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Design of a Fault Tolerant Sampled-Data Fuzzy Observer With Exponential Time-Varying Gains

HAN SOL KIM AND KWANGIL LEE
Department of Control and Automation Engineering, Korea Maritime and Ocean University, 49112 Busan, South Korea

Corresponding author: Kwangil Lee (leeki@kmou.ac.kr)

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ABSTRACT This paper deals with the sampled-data fuzzy observer design problem with time-varying gains under the sensor fault consideration. To this end, a nonlinear system with sensor fault is represented by a Takagi–Sugeno fuzzy model with immeasurable premise variables. The sensor fault considered in this paper is assumed to be a time-varying uncertain matrix included in measurements. The observer is designed to consist of gains varying exponentially between two consecutive sampling instants, by which the equilibrium point of the estimation error dynamics is asymptotically exponentially stabilized. In addition, the observer considered in this paper is assumed not to share the same premise variable with a system. Unlike previous studies, this paper proposes a method handling this mismatched premise problem by using an H-infinity criterion. The proposed observer design condition is formulated in terms of linear matrix inequalities, which is relaxed based on a novel fuzzified Lyapunov–Krasovskii functional and a matrix inequality. Finally, two simulation examples are given to validate the effectiveness of the proposed method.

INDEX TERMS Sampled-data fuzzy observer, sensor fault, Takagi–Sugeno (T–S) fuzzy model, linear matrix inequality (LMI).

I. INTRODUCTION

For stable control, it is important to obtain the exact information about states of a system. However, in general, it is not easy to measure the overall state variables of a system, requiring estimating the remaining state variables form given measurements. For this reason, studies on the state estimation have been actively carried out for several decades [1]–[3]. Among the various approaches, the Takagi–Sugeno (T–S) fuzzy-model-based approach [4] is noteworthy in the state estimation of nonlinear systems because it provides a systematic design procedure. Moreover, it is easy to implement compared to the best known solution, Kalman filter [3], because it dose not even require any stochastic information about the system and measurement noises.

Studies for the state estimation of T–S fuzzy models are mainly based on either the fuzzy filter [5]–[9] or the fuzzy observer [10]–[17] approaches. The fuzzy filter approach has been studied in various fuzzy control systems, including time delays [5], interconnected systems [6], uncertain systems [7], and others. Despite its successful application, the fuzzy filter approach has an limitation in that it is only applicable to asymptotically stable systems, but, very recently, some studies have begun on solving this problem [8]. On the other hand, studies regarding the fuzzy observer-based approach have been actively carried out combined with the controller design [10]–[14]. As a controller and an observer can be designed simultaneously, the observer-based approach is applicable to not only asymptotically stable systems but also unstable systems. Recently, a pioneer study of designing an observer for oscillating systems without a controller was conducted in [15]. As a practical application of the fuzzy observer approach, authors in [16] studied the fuzzy observer-based attitude and heading reference system composed of inertial sensors. Also, in [17], a study of fuzzy observer design for systems with immeasurable premise was conducted based on the concept of imperfect premise matching [18]. However, this method has a disadvantage of complicating the observer design conditions, requiring a less complex solution. This is the first motivation of this paper.

Due to the popularization of digital computer related technologies, a sampled-data system [19] which consists

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of continuous-time systems with digital sensors has been actively researched. In this system configuration, the system operates in the continuous-time domain, while its measurements are available only at discrete sample instants. Since the continuous- and discrete-time signals coexist in sampled-data systems, we cannot directly apply the existing continuous- or discrete-time observer design techniques to this system configuration. To handle this problem, a number of studies have been carried out, and they can be categorized into a discretization method [13], [14], [20], [21] and a time delay method [22]–[28]. In the discretization method, the sampled-data observer is designed based on its discretized model. To this end, the stability of a discretized model is analyzed in the discrete-time domain using a discrete-time Lyapunov function. The complex discretization process makes stability conditions be conservative. In addition, there is no study for guaranteeing the stability of the estimation error system under variable sampling rates.

On the other hand, in the time delay approach, discrete-time measurements are converted into equivalent time-delayed continuous-time measurements. Therefore, the state estimation error system becomes a time delayed continuous-time system, and its stabilization condition is derived based on a Lyapunov-Krasovskii functional (LKF) in the continuous-time domain. In early research, stability conditions suffer from the conservativeness come from a single quadratic LKF [22]. Recently, less conservative results have been obtained based on the fuzzified LKF [24]. In [27] and [28], extended studies to further relax stability conditions by allowing gain matrices to be time-varying between two consecutive sampling instants was conducted. However, to the best of our knowledge, there are few or no studies on designing sampled-data fuzzy observer with time-varying gains, and there is room to further improve the conservatism of existing stability conditions. This is the second motivation of this paper. In addition, using the fuzzified LKF provokes the need for develop another matrix inequality, because a fuzzy integral term makes it difficult to use the conventional matrix inequalities (examples include [23], [32], and others).

In addition, sensors may have faults for a variety of reasons, causing wrong measurements. In order to achieve reliable control, the sensor fault has to be considered. Along this line, authors in [29] designed an observer that detects sensor defects and compensated for it. Similar to the sensor fault, actuator fault is also the serious problem in the control engineering. In [30], fault tolerant control was studied by assuming the actuator fault as a time-varying uncertainty in the gain matrix. This method is simple and powerful to handle the actuator fault. The same approach can be applied to the sensor fault problem, but, such research is still insufficient. This is the last motivation of this paper.

Motivated by the aforementioned analysis, in this paper, we propose a method to design a sampled-data fuzzy observer with time-varying gains for estimating state variables of a nonlinear system under sensor fault consideration. A nonlinear system with immeasurable premise variables is expressed as a T–S fuzzy model. The sensor fault is modeled as a time-varying uncertainty included in measurements. The sampled-data fuzzy observer is allowed not to share the same premise with the system. H_∞ performance criterion is defined to handle both the immeasurable premise and the minimization of the state estimation error. A sufficient condition that guarantees the exponential stability and the H_∞ criterion is derived in terms of linear matrix inequalities (LMIs). We propose the fuzzified LKF and the novel matrix inequality to relax the conservativeness of the derived sufficient condition. Finally, the effectiveness of the proposed method is validated by the simulation examples.

Notations: For any matrix M, $sym\{M\} = (M)^T + M$. The term λ_X represents the maximum eigenvalue of $X^T(t)X(t)$ for all time t and a time-varying matrix $X(t)$. For a positive scalar a, $\mathcal{I}_a$ represents an integer set $\{1, 2, \dots, a\}$.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider a T–S fuzzy system with an external disturbance which is described by the following IF–THEN rules:

$$\mathcal{R}_i: \quad \text{IF } z_1(t) \text{ is } \Gamma_{i1} \text{ AND } \cdots \text{ AND } z_p(t) \text{ is } \Gamma_{ip}$$
$$\text{THEN } \begin{cases} \dot{x}(t) = A_i x(t) + B_i \omega(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $\mathcal{R}_i$ denotes the ith rule with $i \in \mathcal{I}_r$; $z_j(t)$ with $j \in \mathcal{I}_p$ is the jth premise variable; Γ_{ij} is the fuzzy set for $z_j(t)$ in $\mathcal{R}_i$; $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{l\times n}$ are the system matrices; $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^l$, and $\omega(t) \in \mathbb{R}^m$ are a state vector, an output vector, and an external disturbance, respectively.

By applying the singleton fuzzifier, the product inference engine, and the center-average defuzzifier to (1), we obtain the following defuzzified output of the system:

$$\dot{x}(t) = \sum_{i=1}^{r} w_i\big(z(t)\big) \{A_i(x) + B_i\omega(t)\},$$
$$y(t) = Cx(t),$$

where $z(t) = \mathrm{col}\{z_1(t), z_2(t), \dots, z_p(t)\}$ is the premise vector, and $w_i\big(z(t)\big) \in [0, 1]$ is a normalized membership function satisfying the following properties:

$$w_i\big(z(t)\big) = \frac{\mu_i\big(z(t)\big)}{\sum_{j=1}^{r}\mu_j\big(z(t)\big)}, \ \mu_i\big(z(t)\big) = \prod_{j=1}^{p}\Gamma_{ij}\big(z_j(t)\big),$$
$$\sum_{i=1}^{r} w_i\big(z(t)\big) = 1, \ \sum_{i=1}^{r} \dot{w}_i\big(z(t)\big) = 0,$$

in which $\Gamma_{ij}\big(z_j(t)\big) : U_{z_p} \subset \mathbb{R} \to \mathbb{R}_{[0,1]}$ is the membership function of $z_j(t)$ on a compact set U_{z_p}, and $w_i\big(z(t)\big)$ is a differentiable function.

