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A Novel Failure Mode and Effects Analysis Model Using Triangular Distribution-Based Basic Probability Assignment in the Evidence Theory

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ABSTRACT Failure mode and effects analysis (FMEA) is a typical risk assessment and prevention technology, in which different experts provide different assessments on the target system to identify the risk grades of its components. Sometimes, the assessments contain conflict information. How to manage and fuse the conflict assessment information is an open issue. We propose a triangular distribution-based basic probability assignment (TDBPA) method to model and fuse the conflict risk level coming from different experts' assessments in the frame of Dempster-Shafer evidence theory. First, the subjective assessments of risk analysis from domain experts are modeled with belief structure in Dempster-Shafer evidence theory. Then, the assessments are transformed as the TDBPA function. Thirdly, the conflict risk assessments from the FMEA team for failure analysis can be fused with Dempster rule of combination. After that, the modified risk priority number (RPN) model based on fused assessment can be calculated for ranking of failure modes. Finally, recommended actions should be taken for prevention of potential risk items. We verify the rationality and efficiency of the proposed method with a case study in the blades of an aircraft turbine. In short, the presented FMEA methodology procedure in this paper is well organized so that we can apply it in a more simple and understandable way. Utilizing the character of triangular distribution, taking adjacent values into account, TDBPA method can smooth the conflict assessment for information fusion. In addition, the shortcoming of repeating values in classical RPN is eliminated in the proposed method, which improves the ability for risk assessment of FMEA.

INDEX TERMS Failure mode and effects analysis (FMEA), Dempster-Shafer evidence theory, basic probability assignment, triangular distribution, failure analysis, knowledge reasoning.

I. INTRODUCTION

Since it was introduced by NASA in 1960s [1], failure mode and effects analysis (FMEA), as an analytical tool in reliability, has been proved to be remarkably effective and applied extensively in many fields such as risk evaluation [2], decision making [3]–[5] and so on. In these piratical applications, FMEA is generally used for assuring that potential risks have been fully considered and processed properly during the assessment process. A central part of this technology is risk priority number (RPN), the product occurrence (O), severity (S), detection (D), i.e.$RPN = S \times O \times D$. Additionally, its most visible result is the documentation of the collective knowledge of cross-functional teams [6]. In addition, FMEA can facilitate the identification of potential failures in the design or process of products or systems. FMEA method has been applied in diverse industries like medical domain [7], [8], software engineer [9], enterprise [10], production process [11], [12], aviation domain [13], emergency management [3], and so on [4],

In despite of its advantages, some weaknesses of FMEA exists. The three risk factors are scaled by experts with an integer number from 1 to 10 [1]. However, with the increasing complexity of target system, the assessments implies great uncertainty. Moreover, the conventional RPN model also is criticized. To improve the efficiency of the traditional FMEA

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method, many hybrid intelligent methods based on FMEA have been introduced. From the aspect of improving the form of assessment distribution, the linguistic term sets are used to improve the FMEA method in [5], [14]. Besides, a consensus-based multiattribute group decision-making approach in [5] is applied for FMEA in a linguistic context. From the perspective of overcoming the shortcomings of conventional RPN [15], perceptual computing (Per-C) is incorporated into RPN model by analyzing the uncertainties of in words given by experts [16]. In [17]–[19], fuzzy inference system (FIS) is adopted to construct models for decision making using uncertain fuzzy rules. In other direction of research into handling the uncertainty of risk assessments, the cloud model and technique for order preference by similarity to an ideal solution (TOPSIS) method are adopted to improve the risk evaluation efficiency of FMEA in [2] while the fault tree analysis is adopted for a similar purpose in [8]. Though a lot of practical models are introduced to enhance the performance of conventional FMEA, how to model and fuse the conflict assessments from experts is still a key and an open issue.

Some previous works introduce D-S theory to address the uncertain information in FMEA [20]. In [21], the risk levels of risk factors are modeled by membership function in fuzzy sets theory and then transformed as mass function, subsequently, the mass function can be fused with Dempster combination rule to get the final RPN values. To carefully address vague and imprecise risk evaluations, the two-parts RPN values are designed incorporation with D-S theory and belief entropy method in [22].

Dempster-Shafer evidence theory (D-S theory) [23], [24] is a typical tool for information reasoning under uncertainty. It has been widely used in uncertainty modeling and evidence combination because of its efficiency in processing indeterminate information and aggregating the multi-source feature information. In [25]–[27], new methods are proposed in the framework of D-S theory for revealing the data structure in clustering. Besides, a new saliency detection method is designed for image fusion and processing in [28]. In [29], [30], D-S theory is adopted for improving the reliability and efficiency in decision-making process. Futhermore, D-S theory is also effective in classification problems [31]–[33], sensor data fusion [34], [35], decision-making [36], and so on [37]–[39]. After 40 years' development, some open issues in D-S theory and its applications are still worth for paying more efforts [40], [41]. Recent researches on uncertainty measure and management of mass function show a new perspective of developing the theory itself [42]–[44], while the researches on the negation of mass function open a new research direction in D-S theory [45].

Though D-S theory has advantages in decreasing indeterminacy by reserving the common information and fusing multi-source estimation without prior weights, it also has limitations. The open issues in D-S theory include conflict evidence fusion [46]–[49], dependent evidence fusion [50], [51], generation of mass function [52], belief entropy for evidence evaluation [53], [54], decision-making based on mass function [55], [56], incomplete information processing in D-S theory framework [57], approximation of mass function [58] and so on. In general, D-S theory is effective in evidence modeling and uncertain information fusion. Two categories of method are introduced for evidence modeling and fusion. One is evidence preprocessing and the other is modification of the Dempster combination rule. In this paper, we focus on generation of basic probability assignment (BPA) with respect to its application in risk analysis under experts' subjective assessments.

The work in [13] introduces the Gaussian distribution function to improve the modeling of subjective assessment in [20] by generating BPA more precisely. But, in practical engineering, the Gaussian distribution seems to be more complex than the method with fuzzy sets theory for technical engineers in the field. Sometimes, the RPN values coming from subjective assessments of different experts may be conflict integer values. To inherit the advantage of fuzzy sets theory as well as the precise model and fuse the subjective assessments from experts, we propose an improved and simple method for generation of BPA. In detail, we adopt the triangular distribution to model the conflict assessments for further information fusion with Dempster combination rule. With the triangular distribution-based BPA (TDBPA) method, the conflict integer values in risk level coming from subjective assessments of FMEA experts can be modeled and fused for failure analysis and prevention.

Some advantages and features in the TDBPA based-FMEA method in the framework of D-S theory are listed as follows. First of all, due to the values given by domain experts is subjective, the triangular distribution is utilized to model such inaccuracies in the assessment. Secondly, the triangular distribution in the proposed method is constructed with less prior data because the distribution curve is only determined by the assessment values. Thirdly, after fusion of TDBPA-based RPN values, the repeating values in classical RPN are eliminated, which optimizes the ranking results of RPN. Last but not least, the new TDBPA based-FMEA procedure is outlined with simplicity and flexibility. Several numerical examples are used to illustrate that the efficiency and availability of TDBPA and its application in modeling of risk assessment. A case study in the blades of an aircraft turbine is adopted to verify the effectiveness of the new method.

The rest of this paper is organized as follows. Section II shows a literature review on some related works concerning how to improve FMEA. In Section III, D-S theory and triangular distribution are briefly introduced. A novel generation method of BPA based on triangular distribution, named TDBPA, is proposed in Section IV. Section V presents a TDBPA-based FMEA method and its application in a practical engineering. Finally, conclusions and some possible following work are provided in Section VI.

II. LITERATURE REVIEW

A. TERMINOLOGY IN FMEA

Traditionally, during the application process of FMEA, determining the risk priorities of failure modes based on the risk priority number (RPN) is a key step. A failure mode with a higher RPN value is concerned to be more critical than that with a lower RPN.

