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On Selection Combining Diversity in Dual-Hop Relaying Systems Over Double Rice Channels: Fade Statistics and Performance Analysis

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ABSTRACT This paper investigates the performance of selection combining (SC) diversity in dual-hop cooperative networks under double Rice fading channels. Expressions for the probability density function (PDF), cumulative distribution function (CDF), level-crossing rate (LCR), and average duration of fades (ADF) of the SC output fading processes are first investigated. The obtained quantities are in the form of finite and semi-infinite range integrals, which can be easily computed using numerical tools. For the PDF, CDF, and LCR statistics corresponding approximate solutions are determined using the Laplace method's of integration. The PDF and CDF are then applied to derive exact and approximate solutions for the average symbol error probability (SEP) of non-coherent M-ary differential phase-shift keying (M-DPSK) modulation. Moreover, results corresponding to the special cases of mixed Rayleigh×Rice and double Rayleigh fading channels are extracted. The validity of the derived expressions and the accuracy of the approximations have been checked by using computer simulations. The results can be useful in the performance assessment of mobile-to-mobile (M2M) communications with dual-hop cooperative SC diversity and millimeter wave (mmWave) bands where, in general, the line-of-sight (LOS) propagation arises.

INDEX TERMS Double Rice fading, selection combining (SC) diversity, dual-hop cooperative system, level-crossing rate (LCR), average duration of fades (ADF), symbol error probability (SEP), M-ary differential phase-shift keying (M-DPSK) modulation.

I. INTRODUCTION

Multiplicative fading models [1], also referred to as cascaded channels, are suitable for modeling multipath effects in a variety of wireless communication scenarios ranging from relay based systems [2], [3], inter-vehicular communications [4], to keyhole phenomenon in multiple-input multiple-output (MIMO) systems [5]. For this reason, the statistics of the underlying channels as well as the related performance analysis have received a great deal of attention in recent years. For instance, initial studies have been reported in [6] and [1], where the first-order statistics of double Rayleigh channels, constructed as the product of 2 Rayleigh random variables (RVs), have been investigated. The second-order statistics of multi-hop Rayleigh channels have been analyzed in [7], whereas those corresponding to the double Nakagami-m fading model have been investigated in [8], where expressions for the level-crossing rate (LCR) (or equivalently the frequency of outage) and the average duration of fades (ADF) have been obtained. The first- and second-order statistics of double Rice fading channels have been studied in [9], while the corresponding outage probability has been presented in [10]. Recently, novel analytical formulations for many of the fundamental statistics of two independent and non-identically distributed $\kappa - \mu$ RVs have been investigated in [11]. Closed-form expressions for the $n-$th moment, amount of fading, and ergodic capacity of the end-to-end signal-to-noise ratio (SNR) of a dual-hop amplify-and-forward (AF) cooperative communication systems over $\alpha - \eta - \mu$ fading channels have been explored in [12]. In the context of cooperative vehicular ad-hoc networks (VANETs), first- and second-order statistics have been derived together with the outage probability of radio links under the double-generalized Gamma fading channel [13]. In [14], the error rate performance, in terms of the bit error probability (BEP) of M-ary phase shift

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keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM) of AF relaying for an inter-vehicular cooperative scheme, operating over cascaded Nakagami-m fading, has been analyzed and optimized. Bithas *et al.* [15] have studied the impact of co-channel interference (CCI) and outdated channel state information (CSI) on the performance of a vehicular-to-vehicular cooperative system operating over double Nakagami-m fading channels. More recently, exact expressions for the moment generating function (MGF), higher order moments of the SNR, ergodic capacity, and average symbol error probability (SEP) of bidirectional AF based cooperative vehicular systems, operating over cascaded Nakagami-m fading channels, have been investigated in [16]. Considering asymmetrical Rayleigh×Rice fading channels and MIMO relaying, Jayasinghe *et al.* have presented main performance metrics, such as the BEP, moments of the SNR, and the amount of fading, in [17], [18]. Likewise, a study on the statistical properties of double Hoyt fading, with applications to the performance analysis of the average SEP of both coherent M-PSK and square M-QAM schemes, has been presented in [19]. Khattabi [20] recently introduced a new analytical approach to analyze the per-frame-average SEP of CSI-and-noise-assisted dual-hop AF cooperative systems, operating in Rayleigh multipath fading, where the nodes' mobility and imperfect CSI estimates have been considered. More recently, the average SEP of M-QAM modulation of dual-hop CSI-assisted AF cooperative systems, operating in Rayleigh fading environments, and affected by the in-phase and quadrature imbalance, has been studied in [21].

As can be noted, the above survey has been devoted to the conventional multi-hop transmission where the link consists of a series of relaying stations. Apart from this, the impact of multiplicative fading channels has, as well, been addressed in the context of cooperative diversity [3] based systems. For example, the first- and second-order statistics of equal gain combiner (EGC) diversity signals have been investigated in [22] for double Rayleigh fading, and later on in [23] for double Rice propagation scenario. It should be mentioned that, owing to the complexity of the problem that has been dealt with in [22], the results reported there have been obtained by resorting to the approximation of the PDF of the sum of double Rayleigh processes by the gamma distribution [24]. By invoking this same concept of approximation, and considering double Rice fading, the performance of M-PSK modulation with EGC has been addressed in [25]. Recently, Peppas *et al.* [26] approximated the distribution of the sum of generalized normal RVs by invoking the moments matching method, and applied the results to the error performance of EGC for both double Rayleigh and double Nakagami-m fading channels. In addition to the EGC, the maximum ratio combining (MRC) diversity has also been studied. For instance, the performance analysis of M-PSK modulation scheme, with MRC diversity, under double Nakagami-m and double Rice propagation scenarios, has been presented in [27] and [28], respectively. MRC has as well been addressed in [29], where the outage probability and amount of fading of inter-vehicular communications have been investigated, under $n*$Rayleigh fading channels.

Despite the simplicity of implementation of selection combining (SC) diversity, there are few works in the literature dealing with performance analysis issues of cooperative relaying systems under cascaded fading channels. For instance, the authors in [30] have investigated the first- and second-order statistics of SC over double Nakagami-m fading channels. Expressions for the outage probability and error rate of M-PSK modulation have been provided in [31] considering SC diversity over double Rayleigh channels. Paper [32] addressed the derivation of expressions for the BEP performance of binary non-coherent frequency-shift keying (FSK) and differential phase-shift keying (DPSK) modulations under double Rice fading channels.

From the brief summary of the state-of-the-art reported above one can note a lack of contributions on the performance analysis of SC diversity in cooperative relaying systems under double Rice fading channels. The intention of this paper is to contribute to this topic. It builds on our previous study reported in [33], where first- and second-order statistics of double Rice fading channels with SC diversity have been presented.

We specifically consider a dual-hop, multi-relay based cooperative diversity system over which the channel gain on each diversity link is described by a double Rice process. All the sub-channels of the underlying radio system are assumed to be independent and identically distributed (i.i.d.). The relay stations are assumed to transmit the symbols to the destination by using the time division multiple access (TDMA) protocol. Then, the mobile station employs the SC diversity to combine signals received over the TDMA frame. It should be emphasized that this relaying scenario, combined with SC diversity, has also been studied in [34]–[36].

Under the above setting, we provide theoretical results for the first- and second-order statistics of the fading process. We also derive expressions for the SEP of the non-coherent M-ary DPSK (M-DPSK) modulation scheme. Mainly, the major contributions of this paper are summarized as follows:

- We first provide exact and approximate expressions for the cumulative distribution function (CDF) and probability density function (PDF) of the fading envelope available at the output of the SC diversity.
- Considering the underlying envelope, we also present expressions for the LCR and ADF. As is known, these quantities are useful in studying the outage statistics of radio communications.
- Then, we confine our attention to the investigation of the SEP of the non-coherent M-DPSK modulation. This type of modulation is of practical interest owing to its simplicity of implementation. Results corresponding to both Rayleigh×Rice and double Rayleigh fading channels have also been deduced as special cases.
- Furthermore, the validity of the derived expressions has been checked by means of computer simulations.

To the best of our knowledge, we are not aware of any study dealing with SC based cooperative diversity in double Rice fading channels. The obtained results are useful in studying the performance of, for example, mobile-to-mobile (M2M) communications in a dual-hop cooperative SC diversity network using millimeter wave (mmWave) bands, where the radio link is splitted into short hops to compensate for the high path-loss. This fact leads in general to line-of-sight (LOS) propagation scenarios. It is appropriate to mention that the exact, or even, the approximate solutions, reported here, are in the form of integrals, which can, however, be easily evaluated numerically. This work takes a step towards analyzing the performance of cooperative SC diversity in double Rice fading. Deriving closed-form expressions remains, in our opinion, an open complex problem.

The rest of the paper is organized as follows. In Section II, we present the system model, and provide a review of known statistics on double Rice fading channels. Section III is devoted to the presentation of the CDF and PDF of the fading process available at the output of the SC diversity, whereas, in Section IV, expressions for the LCR and ADF of the underlying envelope process have been derived. We investigate in section V the SEP of non-coherent M-DPSK modulation for a multi-relay cooperative SC diversity. Numerical and simulation results are presented in Section VI. Finally, the conclusion is outlined in Section VII.