The system mentioned above holds the following assumption:

Assumption 1: All pairs of (A_i, C) with $i \in \mathcal{I}_r$ are observable. The state and premise variables are not measurable, but *the output vector is measurable only at sampling instances, $t = t_k$ with $k \in \{1, 2, \dots\}$.*

Based on the above assumption, we propose a sampled-data fuzzy observer with exponential time-varying gains whose IF-THEN rules are as follows:

$$\hat{\mathcal{R}}_i\text{: IF } q_1(t) \text{ is } \Upsilon_{i1} \text{ AND } \dots \text{ AND } q_p(t) \text{ is } \Upsilon_{ip} \text{ THEN}$$
$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + e^{-\eta(t-t_k)}L_i\big(\hat{y}(t_k) - y_F(t_k)\big), \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{2}$$

where $\hat{\mathcal{R}}_i$ with $i \in \mathcal{I}_r$ is the observer rule; $q_j(t)$ with $j \in \mathcal{I}_p$ is the jth premise variable; Υ_{ij} is the fuzzy set; $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{y}(t) \in \mathbb{R}^l$ are system and output vectors of an observer, respectively; $e^{-\eta(t-t_k)}L_i \in \mathbb{R}^{n\times l}$ is the exponential time-varying observer gain matrix to be determined in which $\eta \in \mathbb{R}_{\geq 0}$ is a given scalar; t_k is the kth sampling time satisfying $t_{k+1} - t_k = h_k \leq h$, in which h is an allowable maximum sampling period; $y_F(t_k) \in \mathbb{R}^l$ represents an output of the system with measurement faults.

The difference between the output of the observer and the output of the system with measurement fault is represented by the following relationship using the time-varying uncertain matrix:

$$\hat{y}(t_k) - y_F(t_k) := F(t_k)\big(\hat{y}(t_k) - y(t_k)\big),$$

where $F(t_k) \in \mathbb{R}^{l\times l}$ is an unknown time-varying matrix denoting the sensor fault which has the following structure:

$$F(t_k) = \mathrm{diag}\{f_1(t_k), f_2(t_k), \dots, f_l(t_k)\},$$

in which, for $a \in \mathcal{I}_l$, $f_a(t_k) \in [f_a^L,\ f_a^U]$, and f_a^L and f_a^U are constant scalars denoting the lower and upper bounds of admissible failures of the sensor, respectively.

Applying the same process used for the system to (2), we have

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} m_i\big(q(t)\big)\Big\{A_i\hat{x}(t) + e^{-\eta(t-t_k)}L_iF(t_k)\big(\hat{y}(t_k) - y(t_k)\big)\Big\}, \tag{3}$$

where $q(t) = \mathrm{col}\{q_1(t), q_2(t), \dots, q_p(t)\}$ is the premise vector, and $m_i\big(q(t)\big) \in [0, 1]$ is the normalized membership function satisfying the following properties:

$$m_i\big(q(t)\big) = \frac{\nu_i\big(q(t)\big)}{\sum_{j=1}^{r}\nu_j\big(q(t)\big)},\quad \nu_i\big(q(t)\big) = \prod_{j=1}^{p}\Upsilon_{ij}\big(q_j(t)\big),$$
$$\sum_{i=1}^{r} m_i\big(q(t)\big) = 1,\quad \sum_{i=1}^{r}\dot{m}_i\big(q(t)\big) = 0,$$

where $\Upsilon_{ij}\big(q_j(t)\big) : U_{q_p} \subset \mathbb{R} \to \mathbb{R}_{[0,1]}$ is the membership function of $q_j(t)$ on a compact set U_{q_p}, and $m_i\big(q(t)\big)$ is a differentiable function.

Remark 1: As can be seen from the observer model (3), an observer gain used in this paper varies exponentially between two consecutive sampling instants. The advantage of using the time-varying gain is that the state estimation performance is increased by appropriately choosing a parameter, η. In this paper, using the time-varying observer gain is studied for the first time to design the sampled-data fuzzy observer.

Remark 2: In the observer model, $F(t_k)$ represents the sensor failure. $F(t_k)$ is generally a time-varying unknown matrix, but, following the study [30], this paper assumes that both the upper and lower bounds of its norm are known.

For $F(t_k)$, the following is assumed to be satisfied.

Assumption 2: The sensor fault is modeled as a norm-bounded matrix, in the form of

$$\begin{aligned} F(t_k) &= \mathrm{diag}\{f_1(t_k), f_2(t_k), \dots, f_l(t_k)\} \\ &= F_0\big\{I + F_1(t_k)\big\}, \end{aligned}$$

where F_0 is a given known matrix and $F_1(t_k)$ is an unknown matrix satisfying $F_1^T(t_k)F_1(t_k) \leq F_2^TF_2$ with a given matrix F_2. In this case, F_0, $F_1(t_k)$, and F_2 have the following structure:

$$\begin{aligned} F_0 &= \mathrm{diag}\{f_{01}, f_{02}, \dots, f_{0l}\},\ f_{0a} = \big(f_a^L + f_a^U\big)/2, \\ F_1(t_k) &= \mathrm{diag}\{f_{11}, f_{12}, \dots, f_{1l}\},\ f_{1a} = \big\{f_a(t_k) - f_{0a}\big\}/f_{0a}, \\ F_2 &= \mathrm{diag}\{f_{21}, f_{22}, \dots, f_{2l}\},\ f_{2a} = \big(f_a^U - f_a^L\big)/\big(f_a^L + f_a^U\big), \end{aligned}$$

for $a \in \mathcal{I}_l$. Physically, $f_{1a}(t_k)$ is an extracted uncertain term of $f_a(t_k)$ and f_{2a} stands for norm bounds of $f_{1a}(t_k)$.

Remark 3: The observer model in (3) contains an unknown time-varying matrix $F(t_k)$. It models the uncertainty in the system output due to measurement faults, and we only know its upper and lower bounds, represented by Assumption 2. This information is necessary only when determining the observer gain matrix. That is, since the influence of $F(t_k)$ is physically reflected as a measurement fault of $y_F(t_k)$, the information of $F(t_k)$ is not necessary when implementing our observer model. Once the observer gain is determined considering the upper and lower bounds of $F(t_k)$, the following dynamics is used to implement the observer model:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} m_i\big(q(t)\big)\big\{A_i\hat{x}(t) + e^{-\eta(t-t_k)}L_i\big(\hat{y}(t_k) - y_F(t_k)\big)\big\}.$$

From now on, the following shorthand notation is employed in order to enhance the readability.

$$M_w(t) := \sum_{i=1}^{r} w_i\big(z(t)\big)M_i \text{ and } M_m(t) := \sum_{i=1}^{r} m_i\big(q(t)\big)M_i.$$

By defining the estimation error vector as $\tilde{x}(t) := \hat{x}(t) - x(t)$, we obtain the following:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \dot{\hat{x}}(t) - \dot{x}(t) \\ &= A_m(t)\hat{x}(t) + e^{-\eta(t-t_k)}L_m(t)F(t_k)C\big\{\hat{x}(t_k) - x(t_k)\big\} \\ &\quad - A_w(t)x(t) - B_w(t)\omega(t) \\ &= A_m(t)\tilde{x}(t) + e^{-\eta(t-t_k)}L_m(t)F(t_k)C\tilde{x}(t_k) \\ &\quad + \Delta_A(t)x(t) - B_w(t)\omega(t), \end{aligned} \tag{4}$$

where $\Delta_A(t) := A_m(t) - A_w(t)$.

Remark 4: In many cases, premise variables of a T–S fuzzy system consist of its state variables. Thus, an observer generally does not share the same membership function with a *system. This membership function mismatching is expressed as $\Delta_A(t)$ in this paper. In the remainder of the paper, we use λ_Δ to minimize the effect of $\Delta(t)$ on the state estimation performance.*

In addition, we define the following additional vector:

$$\varepsilon(t) = e^{\eta t}\tilde{x}(t),$$

and its time derivative can be obtained as follows:

$$\begin{aligned}\dot{\varepsilon}(t) &= \eta e^{\eta t}\tilde{x}(t) + e^{\eta t}\dot{\tilde{x}}(t)\\ &= \big\{A_m(t)+\eta I\big\}e^{\eta t}\tilde{x}(t) + L_m(t)F(t_k)Ce^{\eta t_k}\tilde{x}(t_k)\\ &\quad +\Delta_A(t)e^{\eta t}x(t) - B_w(t)e^{\eta t}\omega(t)\\ &= \bar{A}_m(t)\varepsilon(t) + L_m(t)F(t_k)C\varepsilon(t_k)\\ &\quad +\Delta_A(t)x_\eta(t) + B_w(t)\omega_\eta(t), \end{aligned} \tag{5}$$

where $\bar{A}_m(t) := A_m(t)+\eta I$; $x_\eta(t) := e^{\eta t}x(t)$; $\omega_\eta(t) := -e^{\eta t}\omega(t)$.

