

Received March 18, 2020, accepted April 6, 2020, date of publication April 8, 2020, date of current version April 23, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2986701

# Velocity Control of a PMSM Fed by an Inverter-DC/DC Buck Power Electronic Converter

VICTOR MANUEL HERNÁNDEZ-GUZMÁN<sup>1</sup>, RAMÓN SILVA-ORTIGOZA<sup>2</sup>,  
AND JORGE ORRANTE-SAKANASSI<sup>3</sup>

<sup>1</sup>Centro Universitario, Facultad de Ingeniería, Universidad Autónoma de Querétaro, Querétaro 76010, México

<sup>2</sup>Laboratorio de Mecatrónica and Energía Renovable, Centro de Innovación y Desarrollo Tecnológico en Computo (CIDETEC), Instituto Politécnico Nacional, Mexico City 07700, México

<sup>3</sup>Tecnológico Nacional de México, Instituto Tecnológico de Matamoros, Matamoros 87490, México

Corresponding author: Victor Manuel Hernández-Guzmán (vmhg@uaq.mx)

This work was supported by the Instituto Politécnico Nacional, México. The work of Victor Manuel Hernández-Guzmán was supported by the SNI, México. The work of Ramón Silva-Ortigoza was supported by the SNI, México, and in part by IPN Programs EDI and SIBE. The work of Jorge Orrante-Sakanassi was supported by the SNI, México.

**ABSTRACT** This paper is concerned with velocity control in a permanent magnet synchronous motor (PMSM) when it is fed by an inverter-DC/DC Buck power converter system as power amplifier. We present, for the first time, a formal local asymptotic stability proof to solve this control problem. We stress that this is the first time that this problem is solved for an AC motor. Our control scheme is simple when compared to differential flatness- and backstepping-based proposals in the literature to solve this problem for DC motors. The key for these achievements is the employment of a novel passivity-based approach which takes advantage of the natural energy exchange among the electrical and mechanical subsystems that compose the inverter-DC/DC Buck power converter-PMSM system. The main features of this novel passivity-based approach are summarized in this paper.

**INDEX TERMS** Energy-based control, inverter-dc/dc buck power converter system, Lyapunov stability, permanent magnet synchronous motors, velocity control.

## I. INTRODUCTION

One common technique that is used to provide power to electromechanical systems is pulse width modulation (PWM). However, the hard commutation that is intrinsic to PWM stresses the actuator (electric motors) inducing abrupt changes in its dynamics which are observed as sudden changes in voltages and electric currents [1]. One manner to avoid this situation is the employment of DC/DC power electronic converters. Since these devices have embedded capacitors and inductors, they provide smooth voltages and electric currents, diminishing noise produced by the hard commutation in PWM-based power amplifiers.

The mathematical models of some DC/DC power electronic converter-DC motor systems were proposed for the first time in [2]. Since then, many works have been reported on the control of different combinations of several DC/DC power electronic converter topologies and DC motors [3]–[13].

The associate editor coordinating the review of this manuscript and approving it for publication was Zheng H. Zhu<sup>1</sup>.

In the recent works [14]–[17], the introduction of an inverter between the DC/DC power electronic converter and the DC motor has rendered possible the bidirectional control of velocity. The problem with the proposed inverter-DC/DC power electronic converter topology is that the hard commutation of the inverter still appears at the DC-motor terminals. Motivated by this drawback, in [18] is proposed a novel inverter-DC/DC power electronic converter topology having the advantage that the hard commutation of the inverter is not present at the DC-motor terminals.

In the present paper we extend the application of the inverter-DC/DC power converter topology introduced in [18] to feed a permanent magnet synchronous motor (PMSM) for velocity regulation purposes. We present a formal stability proof ensuring asymptotic stability when the desired velocity is constant. Our main contribution is that, for the first time, velocity is controlled in an AC motor when it is fed by an inverter-DC/DC power converter. We stress that the previous works in the literature are devoted to control DC-motors whose dynamical model is well known to be linear. Recall that AC motor models, and in particular the PMSM model,

are well known to be nonlinear and multi input-multi output. This renders much more complex the control design task and represents the merit of our contribution.

The features of our proposal are the following.

- The proposed control scheme is composed by four PI control loops. This results in a simple and robust control law as opposite to control laws obtained when applying the control techniques (i.e. differential flatness and back-stepping) that have been proposed to solve this problem in DC-motors. It is the authors belief that this is the very reason why any work solving this problem for AC motors has not been reported until now.
- Our proposal relies on a novel passivity-based approach which exploits the energy exchange that naturally exists among the electrical and mechanical subsystems that compose the inverter-DC/DC Buck power electronic converter-PMSM system. This feature is instrumental to design a simple control law because such energy exchange is represented by the natural cancellation of many terms in the stability analysis. If this was not the case such large amount of terms should be exactly cancelled by computing and feeding back them.
- Contrary to standard passivity-based approaches as that in [19], and other more recent approaches as that in [20] which might be tried to solve this problem, our approach does not require to feedback the time derivative of either the desired electric current nor the desired voltages. This fact allows us to avoid the online computation of a very large amount of terms and, hence, it is also instrumental to obtain a simple control law.
- Our proposal relies on *dominating* many cross terms instead of *cancelling* them. We stress that even the approach in [19] is unable to achieve such terms domination because the proposal in [19] requires to ensure that the electrical subsystem error converges exponentially to zero. Then, this error is used as a vanishing disturbance for the mechanical subsystem dynamics. This procedure is not possible if the electrical subsystem receives the effect of the mechanical subsystem through the existence of cross terms between them.

This paper is organized as follows. In Section II we introduce the plant to be controlled and present its dynamical model. The passivity properties of the plant are described in Section III where we also show how the energy exchange among the system components can be exploited. Our main result is presented in Section IV. In Section V we present a simulation study. Finally, some concluding remarks are given in Section VI.

## II. MATHEMATICAL MODEL

The inverter-DC/DC Buck power electronic converter-PMSM system is depicted in fig. 1. The inverter-DC/DC Buck power converter is composed by four transistors  $Q_{1j}, Q_{2j}, \bar{Q}_{1j}, \bar{Q}_{2j}$ , an inductor  $L$ , a capacitor  $C$  and a resistance  $R_c$ . This arrangement of components is repeated three times to have a

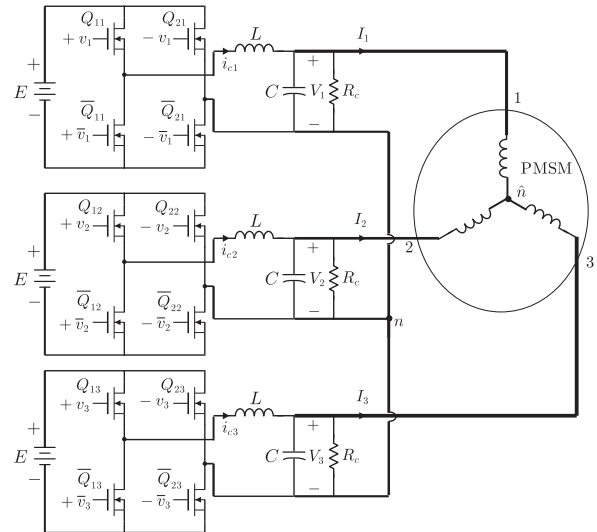


FIGURE 1. The inverter-DC/DC Buck power electronic converter-PMSM system.

three-phase system that feeds the three phases of the PMSM. The symbols  $i_{cj}, V_j, I_j, j = 1, 2, 3$ , represent the electric current through the inductance  $L$ , voltage at the capacitor  $C$  terminals, and electric current through the  $j$ -th phase of the PMSM. Because of the transient responses, the three-phase source voltages are not ensured to be balanced, i.e.  $V_1(t) + V_2(t) + V_3(t) = 0$  is not true for all the time. The balanced voltages assumption is a standard argument to obtain the standard dq dynamical model of a PMSM having an isolated neutral (see [21], Ch. 7). Hence, we also show how to handle the unbalanced voltages assumption to retrieve the standard dq dynamical model of PMSM's. The symbol  $E$  stands for voltage of the DC power supply. The system inputs are  $v_j$  which only take the discrete values  $\{+1, -1, 0\}$  representing the on-positive, the on-negative and the discharging states of transistors  $Q_{1j}, Q_{2j}, \bar{Q}_{1j}, \bar{Q}_{2j}$  [18].

Using the results of [18] we find that the dynamical model of three inverter-DC/DC Buck power converter systems is given as:

$$L \frac{di_{cj}}{dt} = -V_j + Ev_j, \tag{1}$$

$$C \frac{dV_j}{dt} = i_{cj} - I_j - \frac{V_j}{R_c}, \tag{2}$$

where  $j = 1, 2, 3$ . Let  $U = [u_1, u_2, u_3]^T$  represent the average values of  $[v_1, v_2, v_3]^T$ . Also, with some abuse of notation, let  $I_c = [i_{c1}, i_{c2}, i_{c3}]^T$ ,  $I = [I_1, I_2, I_3]^T$ , and  $V = [V_1, V_2, V_3]^T$ , represent the average values of the corresponding variables. Thus, the average model of the above switched dynamical model can be written as:

$$L \dot{I}_c = -V + EU, \tag{3}$$

$$C \dot{V} = I_c - I - \frac{1}{R_c} V. \tag{4}$$

Using the dq transformation [22]:

$$x = T^T x_N, \quad x_N = [x_q, x_d, x_0]^T, \quad (5)$$

$$T = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad (6)$$

where  $x$  stands for  $U, V, I, I_c$ , and  $\theta = n_p q$  with  $q$  the mechanical rotor position and  $n_p$  the motor number of pole pairs, we have that (3) and (4) can be written as (recall that  $T^T = T^{-1}$ ):

$$\begin{aligned} L\dot{I}_{cN} &= -n_p L G_1 I_{cN} \omega - V_N + E U_N, \\ C\dot{V}_N &= -n_p C G_1 V_N \omega + I_{cN} - I_N - \frac{1}{R_c} V_N, \\ G_1 &= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

On the other hand, in fig. 2 we show the star connection of the stator phase windings of a PMSM and the star connection of a three phase voltage source when the motor neutral  $\hat{n}$  and the sources neutral  $n$  are isolated. Let  $V_{1\hat{n}}, V_{2\hat{n}}, V_{3\hat{n}}$  represent the phase to motor neutral voltages, let  $V_1^*, V_2^*, V_3^*$  represent voltages at points 1,2,3, in fig. 2, and let  $V_{\hat{n}}, V_n$  be voltages at the motor neutral and the sources neutral, respectively. Then,  $V_{1\hat{n}} = V_1^* - V_{\hat{n}}, V_{2\hat{n}} = V_2^* - V_{\hat{n}}, V_{3\hat{n}} = V_3^* - V_{\hat{n}}, V_1 = V_1^* - V_n, V_2 = V_2^* - V_n, V_3 = V_3^* - V_n$ , and define  $V_{\hat{n}n} = V_{\hat{n}} - V_n$ . In [21], pp. 422, it is demonstrated that  $V_{1\hat{n}} + V_{2\hat{n}} + V_{3\hat{n}} = 0$  is always true because  $I_1 + I_2 + I_3 = 0$  is always ensured by the star connection with isolated neutral of the motor phase windings. This allows to prove in [21], pp. 423, that the following is always true:

$$V_{\hat{n}n} = \frac{V_1 + V_2 + V_3}{3}. \quad (7)$$

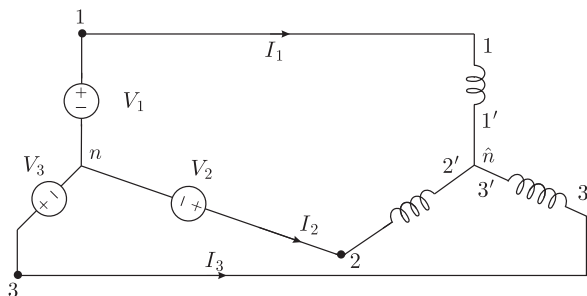


FIGURE 2. The star connection of the motor three phase windings fed by a star connected three phase voltage source.