Definition 1: The RPN consists of three factors: the severity of a failure effect (S), the probability of occurrence of a failure mode (O) and the probability of a failure being detected (D). RPN can be defined as follows:

$$RPN = O \times S \times D. \tag{1}$$

Generally, each risk factor can be measured with 10 ranking levels from 1 to 10. For instance, Table 1 shows the suggested criteria of rating for the occurrence O of a failure in FMEA. Similarly, the severity S of a failure effect and the detectability D of a failure can be mapped to an integer from 1 to 10. More details can be founded in [20].

TABLE 1. Suggested criteria of rating for occurence of a failure in FMEA [20].

Rating	Probability of occurrence	Possible failure rate
10	Extremely high: almost inevitable failure	≥1/2
9	Very high	1/3
8	Repeated failure	1/8
7	High	1/20
6	Moderately high	1/80
5	Moderate	1/400
4	Relatively low	1/2000
3	Low	1/15000
2	Remote	1/150000
1	Nearly impossible	≤1/150000

In [1], the conventional RPN calculation method has been considerably criticized. Considerable research has been conducted to investigate the use of RPN in FMEA. The mean value of RPN(MVRPN) proposed in [20] is an efficient and we utilize it as a substitute to overcome the shortcomings of traditional RPN.

Definition 2: Suppose the RPN value of the n^{th} failure mode has several ratings, which are represented as (RPN_n^1, RPN_n^q), and its corresponding belief value can be described as $(B(RPN_n^1), B(RPN_n^q))$. Then, the mean value of RPN can be defined as follows:

$$MVRPN_n = E(RPN_n) = \sum_q (RPN_n^q) \cdot B(RPN_n^q) \tag{2}$$

B. EVIDENCE THEORY APPLIED IN FMEA

D-S theory, a powerful mathematical tool to reason with uncertain information, is applicable to augment traditional FMEA process, in which the experts' assessments are expressed with probability [59]. D-S theory is capable to aggregate the different evaluation information by considering multiple experts' evaluation opinions. It is usually considered as a proper mathematical framework, of which belief and plausibility distribution are used to fuse different evaluations to make them available for ranking failure modes [60]. To deal with the epistemic uncertainty often affecting the input evaluation, the present paper propose the D-S theory of evidence as a proper mathematical framework in [61]. In [60], a new aggregation method on the basis of D-S theory is introduced to fuse various kind of evaluations, then use the TOPSIS method to prioritize the failure modes. In [62], Li and Chen propose a novel evidential FMEA integrating fuzzy belief structure and grey relational projection method (GRPM), in which they use a new method to transform the experts' fuzzy opinions into BPAs. Besides, a new evidential FMEA using linguistic term is presented in [63]. It transforms the experts' linguistic judgments into BPAs and adopts the Dempster combination rule for fusion. Note that, when experts assign different but precise values to risk factors, the basic probability assignment (BPA) constructed becomes highly conflicting evidence, which cannot be fused by Dempster combination rule and will result in errors in FMEA. There also are many researches focus on resolving this problem in combination with other theories. In [64], Yuan and Deng propose an improved combination rule considering both the uncertainty of evidences and the conflict degree of the system. Also, to address the combination issue, in [65], the evidence distance function and the belief entropy are adopted to assign weights to evidence, which contributes to modifying the conflict evidence. Similarly, the Deng entropy and evidence distance are utilized in [66]. Deng entropy is adopted for uncertain degree modeling of FMEA experts in [67].

The aforementioned methods do not put emphasis on reconstructing the BPAs. To improve the modeling of subjective assessment in [20] by generating BPA more precisely, the work in [13] introduces the Gaussian distribution function, while which seems to be more complex in practical engineering. Furthermore, in a new D-S theory-based fault diagnosis method proposed in [68], the BPAs are constructed in the basis of the triangle fuzzy function of symptoms and the relationship between symptoms and faults. Nevertheless, the corresponding relation between the characteristic values of symptom parameters and the failure modes are difficult to attained with the increasing complexity of target system. In short, the limitation in these research that motivates us to propose a TDBPA method to address the problems remains in FMEA method regarding D-S theory framework.

III. PRELIMINARIES

In this section, the basic concepts of the D-S theory are triangular distribution are introduced.

A. DEMPSTER-SHAFER EVIDENCE THEORY

D-S theory originated in the work of Dempster using probabilities with upper and lower bounds [23] and Shafer established the basic probability assignment function (BPA) on the framework of discernment [24]. Developing on the foundation of the Bayesian theory of probabilities, D-S theory can represent and process uncertain information effectively.

Reasoning and decision-making can be carried out with incomplete or conflicting pieces of evidence even if there is lack of prior information [69]. Formally, the definitions in D-S theory are provided as follows.

Definition 3: Let Ω be a set of mutually exclusive and collectively exhaustive elements H_i, indicated by

$$\Omega = \{H_1, H_2, \ldots, H_i, \ldots, H_N\} \tag{3}$$

The power set of Ω composed with 2^N propositions is called the Frame of Discernment (FOD), denoted as 2^Ω:

$$2^\Omega = \left\{ \begin{array}{l} \emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_N\}, \{H_1, H_2\}, \\ \ldots, \{H_1, H_2, \ldots, H_i\}, \ldots, \Omega \end{array} \right\}, \tag{4}$$

where each element is a proposition and $\emptyset$ is an empty set. In addition, each single set that contains only one element in FOD is called singleton.

Definition 4: A basic probability assignment (BPA) (also called mass function) is a mapping for elements in 2^Ω to the interval [0,1], formally defined by:

$$m : 2^\Omega \rightarrow [0, 1], \tag{5}$$

which satisfies the following conditions:

$$m(\emptyset) = 0, \quad \sum_{A \in \Omega} m(A) = 1, \tag{6}$$

where "A" symbolizes any subset of Ω, which is $A \subseteq \Omega$. If $A \neq \emptyset$, the BPA function $m(A)$ represents how strongly the evidence supports the hypothesis A. If $m(A) > 0$, the A in the frame of discernment is called a focal element and the set of all the focal elements is named a body of evidence (BOE).

Definition 5: A BPA m can also be represented by the belief function Bel or the plausibility function Pl, defined as follows:

$$Bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B), \quad Pl(A) = \sum_{B \cap A \neq \emptyset} m(B). \tag{7}$$

Definition 6: Two pieces of evidence in the frame of discernment Ω indicated as m_1 and m_2. A focal elements of m_1 is described as B and that of m_2 is presented as C. The Dempster's combination rule can be defined as follows:

$$\begin{aligned} m_{1,2}(A) &= m_1(B) \oplus m_2(C) \\ &= \frac{\sum_{B,C \in \Omega, B \cap C = A} m_1(B) \times m_2(C)}{1 - \sum_{B,C \in \Omega, B \cap C = \emptyset} m_1(B) \times m_2(C)} \end{aligned} \tag{8}$$

where a coefficient K is defined as follows:

$$K = \sum_{B,C \in \Omega, B \cap C = \emptyset} m_1(B) \times m_2(C), \tag{9}$$

sometimes, the K is defined as a conflict coefficient between two BOEs.

B. TRIANGULAR DISTRIBUTION

Fuzzy logic technique is commonly used in computer science such as software engineering [70], [71]. As a typical and simple tool, fuzzy triangular distribution is often used as an elementary example of a probability model [72]. Triangular distribution has been used in many fields, such as program evaluation [73] and review technique models [74], [75]. In the case of being absence of data, a triangular distribution is a promising alternative to some of the standard probability distributions [76].

Definition 7: Given the parameters: a = *minimum*, c = *most likely value*, b = *maximum*, the triangular distribution is a continuous probability distribution with a low limit a, a medium value c and the upper limit b. The probability density function of triangular distribution is defined as follows:

$$f(x|a, b, c) = \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)}, & \text{for } a \leq x \leq c, \\ \dfrac{2(b-x)}{(b-a)(b-c)}, & \text{for } c < x \leq b. \end{cases} \tag{10}$$

For $\forall x \in [a, b]$, there always exists $f(x) \in [0, 1]$.