Moreover, the objective of this paper is to solve the following problem.

Problem 1: *For given scalars $h \geq t_{k+1}-t_k$, $\eta \in \mathbb{R}_{>0}$, and $\alpha \in [-\eta, 0)$, find an observer gain matrix L_i with $i \in \mathcal{I}_r$ of (3) that satisfies the following criteria:*

1) *The equilibrium of (4) is exponentially asymptotically stabilized with decay rate of $\eta+\alpha$, when $\omega_\eta(t)=0$ and $x_\eta(t)=0$.*
2) *When $\alpha = 0$ and $\tilde{x}(0)=0$, the following H_∞ criterion is fulfilled:*

$$\int_0^t \varepsilon^T(s)\varepsilon(s)ds - \gamma^2\int_0^t W^T(s)W(s)ds \leq 0, \tag{6}$$

where $\gamma > 0$ is a given scalar; $W(s) = \mathrm{col}\{\omega_\eta(s), x_\eta(s)\}$.

Before closing this section, we provide the following lemma required when developing the main results.

Lemma 1: [31] *For any $s \in [t_k, t]$, $t \in [t_k, t_{k+1})$ with $k > 0$, a symmetric matrix Ξ_{ij} with $(i,j) \in \mathcal{I}_r \times \mathcal{I}_r$, and a given scalar δ_i, if normalized membership functions satisfy $|m_i(q(t)) - m_i(q(s))| \leq \delta_i$, then,*

$$\sum_{i=1}^{r}\sum_{j=1}^{r} m_i\big(q(t)\big)m_j\big(q(s)\big)\Xi_{ij} \succ 0$$

holds if there exist positive definite matrices R_{ij} and N_{ij}, and any matrices $X_{ij} = X_{ji}^T$ and $X_{i(j+r)} = X_{j(i+r)}^T$ with $(i,j) \in \mathcal{I}_r \times \mathcal{I}_r$, such that the following LMIs hold:

$$M_{ij} + M_{ji} \succeq X_{ij} + X_{ji}, \tag{7}$$

$$\Xi_{ij} - 2M_{ij} - \sum_{k=1}^{r}\delta_k\big(M_{ik}^{+} + M_{kj}^{+}\big) \succeq X_{i(j+r)} + X_{(j+r)i}, \tag{8}$$

$$\begin{bmatrix} Y_{11} & Y_{12}\\ * & Y_{11}\end{bmatrix} \succ 0, \tag{9}$$

where $M_{ij} = R_{ij} - N_{ij}$; $M_{ij}^{+} = R_{ij} + N_{ij}$;

$$Y_{11} = \begin{bmatrix} X_{11} & \cdots & X_{1r}\\ \vdots & \ddots & \vdots\\ X_{r1} & \cdots & X_{rr}\end{bmatrix};$$

$$Y_{12} = \begin{bmatrix} X_{1(r+1)} & \cdots & X_{1(2r)}\\ \vdots & \ddots & \vdots\\ X_{r(r+1)} & \cdots & X_{r(2r)}\end{bmatrix}.$$

III. MAIN RESULTS

This section provides a sufficient condition solving Problem 1 based on the following LKF. We modified the time-dependent LKF given in [28] to be less conservative by fuzzifying the LKF as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{10}$$

where

$$\begin{aligned} V_1(t) &= e^{2\alpha t}\varepsilon^T(t)P_m(t)\varepsilon(t),\\ V_2(t) &= (t_{k+1}-t)\int_{t_k}^{t} e^{2\alpha s}\dot{\varepsilon}^T(s)U_m(s)\dot{\varepsilon}(s)ds,\\ V_3(t) &= (t_{k+1}-t)e^{2\alpha t}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}^T\begin{bmatrix}E_{11} & E_{12}\\ * & E_{22}\end{bmatrix}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}, \end{aligned}$$

in which $\alpha \in (-\eta, 0]$ is a given scalar, $P_i \in \mathbb{R}^{n\times n}$ and $U_i \in \mathbb{R}^{n\times n}$ are positive definite matrices to be determined, and $E_{11} = \big(\mathbb{E}_1 + \mathbb{E}_1^T\big)/2$, $E_{12} = -\mathbb{E}_1 + \mathbb{E}_2$, $E_{22} = -\mathbb{E}_2 - \mathbb{E}_2^T + \big(\mathbb{E}_1 + \mathbb{E}_1^T\big)/2$, and $\mathbb{E}_1 \in \mathbb{R}^{n\times n}$ and $\mathbb{E}_2 \in \mathbb{R}^{n\times n}$ are any matrices to be determined.

In the time-derivative of (10), there exists $\dot{m}_i\big(q(t)\big)$. To handle this nonlinearity, we declare the following assumption which is commonly used in existing literatures:

Assumption 3: *For a given positive scalar φ_i, the time derivative of membership function is assumed to satisfy the following condition:*

$$\big|\dot{m}_i\big(q(t)\big)\big| \leq \varphi_i. \tag{11}$$

Next, the following lemma provides a condition of guaranteeing (10) to be an appropriate Lyapunov candidate.

Lemma 2: *The proposed LKF (10) is positive definite if there exists a positive definite matrix $\mathcal{U} \in \mathbb{R}^{n\times n}$ such that the following hold for $i \in \mathcal{I}_r$:*

$$\begin{bmatrix} P_i + \mathcal{U} + hE_{11} & -\mathcal{U} + hE_{12}\\ * & \mathcal{U} + hE_{22}\end{bmatrix} \succ 0, \tag{12}$$

$$U_i - \mathcal{U} \succeq 0. \tag{13}$$

Proof: First, considering that $V_1(t) = \frac{t-t_k+t_{k+1}-t}{t_{k+1}-t_k}V_1(t)$ and $P_m(t) \succ 0$, we can rewrite $V_1(t)$ as

$$\begin{aligned} V_1(t) &= \frac{t-t_k}{h_k}V_1(t) + \frac{t_{k+1}-t}{h_k}V_1(t)\\ &\geq \frac{t-t_k}{h}V_1(t) + \frac{t_{k+1}-t}{h}V_1(t). \end{aligned} \tag{14}$$

Next, if we assume that (13) holds, the following is obvious:

$$\sum_{i=1}^{r} m_i(s)\big(U_i - \mathcal{U}\big) = \sum_{i=1}^{r} m_i(s)U_i - \mathcal{U} \succeq 0. \tag{15}$$

Thus, considering $\alpha \in [-\eta, 0)$ and the above, we get

$$\begin{aligned} V_2(t) &\geq (t_{k+1} - t)\int_{t_k}^{t} e^{2\alpha s}\dot{\varepsilon}^T(s)\mathcal{U}\dot{\varepsilon}(s)ds \\ &\geq (t_{k+1} - t)e^{2\alpha t}\int_{t_k}^{t} \dot{\varepsilon}^T(s)\mathcal{U}\dot{\varepsilon}(s)ds. \end{aligned} \tag{16}$$

Now, applying the well-known Jensen's inequality to the last term of the above inequality, we have

$$\begin{aligned} V_2(t) &\geq \frac{t_{k+1}-t}{t-t_k}e^{2\alpha t}\left\{\int_{t_k}^{t}\dot{\varepsilon}(s)ds\right\}^T \mathcal{U}\left\{\int_{t_k}^{t}\dot{\varepsilon}(s)ds\right\} \\ &\geq \frac{t_{k+1}-t}{h}e^{2\alpha t}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}^T\begin{bmatrix}\mathcal{U} & -\mathcal{U}\\ * & \mathcal{U}\end{bmatrix}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}. \end{aligned} \tag{17}$$

Now, combining $V_3(t)$ with (14) and (17) implies that

$$\begin{aligned} V(t) \geq\ & \frac{t-t_k}{h}e^{2\alpha t}\varepsilon^T(t)P_m(t)\varepsilon(t) \\ &+\frac{t_{k+1}-t}{h}e^{2\alpha t}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}^T \\ &\times\begin{bmatrix}P_m(t)+\mathcal{U}+hE_{11} & -\mathcal{U}+hE_{12}\\ * & \mathcal{U}+hE_{22}\end{bmatrix}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}, \end{aligned}$$

from which we can conclude that LMIs (12) and (13) guarantee that $V(t) \geq 0$. □

The following lemma is useful to relax the conservativeness by introducing some null terms.