II. SYSTEM AND FADING MODELS

As stated above, the problem under investigation is concerned with the performance of selective diversity applied to multi-relay communication systems in double Rice fading. In this section, we briefly provide a description of the system involved, and review known statistics on double Rice channels. The block diagram of the relaying system is shown in Fig. 1. It consists of a transmitter, a receiver, and L relays, each of which is equipped with a single antenna. We assume that there is no direct radio path between the transmitter and the receiver, and the communication is, therefore, considered to be only through the relays. These relays are assumed to operate according to the TDMA fixed gain AF protocols proposed in [37], [38]. Under this transmission scenario, the signals received over the L time slots are combined using SC diversity. This is equivalent to selecting the relay over which we have the highest end-to-end SNR. The fixed gains, introduced in the relays, are taken, without loss of generality, to be one. The multipath complex channel gains over the two hops, denoted by $\eta_i^j(t)$ $(i = 1, 2;\ j = 1, 2, \ldots, L)$, in Fig. 1, are assumed to be described statistically by the Rice flat fading model. It follows that each of the L end-to-end relaying paths is subject to double Rice flat fading. Referring to Fig. 1, a double Rice fading process is simply obtained as the product of two single Rice processes $\eta_1(t)$ and $\eta_2(t)$. That is, by omitting the superscript j $(j = 1, 2, \ldots, L)$, for simplicity, we have [9]

$$\eta(t) = \eta_1(t)\eta_2(t). \tag{1}$$

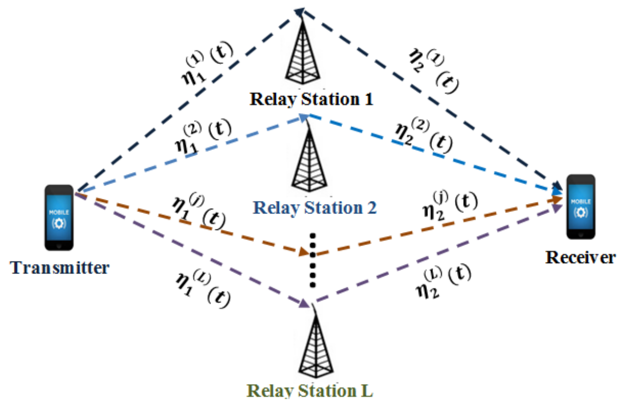

FIGURE 1. Dual-Hop multi-relay communication system.

For the statistics of the SC fading process to be determined, we need to know those corresponding to the double Rice process $\eta(t)$. The PDF, CDF, and LCR of $\eta(t)$ have been derived in [9], and are reviewed subsequently, for the sake of readability. Exploiting the independence assumption of $\eta_1(t)$ and $\eta_2(t)$, the PDF of $\eta(t)$ has been obtained as [9, eq. (12)]

$$p_\eta(z) = \frac{4zk_T}{\Omega_1\Omega_2}\int_0^\infty \frac{1}{y} e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)} e^{-\left(\frac{z^2(1+k_1)}{\Omega_1 y^2}+k_1\right)} \times I_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right) I_0\left(\frac{2z}{y}\sqrt{\frac{k_1(1+k_1)}{\Omega_1}}\right) dy \tag{2}$$

where $I_0(.)$ denotes the zeroth-order modified Bessel function of the first kind [39], k_i $(i = 1, 2)$ stands for the Rice factor of the ith envelope $\eta_i(t)$, Ω_i is the common mean power of the radio signals propagating over the ith hop, and $k_T = (1 + k_1)(1 + k_2)$. Concerning the CDF of the double Rice process $\eta(t)$, it has been determined in [9, eq. (31)] as

$$F_\eta(z) = 1 - \int_0^\infty Q_1\left(\sqrt{2k_1}, \frac{z}{y}\sqrt{\frac{2(1+k_1)}{\Omega_1}}\right) p_{\eta_2}(y)dy \tag{3}$$

where $z \geq 0$, $Q_1(\cdot,\cdot)$ is the first-order Marcum Q-function [40], and $p_{\eta_2}(y)$ stands for the PDF of the single Rice process $\eta_2(t)$ given by [41]

$$p_{\eta_2}(y) = \frac{2y(1+k_2)}{\Omega_2} e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)} I_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right). \tag{4}$$

Finally, in [9, eq. 26] as well, the LCR (or equivalently the frequency of outage) of double Rice processes has been derived to be

$$N_\eta(r) = \frac{4rk_T}{(2\pi)^{5/2}\Omega_1\Omega_2}\int_0^\infty \sqrt{\beta_2\frac{r^2}{y^4}+\beta_1} e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)}$$

$$\times e^{-\left(\frac{r^2(1+k_1)}{y^2\Omega_1}+k_1\right)} \int_{-\pi}^{\pi} e^{\frac{2r\cos\theta_1}{y}\sqrt{\frac{k_1(1+k_1)}{\Omega_1}}}$$
$$\times \int_{-\pi}^{\pi} e^{2y\cos\theta_2\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}} e^{-\frac{K^2(r,y,\theta_1,\theta_2)}{2}}$$
$$\times \left(1+\sqrt{\frac{\pi}{2}}K(r,y,\theta_1,\theta_2)\, e^{\frac{1}{2}K^2(r,y,\theta_1,\theta_2)}\right.$$
$$\left.\times \left\{1+\Phi\left(\frac{K(r,y,\theta_1,\theta_2)}{2}\right)\right\}\right) d\theta_2 d\theta_1 dy \quad (5)$$

where β_i $(i = 1, 2)$ is the variance of the process $\dot{\eta}_i(t)$, in which the upper dot denotes time derivative. In (5) also, $\Phi(\cdot)$ denotes the error function [39, eq. 8.250(1)], and the function $K(\cdot,\cdot,\cdot,\cdot)$ is defined by [9]

$$K(r,y,\theta_1,\theta_2) = \frac{2\pi\sqrt{\frac{k_1\Omega_1}{(1+k_1)}}f_{\rho_1}y^2\sin(\theta_1)}{\sqrt{\beta_1y^4+\beta_2r^2}} + \frac{2\pi\sqrt{\frac{k_2\Omega_2}{(1+k_2)}}f_{\rho_2}r\sin(\theta_2)}{\sqrt{\beta_1y^4+\beta_2r^2}} \quad (6)$$

where f_{ρ_i} $(i = 1, 2)$ is the Doppler frequency shift experienced on the LOS component of the hop i. Here, we point out that none of the semi-infinite range integrals involved in (2), (3), and (5), is known to admit a closed-form solution. These complicated statistical properties constitutes the main drawback of the double Rice fading model. For this reason, and to circumvent this limitation, in [25] for example, Laguerre series based approximation has been resorted to study the performance of EGC diversity. With the reviewed results at hand, we are now in a position to determine the fade statistics of SC in the dual-hop multi-relay system illustrated in Fig. 1. In SC diversity, assuming that all the dual-hop links are subject to the same mean noise power, the best relay can be chosen according to the highest amplitude among the L double Rice fading processes. That is, the fading envelope, available at the output of the SC receiver, can be written as

$$\eta_{SC}(t) = \max\left(\eta^{(1)}(t),\ldots,\eta^{(j)}(t),\ldots,\eta^{(L)}(t)\right) \quad (7)$$

where $\eta^{(j)}(t)$ $(j = 1,\ldots,L)$ stands for the double Rice fading process of the jth relaying path. In the following section, the CDF and PDF of the envelope process $\eta_{SC}(t)$ will be investigated.

III. CDF AND PDF OF THE ENVELOPE PROCESS $\eta_{SC}(t)$

A. CDF OF THE ENVELOPE PROCESS $\eta_{SC}(t)$

1) EXACT EXPRESSION

The CDF of the fading process $\eta_{SC}(t)$ can be obtained according to

$$F_{\eta_{SC}}(z) = \Pr\left[\eta_{SC}(t) \le z\right] = \Pr\left[\eta^{(1)}(t) \le z,\ldots,\eta^{(j)}(t) \le z,\ldots,\eta^{(L)}(t) \le z\right]. \quad (8)$$

where $\Pr[\cdot]$ denotes the probability operator. By assuming that all the L fading processes affecting the L relaying paths of the cooperative diversity system are i.i.d., the CDF of the envelope process $\eta_{SC}(t)$ is expressed by

$$F_{\eta_{SC}}(z) = \left(F_\eta(z)\right)^L = \left(1-\int_0^\infty Q_1\left(\sqrt{2k_1}, \frac{z}{y}\sqrt{\frac{2(1+k_1)}{\Omega_1}}\right)p_{\eta_2}(y)dy\right)^L \quad (9)$$

where, as mentioned earlier, the semi-infinite integral can be computed using numerical techniques.

2) SPECIAL CASES

We discuss here the special cases of the CDF $F_{\eta_{SC}}(z)$. Specifically, we present below simplified expressions for the CDFs of Rayleigh×Rice and double Rayleigh fading channels.

- ***CDF of SC diversity in Rayleigh×Rice fading:*** By letting $k_1 = 0$ in (9), the corresponding CDF $F_{\eta_{SC}}(z)$ is given by

$$F_{\eta_{SC}}(z) = \left(1-\int_0^\infty e^{\left(-\frac{z^2}{y^2\Omega_1}\right)}p_{\eta_2}(y)dy\right)^L. \quad (10)$$

- ***CDF of SC diversity in double Rayleigh fading:*** First fixing $k_2 = 0$ in (10), and then invoking [39, eq. 3.471(9)], the integral in (10) can analytically be solved, and results in the following simplified expression

$$F_{\eta_{SC}}(z) = \left(1-\frac{2z}{\sqrt{\Omega_1\Omega_2}}K_1\left(\frac{2z}{\sqrt{\Omega_1\Omega_2}}\right)\right)^L \quad (11)$$

  where $K_1(.)$ denotes the first-order modified Bessel function of the second kind [39]. Obviously, letting $L = 1$ in (11), yields the CDF of the double Rayleigh fading process given in [6, eq. (4)].

At this point, it is worth mentioning that the CDF can directly be applied to get the outage probability, which is a standard performance metric used in wireless communication systems. This metric is defined as the probability that the fading envelope falls below a given threshold r. Hence, the outage probability is immediately obtained from the CDF as $P_{out}(r) = F_{\eta_{SC}}(r)$.

3) APPROXIMATE SOLUTION

From (9), we have $F_{\eta_{SC}}(z) = (F_\eta(z))^L$, where $F_\eta(z)$ is given in terms of a semi-infinite range integral as can be noted from (3). In other words, seeking an approximation for $F_{\eta_{SC}}(z)$ consists in finding an approximation for $F_\eta(z)$. To this end, we apply the Laplace's method of integration [43] on (3) to get an approximate solution to the underlying integral. The application of this method leads, as shown in Appendix A, to the following approximation for $F_{\eta_{SC}}(z)$

$$F_{\eta_{SC}}(z) \approx \left(\sqrt{2\pi\frac{1+k_2}{\Omega_2}}\exp(-\frac{1}{2}(1+\log(2\frac{1+k_2}{\Omega_2})-k_2)\right. \\ \left.\times I_0(\sqrt{2k_2})\left(1-Q_1(\sqrt{2k_1},2z\sqrt{\frac{k_T}{\Omega_1\Omega_2}})\right)\right)^L. \tag{12}$$

Setting $k_1 = 0$ in (12), gives the following approximate solution for the CDF of SC diversity in Rayleigh×Rice fading channels

$$F_{\eta_{SC}}(z) \approx \left(\sqrt{2\pi\frac{1+k_2}{\Omega_2}}\exp(-\frac{1}{2}(1+\log(2\frac{1+k_2}{\Omega_2})-k_2)\right. \\ \left.\times I_0(\sqrt{2k_2})\left(1-\exp(-2z^2\frac{1+k_2}{\Omega_2})\right)\right)^L. \tag{13}$$

Letting, in addition, $k_2 = 0$ in (13), i.e., the double Rayleigh fading, we obtain

$$F_{\eta_{SC}}(z) \approx \left(\sqrt{\frac{2\pi}{\Omega_2}}\exp(-\frac{1}{2}(1+\log(\frac{2}{\Omega_2}))\right. \\ \left.\times\left(1-\exp(-\frac{2z^2}{\Omega_2})\right)\right)^L. \tag{14}$$

To the best of our knowledge, the approximations presented above are new.