IV. TDBPA FOR SUBJECTIVE ASSESSMENT MODELING

A. PROBLEM DESCRIPTION

In practical engineering, it is common that experts may have different assessments on the same object. The decision will be based on data fusion theory and method. How to model the conflict information or inconsistent opinion for the following data fusion process and subsequently the decision making is still an open issue. Taking the following case as an illustrative example. Example 1 gives a problem in fusing subjective assessment with hard partition based on D-S theory.

Example 1: Two experts give subjective assessments on the risk factor "severity of a failure effect (S)" of failure mode 1 in the framework of D-S theory, the belief structure is shown as follows.

- Expert 1: $m^1_{S1}(6) = 1$,
- Expert 2: $m^1_{S2}(7) = 1$.

According to the statement for criteria of rating in severity of a failure effect, "6" is equivalent to "Significant", which means "operation of system or product is continued and performance of system or product is degraded". The opinion of expert 1 can be represented symbolically as $S1(6, 100\%)$, the BPA is $m^1_{S1}(6) = 1$. Additionally, "7" equates with "Major", which means "operation of the system or product may be continued but performance of system or product is affected". The evaluation result of expert 2 can be represented as $S2(7, 100\%)$. The BPA is $m^1_{S2}(7) = 1$. According to Eq.(9), there is no intersection between the two given estimated value, which can be presented as $\sum_{S1 \cap S2 = \emptyset} m_1(6) \times m_2(7) = 1$, so these two BPAs are highly conflicting evidence and the denominator of the Eq.(8) is 0. In this case, data fusion with Dempster's combination rule is unavailable.

To address the aforementioned conflict information fusion, it is necessary to take the subjectivity of experts when they are

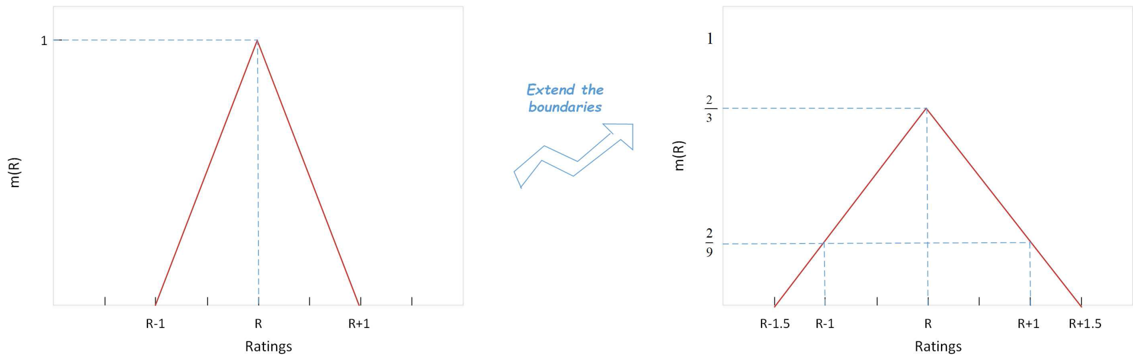

FIGURE 1. Generation of BPA based on the triangular distribution.

giving a precise (with integer) assessment value into consideration. When different assessment with precise values for the risk evaluation factors are given by different experts, we adopt the triangular distribution for constructing basic probability assignment of conflict assessment to make the boundaries become smooth. With fuzzy boundary, there will be no hard partition, consequently, we can solve the combination problem of conflicting evidence coming from integer assessment value. Generally speaking, there is rarely large difference in the estimated values of risk factor given by domain experts. In other words, the risk evaluation opinions of domain experts are usually in neighboring position [13], [20].

B. TDBPA

To smooth the conflict assessment for information fusion, the adjacent value of a risk level is taken into consideration. We construct the BPA by which the situation can be described based on the triangular distribution function.

Definition 8: Assume that $m(X)$ expresses the belief given by an expert on the proposition X, and R denotes the level of failure risk ranging from 1 to 10. If X is a single potential rating of the ith risk factor for the nth failure mode, then the triangular distribution-based BPA (TDBPA) can be constructed as follows:

$$m(R) = f(X|(R-1.5), R, (R+1.5)) = \left\{ \begin{array}{ll} \dfrac{2(2X-2R+3)}{9}, & for\ (R-1.5) \leq X \leq R, \\ \dfrac{2(2R-2X+3)}{9}, & for R < X \leq (R+1.5), \end{array} \right\} \tag{11}$$

where $m(\emptyset) = 0$. The construction of TDBPA is illustrated in Fig. 1. The range of R could be simplified from $(R-1.5)$ to $(R+1.5)$ and the discernment frame is $\left[minX|_{X\subseteq\Omega_i^n} - 1, maxX|_{X\subseteq\Omega_i^n} + 1\right]$, where $i = O, S, D$; $n = 1, 2, 3 \ldots, N$

There are basically three reasons about why the range of R is simplified as $[R-1.5, R+1.5]$ instead of $[R-1, R+1]$: (1) In triangular distribution, as shown in Fig. 1, the minimum and maximum values create sharp boundaries at the edge, which is not allowed, so we enlarge the range of value. (2) The probability density of the central rating far outweighs the ones on both sides. (3) Sometimes, the sum of the probability densities of the three adjacent ratings is not exactly equal to 1, as a normalization strategy, we transform the values of probability density at R and $(R+1)$ directly to TDBPA and the belief value at $(R-1)$ point is defined as follows:

$$m(X-1) = 1 - m(X) - m(X+1). \tag{12}$$

C. APPLY TDBPA IN D-S THEORY

The process of applying TDBPA in framework of D-S theory is depicted in Fig. 2. First, develop a deep understanding for target system or service in order to identify risk items that may occur failure, i.e, $item_1, item_2, \ldots, item_n$. Second, assemble a team of domain experts, i.e., $Expert_{l|l=1,\ldots,L}$, to provide assessments on these risk items. These assessments that are denoted as BPA and some of them that are precise integers, which may become conflict assessments. In this case, we adopt the triangular distribution to construct TDBPA so that data fusion with Dempster combination rule is available. Eventually, fused TDBPAs are obtained for each risk item.

D. NUMERICAL EXAMPLES IN FAILURE MODE ANALYSIS

The following numerical examples in failure analysis are used to illustrate the calculation process of the proposed TDBPA method. To simplify the calculation process, in the following example, according to Eq. (6), Eq.(11), Eq.(12) and Fig. 1, the construction of TDBPA can be reduced to a much simpler form, shown as follows:

$$m(X) = 0.67, \quad m(X-1) = 0.11, \ m(X+1) = 0.22. \tag{13}$$

where $m(\emptyset) = 0$, X is the precise proposition that is equal to numeric R. The specific values can be calculated by Eq.(11), Eq.(12).

Example 2: Two experts give their assessments on the risk factor "severity of a failure effect(S)" of a failure mode 1. Based on prior knowledge in FMEA, "6" is equivalent to "Significant", which means "operation of system or product is continued and performance of system or product is degraded", and "7" equates with "Major", which means