Lemma 3: For a positive definite matrix $U_i \in \mathbb{R}^{n\times n}$, if there exist a positive definite matrix $H_i \in \mathbb{R}^{3n\times 3n}$, and any matrices $W_{1i} \in \mathbb{R}^{n\times n}$, $W_{2i} \in \mathbb{R}^{n\times n}$, and $W_{3i} \in \mathbb{R}^{n\times n}$ with $i \in \mathcal{I}_r$ such that (7)–(9) hold with

$$\Xi_{ij} = \begin{bmatrix}H_i & *\\ W_i^T & U_j\end{bmatrix}, \tag{18}$$

then the following inequality is always satisfied

$$\begin{aligned} -\int_{t_k}^{t}\dot{\varepsilon}^T(s)U_m(s)\dot{\varepsilon}(s)ds \leq\ & (t-t_k)\chi^T(t)H_m(t)\chi(t) \\ &+2\chi^T(t)W_m(t)\big\{\varepsilon(t)-\varepsilon(t_k)\big\}, \end{aligned} \tag{19}$$

where $\chi(t) = \mathrm{col}\{\varepsilon(t), \dot{\varepsilon}(t), \varepsilon(t_k)\}$; $W_m(t) = \mathrm{col}\{W_{1m}(t), W_{2m}(t), W_{3m}(t)\}$.

Proof: It is obvious that the following holds:

$$\begin{aligned} 0 &\preceq \begin{bmatrix}0 & W_m(t)U_m^{-1/2}(s)\\ 0 & U_m^{1/2}(s)\end{bmatrix}\begin{bmatrix}0 & W_m(t)U_m^{-1/2}(s)\\ 0 & U_m^{1/2}(s)\end{bmatrix}^T \\ &= \begin{bmatrix}W_m(t)U_m^{-1}(s)W_m^T(t) & W_m(t)\\ * & U_m(s)\end{bmatrix}. \end{aligned}$$

Multiplying $\mathrm{col}\{\chi(t), \dot{\varepsilon}(s)\}^T$ and $\mathrm{col}\{\chi(t), \dot{\varepsilon}(s)\}$ left- and right-hand sides, respectively, and integrating the above from $s = t_k$ to $s = t$, we have

$$\begin{aligned} 0 &\leq \int_{t_k}^{t}\begin{bmatrix}\chi(t)\\ \dot{\varepsilon}(s)\end{bmatrix}^T\begin{bmatrix}W_m(t)U_m^{-1}(s)W_m^T(t) & W_m(t)\\ * & U_m(s)\end{bmatrix}\begin{bmatrix}\chi(t)\\ \dot{\varepsilon}(s)\end{bmatrix}ds \\ &= \int_{t_k}^{t}\chi^T(t)W_m(t)U_m^{-1}(s)W_m^T(t)\chi(t)ds \\ &\quad +2\int_{t_k}^{t}\chi^T(t)W_m(t)\dot{\varepsilon}(s)ds \\ &\quad +\int_{t_k}^{t}\dot{\varepsilon}^T(s)U_m(s)\dot{\varepsilon}(s)ds. \end{aligned} \tag{20}$$

Now, assuming that there exists $H_m(t) \in \mathbb{R}^{3n\times 3n}$ such that, for $s \in [t_k, t]$,

$$H_m(t) - W_m(t)U_m^{-1}(s)W_m^T(t) \succ 0,$$

and by applying the Schur complements, we have

$$\begin{aligned} 0 &\prec \begin{bmatrix}H_m(t) & *\\ W_m^T(t) & U_m(s)\end{bmatrix} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} m_i\big(q(t)\big)m_j\big(q(s)\big)\begin{bmatrix}H_i & *\\ W_i^T & U_j\end{bmatrix} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} m_i\big(q(t)\big)m_j\big(q(s)\big)\,\Xi_{ij}, \end{aligned}$$

where $\Xi_{ij} := \begin{bmatrix}H_i & *\\ W_i^T & U_j\end{bmatrix}$, then, from Lemma 1, we know that the above matrix inequality guaranteed by the LMIs of (7)–(9).

Then, we can rewrite (20) as follows:

$$\begin{aligned} 0 &\leq \int_{t_k}^{t}\chi^T(t)H_m(t)\chi(t)ds + 2\int_{t_k}^{t}\chi^T(t)W_m(t)\dot{\varepsilon}(s)ds \\ &\quad +\int_{t_k}^{t}\dot{\varepsilon}^T(s)U_m(s)\dot{\varepsilon}(s)ds \\ &= (t-t_k)\chi^T(t)H_m(t)\chi(t) + 2\chi^T(t)W_m(t)\big\{\varepsilon(t)-\varepsilon(t_k)\big\} \\ &\quad +\int_{t_k}^{t}\dot{\varepsilon}^T(s)U_m(s)\dot{\varepsilon}(s)ds, \end{aligned}$$

from which we can conclude that (19) holds. □

Remark 5: It is worth mentioning that, due to $U_m(s)$ of $V_2(t)$, conventional matrix inequalities for handling an integral term cannot be used directly to derive the proposed observer design condition. In other words, if $U_m(s)$ of (20) was a simple matrix U, then the first term of (20) becomes

$$\begin{aligned} &\int_{t_k}^{t}\chi^T(t)W_m(t)U^{-1}W_m^T(t)\chi(t)ds \\ &\quad = (t-t_k)\chi^T(t)W_m(t)U^{-1}W_m^T(t)\chi(t), \end{aligned}$$

which can be found in the conventional approaches. Thus, the advantage of this lemma is to derive the condition for an integral term with a fuzzified matrix $U_m(t)$. Moreover, Lemma 3 allows the additional slack variables in the observer *design condition, which relaxes the conservativeness of a sufficient condition, resulting in improved state estimation performance.*

Summarizing the above, we propose the following theorem that provides a solution to Problem 1.

Theorem 1: For given scalars $h > 0$, $\eta > 0$, $\alpha \in [-\eta, 0)$, $\beta > 0$, $\sigma > 0$, λ_B, λ_Δ, $\varphi_i > 0$, and $\delta_i > 0$ with $i \in \mathcal{I}_r$, the error dynamics (5) satisfies the conditions of Problem 1, if there exist positive definite matrices P_i, U_i, H_i, $\mathcal{U}$, R_{ij}, N_{ij}, a symmetric matrix Z, and any matrices W_{1i}, W_{2i}, W_{3i}, $\bar{L}_i$, M, $\mathbb{E}_1$, $\mathbb{E}_2$, $X_{ij} = X_{ji}^T$, $X_{i(j+r)} = X_{j(i+r)}^T$ of appropriate dimensions, such that the LMIs (7)–(9), (12)–(13),

$$P_i + Z, \succ 0 \tag{21}$$

$$\Psi_{1i} := \begin{bmatrix} \Phi_{1i} + h\Phi_{2i} & * \\ \Omega_i & -\Lambda^{-1} \end{bmatrix} \prec 0, \tag{22}$$

$$\Psi_{2i} := \begin{bmatrix} \Phi_{1i} + hH_i & * \\ \Omega_i & -\Lambda^{-1} \end{bmatrix} \prec 0, \tag{23}$$

simultaneously hold with

$$\Xi_{ij} = \begin{bmatrix} H_i & * \\ W_i^T & U_j \end{bmatrix},$$

where $W_i = \mathrm{col}\{W_{1i}, W_{2i}, W_{3i}\}$;

$$\Phi_{li} = \begin{bmatrix} \phi^l_{11,i} & * & * \\ \phi^l_{21,i} & \phi^l_{22,i} & * \\ \phi^l_{31,i} & \phi^l_{32,i} & \phi^l_{33,i} \end{bmatrix} \text{ with } l \in \{1, 2\}; \tag{24}$$

$$\phi^1_{11,i} = 2\alpha P_i + P_\varphi - E_{11} + sym\{W_{1i} + M^T\bar{A}_i\} + I;$$
$$\phi^1_{21,i} = P_i + W_{2i} - M + \beta M^T\bar{A}_i;$$
$$\phi^1_{22,i} = -\beta(M + M^T);$$
$$\phi^1_{31,i} = -W_{1i}^T + W_{3i} - E_{12}^T + C^T F_0^T \bar{L}_i;$$
$$\phi^1_{32,i} = -W_{2i}^T + \beta C^T F_0^T \bar{L}_i;$$
$$\phi^1_{33,i} = -W_{3i} - W_{3i}^T - E_{22};$$
$$\phi^2_{11,i} = 2\alpha E_{11};$$
$$\phi^2_{21,i} = E_{11}^T;\ \phi^2_{22,i} = U_i;$$
$$\phi^2_{31,i} = 2\alpha E_{12}^T;\ \phi^2_{32,i} = E_{12}^T;\ \phi^2_{33,i} = 2\alpha E_{22};$$

$$\Omega_i = \begin{bmatrix} M & \beta M & 0 \\ M & \beta M & 0 \\ \sigma F_0^T\bar{L}_i & \beta\sigma F_0^T\bar{L}_i & \sigma^{-1}F_2C \end{bmatrix};$$

$$\Lambda^{-1} = \begin{bmatrix} \frac{\gamma^2}{\lambda_B}I & 0 & 0 \\ 0 & \frac{\gamma^2}{\lambda_\Delta}I & 0 \\ 0 & 0 & I \end{bmatrix};\ P_\varphi = \sum_{i=1}^r \varphi_i(P_i + Z).$$

In addition, the observer gain is obtained by $L_i = M^{-T}\bar{L}_i^T$.