B. PDF OF THE ENVELOPE PROCESS $\eta_{SC}(t)$

1) EXACT EXPRESSION

The PDF of the envelope process $\eta_{SC}(t)$ can now be deduced by differentiating the CDF $F_{\eta_{SC}}(z)$ in (9) as follows

$$p_{\eta_{SC}}(z) = \frac{d}{dz}\left(F_{\eta_{SC}}(z)\right) \\ = L\cdot p_\eta(z)\cdot(F_\eta(z))^{L-1} \tag{15}$$

where $p_\eta(z)$ and $F_\eta(z)$ are the PDF and CDF of the double Rice fading given by (2) and (3), respectively. Obviously, by considering the non-diversity case, i.e., $L = 1$, (15) simplifies to the PDF of double Rice processes given in (2). In addition, (15) encompasses the PDFs corresponding to Rayleigh×Rice and double Rayleigh channels. Explicit expressions for both fading scenarios are presented below.

- ***PDF of SC diversity in Rayleigh×Rice fading:*** Letting $k_1 = 0$ in (15), results in

$$p_{\eta_{SC}}(z) = L\left(1-\int_0^\infty e^{\left(-\frac{z^2}{y^2\Omega_1}\right)}p_{\eta_2}(y)dy\right)^{L-1} \\ \times\int_0^\infty\frac{2z}{y^2\Omega_1}e^{\left(-\frac{z^2}{y^2\Omega_1}\right)}p_{\eta_2}(y)dy. \tag{16}$$

- ***PDF of SC diversity in double Rayleigh fading:*** Fixing $k_2 = 0$ in (16), and using [39, eq.3.471(9)], yields the following simplified expression for $p_{\eta_{SC}}(z)$

$$p_{\eta_{SC}}(z) = \frac{4Lz}{\Omega_1\Omega_2}K_0\left(\frac{2z}{\sqrt{\Omega_1\Omega_2}}\right) \\ \times\left(1-\frac{2z}{\sqrt{\Omega_1\Omega_2}}K_1\left(\frac{2z}{\sqrt{\Omega_1\Omega_2}}\right)\right)^{L-1} \tag{17}$$

where $K_0(\cdot)$ is the zeroth-order modified Bessel function of the second kind [39]. Finally, for $L = 1$, (17) reduces to the PDF of double Rayleigh processes [6].

2) APPROXIMATE SOLUTION

In a similar manner as for the CDF, we provide below a simplified approximate solution for the PDF $p_{\eta_{SC}}(z)$ by finding an approximation for the PDF $p_\eta(z)$. Again, this is achieved by invoking the Laplace's method of integration. The application of this method yields, as detailed in Appendix B, the following approximate quantity for $p_{\eta_{SC}}(z)$

$$p_{\eta_{SC}}(z) \approx 2L\sqrt{\pi z}\left(\frac{k_T}{\Omega_1\Omega_2}\right)^{\frac{3}{4}}I_0\left(2\sqrt{zk_2}\left(\frac{k_T}{\Omega_1\Omega_2}\right)^{\frac{1}{4}}\right) \\ \times I_0\left(2\sqrt{zk_1}\left(\frac{k_T}{\Omega_1\Omega_2}\right)^{\frac{1}{4}}\right) \\ \times e^{-\left(2z\sqrt{\frac{k_T}{\Omega_1\Omega_2}}+k_2+k_1\right)}\left(F_\eta(z)\right)^{L-1}. \tag{18}$$

Again, (18) is generic, and contains approximate results for Rayleigh×Rice and double Rayleigh channels as special cases. These results are provided below.

- ***Rayleigh×Rice fading:*** Letting $k_1 = 0$ in (18), we obtain

$$p_{\eta_{SC}}(z) \approx 2L\sqrt{\pi z}\left(\frac{1+k_2}{\Omega_1\Omega_2}\right)^{\frac{3}{4}} \\ \times I_0\left(2\sqrt{zk_2}\left(\frac{1+k_2}{\Omega_1\Omega_2}\right)^{\frac{1}{4}}\right) \\ \times e^{-\left(2z\sqrt{\frac{1+k_2}{\Omega_1\Omega_2}}+k_2\right)}\left(1-\int_0^\infty e^{\left(-\frac{z^2}{y^2\Omega_1}\right)}p_{\eta_2}(y)dy\right)^{L-1}. \tag{19}$$

- ***Double Rayleigh fading:*** Setting $k_2 = 0$ in (19), and using [39, eq. 3.471(9)], results in the following PDF $p_{\eta_{SC}}(z)$

$$p_{\eta_{SC}}(z) \approx 2L\sqrt{\pi z}\left(\frac{1}{\Omega_1\Omega_2}\right)^{\frac{3}{4}}e^{-\frac{2z}{\sqrt{\Omega_1\Omega_2}}} \\ \times\left(1-\frac{2z}{\sqrt{\Omega_1\Omega_2}}K_1\left(\frac{2z}{\sqrt{\Omega_1\Omega_2}}\right)\right)^{L-1}. \tag{20}$$

As an other point, we show in the following that the approximate PDF given in (18) permits to find a rather alternative approximation for the CDF $F_{\eta_{SC}}(z)$. Indeed, $F_{\eta_{SC}}(z)$ can be obtained from (18) according to [42]

$$\begin{aligned} F_{\eta_{SC}}(z) &= \int_0^z p_{\eta_{SC}}(x)dx \\ &\approx 2L\sqrt{\pi}(\frac{k_T}{\Omega_1\Omega_2})^{\frac{3}{4}} \exp(-k_1-k_2) \\ &\quad \times \int_0^z \sqrt{x} I_0(2\sqrt{xk_2}(\frac{k_T}{\Omega_1\Omega_2})^{\frac{1}{4}}) I_0(2\sqrt{xk_1}(\frac{k_T}{\Omega_1\Omega_2})^{\frac{1}{4}}) \\ &\quad \times \exp(-2x\sqrt{\frac{k_T}{\Omega_1\Omega_2}})(F_\eta(x))^{L-1}dx. \end{aligned} \tag{21}$$

Obviously, approximate CDFs can be extracted for Rayleigh×Rice and double Rayleigh fading by letting, in (21), $k_1 = 0$ and $k_1 = k_2 = 0$, respectively.

IV. SECOND-ORDER STATISTICS OF THE ENVELOPE PROCESS $\eta_{SC}(t)$

In this section, we focus on the derivation of the second order statistics of the fading process $\eta_{SC}(t)$ in terms of the LCR and ADF. Recall that the LCR and ADF are useful in determining, respectively, the frequency and the average time duration of outage of radio links.

A. LCR OF THE ENVELOPE PROCESS $\eta_{SC}(t)$

In problems dealing with the investigation of the LCR of selective diversity, where the diversity paths are subject to independent fading, the results reported in [44, eq. (12)] can be applied to yield

$$N_{\eta_{SC}}(r) = \sum_{l=1}^{L} N_{\eta^{(l)}}(r) \prod_{\substack{k=1 \\ k\neq l}}^{L} F_{\eta^{(k)}}(r) \tag{22}$$

where $N_{\eta^{(l)}}(r)$ $(l = 1, \ldots, L)$ represents the LCR of the lth SC diversity path available in (5), while $F_{\eta^{(k)}}(r)$ $(k = 1\ldots, L)$ is the CDF of the kth relaying path given in (3). Since we are studying the case where the fading processes of the cooperative diversity system are i.i.d., (22) simplifies to

$$N_{\eta_{SC}}(r) = L \cdot N_\eta(r) \cdot (F_\eta(r))^{L-1}. \tag{23}$$

Upon replacement of the appropriate quantities in (23), we get the expression (24), as shown at the bottom of the next page, for the LCR $N_{\eta_{SC}}(r)$, which is shown at the bottom of the next page. Unfortunately, the expression involves four integrals that can be evaluated only by using numerical techniques. For the special cases given below, simplification of (24) is possible.

1) SPECIAL CASES

For the special case where the Doppler frequencies of the LOS components are equal to zero, i.e., $f_{\rho_1} = f_{\rho_2} = 0$, (24) simplifies, and yields the equation (25), which is given at the bottom of the next page. Regarding the special case of Rayleigh×Rice fading channels, i.e., the case where $k_1 = 0$, the finite-range integral with respect to the variable θ_1 in (24) can be evaluated and the result is provided in (26), as shown at the bottom of the next page, where the function $F(\cdot,\cdot,\cdot)$ is given by

$$F(r, y, \theta_2) = \frac{2\pi\sqrt{k_2\Omega_2} f_{\rho_2} r \sin(\theta_2)}{\sqrt{(1+k_2)\left(\beta_1 y^4 + \beta_2 r^2\right)}}. \tag{27}$$

Also, for the special case corresponding to $f_{\rho_2} = 0$, (26) reduces to

$$\begin{aligned} N_{\eta_{SC}}(r) &= \frac{4Lr(1+k_2)}{\sqrt{2\pi}\Omega_1\Omega_2} \int_0^\infty \sqrt{\beta_2 \frac{r^2}{y^4} + \beta_1} \\ &\quad \times e^{-\left(\frac{y^2(1+k_2)}{\Omega_2} + k_2\right)} \\ &\quad \times e^{-\left(\frac{r^2}{\Omega_1 y^2}\right)} I_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right) dy \\ &\quad \times \left\{1 - \int_0^\infty e^{\left(-\frac{r^2}{z^2\Omega_1}\right)} p_{\eta_2}(z)dz\right\}^{L-1}. \end{aligned} \tag{28}$$

Additionally, by fixing $k_1 = k_2 = 0$ in (24), a simplified expression for the LCR of SC diversity in double Rayleigh fading channels can be deduced as

$$\begin{aligned} N_{\eta_{SC}}(r) &= \frac{4Lr}{\sqrt{2\pi}\Omega_1\Omega_2} \\ &\quad \times \left(1 - 2r\sqrt{\frac{1}{\Omega_1\Omega_2}} K_1\left(2r\sqrt{\frac{1}{\Omega_1\Omega_2}}\right)\right)^{L-1} \\ &\quad \times \int_0^\infty \sqrt{\beta_2 \frac{r^2}{y^4} + \beta_1} e^{-\left(\frac{y^2}{\Omega_2} + \frac{r^2}{\Omega_1 y^2}\right)} dy. \end{aligned} \tag{29}$$

Setting $L = 1$ in (29), i.e., the non-diversity case, we obtain the LCR of double Rayleigh fading given in [45, eq. (17)].