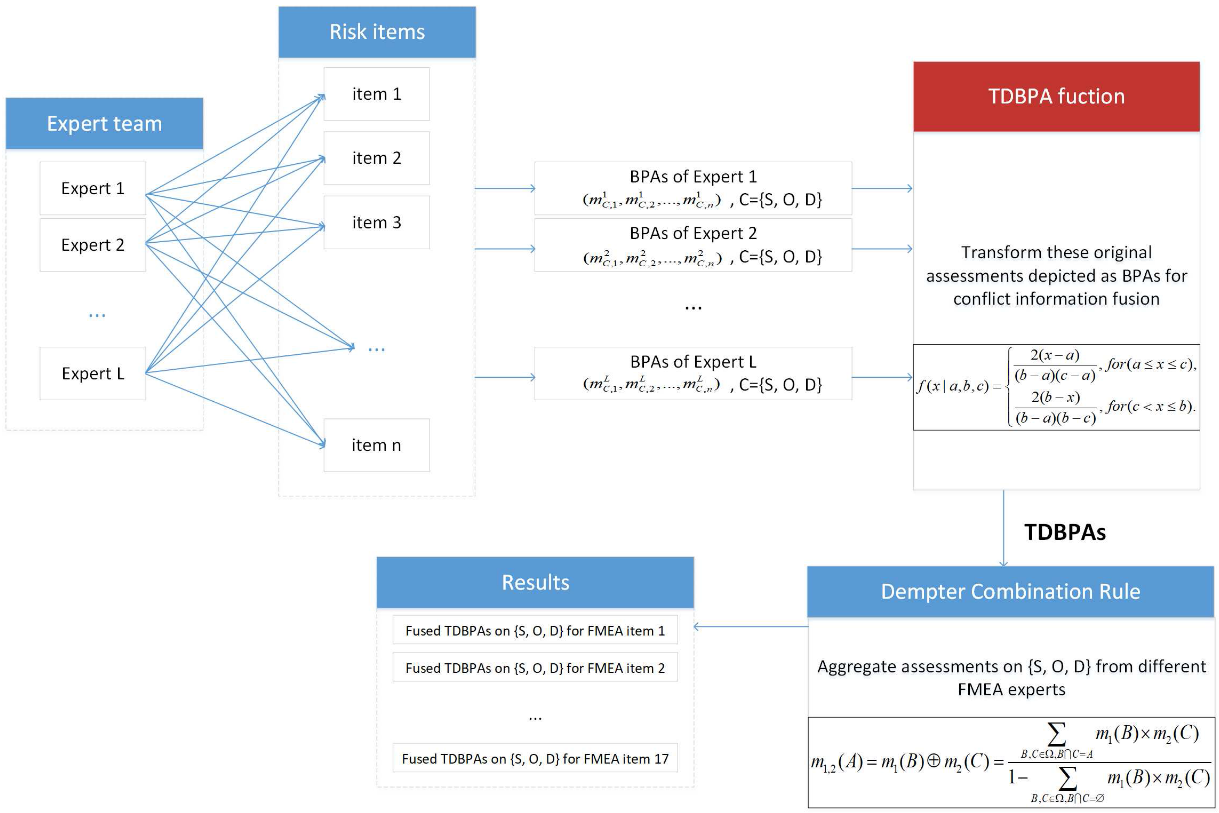

FIGURE 2. **Applying the TDBPA method in framework of D-S theory for multiple risk items.**

"operation of the system or product may be continued but performance of system or product is affected". In this case, the FOD of assessments can be constructed as

$$\Omega_S^1 = (5, 6, 7, 8),$$

and the TDBPA function for each expert can be attained by using Eq.(13):

- Expert 1:

$$m_{S1}^1(5) = 0.11, \quad m_{S1}^1(6) = 0.67, \; m_{S1}^1(7) = 0.22,$$

- Expert 2:

$$m_{S2}^1(6) = 0.11, \quad m_{S2}^1(7) = 0.67, \; m_{S2}^1(8) = 0.22.$$

The combined BPA of Expert 1 and Expert 2 can be obtained by using Eq.(8) and the result is shown as follows:

$$m_{S,12}^1(5) = m_{S1}^1 \oplus m_{S2}^1 = \frac{\sum_{X\cap Y=5,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)} = 0,$$

$$m_{S,12}^1(6) = m_{S1}^1 \oplus m_{S2}^1 = \frac{\sum_{X\cap Y=6,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)} = 0.33,$$

$$m_{S,12}^1(7) = m_{S1}^1 \oplus m_{S2}^1 = \frac{\sum_{X\cap Y=7,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)} = 0.67,$$

$$m_{S,12}^1(8) = m_{S1}^1 \oplus m_{S2}^1 = \frac{\sum_{X\cap Y=8,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega_S^1} m_{S1}^1(X)\times m_{S2}^1(Y)} = 0.$$

The fusion result shows that a higher belief will assigned to the risk level '7', which is helpful for failure risk prevention. The numerical example also verifies the validity of the proposed method.

Example 3: The evaluation results of two experts on the risk factor "detection of a failure (D)" of the failure mode 1 can be expressed as follows. According to [20], "6" is equivalent to "Low", which means "the possibility of detecting the potential occurring of failure mode is low"; "8" means "Remote", which means "The possibility of detecting the potential occurring of failure mode is remote". The results of symbolization are $D1(6, 100\%)$, $D2(8, 100\%)$. In this case, the FOD of assessments can be constructed as

$$\Omega_D^1 = (5, 6, 7, 8, 9),$$

consequently, the TDBPA function for each expert can be obtained by using Eq.(13):

- Expert 1:

$$m_{D2}^1(5) = 0.11, \quad m_{D2}^1(6) = 0.67, \; m_{D2}^1(7) = 0.22$$

- Expert 2:

$$m^1_{D2}(7) = 0.11, \quad m^1_{D2}(8) = 0.67, \; m^1_{D2}(9) = 0.22$$

The combined BPA function of Expert 1 and Expert 2 can be obtained by using Eq.(8), the result is shown as follows:

$$\begin{aligned} m^1_{D,12}(5) &= m^1_{D1} \oplus m^1_{D2} \\ &= \frac{\sum_{X\cap Y=5,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)} = 0, \\ m^1_{D,12}(6) &= m^1_{D1} \oplus m^1_{D2} \\ &= \frac{\sum_{X\cap Y=6,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)} = 0, \\ m^1_{D,12}(7) &= m^1_{D1} \oplus m^1_{D2} \\ &= \frac{\sum_{X\cap Y=7,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)} = 1, \\ m^1_{D,12}(8) &= m^1_{D1} \oplus m^1_{D2} \\ &= \frac{\sum_{X\cap Y=8,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_D} m^1_{D1}(X)\times m^1_{D2}(Y)} = 0. \end{aligned}$$

As we can see from Example 2 and 3, the TDBPA method can solve the conflict brought by different integer values in the risk evaluation factor given by different experts. Utilizing the triangular distribution, the constructed TDBPAs can be fused by the Dempster's combination rule. The fusion result is consistent with reality.

Example 4: Three experts assess on the risk factor ''severity of a failure (S)'' for failure mode 1 with $S(7, 100\%)$, which means that the three experts estimated the corresponding failure risk as the ''Major'' level and ''operation of system or product is continued and performance of system or product is degraded''. The FOD of assessments can be constructed as

$$\Omega^1_S = (6, 7, 8),$$

and the TDBPA function for the expert can be calculated by utilizing Eq.(13):

- Expert k:

$$m^1_{Sk}(6) = 0.11, \quad m^1_{Sk}(7) = 0.67, \; m^1_{Sk}(8) = 0.22,$$

where $k = 1, 2, 3$. The combined BPA of Expert 1 and Expert 2 can be obtained by using Eq.(8) and the result is shown as follows:

$$\begin{aligned} m^1_{S,12}(6) &= m^1_{S1} \oplus m^1_{S2} \\ &= \frac{\sum_{X\cap Y=6,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)} = 0.024, \\ m^1_{S,12}(7) &= m^1_{S1} \oplus m^1_{S2} \\ &= \frac{\sum_{X\cap Y=7,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)} = 0.88, \\ m^1_{S,12}(8) &= m^1_{S1} \oplus m^1_{S2} \\ &= \frac{\sum_{X\cap Y=8,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)} = 0.096. \end{aligned}$$

Subsequently, the final fusing result can be obtained by involving in the assessment from Expert 3, the result is shown as follows:

$$\begin{aligned} m^1_{S,123}(6) &= m^1_{S,12} \oplus m^1_{S3} \\ &= \frac{\sum_{X\cap Y=6,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)} \\ &= 0.0043, \end{aligned}$$

$$\begin{aligned} m^1_{S,123}(7) &= m^1_{S,12} \oplus m^1_{S3} \\ &= \frac{\sum_{X\cap Y=7,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)} \\ &= 0.9600, \end{aligned}$$

$$\begin{aligned} m^1_{S,123}(8) &= m^1_{S,12} \oplus m^1_{S3} \\ &= \frac{\sum_{X\cap Y=8,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)} \\ &= 0.0357. \end{aligned}$$

According to the fusion result, if two or more experts give the same estimated values, the proposed method can contribute to a convergency result on the belief incorporation with Dempster rule of combination, which is helpful for decision-making in practical engineering.

