} & 0 & 0 \\ * & * & * & \Upsilon_{44} & 0 \\ * & * & * & * & \Upsilon_{55} \end{bmatrix} < 0 \quad (46)$$

$$\tilde{\Upsilon}_{11} = \begin{bmatrix} \tilde{\varphi}^i_1 & \bar{\alpha}^{\tilde{d}}_i\bar{G}^i_3 & 0 \\ \bar{\alpha}^{\tilde{d}}_i\bar{G}^i_3 & -\bar{\alpha}^{\tilde{d}}_i\bar{M}^i_3 - \bar{\alpha}^{\tilde{d}}_i\bar{G}^i_2 & 0 \\ 0 & 0 & -\theta^i\eta^i \end{bmatrix}$$

$$\tilde{\Upsilon}_{12} = \begin{bmatrix} \bar{P}^i_2\bar{A}^{iT} + \bar{Y}^{iT}_1\beta^i\bar{B}^{iT} & \bar{P}^i_2\bar{A}^{iT} + \bar{Y}^{iT}_1\beta^i\bar{B}^{iT} - \bar{P}^i_2 \\ \bar{T}^i_2\bar{A}^{iT} & \bar{T}^i_2\bar{A}^{iT} \\ \theta^i\bar{G}^{iT} & \theta^i\bar{G}^{iT} \end{bmatrix}$$

$$\tilde{\Upsilon}_{13} = \begin{bmatrix} \bar{P}^i_2\bar{E}^{iT} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{\Upsilon}_{14} = \begin{bmatrix} \bar{Y}^{iT}_1\beta^i & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Upsilon}_{15} = \begin{bmatrix} \bar{Y}^{iT}_1\bar{R}^{\frac{i}{2}} & \bar{P}^i_2Q^{\frac{i}{2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Upsilon_{22} = \begin{bmatrix} -\bar{P}^i_2 & 0 \\ 0 & -\tilde{d}^{-2}\bar{G}^i_2 \end{bmatrix}, \quad \Upsilon_{23} = \begin{bmatrix} 0 & \bar{\varepsilon}^i_a\bar{D}^i \\ 0 & \bar{\varepsilon}^i_a\bar{D}^i \end{bmatrix},$$

$$\Upsilon_{24} = \begin{bmatrix} 0 & \bar{\varepsilon}^i_b\bar{B}^i\beta^i_0 \\ 0 & \bar{\varepsilon}^i_b\bar{B}^i\beta^i_0 \end{bmatrix}$$

$$\Upsilon_{33} = \begin{bmatrix} -\varepsilon_a^i & 0 \\ 0 & -\varepsilon_b^i \end{bmatrix}, \quad \Upsilon_{44} = \begin{bmatrix} -\varepsilon_a^i & 0 \\ 0 & -\varepsilon_b^i \end{bmatrix},$$

$$\Upsilon_{55} = \begin{bmatrix} -\theta^i I & 0 \\ 0 & -\theta^i I \end{bmatrix},$$

$$V^i < \mu^i V^{i-1} \tag{47}$$

$$\begin{bmatrix} -1 & \bar{x}_L^i \\ \bar{x}_1^i & -\bar{\Phi}_l^i \end{bmatrix} < 0 \tag{48}$$

$$\begin{bmatrix} -\Delta u_m^2 & \bar{Y}_1^i \\ \bar{Y}_1^i & -\bar{\Phi}_l^i \end{bmatrix} < 0 \tag{49}$$

$$\begin{bmatrix} -\Delta y_m^2 \tilde{\Phi}_l^i & \tilde{\Phi}_l^i \bar{C}^{iT} \\ \bar{C}^i \tilde{\Phi}_l^i & -I \end{bmatrix} < 0 \tag{50}$$

where $\tilde{\varphi}_1^i = -\bar{\alpha}_i \bar{P}_2^i + \bar{T}_3^i - \bar{\alpha}_i^{\tilde{d}} \bar{G}_3^i$, $\tilde{\Psi}_l^{-i}\theta = \tilde{\Phi}_l$, and $\tilde{\Psi}_l^i = \bar{P}_1^i + \tilde{d}\bar{T}_1^i + \frac{\tilde{d}^2(1+\tilde{d})}{2}\bar{G}_1^i$, then the robust model predictive fault-tolerant control problem with constant delays is solvable, and the stable fault-tolerant controller gain in the system is $\bar{K}^i = \bar{Y}_1^i \bar{P}_2^{-i}$.