2) APPROXIMATE LCR IN DOUBLE RAYLEIGH FADING

It can be noted that the semi-infinite range integral appearing in (29), i.e., the LCR $N_{\eta_{SC}}(r)$ in double Rayleigh fading, has the form of the Laplace type integral in one dimension, as described in Appendix A. Thus, the application of the Laplace's method of integration [7], [43] on (29) allows us to get the following approximate solution for the LCR of double Rayleigh fading channels

$$\begin{aligned} N_{\eta_{SC}}(r) &\approx \frac{Lr}{\Omega_2}\sqrt{\frac{2}{\Omega_1}}\sqrt{\beta_2\frac{\Omega_1}{\Omega_2} + \beta_1} e^{-\left(2r\sqrt{\frac{1}{\Omega_1\Omega_2}}\right)} \\ &\quad \times \left(1 - 2r\sqrt{\frac{1}{\Omega_1\Omega_2}} K_1\left(2r\sqrt{\frac{1}{\Omega_1\Omega_2}}\right)\right)^{L-1}. \end{aligned} \tag{30}$$

Again, for $L = 1$, (30) coincides with the already known result presented in [7, eq. (33)].

B. ADF OF THE ENVELOPE PROCESS $\eta_{SC}(t)$

Complementary to the LCR, we here give the ADF corresponding to $\eta_{SC}(t)$. Recalling the definition given in [46], this statistics can be determined as

$$T_{\eta_{SC}}(r) = \frac{F_{\eta_{SC}}(r)}{N_{\eta_{SC}}(r)} = \frac{F_{\eta}(z)}{LN_{\eta}(z)}. \tag{31}$$

That is, using (3) and (5), $T_{\eta_{SC}}(r)$ can be determined from (31). Consequently, the approximations investigated for the LCR and CDF apply also in getting an approximation for the ADF. Besides, in line with the above, (31) degenerates to the ADF of Rayleigh×Rice ($k_1 = 0$), and double Rayleigh ($k_1 = k_2 = 0$) channels.

V. SEP OF M-DPSK MODULATION SCHEME

In this section, we focus on the determination of the SEP of M-DPSK modulation SC diversity. To this aim, we confine our attention to the case of noiseless relays. This scenario can be encountered in many practical cases as reported in [47], [48]. Under such condition, the SEP to be presented can be seen as a lower bound of that corresponding to the case where the noise of the relays cannot be ignored. Besides, the subsequent performance analysis applies also if the overall additive noise, corresponding to the relaying path, is Gaussian [49]. To proceed with the derivation of the SEP, we invoke the CDF approach reported in [51, eq. (32)]. According to this approach, the SEP performance $\bar{P}_s$ is defined by

$$\bar{P}_s = -\int_0^{\infty} P_s{}'(E\,|\gamma\,)F_{\gamma_{sc}}(\gamma)\,d\gamma \tag{32}$$

where $P_s{}'(E\,|\gamma\,)$ stands for the first-order derivative of the conditional SEP of the non-coherent M-DPSK modulation over an additive white Gaussian noise (AWGN) channel given by [52, eq. (3)]

$$P_s'(E\,|\gamma\,) = -\frac{\sin\left(\frac{\pi}{M}\right)}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\left(-\gamma\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)\right)}d\theta \tag{33}$$

where M is the number of transmitted information symbols. In (33) also, $F_{\gamma_{SC}}(\gamma)$ stands for the CDF of the instantaneous SNR per symbol of the SC diversity. Considering the same noise power over the paths, the underlying SNR is defined by $\gamma_{SC}(t) = \eta_{SC}^2(t)\frac{E_s}{N_0}$, in which E_s stands for the average symbol energy, and N_0 is the one-sided power spectral density of the receiver noise. An expression for $F_{\gamma_{SC}}(\gamma)$ can directly be determined from $F_{\eta_{SC}}(z)$ according to

$$F_{\gamma_{SC}}(\gamma) = F_{\eta_{SC}}\left(\sqrt{\frac{\gamma}{E_s/N_0}}\right). \tag{34}$$

Substituting (9) in (34), we obtain

$$F_{\gamma_{SC}}(\gamma) = \left(1 - \int_0^{\infty} Q_1\left(\sqrt{2k_1}, \frac{1}{y}\sqrt{2\Omega_2(1+k_1)\frac{\gamma}{\bar{\gamma}}}\right) \times \frac{2y(1+k_2)}{\Omega_2}e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)} I_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)dy\right)^L \tag{35}$$

$$N_{\eta_{SC}}(r) = \frac{Lrk_T}{\sqrt{2\pi^5}\Omega_1\Omega_2}\int_0^{\infty}\sqrt{\beta_2\frac{r^2}{y^4}+\beta_1}\,e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)}e^{-\left(\frac{r^2(1+k_1)}{y^2\Omega_1}+k_1\right)}\int_{-\pi}^{\pi} e^{\frac{2r\cos\theta_1}{y}\sqrt{\frac{k_1(1+k_1)}{\Omega_1}}}\int_{-\pi}^{\pi} e^{2y\cos\theta_2\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}}e^{-\frac{K^2(r,y,\theta_1,\theta_2)}{2}}$$
$$\times\left(1+\sqrt{\frac{\pi}{2}}K(r,y,\theta_1,\theta_2)\,e^{\frac{1}{2}K^2(r,y,\theta_1,\theta_2)}\left\{1+\Phi\left(\frac{K(r,y,\theta_1,\theta_2)}{2}\right)\right\}\right)d\theta_2 d\theta_1 dy.$$
$$\times\left(1-\int_0^{\infty}Q_1\left(\sqrt{2k_1},\frac{r}{z}\sqrt{\frac{2(1+k_1)}{\Omega_1}}\right)p_{\eta_2}(z)dz\right)^{L-1}. \tag{24}$$

$$N_{\eta_{SC}}(r) = \frac{4Lrk_T}{\sqrt{2\pi}\Omega_1\Omega_2}\int_0^{\infty}\sqrt{\beta_2\frac{r^2}{y^4}+\beta_1}\,e^{-\left(\frac{r^2(1+k_1)}{\Omega_1 y^2}+k_1\right)}e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)}I_0\left(\frac{2r}{y}\sqrt{\frac{k_1(1+k_1)}{\Omega_1}}\right)I_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)dy$$
$$\times\left(1-\int_0^{\infty}Q_1\left(\sqrt{2k_1},\frac{r}{z}\sqrt{\frac{2(1+k_1)}{\Omega_1}}\right)p_{\eta_2}(z)dz\right)^{L-1}. \tag{25}$$

$$N_{\eta_{SC}}(r) = \frac{4Lr(1+k_2)}{(2\pi)^{\frac{3}{2}}\Omega_1\Omega_2}\int_0^{\infty}\sqrt{\beta_2\frac{r^2}{y^4}+\beta_1}\,e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)}e^{-\frac{r^2}{\Omega_1 y^2}}\int_{-\pi}^{\pi}e^{2y\cos\theta_2\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}}e^{-\frac{F^2(r,y,\theta_2)}{2}}\left(1+\sqrt{\frac{\pi}{2}}F(r,y,\theta_2)\,e^{\frac{F^2(r,y,\theta_2)}{2}}\right.$$
$$\left.\times\left\{1+\Phi\left(\frac{F(r,y,\theta_2)}{2}\right)\right\}\right)d\theta_2 dy\left\{1-\int_0^{\infty}e^{\left(-\frac{r^2}{z^2\Omega_1}\right)}p_{\eta_2}(z)dz\right\}^{L-1} \tag{26}$$

where $\bar{\gamma} = \Omega_1\Omega_2\frac{E_s}{N_0}$ [27] stands for the average SNR per symbol. Letting, in (35), $k_1 = 0$ and $k_1 = k_2 = 0$, yields, the CDFs of the SNR in Rayleigh×Rice and double Rayleigh fading, respectively. In the sequel, we derive an exact expression for $\bar{P}_s$.

A. EXACT EXPRESSION

1) MULTI-RELAY CASE

The substitution of (33) and (35) in (32) yields the following generic expression for $\bar{P}_s$

$$\bar{P}_s = \frac{\sin\left(\frac{\pi}{M}\right)}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^\infty e^{\left(-\gamma\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)\right)} \times\left(1-\int_0^\infty\frac{2y\left(1+k_2\right)}{\Omega_2}e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)} \times Q_1\left(\sqrt{2k_1},\frac{1}{y}\sqrt{2\Omega_2\left(1+k_1\right)\frac{\gamma}{\bar{\gamma}}}\right) \times I_0\left(2y\sqrt{\frac{k_2\left(1+k_2\right)}{\Omega_2}}\right)dy\right)^L d\gamma d\theta. \quad (36)$$

Unfortunately, the integrals involved in (36) can not, again, be evaluated analytically. The complexity of the expression in (36) can, slightly, be simplified in the special cases given below.