Proof: First, the time derivative of $V_1(t)$ is as follows:

$$\begin{aligned}\dot{V}_1(t) &= 2e^{2\alpha t}\varepsilon^T(t)P_m(t)\dot{\varepsilon}(t) + 2\alpha e^{2\alpha t}\varepsilon^T(t)P_m(t)\varepsilon(t) \\ &\quad + e^{2\alpha t}\varepsilon^T(t)\dot{P}_m(t)\varepsilon(t),\end{aligned} \tag{25}$$

where $\dot{P}_m(t) = \sum_{i=1}^r \dot{m}_i(q(t))P_i$.

Moreover, it is obvious from the property of a membership function that, for any symmetric matrix $Z \in \mathbb{R}^{n\times n}$, the following holds

$$\sum_{i=1}^r \dot{m}_i(q(t))Z = 0.$$

Then, assuming that (11) and (21) hold, we have

$$\dot{P}_m(t) = \sum_{i=1}^r \dot{m}_i(q(t))(P_i + Z) \le \sum_{i=1}^r \varphi_i(P_i + Z). \tag{26}$$

Considering (26), we can rewrite (25) as follows:

$$\begin{aligned}\dot{V}_1(t) &\le 2e^{2\alpha t}\varepsilon^T(t)P_m(t)\dot{\varepsilon}(t) + 2\alpha e^{2\alpha t}\varepsilon^T(t)P_m(t)\varepsilon(t) \\ &\quad + e^{2\alpha t}\varepsilon^T(t)P_\varphi\varepsilon(t),\end{aligned} \tag{27}$$

where $P_\varphi = \sum_{i=1}^r \varphi_i(P_i + Z)$.

Next, $\dot{V}_2(t)$ becomes

$$\begin{aligned}\dot{V}_2(t) &= (t_{k+1} - t)e^{2\alpha t}\dot{\varepsilon}^T(t)U_m(t)\dot{\varepsilon}(t) \\ &\quad - \int_{t_k}^t e^{2\alpha s}\dot{\varepsilon}^T(s)U_m(s)\dot{\varepsilon}(s)ds \\ &\le (t_{k+1} - t)e^{2\alpha t}\dot{\varepsilon}^T(t)U_m(t)\dot{\varepsilon}(t) \\ &\quad - e^{2\alpha t}\int_{t_k}^t \dot{\varepsilon}^T(s)U_m(s)\dot{\varepsilon}(s)ds,\end{aligned} \tag{28}$$

because $\alpha \in (-\eta, 0]$.

Assuming that (7)–(9) hold with (18), from (19) of Lemma 3, we can majorize the last term of (28) as follows:

$$\begin{aligned}\dot{V}_2(t) &\le (t_{k+1} - t)e^{2\alpha t}\dot{\varepsilon}^T(t)U_m(t)\dot{\varepsilon}(t) \\ &\quad + (t - t_k)e^{2\alpha t}\chi^T(t)H_m(t)\chi(t) \\ &\quad + 2e^{2\alpha t}\chi^T(t)W_m(t)\{\varepsilon(t) - \varepsilon(t_k)\}.\end{aligned} \tag{29}$$

In addition, the time derivative of $V_3(t)$ becomes

$$\begin{aligned}\dot{V}_3(t) &= 2(t_{k+1} - t)e^{2\alpha t}\begin{bmatrix}\varepsilon(t) \\ \varepsilon(t_k)\end{bmatrix}^T \begin{bmatrix} E_{11} & E_{12} \\ * & E_{22}\end{bmatrix}\begin{bmatrix}\dot{\varepsilon}(t) \\ 0\end{bmatrix} \\ &\quad + (t_{k+1} - t)2\alpha e^{2\alpha t}\begin{bmatrix}\varepsilon(t) \\ \varepsilon(t_k)\end{bmatrix}^T \begin{bmatrix} E_{11} & E_{12} \\ * & E_{22}\end{bmatrix}\begin{bmatrix}\varepsilon(t) \\ \varepsilon(t_k)\end{bmatrix} \\ &\quad - e^{2\alpha t}\begin{bmatrix}\varepsilon(t) \\ \varepsilon(t_k)\end{bmatrix}^T \begin{bmatrix} E_{11} & E_{12} \\ * & E_{22}\end{bmatrix}\begin{bmatrix}\varepsilon(t) \\ \varepsilon(t_k)\end{bmatrix}.\end{aligned} \tag{30}$$

On the other hand, following Assumption 2, the closed-loop of the system (5) becomes

$$\begin{aligned}\dot{\varepsilon}(t) &= \bar{A}_m(t)\varepsilon(t) + L_m(t)F_0C\varepsilon(t_k) \\ &\quad + L_m(t)F_0F_1(t_k)C\varepsilon(t_k) \\ &\quad + \Delta_A(t)x_\eta(t) + B_w(t)\omega_\eta(t),\end{aligned}$$

from which we know that the following always holds for any matrix $M \in \mathbb{R}^{n\times n}$ and a given positive scalar β:

$$\begin{aligned}0 &= 2e^{2\alpha t}\{M\varepsilon(t) + \beta M\dot{\varepsilon}(t)\}^T \\ &\quad \times \{-\dot{\varepsilon}(t) + \bar{A}_m(t)\varepsilon(t) + L_m(t)F_0C\varepsilon(t_k) \\ &\quad + L_m(t)F_0F_1(t_k)C\varepsilon(t_k) \\ &\quad + \Delta_A(t)x_\eta(t) + B_w(t)\omega_\eta(t)\} \\ &= \Theta_1(t) + \Theta_2(t) + \Theta_3(t),\end{aligned} \tag{31}$$

where

$$\Theta_1(t) = 2e^{2\alpha t}\Big\{M\varepsilon(t)+\beta M\dot{\varepsilon}(t)\Big\}^T \times\Big\{-\dot{\varepsilon}(t)+\bar{A}_m(t)\varepsilon(t)+L_m(t)F_0C\varepsilon(t_k)\Big\},$$

$$\Theta_2(t) = 2e^{2\alpha t}\Big\{M\varepsilon(t)+\beta M\dot{\varepsilon}(t)\Big\}^T \times\Big\{L_m(t)F_0F_1(t_k)C\varepsilon(t_k)\Big\},$$

$$\Theta_3(t) = 2e^{2\alpha t}\Big\{M\varepsilon(t)+\beta M\dot{\varepsilon}(t)\Big\}^T \times\Big\{\Delta_A(t)x_\eta(t)+B_w(t)\omega_\eta(t)\Big\}.$$

By defining the column vector $\chi(t) = \mathrm{col}\{\varepsilon(t), \dot{\varepsilon}(t), \varepsilon(t_k)\}$, $\Theta_1(t)$ can be rewritten as

$$\Theta_1(t) = e^{2\alpha t}\chi^T(t)\hat{\Theta}_{1m}(t)\chi(t), \tag{32}$$

where

$$\hat{\Theta}_{1m}(t) = \begin{bmatrix} sym\{\bar{A}_m^T(t)M\} & * & * \\ -M+\beta M^T\bar{A}_m(t) & -\beta(M+M^T) & * \\ C^TF_0^TL_m^T(t)M & \beta C^TF_0^TL_m^T(t)M & 0\end{bmatrix}.$$

Moreover, we can rewrite $\Theta_2(t)$ as follows:

$$\Theta_2(t) = 2e^{2\alpha t}\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\end{bmatrix}^T\begin{bmatrix}M^TL_m(t)F_0\\ \beta M^TL_m(t)F_0\end{bmatrix}F_1(t_k)C\varepsilon(t_k). \tag{33}$$

Applying the well-known matrix inequality of

$$X^TY+Y^TX \le \sigma X^TX+\sigma^{-1}Y^TY, \tag{34}$$

where X and Y are matrices, and σ is a given scalar, (33) becomes

$$\begin{aligned}\Theta_2(t) &\le e^{2\alpha t}\Bigg\{\sigma\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\end{bmatrix}^T\begin{bmatrix}M^TL_m(t)F_0\\ \beta M^TL_m(t)F_0\end{bmatrix} \times\begin{bmatrix}M^TL_m(t)F_0\\ \beta M^TL_m(t)F_0\end{bmatrix}^T\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\end{bmatrix} \\ &\quad +\sigma^{-1}\varepsilon^T(t_k)C^TF_1^T(t_k)F_1(t_k)C\varepsilon(t_k)\Bigg\} \\ &\le e^{2\alpha t}\left\{\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\\ \varepsilon(t_k)\end{bmatrix}^T\begin{bmatrix}\sigma^{\frac{1}{2}}M^TL_m(t)F_0\\ \sigma^{\frac{1}{2}}\beta M^TL_m(t)F_0\\ \sigma^{-\frac{1}{2}}C^TF_2^T\end{bmatrix} \times\begin{bmatrix}\sigma^{\frac{1}{2}}M^TL_m(t)F_0\\ \sigma^{\frac{1}{2}}\beta M^TL_m(t)F_0\\ \sigma^{-\frac{1}{2}}C^TF_2^T\end{bmatrix}^T\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\\ \varepsilon(t_k)\end{bmatrix}\right\},\end{aligned} \tag{35}$$

here we employed the condition $F_1^T(t_k)F_1(t_k) \le F_2^TF_2$ given in Assumption 2.