## C. OPTIMIZATION ALGORITHM

In this part, we seek the controller design with the minimum upper bound under the maximum disturbance. The optimization problem at time $k$ can be solved using the following formula:

$$\min_{\Delta u(t+i|t, k+j|k), i, j \geq 0} \theta^i \tag{51}$$

subject to $V^i(k|k) \leq \theta^i$, namely, subject to $\begin{bmatrix} -1 & \bar{x}_l^i \\ \bar{x}_1^i & -\Phi_l^i \end{bmatrix} < 0$

In (18), $\eta^i$ can be optimized. Since (18) is a bilinear inequality, let $\varsigma^i = \theta^i \eta^i$; thus, $\varsigma^i$ is optimized instead.

$$\min_{\substack{\bar{P}_2^i, \bar{M}_2^i, \bar{T}_2^i, \bar{G}_2^i, \bar{T}_3^i, \bar{M}_3^i, \\ \bar{M}_3^i, \bar{G}_3^i, \bar{Y}_1^i, \varepsilon_a, \varepsilon_b}} \varsigma^i$$

$$\text{subject to } (18 - 22) \tag{52}$$

## IV. SIMULATION

In this paper, fault-tolerant control for an injection molding process, which is a representative multi-phase batch process, is simulated. The injection molding process consists of five phases: mold closure, injection, packing, cooling and mold opening. First, during the injection phase, the molten material is injected into the mold cavity until the cavity is filled. Then, the system is switched to the packing phase, and the polymer is filled into the contractions that are caused by cooling and curing to realize the objective of packing. After the packing phase, the cooling and mold opening phases begin, in which the polymer in the mold cavity cools until it is fully cured, and then the final product is ejected. The injection rate and the packing pressure are the two main variables to control because they have the largest impacts on the control efficiency in the corresponding phases, and errors tend to occur during these two phases. The injection rate and the packing pressure are controlled by the degrees of opening of the corresponding valves. When the mold cavity pressure

reaches a threshold level at the injection phase, the system will switch to the next phase, namely, the packing phase.

In this paper, the model is transformed into a switched system, and the injection phase and packing phase of the injection molding process are controlled based on the principle of predictive fault-tolerant control under the conditions of time delay, perturbations and actuator faults.

The injection phase is defined as the first phase, and the packing phase is defined as the second phase. For the injection rate IV at the injection phase, the packing pressure NP at the packing phase and the valve opening degree VO, the model is expressed as:

$$(1-0.9291z^{-1}-0.03191z^{-2})IV = (8.687z^{-1}-5.617z^{-2})VO$$
$$(1-1.317z^{-1}+0.3259z^{-2})NP = (171.8z^{-1}-156.8z^{-2})VO \tag{53}$$

The model for the relationship between the mold cavity pressure NP and the injection rate at the injection phase is expressed as:

$$NP(1 - z^{-1}) = 0.1054z^{-1}IV \tag{54}$$

For the injection rate IV, packing pressure NP and valve opening degree VO in systems with real actuator faults, the model is expressed as:

$$(1-0.9291z^{-1}-0.03191z^{-2})IV = (6.950z^{-1}-4.494z^{-2})VO$$
$$(1-1.317z^{-1}+0.3259z^{-2})NP = (137.4z^{-1}-125.4z^{-2})VO \tag{55}$$

Denote $x_1^1(k) = 0.03191IV(k-1) - 4.4936VO(k-1)$, $u^1(k) = VO(k)$, $y^1(k) = NP(k)$, $x_1^2(k) = 0.3259NP(k-1) - 125.44(k-1)$, $u^2(k) = VO(k)$, $y^2(k) = NP(k)$.