- ***Rayleigh×Rice fading:*** Letting $k_1 = 0$ in (36), results in

$$\bar{P}_s = \frac{\sin\left(\frac{\pi}{M}\right)}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^\infty e^{\left(-\gamma\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)\right)}, \times\left(1-\int_0^\infty\frac{2y\left(1+k_2\right)}{\Omega_2}e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)} \times e^{-\left(\frac{\Omega_2}{y^2}\frac{\gamma}{\bar{\gamma}}\right)} I_0\left(2y\sqrt{\frac{k_2\left(1+k_2\right)}{\Omega_2}}\right)dy\right)^L d\gamma d\theta. \quad (37)$$

- ***Double Rayleigh fading:*** Putting $k_1 = k_2 = 0$ in (36), yields

$$\bar{P}_s = \frac{\sin\left(\frac{\pi}{M}\right)}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^\infty e^{\left(-\gamma\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)\right)} \times\left(1-2\sqrt{\frac{\gamma}{\bar{\gamma}}}K_1\left(2\sqrt{\frac{\gamma}{\bar{\gamma}}}\right)\right)^L d\gamma d\theta. \quad (38)$$

- ***Binary Case:*** Setting $M = 2$ in (36), (37), and (38), we get known results for, respectively, double Rice [32, eq. (7)], Rayleigh×Rice [32, eq. (8)], and double Rayleigh [32, eq. (9)] channels, as required.

2) TWO RELAY CASE (L = 2)

Here, we show that it is possible to simplify the generic expression for the SEP given in (36). Letting $L = 2$ in (36), performing an integration-by-part, and doing lengthy algebraic manipulations, results in an expression that can, for convenience, be written as

$$\bar{P}_s = \frac{2k_T}{\bar{\gamma}\pi}sin\left(\frac{\pi}{M}\right)\left[P_1 - P_2\right] \quad (39)$$

where P_1 and P_2 are given by

$$P_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^\infty\frac{I_0(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}})}{y(1-\cos(\frac{\pi}{M})\cos(\theta))}e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2+k_1\right)} \times\int_0^\infty e^{-\left(\gamma\left(\frac{\Omega_2(1+k_1)}{y^2\bar{\gamma}}+(1-\cos(\frac{\pi}{M})\cos(\theta))\right)\right)} \times I_0(\frac{2}{y}\sqrt{\Omega_2 k_1(1+k_1)\frac{\gamma}{\bar{\gamma}}})d\gamma dy d\theta \quad (40)$$

and

$$P_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^\infty\int_0^\infty\frac{I_0(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}})I_0(2x\sqrt{\frac{k_2(1+k_2)}{\Omega_2}})}{1-\cos(\frac{\pi}{M})\cos(\theta)} \times\frac{2x(1+k_2)}{y\Omega_2}e^{-\left(\frac{(y^2+x^2)(1+k_2)}{\Omega_2}+2k_2+k_1\right)} \times\int_0^\infty I_0(\frac{2}{y}\sqrt{\Omega_2 k_1(1+k_1)\frac{\gamma}{\bar{\gamma}}}) \times e^{-\left(\gamma\left(\frac{\Omega_2(1+k_1)}{y^2\bar{\gamma}}+(1-\cos(\frac{\pi}{M})\cos(\theta))\right)\right)} \times Q_1\left(\sqrt{2k_1},\frac{2}{x}\sqrt{2\Omega_2(1+k_1)\frac{\gamma}{\bar{\gamma}}}\right)d\gamma dx dy d\theta, \quad (41)$$

respectively. Then, in (40), the semi-infinite integral with respect to the variable γ can be solved. Indeed, using [39, eq. (6.614.3)], [53, eqs. (07.44.26.0008.01) and (07.34.03.0006.01)] and doing some algebraic manipulations, the quantity P_1 reduces to (42), which is shown at the bottom of the next page. Similarly, using [40, eq. (46)] in (41), the quantity P_2 is simplified to yield (43), as shown at the bottom of the next page, where the quantities m and n are given by

$$m = \frac{\sqrt{2k_1}\sqrt{\bar{\gamma}+\frac{\Omega_2}{y^2}\left(1+k_1\right)-\cos\left(\frac{\pi}{M}\right)\cos\left(\theta\right)\bar{\gamma}}}{\sqrt{\bar{\gamma}+\Omega_2\left(1+k_1\right)\left(\frac{x^2+y^2}{y^2x^2}\right)-\cos\left(\frac{\pi}{M}\right)\cos\left(\theta\right)\bar{\gamma}}} \quad (44)$$

and

$$n = \frac{\sqrt{2k_1}\Omega_2\left(1+k_1\right)}{\sqrt{\bar{\gamma}+\frac{\Omega_2(1+k_1)}{y^2}-\cos\left(\frac{\pi}{M}\right)\cos\left(\theta\right)\bar{\gamma}}} \times\sqrt{\frac{\frac{1}{y^2x^2}}{\bar{\gamma}+\Omega_2\left(1+k_1\right)\left(\frac{x^2+y^2}{y^2x^2}\right)-\cos\left(\frac{\pi}{M}\right)\cos\left(\theta\right)\bar{\gamma}}} \quad (45)$$

respectively. In summary, for the important case of two-relay based cooperative SC diversity, the SEP of M-DPSK is obtained by substituting (42) and (43) in (39). Now, for Rayleigh×Rice fading, i.e., $k_1 = 0$, the application of [53, eq. (07.20.03.0001.01)], allows us to write

$$\bar{P}_s = 2\frac{\sin\left(\frac{\pi}{M}\right)}{\pi}(1+k_2)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\infty}\frac{e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)}}{1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)} \times\frac{yI_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)}{y^2\bar{\gamma}+\Omega_2-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\,y^2\bar{\gamma}}dyd\theta$$
$$-4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\infty}\int_0^{\infty}\frac{I_0\left(2x\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)I_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)}{\pi\Omega_2\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)} \times\frac{x\sin\left(\frac{\pi}{M}\right)(1+k_2)^2 e^{-\left(\frac{(y^2+x^2)(1+k_2)}{\Omega_2}+2k_2\right)}}{y\left(\bar{\gamma}+\Omega_2\left(\frac{x^2+y^2}{y^2x^2}\right)-\cos\left(\frac{\pi}{M}\right)cos(\theta)\,\bar{\gamma}\right)}dydxd\theta. \tag{46}$$

Additionally, by setting $k_1 = k_2 = 0$ in (39), and using [39, eqs. (3.382.4) and (3.383.10)] and [53, eq. (07.45.03.0004.01)], yields the following result for the SEP in double Rayleigh fading

$$\bar{P}_s = \frac{\sin\left(\frac{\pi}{M}\right)}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{e^{\left(\frac{1}{\bar{\gamma}\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)}\right)}}{\bar{\gamma}\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)^2} \times\Gamma\left(0,\frac{1}{\bar{\gamma}\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)}\right)d\theta$$
$$-2\frac{\sin\left(\frac{\pi}{M}\right)}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\infty}\frac{e^{-y^2\left(\frac{1}{\Omega_2}-\frac{1}{y^2\bar{\gamma}+\Omega_2-\cos\left(\frac{\pi}{M}\right)\cos(\theta)y^2\bar{\gamma}}\right)}}{\left(1-\cos\left(\frac{\pi}{M}\right)cos(\theta)\right)} \times\frac{y^3\Gamma\left(-1,\frac{y^2}{y^2\bar{\gamma}+\Omega_2-cos\left(\frac{\pi}{M}\right)\cos(\theta)y^2\bar{\gamma}}\right)}{\left(y^2\bar{\gamma}+\Omega_2-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\,y^2\bar{\gamma}\right)^2}dyd\theta \tag{47}$$

where $\Gamma(\cdot,\cdot)$ denotes the incomplete gamma function [39]. Finally, for $M = 2$, (39) reduces, as required, to [32, eq. (10)], [32, eq. (17)] and [32, eq. (18)], which are known results for double Rice, Rayleigh×Rice, and double Rayleigh fading channels, respectively.

3) SINGLE RELAY CASE (L = 1)

To the best of our knowledge, even for the non-diversity case, i.e., $L = 1$, the SEP of non-coherent M-DPSK, over double Rice fading, has not been reported previously. The result corresponding to this case is obtained by fixing $L = 1$ in (36), performing an integration-by-part, and doing lengthy algebraic manipulations. This yields

$$\bar{P}_s = \frac{\sin\left(\frac{\pi}{M}\right)k_T}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta\int_0^{\infty}yI_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right) \times\frac{e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2+k_1\right)}e^{-\left(\frac{k_1\Omega_2(1+k_1)}{y^2\bar{\gamma}+\Omega_2(1+k_1)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)y^2\bar{\gamma}}\right)}}{\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)} \times\frac{1}{y^2\bar{\gamma}+\Omega_2(1+k_1)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\,y^2\bar{\gamma}}dy. \tag{48}$$

Similarly, for Rayleigh×Rice ($k_1 = 0$) and double Rayleigh fading ($k_1 = k_2 = 0$), (48) reduces to

$$\bar{P}_s = \frac{\sin\left(\frac{\pi}{M}\right)}{\pi}(1+k_2)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\infty}yI_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right) \times\frac{1}{y^2\bar{\gamma}+\Omega_2-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\,y^2\bar{\gamma}} \times\frac{e^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2\right)}}{1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)}dyd\theta \tag{49}$$

$$P_1 = \bar{\gamma}\int_0^{\infty}ye^{-\left(\frac{y^2(1+k_2)}{\Omega_2}+k_2+k_1\right)}I_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{e^{-\left(\frac{\Omega_2k_1(1+k_1)}{y^2\bar{\gamma}+\Omega_2(1+k_1)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)y^2\bar{\gamma}}\right)}}{\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)\left(y^2\bar{\gamma}+\Omega_2(1+k_1)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\,y^2\bar{\gamma}\right)}d\theta dy \tag{42}$$

$$P_2 = \frac{2\bar{\gamma}(1+k_2)}{\Omega_2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\infty}\int_0^{\infty}\frac{xyI_0\left(2y\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)I_0\left(2x\sqrt{\frac{k_2(1+k_2)}{\Omega_2}}\right)e^{-\left(\frac{(y^2+x^2)(1+k_2)}{\Omega_2}+2k_2+k_1\right)}}{\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)\left(y^2\bar{\gamma}+\Omega_2(1+k_1)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\,y^2\bar{\gamma}\right)}\left[e^{\frac{\Omega_2k_1(1+k_1)}{\left(y^2\bar{\gamma}+\Omega_2(1+k_1)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)y^2\bar{\gamma}\right)}}\right.$$
$$\left.\times Q_1(m,n)-\frac{I_0\left(\frac{\frac{2\Omega_2k_1(1+k_1)}{xy\bar{\gamma}}}{1+\Omega_2(1+k_1)\left(\frac{x^2+y^2}{x^2y^2\bar{\gamma}}\right)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)}\right)}{\frac{x^2\bar{\gamma}+\Omega_2(1+k_1)\left(\frac{x^2+y^2}{y^2}\right)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)x^2\bar{\gamma}}{\Omega_2(1+k_1)}}e^{\frac{k_1cos\left(\frac{\pi}{M}\right)\cos(\theta)x^2y^2\bar{\gamma}-k_1x^2y^2\bar{\gamma}}{x^2y^2\bar{\gamma}+\Omega_2(1+k_1)(x^2+y^2)-\cos\left(\frac{\pi}{M}\right)\cos(\theta)x^2y^2\bar{\gamma}}}\right]dydxd\theta \tag{43}$$

and

$$\bar{P}_s = \frac{\sin\left(\frac{\pi}{M}\right)}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{\left(\frac{1}{\bar{\gamma}(1-\cos(\frac{\pi}{M})\cos(\theta))}\right)}}{\bar{\gamma}\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)^2} \times \Gamma\left(0, \frac{1}{\bar{\gamma}\left(1-\cos\left(\frac{\pi}{M}\right)\cos(\theta)\right)}\right) d\theta \quad (50)$$

respectively. In addition, for DPSK modulation ($M = 2$), the results obtained from (48), (49), and (50), are, as expected, in agreement with [32, eq. (19)], [32, eq. (20)], and [32, eq. (21)], respectively.