Hence, using Kirchoff's Voltage Law, Faraday's Law and Ohm's Law to each phase winding we have that [21]:

$$\begin{aligned} \dot{\Lambda} + R I &= \hat{V} = [V_1 - V_{\hat{n}n}, V_2 - V_{\hat{n}n}, V_3 - V_{\hat{n}n}]^T, \\ \hat{V} &= [V_{1\hat{n}}, V_{2\hat{n}}, V_{3\hat{n}}]^T, \quad \Lambda = \bar{L} I + \Gamma, \end{aligned} \quad (8)$$

where  $R$  stands for the phase windings resistance,  $\Lambda$  represents the flux linkages at the stator phase windings,  $\bar{L}$  is the motor inductance matrix and  $\Gamma$  is the flux linkages due to the motor permanent magnet. Then applying the dq transformation defined in (5) to (8) and using (7) we find (see [22] for further details):

$$L_q \dot{I}_q = -R I_q - n_p L_d I_d \omega - \Phi_M \omega + V_q, \quad (9)$$

$$L_d \dot{I}_d = -R I_d + n_p L_q I_q \omega + V_d, \quad (10)$$

$$L_0 \dot{I}_0 = -R I_0, \quad (11)$$

$$J \dot{\omega} = -b \omega + n_p (L_d - L_q) I_d I_q + \Phi_M I_q - \tau_L, \quad (12)$$

where the positive constant scalars  $L_d, L_q, L_0$  stand for the dq0 phase inductances,  $\omega = \dot{q}$  is the motor velocity,  $\Phi_M, b, J$ , are positive constants standing for torque constant, viscous friction coefficient and rotor inertia, respectively, and  $\tau_L$  is load torque. Notice that, according to (5), (6),  $I_0 = \sqrt{\frac{1}{3}}(I_1 + I_2 + I_3)$  and  $V_0 = \sqrt{\frac{1}{3}}(V_1 + V_2 + V_3)$ . Moreover, the star connection with isolated neutral of the motor phase windings ensures that  $I_0 = 0$  for all time. This is consistent with the fact that (11) has no input and it is exponentially stable. We also stress that in the case of balanced source voltages (11) converts into  $L_0 \dot{I}_0 = -R I_0 + V_0$ , which is consistent with the balanced current condition, i.e.  $I_0 = 0$ , since  $V_0 = 0$  when the source voltages are balanced. Thus, (11) is also consistent with the unbalanced source voltages condition because  $V_0 \neq 0$  is not present, ensuring  $I_0 = 0$  for all time.

Thus, the dq model of a PMSM fed by three inverter-DC/DC Buck power converter systems is given by (9)-(12) and:

$$L \dot{I}_{c q} = -n_p L I_{c d} \omega - V_q + E U_q, \quad (13)$$

$$L \dot{I}_{c d} = n_p L I_{c q} \omega - V_d + E U_d, \quad (14)$$

$$L \dot{I}_{c 0} = -V_0 + E U_0, \quad (15)$$

$$C \dot{V}_q = -n_p C V_d \omega + I_{c q} - I_q - \frac{1}{R_c} V_q, \quad (16)$$

$$C \dot{V}_d = n_p C V_q \omega + I_{c d} - I_d - \frac{1}{R_c} V_d, \quad (17)$$

$$C \dot{V}_0 = I_{c 0} - I_0 - \frac{1}{R_c} V_0. \quad (18)$$

Important for our purposes is the following class of saturation functions.

**Definition 1:** Given positive constants  $L^*$  and  $M$ , with  $L^* < M$ , a function  $\sigma : \mathcal{R} \rightarrow \mathcal{R} : \zeta \mapsto \sigma(\zeta)$  is said to be a strictly increasing linear saturation for  $(L^*, M)$  if it is locally Lipschitz, strictly increasing, and satisfies [23]:

$$\sigma(\zeta) = \zeta, \text{ when } |\zeta| \leq L^*,$$

$$|\sigma(\zeta)| < M, \quad \forall \zeta \in \mathcal{R}.$$

### III. OPEN LOOP ENERGY EXCHANGE

Consider the dynamical model in (9)-(18), excepting (11), (15), (18). The total energy stored in the system is

given as:

$$\begin{aligned}
 & V(V_d, V_q, I_{cd}, I_{cq}, I_q, I_d, \omega) \\
 &= \frac{C}{2}(V_d^2 + V_q^2) \\
 &+ \frac{L}{2}(I_{cd}^2 + I_{cq}^2) + \frac{1}{2}(L_q I_q^2 + L_d I_d^2) + \frac{1}{2}J\omega^2. \quad (19)
 \end{aligned}$$

The terms  $C(V_d^2 + V_q^2)/2$  stand for electric energy stored in capacitors of the Buck power converters, whereas  $L(I_{cd}^2 + I_{cq}^2)/2$  represent the magnetic energy stored in inductances of the Buck power converters, and  $(L_q I_q^2 + L_d I_d^2)/2$  stand for the magnetic energy stored in the electrical subsystem of the PMSM. Finally,  $\frac{1}{2}J\omega^2$  is the kinetic energy stored in the mechanical subsystem of the PMSM. The time derivative of  $V$  along the trajectories of system in (9)-(18) is given as:

$$\begin{aligned}
 \dot{V} = & V_d[n_p C V_q \omega + I_{cd} - I_d - \frac{1}{R_c} V_d] \\
 & + V_q[-n_p C V_d \omega + I_{cq} - I_q - \frac{1}{R_c} V_q] \\
 & + I_{cd}[n_p L I_{cq} \omega - V_d + E U_d] \\
 & + I_{cq}[-n_p L I_{cd} \omega - V_q + E U_q] \\
 & + I_q[-R I_q - n_p L_d I_d \omega - \Phi_M \omega + V_q] \\
 & + I_d[-R I_d + n_p L_q I_q \omega + V_d] \\
 & + \omega[-b\omega + n_p(L_d - L_q)I_d I_q + \Phi_M I_q - \tau_L].
 \end{aligned}$$

Notice that several terms cancel to obtain:

$$\begin{aligned}
 \dot{V} = & -\frac{1}{R_c}(V_q^2 + V_d^2) - R(I_q^2 + I_d^2) - b\omega^2 - \omega\tau_L \\
 & + E I_{cd} U_d + E I_{cq} U_q.
 \end{aligned}$$

We stress that these term cancellations represent 1) energy exchange between the electrical and the mechanical subsystems of the PMSM, 2) energy exchange between the capacitor and the electrical subsystem of the PMSM, 3) energy exchange between the capacitor and the inductance of the Buck power electronic converter, 4) energy exchange between the dq phases of the inductor of the Buck power electronic converter, and 5) energy exchange between the dq phases of the capacitor of the Buck power electronic converter.

Hence, if we define the input  $[E U_q, E U_d, -\tau_L]^T$  and the output  $[I_{cq}, I_{cd}, \omega]^T$ , then the dynamical model in (9)-(18) is passive. These properties are exploited in this paper to design a velocity controller for the inverter-DC/DC Buck power converter-PMSM system.

#### IV. MAIN RESULT

Our main result is stated in the following proposition.

*Proposition 1:* Consider the mathematical model in (9)-(18) in closed-loop with the following controller:

$$U_q = \frac{1}{E} \left( V_q^* - K_{pcq} \tilde{I}_{cq} - K_{icq} \int_0^t \tilde{I}_{cq} dr \right), \quad (20)$$

$$U_d = \frac{1}{E} \left( V_d^* - K_{pcd} \tilde{I}_{cd} - K_{icd} \int_0^t \tilde{I}_{cd} dr \right), \quad (21)$$

$$U_0 = 0, \quad (22)$$

$$\begin{aligned}
 I_{cq}^* = & \frac{1}{R_c} V_q^* - K_{pvq} \tilde{V}_q + I_q^* \\
 & - K_{ivq} \int_0^t \left( \tilde{V}_q + n_p L \omega \tilde{I}_{cd} + \frac{L K_{pvq}}{C} \tilde{I}_{cq} \right) dr, \quad (23)
 \end{aligned}$$

$$\begin{aligned}
 I_{cd}^* = & \frac{1}{R_c} V_d^* - K_{pvd} \tilde{V}_d + I_d^* \\
 & - K_{ivd} \int_0^t \left( \tilde{V}_d - n_p L \omega \tilde{I}_{cq} + \frac{L K_{pvd}}{C} \tilde{I}_{cd} \right) dr, \quad (24)
 \end{aligned}$$

$$I_d^* = 0, \quad I_q^* = \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \sigma(z)), \quad z = \int_0^t \tilde{\omega} dr, \quad (25)$$

$$\begin{aligned}
 V_d^* = & -\alpha_{pd} I_d - \alpha_{id} \int_0^t \left( I_d - n_p C \omega \tilde{V}_q - \frac{n_p L}{R_c} \tilde{I}_{cq} \omega \right. \\
 & \left. + \frac{L \alpha_{pd}}{L_d} \left( \frac{1}{R_c} + K_{pv} \right) \tilde{I}_{cd} \right. \\
 & \left. - L n_p K_{pvq} \omega \tilde{I}_{cq} + \frac{C \alpha_{pd}}{L_d} \tilde{V}_d \right) dr, \quad (26)
 \end{aligned}$$

$$\begin{aligned}
 V_q^* = & -\alpha_{pq} \tilde{I}_q - \alpha_{iq} \int_0^t \left( \tilde{I}_q + n_p C \omega \tilde{V}_d + \frac{n_p L}{R_c} \omega \tilde{I}_{cd} \right. \\
 & \left. + \frac{L \alpha_{pq}}{L_q} \left( \frac{1}{R_c} + K_{pv} \right) \tilde{I}_{cq} \right. \\
 & \left. + L n_p K_{pvd} \omega \tilde{I}_{cd} + \frac{C \alpha_{pq}}{L_q} \tilde{V}_q \right) dr, \quad (27)
 \end{aligned}$$

where:

$$\tilde{I}_{ci} = I_{ci} - I_{ci}^*, \quad \tilde{V}_i = V_i - V_i^*, \quad (28)$$

$$\tilde{\omega} = \omega - \omega^*, \quad \tilde{I}_i = I_i - I_i^*, \quad (29)$$

with subindex  $i$  standing for d and q and  $I_d^* = 0$ ,  $\omega^*$  is a real constant standing for the desired velocity,  $\sigma(z)$  is a strictly increasing linear saturation function for some  $(L^*, M)$  (see Definition 1). Furthermore, it is also required that function  $\sigma(z)$  be continuously differentiable such that:

$$0 < \frac{d\sigma(z)}{dz} \leq 1, \quad \forall z \in \mathcal{R}. \quad (30)$$

There always exist positive constants  $k_p, k_i, \alpha_{pd}, \alpha_{pq}, K_{pvq}, K_{pvd}, K_{pcq}, K_{pcd}, K_{icq}, K_{icd}, K_{ivq}, K_{ivd}, \alpha_{id}, \alpha_{iq}, \beta$ , such that the origin of the closed-loop system is locally asymptotically stable.

#### A. CLOSED-LOOP DYNAMICS

First notice that the zero sequence dynamics (11), (15), (18), is globally exponentially stable. Also notice that the remaining expressions in the model (9)-(18) are independent from the zero sequence variables, i.e.  $I_{c0}, V_0, I_0$ . Thus, choosing  $U_0 = 0$  in (22) is enough to ensure that  $I_0 = 0$  and both  $I_{c0} \rightarrow 0, V_0 \rightarrow 0$  exponentially as time grows. This means that both  $I_c = [i_{c1}, i_{c2}, i_{c3}]^T$  and  $V = [V_1, V_2, V_3]^T$  become balanced as time grows. Moreover, according to (5), (6),  $U_0 = \sqrt{\frac{1}{3}}(u_1 + u_2 + u_3)$  and, thus,  $U_0 = 0$  is consistent with the values of  $u_1, u_2, u_3$ , computed using the inverse dq transformation  $U = T^T U_N$ .