At the injection phase, the injection rate IV is set to 40 mm/s; at the packing phase, the packing pressure NP is set to 300 bar. Under the conditions of an actuator fault and the constraint conditions, the time-delayed extended state-space fault model for the injection phase is:

$$\begin{cases} x^1(k+1) = \left( \begin{bmatrix} 0.9291 & 1 & 0 \\ 0.03191 & 0 & 0 \\ 0.1054 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.02\delta(k) & 0 & 0 \\ 0.01\delta(k) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) x^1(k) \\ + \begin{bmatrix} 0.002 & 0 & 0 \\ 0.001 & 0 & 0 \\ -0.0021 & 0 & 0 \end{bmatrix} x^1(k-d(k)) + \begin{bmatrix} 8.687 \\ -5.617 \\ 0 \end{bmatrix} \alpha u^1(k) + \omega^1(k) \\ y^1(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x^1(k) \end{cases} \tag{56}$$

and the input and output constraints in this phase are selected as:

$$\begin{cases} |y^1(t, k)| \leq 45 \\ |u^1(t, k)| \leq 6 \end{cases} \tag{57}$$

The packing phase model is:

$$
\begin{cases}
x^2(k+1) = \left( \begin{bmatrix} 1.317 & 1 \\ -0.3259 & 0 \end{bmatrix} + \begin{bmatrix} 0.02\delta(t,k) & 0 \\ 0.01\delta(t,k) & 0 \end{bmatrix} \right) x^2(k) \\
\quad + \begin{bmatrix} 0.002 & 0 \\ 0.001 & 0 \end{bmatrix} x^2(k-d(k)) + \begin{bmatrix} 171.8 \\ -156.8 \end{bmatrix} \alpha u^2(k) + \omega^2(k) \\
y^2(k) \;=\; \begin{bmatrix} 1 & 0 \end{bmatrix} x^2(k)
\end{cases}
$$

(58)

and the input and output constraints in this phase are:

$$
\begin{cases}
\left| y^2(t,k) \right| \le 302 \\
\left| u^2(t,k) \right| \le 0.6
\end{cases}
$$

(59)

where $\delta(k)$ is a random variable that is within the range $[0, 1]$, $\alpha = 0.8$, and the switching condition is $G^1(x(k)) = 350 - \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x^1(k) < 0$; hence, once the mold cavity pressure exceeds 350 Pa, the system will be switched from the injection phase to the packing phase. To examine the design performance in this paper, we determine the initial control law parameters. Then, MATLAB software is used for simulation. The initial control law is obtained via Theorem 1 and its optimization algorithm, and the controller gain is

$$
\bar{K}^1 = [-0.1060, -0.1141, 0, 0.0433, 0.1323]
$$
$$
\bar{K}^2 = [-0.0015, -0.0012, 0.0004, 0.0013] \qquad (60)
$$

To evaluate the performance of the proposed method, we compare it with that of the traditional control method [7]. In the traditional method, the controller design is of the following form: $u(k+j|k) = K^i \begin{bmatrix} x(k+j|k) \\ \hat{x}^i(k+j|k) \end{bmatrix}$ (where $\hat{x}^i(k+j|k)$ is obtained from (7)). The sampling time of each step is 5 ms. Two kinds of faults are selected here to analyze the influence of faults on the control performance of the system. One is a constant fault (case 1); the other is a time-varying fault (case 2). The comparison results are as follows.
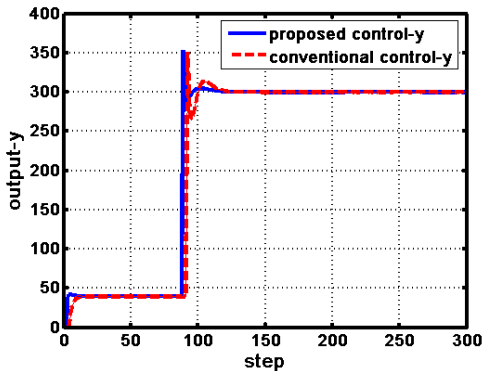


**FIGURE 1.** An output comparison between the proposed control and the traditional control strategy under constant faults.