B. APPROXIMATE SOLUTIONS

Here, we exploit the approximate solutions for $F_{\eta_{SC}}(r)$, given in (12) and (21), to get approximate solutions for $\bar{P}_s$.

1) APPROXIMATE SOLUTION BASED ON (12)

Substituting (12) in (34), yields the following expression for the CDF $F_{\gamma_{SC}}(\gamma)$

$$F_{\gamma_{SC}}(\gamma) \approx \left(\sqrt{2\pi\frac{1+k_2}{\Omega_2}} \exp(-\frac{1}{2}(1+\log(2\frac{1+k_2}{\Omega_2})-k_2) \times I_0(\sqrt{2k_2})\left(1-Q_1(\sqrt{2k_1}, 2\sqrt{k_T\frac{\gamma}{\bar{\gamma}}})\right)\right)^L. \quad (51)$$

Then, replacing (51) and (33) in (32), results in

$$\bar{P}_s \approx \frac{\sin(\frac{\pi}{M})}{2\pi}(I_0(\sqrt{2k_2}))^L \times \left(\sqrt{2\pi\frac{1+k_2}{\Omega_2}} \exp(-\frac{1}{2}(1+\log(2\frac{1+k_2}{\Omega_2})-k_2)\right)^L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{\infty} \exp(-\gamma(1-\cos(\frac{\pi}{M})\cos(\theta))) \times \left(1-Q_1(\sqrt{2k_1}, 2\sqrt{k_T\frac{\gamma}{\bar{\gamma}}})\right)^L d\gamma. \quad (52)$$

For the particular case corresponding to $L = 1$, the semi-infinite range integral in (52) can be solved using the results of [50]. This gives, as detailed in Appendix C,

$$\bar{P}_s \approx \sin(\frac{\pi}{M})\sqrt{\frac{1+k_2}{2\pi\Omega_2}} \exp(-\frac{1}{2}(1+\log(2\frac{1+k_2}{\Omega_2})-k_2) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{I_0(\sqrt{2k_2})}{1-\cos(\frac{\pi}{M})\cos(\theta)} \left(1-\Gamma\left(1, 2\frac{k_T}{\bar{\gamma}}\right) +\gamma\left(1, \frac{2\frac{k_1k_T}{\bar{\gamma}}}{k_1+1-\cos(\frac{\pi}{M})\cos(\theta)}\right) \times \exp\left(-\frac{2\frac{k_T}{\bar{\gamma}}(1-\cos(\frac{\pi}{M})\cos(\theta))}{k_1+1-\cos(\frac{\pi}{M})\cos(\theta)}\right)\right) d\theta. \quad (53)$$

Fixing, in addition, $k_1 = 0$ in (53), and using [54, eq. (4.3.133)], the finite-range integral is solved, to yield the following closed-form expression for $\bar{P}_s$ in Rayleigh×Rice fading

$$\bar{P}_s \approx 2\sqrt{2\frac{1+k_2}{\pi\Omega_2}} \exp(-\frac{1}{2}(1+\log(2\frac{1+k_2}{\Omega_2})-k_2) \times I_0(\sqrt{2k_2}) \tan^{-1}\left(\frac{1+\cos(\frac{\pi}{M})}{\sin(\frac{\pi}{M})}\right)(1-\exp(-2\frac{1+k_2}{\bar{\gamma}})). \quad (54)$$

where $\tan^{-1}(\cdot)$ denotes the inverse of the tangent function. Similarly, the approximate SEP in double Rayleigh fading channels is deduced by putting $k_2 = 0$ in (54). This gives

$$\bar{P}_s \approx 2\sqrt{\frac{2}{\pi\Omega_2}} \exp(-\frac{1}{2}(1+\log(\frac{2}{\Omega_2})) \times \tan^{-1}\left(\frac{1+\cos(\frac{\pi}{M})}{\sin(\frac{\pi}{M})}\right)(1-\exp(-\frac{2}{\bar{\gamma}})). \quad (55)$$

2) APPROXIMATE SOLUTION BASED ON (21)

By proceeding similarly as above, the approximate solution for $\bar{P}_s$, obtained by making use of the CDF in (21), is found to be given by

$$\bar{P}_s \approx L\frac{\sin(\frac{\pi}{M})}{\sqrt{\pi}}(\frac{k_T}{\Omega_1\Omega_1})^{\frac{3}{4}} \exp(-k_1-k_2) \int_0^{\infty} d\gamma \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{\sqrt{\frac{\gamma}{\bar{\gamma}}\Omega_1\Omega_1}} \exp(-\gamma(1-\cos(\frac{\pi}{M})\cos(\theta))) \times \sqrt{x}I_0(2\sqrt{xk_2}(\frac{k_T}{\Omega_1\Omega_1})^{\frac{1}{4}})I_0(2\sqrt{xk_1}(\frac{k_T}{\Omega_1\Omega_1})^{\frac{1}{4}}) \times \exp(-2x\sqrt{\frac{k_T}{\Omega_1\Omega_1}})(F_\eta(x))^{L-1}dx. \quad (56)$$

Numerical and simulation results are given for the purpose of verifying the validity of the derivations and the accuracy of the approximations. This will be the topic of the next section.

VI. NUMERICAL AND SIMULATION RESULTS

In this section, the validity of the exact theoretical results, for the PDF, CDF, LCR and SEP, is checked, and the accuracy of the proposed approximations is assessed.

The simulation of the fading process $\eta_{SC}(t)$ is achieved by using the concept of Rice's sum-of-sinusoids [46]. The method of exact Doppler spread [55] is employed in the determination of the parameters of the sinusoids. We also use Clarke's isotropic scattering model [56] for which $\beta_i = \frac{\Omega_i}{(1+k_i)}(\pi f_{\max_i})^2$ $(i = 1, 2)$, where $f_{\max_1}$ ($f_{\max_2}$) denotes the maximum Doppler frequency caused by the motion of the

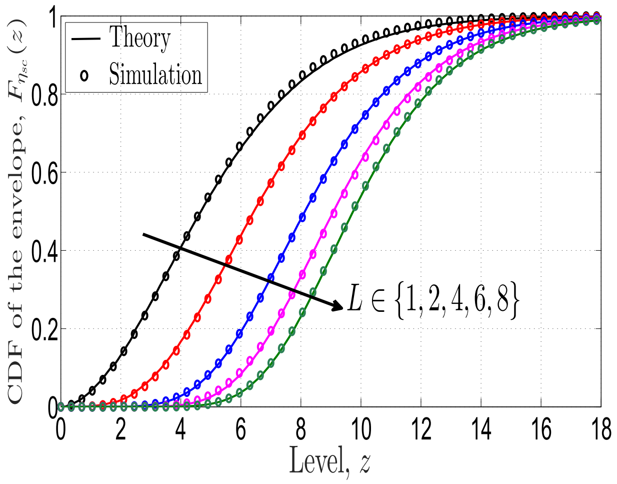

FIGURE 2. The CDF $F_{\eta_{SC}}(z)$ of $\eta_{SC}(t)$ over double Rice fading channels for different values of L.

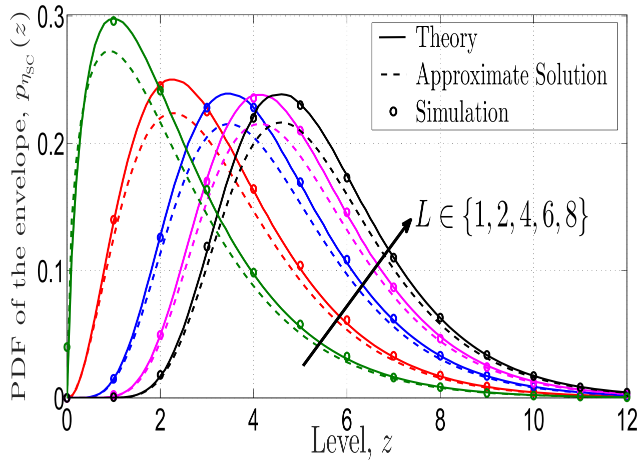

FIGURE 3. The PDF $p_{\eta_{SC}}(z)$ of $\eta_{SC}(t)$ over double Rice fading channels for different values of L.

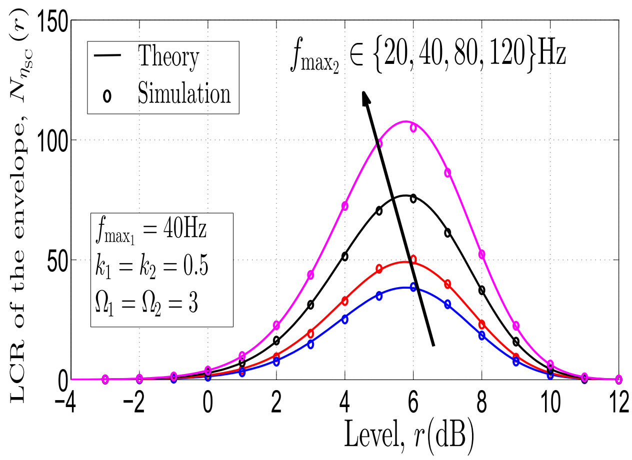

FIGURE 4. The LCR $N_{\eta_{SC}}(r)$ of $\eta_{SC}(t)$ over double Rice fading channels for $L = 4$ and different values of the maximum Doppler frequencies.