Example 5: Three experts give their opinions on the risk factor ''severity of a failure (S)'' of a failure mode 1. According to the statement for criteria of rating in severity of a failure effect, ''6'' is equivalent to ''Significant''; ''7'' equates with ''Major''; ''8'' means ''Extreme''. Then, the evaluation results can be symbolized as $S1((6, 7), 100\%)$, $S2(8, 100\%)$, $S3(7, 30\%; 8, 70\%)$. The FOD of assessments can be constructed as

$$\Omega^1_S = (6, 7, 8, 9),$$

and the TDBPA function for each expert can be obtained by using Eq.(13):

- Expert 1:

$$m^1_{S1}(6, 7) = 1,$$

- Expert 2:

$$m^1_{S2}(7) = 0.11, \quad m^1_{S2}(8) = 0.67, \; m^1_{S2}(9) = 0.22,$$

- Expert 3:

$$m^1_{S3}(7) = 0.3, \quad m^1_{S3}(8) = 0.7.$$

The fusion result for Expert 1 and 2 can be attained by using Eq.(8), shown as follows:

$$m^1_{S,12}(6) = m^1_{S1} \oplus m^1_{S2} = \frac{\sum_{X\cap Y=6,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)} = 0,$$

$$m^1_{S,12}(7) = m^1_{S1} \oplus m^1_{S2} = \frac{\sum_{X\cap Y=7,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)} = 1,$$

$$m^1_{S,12}(8) = m^1_{S1} \oplus m^1_{S2} = \frac{\sum_{X\cap Y=8,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)} = 0,$$

$$m^1_{S,12}(9) = m^1_{S1} \oplus m^1_{S2} = \frac{\sum_{X\cap Y=9,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S1}(X)\times m^1_{S2}(Y)} = 0.$$

Subsequently, the final fusion result can be obtained by involving the assessment from Expert 3, the calculation result is shown as follows:

$$m^1_{S,123}(7) = m^1_{S,12} \oplus m^1_{S3} = \frac{\sum_{X\cap Y=7,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)} = 1,$$

$$m^1_{S,123}(8) = m^1_{S,12} \oplus m^1_{S3} = \frac{\sum_{X\cap Y=8,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Omega^1_S} m^1_{S,12}(X)\times m^1_{S3}(Y)} = 0.$$

Example 2 to 5 shows the validity and practicability of TDBPA. The conflict assessment from different experts can be fused with Dempster rule of combination after preprocessing with TDBPA method.

E. DISCUSSION

The triangular distribution is practical in statistic and the most likely outcome can often be estimated without the prior data such as the mean and standard deviation. Compared with the probability values generated by the normal distribution function, the result produced by triangular distribution is asymmetric, because it assigns a higher probability to the larger integer rating, which can be regarded as a pessimistic strategy for a strict prevention on failure risks. Correspondingly, the result gained by normal distribution is symmetric, which is a neutral strategy.

Three desirable features of TDBPA can be concluded as follows. Firstly, adjacent values of the assessed precise integer are utilized to smooth the conflict assessments for information fusion. Secondly, TDBPA only relates to corresponding original BPA values. Essentially, the triangular distribution in the proposed method is constructed with less prior data because it is determined by the assessment values. Last but not least, the calculation of triangular distribution is simple for practical application in engineering in comparison with other distribution function.

V. AN IMPROVED FMEA METHOD BASED ON TDBPA

A. TDBPA-BASED FMEA METHOD

Many theories have been applied to FMEA to make it more effective in failure analysis of practical engineering. However, how to address the conflict assessments from different FMEA experts is still an open issue. To manage the conflict among different risk levels assessed by different experts, we propose an improved FMEA method based on TDBPA in the framework of D-S theory. The TDBPA is adopted to model the difference and uncertainty of evaluation information received from multiple experts, subsequently, the Dempster rule of combination is used to combine the TDBPA.

In the proposed method, a simplified FOD is applied according to the practical application. In addition, MVRPN is used to determine the risk priority order of multiple failure modes. The flowchart in Fig.2 shows the novel approach for the evidential FMEA process incorporating TDBPA-based conflict management method. The main process for carrying out this novel FMEA method can be divided into several steps which are briefly explained as follows:

- **Step 1.** To utilize FMEA, a specific methodology, estimate a system or service effectively, it is essential to investigate target system or service for ways of failure occurrence. Then potential failure modes can be determined as soon as possible.
- **Step 2.** Determine the effect of each failure according to experts' personal experience or historical data. Note that the amount of effect depends.
- **Step 3.** Recognize operational and environmental stresses that contribute to the failure of some components of target system or service and categorise them.
- **Step 4.** After advance preparation, an expert team should be built to estimate the possible failure items. In addition, express these estimations as BPA for further processing.
- **Step 5.** Transform original BPAs to TDBPAs for conflict information fusion. In the first place, simplify the FOD for failure risk. Generally, assume that there are L experts $(E_1, \ldots, E_L)$ in a FMEA team, and N failure modes $(F_1, \ldots, F_N)$ are considered. The FOD Ω^n_i of the n^{th} failure mode with respect to the i^{th} risk factor can be expressed as:

$$\Omega^n_i = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10), \quad i = O, S, D;\ n = 1, 2, 3, \ldots, N. \tag{14}$$

While in practical application, the FOD can be simplified as:

$$\Omega^n_i = (minX|_{X\subseteq\Omega^n_i}, minX|_{X\subseteq\Omega^n_i} + 1, \ldots, maxX|_{X\subseteq\Omega^n_i}) \tag{15}$$

where $i = O, S, D$, $n = 1, 2, \ldots, N$. $minX|_{X\subseteq\Omega^n_i}$ and $maxX|_{X\subseteq\Omega^n_i}$ are the minimum and maximum rank of