Under case 1, $\alpha = 0.8$. The results are presented in Figs. 1 and 2. As shown in Figs. 1, under the proposed control method, the system fluctuates less, output tends to stabilize more quickly at the second phase, and the phase running time is shorter. The operation time of the first phase is 88,
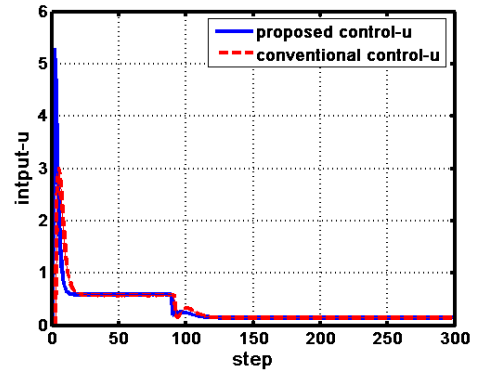


**FIGURE 2.** An input comparison between the proposed control and the traditional control strategy under constant faults.
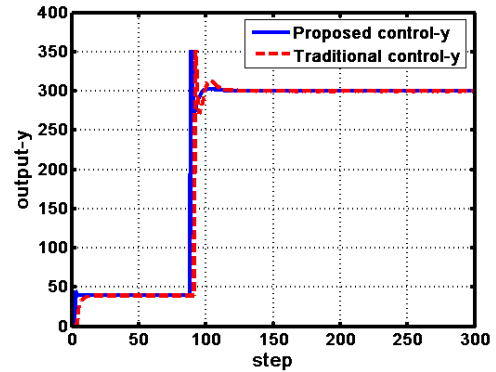


**FIGURE 3.** An output comparison between the proposed control and the traditional control strategy under time-varying faults.
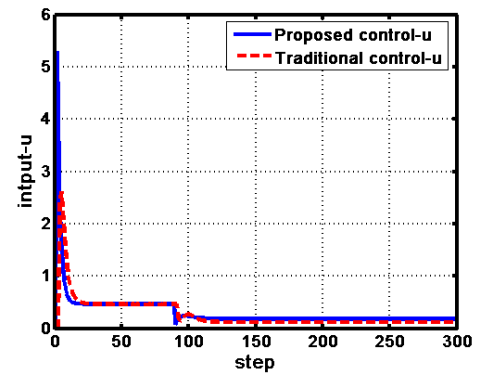


**FIGURE 4.** An input comparison between the proposed control and the traditional control strategy under time-varying faults.

while for the traditional control strategy, the operation time of the first phase is 93. In addition, According to Fig. 2, under random perturbations, both the predictive fault-tolerant control strategy and the traditional control strategy can enable the system to stabilize at the injection phase. However, from the 94th step, according the input chart, the system that adopts the predictive fault-tolerant control strategy is switched more stably at an earlier time, and the range of the curve's fluctuation is smaller. Figs. 1 and 2 compare the output and input for a fault of 0.8. We present the comparison results under other fault conditions in Figs. 3 and 4. Here, the time-varying fault
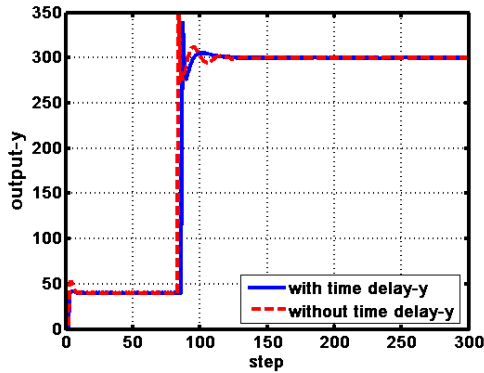
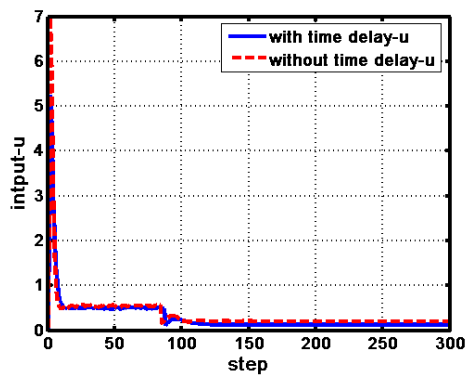**FIGURE 5.** An output comparison with and without time delay under the proposed method.