On the other hand, define:

$$z_d = \int_0^t \left( I_d - n_p C \omega \tilde{V}_q - \frac{n_p L}{R_c} \tilde{I}_{cq} \omega + \frac{L \alpha_{pd}}{L_d} \left( \frac{1}{R_c} + K_p V_d \right) \tilde{I}_{cd} - L n_p K_p V_q \omega \tilde{I}_{cq} + \frac{C \alpha_{pd}}{L_d} \tilde{V}_d \right) dr - \frac{n_p L_q \omega^*}{\alpha_{id} \Phi_M} (b \omega^* + \tau_L). \quad (31)$$

$$z_q = \int_0^t \left( \tilde{I}_q + n_p C \omega \tilde{V}_d + \frac{n_p L}{R_c} \omega \tilde{I}_{cd} + \frac{L \alpha_{pq}}{L_q} \left( \frac{1}{R_c} + K_p V_q \right) \tilde{I}_{cq} + L n_p K_p V_d \omega \tilde{I}_{cd} + \frac{C \alpha_{pq}}{L_q} \tilde{V}_q \right) dr + \frac{1}{\alpha_{iq}} \left( \frac{R}{\Phi_M} (b \omega^* + \tau_L) + \Phi_M \omega^* \right). \quad (32)$$

Hence, from (26) and (27) we have:

$$V_d^* = -\alpha_{pd} I_d - \alpha_{id} z_d - \delta, \quad \delta = \frac{n_p L_q \omega^*}{\Phi_M} (b \omega^* + \tau_L), \quad (33)$$

$$V_q^* = -\alpha_{pq} \tilde{I}_q - \alpha_{iq} z_q + \xi, \quad \xi = \left( \frac{R}{\Phi_M} (b \omega^* + \tau_L) + \Phi_M \omega^* \right). \quad (34)$$

Adding and subtracting  $I_{cq}^*$ ,  $I_{cd}^*$ ,  $C \dot{V}_q^*$ ,  $n_p C V_d \omega^*$ ,  $n_p C V_d^* \tilde{\omega}$ ,  $n_p C V_d^* \omega^*$ ,  $C \dot{V}_d^*$ ,  $n_p C V_q \omega^*$ ,  $n_p C V_q^* \tilde{\omega}$ ,  $n_p C V_q^* \omega^*$ , and replacing (23) and (24) in (16) and (17), respectively, we find:

$$C \dot{V}_q = -n_p C \tilde{V}_d \tilde{\omega} - n_p C V_d^* \tilde{\omega} - n_p C \tilde{V}_d \omega^* - n_p C (-\alpha_{pd} I_d - \alpha_{id} z_d) \omega^* + \tilde{I}_{cq} - \left( \frac{1}{R_c} + K_p V_q \right) \tilde{V}_q - K_{ivq} \zeta_{vq} - \tilde{I}_q - C \dot{V}_q^*, \quad (35)$$

$$C \dot{V}_d = n_p C \tilde{V}_q \tilde{\omega} + n_p C V_q^* \tilde{\omega} + n_p C \tilde{V}_q \omega^* + n_p C (-\alpha_{pq} \tilde{I}_q - \alpha_{iq} z_q) \omega^* + \tilde{I}_{cd} - \left( \frac{1}{R_c} + K_p V_d \right) \tilde{V}_d - K_{ivd} \zeta_{vd} - \tilde{I}_d - C \dot{V}_d^*, \quad (36)$$

$$\zeta_{vq} = \int_0^t \left( \tilde{V}_q + n_p L \omega \tilde{I}_{cd} + \frac{L K_p V_q}{C} \tilde{I}_{cq} \right) dr - \frac{n_p C}{K_{ivq}} \delta \omega^*, \quad (37)$$

$$\zeta_{vd} = \int_0^t \left( \tilde{V}_d - n_p L \omega \tilde{I}_{cq} + \frac{L K_p V_d}{C} \tilde{I}_{cd} \right) dr - \frac{n_p C}{K_{ivd}} \xi \omega^*. \quad (38)$$

On the other hand, notice that:

$$I_q^* = \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \chi(z) + b \omega^* + \tau_L), \quad (39)$$

if we define:

$$\chi(z) = \sigma(z) + \frac{1}{k_i} (b \omega^* + \tau_L). \quad (40)$$

Hence, from (23), (24), (33), (34), (37), (38), (39), we can write:

$$I_{cq}^* = \frac{1}{R_c} (-\alpha_{pq} \tilde{I}_q - \alpha_{iq} z_q) - K_p V_q \tilde{V}_q - K_{ivq} \zeta_{vq} + \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \chi(z)) + \xi_1, \quad (41)$$

$$\xi_1 = \frac{1}{R_c} \xi - n_p C \delta \omega^* + \frac{1}{\Phi_M} (b \omega^* + \tau_L), \quad I_{cd}^* = \frac{1}{R_c} (-\alpha_{pd} I_d - \alpha_{id} z_d) - K_p V_d \tilde{V}_d - K_{ivd} \zeta_{vd} + \xi_2, \quad \xi_2 = -\frac{1}{R_c} \delta - n_p C \xi \omega^*. \quad (42)$$

Thus, replacing (20) and (21) in (13) and (14), respectively, and adding and subtracting  $n_p L I_{cq} \omega^*$ ,  $L \dot{I}_{cq}^*$ ,  $L \dot{I}_{cd}^*$ ,  $n_p L I_{cq}^* \omega^*$ ,  $n_p L I_{cq}^* \tilde{\omega}$ ,  $n_p L I_{cd} \omega^*$ ,  $n_p L I_{cd}^* \tilde{\omega}$ , we find:

$$L \dot{I}_{cq} = -n_p L \tilde{I}_{cd} \tilde{\omega} - n_p L I_{cd}^* \tilde{\omega} - n_p L \tilde{I}_{cd} \omega^* - \tilde{V}_q - K_{pcq} \tilde{I}_{cq} - K_{icq} \zeta_q - L \dot{I}_{cq}^* - n_p L \left( \frac{1}{R_c} (-\alpha_{pd} I_d - \alpha_{id} z_d) - K_p V_d \tilde{V}_d - K_{ivd} \zeta_{vd} \right) \omega^*, \quad (43)$$

$$L \dot{I}_{cd} = n_p L \tilde{I}_{cq} \tilde{\omega} + n_p L I_{cq}^* \tilde{\omega} + n_p L \tilde{I}_{cq} \omega^* - \tilde{V}_d - K_{pcd} \tilde{I}_{cd} - K_{icd} \zeta_d - L \dot{I}_{cd}^* + n_p L \left( \frac{1}{R_c} (-\alpha_{pq} \tilde{I}_q - \alpha_{iq} z_q) - K_p V_q \tilde{V}_q - K_{ivq} \zeta_{vq} + \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \chi(z)) \right) \omega^*, \quad (44)$$

$$\zeta_q = \int_0^t \tilde{I}_{cq} dr + \frac{1}{K_{icq}} n_p L \xi_2 \omega^*, \quad \zeta_d = \int_0^t \tilde{I}_{cd} dr - \frac{1}{K_{icd}} n_p L \xi_1 \omega^*. \quad (45)$$

Replacing (25) and (27) in (9) and adding and subtracting  $V_q^*$ ,  $L_q \dot{I}_q^*$ ,  $\Phi_M \omega^*$ ,  $R I_q^*$ ,  $n_p L_d I_d \omega^*$  we obtain:

$$L_q \dot{I}_q = -(R + \alpha_{pq}) \tilde{I}_q - n_p L_d I_d \tilde{\omega} - \Phi_M \tilde{\omega} - \alpha_{iq} z_q - n_p L_d I_d \omega^* + \frac{R k_p}{\Phi_M} \tilde{\omega} + \frac{R k_i}{\Phi_M} \chi(z) - L_q \dot{I}_q^* + \tilde{V}_q. \quad (46)$$

On the other hand, replacing (25) and (26) in (10) and adding and subtracting  $V_d^*$ ,  $n_p L_q I_q \omega^*$ ,  $n_p L_q I_q^* \tilde{\omega}$  yields:

$$L_d \dot{I}_d = -(R + \alpha_{pd}) I_d + n_p L_q \tilde{I}_q \tilde{\omega} + n_p L_q I_q^* \tilde{\omega} + \tilde{V}_d + n_p L_q \tilde{I}_q \omega^* - \frac{n_p L_q k_i}{\Phi_M} \omega^* \chi(z) - \frac{n_p L_q k_p}{\Phi_M} \tilde{\omega} \omega^* - \alpha_{id} z_d. \quad (47)$$

Finally, adding and subtracting  $n_p (L_d - L_q) I_d I_q^*$ ,  $\Phi_M I_q^*$ ,  $b \omega^*$ , using (39), and taking advantage from the fact that  $J \dot{\omega}^* = 0$ , we can write (12) as:

$$J \dot{\omega} = -(b + k_p) \tilde{\omega} + n_p (L_d - L_q) I_d \tilde{I}_q + n_p (L_d - L_q) I_d I_q^* + \Phi_M \tilde{I}_q - k_i \chi(z). \quad (48)$$

In the above dynamical equations we have that, from (39), (27), (26), (23), (24):

$$\dot{i}_q^* = \frac{1}{\Phi_M} \left( -k_p \dot{\tilde{\omega}} - k_i \frac{d\chi(z)}{dz} \tilde{\omega} \right), \quad (49)$$

$$\begin{aligned} \dot{V}_q^* &= -\alpha_{pq} \dot{\tilde{I}}_q - \alpha_{iq} \left( \tilde{I}_q + n_p C \omega \tilde{V}_d + \frac{n_p L}{R_c} \omega \tilde{I}_{cq} \right. \\ &\quad \left. + \frac{L \alpha_{pq}}{L_q} \left( \frac{1}{R_c} + K_{pvq} \right) \tilde{I}_{cq} \right. \\ &\quad \left. + L n_p K_{pv} \omega \tilde{I}_{cd} + \frac{C \alpha_{pq}}{L_q} \tilde{V}_q \right), \quad (50) \end{aligned}$$

$$\begin{aligned} \dot{V}_d^* &= -\alpha_{pd} \dot{I}_d - \alpha_{id} \left( I_d - n_p C \omega \tilde{V}_q - \frac{n_p L}{R_c} \tilde{I}_{cq} \omega \right. \\ &\quad \left. + \frac{L \alpha_{pd}}{L_d} \left( \frac{1}{R_c} + K_{pvd} \right) \tilde{I}_{cd} \right. \\ &\quad \left. - L n_p K_{pv} \omega \tilde{I}_{cq} + \frac{C \alpha_{pd}}{L_d} \tilde{V}_d \right), \quad (51) \end{aligned}$$

$$\begin{aligned} \dot{i}_{cq}^* &= \frac{1}{R_c} \dot{V}_q^* - K_{pvq} \dot{\tilde{V}}_q + \dot{i}_q^* \\ &\quad - K_{ivq} \left( \tilde{V}_q + n_p L \omega \tilde{I}_{cd} + \frac{L K_{pvq}}{C} \tilde{I}_{cq} \right), \quad (52) \end{aligned}$$

$$\begin{aligned} \dot{i}_{cd}^* &= \frac{1}{R_c} \dot{V}_d^* - K_{pvd} \dot{\tilde{V}}_d \\ &\quad - K_{ivd} \left( \tilde{V}_d - n_p L \omega \tilde{I}_{cq} + \frac{L K_{pvd}}{C} \tilde{I}_{cd} \right). \quad (53) \end{aligned}$$