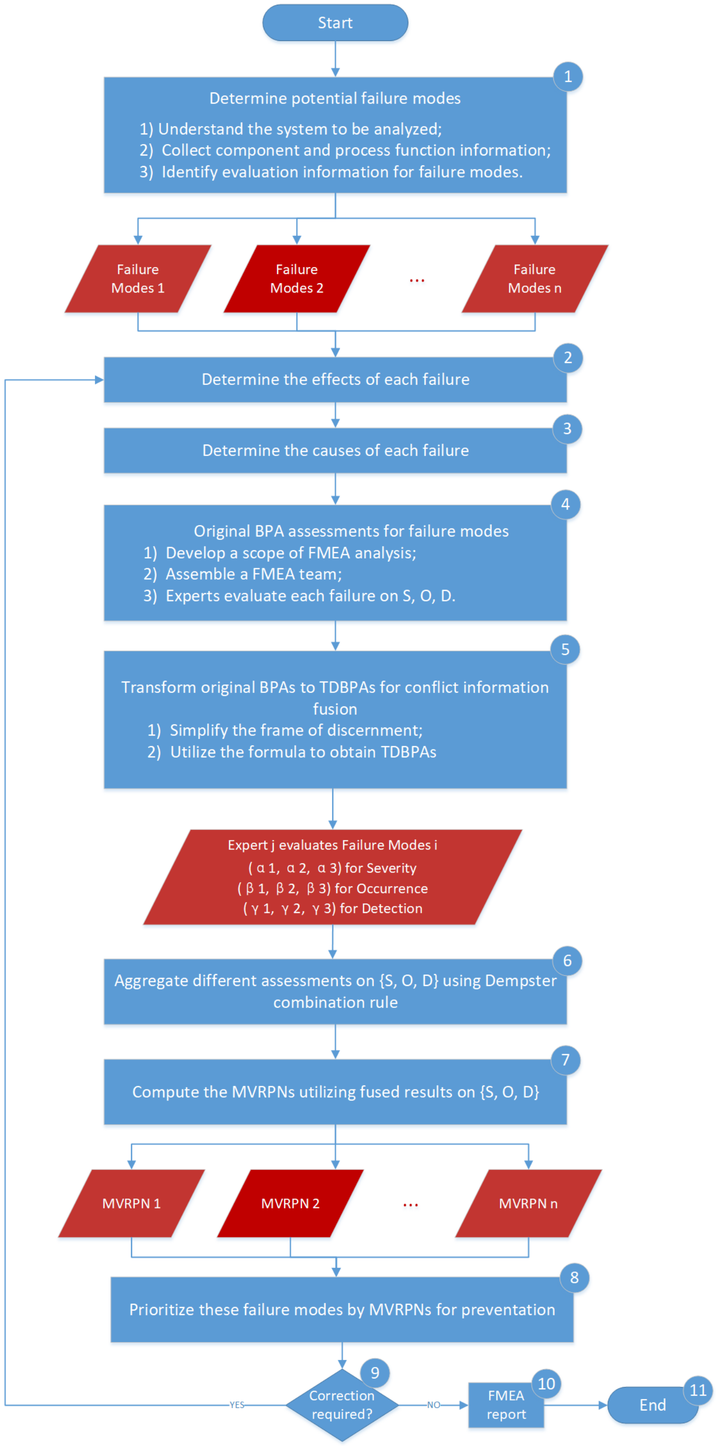

FIGURE 3. The flowchart of the proposed TDBPA based-FMEA method.

the n^{th} failure mode with respect to the i^{th} risk factor (O, S, D) from the evaluation of the L experts, respectively. After that, process some assessments denoted as BPA that are precise integers with Eq. 11 and Eq. 12 to generate respective TDBPAs.

- **Step 6.** Aggregate different assessments from experts on S, O, D with Dempster combination rule.
- **Step 7.** Compute the MVRPN proposed in [20] with fused results on S, O, D.
- **Step 8.** Prioritize failure modes by MVRPNs. Actions on FMEA items with the priorities should be taken for risk prevention.

B. APPLICATION AND EXPERIMENTAL RESULT

Rotor blades, classified into two categories: compressor rotor blades and turbo rotor blades, are the major components of an aircraft turbine. They work for energy conversion. Since they are thin-form and they rotate in a high speed, rotor

TABLE 2. Original evaluation information on the three aspects of 17 failure modes.

Item	Rating of risk factor								
	Expert 1			Expert 2			Expert 3		
	O	S	D	O	S	D	O	S	D
1	3:40% 4:60%	7	2	3:90% 4:10%	7	2	3:80% 4:20%	7	2
2	2	8	4	2	8:70% 9:30%	4	2	8	4
3	1	10	3	1	10	3	1	10	3
4	1	6:80% 7:20%	3	1	6	2:30% 3:70%	1	6	3
5	1	3	1:50% 2:50%	1	3	1:70% 2:30%	1	2:40% 3:60%	1
6	2	6	5	2	6	5	2	6	5
7	1	7	3	1	7	3	1	7	3
8	3	5:60% 6:40%	1	3	5:80% 6:20%	1	3	5:80% 7:20%	1
9	1:10% 2:90%	m(9)=0.4 m(10)=0.6	4	1:25% 2:75%	9:10% 10:90%	4	1:20% 2:80%	9:10% 10:90%	4
10	1	10	6	1	10	6	1	10	6
11	1	10	5	1	10	5	1	10	5
12	1	10	5:40% 6:60%	1	10	4:20% 5:80%	1	10	5:30% 6:70%
13	1	10	4:20% 5:80%	1	10	5	1	10	5
14	1	10	6	1	10	6:80% 7:20%	1	10	6
15	2	6:5% 7:95%	3	2	7	3	2	7	3:70% 4:30%
16	1:10% 2:90%	4	3	1:25% 2:75%	4	3	1:80% 2:20%	4	2:20% 3:80%
17	2	5:90% 6:10%	3	2	5:90% 6:10%	3	2	5:60% 6:40%	3

blades are regarded as one of the components of the highest failure rates in aircraft turbines. A single failure may be fatal. Therefore, in order to ensure their operating status is under control and prevent failures, risk analysis is prerequisite in their design [20].

To demonstrate the effectiveness of improved FMEA method, the application analysis for the rotor blades of an aircraft turbine is adopted. In the case study, there are nine potential failure modes in the turbo rotor blades and eight failure modes of the compressor rotor blades [20].

The original evaluated values on the three aspects of 17 failure modes are given by three experts in Table 2.

Take the first FMEA item (denoted as item1) as example, of which the calculation process will be displayed in detail. From the original data shown in Table 2, three experts gave precise integer values for S, D. In order to solve the combination problem of conflicting evidence coming from integer assessment value, we process them with Eq.11 as follows.

Expert 1, Expert 2, Expert3 all evaluate that the risk level of item1 in aspect of two risk factors, S, D, are 7 and 2 with 100% confirmation subjectively. According to Eq.11, assume R correspondingly equals to 7 and 2, TDBPAs can be constructed as:

$$f(X|5.5, 7, 8.5) = \begin{cases} \dfrac{2(2X-11)}{9}, & 5.5 \le X \le 7, \\ \dfrac{2(17-2X)}{9}, & 7 < X \le 8.5. \end{cases} \quad (16)$$

$$f(X|0.5, 2, 3.5) = \begin{cases} \dfrac{2(2X-1)}{9}, & 0.5 \le X \le 2, \\ \dfrac{2(7-2X)}{9}, & 2 < X \le 3.5. \end{cases} \quad (17)$$

Then, we transform the values of probability density at 7, 8 and 2, 3 directly to TDBPA and attain the belief value at 6 and 1 by Eq.12. Therefore, the TDBPA generated of three experts for item1 are

$$m^1_{S1}(6) = 0.11, \quad m^1_{S1}(7) = 0.67, \; m^1_{S1}(8) = 0.22,$$
$$m^1_{D1}(1) = 0.11, \quad m^1_{D1}(2) = 0.67, \; m^1_{D1}(3) = 0.22.$$

Similarly, with Eq.11 and Eq.12, the TDBPAs for other experts and FMEA items can be calculated respetively. The management of conflict information constructed as TDBPAs is shown in Table 4. Then, aggregate the TDBPAs from *Expert*1, *Expert* 2, *Expert*3 on S, O, D with Dempster combination rule. First, fusion results with Dempster combination rule for *Expert*1, *Expert*2 in item1 according to Eq. 8 are denoted as:

$$m^1_{O,12}(3) = \frac{0.4\times0.9}{0.4\times0.9+0.6\times0.1} = 0.8571,$$
$$m^1_{O,12}(4) = \frac{0.6\times0.1}{0.4\times0.9+0.6\times0.1} = 0.1429.$$
$$m^1_{S,12}(6) = \frac{0.11\times0.11}{0.11\times0.11+0.67\times0.67+0.22\times0.22} = 0.0237,$$
$$m^1_{S,12}(7) = \frac{0.67\times0.67}{0.11\times0.11+0.67\times0.67+0.22\times0.22} = 0.8812,$$

TABLE 3. Fused TDBPAs for three risk factors in 17 FMEA items.