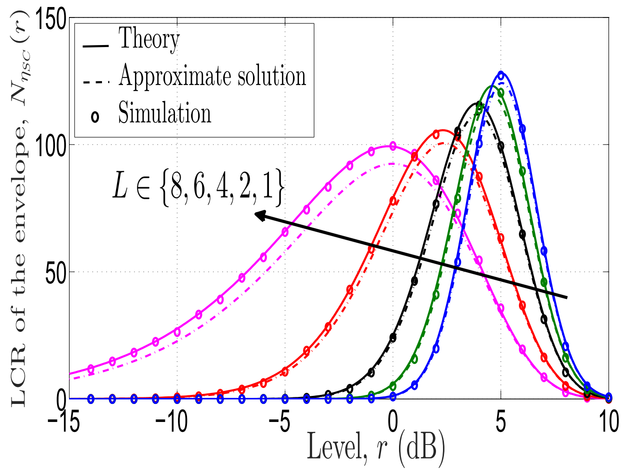

FIGURE 5. The LCR $N_{\eta_{SC}}(r)$ of $\eta_{SC}(t)$ over double Rayleigh fading channels for different values of L.

mobile transmitter (mobile receiver) station. All the results to be shown are obtained for the Doppler frequencies $f_{\max_1} = f_{\max_2} = 80$ Hz, except that of Fig. 4. Also, for simplicity, the Doppler frequencies of the LOS components are set to zero, i.e., $f_{\rho_i} = 0$ $(i = 1, 2)$.

The results, derived for the CDF, PDF, ADF, and SEP, are plotted together with corresponding simulation data in Figs. 2–9. It can be observed from these figures that the simulation data match the exact theory quite well, thereby confirming the validity of the exact derived expressions. Fig. 2 illustrates the behavior of the CDF (the outage probability) of SC diversity for different values of L. For a fixed fading level z, an increase in the number of diversity paths L results, as expected, in a decrease in the outage probability. It should be pointed out that the slightly poor fit between theory and simulation, for $L = 1$ and over the range $z > 6$, could be caused by the lack of averaging over different simulation trials. Fig. 3 shows a comparison between the theoretical, approximate, and simulated PDFs of the fading process $\eta_{SC}(t)$ for $k_1 = k_2 = 0.5$, $\Omega_1 = \Omega_2 = 3$, and different values of L. The shape correspondence between the approximate and theoretical results reveals the validity of (18) as a simple approximate solution for (15). It also appears that the accuracy of the approximate PDF is slightly improved when the number of paths L increases. Indeed, this observation can be confirmed from the Kullback-Leibler divergence (KL) measure, between the approximate and theoretical PDFs, shown in Table 1.

Fig. 4 illustrates the shape of the LCR $N_{\eta_{SC}}(r)$ for different values of the maximum Doppler frequencies. The curves in this figure manifest the expected behavior of the LCR as a function of the Doppler frequencies and the crossing level. Likewise, the accuracy of the approximate solution in (30), considering the scenario of double Rayleigh channels, can be interpreted from Fig. 5. The results reveal that the accuracy of (30) improves as the number of relays L increases. Hence, it appears that, in this case, (30) is computationally advantageous over (29). Moreover, to provide an accurate assessment for the LCR approximation in (30), the mean square error (MSE) between approximate and exact LCR

TABLE 1. **The Kullback-Leibler divergence measure between the exact PDF expression in (15) and the corresponding approximate solution in (18) for different values of *L*.**

KL	L				
	1	2	4	6	8
PDF	0.0014	$2.4122 \cdot 10^{-4}$	$5.8910 \cdot 10^{-5}$	$5.8734 \cdot 10^{-5}$	$5.8611 \cdot 10^{-5}$

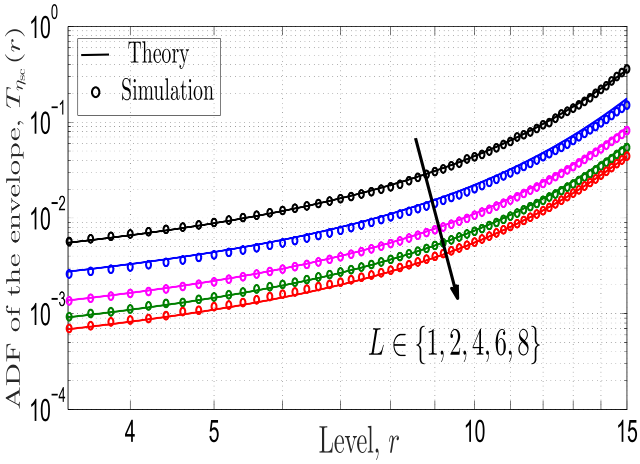

FIGURE 6. **The ADF $T_{\eta_{SC}}(r)$ of $\eta_{SC}(t)$ over double Rice fading channels for different values of *L*.**

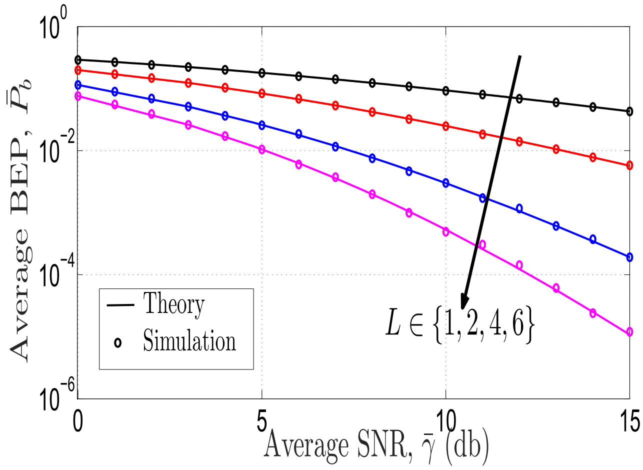

FIGURE 8. **The BEP of binary non-coherent DPSK modulation over double Rice fading channel for various values of *L*.**

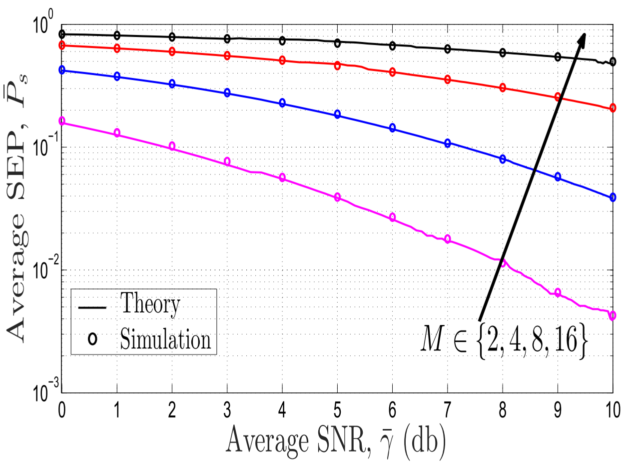

FIGURE 7. **The SEP of non-coherent M-DPSK modulation for two relay cooperative SC diversity receivers over double Rice fading channel for various values of *M*.**

TABLE 2. **Mean square errors between the exact analytical expression and approximate solution for the LCR of double Rayleigh fading channels for different values of *L*.**

MSE	L				
	1	2	4	6	8
LCR	7.9297	2.6152	1.9271	1.6851	1.5527

solutions is computed and reported in Table 2. As can be observed, the MSE declines as the number of paths L increases. Concerning the behavior of the ADF $T_{\eta_{SC}}(r)$, it is illustrated in Fig. 6 for various values of L.

Figs. 7–9 show both analytical and simulated curves of the SEP $\bar{P}_s$ for M-DPSK modulations. Fig. 7 presents $\bar{P}_s$ for the scenario of two relays based cooperative SC diversity, considering $k_1 = k_2 = 4$, $\Omega_1 = \Omega_2 = 10$, and various values of M. As expected, by increasing M, the SEP increases, and thus, the quality of the transmission deteriorates. It should also be mentioned that the numerical evaluation of $\bar{P}_s$, expressed in (36), has been accomplished by using Mathematica tool and an ordinary laptop computer. This operation has taken a time period ranging from 19.5 s (for $M = 2$) to 109.2 s (for $M = 16$). The BEP, corresponding to binary DPSK modulation, is depicted in Fig. 8 for $L \in \{1, 2, 4, 6\}$. As expected, the quality of the wireless transmission improves as the number of paths L increases. Finally, the impact of the Rice factors k_1 and k_2 on the SEP $\bar{P}_s$ is illustrated in Fig. 9 for $L = 1$. As expected, the best SEP performance is obtained in the case of double Rice fading ($k_1 = k_2 = 4$), while the worst one corresponds to the double Rayleigh fading ($k_1 = k_2 = 0$).

In Fig. 10, we examine the accuracy of the proposed approximate solutions for the CDF $F_{SC}(z)$ in (21), considering $k_1 = k_2 = 2$, $\Omega_1 = \Omega_2 = 6$, and $L \in \{1, 2, 4\}$. The results in this figure reveal that (21) is a good approximation for (9) over the range of low values of the fading amplitude z. Moreover, it can be observed that the error between the exact solution, given in (9), and approximate quantity, shown in (21), decreases as L increases. This tends to indicate that the approximation in (21) is useful in evaluating, efficiently, low-SNR outage of radio links. Finally, the quality of the approximate BEP, provided in (56), can be studied from the content of Fig. 11, for $k_1 = k_2 = 2$, $\Omega_1 = \Omega_2 = 6$,

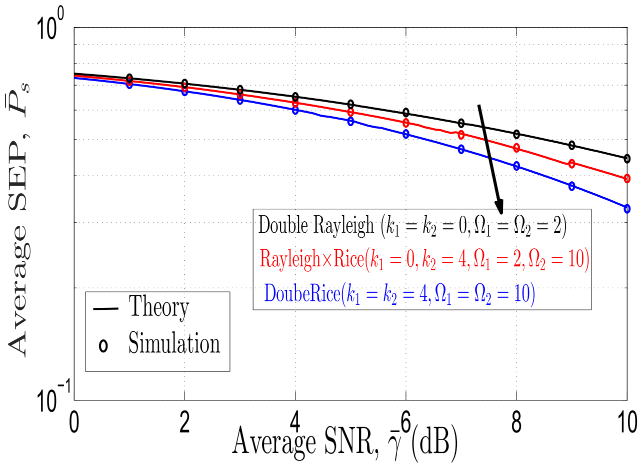

FIGURE 9. The SEP of non-coherent 8-DPSK modulation for $L = 1$ (single relay case) and various values of k_1 and k_2.