The closed-loop dynamics is given by (31)-(53) and  $z = \int_0^t \tilde{\omega} dr$ . The equilibria of this dynamics are found as follows. From  $z = \int_0^t \tilde{\omega} dr$  and  $\dot{z} = 0$ , we conclude that  $\tilde{\omega} = 0$ . From (45) and  $\dot{\zeta}_q = \dot{\zeta}_d = 0$  we have that  $\tilde{I}_{cq} = \tilde{I}_{cd} = 0$ . Hence, from (37), (38), and  $\dot{\zeta}_{vq} = \dot{\zeta}_{vd} = 0$  we find  $\tilde{V}_q = \tilde{V}_d = 0$ . Proceeding analogously in (31), (32), we have that  $I_d = \tilde{I}_q = 0$ . Using these results and  $\tilde{\omega} = 0$  in (48), we find that  $\chi(z) = \sigma(z) + \frac{1}{k_i}(b\omega^* + \tau_L) = 0$ , i.e.  $z = -\frac{1}{k_i}(b\omega^* + \tau_L)$  if:

$$L^* > \frac{1}{k_i} |b\omega^* + \tau_L|. \quad (54)$$

Moreover, from (49) we have  $\dot{i}_q^* = 0$ . From (50), (51) and  $\dot{\tilde{I}}_q = \dot{I}_d = 0$  we have  $\dot{V}_q^* = 0$  and  $\dot{V}_d^* = 0$ . Furthermore, from  $\tilde{V}_q = \tilde{V}_d = 0$  and (52), (53) we have that  $\dot{i}_{cq}^* = \dot{i}_{cd}^* = 0$ . Thus, using the conditions  $\dot{\tilde{I}}_q = 0$  and  $\dot{I}_d = 0$  in (46), (47), we find that  $z_q = 0$ ,  $z_d = 0$ . Proceeding similarly in (35), (36), yields  $\zeta_{vq} = 0$  and  $\zeta_{vd} = 0$ . Finally, from (43), (44), and  $\dot{\tilde{I}}_{cq} = 0$ ,  $\tilde{I}_{cd} = 0$ , we find  $\zeta_q = 0$  and  $\zeta_d = 0$ . Thus, the only equilibrium point is:

$$y = 0, \text{ where } y = [\tilde{\omega}, z + \frac{1}{k_i}(b\omega^* + \tau_L), \tilde{I}_q, I_d, z_q, z_d, \tilde{V}_q, \tilde{V}_d, \zeta_{vq}, \zeta_{vd}, \tilde{I}_{cq}, \tilde{I}_{cd}, \zeta_q, \zeta_d]^T \in \mathcal{R}^{14}. \quad (55)$$

## B. STABILITY ANALYSIS

The closed-loop dynamics (35), (36), (43), (44), (46), (47), (48), can be rewritten as:

$$\begin{aligned} C \dot{\tilde{V}}_q &= -n_p C \tilde{V}_d \tilde{\omega} + \tilde{I}_{cq} - \tilde{I}_q - \left( \frac{1}{R_c} + K_{pvq} \right) \tilde{V}_q \\ &\quad - n_p C V_d^* \tilde{\omega} - n_p C \tilde{V}_d \omega^* \\ &\quad - n_p C (-\alpha_{pd} I_d - \alpha_{id} z_d) \omega^* - K_{ivq} \zeta_{vq} - C \dot{V}_q^*, \quad (56) \end{aligned}$$

$$\begin{aligned} C \dot{\tilde{V}}_d &= n_p C \tilde{V}_q \tilde{\omega} + \tilde{I}_{cd} - \tilde{I}_d - \left( \frac{1}{R_c} + K_{pvd} \right) \tilde{V}_d \\ &\quad + n_p C V_q^* \tilde{\omega} + n_p C \tilde{V}_q \omega^* \\ &\quad + n_p C (-\alpha_{pq} \tilde{I}_q - \alpha_{iq} z_q) \omega^* - K_{ivd} \zeta_{vd} - C \dot{V}_d^*, \quad (57) \end{aligned}$$

$$\begin{aligned} L \dot{\tilde{I}}_{cq} &= -n_p L \tilde{I}_{cd} \tilde{\omega} - \tilde{V}_q - K_{pcq} \tilde{I}_{cq} - K_{icq} \zeta_q \\ &\quad - n_p L \left( \frac{1}{R_c} (-\alpha_{pd} I_d - \alpha_{id} z_d) \right. \\ &\quad \left. - K_{pvd} \tilde{V}_d - K_{ivd} \zeta_{vd} \right) \omega^* \\ &\quad - n_p L i_{cd}^* \tilde{\omega} - n_p L \tilde{I}_{cd} \omega^* - L \dot{i}_{cq}^*, \quad (58) \end{aligned}$$

$$\begin{aligned} L \dot{\tilde{I}}_{cd} &= n_p L \tilde{I}_{cq} \tilde{\omega} - \tilde{V}_d - K_{pcd} \tilde{I}_{cd} - K_{icd} \zeta_d \\ &\quad + n_p L \left( \frac{1}{R_c} (-\alpha_{pq} \tilde{I}_q - \alpha_{iq} z_q) - K_{pvq} \tilde{V}_q \right. \\ &\quad \left. - K_{ivq} \zeta_{vq} + \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \chi(z)) \right) \omega^* \\ &\quad + n_p L i_{cq}^* \tilde{\omega} + n_p L \tilde{I}_{cq} \omega^* - L \dot{i}_{cd}^*, \quad (59) \end{aligned}$$

$$\begin{aligned} L_q \dot{\tilde{I}}_q &= -(R + \alpha_{pq}) \tilde{I}_q - n_p L_d I_d \tilde{\omega} - \Phi_M \tilde{\omega} + \tilde{V}_q \\ &\quad - n_p L_d I_d \omega^* + \frac{R k_p}{\Phi_M} \tilde{\omega} + \frac{R k_i}{\Phi_M} \chi(z) - \alpha_{iq} z_q - L_q \dot{i}_q^*. \quad (60) \end{aligned}$$

$$\begin{aligned} L_d \dot{I}_d &= -(R + \alpha_{pd}) I_d + n_p L_q \tilde{I}_q \tilde{\omega} + \tilde{V}_d \\ &\quad + n_p L_q I_q^* \tilde{\omega} + n_p L_q \tilde{I}_q \omega^* - \frac{n_p L_q k_i}{\Phi_M} \omega^* \chi(z) \\ &\quad - \frac{n_p L_q k_p}{\Phi_M} \tilde{\omega} \omega^* - \alpha_{id} z_d. \quad (61) \end{aligned}$$

$$\begin{aligned} J \dot{\tilde{\omega}} &= -(b + k_p) \tilde{\omega} + n_p (L_d - L_q) I_d \tilde{I}_q + \Phi_M \tilde{I}_q \\ &\quad + n_p (L_d - L_q) I_d I_q^* - k_i \chi(z). \quad (62) \end{aligned}$$

Notice that, excepting some changes of variables, the first rows of the closed-loop dynamics in (56)-(62) are almost identical to the open-loop dynamics in (9)-(18), excepting (11), (15), (18). One important difference is that the coefficients of the damping injection terms have been enlarged. Hence, improving stability, as shown below. The remaining terms in (56)-(62) will be dominated by sign definite terms in the stability analysis that we present in the following. We stress that *dominating these terms*, instead of *cancelling these terms*, improves the closed-loop performance because this avoids numerical errors and noise amplification produced when including a large amount of online computations.

Another important difference is that six nonstandard PI control loops have been included. See (31), (32), (37), (38), (45). This feature is intended to improve robustness with respect to parametric uncertainties and external disturbances.

These observations motivate the use of the following “energy” storage function for the closed-loop dynamics (see (19)):

$$W(y) = \frac{C}{2}(\tilde{V}_d^2 + \tilde{V}_q^2) + \frac{L}{2}(\tilde{I}_{cd}^2 + \tilde{I}_{cq}^2) + \frac{1}{2}(L_q \tilde{I}_q^2 + L_d \tilde{I}_d^2) + \frac{1}{2}(\alpha_{iq} z_q^2 + \alpha_{id} z_d^2) + \frac{1}{2}(K_{ivd} \zeta_{vd}^2 + K_{ivq} \zeta_{vq}^2) + \frac{1}{2}(K_{icd} \zeta_d^2 + K_{icq} \zeta_q^2) + V_\omega(\tilde{\omega}, z + \underbrace{\frac{\tau_L + b\omega^*}{k_i}}_{:=\kappa}), \quad (63)$$

where:

$$V_\omega(\tilde{\omega}, z + \frac{\tau_L + b\omega^*}{k_i}) = \frac{1}{2} J \tilde{\omega}^2 + [k_i + \beta(b + k_p)] \int_{-\frac{\tau_L + b\omega^*}{k_i}}^z \chi(r) dr + \beta J \chi(z) \tilde{\omega}.$$

We stress that function  $V_\omega(\tilde{\omega}, z + \frac{\tau_L + b\omega^*}{k_i})$ , defined in (63), is proven to be positive definite and radially unbounded in appendix if  $k_p > 0, k_i > 0, \beta > 0$  such that (69) is satisfied.

Taking advantage from the following cancellations, which are a direct consequence of the features explained in the paragraph after (62):

$$\begin{aligned} n_p L \tilde{I}_{cd} \tilde{I}_{cq} \omega^* - n_p L \tilde{I}_{cq} \tilde{I}_{cd} \omega^* &= 0, \\ -n_p C \tilde{V}_q \tilde{V}_d \omega^* + n_p C \tilde{V}_d \tilde{V}_q \omega^* &= 0, \\ \tilde{V}_d n_p C \tilde{V}_q \tilde{\omega} - \tilde{V}_q - n_p C \tilde{V}_d \tilde{\omega} &= 0, \quad \tilde{V}_d \tilde{I}_{cd} - \tilde{I}_{cd} \tilde{V}_d = 0, \\ -\tilde{V}_d \tilde{I}_d + \tilde{I}_d \tilde{V}_d &= 0, \quad \tilde{V}_q \tilde{I}_{cq} - \tilde{I}_{cq} \tilde{V}_q = 0, \\ -\tilde{V}_q \tilde{I}_q + \tilde{I}_q \tilde{V}_q &= 0, \quad \tilde{I}_{cd} n_p L \tilde{I}_{cq} \tilde{\omega} - \tilde{I}_{cq} n_p L \tilde{I}_{cd} \tilde{\omega} = 0, \\ \tilde{I}_q [-n_p L_d \tilde{I}_d \tilde{\omega} - \Phi_M \tilde{\omega}] + I_d [n_p L_q \tilde{I}_q \tilde{\omega} + n_p L_q \tilde{I}_q^* \tilde{\omega}] \\ + \tilde{\omega} [n_p (L_d - L_q) I_d \tilde{I}_q - n_p L_q I_d \tilde{I}_q^* + \Phi_M \tilde{I}_q] &= 0, \\ -\tilde{I}_q \alpha_{iq} z_q + \alpha_{iq} z_q \tilde{I}_q &= 0, \quad -I_d \alpha_{id} z_d + \alpha_{id} z_d I_d = 0, \\ -\tilde{\omega} k_i \chi(z) + [k_i + \beta(b + k_p)] \chi(z) \tilde{\omega} \\ - \beta \chi(z) (b + k_p) \tilde{\omega} &= 0, \end{aligned} \quad (64)$$

we find that the time derivative of  $W$  along the trajectories of the closed-loop system (56)-(62), (31), (32), (37), (38), (45), is given as:

$$\begin{aligned} \dot{W} &= \tilde{I}_{cd} \left[ n_p L \left( \frac{1}{R_c} (-\alpha_{pq} \tilde{I}_q) - K_{pvq} \tilde{V}_q \right. \right. \\ &+ \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \chi(z)) + \xi_1 \left. \right) \tilde{\omega} \\ &+ n_p L \tilde{I}_{cq} \omega^* - K_{pcd} \tilde{I}_{cd} - L \dot{I}_{cd}^* \\ &+ n_p L \left( \frac{1}{R_c} (-\alpha_{pq} \tilde{I}_q) - K_{pvq} \tilde{V}_q \right. \\ &+ \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \chi(z)) \left. \right) \omega^* \left. \right] \\ &+ \tilde{I}_{cq} \left[ -n_p L \left( \frac{1}{R_c} (-\alpha_{pd} I_d) - K_{pvd} \tilde{V}_d + \xi_2 \right) \tilde{\omega} \right. \end{aligned}$$

$$\begin{aligned} &- n_p L \tilde{I}_{cd} \omega^* - n_p L \left( \frac{1}{R_c} (-\alpha_{pd} I_d) \right. \\ &- K_{pvd} \tilde{V}_d \left. \right) \omega^* - K_{pcq} \tilde{I}_{cq} - L \dot{I}_{cq}^* \left. \right] \\ &+ \tilde{V}_d \left[ n_p C (-\alpha_{pq} \tilde{I}_q + \xi) \tilde{\omega} + n_p C (-\alpha_{pq} \tilde{I}_q) \omega^* \right. \\ &- \left( \frac{1}{R_c} + K_{pvd} \right) \tilde{V}_d - C \dot{V}_d^* \left. \right] \\ &+ \tilde{V}_q \left[ -n_p C (-\alpha_{pd} I_d - \delta) \tilde{\omega} - n_p C (-\alpha_{pd} I_d) \omega^* \right. \\ &- \left( \frac{1}{R_c} + K_{pvq} \right) \tilde{V}_q - C \dot{V}_q^* \left. \right] \\ &+ I_d [- (R + \alpha_{pd}) I_d + n_p L_q \tilde{I}_q \omega^* \\ &- \frac{n_p L_q k_i}{\Phi_M} \omega^* \chi(z) - \frac{n_p L_q k_p}{\Phi_M} \tilde{\omega} \omega^*] \\ &+ \tilde{I}_q [- (R + \alpha_{pq}) \tilde{I}_q - n_p L_d I_d \omega^* + \frac{R k_p}{\Phi_M} \tilde{\omega} \\ &+ \frac{R k_i}{\Phi_M} \chi(z) - L_q \dot{I}_q^*] \\ &+ \beta \chi(z) [n_p (L_d - L_q) I_d \tilde{I}_q \\ &+ n_p (L_d - L_q) I_d \frac{1}{\Phi_M} (-k_p \tilde{\omega} - k_i \chi(z)) \\ &+ b \omega^* + \tau_L] + \Phi_M \tilde{I}_q - k_i \chi(z) \\ &+ \tilde{\omega} [- (b + k_p) \tilde{\omega} + n_p L_d I_d \frac{1}{\Phi_M} (-k_p \tilde{\omega} \\ &- k_i \chi(z) + b \omega^* + \tau_L)] + \beta J \frac{d \chi(z)}{dz} \tilde{\omega}^2 \\ &+ K_{ivd} \zeta_{vd} \left[ \frac{L K_{pvd}}{C} \tilde{I}_{cd} \right] + K_{ivq} \zeta_{vq} \left[ \frac{L K_{pvq}}{C} \tilde{I}_{cq} \right] \\ &+ \alpha_{iq} z_q \left[ \frac{L \alpha_{pq}}{L_q} \left( \frac{1}{R_c} + K_{pvq} \right) \tilde{I}_{cq} + L n_p K_{pvd} \omega \tilde{I}_{cd} \right. \\ &+ \frac{C \alpha_{pq}}{L_q} \tilde{V}_q \left. \right] \\ &+ \alpha_{id} z_d \left[ \frac{L \alpha_{pd}}{L_d} \left( \frac{1}{R_c} + K_{pvd} \right) \tilde{I}_{cd} - L n_p K_{pvq} \omega \tilde{I}_{cq} \right. \\ &+ \frac{C \alpha_{pd}}{L_d} \tilde{V}_d \left. \right]. \end{aligned}$$

Taking into account (30), it is found that  $\dot{W}$  can be upper bounded as:

$$\begin{aligned} \dot{W} &\leq -v^T Q v + \tilde{I}_{cd} \left[ -L \dot{I}_{cd}^* \right] + \tilde{I}_{cq} \left[ -L \dot{I}_{cq}^* \right] \\ &+ \tilde{V}_d \left[ -C \dot{V}_d^* \right] + \tilde{V}_q \left[ -C \dot{V}_q^* \right] + \tilde{I}_q [-L_q \dot{I}_q^*] \\ &+ K_{ivd} \zeta_{vd} \left[ \frac{L K_{pvd}}{C} \tilde{I}_{cd} \right] + K_{ivq} \zeta_{vq} \left[ \frac{L K_{pvq}}{C} \tilde{I}_{cq} \right] \\ &+ \alpha_{iq} z_q \left[ \frac{L \alpha_{pq}}{L_q} \left( \frac{1}{R_c} + K_{pvq} \right) \tilde{I}_{cq} \right. \\ &+ L n_p K_{pvd} \omega \tilde{I}_{cd} + \frac{C \alpha_{pq}}{L_q} \tilde{V}_q \left. \right] \\ &+ \alpha_{id} z_d \left[ \frac{L \alpha_{pd}}{L_d} \left( \frac{1}{R_c} + K_{pvd} \right) \tilde{I}_{cd} \right. \\ &- L n_p K_{pvq} \omega \tilde{I}_{cq} + \frac{C \alpha_{pd}}{L_d} \tilde{V}_d \left. \right], \end{aligned} \quad (65)$$

where:

$$v = [|\tilde{\omega}|, |\chi(z)|, |\tilde{I}_q|, |I_d|, |\tilde{V}_q|, |\tilde{V}_d|, |\tilde{I}_{cq}|, |\tilde{I}_{cd}|]^T \in \mathcal{R}^8, \quad (66)$$

and the entries of the symmetric matrix  $Q$  are given as:

$$\begin{aligned} Q_{11} &= b + k_p - \beta J - \frac{n_p L_d k_p}{\Phi_M} |I_d| - \frac{n_p L k_p}{\Phi_M} |\tilde{I}_{cd}|, \\ Q_{22} &= \beta k_i - \frac{\beta n_p L_d k_i}{\Phi_M} |I_d|, \\ Q_{33} &= R + \alpha_{pq}, \quad Q_{44} = R + \alpha_{pd}, \\ Q_{55} &= \frac{1}{R_c} + K_{pVq}, \quad Q_{66} = \frac{1}{R_c} + K_{pVd}, \\ Q_{77} &= K_{pcq}, \quad Q_{88} = K_{pcd}, \\ Q_{12} &= Q_{21} = -\frac{n_p L_d k_i}{2\Phi_M} |I_d| - \frac{\beta n_p k_p |L_d - L_q|}{2\Phi_M} |I_d| \\ &\quad - \frac{n_p L k_i}{2\Phi_M} |\tilde{I}_{cd}|, \\ Q_{13} &= Q_{31} = -\frac{R k_p}{2\Phi_M}, \\ Q_{14} &= Q_{41} = -\frac{n_p L_d}{2\Phi_M} |b\omega^* + \tau_L| - \frac{n_p L_q k_p}{2\Phi_M} |\omega^*|, \\ Q_{24} &= Q_{42} = -\frac{\beta n_p |L_d - L_q|}{2\Phi_M} |b\omega^* + \tau_L| - \frac{n_p L_q k_i}{2\Phi_M} |\omega^*|, \\ Q_{23} &= Q_{32} = -\frac{\beta n_p |L_d - L_q|}{2} |I_d| - \frac{R k_i}{2\Phi_M}, \\ Q_{34} &= Q_{43} = -\frac{n_p L_d |\omega^*|}{2} - \frac{1}{2} n_p L_q |\omega^*|, \\ Q_{15} &= Q_{51} = -\frac{n_p C \alpha_{pd}}{2} |I_d| - \frac{n_p C \delta}{2}, \\ Q_{45} &= Q_{54} = -\frac{n_p C \alpha_{pd} |\omega^*|}{2}, \\ Q_{56} &= Q_{65} = 0, \quad Q_{16} = Q_{61} = -\frac{n_p C \alpha_{pq}}{2} |\tilde{I}_q| - \frac{n_p C \xi}{2}, \\ Q_{36} &= Q_{63} = -\frac{n_p C \alpha_{pq} |\omega^*|}{2}, \\ Q_{58} &= Q_{85} = -\frac{n_p L K_{pVq} |\omega^*|}{2}, \\ Q_{17} &= Q_{71} = -\frac{n_p L}{2} \left( \frac{1}{R_c} \alpha_{pd} |I_d| + K_{pVd} |\tilde{V}_d| + \xi_2 \right), \\ Q_{47} &= Q_{74} = -\frac{n_p L \alpha_{pd}}{2 R_c} \omega^*, \\ Q_{67} &= Q_{76} = -\frac{n_p L K_{pVd} |\omega^*|}{2}, \\ Q_{18} &= Q_{81} = -\frac{n_p L \xi_1}{2\Phi_M} - \frac{n_p L K_{pVq}}{2} |\tilde{V}_q| \\ &\quad - \frac{n_p L \alpha_{pq}}{2 R_c} |\tilde{I}_q| - \frac{n_p L k_p |\omega^*|}{2\Phi_M}, \\ Q_{28} &= Q_{82} = -\frac{n_p L k_i |\omega^*|}{2\Phi_M}, \\ Q_{38} &= Q_{83} = -\frac{n_p L \alpha_{pq} |\omega^*|}{2 R_c}. \end{aligned} \quad (67)$$

Notice that matrix  $Q$  can always be rendered positive definite, if the entries of  $v$  remain small, by using large enough

positive constants  $k_p, k_i, \alpha_{pd}, \alpha_{pq}, K_{pVq}, K_{pVd}, K_{pcq}, K_{pcd}$ , and a small enough  $\beta > 0$ . This is proven by showing that the eight leading principal minors of  $Q$  are positive, which is explained as follows. We have that  $Q_{11} > 0$  can be rendered true by using suitable values for  $k_p$  and  $\beta$  if  $|I_d|, |\tilde{I}_{cd}|$  are small. Once this is achieved, the second leading principal minor can be rendered positive if  $k_i$  is large,  $\beta$  small and  $|I_d|, |\tilde{I}_{cd}|$ , are small. Notice that, although  $Q_{12}$  and  $Q_{21}$  depend on  $k_i$ , the entry  $Q_{22}$  can be enlarged without enlarging  $Q_{12}$  nor  $Q_{21}$  if  $|I_d|, |\tilde{I}_{cd}|$  remain small.

Following this line of ideas, the remaining leading principal minors can always be rendered positive, if the entries of  $v$  are small, because the entry  $Q_{ii}$  can be enlarged without enlarging any of the entries of  $Q$  laying in the rows and columns 1 to  $i$ . It is important to stress, however, that in these remaining cases we have no need to resort to the trick explained in the last sentence of the previous paragraph.