Similarly, by applying the inequality (34) to $\Theta_3(t)$, we have

$$\begin{aligned}\Theta_3(t) &= 2e^{2\alpha t}\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\end{bmatrix}^T\begin{bmatrix}M^T\\ \beta M^T\end{bmatrix}\begin{bmatrix}\Delta_A^T(t)\\ B_w^T(t)\end{bmatrix}^T\begin{bmatrix}x_\eta(t)\\ \omega_\eta(t)\end{bmatrix} \\ &\le e^{2\alpha t}\Bigg\{\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\end{bmatrix}^T\begin{bmatrix}M^T & M^T\\ \beta M^T & \beta M^T\end{bmatrix} \times\begin{bmatrix}\rho_1 I & 0\\ 0 & \rho_2 I\end{bmatrix}\begin{bmatrix}M & \beta M\\ M & \beta M\end{bmatrix}\begin{bmatrix}\varepsilon(t)\\ \dot{\varepsilon}(t)\end{bmatrix} \\ &\quad +\rho_1^{-1}x_\eta^T(t)\Delta_A^T(t)\Delta_A(t)x_\eta(t) \\ &\quad +\rho_2^{-1}\omega_\eta^T(t)B_w^T(t)B_w(t)\omega_\eta(t)\Bigg\},\end{aligned} \tag{36}$$

where ρ_1 and ρ_2 are given positive scalars.

Summarizing the result of (31) with (32), (35), and (36), we have

$$\begin{aligned}0 &= \Theta_1(t)+\Theta_2(t)+\Theta_3(t) \\ &\le e^{2\alpha t}\Big[\chi^T(t)\Big\{\hat{\Theta}_{1m}(t)+\Omega_m^T(t)\Lambda\Omega_m(t)\Big\}\chi(t) \\ &\quad +\rho_1^{-1}x_\eta^T(t)\Delta_A^T(t)\Delta_A(t)x_\eta(t) \\ &\quad +\rho_2^{-1}\omega_\eta^T(t)B_w^T(t)B_w(t)\omega_\eta(t)\Big],\end{aligned} \tag{37}$$

where $\Lambda = \mathrm{diag}\{\rho_1 I, \rho_2 I, I\}$;

$$\Omega_m(t) = \begin{bmatrix} M & \beta M & 0\\ M & \beta M & 0\\ \sigma^{\frac{1}{2}}F_0^TL_m^T(t)M & \sigma^{\frac{1}{2}}\beta F_0^TL_m^T(t)M & \sigma^{-\frac{1}{2}}F_2C\end{bmatrix}.$$

To guarantee the H_∞ performance criterion (6), adding $e^{2\alpha t}\{\varepsilon^T(t)\varepsilon(t)-\gamma^2W^T(t)W(t)\}$ to $\dot{V}(t)$ and summarizing (27), (29), (30), and (37) yield

$$\begin{aligned}&\dot{V}(t)+e^{2\alpha t}\Big\{\varepsilon^T(t)\varepsilon(t)-\gamma^2W^T(t)W(t)\Big\} \\ &\le e^{2\alpha t}\Big[2\varepsilon^T(t)P_m(t)\dot{\varepsilon}(t)+2\alpha\varepsilon^T(t)P_m(t)\varepsilon(t)+\varepsilon^T(t)P_\varphi\varepsilon(t) \\ &\quad +2\chi^T(t)W_m(t)\{\varepsilon(t)-\varepsilon(t_k)\}+\varepsilon^T(t)I\varepsilon(t) \\ &\quad -\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}^T\begin{bmatrix}E_{11} & E_{12}\\ * & E_{22}\end{bmatrix}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix} \\ &\quad +\chi^T(t)\Big\{\hat{\Theta}_{1m}(t)+\Omega_m^T(t)\Lambda\Omega_m(t)\Big\}\chi(t) \\ &\quad +x_\eta^T(t)\Big\{\rho_1^{-1}\Delta_A^T(t)\Delta_A(t)-\gamma^2 I\Big\}x_\eta(t) \\ &\quad +\omega_\eta^T(t)\Big\{\rho_2^{-1}B_w^T(t)B_w(t)-\gamma^2 I\Big\}\omega_\eta(t)\Big] \\ &\quad +(t_{k+1}-t)e^{2\alpha t}\Bigg\{2\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}^T\begin{bmatrix}E_{11} & E_{12}\\ * & E_{22}\end{bmatrix}\begin{bmatrix}\dot{\varepsilon}(t)\\ 0\end{bmatrix} \\ &\quad +2\alpha\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix}^T\begin{bmatrix}E_{11} & E_{12}\\ * & E_{22}\end{bmatrix}\begin{bmatrix}\varepsilon(t)\\ \varepsilon(t_k)\end{bmatrix} \\ &\quad +\dot{\varepsilon}^T(t)U_m(t)\dot{\varepsilon}(t)\Bigg\} \\ &\quad +(t-t_k)e^{2\alpha t}\chi^T(t)H_m(t)\chi(t) \\ &= e^{2\alpha t}\chi^T(t)\Big\{\Phi_{1m}(t)+\Omega_m^T(t)\Lambda\Omega_m(t)\Big\}\chi(t) \\ &\quad +e^{2\alpha t}(t_{k+1}-t)\chi^T(t)\Phi_{2m}(t)\chi(t) \\ &\quad +e^{2\alpha t}(t-t_k)\chi^T(t)H_m(t)\chi(t) \\ &\quad +e^{2\alpha t}x_\eta^T(t)\Big\{\rho_1^{-1}\Delta_A^T(t)\Delta_A(t)-\gamma^2 I\Big\}x_\eta(t) \\ &\quad +e^{2\alpha t}\omega_\eta^T(t)\Big\{\rho_2^{-1}B_w^T(t)B_w(t)-\gamma^2 I\Big\}\omega_\eta(t),\end{aligned}$$

where $\Phi_{1m}(t)$ and $\Phi_{2m}(t)$ are defined in (24).

From the above, we know that $\dot{V}(t) + e^{2\alpha t}\big\{\varepsilon^T(t)\varepsilon(t) - \gamma^2 W^T(t)W(t)\big\} \leq 0$ for $t \in [t_k, t_{k+1})$ is guaranteed by the following conditions:

$$e^{2\alpha t}\chi^T(t)\big\{\Phi_{1m}(t) + \Omega_m^T(t)\Lambda\Omega_m(t) \\ +(t_{k+1} - t)\Phi_{2m}(t) + (t - t_k)H_m(t)\big\}\chi(t) \leq 0 \quad (38)$$

and

$$e^{2\alpha t}x_\eta^T(t)\Big\{\rho_1^{-1}\Delta_A^T(t)\Delta_A(t) - \gamma^2 I\Big\}x_\eta(t) = 0, \quad (39)$$

$$e^{2\alpha t}\omega_\eta^T(t)\Big\{\rho_2^{-1}B_w^T(t)B_w(t) - \gamma^2 I\Big\}\omega_\eta(t) = 0. \quad (40)$$

The conditions (39) and (40) are satisfied when $\rho_1^{-1} = \frac{\gamma^2}{\lambda_\Delta}$ and $\rho_2^{-1} = \frac{\gamma^2}{\lambda_B}$. Moreover, we can rewrite (38) as follows:

$$(38) \leq e^{2\alpha t}\frac{t_{k+1} - t}{h}\chi^T(t)\Big\{\Phi_{1m}(t) + \Omega_m^T(t)\Lambda\Omega_m(t) \\ +h\Phi_{2m}(t)\Big\}\chi(t) + e^{2\alpha t}\frac{t - t_k}{h}\chi^T(t)\Big\{\Phi_{1m}(t) \\ +\Omega_m^T(t)\Lambda\Omega_m(t) + hH_m(t)\Big\}\chi(t) \leq 0,$$

which is guaranteed by the following inequalities:

$$\Phi_{1m}(t) + \Omega_m^T(t)\Lambda\Omega_m(t) + h\Phi_{2m}(t) \prec 0, \quad (41)$$

$$\Phi_{1m}(t) + \Omega_m^T(t)\Lambda\Omega_m(t) + hH_m(t) \prec 0. \quad (42)$$

Finally, by applying the Schur complements on (41) and (42), we have

$$\sum_{i=1}^{r} m_i\big(q(t)\big)\Psi_{li} \prec 0 \text{ with } l \in \mathcal{I}_2,$$

where

$$\Psi_{1i} = \begin{bmatrix}\Phi_{1i}+h\Phi_{2i} & * \\ \Omega_i & -\Lambda^{-1}\end{bmatrix};\ \Psi_{2i} = \begin{bmatrix}\Phi_{1i}+hH_i & * \\ \Omega_i & -\Lambda^{-1}\end{bmatrix},$$

which is guaranteed by the LMIs (22) and (23).