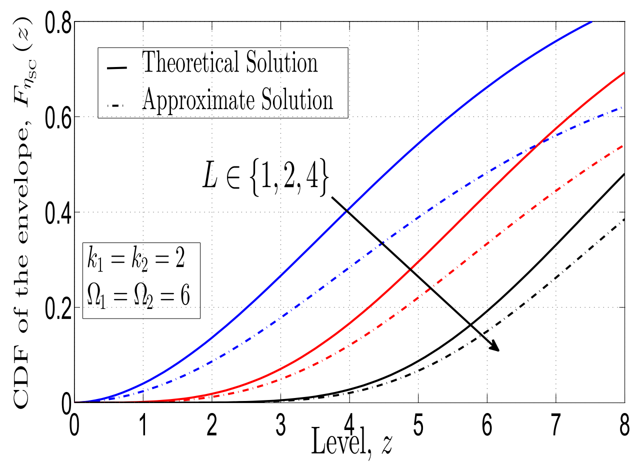

FIGURE 10. The approximate CDF $F_{\eta_{SC}}(z)$ in (21) for different values of L.

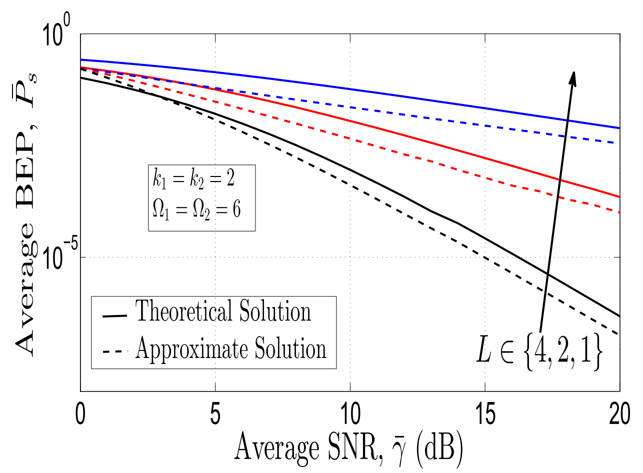

FIGURE 11. The approximate BEP of non-coherent DPSK in (56) for various values of L.

and $L \in \{1, 2, 4\}$. We observe from this figure that (56) appears to be a reasonably good lower bound for the exact BEP performance given in (36).

VII. CONCLUSION

This paper has presented a theoretical analysis of fade statistics and SEP performance of dual-hop AF multi-relay based cooperative selection diversity in double Rice fading channels. Exact and approximate analytical expressions for the CDF, PDF, LCR, and ADF of the fading process have first been derived. By resorting to the CDF approach, the SEP performance of non-coherent M-DPSK modulation has thereafter been investigated. Additionally, corresponding results have been extracted for RayleighxRice and double Rayleigh channels, which appear as special cases of the double Rice fading channel. Despite that all the statistical quantities and the performance metrics, reported in this work, are expressed in terms of integrals, numerical computation and simulation results have proved their usefulness in assessing the performance of the considered cooperative selection diversity. Deriving exact closed-form expressions for these quantities remains an open problem.

APPENDIXES

APPENDIX A

In this appendix, we detail the Laplace's method of integration applied to approximate the CDF $F_{\eta_{SC}}(z)$ given in (12). This method uses the following result [7], [43]

$$\int_0^{\infty} g(y)e^{-\lambda f(y)}dy \approx \sqrt{\frac{2\pi}{\lambda}}\frac{g(y_0)}{\sqrt{f''(y_0)}}e^{-\lambda f(y_0)} \tag{57}$$

where λ is a positive parameter that can be large or small [7], [43], $f(y)$ and $g(y)$ represent two real-valued functions that are assumed to be infinitely differentiable, and the parameter y_0 denotes the critical point of the function $f(y)$. Also in (57), $f''(y)$ denotes the second derivative of the function $f(y)$ with respect to the variable y. By observing the semi-infinite integral in (3), it is clear that this integral satisfies all conditions of the Laplace's theorem of integration. Hence, by identifying (3) and (57) we can write

$$\begin{cases} f(y) = \dfrac{1+k_1}{\Omega_1}y^2 - \log(y) \\ g(y) = 2\dfrac{1+k_2}{\Omega_2}\exp(-k_2)I_0(2y\sqrt{\dfrac{1+k_2}{\Omega_2}k_2}) \\ \qquad \times\left(1 - Q_1(\sqrt{2k_1}, \dfrac{z}{y}\sqrt{2\dfrac{1+k_1}{\Omega_1}})\right) \\ f''(y) = 2\dfrac{1+k_2}{\Omega_2} - \dfrac{1}{y^2} \\ y_0 = \sqrt{\dfrac{\Omega_1}{2(1+k_1)}} \end{cases} \tag{58}$$

Substituting (58) in (57), and performing some algebraic manipulations, the CDF $F_\eta(z)$ of the double Rice process $\eta(t)$ can be approximated by

$$F_\eta(z) \approx \sqrt{2\pi\frac{1+k_2}{\Omega_2}}\exp(-\frac{1}{2}(1+\log(2\frac{1+k_2}{\Omega_2}) - k_2) \times I_0(\sqrt{2k_2})\left(1 - Q_1(\sqrt{2k_1}, 2z\sqrt{\frac{k_T}{\Omega_1\Omega_2}})\right). \tag{59}$$

Finally, inserting (59) in (9), results in the approximate solution of the CDF $F_{\eta_{SC}}(z)$ and gives (12).

APPENDIX B

In this appendix, we again apply the Laplace's method of integration to get (18). To this end, we proceed by identifying (2) and (57) to get

$$\begin{cases} f(y) = \dfrac{y^2(1+k_2)}{\Omega_2} + \dfrac{z^2(1+k_1)}{\Omega_1 y^2} + k_1 + k_2 \\ g(y) = \dfrac{4zk_T}{\Omega_2\Omega_1 y} I_0\Big(2y\sqrt{\dfrac{k_2(1+k_2)}{\Omega_2}}\Big) I_0\left(\dfrac{2z}{y}\sqrt{\dfrac{k_1(1+k_1)}{\Omega_1}}\right) \\ f''(y) = \dfrac{2(1+k_2)}{\Omega_2} + \dfrac{6z^2(1+k_1)}{\Omega_1 y^4} \\ y_0 = \sqrt{z\sqrt{\dfrac{\Omega_2(1+k_1)}{\Omega_1(1+k_2)}}}. \end{cases} \tag{60}$$

Then, substituting (60) in (57), and doing some algebraic manipulations, gives

$$p_\eta(z) \approx 2\sqrt{\pi z\Big(\frac{k_T}{\Omega_1\Omega_2}\Big)^{\frac{3}{2}}} I_0\left(2\sqrt{zk_2}\Big(\frac{k_T}{\Omega_1\Omega_2}\Big)^{\frac{1}{4}}\right) \times I_0\left(2\sqrt{zk_1}\Big(\frac{k_T}{\Omega_1\Omega_2}\Big)^{\frac{1}{4}}\right) e^{-\left(2z\sqrt{\frac{k_T}{\Omega_1\Omega_2}}+k_1+k_2\right)}. \tag{61}$$

Finally, the desired quantity in (18) is obtained by replacing (61) in (15).

APPENDIX C

This appendix explains the derivation of (53). Specifically, (53) is obtained by solving the following integral given in (52) for the case of $L = 1$

$$I = \int_0^\infty \exp(-\gamma(1-\cos(\frac{\pi}{M})\cos(\theta))) \times \left(1 - Q_1(\sqrt{2k_1}, 2\sqrt{k_T\frac{\gamma}{\bar{\gamma}}})\right) d\gamma. \tag{62}$$

This integral can easily be simplified to

$$I = \frac{1}{1-\cos(\frac{\pi}{M})\cos(\theta)} - J \tag{63}$$

where J is a semi-infinite range integral expressed by

$$J = \int_0^\infty \exp(-\gamma(1-\cos(\frac{\pi}{M})\cos(\theta))) \times Q_1(\sqrt{2k_1}, 2\sqrt{k_T\frac{\gamma}{\bar{\gamma}}}) d\gamma. \tag{64}$$

Next, using [50, eq. (19)], a closed-form expression for J is obtained to be

$$J = \frac{1}{1-\cos(\frac{\pi}{M})\cos(\theta)} \left(\Gamma\left(1, 2\frac{k_T}{\bar{\gamma}}\right) + \gamma\left(1, \frac{2\frac{k_1 k_T}{\bar{\gamma}}}{k_1+1-\cos(\frac{\pi}{M})\cos(\theta)}\right) \times \exp\left(-\frac{2\frac{k_T}{\bar{\gamma}}(1-\cos(\frac{\pi}{M})\cos(\theta))}{k_1+1-\cos(\frac{\pi}{M})\cos(\theta)}\right)\right) \tag{65}$$

where $\gamma(\cdot,\cdot)$ denotes the lower incomplete gamma function [39]. Finally, replacing (65) in (63), a closed-form expression for I is given by

$$I = \frac{1}{1-\cos(\frac{\pi}{M})\cos(\theta)} \left(1 - \Gamma\left(1, 2\frac{k_T}{\bar{\gamma}}\right) + \gamma\left(1, \frac{2\frac{k_1 k_T}{\bar{\gamma}}}{k_1+1-\cos(\frac{\pi}{M})\cos(\theta)}\right) \times \exp\left(-\frac{2\frac{k_T}{\bar{\gamma}}(1-\cos(\frac{\pi}{M})\cos(\theta))}{k_1+1-\cos(\frac{\pi}{M})\cos(\theta)}\right)\right). \tag{66}$$

This concludes the derivation of (53).

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