On the other hand, using (49), (50), (51), (52), (53), (35), (36), (46), (47), (48), (33), (34), (39), we can expand the terms containing time derivatives in (65) to obtain their complete expressions. Then, it is not difficult to realize that the last five rows in (65) cancel with several terms in the above described expansion of terms. Thus, we realize that after a straightforward although tedious procedure, (65) can be written as:

$$\dot{W} \leq -v^T P v, \quad (68)$$

where  $P$  is a symmetric matrix which is built by following the same procedure used to build matrix  $Q$ . This means that, if the entries of  $v$  remain small, matrix  $P$  can always be rendered positive definite using large enough positive constants  $k_p, k_i, \alpha_{pd}, \alpha_{pq}, K_{pVq}, K_{pVd}, K_{pcq}, K_{pcd}$ , and a small enough  $\beta > 0$ . This is proven by showing that the eight leading principal minors of  $P$  are positive by using the procedure explained in the paragraphs after (67). This means that  $\dot{W} \leq -v^T P v \leq 0$ , if  $y$  is small, and the closed-loop system is stable. Finally, since the closed-loop system is autonomous, we can invoke the LaSalle invariance principle [24], Ch. 4, to conclude that the origin  $y = 0$  is locally asymptotically stable. This completes the proof of Proposition 1.

Conditions for this stability result are summarized by  $k_p, k_i, \alpha_{pd}, \alpha_{pq}, K_{pVq}, K_{pVd}, K_{pcq}, K_{pcd}, K_{icq}, K_{icd}, K_{iVq}, K_{iVd}, \alpha_{id}, \alpha_{iq}, \beta$ , are positive, (69), (54) are satisfied, and the eight leading principal minors of matrix  $P$  introduced in (68) are positive.

*Remark 1:* The control scheme in Proposition 1 is made up of four main loops: 1) a PI controller for electric current through the inductor of the DC/DC Buck power converter, 2) a PI controller for voltage at the DC/DC Buck power converter output (at the capacitor terminals), 3) a PI controller for electric current through the motor stator phase windings, and 4) a PI controller, with a saturated integral part, for motor velocity; the saturated integral part is employed in order to render possible to dominate some third order terms, where this integral part appears, with some negative



definite second order terms. Thus, our proposal contains the fundamental components in industrial applications and, hence, it is expected to be robust with respect to parametric uncertainties and external disturbances.

*Remark 2:* The novel passivity-based approach that is employed in this paper has the following properties.

- The cancellation of terms presented in (64) represents the energy exchange among the magnetic energy stored in the motor electrical subsystem, the kinetic energy stored in the motor mechanical subsystem, the electrical energy stored in the Buck power converter capacitor, and the magnetic energy stored in the Buck power converter inductor.

This property is a direct consequence of the fact that the first rows of the closed-loop dynamics in (56)-(62) are almost identical to the open-loop dynamics in (9)-(18), excepting (11), (15), (18). Thus, the passivity property that was established in Section III is instrumental to achieve this step.

These term cancellations are instrumental to obtain a simpler control law. This is because, if not cancelled naturally, these terms must be cancelled using additional terms in the control law. Also instrumental for a simple control law is the fact that our design relies on *dominating* many cross terms instead of *cancelling* them as usual in differential flatness- and backstepping-based designs. Moreover, the standard passivity-based approach introduced by [19] requires to complete isolated error equations for the electrical subsystems by computing and feeding back online the time derivative of the desired electrical currents and voltages. As it is clear in the above proof, this would require a large amount of computations which, as it is also remarked by [19], would deteriorate performance because of numerical errors and noise amplification. Finally, let us point out that similar problems would arise with the control design approach introduced by [20] because such technique also requires to compute and to feedback on line the time derivative of the desired electric currents and voltages.

- A nested-loop passivity-based control approach is exploited in [19]. This means that the electric current error is first proven to converge exponentially to zero and this allows to use this variable as a vanishing perturbation for the mechanical subsystem. This, however, requires the online computation of the time derivative of the desired electric current. Instead of that, we use an approach which is similar to what was called in [19] passivity-based control with total energy shaping. Although the latter approach has been disregarded in [19] arguing that it results in more complex controllers, we prove the opposite in the present paper.
- The previous features of our approach allow to naturally include PI internal loops, which are important to improve the robustness properties of the control scheme with respect to both parametric uncertainties and external disturbances.

## V. SIMULATION RESULTS

In order to give some insight on the achievable performance by the closed-loop system in Proposition 1, in this section we present a numerical example. For this, we employ the PMSM model Estun EMJ-04APB22, whose numerical parameters were identified in [25], i.e.  $n_p = 4$ ,  $R = 2.7$  [Ohm],  $L_d = L_q = 8.5$  [mH],  $\Phi_M = 0.301$  [Nm/A],  $J = 31.69 \times 10^{-6}$  [Kgm<sup>2</sup>],  $b = 52.79 \times 10^{-6}$  [Nm/(rad/s)]. For the inverter-Buck DC/DC electronic power converter system we choose the numerical values employed in [18], i.e.  $L = 4.94$  [mH],  $E = 150$  [V],  $C = 114.4 \times 10^{-6}$  [F],  $R_c = 48$  [Ohm]. The controller gains are  $\alpha_{pd} = 10$ ,  $\alpha_{id} = 30$ ,  $\alpha_{pq} = 10$ ,  $\alpha_{iq} = 30$ ,  $k_p = 0.1$ ,  $k_i = 20$ ,  $K_{pVq} = 40$ ,  $K_{iVq} = 80$ ,  $K_{pVd} = 40$ ,  $K_{iVd} = 80$ ,  $K_{pcq} = 3000$ ,  $K_{pcd} = 3000$ ,  $K_{icq} = 2000$ ,  $K_{icd} = 2000$ . Inspired by [23], we employ the following linear saturation function:

$$\sigma(x) = \begin{cases} -L^* + (M - L^*) \tanh\left(\frac{x + L^*}{M - L^*}\right), & \text{if } x < -L^* \\ x, & \text{if } |x| \leq L^* \\ L^* + (M - L^*) \tanh\left(\frac{x - L^*}{M - L^*}\right), & \text{if } x > L^*, \end{cases}$$

where  $M = 43.49$ ,  $L^* = 42.26$ . The desired velocity is defined as follows. Given:

$$\begin{aligned} t_{i1} &= 0.01[\text{s}], \quad t_{f1} = 0.012[\text{s}], \quad a = \frac{t - t_{i1}}{t_{f1} - t_{i1}}, \\ u_{i1} &= 0, \quad u_{f1} = \frac{450 \times 2\pi}{60} [\text{rad/s}], \\ \varphi_1 &= a^5 (252 - 1050a + 1800a^2 - 1575a^3 + 700a^4 - 126a^5), \\ t_{i2} &= 0.06[\text{s}], \quad t_{f2} = 0.063[\text{s}], \quad b = \frac{t - t_{i2}}{t_{f2} - t_{i2}}, \\ u_{i2} &= \frac{450 \times 2\pi}{60} [\text{rad/s}], \quad u_{f2} = \frac{-450 \times 2\pi}{60} [\text{rad/s}], \\ \varphi_2 &= b^5 (252 - 1050b + 1800b^2 - 1575b^3 + 700b^4 - 126b^5), \end{aligned}$$

we have that:

$$\omega^* = \begin{cases} 0, & t < t_{i1} \\ u_{i1} + (u_{f1} - u_{i1})\varphi_1, & t_{i1} \leq t < t_{f1} \\ u_{f1}, & t_{f1} \leq t < t_{f2} \\ u_{i2} + (u_{f2} - u_{i2})\varphi_2, & t_{i2} \leq t < t_{f2} \\ u_{f2}, & t_{f2} \leq t. \end{cases}$$

We also consider a step torque disturbance  $\tau_L = 0.6$  [Nm] which is applied at  $t = 0.03$  [s] and disappears at  $t = 0.08$  [s]. Although the electric currents through the motor phase windings are always balanced it is interesting to observe the effects of some unbalanced condition produced by some disturbance. Moreover, as stated earlier, the source voltages (at the converters capacitors) and electric currents (through the converters inductors) may be unbalanced during the transient response. These are the reasons why all of the initial conditions are chosen to be zero excepting  $V_0(0) = 5$  [V],  $I_0(0) = 0.5$  [A],  $I_{c0}(0) = 1$  [A]. Finally, we have employed  $L_0 = 3$  [mH]  $< L_q = L_d$ .

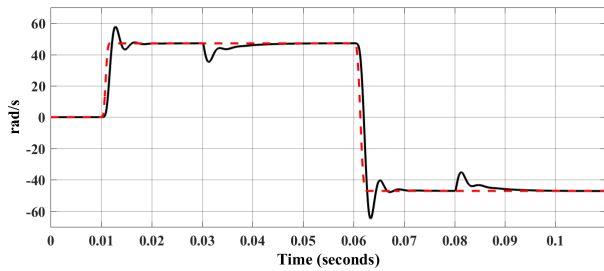


FIGURE 3. Continuous: actual velocity response  $\omega$ . Dashed: desired velocity  $\omega^*$ .

In fig. 3 we present the actual motor velocity  $\omega$  and the desired velocity  $\omega^*$ . We observe that  $\omega$  reaches  $\omega^*$  in steady state which was expected because  $\omega^*$  is assumed to be constant in Proposition 1. Moreover, this is also achieved in steady state despite a constant torque disturbance appears or disappears. We have performed several additional simulations which allow us to conclude that the transient response can be easily modified by suitably selecting  $k_p$  and  $k_i$ . We present the transient response in fig. 3 just because the transient response there can be easily observed.

In fig. 4 we present the three voltages at the converters capacitors. We observe that these three voltages are identical for  $0 \leq t < 0.01[s]$  which clearly shows that these voltages are not balanced, i.e. that  $V_1 + V_2 + V_3 \neq 0$ , in this time interval. This is because we have assumed that  $V_0(0) \neq 0$  and, since the zero sequence dynamics is exponentially stable, these three voltages become balanced, i.e.  $V_0 \rightarrow 0$  exponentially, after a while, i.e. after  $t = 0.01[s]$  in fig. 4. We also observe that any of these three voltages are not larger than  $\pm 60[V]$ . We recall that the rated voltage for this motor is  $\pm 200[V]$  rms (see [25]).

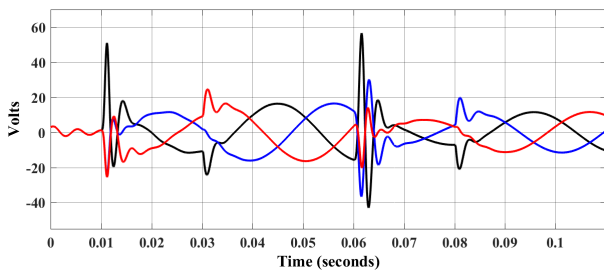


FIGURE 4. Three phase voltages at the converters capacitors  $V_1, V_2, V_3$ .

In fig. 5 we can also observe that electric currents through the motor phases are unbalanced for  $0 \leq t < 0.01[s]$ . Recall that this is because we have chosen  $I_0(0) \neq 0$ . We observe that the steady state values of these electric current are less than  $\pm 2[A]$  with some isolated peaks within  $\pm 5[A]$ , which appear because of the sudden velocity reference changes. These electric current values are consistent with the rated current for this motor which is  $2.7[A]$  rms (see [25]).

In fig. 6 we present the electric currents through the converters inductors. We observe the same unbalanced condition as in figs. 4 and 5. As an important additional observation, we realize that electric currents through the converters

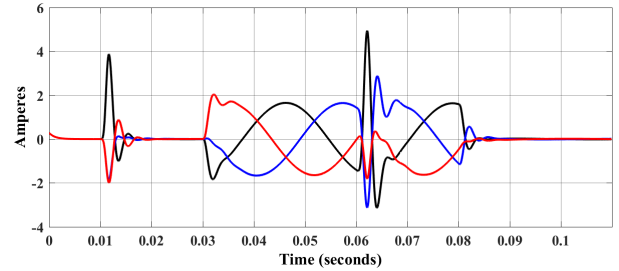


FIGURE 5. Electric currents through the motor phases  $I_1, I_2, I_3$ .