So far, we have derived the sufficient condition for guaranteeing $\dot{V}(t) + e^{2\alpha t}\left\{\varepsilon^T(t)\varepsilon(t) - \gamma^2 W^T(t)W(t)\right\} \leq 0$. From now on, we will prove whether the derived condition solves Problem 1.

First, integrating $\dot{V}(t)+e^{2\alpha t}\big\{\varepsilon^T(t)\varepsilon(t)-\gamma^2 W^T(t)W(t)\big\} \leq 0$ yields

$$V(t) - V(0) + \int_0^t e^{2\alpha s}\left\{\varepsilon^T(s)\varepsilon(s) - \gamma^2 W^T(s)W(s)\right\}ds \leq 0.$$

Considering $V(t) \geq 0$ and assuming $\alpha = 0$ and $\tilde{x}(0) = 0$, the above becomes

$$\int_0^t \varepsilon^T(s)\varepsilon(s)ds - \gamma^2\int_0^t W^T(s)W(s)ds \leq 0,$$

from which we know that the second condition of Problem 1 is satisfied.

Next, if we assume that $\omega_\eta(t) = 0$ and $x_\eta(t) = 0$, then $W(t) = 0$, which means

$$0 \geq \dot{V}(t) + e^{2\alpha t}\left\{\varepsilon^T(t)\varepsilon(t) - \gamma^2 W^T(t)W(t)\right\} \\ = \dot{V}(t) + e^{2\alpha t}\varepsilon^T(t)\varepsilon(t) \geq \dot{V}(t) \text{ for } t \in [t_k, t_{k+1}).$$

Thus, from $\int_0^t \dot{V}(s)ds \leq 0$, we have

$$V(t) - V(0) \leq 0,$$

which implies

$$\lambda_{\min}(P_i)e^{2\alpha t}\|\varepsilon(t)\|^2 = \lambda_{\min}(P_i)e^{2(\eta+\alpha)t}\|\tilde{x}(t)\|^2 \leq V(t) \leq V(0).$$

From the above, we know that the following holds

$$\|\tilde{x}(t)\| \leq \sqrt{\frac{V(0)}{\lambda_{\min}(P_i)}}e^{-(\eta+\alpha)t}.$$

Thus, we can say that the first condition of Problem 1 is also satisfied. Summarizing the above, we conclude that if there exists a solution of LMIs (7)–(9), (12)–(13), (22)–(23), then Problem 1 is solved. This completes the proof. □

Remark 6: Recently, sampled-data fuzzy observer design problem for oscillating systems has been actively studied. The main distinguishable features of this study are as follows: First, by using a time-varying observer gain and fuzzified LKF, improved state estimation performance is obtained. Secondly, the problem of membership function mismatching is easily solved using the H_∞ condition. Finally, by considering the sensor fault, robust state estimation performance is guaranteed even in the presence of sensor faults.

IV. NUMERICAL EXAMPLES

In this section, two numerical examples are provided to show the validity and superiority of the proposed method. The first example shows the comparison results of the state estimation performance according to various sampling periods. In the second example, the robust state estimation performance of the proposed method with respect to the sensor fault is validated.

A. EXAMPLE 1

In this example, the following nonlinear mass-spring system is employed [33]:

$$\ddot{\xi}(t) = -0.01\xi(t) - 0.67\xi^3(t) + 0.1\omega(t), \\ y(t_k) = \xi(t_k).$$

By choosing the state and premise variables as $x(t) = \text{col}\{x_1(t), x_2(t)\} = \text{col}\{\xi(t), \dot{\xi}(t)\}$ and $z(t) = x_1^2(t)$, respectively, we constructed the T–S fuzzy model as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} w_i\big(z(t)\big)\big\{A_i x(t) + B_i\omega(t)\big\}, \\ y(t_k) = Cx(t_k)$$

where $w_1\big(z(t)\big) = 1 - z(t)$; $w_2\big(z(t)\big) = 1 - w_1\big(z(t)\big)$;

$$A_1 = \begin{bmatrix}0 & 1 \\ -0.01 & 0\end{bmatrix};\ A_2 = \begin{bmatrix}0 & 1 \\ -0.68 & 0\end{bmatrix};$$

$$B_1 = B_2 = \begin{bmatrix}0.1 \\ 0\end{bmatrix};\ C = \begin{bmatrix}1 & 0\end{bmatrix}.$$

Next, the normalized membership function of the sampled-data fuzzy observer is chosen as $m_1\big(q(t)\big) = 1 - q(t)$

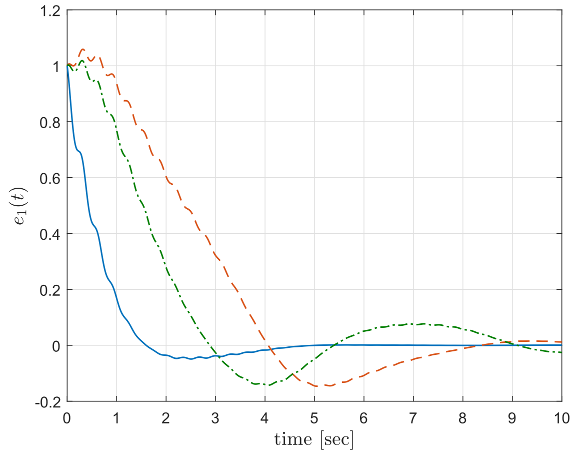

FIGURE 1. The time responses of $\tilde{x}_1(t)$: Theorem 1 with $\eta = 2$ (solid), [16] (dashed), and [17] (dash-dotted).

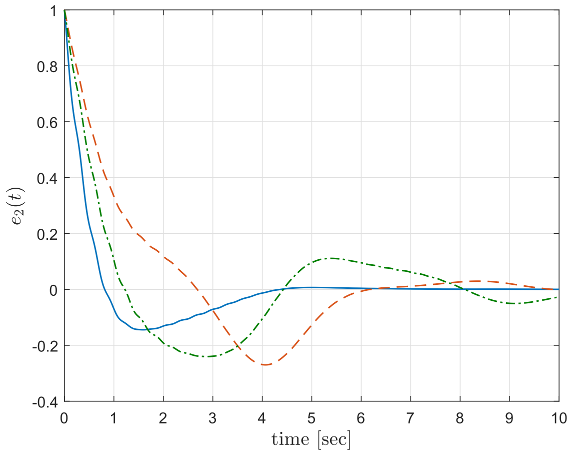

FIGURE 2. The time responses of $\tilde{x}_2(t)$: Theorem 1 with $\eta = 2$ (solid), [16] (dashed), and [17] (dash-dotted).

and $m_2\big(q(t)\big) = 1 - m_1\big(q(t)\big)$ with $q(t) = \hat{x}_1^2(t)$. In addition, the external disturbance is chosen as $\omega(t) = 0.1e^{-0.5t}\sin(3t)$, and the sensor fault is modeled as follows:

$$F(t_k) = (f_1^U - f_1^L)\frac{\sin(20t) + 1}{2} + f_1^L,$$

where $f_1^U = 0.7$ and $f_1^L = 0.2$.

Assuming the maximum allowable sampling period as $h = 0.4$ and setting the parameters as $(\beta, \sigma, \varphi_i, \delta_i) = (1, 0.1, 1, 0.5)$, the following observer gains are obtained by solving the corresponding LMIs of Theorem 1:

$$L_1 = \begin{bmatrix} -3.6663 \\ -2.2303 \end{bmatrix}, \ L_2 = \begin{bmatrix} -3.5819 \\ -1.5105 \end{bmatrix}.$$

Moreover, the conventional approaches were employed to compare the performance. For the same maximum allowable sampling periods, solving the corresponding LMIs of each method provides the following observer gains:

$$L_1 = \begin{bmatrix} 1.4996 \\ 0.3445 \end{bmatrix}, \ L_2 = \begin{bmatrix} 1.5302 \\ -0.2204 \end{bmatrix}, \tag{43}$$

and

$$L_1 = \begin{bmatrix} 1.7175 \\ 0.9020 \end{bmatrix}, \ L_2 = \begin{bmatrix} 1.7175 \\ 0.2377 \end{bmatrix}, \tag{44}$$

where (43) and (44) are the results of [16] and [17], respectively.