Component	Compressor rotor blades							
Failure mode items	1	2	3	4	5	6	7	8
O	m(3)=0.96 m(4)=0.04	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(1)=0.9781 m(2)=0.0219	m(1)=0.9781 m(2)=0.0219	m(1)=0.9781 m(2)=0.0219	m(1)=0.0044 m(2)=0.9600 m(3)=0.0356	m(1)=0.9781 m(2)=0.0219	m(1)=0.0044 m(2)=0.96 m(3)=0.0356
S	m(6)=0.0044 m(7)=0.96 m(8)=0.0356	m(7)=0 m(8)=0.955 m(9)=0.045	m(9)=0.0019 m(10)=0.9981	m(5)=0.0000 m(6)=0.9730 m(7)=0.0270	m(2)=0.0182 m(3)=0.9818 m(4)=0.0000	m(1)=0.0044 m(2)=0.9600 m(3)=0.0356	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(5)=1.0000 m(6)=0.0000 m(7)=0.0000
D	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(2)=0.0044 m(3)=0.96 m(4)=0.0356	m(2)=0.0118 m(3)=0.9882 m(4)=0.0000	m(1)=0.8922 m(2)=0.1078	m(1)=0.0044 m(2)=0.9600 m(3)=0.0356	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(1)=0.9781 m(2)=0.0219

Component	Turbo rotor blades								
Failure mode items	9	10	11	12	13	14	15	16	17
O	m(1)=0.0092 m(2)=0.9908	m(1)=0.9781 m(2)=0.0219	m(1)=0.9781 m(2)=0.0219	m(1)=0.9781 m(2)=0.0219	m(1)=0.9781 m(2)=0.0219	m(1)=0.9781 m(2)=0.0219	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(1)=0.0092 m(2)=0.9908	m(1)=0.0044 m(2)=0.96 m(3)=0.0356
S	m(9)=0.0082 m(10)=0.9918	m(9)=0.0019 m(10)=0.9981	m(9)=0.0019 m(10)=0.9981	m(9)=0.0019 m(10)=0.9981	m(9)=0.0019 m(10)=0.9981	m(9)=0.0019 m(10)=0.9981	m(6)=0.0015 m(7)=0.9985 m(8)=0.0000	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(5)=0.9918 m(6)=0.0082
D	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(1)=0.0044 m(2)=0.96 m(3)=0.0356	m(4)=0.0000 m(5)=1.0000 m(6)=0.0000	m(4)=0.0069 m(5)=0.9931 m(6)=0.0000	m(5)=0.0000 m(6)=0.9730 m(7)=0.0270	m(2)=0.0000 m(3)=0.9545 m(4)=0.0455	m(2)=0.0069 m(3)=0.9931 m(4)=0.0000	m(1)=0.0044 m(2)=0.96 m(3)=0.0356

$$m^1_{S,12}(8)=\frac{0.22\times0.22}{0.11\times0.11+0.67\times0.67+0.22\times0.22}=0.095.$$
$$m^1_{D,12}(1)=\frac{0.11\times0.11}{0.11\times0.11+0.67\times0.67+0.22\times0.22}=0.0237,$$
$$m^1_{D,12}(2)=\frac{0.67\times0.67}{0.11\times0.11+0.67\times0.67+0.22\times0.22}=0.8812,$$
$$m^1_{D,12}(3)=\frac{0.22\times0.22}{0.11\times0.11+0.67\times0.67+0.22\times0.22}=0.095.$$

Then, fusion results with Dempster combination rule for *Expert*1, *Expert* 2, *Expert*3 in item1 according to Eq. 8 are as follows:

$$m^1_{O,123}(3)=\frac{0.8571\times0.8}{0.8571\times0.8+0.1429\times0.2}=0.96,$$
$$m^1_{O,123}(4)=\frac{0.1429\times0.2}{0.8571\times0.8+0.1429\times0.2}=0.04.$$
$$m^1_{S,123}(6)=\frac{0.0237\times0.11}{0.0237\times0.11+0.8812\times0.67+0.095\times0.22}=0.0042,$$
$$m^1_{S,123}(7)=\frac{0.8812\times0.67}{0.0237\times0.11+0.8812\times0.67+0.095\times0.22}=0.9617,$$
$$m^1_{S,123}(8)=\frac{0.095\times0.22}{0.0237\times0.11+0.8812\times0.67+0.095\times0.22}=0.034.$$
$$m^1_{D,123}(1)=\frac{0.0237\times0.11}{0.0237\times0.11+0.8812\times0.67+0.095\times0.22}=0.0042,$$
$$m^1_{D,123}(2)=\frac{0.8812\times0.67}{0.0237\times0.11+0.8812\times0.67+0.095\times0.22}=0.9617,$$
$$m^1_{D,123}(3)=\frac{0.095\times0.22}{0.0237\times0.11+0.8812\times0.67+0.095\times0.22}=0.034.$$

And other fused TDBPAs for three risk factors in 17 FMEA items are list in Table 3.

Next, the rating values of integrated FMEA item assessments for each risk factor, i.e.S_i, O_i, D_i, can be attained as follows:

$$O_i = \sum_{R=minX}^{maxX} Rm(S_i),$$
$$S_i = \sum_{R=minX}^{maxX} Rm(O_i),$$
$$D_i = \sum_{R=minX}^{maxX} Rm(D_i) \tag{18}$$

where $minX$ and $maxX$ are defined in simplified FOD. Besides, R is the relative rating value as defined in Eq. 15 i.e.($minX, R, \ldots, maxX$). Compute the rating values of S, O, D and utilize Eq. 2 to attain MVRPN for item1:

$$\begin{aligned}
O_1 &= 3 \times 0.96 + 4 \times 0.04 = 3.04 \\
S_1 &= 6 \times 0.0042 + 7 \times 0.9617 + 8 \times 0.034 \\
&= 7.0291 \\
D_1 &= 1 \times 0.0042 + 2 \times 0.9617 + 3 \times 0.034 \\
&= 2.0296 \\
MVRPN_1 &= 3.04 \times 7.0291 \times 2.0296 = 43.42
\end{aligned}$$

Similarly, the MVRPN values of other FMEA items are listed in Table 5.

In [20], a method, including the modified D-S and a simplified discernment frame, is used to deal with the risk priority evaluation of the failure modes of rotor blades of an aircraft turbine based on multiple sources of evaluation information. Moreover, Su *et al.* present an improved method to aggregate different risk evaluations given by multiple experts as a modification of Yang *et al.*'s method [20] in [13]. Note that, three papers focus on overcoming the shortcomings of traditional FMEA in piratical engineering and use the same data set to

TABLE 4. The constructed TDBPA basing on the evaluation information of 17 failure modes in the rotor blades of an aircraft turbine.

Item	Rating of risk factor								
	Expert 1			Expert 2			Expert 3		
	O	S	D	O	S	D	O	S	D
1	m(3)=0.4 m(4)=0.6	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(3)=0.9 m(4)=0.1	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(3)=0.8 m(4)=0.2	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22
2	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(7)=0.11 m(8)=0.67 m(9)=0.22	m(3)=0.11 m(4)=0.67 m(5)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(8)=0.7 m(9)=0.3	m(3)=0.11 m(4)=0.67 m(5)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(7)=0.11 m(8)=0.67 m(9)=0.22	m(3)=0.11 m(4)=0.67 m(5)=0.22
3	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(2)=0.11 m(3)=0.67 m(4)=0.22
4	m(1)=0.78 m(2)=0.22	m(6)=0.8 m(7)=0.2	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.78 m(2)=0.22	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(2)=0.3 m(3)=0.7	m(1)=0.78 m(2)=0.22	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22
5	m(1)=0.78 m(2)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.5 m(2)=0.5	m(1)=0.78 m(2)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.7 m(2)=0.3	m(1)=0.78 m(2)=0.22	m(2)=0.4 m(3)=0.6	m(1)=0.78 m(2)=0.22
6	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(4)=0.11 m(5)=0.67 m(6)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(4)=0.11 m(5)=0.67 m(6)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(4)=0.11 m(5)=0.67 m(6)=0.22
7	m(1)=0.78 m(2)=0.22	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.78 m(2)=0.22	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.78 m(2)=0.22	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22
8	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(5)=0.6 m(6)=0.4	m(1)=0.78 m(2)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(5)=0.8 m(6)=0.2	m(1)=0.78 m(2)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(5)=0.8 m(7)=0.2	m(1)=0.78 m(2)=0.22
9	m(1)=0.1 m(2)=0.9	m(9)=0.4 m(10)=0.6	m(3)=0.11 m(4)=0.67 m(5)=0.22	m(1)=0.25 m(2)=0.75	m(9)=0.1 m(10)=0.9	m(3)=0.11 m(4)=0.67 m(5)=0.22	m(1)=0.2 m(2)=0.8	m(9)=0.1 m(10)=0.9	m(3)=0.11 m(4)=0.67 m(5)=0.22
10	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(5)=0.11 m(6)=0.67 m(7)=0.22
11	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(4)=0.11 m(5)=0.67 m(6)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(4)=0.11 m(5)=0.67 m(6)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(4)=0.11 m(5)=0.67 m(6)=0.22
12	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(5)=0.4 m(6)=0.6	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(4)=0.2 m(5)=0.8	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(5)=0.3 m(6)=0.7
13	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(4)=0.2 m(5)=0.8	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(4)=0.11 m(5)=0.67 m(6)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(4)=0.11 m(5)=0.67 m(6)=0.22
14	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(5)=0.11 m(6)=0.67 m(7)=0.22	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(6)=0.8 m(7)=0.2	m(1)=0.78 m(2)=0.22	m(9)=0.11 m(10)=0.89	m(5)=0.11 m(6)=0.67 m(7)=0.22
15	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(6)=0.05 m(7)=0.95	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(6)=0.11 m(7)=0.67 m(8)=0.22	m(3)=0.7 m(4)=0.3
16	m(1)=0.1 m(2)=0.9	m(3)=0.11 m(4)=0.67 m(5)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.25 m(2)=0.75	m(3)=0.11 m(4)=0.67 m(5)=0.22	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.2 m(2)=0.8	m(3)=0.11 m(4)=0.67 m(5)=0.22	m(2)=0.2 m(3)=0.8
17	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(5)=0.9 m(6)=0.1	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(5)=0.9 m(6)=0.1	m(2)=0.11 m(3)=0.67 m(4)=0.22	m(1)=0.11 m(2)=0.67 m(3)=0.22	m(5)=0.6 m(6)=0.4	m(2)=0.11 m(3)=0.67 m(4)=0.22