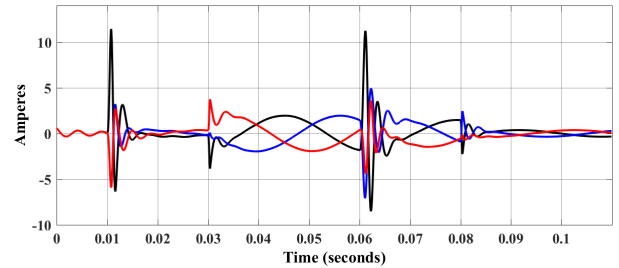


FIGURE 6. Electric currents through the converters inductors  $i_{c1}, i_{c2}, i_{c3}$ .

inductors reach larger values than electric currents through the motor phases. This can be easily explained using fig. 1 where we have that  $I_{cj} = I_j + I_{Cj} + I_{Rcj}$  where  $j = 1, 2, 3$ , and  $I_{Cj}, I_{Rcj}$  are electric currents through the converters capacitor and through  $R_c$ , respectively.

In fig. 7 we present  $u_1, u_2, u_3$ , i.e. the average on-off signals applied at the power transistor inputs. Recall that this signals take continuous values in the range  $[-1, +1]$ . In fig. 7 it is corroborated that all of the three signals  $u_1, u_2, u_3$  remain within this range all the time, despite the transient periods when changes in the reference velocity are commanded and when a torque disturbance appears and disappears.

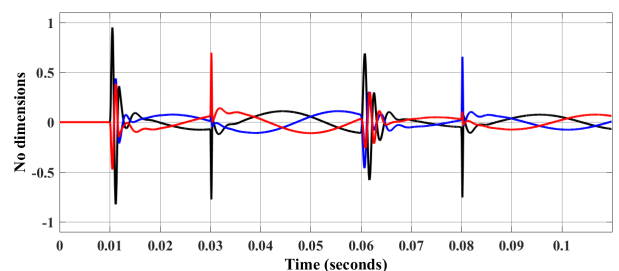


FIGURE 7. Average signals applied at the transistors inputs  $u_1, u_2, u_3$ .

The three phase variables plotted in figs. 4, 5, 6, and 7, have been computed using the dq transformation in (5) where  $x$  takes the values  $[V_1, V_2, V_3], [I_1, I_2, I_3], [i_{c1}, i_{c2}, i_{c3}]$  and  $[u_1, u_2, u_3]$  whereas  $x_N$  takes the values  $[V_q, V_d, V_0], [I_q, I_d, I_0], [I_{cq}, I_{cd}, I_{c0}]$  and  $[U_q, U_d, U_0]$ , respectively. Although it is usual in the control literature on electric machines to report only the behavior of the dq variables, we believe that more insight is given by observing the three phase variables in this particular control problem.

Finally, in figs. 8, 9, and 10 we present results obtained when there are uncertainties only in  $\Phi_M$ , only in  $R_c$ , and only in  $L_d, L_q$ , respectively. In these simulations we employ all of the above plant and controller parameters but we consider the following changes in parameters of the plant: 1) in fig. 8 we use  $\Phi_M = 0.15$ [Nm/A], in fig. 9 we use  $R_c = 20$ [Ohm] whereas in fig. 10 we use  $L_d = L_q = 12.5$ [mH]. These parameters are used for all the simulation time. We stress that only fig. 10 has a small difference in the range of the vertical axis with respect to figs. 3, 8 and 9. We observe that uncertainties in the motor torque constant  $\Phi_M$  and inductances  $L_d, L_q$ , have the largest effects on the velocity response. However, these results also show that the control scheme is robust with respect to uncertainties in these parameters since the closed-loop system remains to be asymptotically stable and performance deterioration is not observed to be large. The plant parameters  $\Phi_M, R_c, L_d, L_q$  are the most likely to present changes or uncertainties during normal operation of the plant, and this is the reason why we only consider uncertainties in these parameters. Moreover, these uncertainties are not expected to appear as abrupt (step) changes during normal operation of plant.

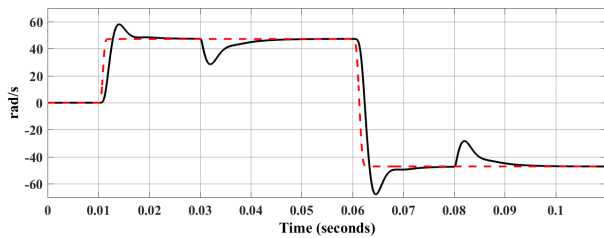


FIGURE 8. Continuous: actual velocity response  $\omega$ . Dashed: desired velocity  $\omega^*$ . Uncertainty in parameter  $\Phi_M$  is present.

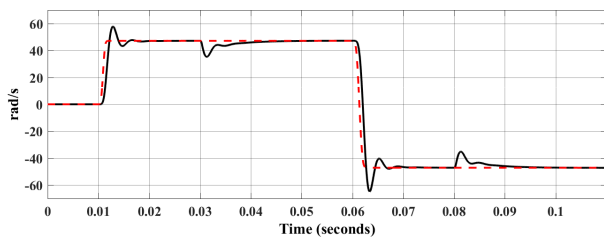


FIGURE 9. Continuous: actual velocity response  $\omega$ . Dashed: desired velocity  $\omega^*$ . Uncertainty in parameter  $R_c$  is present.

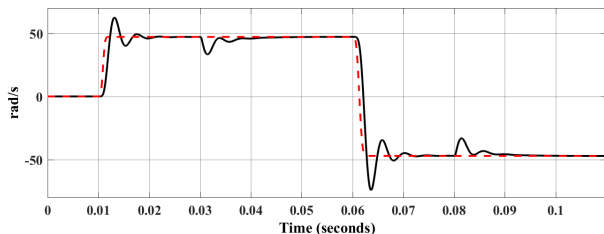


FIGURE 10. Continuous: actual velocity response  $\omega$ . Dashed: desired velocity  $\omega^*$ . Uncertainties in parameters  $L_d$  and  $L_q$  are present.

## VI. CONCLUSION

We have presented, for the first time, a velocity controller for a permanent magnet synchronous motor when it is fed by

and inverter-Buck DC/DC power converter system. Although our stability proof only ensures local asymptotic stability, the merit of our proposal is that this is the first time that such a problem is solved for an AC motor. Moreover, our controller is much simpler when compared to proposals in the literature for DC motors which are designed using differential flatness or backstepping.

One difficulty to formally solve this problem is that the dq model of the AC motor is required for stability proof purposes whereas formal studies on inverter-Buck DC/DC power converter systems are performed in original coordinates. Hence, we propose, also for the first time, to perform a dq coordinate transformation on the inverter-Buck DC/DC power converter system in order to succeed to present a stability proof. We believe that this idea may pave the way to control other classes of AC motors, i.e. induction motors, and we will present such a result elsewhere.

Another source of difficulty that was found is the fact that the three phase source voltages might be unbalanced during the transient periods. Recall that the standard model of permanent magnet synchronous motors is derived in the control literature by assuming that such voltages are balanced. Hence, we reviewed the permanent magnet synchronous motor modeling literature to find how to take into account the fact that the three phase source voltages are unbalanced.

## APPENDIX

### POSITIVE DEFINITENESS OF $\kappa$ INTRODUCED IN (63)

The scalar function  $\kappa = V_\omega(\tilde{\omega}, z + \frac{\tau_L + b\omega^*}{k_i})$  introduced in (63) can be written as:

$$V_\omega(\tilde{\omega}, z + \frac{\tau_L + b\omega^*}{k_i}) = \frac{1}{2}J(\tilde{\omega} + \beta\chi(z))^2 - \frac{1}{2}J\beta^2\chi^2(z) + [k_i + \beta(b + k_p)] \int_{-\frac{\tau_L + b\omega^*}{k_i}}^z \chi(r)dr.$$

According to Definition 1:

$$|\sigma(z)| \geq \begin{cases} |z|, & |z| \leq L^* \\ L^*, & |z| > L^* \end{cases}$$

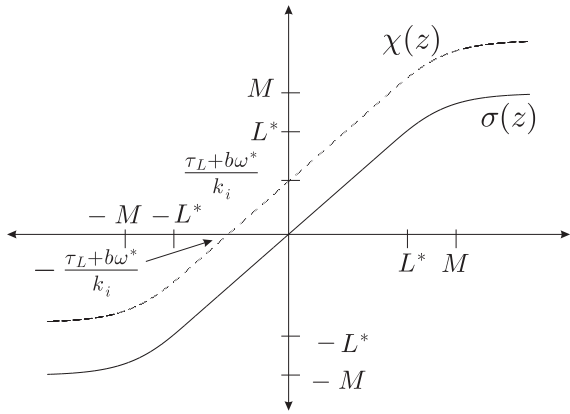
and, hence, by direct integration we find that (see fig. 11):

$$\int_{-\frac{\tau_L + b\omega^*}{k_i}}^z \chi(r)dr \geq G(z),$$

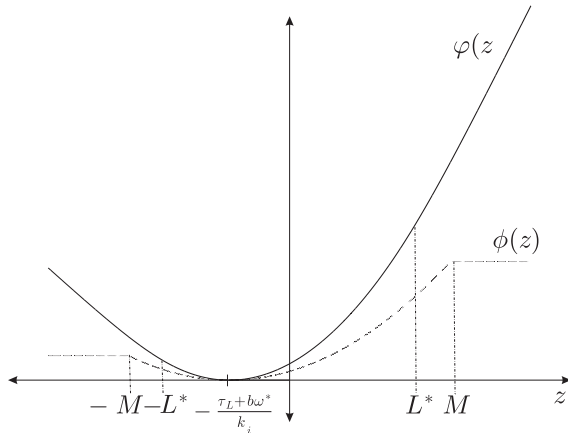
where:

$G(z)$

$$= \begin{cases} \frac{1}{2} \left( z + \frac{\tau_L + b\omega^*}{k_i} \right)^2, & |z| \leq L^* \\ \frac{1}{2} \left( L^* + \frac{\tau_L + b\omega^*}{k_i} \right)^2 + \left( L^* + \frac{\tau_L + b\omega^*}{k_i} \right) (z - L^*), & z > L^* \\ \frac{1}{2} \left( -L^* + \frac{\tau_L + b\omega^*}{k_i} \right)^2 + \left( -L^* + \frac{\tau_L + b\omega^*}{k_i} \right) (z + L^*), & z < -L^*. \end{cases}$$



(a)  $\sigma(z)$  and  $\chi(z)$ .



(b)  $\phi(z) = \frac{1}{2}J\beta^2 H(z)$  and  $\varphi(z) = (k_i + \beta(b + k_p))G(z)$ .

**FIGURE 11.** Graphical verification of (69). (a)  $\sigma(z)$  and  $\chi(z)$ .

Also notice that:

$$-\frac{1}{2}J\beta^2\chi^2(z) \geq -\frac{1}{2}J\beta^2H(z),$$

$$H(z) = \begin{cases} \frac{1}{2}\left(z + \frac{\tau_L + b\omega^*}{k_i}\right)^2, & |z| \leq M \\ \frac{1}{2}\left(M + \frac{\tau_L + b\omega^*}{k_i}\right)^2, & z > M \\ \frac{1}{2}\left(-M + \frac{\tau_L + b\omega^*}{k_i}\right)^2, & z < -M. \end{cases}$$

Hence, it is clear that it is always possible to find large enough constants  $k_i > 0$  and  $k_p > 0$  and a small enough constant  $\beta > 0$  such that:

$$[k_i + \beta(b + k_p)] \int_{-\frac{\tau_L + b\omega^*}{k_i}}^z \chi(r)dr - \frac{1}{2}J\beta^2\chi^2(z) \geq [k_i + \beta(b + k_p)]G(z) - \frac{1}{2}J\beta^2H(z) > 0, \quad (69)$$

which can be verified graphically (see fig. 11), i.e. that:

$$[k_i + \beta(b + k_p)] \int_{-\frac{\tau_L + b\omega^*}{k_i}}^z \chi(r)dr - \frac{1}{2}J\beta^2\chi^2(z),$$

is a positive definite radially unbounded function in  $z + \frac{\tau_L + b\omega^*}{k_i}$ . Thus, since term  $\frac{1}{2}J(\tilde{\omega} + \beta\chi(z))^2 \geq 0$  is zero only when  $\tilde{\omega} + \beta\chi(z) = 0$ , property in (69) ensures that  $V_\omega(\tilde{\omega}, z + \frac{\tau_L + b\omega^*}{k_i})$  is zero only when both  $\tilde{\omega}$  and  $z + \frac{\tau_L + b\omega^*}{k_i}$  are zero, i.e. when  $\chi(z) = 0$ . This proves that  $V_\omega$  is positive definite and radially unbounded.