**FIGURE 6.** An input comparison with and without time delay under the proposed method.



**FIGURE 7.** An output comparison between the proposed control and 2D-ILC.
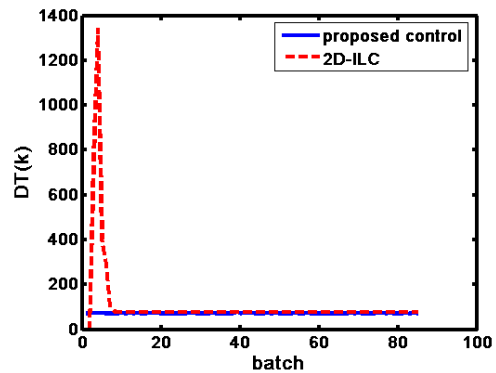


**FIGURE 8.** Tracking performance comparison between the proposed control and 2D-ILC.

is chosen, and $\alpha = 0.6 + 0.2 \sin(k)$ (case 2). According to the two figures, although the control performance of the system has decreased, the control target can still be attained. In addition, to analyze the impact of a time delay on the system control performance, the parameters of systems with a time delay and those without a time delay are compared under the predictive fault-tolerant control strategy that is proposed in this paper, as shown in Figs. 5 and 6. The two figures present the comparison results of output $y$ and input $u$ for systems with and without a time delay. The system with a time delay is characterized by lower control performance and larger fluctuations in the output and input at the time of system switching; however, the system ultimately stabilizes under the predictive fault-tolerant control strategy. The comparison results demonstrate that the time delay affects the control performance of the system.

In the common control method for a batch process, the process is regarded as a two-dimensional (2D) system, and its tracking control via the iterative learning control (ILC) method is studied. Here, we compare our proposed method with this method (2D-ILC). The tracking error is selected as $DT(k) = \sqrt{\sum_{k=0}^{\infty} e^T(k)e(k)}$. The comparison results demonstrate that the output has a short running time in each phase when the method proposed in this paper is used, but the
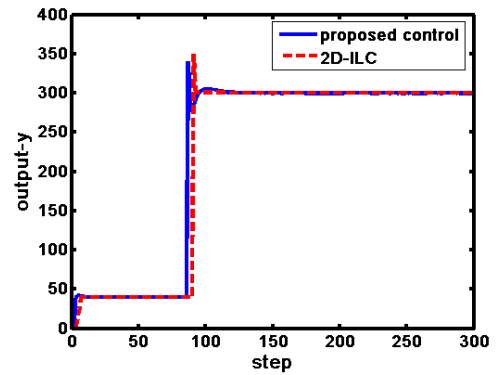
fluctuations are larger at the initial time and the switching time. According to the comparison of the tracking performance, the tracking error of our proposed method is small, as shown in Figs. 7 and 8. The predictive fault-tolerant control strategy proposed in this paper can ensure high system stability and satisfactory control performance.

## V. CONCLUSION

In this paper, an $H_\infty$-model predictive fault-tolerant control strategy is proposed for multi-phase batch processes with interval time delays and actuator faults. The solvability condition for ensuring that the system's output tracks the specified output is constructed in the form of LMIs. In addition, a switching law has been proposed. The results of the injection molding process simulation demonstrate that a time delay impacts the system stability; however, the control strategy that is proposed in this paper ensures that the system can still operate stably under these conditions. In addition, compared with traditional control methods, the simulation proves that the control strategy that is proposed in this paper can realize reduced fluctuations in output, input and their increment; faster convergence; shorter operating time of the first phase; energy conservation; and emission reduction. From a long-term perspective, the method that is proposed in this paper can serve as a reference for the design of energy-saving controllers.

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