For the initial conditions of $x(0) = \mathrm{col}\{1, 0\}$ and $\hat{x}(0) = \mathrm{col}\{0, -1\}$, Figs. 1 and 2 show the time responses of the estimation error $\tilde{x}(t) = x(t) - \hat{x}(t)$ of each method. As can be seen from the figures, the observer designed by the proposed method provides better performance than others.

The following H_∞ performance along with various maximum sampling periods is also measured under the initial condition of $x(0) = \hat{x}(0) = \mathrm{col}\{1, 0\}$.

$$\gamma^* = \frac{\sqrt{\int_0^{10} \varepsilon^T(t)\varepsilon(t)dt}}{\sqrt{\int_0^{10} \{\omega^T(t)\omega(t) + x^T(t)x(t)\}dt}}.$$

TABLE 1. Comparison of the H_∞ performance. Each performance value is scaled as 100 × γ^*, and "INF" means infeasible solution.

h	0.2	0.4	0.6	0.8	1.2
[16]	0.1856	0.2645	INF	INF	INF
[17]	0.2019	0.2334	0.2927	0.3757	INF
Proposed ($\eta = 0$)	0.1707	0.1866	0.2041	0.2250	0.2987
Proposed ($\eta = 1$)	0.1654	0.1803	0.1950	0.2133	INF
Proposed ($\eta = 2$)	0.1623	0.1741	0.1879	INF	INF

The result is summarized in Table 1. As can be seen from the result, the proposed method provides better robust state estimation performance in the presence of disturbance and sensor defects compared to the existing methods. In addition, the proposed method provides a feasible solution at largest sampling periods in this example. The main reason is that Theorem 1 is numerically relaxed because it is derived based on the novel time-dependent fuzzified LKF. That means, numerically relaxed condition provides smaller γ. In addition, considering the sensor fault and immeasurable premise variable also helps improve the estimation performance. Moreover, The larger η, the better the estimation performance, but the maximum allowable h decreases.

B. EXAMPLE 2

This example shows the robustness of the proposed method with respect to the sensor fault. Consider the Van der Pol oscillator of the following form [34]:

$$\ddot{\zeta}(t) - \mu\big(1 - \zeta^2(t)\big)\dot{\zeta}(t) + \zeta(t) = \omega(t),$$
$$y(t_k) = \zeta(t_k),$$

where $\zeta(t) \in [-M_\zeta, M_\zeta]$ with $M_\zeta = 2.5$; $\mu = 1$; $\omega(t) = 0.1e^{-0.5t}\sin(3t)$.

As done in Example 1, by choosing the state and premise variables as $x(t) = \mathrm{col}\{x_1(t), x_2(t)\} = \mathrm{col}\{\zeta(t), \dot{\zeta}(t)\}$ and

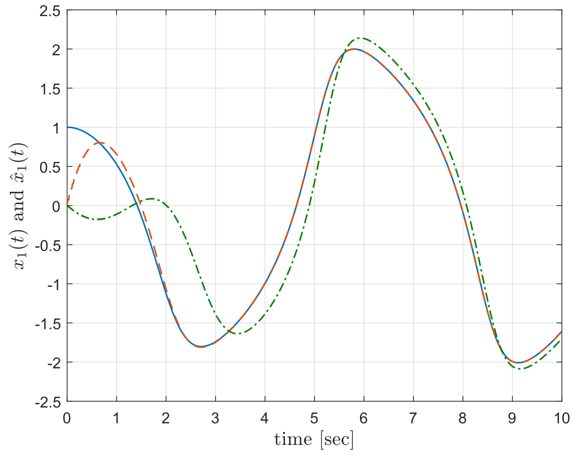

FIGURE 3. **The state responses of $x_1(t)$ and $\tilde{x}_1(t)$: $x_1(t)$ (solid), $\hat{x}_1(t)$ of the proposed method (dashed), $\hat{x}_1(t)$ of [17] (dash-dotted).**

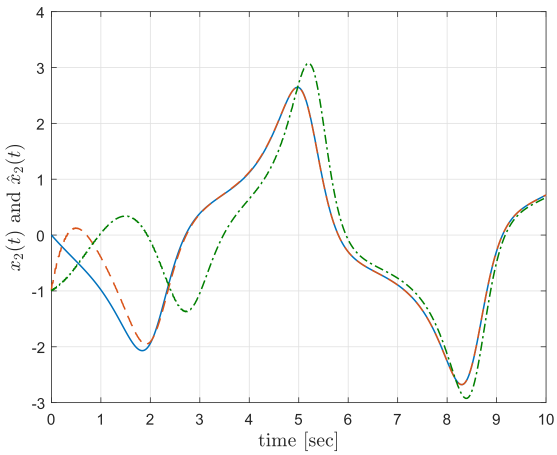

FIGURE 4. **The state responses of $x_2(t)$ and $\tilde{x}_2(t)$: $x_2(t)$ (solid), $\hat{x}_2(t)$ of the proposed method (dashed), $\hat{x}_2(t)$ of [17] (dash-dotted).**

$z(t) = x_1(t)$, the following T–S fuzzy model can be obtained:

$$\dot{x}(t) = \sum_{i=1}^{2} w_i\big(z(t)\big)\big\{A_i x(t) + B_i\omega(t)\big\},$$
$$y(t_k) = Cx(t_k)$$

where $w_1\big(z(t)\big) = z^2(t)/M_\zeta^2$; $w_2\big(z(t)\big) = 1 - w_1\big(z(t)\big)$;

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & \mu\big(1 - M_\zeta^2\big) \end{bmatrix};\ A_2 = \begin{bmatrix} 0 & 1 \\ -1 & \mu \end{bmatrix};$$
$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

In this example, we assume that there exists a severe sensor fault which is modeled as

$$F(t_k) = (f_1^U - f_1^L)\frac{\sin(200t) + 1}{2} + f_1^L,$$

where $f_1^U = 0.1$ and $f_1^L = 0$. This means that only maximum 10% amount of its true value is measured.

Now, using the same parameters with those in Example 1 and solving the corresponding LMIs of Theorem 1, we obtained the following observer gain matrix at $h = 0.1$:

$$L_1 = \begin{bmatrix} -67.6503 \\ -98.2711 \end{bmatrix},\ L_2 = \begin{bmatrix} -69.6390 \\ -117.7402 \end{bmatrix},$$

where $\eta = 2$ is used. On the other hand, by solving the LMI conditions given in [17] at the same sampling period, we have

$$L_1 = \begin{bmatrix} 8.6710 \\ -22.8237 \end{bmatrix},\ L_2 = \begin{bmatrix} 8.6710 \\ 29.3268 \end{bmatrix}.$$

When the initial conditions are $x(0) = \text{col}\{1, 0\}$ and $\hat{x}(0) = \text{col}\{0, -1\}$, the state responses of the system and sampled-data fuzzy observer are shown in Figs. 3 and 4. As can be seen from the figures, the sampled-data fuzzy observer designed by the proposed method gives better state estimation performance even in the severe sensor fault circumstance compared to the existing method. Therefore, we can conclude that the proposed sensor fault compensation method is valid.

V. CONCLUSION

In this paper, we proposed a method to design a sampled-data fuzzy observer with time-varying gains for estimating state variables of a nonlinear system under the sensor fault consideration. To do this, a nonlinear system with immeasurable premise variables was represented by a T–S fuzzy model. H_∞ performance criterion was defined to handle both the membership function mismatching problem and the minimization of the state estimation error. A sufficient condition to ensure exponential stability and H_∞ criterion was derived in terms of LMIs. The fuzzified LKF and the novel matrix inequality were proposed to relax the conservativeness of the derived LMI-based sufficient condition. Finally, the simulation examples were given to show the superiority of the proposed method.

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HAN SOL KIM received the B.S. degree in electronic and computer engineering from Hanyang University, South Korea, in 2011, and the M.S. and Ph.D. degrees in electrical and electronic engineering from Yonsei University, South Korea, in 2012 and 2018, respectively. He worked with the Samsung Electronics Company, South Korea, from 2018 to 2019, as a Senior Engineer. Since 2019, he has been with the Department of Control and Automation Engineering, Korea Maritime and Ocean University, where he is currently an Assistant Professor. His current research interests include intelligent robot, intelligent control, and unmanned aerial vehicle.

KWANGIL LEE received the B.S., M.S., and Ph.D. degrees in computer science from Chungnam National University, South Korea, in 1993, 1996, and 2001, respectively. He worked with the Electronics and Telecommunications Research Institute, South Korea, from 2006 to 2017, as a Senior Engineer. Since 2017, he has been with the Department of Control and Automation Engineering, Korea Maritime and Ocean University, where he is currently an Assistant Professor. His current research interests include smart ship, e-navigation, and marine cyber security.