**REFERENCES**

- [1] F. Anritter, P. Maurer, and J. Reger, “Flatness based control of a buck-converter driven DC motor,” in *Proc. 4th IFAC Symp. Mech. Syst.*, Heidelberg, Germany, 2006, pp. 1–6.
- [2] S. E. Lyshevski, *Electromechanical Systems, Electric Machines, and Applied Mechatronics*. Boca Raton, FL, USA: CRC Press, 1999.
- [3] I. Boldea and S. A. Nasar, *Electric Drives*. Boca Raton, FL, USA: CRC Press, 1999.
- [4] H. Fadil and F. Giri, “Accounting of DC-DC power converter dynamics in DC motor velocity adaptive control,” in *Proc. IEEE Int. Conf. Control Appl.*, Munich, Germany, Oct. 2006, pp. 3157–3162.
- [5] V. M. Hernández-Guzmán, R. Silva-Ortigoza, D. Muñoz-Carrillo, “Velocity control of a brushed DC-motor driven by a DC to DC buck power converter,” *Int. J. Innov. Comput., Inf. Control*, vol. 11, no. 2, pp. 509–521, 2015.
- [6] J. Linares-Flores and H. Sira-Ramirez, “A smooth starter for a DC machine: A flatness based approach,” in *Proc. 1st Int. Conf. Electr. Electron. Eng. (ICEEE)*, Acapulco, Mexico, Sep. 2004, pp. 589–594.
- [7] J. Linares-Flores and H. Sira-Ramirez, “Sliding mode-delta modulation GPI control of a DC motor through a buck converter,” in *Proc. 2nd IFAC Symp. Syst. Struct. Control*, Oaxaca, Mexico, Dec. 2004, pp. 405–409.
- [8] J. Linares-Flores and H. Sira-Ramirez, “DC motor velocity control through a DC-to-DC power converter,” in *Proc. 43rd IEEE Conf. Decis. Control (CDC)*, Atlantis, The Bahamas, Dec. 2004, pp. 5297–5302.
- [9] R. Silva-Ortigoza, J. R. García-Sánchez, J. M. Alba-Martínez, V. M. Hernández-Guzmán, M. Marcelino-Aranda, H. Taud, and R. Bautista-Quintero, “Two-stage control design of a buck converter/DC motor system without velocity measurements via a  $\Sigma - \Delta$ -modulator,” *Math. Problems Eng.*, vol. 2013, Jun. 2013, Art. no. 929316. [Online]. Available: <https://www.hindawi.com/journals/mpe/2013/929316/>
- [10] R. Silva-Ortigoza, C. Márquez-Sánchez, F. Carrizosa-Corral, M. Antonio-Cruz, J. M. Alba-Martínez, and G. Saldaña-González, “Hierarchical velocity control based on differential flatness for a DC/DC buck converter-DC motor system,” *Math. Problems Eng.*, vol. 2014, pp. 1–12, Apr. 2014. [Online]. Available: <https://www.hindawi.com/journals/mpe/2014/912815/>
- [11] R. Silva-Ortigoza, V. M. Hernandez-Guzman, M. Antonio-Cruz, and D. Munoz-Carrillo, “DC/DC buck power converter as a smooth starter for a DC motor based on a hierarchical control,” *IEEE Trans. Power Electron.*, vol. 30, no. 2, pp. 1076–1084, Feb. 2015.
- [12] H. Sira-Ramirez and M. A. Oliver-Salazar, “On the robust control of buck-converter DC-motor combinations,” *IEEE Trans. Power Electron.*, vol. 28, no. 8, pp. 3912–3922, Aug. 2013.
- [13] R. Sureshkumar and S. Ganeshkumar, “Comparative study of proportional integral and backstepping controller for buck converter,” in *Proc. Int. Conf. Emerg. Trends Electr. Comput. Technol.*, Nagercoil, India, Mar. 2011, pp. 375–379.
- [14] E. Hernández-Márquez, J. R. García-Sánchez, R. Silva-Ortigoza, M. Antonio-Cruz, V. M. Hernández-Guzmán, H. Taud, and M. Marcelino-Aranda, “Bidirectional tracking robust controls for a DC/DC buck converter-DC motor system,” *Complexity*, vol. 2018, pp. 1–10, Aug. 2018. [Online]. Available: <https://www.hindawi.com/journals/complexity/2018/1260743>
- [15] E. Hernandez-Marquez, R. Silva-Ortigoza, J. R. Garcia-Sanchez, M. Marcelino-Aranda, and G. Saldana-Gonzalez, “A DC/DC buck-boost converter-inverter-DC motor system: Sensorless passivity-based control,” *IEEE Access*, vol. 6, pp. 31486–31492, 2018. [Online]. Available: <https://ieeexplore.ieee.org/document/8382160>
- [16] R. S. Ortigoza, J. N. A. Juárez, J. R. G. Sánchez, V. M. H. Guzmán, C. Y. S. Cervantes, and H. Taud, “A sensorless passivity-based control for the DC/DC buck converter-inverter-DC motor system,” (in Spanish), *IEEE Latin Amer. Trans.*, vol. 14, no. 10, pp. 4227–4234, Oct. 2016.

- [17] R. Silva Ortigoza, J. N. Alba Juarez, J. R. Garcia Sanchez, M. Antonio Cruz, V. M. Hernandez Guzman, and H. Taud, "Modeling and experimental validation of a bidirectional DC/DC buck power electronic converter-DC motor system," (in Spanish), *IEEE Latin Amer. Trans.*, vol. 15, no. 6, pp. 1043–1051, Jun. 2017.
- [18] E. Hernández-Márquez, "DC/DC electronic power converter-based AC generation and its employment as motor drives: Control design and experimental implementation," Ph.D. dissertation, IPN-CIDETEC, Mexico City, Mexico, 2019.
- [19] R. Ortega, A. Loria, P. Nicklasson, and H. Sira-Ramirez, *Passivity-based Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications*, London, U.K.: Springer, 1998.
- [20] A. Astolfi and R. Ortega, "Immersion and invariance: A new tool for stabilization and adaptive control of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 48, no. 4, pp. 590–606, Apr. 2003.
- [21] J. Chiasson, *Modeling and High Performance Control of Electric Machines*. Hoboken, NJ, USA: Wiley, 2005.
- [22] D. M. Dawson, J. Hu, and T. C. Burg, *Nonlinear Control of Electric Machinery*. New York, NY, USA: Marcel Dekker, 1998.
- [23] A. Zavala-Rio and V. Santibanez, "A natural saturating extension of the PD-with-desired-gravity-compensation control law for robot manipulators with bounded inputs," *IEEE Trans. Robot.*, vol. 23, no. 2, pp. 386–391, Apr. 2007.
- [24] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.
- [25] F. Mendoza-Mondragon, V. M. Hernandez-Guzman, and J. Rodriguez-Resendiz, "Robust speed control of permanent magnet synchronous motors using Two-Degrees-of-Freedom control," *IEEE Trans. Ind. Electron.*, vol. 65, no. 8, pp. 6099–6108, Aug. 2018.



#### VICTOR MANUEL HERNÁNDEZ-GUZMÁN

was born in Querétaro, Mexico. He received the B.S. degree in electrical engineering from the Instituto Tecnológico de Querétaro, Querétaro, in 1988, the M.S. degree in electrical engineering from the Instituto Tecnológico de la Laguna, Torreon, Mexico, in 1991, and the Ph.D. degree in electrical engineering from the Centro de Investigación y Estudios Avanzados (CINVESTAV), Instituto Politécnico Nacional (IPN), Mexico City,

Mexico, in 2003.

He has been a Professor with the Universidad Autónoma de Querétaro, Querétaro, since 1995, where he teaches classical and nonlinear control. He has coauthored the book *Automatic Control: Design Theory, Prototype Construction, Modeling, Identification, and Experimental Tests* (in Spanish) (Mexico: Coleccion, CIDETEC, IPN, 2013) and the book *Automatic Control With Experiments* (Cham, Switzerland: Springer, 2019). He has published over 45 articles in refereed journals. His research interests include the control of mechatronic systems, mobile robots, electromechanical systems, and different classes of ac electric motors when actuating on complex nonlinear mechanical loads. He has a particular interest in prototype construction to teach classical and nonlinear control techniques.



**RAMÓN SILVA-ORTIGOZA** received the B.S. degree in electronics from the Benemerita Universidad Autónoma de Puebla, Puebla, Mexico, in 1999, and the M.S. and Ph.D. degrees in electrical engineering (mechatronics) from the Centro de Investigación y Estudios Avanzados (CINVESTAV), Instituto Politécnico Nacional (IPN), Mexico City, Mexico, in 2002 and 2006, respectively.

He has been a Researcher with the Department of Mechatronics and Renewable Energy, Centro de Innovación y Desarrollo Tecnológico en Computo (CIDETEC), IPN, since 2006, and belongs to SNI-CONACYT, Mexico. He has coauthored the book *Control Design Techniques in Power Electronics Devices* (London, U.K.: Springer-Verlag, 2006), the book *Automatic Control: Design Theory, Prototype Construction, Modeling, Identification, and Experimental Tests* (in Spanish) (Mexico: CIDETEC, IPN, 2013), and the book *Automatic Control With Experiments* (Cham, Switzerland: Springer, 2019). He has published over 55 articles in JCR indexed journals and three chapters in international books, and he has presented over 40 papers in international conferences. He has been an advisor of over 25 master's degree students and two B.S. degree students. He has been the leader of more than ten research projects, and he has been in collaboration with nine additional research projects. His research interests include mechatronic control systems, mobile robotics, control in power electronics, and the development of educational technology. His research work has been cited over 1000 times. Five of his students have been honored with the Presea Lazaro Cardenas Award, the most important prize, granted by the Instituto Politécnico Nacional, in 2012, 2015, 2016, 2018, and 2019. Also, two of these students have been honored with the Best Master Thesis Award from the Instituto Politécnico Nacional, in 2015 and 2017. He has also been a referee of several awards of research and engineering in Mexico, the National Program of Quality Postgraduate, and research projects of CONACYT. He was an Editor of the book *Mechatronics* (in Spanish) (Mexico: CIDETEC, IPN, 2010). He has been a Reviewer of several JCR indexed journals.



**JORGE ORRANTE-SAKANASSI** was born in Torreon, Mexico, in 1983. He received the B.Sc., M.Sc., and Ph.D. degrees from the Instituto Tecnológico de La Laguna, in 2006, 2008, and 2012, respectively.

He was a Postdoctoral Fellow of the Universidad Autónoma de Querétaro, from 2013 to 2015, and he was a Professor of CONACYT commissioned at the Tecnológico Nacional de México, Instituto Tecnológico de Hermosillo, from 2015 to 2018. He has been attached with the Tecnológico Nacional de México, División de Estudios de Posgrado e Investigación, Instituto Tecnológico de Matamoros, since 2018. His research interests are in the control of mechanical systems, robot manipulators, electromechanical systems, modeling, nonlinear systems, and stability analysis.

...