verify the efficiency of proposed method. Thus, The RPN values obtained by TDBPA-based FMEA method as well as the methods in [13], [20] are presented in Fig.3.

Observing the RPN values of the TDBPA-based FMEA method in Table 5, failure mode 2 gains the largest mean value of RPN among the failure modes of compressor rotor blades and failure mode 5 gains the smallest value. The larger the RPN mean value is, the higher the corresponding risk priority should be. So, analyzing the failure modes of compressor, the priorities of failure modes are sorted from high to low, shown as follows: failure mode 2 $\succ$ failure mode 6 $\succ$ failure mode 1 $\succ$ failure mode 3 $\succ$ failure mode 7 $\succ$ failure mode 4 $\succ$ failure mode 8 $\succ$ failure mode 5, where $\succ$ means a higher priority. Similarly, among the failure modes for the engine

TABLE 5. MVRPN of 17 failure modes.

Component	Methods	Compressor rotor blades								Turbo rotor blades								
Failure mode		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
MVRPN	Yang et al.'s method	42.56	64	30	18	4.17	60	21	15	78.92	60	50	50	60	60	42	23.88	50.09
	Su et al.'s method	42.56	64	30	17.97	3.14	60	21	15	79.57	60	50	50	50	60.04	42.09	23.86	30.05
	TDBPA-based FMEA	43.42	65.92	30.91	18.38	3.38	61.65	21.74	15.46	80.19	61.51	51.31	50.99	50.92	61.46	43.29	24.02	30.85

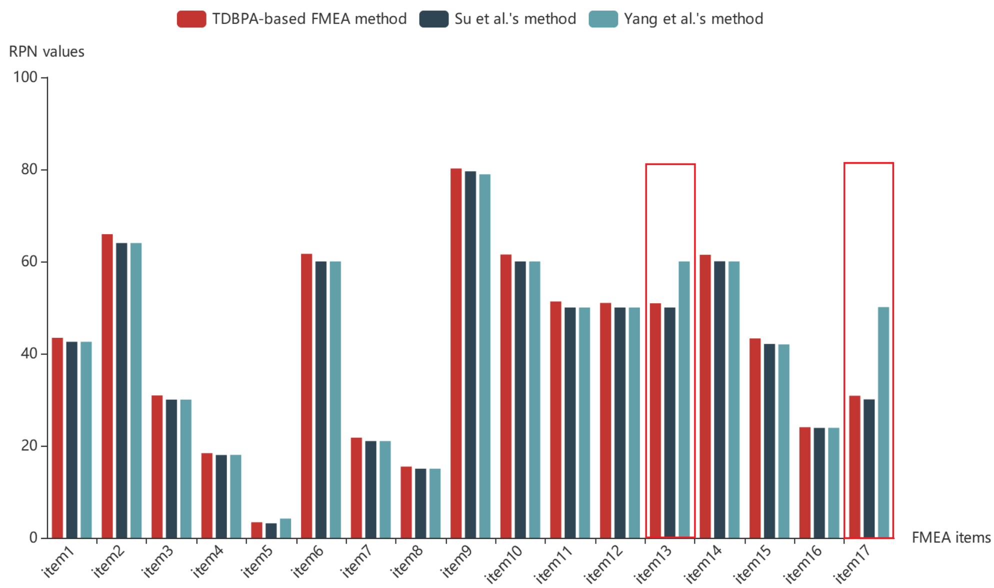

FIGURE 4. The RPN values of failure modes based on the TDBPA-based RPN and other methods.

rotor blades, the priorities of failure modes are sorted as follows: failure mode 9 $\succ$ failure mode 14 $\succ$ failure mode 10 $\succ$ failure mode 11 $\succ$ failure mode 12 $\succ$ failure mode 13 $\succ$ failure mode 15 $\succ$ failure mode 17 $\succ$ failure mode 16.

From the Fig. 3 basing on Table 5, we can see that the shortcomings of repetitive RPN values in Yang *et al.*'s method and Su *et al.*'s method have been overcome with the TDBPA-based FMEA method. The three RPN values in 13th and 17th items of FMEA, marked with red boxes, exist an unignorable difference. The RPN vaules attained by Yang *et al.*'s method are higher than the other two, which are incorrect reported according to [13]. While the proposed method has a similar tendency with Su *et al.*'s method, which shows the validity and practicability of the proposed method. RPN values are used for ranking of failure modes. However6, the repetitive RPN value is one of the key shortcomings in classical FMEA method because it assigns a repeated priority to corresponding failure modes rather than a distinguishable ranking result. In addition, each RPN value calculated by the proposed method is larger in some degree than that obtained by Su *et al.*'s method, which shows that the proposed method is a pessimistic strategy in comparison with Su *et al.*'s method while Su *et al.*'s method seems to be a neutral strategy.

VI. CONCLUSION

In this paper, in the framework of D-S theory, triangular distribution is adopted to construct the BPA for conflict management of risk levels with different precise integer values. The TDBPA method can smooth the conflict assessment for information fusion by taking into consideration of the adjacent value of a risk level assessed by an expert. Therefore, it can model the assessed data in the form of belief function for further information fusion by using Dempster rule of combination. The improved FMEA incorporating TDBPA method is then validated to be efficient in the blades of an aircraft turbine. Meanwhile, three desirable features of presented FMEA approach are worth noticing. Firstly, the overall ranking result of failure modes based on the proposed method is consistent with other methods. Furthermore, the experiment result shows a higher value of RPN value in comparison with the literature, which shows that the proposed method may be a pessimistic strategy for risk analysis in some cases. More importantly, the TDBPA-based MVRPN eliminates the repeating values in classical RPN, which optimizes the ranking results of RPN. Last, the calculation of TDBPA-based MVRPN is more simple for practical application in engineering.

For further work, inspired by the studies on synthesizing clustering and visualization with FMEA, more attention should be paid on these methods [77], [78]. Since evolving tree (ET) and fuzzy adaptive resonance theory (ART) are proved to be effective in addressing uncertainty, more related visualization and intelligent technologies such as complex network [79], neural network [80] will be investigated.

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